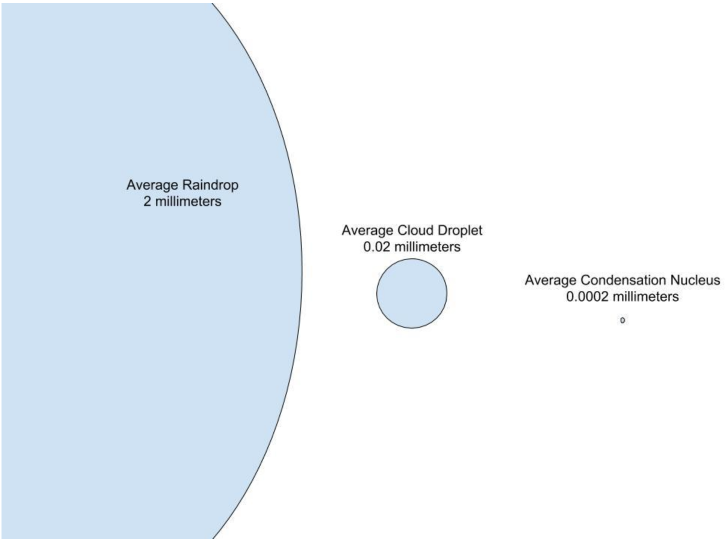
Cloud Condensation to Precipitation

November 10th, 2021

Western

Necessary steps towards Cloud Formation to Precipitation



1. Moist air rises after heating
2. Supersaturation occurs
3. Reaching critical radius
4. Convection/turbulence pushes air through cloud

Figure: The comparison of Raindrop to a Cloud drop to CCN (From Nugent et al)

Saturation Vapour Pressure

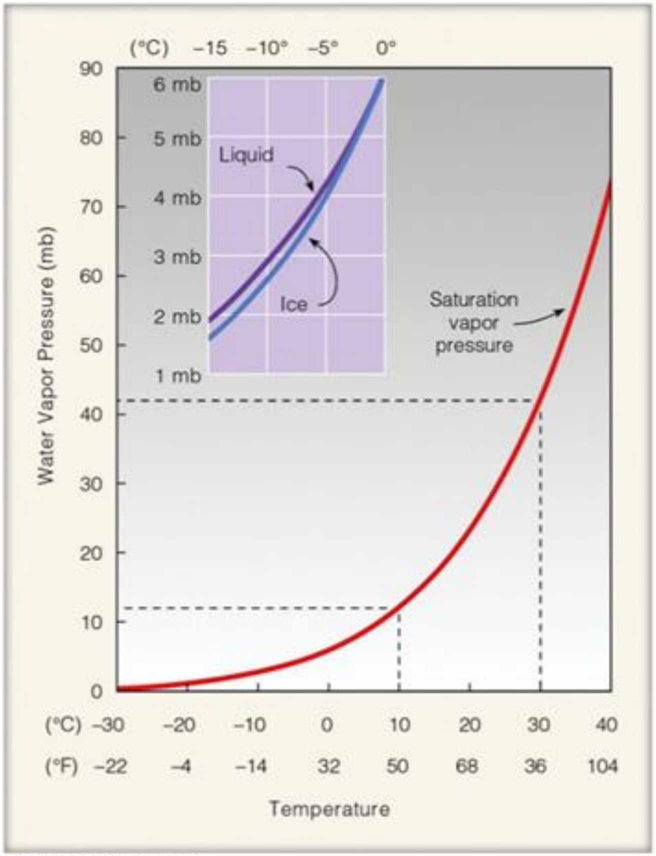


Figure: Saturation Vapour increases with temperature. (From Ahrens, 2005, pg. 81)

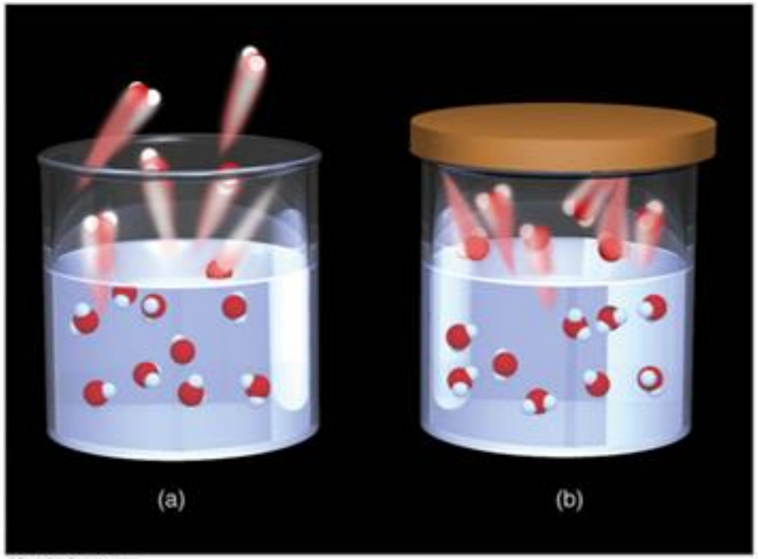


Figure: The representation of an open (a) and a closed(b) system for the equilibrium of vapour to water. (Taken from Dominguez)

The Clausius-Clapeyron Equation

Clausius- Clapeyron is the change in pressure over change in Temperature

e_s- Saturation Vapour Pressure
L_v- Latent heat of Vaporization
ρ_v- water vapour density
ρ_L- liquid water density

$$\frac{\mathrm{dP}}{\mathrm{dT}} = \frac{\Delta \mathrm{s}}{\Delta \mathrm{v}} = \frac{\mathrm{L_v}}{\mathrm{T}\Delta \mathrm{v}}$$

$$\frac{\mathrm{dP}}{\mathrm{dT}} = \frac{\mathrm{de_s}}{\mathrm{dT}} = \frac{\mathrm{L_v}}{\mathrm{T}}\left[\frac{1}{\rho_\mathrm{v}} - \frac{1}{\rho_\mathrm{L}}\right]^{-1}$$

$$\frac{\mathrm{de_s}}{\mathrm{dT}} \cong \frac{\mathrm{L_v}}{\mathrm{T}}\rho_\mathrm{v}$$

Water vapour acts as an Ideal Gas, so the Saturation vapour pressure can be represented as:

$$\mathrm{e_s} = \mathrm{R_v T}\rho_\mathrm{v}$$

The Clausius-Clapeyron Equation

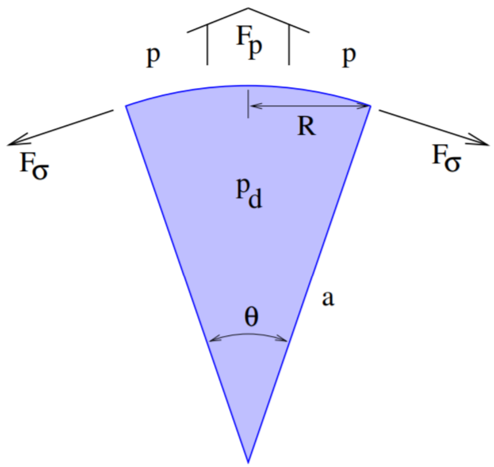
By rearranging the equation, a simple ODE is formed:

$$\frac{de_s}{e_s} = \frac{L_v}{R_v T^2} dT$$

$$\int_{e_0}^{e_s} \frac{de_s}{e_s} = \int_{T_0}^{T} \frac{L_v dT}{R_v\, T^2}$$

$$\ln\left(\frac{e_s}{e_0}\right) = \frac{L_v}{R_v}\left[\frac{1}{T_0} - \frac{1}{T}\right]$$

Kelvin Effect



(From Cloud Microphysics)

$$\text{Laplace Formula}: p_w - p_v = \frac{2\sigma_v}{a}$$

$$\frac{dP}{dT} = \frac{de_s}{dT} = \frac{de_{sat,w}}{dT} + \frac{2v_w}{v_v - v_w}\frac{d(\frac{\sigma_v}{a})}{dT}$$

$$v_w = 1/n_w \qquad \frac{e_s}{e_0} = \exp\left(\frac{2\sigma}{n_w RT\, a}\right)$$

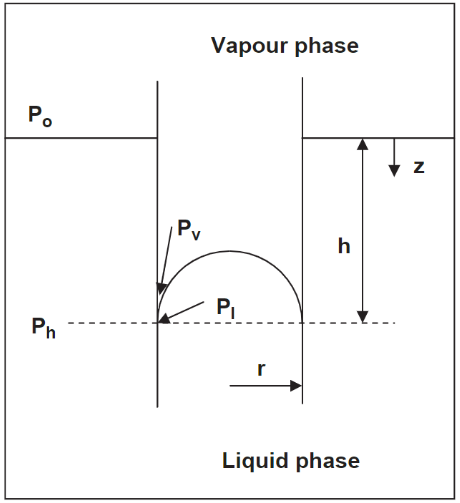


Figure: Experimental set up for simplistic derivation of the Kelvin law (From Galvin, 2005)

Gibbs Free Energy

Equation 1 is the Gibbs free energy as a balance between Evaporation and Condensation.

Applying gas law and system parameters produces Equation 4

$$\Delta E = A\sigma - nV(\mu_v - \mu_l) \quad (1)$$

$$(\mu_v - \mu_l) = kT \ln\left(\frac{e_s}{e_0}\right) \quad (2)$$

$$\Delta E = A\sigma - nVkT \ln\left(\frac{e_s}{e_0}\right) \quad (3)$$

$$\Delta E = 4\pi R^2\sigma - \frac{4}{3}\pi R^3 nkT \ln\left(\frac{e_s}{e_0}\right) \quad (4)$$

$$\frac{e_s}{e_0} = \exp\left(\frac{2\sigma}{n_L RT\, r_d}\right) \quad (5)$$

$\mu_v - \mu_l =$ *Gibbs free energies per molecule*

Gibbs Free Energy

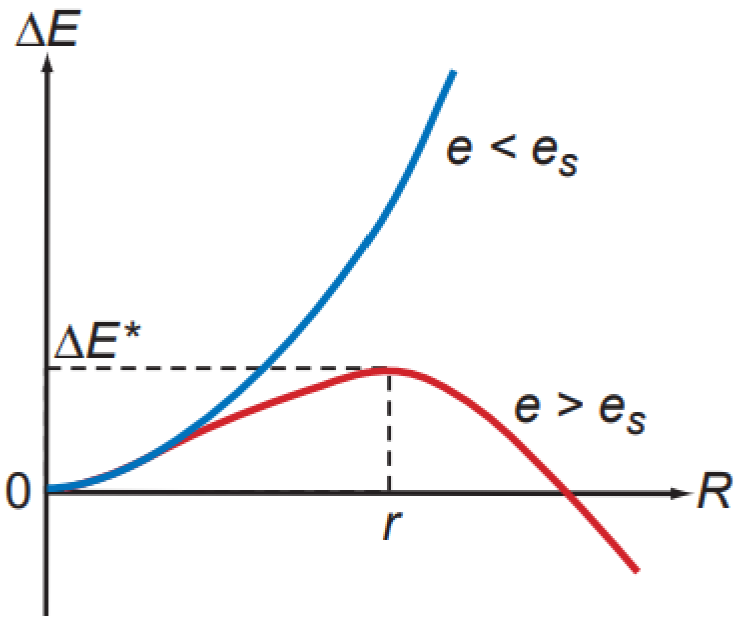


Figure: Blue is droplet in a subsaturated environment, Red is a droplet in a supersaturated environment. The change in energy as a function of droplet radius (From Wallace and Hobbs, 2006)

Critical Radius: $r_d = \frac{2\sigma}{n_L RT \ln(\frac{e_s}{e_0})}$

- *Cloud formation cannot be explained by homogenous nucleation*

Raoult's Solution and Kohler's Curves

$$\frac{e'}{e_s(\infty)} = \frac{n_w}{n_w + n_d}$$

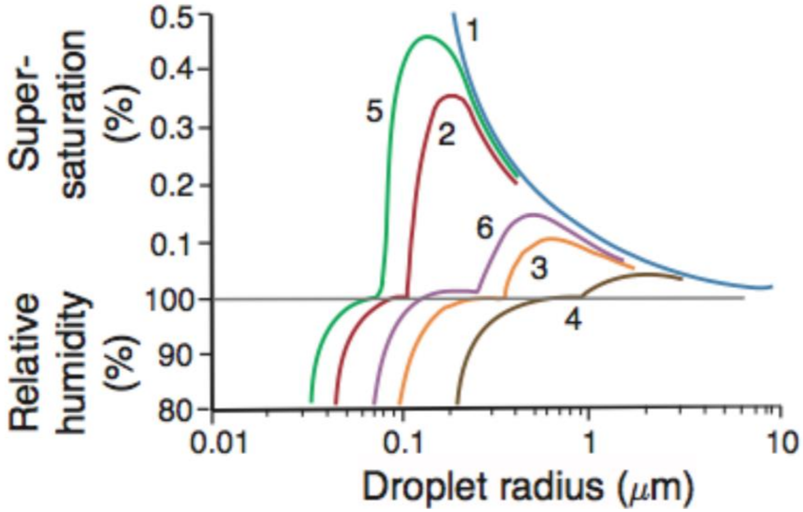


Figure: Combination solution for Raoult and Kelvin (From Dominguez)

Goff –Gratch

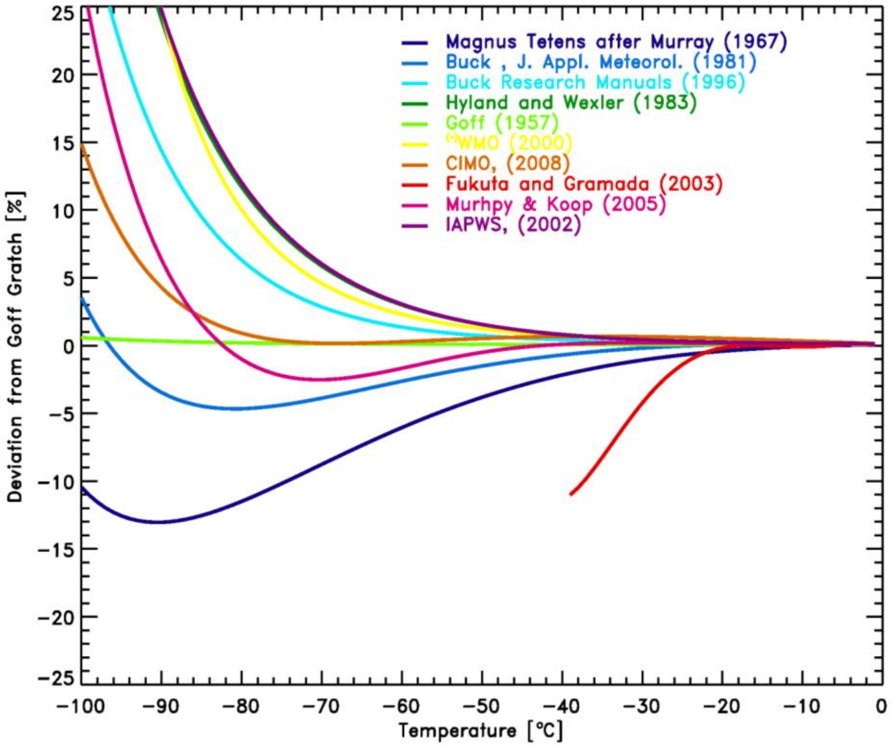


Figure: (From Vomel 2013)

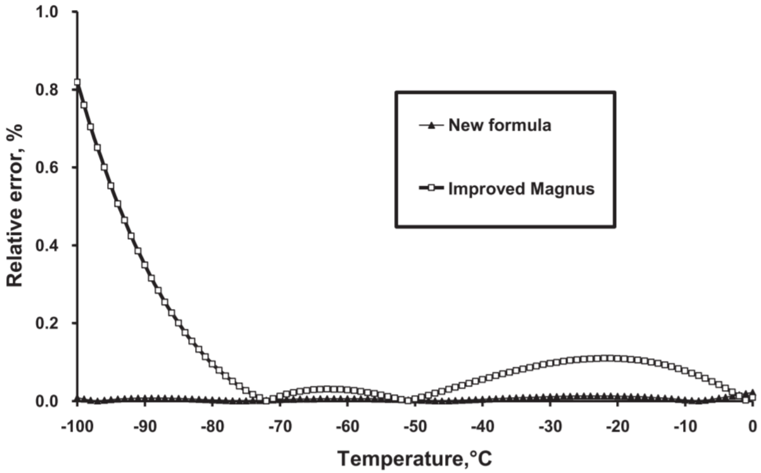


Figure: (From Huang 2017)

Not Condensation?

Must be Collision

Collisional Coalescence

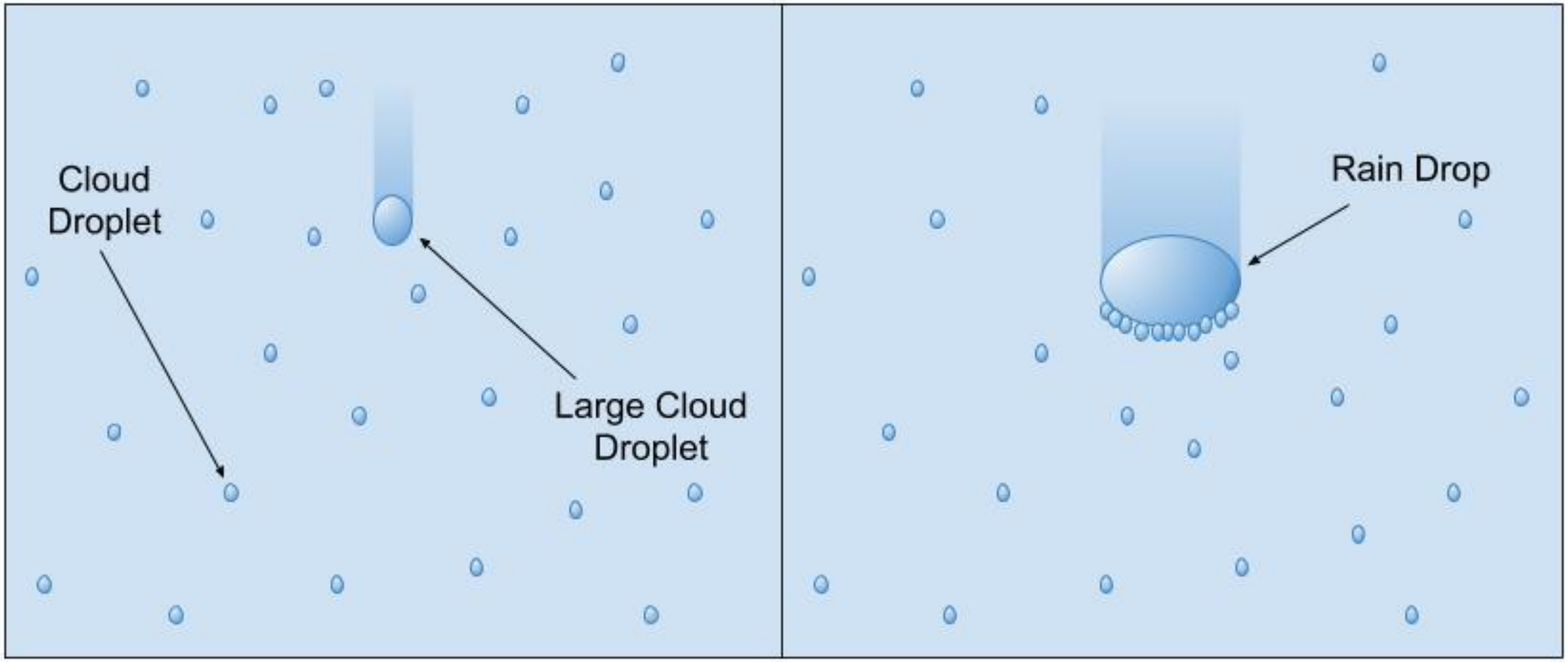
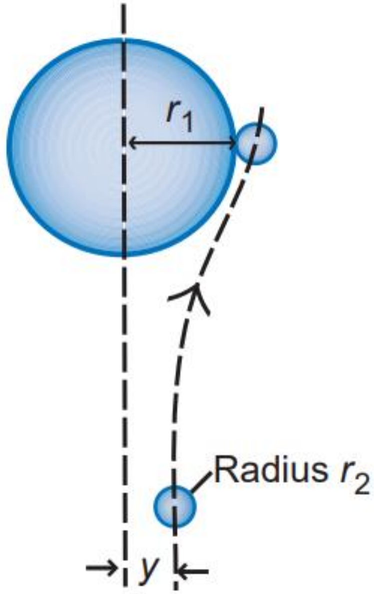


Figure: Process as large cloud nuclei fall through and collide with smaller droplets. Achieving coalescence. (From Nugent et al)

Collisional Coalescence



- Primarily occurs in warm clouds
- If collision occurs with ice, called aggregation

$$\text{Collisional Efficiency} : E = \frac{y^2}{(r_1+r_2)^2}$$

Figure: Collisional setup. The droplet below misses the falling drop unless within the critical distance due to drag. (From Wallace and Hobb, 2006)

Collisional Coalescence

Continuous Collection model

- Assume uniform distribution through space
- Uniform collection rate
- Uniform size

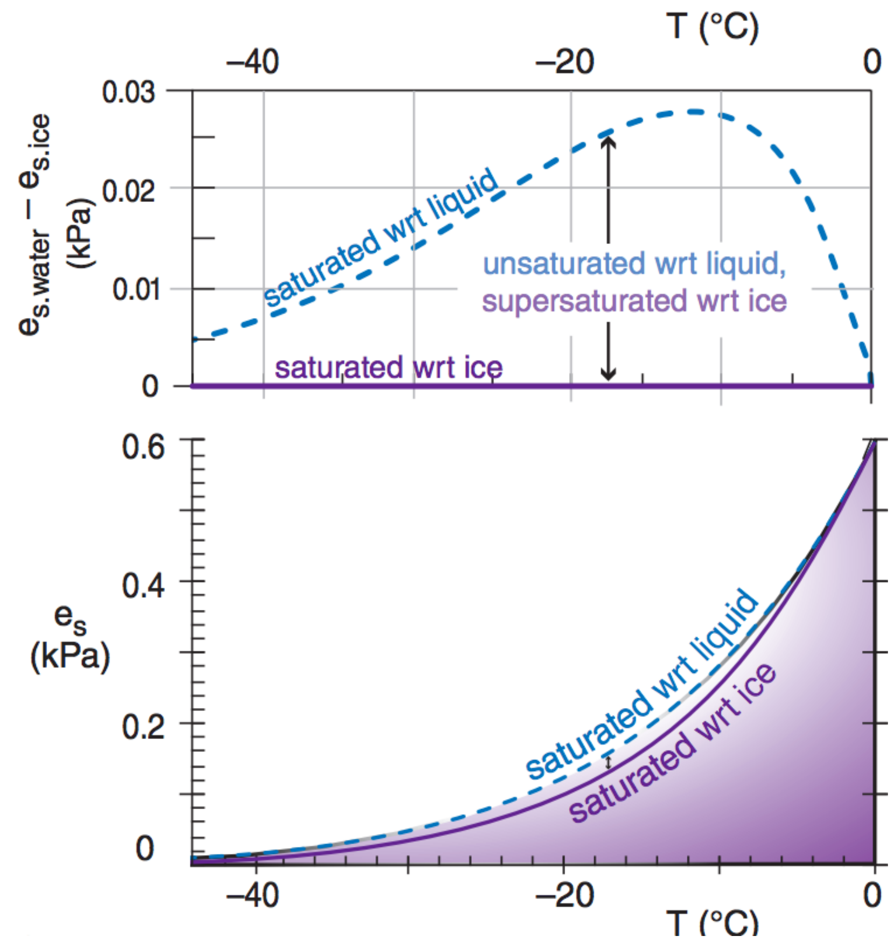
$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_l E_c$$

$$\frac{dr_1}{dt} = \frac{(v_1 - v_2) w_l E_c}{4\rho_l}$$

$$\frac{dr_1}{dt} = \frac{(v_1) w_l E}{4\rho_l}$$

Bergeron Process

- Accretion of ice particles onto CCN
- In warm columns, produces rain
- Cool columns allow for Snow, Slush and Freezing rain
- Much easier to condense on surface of Ice vs Liquid



The End