Planetary Motion Assignment 1

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1 Question 1

Three functions which convert inputs into a given output. The link to the github repository for the entire assignment is: https://github.com/Vpinnegar/PlanetaryMotionA1

1.1 README

1.1.1 julian_to_gst

accepts a time in Julian Date format and outputs the angle between the First point of Aries and Greenwich meridian in degrees as given in equation 1. This is also called the sidereal time.

$$t = (JD - 2451545.0) \tag{1}$$

$$T = t/36525 \tag{2}$$

$$\theta = 280.46061837 + 360.98564736629 * (JD - JD2000) + 0.000387933 * T * *2 - (T * *3)/38710000.0 \tag{3}$$

$$\theta = mod(\theta, 360.)$$

(4)

1.1.2 cartesian_to_ra_dec

accepts the Cartesian x, y, z coordinates of a body in the geocentric equatorial coordinate system above and the Julian date, and returns the Cartesian x, y, z coordinates in a geocentric cartographic coordinate system with the same origin but with its z-axis along the north pole and its x-axis out along the Greenwich meridian.

$$x_g = r_g cos\delta cos\alpha$$

(5)

$$y_g = r_g cos\delta sin\alpha$$

(6)

$$z_g = r_g sin\delta$$

(7)

rearranged for returning Right ascension and Declination as

$$r_g = \sqrt{x_g^2 + y_g^2 + z_g^2} \tag{8}$$

$$\delta = arcsin(z_g/r_g) \tag{9}$$

$$\alpha = atan2(y_g, x_g) \tag{10}$$

1.1.3 equatorial_to_grenwichnorth

accepts the Cartesian x, y, z coordinates of a body in the geocentric equatorial coordinate system above and the Julian date, and returns the Cartesian x, y, z coordinates in a geocentric cartographic coordinate system with the same origin but with its z-axis along the north pole and its x-axis out along the Greenwich meridian.

$$M = \begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.1.4 cartesian_to_geodetic

accepts the Cartesian coordinates in meters and outputs the geodetic coordinates Latitude, Longitude and altitude. This is dictated by Figure 1 taken from Chapter 1. Additionally, it checks for if the input is numeric.

$$\begin{aligned} p &= \sqrt{x^2 + y^2} && (1.6) \\ u &= \operatorname{atan}(za/pb) && (1.7) \\ \phi &= \operatorname{atan2}\left(z + e'^2 b \sin^3 u, p - e^2 a \cos^3 u\right) && (1.8) \\ \lambda &= \operatorname{atan2}(y, x) && (1.9) \\ h &= p/\cos\phi - a/\sqrt{1 - e^2 \sin^2 \phi} && (1.10) \end{aligned}$$

Figure 1: The equations for the conversion from Cartesian coordinates to the latitude, longitude and altitude. Bowring B, 1976, Transformation from spatial to geographical coordinates, Survey Review, 23, p 323-327 .

1.2 Example: Moon

The moon on Sept 1, 2024 at 00:00:0000 UTC has a Right ascension of 09h 17m 12.10s which is 139.30deg and Declination of +20deg 02' 21.2".

The skyfield api (https://rhodesmill.org/skyfield/almanac.html) as seen in Figure 2 outputs the Cartesian/geocentric equatorial plane coordinates as $(x, y, z) = (-284299.37182268m, 244532.35645361m, 136778.25959189m)$. Putting this value into the cartesian_to_ra_dec outputs right ascension as 139.3 deg and declination as 20.04 deg. This means that the conversion within the code is accurate. Running these values numerically through the equations in figure 1 gives the latitude as 20.04 and Longitude as 139.3 Additionally, this is equivalent to both the declination and right ascension in degree format respectively. Figure 3 is an example analysis.

```python
from skyfield.api import load

# Load the ephemeris data
eph = load('de421.bsp')
earth, moon = eph['earth'], eph['moon']

# Specify the date and time (UTC)
ts = load.timescale()
time = ts.utc(2024, 9, 1, 0, 0, 0)

# Calculate position
astrometric = earth.at(time).observe(moon).apparent()
ra, dec, distance = astrometric.radec()
geocentric = astrometric.position.km
```

Figure 2: The example code for testing on the moon on September 1, 2024 00:00:0000 UTC

2 Question 2

ISS has an approximately circular orbit with an altitude of 300-420 km. It's orbit is tilted at 51.6 degrees to the earths equator.

Prograde circle with radius 6680 km rotated 51.6 degrees from the first point of aries takes 92 minutes to circle the earth in the geocentric equatorial reference frame .

$a = 6378$ Km

$b = 6356$ Km

$e^2 = \frac{(a^2 - b^2)}{a^2} = 0.066694478\ldots$

$p = \sqrt{x^2 + y^2} = 374996$

$u = \text{atan2}(z \cdot a, p \cdot b) = 0.351$

$\phi = \text{atan2}(z + e^2 b \sin^3(u), p - e^2 a \cos(u))$

$= 0.3497848$ rads

$= 20.04°$

$\lambda = \text{atan2}(y, x)$

$= 2.431$

$= 139.3°$

Figure 3: Calculation of Latitude and Longitude, which is equivalent to the degree form of the right ascension and declination

2.1 a

The declination on the celestial sphere is the angle north or south of the celestial equator. Orbit inclination is the angle in relation to the stationary body's equatorial plane with the plane of a orbiting body. Given an observer at

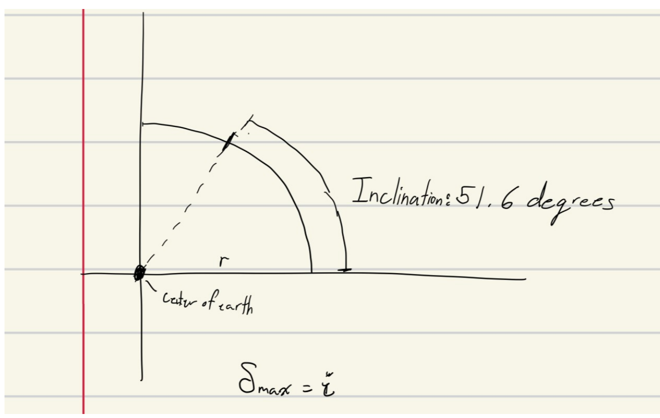

Figure 4: Enter Caption

the center of the earth and the Equator is on the same plane as the celestial equator, the most northerly declination of the ISS is 51.6 degrees. Equivalent to the orbital inclination.

2.2 b

Orbital inclination/Maximum Declination: 51.6 degrees
Period of orbit: 92 minutes
Julian Date: 256674.5
RA: 0 degrees
Declination: 0 degrees Rotates with the earth Using the given information, ISS is travelling "northward" on it's orbit and passes the first point of aries on 2456674.5 in the Julian Date format or 2014-01-17 00:00:00.000, the Right Ascension and Declination will both be 0, as the First point of aries defines the positive x axis in the celestial coordinate system.

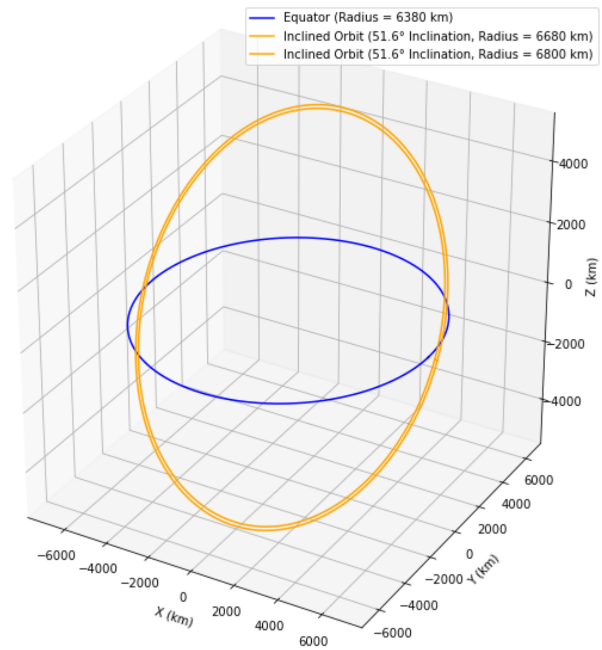

Figure 5: The orbit of the ISS plotted at both minimum height of 300 km and maximum of 480 km at an inclination of 51.6 degrees

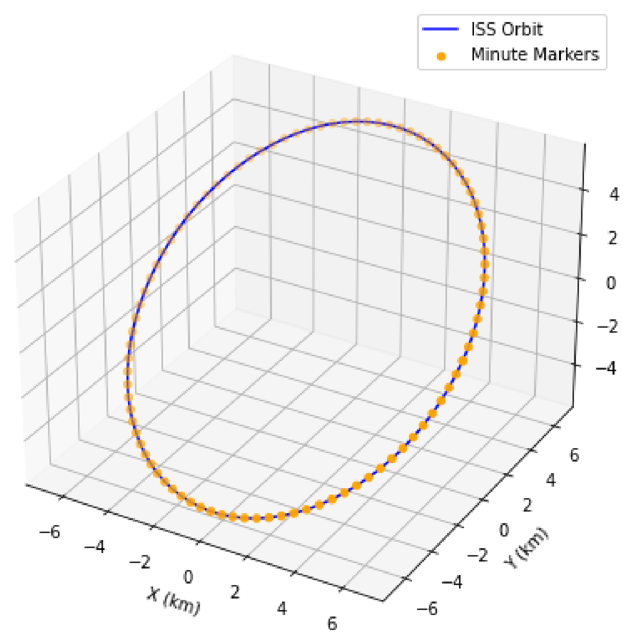

Figure 6: Every minute plotted on the orbit of the ISS

2.3 C

The geodetic latitude at Julian date 2456674.5 is:
Geodetic latitude: 0.0
Geodetic longitude: -116.33879140554926
altitude : 301863.0

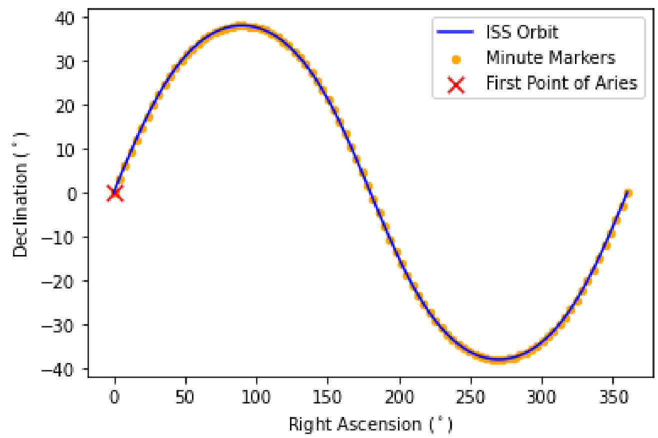

Figure 7: Starting from the First point of Aries, the orbit of the ISS over one period of 92 minutes as it varies in Declination and Right Ascension

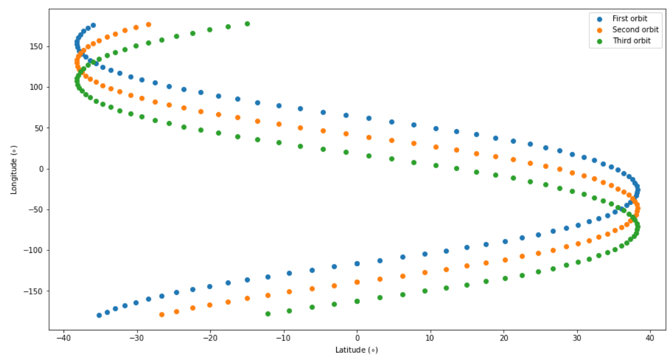

Figure 8: The Latitude vs the Longitude of the first three orbits plotted.

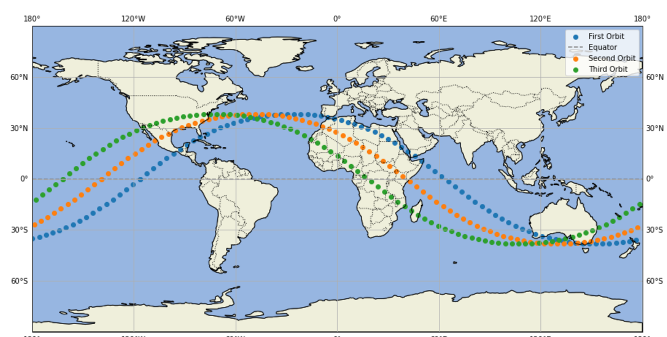

Figure 9: Using cartopy, the first three orbits of the ISS from the specified Julian date

3 3

3.1 ReadMe

This module is used for the conversion of orbital elements eccentricity, true anomaly and eccentric anomaly. All of which are related by:

$$tan(\frac{f}{2}) = \sqrt{\frac{1+e}{1-e}}tan(\frac{E}{2}) \tag{11}$$

where f is the true anomaly, e is the eccentricity and E is the eccentric anomaly.

3.2 ta_to_eccentric_a

This function converts true anomaly and eccentricity into Eccentric anomaly based on the above equation. The rearranged final form of the equation is

$$E = 2 * tan^{-1}(\sqrt{\frac{1-e}{1+e}}tan(\frac{f}{2})) \tag{12}$$

Within this function, true anomaly cannot be larger than 2π. This function also checks for a numeric answer, which when receiving non-numeric outputs an error.

3.3 eccentricity_to_ta

This function takes in inputs eccentricity in radians and Eccentric anomaly in radians and outputs the True Anomaly in radians as dictated by

$$f = 2 * tan^{-1}(\sqrt{\frac{1+e}{1-e}}tan(\frac{E}{2})) \tag{13}$$

. The function also checks if the input is numeric, outputting an error if it is in the wrong format.

3.3.1 Example:

Table 1 represents orbital elements for the dwarf planets as taken from https://minorplanetcenter.net/dwarf_planets

Object	ω (°)	Ω (°)	i (°)	e	q (AU)	a (AU)	M (°)	n (°/day)	Q (AU)	Epoch
(134340) Pluto	113.7	110.3	17.2	0.25	29.69	39.40	51.2	0.004	49.11	1989-08-20.8

Table 1: Orbital Parameters of Pluto

Taking the mean orbital anomaly for pluto and the eccentricity, I calculated the Eccentric anomaly to be 64.1° or 1.12 in radians. Using this eccentric anomaly, and the eccentricity of 0.25, calling the function as:

$$eccentricity\_to\_ta(0.25, 1.12) \tag{14}$$

where the first position is the eccentricity and the second position is the Eccentric anomaly, the calculated true anomaly is equal to 77.9°.

4 Question 4

4.1 ReadMe

This module takes in the orbital element $\mu = G(M_1 + m_2)$, a, e and f, and utilizes the equations defined in figure to produce the x, y postions and velocities of the orbiting body around the central mass.

4.2 orbitposition_velocity

Inputs for this function are $\mu = G(M_1+m_2)$, a, e and f. μ is the Standard Gravitational Parameter, and is inputted as a float. In Au, Solar Mass and year units, it is equivalent to $4 * \pi^2$. a is the Semi-major axis, and is a float. This will be in AU units. e is the eccentricity and is a float. f is the true anomaly and is a float in units of radians.

This routine checks that f is within the accepted values for true anomaly ($0 < f < 2 * \pi$) and that the inputs are numeric.

5 Question 5

5.1 ReadMe

This module multiplies the position and velocity vectors through the angles ω, i and Ω, transforming it to the heliocentric form. The rotation matrices must be multiplied in that order and are represented by:

$$
\begin{aligned}
x &= a(\cos E - e) \\
y &= a\sqrt{1-e^2}\sin E \\
z &= 0 \\
\dot{x} &= -\frac{an\sin E}{1-e\cos E} \\
\dot{y} &= \frac{a\sqrt{1-e^2}n\cos E}{1-e\cos E} \\
\dot{z} &= 0
\end{aligned}
$$

Figure 10: The equations for position and velocity as seen in the text.

$$
\begin{aligned}
\mathrm{R}_\omega &= \begin{bmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\mathrm{R}_i &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} \\
\mathrm{R}_\Omega &= \begin{bmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

where the argument of perihelion ω rotates around the z axis, the inclination i rotates around the x axis and the Longitude of the Ascending node Ω rotates around the z axis.

5.2 rotate_oiO

This function rotates through the ω, i and Ω. Checks if numeric (int or float) and checks if inclination is within 180 degrees.

6 Question 6

6.1 ReadMe

This module outputs the rotated heliocentric coordinates and velocities when given the semi major axis, inclination, eccentricity, Longitude of Ascending node, argument of perihelion, true anomaly and Standard gravitational parameter. https://github.com/Vpinnegar/PlanetaryMotionA1/blob/main/Question6.py

7 Question 7

7.1 A

7.2 B

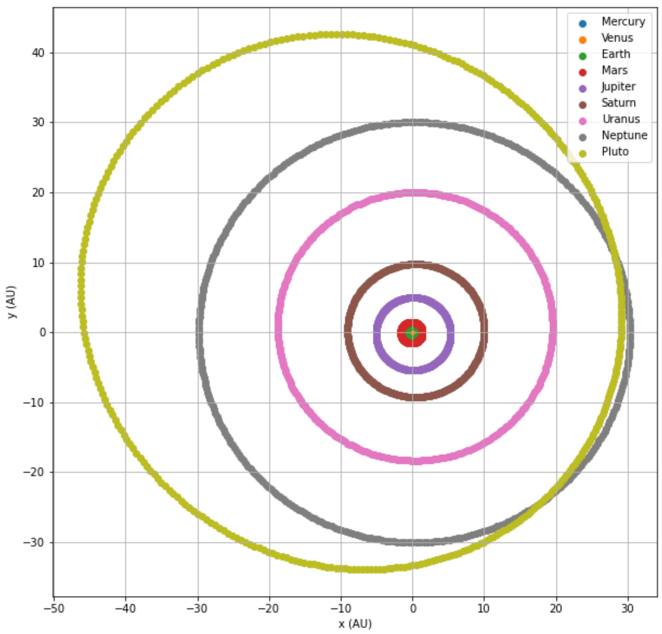

Figure 11: All fo the Planets and Pluto plotted on the x-y plane at intervals of 1 degree in the true anomaly

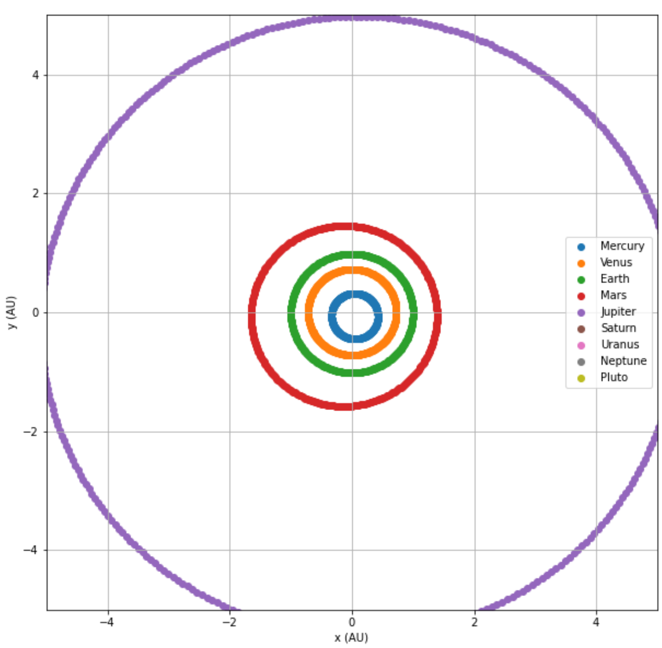

Figure 12: The inner Planets plotted on the x-y plane at intervals iof 1 degree in the true anomaly, bounded at 5 AU and -5 AU

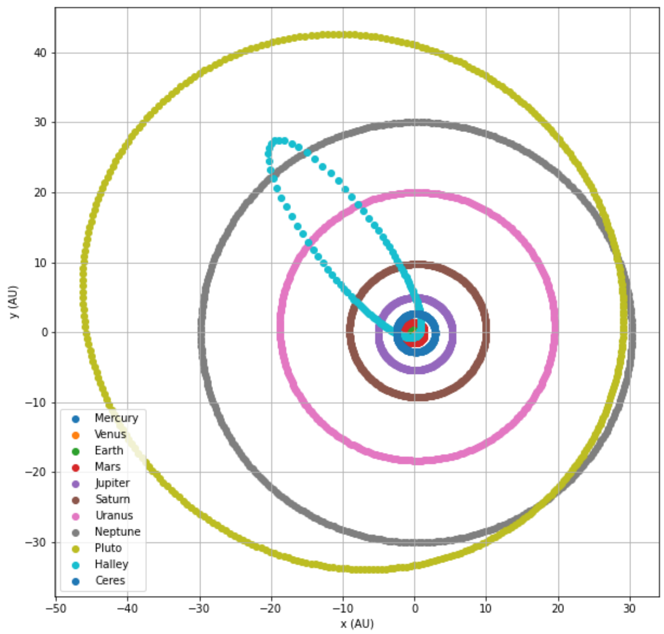

Figure 13: The Planets plotted with the addition of Halley's comet and Ceres. Data for Halley's comet taken from https://in-the-sky.org/data/object.php?id=0001p . Data for ceres taken from https://www.princeton.edu/ willman/planetary$_{s}ystems/Sol/Ceres/$

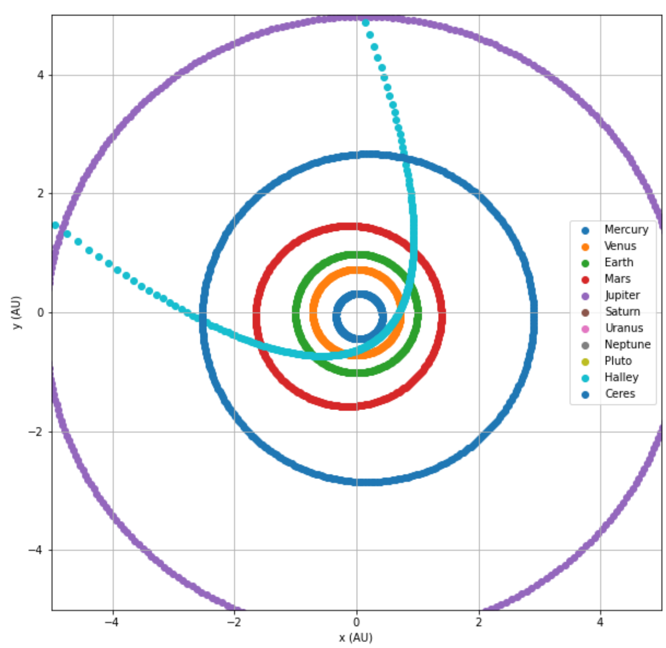

Figure 14: The inner Solar System plotted with the addition of Halley and Ceres