

Tutorial Sheet 3 (Convergence of Infinite Series)

1. Test the convergence of the following series:

- (a) $\sum \frac{1}{(2n-1)^p}$, (b) $\sum \left[1 + \frac{1}{\sqrt{n}}\right]^{-n^{3/2}}$, (c) $\sum \frac{n^n x^n}{n!}$,
 (d) $\sum \frac{1}{n^{1+(1/n)^p}}$, (e) $\sum \frac{x^n}{n(n+1)}$, (f) $\sum [(n^3 + 1)^{1/3} - n]$,
 (g) $\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1.3}{2.4} \cdot \frac{x^5}{5} + \frac{1.3.5}{2.4.6} \cdot \frac{x^7}{7} + \dots$
 (h) $\frac{1^2}{4^2} + \frac{5^2}{8^2} + \frac{9^2}{12^2} + \frac{13^2}{16^2} + \dots$
 (i) $1 + \frac{x}{2} + \frac{2!}{3^2} \cdot x^2 + \frac{3!}{4^3} \cdot x^3 + \frac{4!}{5^4} \cdot x^4 + \dots$

2. Test the convergence of the series whose n^{th} term is:

- (a) $\frac{a^n}{x^{n+a^n}}$, (b) $\frac{1}{\sqrt{n} + \sqrt{n+1}}$, (c) $\frac{\sqrt{n+1}-1}{(n+2)^3-1}$,
 (d) $\frac{(a+nx)^n}{n!}$, (e) $\frac{1}{\sqrt{n}} \tan \frac{1}{n}$, (f) $3^{-n-(-1)^n}$,

Answers:

1	(a) $p > 1$ Conv., $p \leq 1$ Div.	(b) Conv.	(c) $x < \frac{1}{e}$ Conv., $x \geq \frac{1}{e}$ Div.
	(d) Div.	(e) $x \leq 1$ Conv., $x > 1$ Div.	(f) Conv.
	(g) $x^2 \leq 1$ Conv., $x^2 > 1$ Div.	(h) Div.	(i) $x < e$ Conv., $x \geq e$ Div.
2	(a) $\frac{x}{a} \leq 1$ Div., $\frac{x}{a} > 1$ Conv.	(b) Div.	(c) Conv.
	(d) $x < \frac{1}{e}$ Conv., $x \geq \frac{1}{e}$ Div.	(e) Conv.	(f) Conv.