Funciones Trigonométricas Hiperbólicas

Son combinaciones especiales de funciones exponenciales. Se les llama así porque tienen algunas características similares a las funciones trigonométricas circulares.

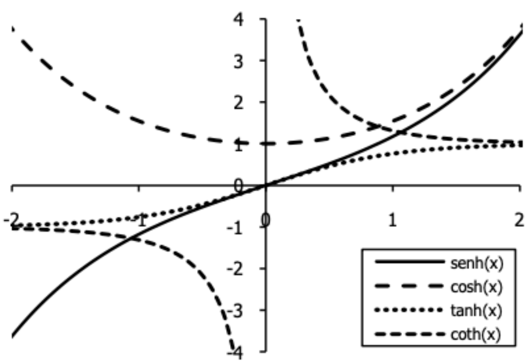
DEFINICIONES

$$\operatorname{senh} x \equiv \frac{e^x - e^{-x}}{2} \qquad \cosh x \equiv \frac{e^x + e^{-x}}{2}$$

$$\tanh x \equiv \frac{\operatorname{senh} x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$\coth x \equiv \frac{\cosh x}{\operatorname{senh} x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1}$$

GRÁFICA



VALORES LÍMITE

	$x \to 0$	$x \to -\infty$	$x \to \infty$
$\operatorname{senh} x =$	0	$-\infty$	∞
$\cosh x =$	1	∞	∞
$\tanh x =$	0	-1	1
$\coth x =$	$\pm\infty$	-1	1

ARGUMENTO NEGATIVO

$$\operatorname{senh}(-x) = -\operatorname{senh} x \qquad \cosh(-x) = \cosh x$$

$$\tanh(-x) = -\tanh x \qquad \coth(-x) = -\coth x$$

IDENTIDADES

$$\sinh x + \cosh x = e^x \qquad \cosh x - \sinh x = e^{-x}$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \tanh x \coth x = 1$$

$$1 - \tanh^2 x = \frac{1}{\cosh^2 x} \qquad 1 - \coth^2 x = \frac{-1}{\sinh^2 x}$$

$$\operatorname{senh}(2x) = 2 \operatorname{senh} x \cosh x \qquad \cosh(2x) = 2\cosh^2 x - 1$$

RELACIONES MUTUAS

$$\operatorname{senh} x = \sqrt{\cosh^2 x - 1} = \frac{\tanh x}{\sqrt{1 - \tanh^2 x}} = \frac{1}{\sqrt{\coth^2 x - 1}}$$

$$\cosh x = \sqrt{\operatorname{senh}^2 x + 1} = \frac{1}{\sqrt{1 - \tanh^2 x}} = \frac{\coth x}{\sqrt{\coth^2 x - 1}}$$

$$\tanh x = \frac{\sinh x}{\sqrt{\sinh^2 x + 1}} = \frac{\sqrt{\cosh^2 x - 1}}{\cosh x} = \frac{1}{\coth x}$$

$$\coth x = \frac{\sqrt{\sinh^2 x + 1}}{\sinh x} = \frac{\cosh x}{\sqrt{\cosh^2 x - 1}} = \frac{1}{\tanh x}$$

TEOREMAS DE ADICIÓN

$$\sinh(a \pm b) = \sinh a \cosh b \pm \cosh a \sinh b$$

$$\cosh(a \pm b) = \cosh a \cosh b \pm \sinh a \sinh b$$

$$\tanh(a \pm b) = \frac{\tanh a \pm \tanh b}{1 \pm \tanh a \tanh b}$$

$$\coth(a \pm b) = \frac{\coth a \coth b \pm 1}{\coth a \coth b}$$

DERIVADAS

$$\frac{d}{dx} \operatorname{senh} u = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx} \cosh u = \operatorname{senh} u \frac{du}{dx}$$

$$\frac{d}{dx} \tanh u = \left(1 - \tanh^2 u\right) \frac{du}{dx}$$

$$\frac{d}{dx} \coth u = \left(1 - \coth^2 u\right) \frac{du}{dx}$$

INTEGRALES

$$\int \operatorname{senh} u\, du = \cosh u + C$$

$$\int \cosh u\, du = \operatorname{senh} u + C$$

$$\int \tanh u\, du = \ln(\cosh u) + C$$

$$\int \coth u\, du = \ln(\operatorname{senh} u) + C$$