Practica :

① Resolver los siguientes sistema de ecuaciones lineales

(i) Verificar que la matriz ampliada tenga inversa

(ii) Calcular A^{-1} y $AX = B$

(iii) Aplicar el método de Gauss-Jordan.

(iv) Resolver el sistema por el método de Cramer.

(v) Resolver el sistema por sustitución.

(a) $\begin{cases} 2x + y - z = 1 \\ x - 2y + 2z = 3 \\ 3x - 2y + z = 2 \end{cases}$

(b) $\begin{aligned} x + y - z &= 1 \\ 2x + 2y - 3z &= 1 \\ 4x - 2y - z &= 1 \end{aligned}$

ⓐ $\begin{cases} 2x + y - z = 1 \\ x - 2y + 2z = 3 \\ 3x - 2y + z = 2 \end{cases}$

Solución: (i) Verificar si la matriz ampliada Tiene inversa

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -2 & 1 \end{bmatrix}; \quad \det(A) \neq 0$$

$$\Rightarrow \det(A) = \left| \begin{array}{ccc|cc} 2 & 1 & -1 & 2 & 1 \\ 1 & -2 & 2 & 1 & -2 \\ 3 & -2 & 1 & 3 & -2 \end{array} \right| =$$

$$= -4 + 6 + 2 - (6 + (-8) + 1) = 4 - (-1) = 5$$

$\Rightarrow \det(A) = 5 \neq 0$. Por lo tanto A tiene inversa.

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -2 & 1 \end{pmatrix}$$

(ii) ⓐ $(A \mid I)$

ⓑ $A^{-1} = \frac{1}{\det(A)} \cdot (Adj(A))^t$; dando $Adj(A) = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$

$\alpha_{ij} = (-1)^{i+j} |M_{ij}|$. Calculemos

- $\alpha_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 2 \\ -2 & 1 \end{vmatrix} = 2$; $\alpha_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 5$

- $\alpha_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix} = 4$; $\alpha_{21} = (-1)^{2+1} \begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} = 1$

- $\alpha_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 5$; $\alpha_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} = 7$

- $\alpha_{31} = (-1)^{3+1} \begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} = 0$; $\alpha_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = -5$

$\alpha_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -5$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} =$

$= ad - bc$.

$$\mathrm{Adj}(A) = \begin{pmatrix} 2 & 5 & 4 \\ 1 & 5 & 7 \\ 0 & -5 & -5 \end{pmatrix} ; \text{ entonces } (\mathrm{Adj}(A))^t = \begin{pmatrix} 2 & 1 & 0 \\ 5 & 5 & -5 \\ 4 & 7 & -5 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{(\mathrm{Adj}(A))^t}{\det(A)} = \frac{1}{5} \cdot \begin{pmatrix} 2 & 1 & 0 \\ 5 & 5 & -5 \\ 4 & 7 & -5 \end{pmatrix} = \begin{pmatrix} 2/5 & 1/5 & 0 \\ 1 & 1 & -1 \\ 4/5 & 7/5 & -1 \end{pmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B.$$

Así que;

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2/5 & 1/5 & 0 \\ 1 & 1 & -1 \\ 4/5 & 7/5 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(iii) Aplicar el Método de Gauss

$$\begin{cases} 2x + y - z = 1 \\ x - 2y + 2z = 3 \\ 3 - 2y + z = 2 \end{cases} \Rightarrow AX = B \Leftrightarrow \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & -2 & 2 & 3 \\ 3 & -2 & 1 & 2 \end{array}\right) \xrightarrow{f_2 : f_2 \leftrightarrow f_1} \left(\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -2 & 1 & 2 \end{array}\right) \xrightarrow[f_3 : f_3 - 3f_1]{f_2 : f_2 - 2f_1} \left(\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 5 & -5 & -5 \\ 0 & 4 & -5 & -7 \end{array}\right)$$

$$f_2 : \frac{1}{5} f_2 \left(\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 4 & -5 & -7 \end{array}\right) \xrightarrow{f_3 : f_3 - 4f_2} \left(\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & -3 \end{array}\right)$$

$$\begin{aligned} x - 2y + 2z &= 3 & &\Rightarrow \boxed{x = 1} \\ y - z &= -1 & &\Rightarrow \boxed{y = 2} \\ -z &= -3 & &\Rightarrow \boxed{z = 3} \end{aligned}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & -2 & 2 & 3 \\ 3 & -2 & 1 & 2 \end{array}\right) \end{array}$$

Método de Cramer.

$$x = \frac{\det(A_1)}{\det(A)} \; ; \quad y = \frac{\det(A_2)}{\det(A)} \; ; \quad z = \frac{\det(A_3)}{\det(A)}$$

$$\det(A_1) = \left|\begin{array}{ccc|cc} 1 & 1 & -1 & 1 & 1 \\ 3 & -2 & 2 & 3 & -2 \\ 2 & -2 & 1 & 2 & -2 \end{array}\right| = -2 + 4 + 6 - (4 - 4 + 3)$$

$$= 8 - 3 = 5$$

$$x = \frac{\det(A_1)}{\det(A)} = \frac{5}{5} = 1 \Rightarrow \boxed{x = 1}$$

$$\det(A_2) = \left|\begin{array}{ccc|cc} 2 & 1 & -1 & 2 & 1 \\ 1 & 3 & 2 & 1 & 3 \\ 3 & 2 & 1 & 3 & 2 \end{array}\right| = 6 + 6 - 2 - (-9 + 8 + 1)$$

$$10 - 0 = 10$$

$$y = \frac{\det(A_2)}{\det(A)} = \frac{10}{5} \Rightarrow \boxed{y = 2}$$

$\det(A_3)$

$$\boxed{z = 3}$$