MID - TERM EXAM

RBE 501

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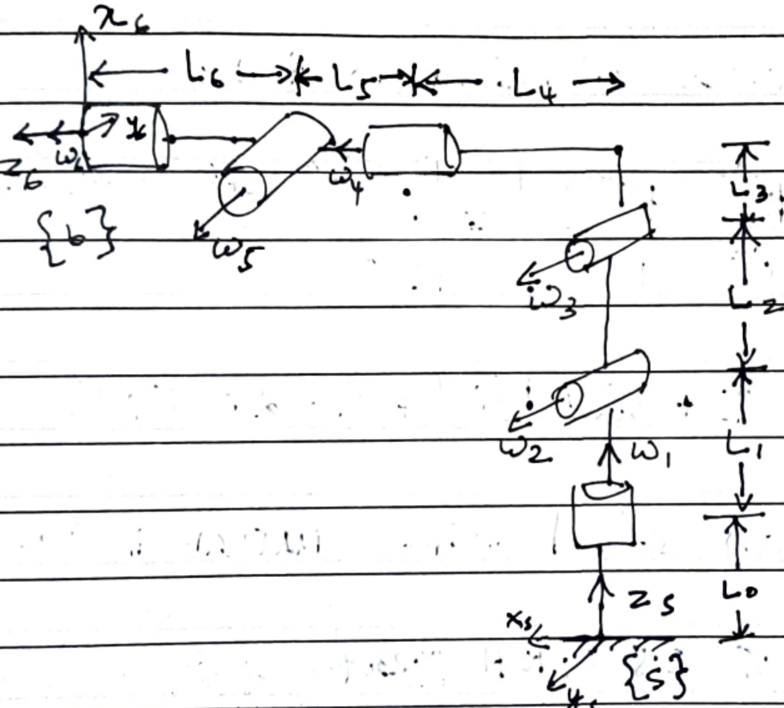
Q·1



$L_0 = 165$ mm $\quad L_1 = 125$ mm $\quad L_2 = 270$ mm $\quad L_3 = 70$ mm

$L_4 = 134$ mm $\quad L_5 = 168$ mm $\quad L_6 = 72$ mm

i	ω_i	p_i	v_i
1	$[0, 0, 1]$	$[0, 0, l_0]$	$[0, 0, 0]$
2	$[0, 1, 0]$	$[0, 0, l_1+l_0]$	$[+(l_1+l_0), 0, 0]$
3	$[0, 1, 0]$	$[0, 0, l_0+l_1+l_2+l_3]$	$[+l_0+l_1+l_2, 0, 0]$
4	$[1, 0, 0]$	$[l_4, 0, l_0+l_1+l_2+l_3]$	$[0, +(l_0+l_1+l_2+l_3), 0]$
5	$[0, 1, 0]$	$[l_4+l_5, 0, l_0+l_1+l_2+l_3]$	$[+(l_0+l_1+l_2+l_3), 0, +(l_4+l_5)]$
6	$[1, 0, 0]$	$[l_4+l_5+l_6, 0, l_0+l_1+l_2+l_3]$	$[0, +(l_0+l_1+l_2+l_3), 0]$

$$M = \begin{bmatrix} 0 & 0 & 1 & l_4+l_5+l_6 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & l_0+l_1+l_2+l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q.2 $q = [\underset{\theta_1}{\pi/4}, \underset{\theta_2}{0}, \underset{\theta_3}{\pi/4}, \underset{\theta_4}{0}, \underset{\theta_5}{-\pi/4}, \underset{\theta_6}{\pi/4}]$

Forward Kinematics:

$$= e^{[S_1]\theta_1} \times e^{[S_2]\theta_2} \times e^{[S_3]\theta_3} \times e^{[S_4]\theta_4} \times e^{[S_5]\theta_5} \times e^{[S_6]\theta_6} \times M$$

All S values were calculated previously.

$$e^{[S]\theta} = \begin{bmatrix} R^{3\times 3} & p^{3\times 1} \\ 0^{1\times 3} & 1 \end{bmatrix} = \begin{bmatrix} e^{[\omega]\theta} & (I\theta + (1-\cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2) \times v \\ 0 & 1 \end{bmatrix}$$

$$\hookrightarrow I + \sin\theta[\omega] + (1-\cos\theta)[\omega]^2$$

The question was solved using MATLAB in 'q2.m'

$$T_{sb} = \begin{bmatrix} 0.5 & 0.5 & 0.7071 & 0.2369 \\ -0.5 & -0.5 & 0.7071 & 0.2369 \\ 0.7071 & -0.7071 & 0 & 0.396 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q.3. Jacobian wrt s frame is $= \left[S_1 \quad Ad_{e^{[S_1]\theta_2}} S_2 \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3 \quad \cdots \right]$

All the S values are calculated in question 1.

Each $e^{[S]\theta}$ term is basically: $\begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$

$$T = \begin{bmatrix} e^{[\omega]\theta \; 3\times3} & (I\theta - (1-\cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2) \times v^{\,3\times1} \\ 0^{1\times3} & 1 \end{bmatrix}$$

The adjoint is $(Ad_{e^{[S]\theta}})$: $\begin{bmatrix} R^{3\times3} & 0^{3\times3} \\ [P]R & R^{3\times3} \end{bmatrix}$

where R is the rotation matrix of T.
P is the translation vector.

~~The question was solved~~

Jacobian wrt body frame $= Ad_{T_{bs}} J_s$

$T_{bs} = T_{sb}^{-1}$ ($\because$ Calculated in Question 2)

The question was solved using MATLAB in 'q3.m'

$$J_b = \begin{bmatrix} \overset{-0.7071}{\cancel{+0.2369}} & -0.7071 & -0.7071 & -0.5 & -0.7071 & 0 \\ -0.7071 & -0.7071 & -0.7071 & 0.5 & -0.7071 & 0 \\ 0 & 0 & 0 & 0.7071 & 0 & 1 \\ -0.2369 & -0.2369 & -0.2369 & 0.036 & -0.0509 & 0 \\ -0.2369 & 0.2369 & 0.2369 & 0.036 & 0.0509 & 0 \\ 0 & 0.1060 & -0.164 & 0 & 0 & 0 \end{bmatrix}$$

Q4: The analytical Jacobian is

$$J_a = R \times J_{v_b}$$

$J_{v_b}^{3\times6}$ = The linear velocity components of J_b
($\because$ J_b was solved in question 4)

$R_{sb}^{3\times3}$ = Obtained from T_{sb}
($\because$ T_{sb} was calculated in question 2)

This was solved using MATLAB in 'q_4.m'

$J_{\cancel{analytical}} = R \times J_{v_b}$ =

$$R_{sb} = \begin{bmatrix} 0.5 & 0.5 & 0.7071 \\ -0.5 & -0.5 & 0.7071 \\ 0.7071 & -0.7071 & 0 \end{bmatrix} \times J_{v_b}$$

$$= \begin{bmatrix} -0.2369 & 0.075 & -0.116 & 0.036 & 0 & 0 \\ 0.2369 & 0.075 & -0.116 & -0.036 & 0 & 0 \\ 0 & -0.335 & -0.335 & 0 & -0.072 & 0 \end{bmatrix}$$

//

Q.5 To check for singularity and manipulability :

We split the previously calculated J_b into

$$J_b = \begin{bmatrix} J_{\omega_b} \\ J_{v_b} \end{bmatrix}$$

$$A_\omega = J_{\omega_b} \times J_{\omega_b}^T$$

$$A_v = J_{v_b} \times J_{v_b}^T$$

Find Eigen values (λ) of A and check if

$$\mu = \frac{\lambda_{max}}{\lambda_{min}} \sim 1 \text{ or } \sim \infty$$

if it is close to 1, then the configuration is not at singularity

With increase in μ the configuration becomes more closer to a singularity.

Calculated from 'q5.m'.

The rank of $J_b = 6$

Since the matrix rank of the Jacobian = Full rank,

The robot is not at singularity.

Rotational conditional number $= \frac{\lambda_{max}}{\lambda_{min}} = 3$ //

Linear conditional number $= \frac{\lambda_{max}}{\lambda_{min}} = 6.4018$ //

If the conditional numbers $\to \infty$ they signify singularity which is not the case here.

Q6: Since the force is applied in the vertical direction wrt $\{B\}$, it is in the x direction.

$$\therefore \quad F_{tip} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -100 \\ 0 \\ 0 \end{bmatrix}$$

$$\underbrace{\tau}_{\text{Torques on each joint.}} = \underbrace{J_b'}_{\text{Transpose of Jacobian wrt body frame}} \times \underbrace{F\_tip}_{\text{Force}}$$

The question was calculated in MATLAB via 'q6.m'.

$$= \begin{bmatrix} 23.69 \\ 23.69 \\ 23.69 \\ -3.6 \\ 5.090 \\ 0 \end{bmatrix}$$

Q.7 To calculate twist in the body frame:

$$T_{bd}(\theta^{\circ}) = \left(T_{sb}(\theta^{\circ})\right)^{-1} T_{sd}$$

T_{sd} → Given in the question

$\left(T_{sb}(\theta^{\circ})\right)^{-1}$ → calculated previously

$$[V_b] = \log\left(T_{bd}(\theta^{\circ})\right)$$

log → Matrix logarithm

The question was solved on MATLAB in 'q7.m'

$$V_b = \cancel{\begin{bmatrix} 23.69 \\ 23.69 \\ 23.69 \end{bmatrix}} = \begin{bmatrix} 1.76 \\ -0.729 \\ 1.76 \\ -0.0418 \\ -0.1437 \\ 0.0341 \end{bmatrix}$$

Q.8 To calculate θ^{i+1} using Newton Raphson,

$$\theta^{i+1} = \theta^{i} + J_b(\theta^{i})^{-1} V_b$$

V_b → calculated in question 7

$J_b(\theta^{i})^{-1}$ → Calculated in question 3

θ^{i} → Given initial configuration

The question was solved in MATLAB in 'q8.m' //

$$\theta^{i+1} = \begin{bmatrix} 0.8023 \\ 0.0811 \\ 0.6299 \\ 2.4651 \\ -1.44 \\ 4.288 \end{bmatrix}$$ //