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MATRICES

Matrix :- It is a rectangular array of members. The no.s in array are said to be entries of the matrices

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

3 elementary row operations

⇒ Inverse of $A_{n \times n}$:

→ If ∃ a matrix $B_{n \times n}$ such that $AB = BA = I_{n \times n}$.
$B = A^{-1}$

If A is sq. matrix and if B of same order can be found such that $AB = BA = I$, then B is said to be inverse of A. and we write $B = A^{-1}$.

→ Inverse only exist when $|A| \neq 0$.

If A^{-1} exist then A is said to be non-singular matrix.

If A^{-1} doesn't exist then A is said to be singular matrix.

Methods of finding A^{-1}.

We define 3 elementary row operation on A.

1) Multiply a row with non zero constant.

2) Interchange two rows.

3) Add a constant times to one row to another row.

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

$[A|I]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$R_1 - 2R_2$
$R_3 \rightarrow -R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$R_1 \longrightarrow R_1 - 9R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$R_2 \longrightarrow R_2 + 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Q. Find inverse of following matrices

i) $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$$

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$R_1 - R_2$ $\left[\begin{array}{cc|cc} 1 & 1 & 1 & -1 \\ 2 & 3 & 0 & 1 \end{array}\right]$

$R_2 - 2R_1$ $\left[\begin{array}{cc|cc} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & 3 \end{array}\right]$

$R_1 - R_2$ $\left[\begin{array}{cc|cc} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 3 \end{array}\right]$

(ii) $B = \begin{bmatrix} 0 & -3 \\ 7 & 2 \end{bmatrix}$

$\left[\begin{array}{cc|cc} 0 & -3 & 1 & 0 \\ 7 & 2 & 0 & 1 \end{array}\right]$

$R_2 \to 2R_2 + R_1$ $\left[\begin{array}{cc|cc} 0 & -3 & 1 & 0 \\ 14 & 1 & 1 & 2 \end{array}\right]$

$R_1 \to R_1 + 3R_2$ $\left[\begin{array}{cc|cc} 42 & 0 & 4 & 6 \\ 14 & 1 & 1 & 2 \end{array}\right]$

$R_1 \to \frac{R_1}{42}$ $\left[\begin{array}{cc|cc} 1 & 0 & 2/21 & 1/7 \\ 14 & 1 & 1 & 2 \end{array}\right]$

$R_2 \to R_2 - 14R_1$ $\left[\begin{array}{cc|cc} 1 & 0 & 2/21 & 1/7 \\ 0 & 1 & -1/3 & 0 \end{array}\right]$

OR.

$R_1 \leftrightarrow R_2$ $\left[\begin{array}{cc|cc} 7 & 2 & 0 & 1 \\ 0 & -3 & 1 & 0 \end{array}\right]$

$R_1 : \frac{1}{7} R_2$ $\left[\begin{array}{cc|cc} 1 & 2/7 & 0 & 1/7 \\ 0 & -3 & 1 & 0 \end{array}\right]$

$R_2 : \frac{-1}{3} R_2$ $\left[\begin{array}{cc|cc} 1 & 2/7 & 0 & 1/7 \\ 0 & 1 & -1/3 & 0 \end{array}\right]$

$R_1 : R_1 - \frac{2}{7} R_2$ $\left[\begin{array}{cc|cc} 1 & 0 & 2/21 & 1/7 \\ 0 & 1 & -1/3 & 0 \end{array}\right]$

(iii) $A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$

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$R_2 \rightarrow R_2 - 2R_1$ $\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ -1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$

$R_3 \rightarrow R_1 + R_3$ $\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right]$

This is singular matrix because $R_2 + R_3 = 0$.

Reduced Row Echelon form :-

1) If a row does not consist entirely of zeroes then first non-zero no. in the row is a 1. We call this a leading 1.

2) If there are any two rows that consist entirely of zeroes, then they are grouped together at the bottom of the matrix.

3) In any two successive rows that do not consist entirely of zeroes, the leading 1 in the lower row occurs further to the right then the leading 1 in the higher row.

4) Each column that contains a leading 1 has zeroes everywhere else in that column.

$$A = \begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

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$R_1 \leftrightarrow R_2$ $\begin{bmatrix} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$

$R_1 \to \frac{R_1}{2}$ $\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$

$R_3 - 2R_1$ $\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{bmatrix}$

$\frac{-R_2}{2}$ $\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{bmatrix}$

$R_3 - 5R_2$ $\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1/2 & -1 \end{bmatrix}$

$R_1 + 5R_2$
$2R_3$ $\begin{bmatrix} 1 & 2 & 0 & 0 & -23/2 & -16 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$

Rank of matrix $= 3$

Q-1 Find the rank of matrix :-

$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

$R_1 \to R_1 - R_3$ $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

$R_2 \to R_1 + R_2$
$R_3 \to R_2 + R_3$ $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

$R_2 \to R_2 - 3R_2$ $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -7 \\ 0 & 1 & 2 \end{bmatrix}$

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$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -7 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_2 - R_3$ $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & -9 \end{bmatrix}$

$R_1 \to R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & +13 \\ 0 & 1 & -7 \\ 0 & 0 & -9 \end{bmatrix}$

$R_3 \to \frac{-R_3}{+9}$ $\begin{bmatrix} 1 & 0 & +13 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 + 7R_3$
$R_1 \to R_1 - 13R_3$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rank = 3

Q-2 find rank of matrix

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 1 & 4 & 3 \\ 2 & 7 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$ $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 5 \\ 2 & 7 & 1 \end{bmatrix}$

$R_3 \to R_3 - 2R_1$ $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 5 \\ 0 & -1 & -5 \end{bmatrix}$

$R_3 \to R_2 + R_3$ $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

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$R_1 \rightarrow R_1 - 4R_2$
$$\begin{bmatrix} 1 & 0 & -17 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank $= 2$

Rank is not equal to order of matrix as determinant is 0.

(ii)
$$A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$R_1 \rightarrow \frac{R_1}{3}$
$$\begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

$R_3 \rightarrow 3R_1 - R_3$
$$\begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & 2 & -4 & 4 & 2 & -6 \end{bmatrix}$$

$R_2 \rightarrow \frac{R_2}{3}$
$$\begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 1 & -2 & +2 & 4/3 & -5/3 \\ 0 & 2 & -4 & 4 & 2 & -6 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$
$$\begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 1 & -2 & 2 & 4/3 & -5/3 \\ 0 & 0 & 0 & 0 & 2-8/3 & -6+10/3 \\ & & & & -2/3 & -8/3 \end{bmatrix}$$

$R_3 \rightarrow \frac{-3}{2} R_3$
$$\begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 1 & -2 & 2 & 4/3 & -5/3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Rank $= 3$

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Q

$$\begin{aligned} 2x + y + z &= 2 \\ -x + 2y + 0z &= 1 \\ x - y + 2z &= 2 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$\cancel{2x}$ $Ax = b$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ -1 & 2 & 0 & 1 \\ 1 & -1 & 2 & -2 \end{array}\right]$$

$R_1 \leftrightarrow R_3$ $\left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ -1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{array}\right]$

$R_2 : R_2 + R_1$
$R_3 : R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 3 & -3 & 6 \end{array}\right]$$

$R_1 : R_1 + R_2$
$R_3 : \frac{R_3}{3} - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -2 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & +1 & -1 \end{array}\right]$$

$R_2 : R_2 - 2R_3$
$R_1 : R_1 - 4R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & +1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$ find as.

$x = 1 \quad y = 1 \quad z = -1$

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Q
$$\begin{bmatrix} 0 & 1 & 5 \\ 1 & 4 & 3 \\ 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ -1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$
$$\left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -1 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$
$$\left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 3 \end{array}\right]$$

$R_3 \rightarrow R_2 + R_3$
$$\left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

This system has no sol^n.

$\Rightarrow$ When a complete row is 0 Then eq^n has ∞ sol^n.

Q Determine rank of matrices:-

1) $A = \begin{bmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 10 & 11 & 12 & 13 \end{bmatrix}$

$R_1 \rightarrow R_2 - R_1$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 10 & 11 & 12 & 13 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 4R_1$
$R_3 \rightarrow R_3 - 5R_1$
$R_4 \rightarrow R_4 - 10R_1$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

Rank = 2.

Q-2 $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 0 & 3 & 1 \\ 2 & -4 & -2 & -4 \\ -1 & -6 & -9 & -8 \end{bmatrix}$

A/F $R_2 \to R_2 - R_3$
~~$R_3 \to R$~~
$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 4 & 5 & 5 \\ 2 & -4 & -2 & -4 \\ -1 & -6 & -9 & -8 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 + R_1$
$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 4 & 5 & 5 \\ 0 & -8 & -10 & -10 \\ 0 & -4 & -5 & -5 \end{bmatrix}$$

$R_4 \to R_2 + R_4$
$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 4 & 5 & 5 \\ 0 & -8 & -10 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$
$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 4 & 5 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

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Q-3. $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

$R_3 \to R_3 - R_1$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

$R_2 \to -\frac{R_2}{6}$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Q-4 $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -4 \end{bmatrix}$

$R_2 \to -R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -2 & -4 \end{bmatrix}$

$R_3 \to \frac{R_3}{2} + R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $\rho(A) = 2$

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Q-5 $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \end{bmatrix}$

$R_3 \to R_1 + R_3$ $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ 0 & 5 & 8 \end{bmatrix}$

$R_1 \to \frac{R_1}{3}$ $\begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 5 & 8 \\ 0 & 5 & 8 \end{bmatrix}$

$R_3 \to R_2 - R_3$ $\begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 5 & 8 \\ 0 & 0 & 0 \end{bmatrix}$

$R_2 \to \frac{R_2}{5}$

$\begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 1 & 8/5 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2$

Q-6 $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_2$ $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$ $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$

$\rho(A) = 2$

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Solution of Linear System of Eq^n:-

(all B's = 0) **Homogeneous System**

- Consistent
  - Unique Sol^n (or trivial Sol^n)
  - Infinite

(atleast 1 B ≠ 0) **Non-Homogeneous System**

- Inconsistent (No-Solution)
- (have sol^n) Consistent
  - Unique Sol^n
  - Infinite many Sol^n.

Q

$$2x + 3y + 4z = 6$$
$$x + 2y + 5z = 8$$
$$3x + 5y + z = 10$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 5 \\ 3 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}$$

$$AX = B$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 6 \\ 1 & 2 & 5 & 8 \\ 3 & 5 & 1 & 10 \end{array}\right]$$

$R_1 \leftrightarrow R_2$
$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 8 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 1 & 10 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 8 \\ 0 & -1 & -6 & -10 \\ 0 & -1 & -14 & -14 \end{array}\right]$$

$R_2 \rightarrow -R_2$
$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 8 \\ 0 & 1 & 6 & 10 \\ 0 & -1 & -14 & -14 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 8 \\ 0 & 1 & 6 & 10 \\ 0 & -1 & -14 & -14 \end{array}\right]$$

$R_3 \to R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 8 \\ 0 & 1 & 6 & 10 \\ 0 & 0 & -8 & -4 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & \cancel{8}\,-7 & -12 \\ 0 & 1 & 6 & 10 \\ 0 & 0 & -8 & -4 \end{array}\right]$$

$R_3 \to \frac{-R_2}{8}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -7 & -12 \\ 0 & 1 & 6 & 10 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

$R_1 \longrightarrow R_1 + 7R_3$

$R_2 \longrightarrow R_2 + (-6R_2)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7/2 + (-12) \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -17/2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

$\Rightarrow$ Check.

$\cancel{2}\left(\frac{-17}{\cancel{2}}\right) + 3(7) + \cancel{4}^{2}\left(\frac{1}{\cancel{2}}\right) = 6$

$-17 + 21 + 2$

$23 - 17 = 6$

$6 = 6$

$x = \frac{-17}{2}$, $y = 7$, $z = \frac{1}{2}$ Ans.

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⇒ Augmented Matrix

$$[A|B] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ a_{31} & \vdots & & \vdots & b_3 \\ \vdots & \vdots & & \vdots & b_4 \\ a_{n1} & & & a_{nn} & \vdots \\ & & & & b_n \end{array}\right]$$

C-1 Non-homogeneous

$\rho(A) = \rho(A|B) = n$ (no. of variables)

↑

System is consistent

C-2 $\rho(A) = \rho(A|B) < n$ (no. of variable)

↑

System is consistent but have infinite many sol^n.

C-3 $\rho(A) \neq \rho(A|B)$ ~~> n (no. of variables)~~

↑

System is inconsistent means no sol^n.

Q2 Show that the system of eq^n:

$x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30$ are consistent & solve them.

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & \cancel{14}\ 8 \\ 0 & 3 & 6 & \cancel{30}\ 24 \end{array}\right]$$

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$R_3 \rightarrow R_3 - 3R_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A|B) = 2 < 3$ (no. of variables)
System is consistent but infinitely many sol^n.

$$x + y + z = 6$$
$$0x + y + 2z = 8$$

Let $\boxed{z = k}$
$$\boxed{y = 8 - 2k}$$

$$x + 8 - 2k + k = 6$$
$$\boxed{x = k - 2}$$

$k = 1, 2, \dots$

Q.2. Solve
$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$
$$2x - 2y + 3z = 7$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 2R_1$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & -4 & 1 & -5 \end{array}\right]$$

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$R_2 \rightarrow \frac{-R_2}{2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & -2 & -2 & -10 \\ 0 & -4 & 1 & -5 \end{array}\right]$

$R_1 \rightarrow R_1 - R_2$
$R_3 \rightarrow R_3 + 2R_2$
$R_4 \rightarrow R_4 + 4R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 11/2 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & -1 & -3 \end{array}\right]$

$R_3 \longrightarrow \frac{-R_3}{3}$ $\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 11/2 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & +1 & +3 \\ 0 & 0 & -1 & -3 \end{array}\right]$

$R_4 \rightarrow R_3 + R_4$ $\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 11/2 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\cancel{R} \quad \cancel{\rho(A} \quad \cancel{R_4 \rightarrow \frac{R_4}{-5}}$ $\left[\begin{array}{ccc|c} 1 & 0 & 3/2 & 11/2 \\ 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\cancel{\rho(A) \neq \rho(A|B)}$ $\rho(A) = \rho(A|B) = 3$

$0x + 0y + z = -3$

$\boxed{z = -3}$

$0x + y + \left(\frac{-1}{2}\right) z = -3$

$y + \frac{3}{2} = \frac{1}{2}$

$\boxed{y = -1}$ $\cancel{y = \frac{-3}{2} = \frac{-2}{2}}$..

$x + \frac{3}{2}(-3) = \frac{11}{2}$

$x - \frac{9}{2} = \frac{11}{2}$

$x = \frac{20}{2} = 10$

$x = 10,\ y = -1,\ z = -3$

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Q For what values of n the eq^n

$$x + y + z = 1$$
$$x + 2y + 4z = n$$
$$x + 4y + 10z = n^2$$

are consistent & solve them completely in each case

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & n \\ 1 & 4 & 10 & n^2 \end{array}\right]$$

$R_2 \longrightarrow R_2 - R_1$
$R_3 \longrightarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & n-1 \\ 0 & 3 & 9 & n^2-1 \end{array}\right]$$

$R_3 \longrightarrow R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & n-1 \\ 0 & 0 & 0 & n^2-1-3n+3 \end{array}\right]$$

$$n^2 - 3n + 2 = 0$$
$$n^2 - 2n - n + 2 = 0$$
$$n(n-2) - 1(n-2) = 0$$
$$(n-1)(n-2) = 0$$
$$n = 1, 2$$

for $n = 1$ or $n = 2$ the system of eqn^s is consistent

Case 1 $\quad n = 1$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y + z = 1$$
$$y + 3z = 0$$

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Let $\boxed{z = k_1}$

$\boxed{y = -3k_1}$

$x = 1 - y - z$

$x = 1 + 3k_1 - k_1$

$\boxed{x = 1 + 2k_1}$

Case-2. when $n = 2$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + y + z = 1$

$y + 3z = 1$

$\boxed{z = k_2}$ $\boxed{y = 1 - 3k_2}$

$x = 1 + 3k_2 - 1 - k_2$

$\boxed{x = 2k_2}$

Q-2 Find the values of λ & μ so that the system of equations

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$ has

1) Unique sol^n
2) no sol^n
3) ∞ many sol^n.

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

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$R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 1 & -6 & -17 & -19 \\ 0 & 0 & 5-\lambda & \mu-9 \end{array}\right]$$

$R_1 \leftrightarrow R_2$ $$\left[\begin{array}{ccc|c} 1 & -6 & -17 & -19 \\ 2 & 3 & 5 & 9 \\ 0 & 0 & 5-\lambda & \mu-9 \end{array}\right]$$

Case-1.
for unique solⁿ.

$\lambda - 5 \neq 0$ $\quad$ $\mu - 9 \neq 0$
$\lambda \neq 5$ $\quad$ $\mu \neq 9$

$\rho(A) = \rho(A|B) = 3$ (no. of variables)

Case-2. $\lambda - 5 = 0$ & $\mu - 9 \neq 0$
$\lambda = 5$ $\quad$ $\mu \neq 9$ $\quad$ $(\rho(A)) \neq \rho(A|B)$

Case-3. $\lambda - 5 = 0$ $\quad$ $\mu - 9 = 0$
$\lambda = 5$ $\quad$ $\mu = 9$

$\rho(A) = \rho(A|B) < 3$ (no. of variables)

Q-3. Solve the system of $eq^n s$:-

$x + 3y - 2z = 0$
$2x - y + 4z = 0$
$x - 11y + 14z = 0$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array}\right]$$

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$R_3 \rightarrow R_3 - 2R_2$ $\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\rho(A) = \rho(A|B) = 2 < 3$ (no. of variables)

$x + 3y - 2z = 0$

$-7y + 8z = 0$

$\boxed{z = k_1}$

$-7y + 8k_1 = 0$

$\boxed{y = \frac{8}{7} k_1}$

$x = -3y + 2z$

$= -3 \times \frac{8}{7} k_1 + 2k_1$

$= 2k_1 - \frac{24}{7} k_1$

$\boxed{x = \frac{-10}{7} k_1}$

Q-4 Solve :

$x + 2y + 3z = 0$

$3x + 4y + 4z = 0$

$7x + 10y + 12z = 0$

$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 4 & 4 & 0 \\ 7 & 10 & 12 & 0 \end{array}\right]$

$R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - 7R_1$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right]$

$R_3 \rightarrow R_3 - 2R_2$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

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$\rho(A) = \rho(A|B) = 3$

$x + 2y + 3z = 0$

$-2y - 5z = 0$

$z = 0$

$x = y = z = 0$

Q Solve :-

$x + y + z = 6$

$x - y + 2z = 5$

$3x + y + z = 8$

$2x - 2y + 3z = 7$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{array}\right]$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & -2 & -2 & -10 \\ 0 & -4 & 1 & -5 \end{array}\right]$$

$R_3 \to \frac{-R_3}{2}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -1 \\ 0 & 1 & 1 & 5 \\ 0 & -4 & 1 & -5 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & -2 & 1 & -1 \\ 0 & -4 & 1 & -5 \end{array}\right]$$

$R_3 \to R_3 + 2R_2$
$R_4 \to R_4 + 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 5 & 15 \end{array}\right]$$

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$R_3 \to \frac{R_3}{3}$

$R_4 \to \frac{R_4}{5}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_4 \to R_3 - R_4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + y + z = 6$

$y + z = 5$

$\boxed{z = 3}$ $\boxed{y = 2}$ $\boxed{x = 1}$

Q2

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

$$(A|B) = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_2 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 8 & 29 & 9 & 13 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 1 & 27 & -1 & 8 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_1 \to R_1 - 5R_2$

$R_3 \to R_3 - 7R_2$

$$\left[\begin{array}{ccc|c} 0 & -132 & 12 & -36 \\ 1 & 27 & -1 & 8 \\ 0 & -187 & 17 & -51 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 0 & -132 & 12 & -36 \\ 0 & -187 & 17 & -51 \end{array}\right]$$

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$R_2 \rightarrow \frac{R_2}{12}$

$R_3 \rightarrow \frac{R_3}{17}$

$$\left[\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 0 & -11 & 1 & -3 \\ 0 & -11 & 1 & -3 \end{array}\right]$$

$R_3 \rightarrow R_3 + (-R_2)$

$$\left[\begin{array}{ccc|c} 1 & 27 & -1 & 8 \\ 0 & -11 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A|B) < 3$ no. of variables

$-11y + z = -3$

$x + 27y - z = 8$

$\boxed{z = k}$

$-11y + k = -3$

$y = \frac{-k-3}{-11} = \frac{k+3}{11}$

$x + 27\left(\frac{k+3}{11}\right) - k = 8$

$x = 8 + k - \frac{27(k+3)}{11}$

$= \frac{88 + 11k - 27k - 81}{11}$

$\boxed{x = \frac{7 - 16k}{11}}$ $\boxed{y = \frac{k+3}{11}}$

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Eigen values & Eigen vectors.

Let A be $n \times n$ matrix,

Suppose the linear transformation $Ax = y$ transforms x into scalar multiple of itself. i.e $Ax = y = \lambda x$
Then the unknown scalar λ is known as eigen value of the matrix A. and the corresponding non-zero vector x is known as eigen vector of A. corresponding to the eigen value λ.

Thus the eigen values are scalar's λ ~~wil~~ which satisfies the equation

$$Ax = \lambda x$$

or $$Ax - \lambda x = 0$$

or $$[A - \lambda I] x = 0$$

This system of equation has non-trivial solution if the coefficient matrix $(A - \lambda I)$ is singular

i.e $|A - \lambda I| = 0$

Q. Find the eigen values & corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

for find eigen values
$|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

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~~$8-\lambda\{(7-\lambda)(+3\mp\lambda)+8\{$~~

$$(8-\lambda)\{(7-\lambda)(3-\lambda)-16\}+6\{-6(3-\lambda)+8\}+2\{24-2(7-\lambda)\}=0$$

$$(8-\lambda)\{21+\lambda^2-10\lambda-16\}+6\{-18+\lambda+8\}+2\{24-14+2\lambda\}=0$$

$$-\lambda^3+18\lambda^2-45\lambda=0$$

$$\lambda^3-18\lambda^2+45\lambda=0$$

$$\lambda(\lambda-3)(\lambda-15)=0$$

$$\boxed{\lambda=3,0,15}$$

$\Rightarrow$ Now for finding Eigen vectors

$$[A-\lambda I][x]=0$$

① for $\lambda=0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$x_1 8-6x_2+2x_3=0$$

$$-6x_1+7x_2-4x_3=0$$

$$2x_1-4x_2+3x_3=0$$

$$\frac{x_1}{24-14}=\frac{-x_2}{-32+12}=\frac{x_3}{56-36}=k_1$$

$$\frac{x_1}{10}=\frac{-x_2}{-20}=\frac{x_3}{20}=k_1$$

$$x_1=k_1,\quad x_2=2k_1,\quad x_3=2k_1$$

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$$X_1 = \begin{bmatrix} k_1 \\ 2k_1 \\ 3k_1 \end{bmatrix} \qquad k_1 = 1, 2, 3, \ldots$$

for $\lambda = 3$

$$\begin{bmatrix} 8-3 & -6 & 2 \\ -6 & 7-3 & -4 \\ 2 & -4 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$5x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 + 4x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 = 0$$

$$\frac{x_1}{24-8} = \frac{-x_2}{-20+12} = \frac{x_3}{20-36} = k_2$$

$$\frac{x_1}{16} = \frac{x_2}{08} = \frac{x_3}{-16} = k_2$$

$$x_1 = 2k_2, \quad x_2 = k_2 = x_3 = -2k_2$$

$$X_2 = \begin{bmatrix} 2k_2 \\ k_2 \\ -2k_2 \end{bmatrix} \qquad k_2 = 1, 2, 3 \ldots$$

if $k_2 = k$ $\quad X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

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for $\lambda = 15$

$$\begin{bmatrix} 8-15 & -6 & 2 \\ -6 & 7-5 & -4 \\ 2 & -4 & 3-15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-7x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 - 8x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 - 12x_3 = 0$$

$$\frac{x_1}{24+16} = \frac{-x_2}{28+12} = \frac{x_3}{+56 - 36} = k_3$$

$$\frac{x_1}{40 \cancel{20}} = \frac{-x_2}{40} = \frac{x_3}{20} = k_3$$

$$X_3 = \begin{bmatrix} 2k_3 \\ -2k_3 \\ k_3 \end{bmatrix}, \quad k_3 = 1, 2, \ldots$$

Hence eigen ~~value~~ vector.

$$X = \begin{array}{c} \begin{matrix} X_1 & X_2 & X_3 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \end{array}$$

Caley Hamilton Theorem :-

Every square matrix satisfies its characteristic eq^n.

Q Find the characteristic eq^n from matrix →

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Show this eq^n is satisfied by A & Hence find A^{-1}

$$A = \begin{bmatrix} 0-\lambda & 1 & 2 \\ 1 & 2-\lambda & 3 \\ 3 & 1 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$A = \begin{vmatrix} 0-\lambda & 1 & 2 \\ 1 & 2-\lambda & 3 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$= -\lambda\{(2-\lambda)(1-\lambda) - 3\} - 1\{(1-\lambda) - 9\} + 2\{(1) - 3(2-\lambda)\}$$

$$= -\lambda(2 - 3\lambda + \lambda^2 - 3) - 1(1 - \lambda - 9) + 2(1 - 6 + 3\lambda)$$

$$= -2\lambda + 3\lambda^2 - \lambda^3 + 3\lambda - 1 + \lambda + 9 + 2 - 12 + 6\lambda$$

$$0 = +\lambda^3 - 3\lambda^2 - 8\lambda + 2$$

$$\lambda^3 - 3\lambda^2 - 8\lambda + 2 = 0$$ (by Cayley Hamilton Theorm)

$$A^3 - 3A^2 - 8A + 2I = 0$$

for verificat^n

$$A^3 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 20 & 31 \\ 41 & 38 & 57 \\ 36 & 26 & 36 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 19 & 20 & 31 \\ 41 & 38 & 57 \\ 36 & 26 & 36 \end{bmatrix} - 3\begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix} - 8\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 20 & 31 \\ 41 & 38 & 57 \\ 36 & 26 & 36 \end{bmatrix} - \begin{bmatrix} 21 & 12 & 15 \\ 33 & 24 & 33 \\ 12 & 18 & 30 \end{bmatrix} - \begin{bmatrix} 0 & 8 & 16 \\ 8 & 16 & 24 \\ 24 & 8 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ Ans. Hence verified.

To find Inverse.

$$A^3 - 3A^2 - 8A + 2I = 0$$

$\cancel{A^2(A-3I)} -$ multiply A^{-1} on both sides

$$A^2 - 3A - 8I + 2A^{-1} = 0$$

$$2A^{-1} = -A^2 + 3A + 8I$$

OR.

$$A^3 = 3A^2 + 8A - 2I$$

$$2A^{-1} = -\begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix} + 3\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} - 8\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix}$$

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$$= -\begin{bmatrix} 7 & 4 & 5 \\ 11 & 8 & 11 \\ 4 & 6 & 10 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 \\ -8 & 6 & \cancel{13}\ -2 \\ 5 & -3 & 1 \end{bmatrix}$$

$$A^{-1} = \cancel{\begin{bmatrix} 5/2 & 2 & 11/2 \\ -3/2 & 3 & 13/2 \\ 10 & 1 & -2 \end{bmatrix}} \begin{bmatrix} 1/2 & -1/2 & -1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & -1/2 \end{bmatrix}$$

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