Prime Number

A **prime number** is a whole number greater than 1, which is only divisible by 1 and itself. First few prime numbers are : 2 3 5 7 11 13 17 19 23 ....

Naive Method to Check if a number is Prime: Since a number is prime only if it is divisible by 1 and the number itself, the naive method to check for primality of a number would be to iterate from 1 to N and check if there aren't any factors of N except and 1 and N itself.

Algorithm:

1. If, N is less than 2. It is not prime, return False.
2. Else:
   - Iterate from 2 to N-1 and check if any of the numbers between 2 and N-1 (both inclusive) divides N or not. If yes, then N is not prime, return False.
   - Otherwise, return True.

Analysis of the above algorithm: Since we are traversing linearly from 2 to N-1, the time complexity of the above algorithm will be linear **O(N)**.

Sieve of Eratosthenes

Using **Sieve of Eratosthenes** is the most efficient way of generating prime numbers upto a given number N.

Following is the algorithm to find all the prime numbers less than or equal to a given integer n by Eratosthenes' method:

1. Create a list of consecutive integers from 2 to n: (2, 3, 4, ..., n).
2. Initially, let p equal 2, the first prime number.
3. Starting from p^2, count up in increments of p and mark each of these numbers greater than or equal to p^2 itself in the list. These numbers will be $p(p+1)$, $p(p+2)$, $p(p+3)$, etc..
4. Find the first number greater than p in the list that is not marked. If there was no such number, stop. Otherwise, let p now equal this number (which is the next prime), and repeat from step 3.

Explanation with Example: Let us take an example when n = 50. So we need to print all print numbers smaller than or equal to 50.

We create a list of all numbers from 2 to 50.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

According to the algorithm we will mark all the numbers which are divisible by 2 and are greater than or equal to the square of it.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Now we move to our next unmarked number 3 and mark all the numbers which are multiples of 3 and are greater than or equal to the square of it.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

We move to our next unmarked number 5 and mark all multiples of 5 and are greater than or equal to the square of it.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

We continue this process and our final table will look like below:

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

So the prime numbers are the unmarked ones: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.