Quick Sort

QuickSort is a Divide and Conquer algorithm. It picks an element as pivot and partitions the given array around the picked pivot. There are many different versions of quickSort that pick pivot in different ways.

1. Always pick first element as pivot.
2. Always pick last element as pivot (implemented below)
3. Pick a random element as pivot.
4. Pick median as pivot.

The key process in quickSort is partition(). Target of partitions is, given an array and an element x of array as pivot, put x at its correct position in sorted array and put all smaller elements (smaller than x) before x, and put all greater elements (greater than x) after x. All this should be done in linear time.

Pseudo Code for recursive QuickSort function :

```
/* low  --> Starting index,  high  --> Ending index */
quickSort(arr[], low, high)
{
    if (low < high)
    {
        /* pi is partitioning index, arr[pi] is now
           at right place */
        pi = partition(arr, low, high);

        quickSort(arr, low, pi - 1);  // Before pi
        quickSort(arr, pi + 1, high); // After pi
    }
}
```

Partition Algorithm:
There can be many ways to do partition, following pseudo code adopts the method given in CLRS book. The logic is simple, we start from the leftmost element and keep track of index of smaller (or equal to) elements as i. While traversing, if we find a smaller element, we swap current element with arr[i]. Otherwise we ignore current element.

```
/* low  --> Starting index,  high  --> Ending index */
quickSort(arr[], low, high)
{
    if (low < high)
    {
        /* pi is partitioning index, arr[pi] is now
           at right place */
        pi = partition(arr, low, high);

        quickSort(arr, low, pi - 1);  // Before pi
        quickSort(arr, pi + 1, high); // After pi
    }
}
```

Pseudo code for partition()

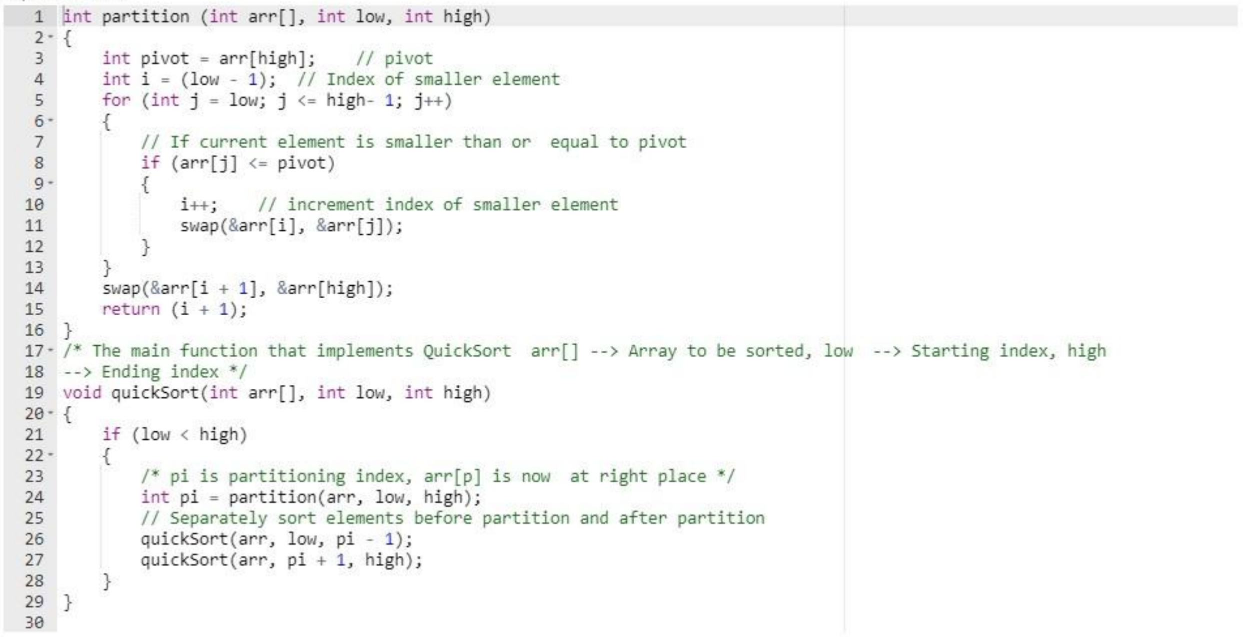
```
/* This function takes last element as pivot, places
   the pivot element at its correct position in sorted
   array, and places all smaller (smaller than pivot)
   to left of pivot and all greater elements to right
   of pivot */
partition (arr[], low, high)
{
    // pivot (Element to be placed at right position)
    pivot = arr[high];

    i = (low - 1)  // Index of smaller element

    for (j = low; j <= high- 1; j++)
    {
        // If current element is smaller than or
        // equal to pivot
        if (arr[j] <= pivot)
        {
            i++;    // increment index of smaller element
            swap arr[i] and arr[j]
        }
    }

    swap arr[i + 1] and arr[high])
    return (i + 1)
}
```

Illustration of partition() :

```
arr[] = {10, 80, 30, 90, 40, 50, 70}

Indexes:  0   1   2   3   4   5   6



low = 0, high =  6, pivot = arr[h] = 70

Initialize index of smaller element, i = -1


Traverse elements from j = low to high-1

j = 0 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])

i = 0
arr[] = {10, 80, 30, 90, 40, 50, 70} // No change as i and j

                                     // are same



j = 1 : Since arr[j] > pivot, do nothing

// No change in i and arr[]



j = 2 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])

i = 1
arr[] = {10, 30, 80, 90, 40, 50, 70} // We swap 80 and 30
```

```
j = 3 : Since arr[j] > pivot, do nothing

// No change in i and arr[]



j = 4 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])

i = 2
arr[] = {10, 30, 40, 90, 80, 50, 70} // 80 and 40 Swapped

j = 5 : Since arr[j] <= pivot, do i++ and swap arr[i] with arr[j]

i = 3
arr[] = {10, 30, 40, 50, 80, 90, 70} // 90 and 50 Swapped



We come out of loop because j is now equal to high-1.

Finally we place pivot at correct position by swapping

arr[i+1] and arr[high] (or pivot)

arr[] = {10, 30, 40, 50, 70, 90, 80} // 80 and 70 Swapped



Now 70 is at its correct place. All elements smaller than

70 are before it and all elements greater than 70 are after

it.
```

Implementation:



```
int partition (int arr[], int low, int high)
{
    int pivot = arr[high];    // pivot
    int i = (low - 1);  // Index of smaller element
    for (int j = low; j <= high- 1; j++)
    {
        // If current element is smaller than or  equal to pivot
        if (arr[j] <= pivot)
        {
            i++;    // increment index of smaller element
            swap(&arr[i], &arr[j]);
        }
    }
    swap(&arr[i + 1], &arr[high]);
    return (i + 1);
}
/* The main function that implements QuickSort  arr[] --> Array to be sorted, low  --> Starting index, high
--> Ending index */
void quickSort(int arr[], int low, int high)
{
    if (low < high)
    {
        /* pi is partitioning index, arr[p] is now  at right place */
        int pi = partition(arr, low, high);
        // Separately sort elements before partition and after partition
        quickSort(arr, low, pi - 1);
        quickSort(arr, pi + 1, high);
    }
}

```

Analysis of QuickSort

Time taken by QuickSort in general can be written as following.

```
T(n) = T(k) + T(n-k-1) + θ(n)
```

The first two terms are for two recursive calls, the last term is for the partition process. k is the number of elements which are smaller than pivot.
The time taken by QuickSort depends upon the input array and partition strategy. Following are three cases.

Worst Case: The worst case occurs when the partition process always picks greatest or smallest element as pivot. If we consider above partition strategy where last element is always picked as pivot, the worst case would occur when the array is already sorted in increasing or decreasing order. Following is recurrence for worst case.

```
 T(n) = T(0) + T(n-1) + θ(n)
which is equivalent to
 T(n) = T(n-1) + θ(n)
```

The solution of above recurrence is $\Theta(n^2)$.

Best Case: The best case occurs when the partition process always picks the middle element as pivot. Following is recurrence for best case.

```
 T(n) = 2T(n/2) + θ(n)
```

The solution of above recurrence is **Θ(nLogn)**. It can be solved using case 2 of Master Theorem.

Average Case: To do average case analysis, we need to consider all possible permutation of array and calculate time taken by every permutation which doesn't look easy.
We can get an idea of average case by considering the case when partition puts O(n/9) elements in one set and O(9n/10) elements in other set. Following is recurrence for this case.

```
 T(n) = T(n/9) + T(9n/10) + θ(n)
```

Solution of above recurrence is also O(nLogn)

Although the worst case time complexity of QuickSort is $O(n^2)$ which is more than many other sorting algorithms like **Merge Sort** and **Heap Sort**, QuickSort is faster in practice, because its inner loop can be efficiently implemented on most architectures, and in most real-world data. QuickSort can be implemented in different ways by changing the choice of pivot, so that the worst case rarely occurs for a given type of data. However, merge sort is generally considered better when data is huge and stored in external storage.