

Permutation & Combination

Permutation

Permutation is the different arrangements of a given number of elements taken one by one, or some, or all at a time. For example, if we have two elements A and B, then there are two possible arrangements, AB and BA.

Number of permutations when 'r' elements are arranged out of a total of 'n' elements is ${}^n P_r = \frac{n!}{(n-r)!}$. For example, let $n = 4$ (A, B, C and D) and $r = 2$ (All permutations of size 2). The answer is $4!/((4-2)!) = 12$. The twelve permutations are AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB and DC.

Important Properties of Permutation:

1. ${}^n P_n = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 1 = n!$.
2. ${}^n P_0 = n! / n! = 1$.
3. ${}^n P_1 = n$.
4. ${}^n P_{n-1} = n!$.
5. ${}^n P_r / {}^n P_{r-1} = n - r + 1$.

Permutation with repetition allowed: The number of permutation or arrangements of N numbers with repetition allowed will be N^N . For Example, permutations of {1,2} with repetitions will be {{1,1}, {1,2}, {2,1},{2,2}}.

Permutation with duplicates: The number of permutations or arrangements of N objects of which p_1 are of one kind, p_2 are of second kind, ..., p_k are of k-th kind and the rest if any, are of different kinds is: $N! / (p_1! \cdot p_2! \cdot \dots \cdot p_k!)$.

Combination

Combination is the different selections of a given number of elements taken one by one, or some, or all at a time. For example, if we have two elements A and B, then there is only one way to select two items, we select both of them.

Number of combinations when 'r' elements are selected out of a total of 'n' elements is ${}^n C_r = \frac{n!}{(r!) \cdot (n-r)!}$. For example, let $n = 4$ (A, B, C and D) and $r = 2$ (All combinations of size 2). The answer is $4!/((2!) \cdot (4-2)!) = 6$. The six combinations are AB, AC, AD, BC, BD, CD.

Important Properties of Combination:

1. ${}^n C_0 = {}^n C_n = 1$.
2. ${}^n C_r = {}^n C_{n-r}$.
3. ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$.
4. $n \cdot {}^{n-1} C_{r-1} = (n-r+1) \cdot {}^n C_{r-1}$.