Matrix Operations (+,-,*)

Matrices Addition

The addition of two matrices A_{m*n} and B_{m*n} gives a matrix C_{m*n}. Here, m and n represents the number of rows and columns in the matrix respectively. The elements of C are sum of corresponding elements in A and B which can be shown as:

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 12 & 14 \end{bmatrix}$$

The algorithm for addition of matrices can be written as:

```
for i in 1 to m
   for j in 1 to n
      c_ij = a_ij + b_ij
```

Key points:

- Addition of matrices is commutative which means A+B = B+A
- Addition of matrices is associative which means A+(B+C) = (A+B)+C
- The order of matrices A, B and A+B is always same
- If order of A and B is different, A+B can't be computed
- The complexity of addition operation is O(m*n) where m*n is order of matrices

Matrices Subtraction

The subtraction of two matrices A_{m*n} and B_{m*n} gives a matrix C_{m*n}. Here, m and n represents the number of rows and columns in the matrix respectively. The elements of C are difference of corresponding elements in A and B which can be represented as:

$$\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

The algorithm for subtraction of matrices can be written as:

```
for i in 1 to m
   for j in 1 to n
      c_ij = a_ij-b_ij
```

Key points:

- Subtraction of matrices is non-commutative which means A-B ≠ B-A
- Subtraction of matrices is non-associative which means A-(B-C) ≠ (A-B)-C
- The order of matrices A, B and A-B is always same
- If order of A and B is different, A-B can't be computed
- The complexity of subtraction operation is O(m*n) where m*n is order of matrices

Matrices Multiplication

The multiplication of two matrices A_{m*n} and B_{n*p} gives a matrix C_{m*p}. It means number of columns in A must be equal to number of rows in B to calculate C=A*B. To calculate element c_{11}, multiply elements of 1st row of A with 1st column of B and add them (5*1+6*4) which can be shown as:

$$\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 29 & 40 \\ 44 & 61 \end{bmatrix}$$

The algorithm for multiplication of matrices A with order m*n and B with order n*p can be written as:

```
for i in 1 to m
   for j in 1 to p
      c_ij = 0
      for k in 1 to n
         c_ij += a_ik*b_kj
```

Key points:

- Multiplication of matrices is non-commutative which means A*B ≠ B*A
- Multiplication of matrices is associative which means A*(B*C) = (A*B)*C
- For computing A*B, the number of columns in A must be equal to number of rows in B
- Existence of A*B does not imply existence of B*A
- The complexity of multiplication operation (A*B) is O(m*n*p) where m*n and n*p are order of A and B respectively
- The order of matrix C computed as A*B is O(m*p) where m*n and n*p are order of A and B respectively