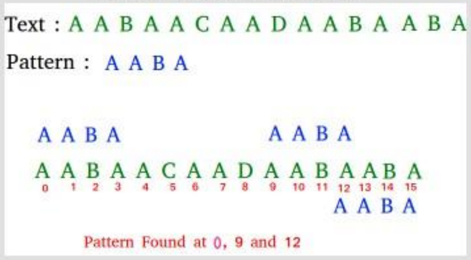
KMP Algorithm for pattern Searching

Given a text *txt[0..n-1]* and a pattern *pat[0..m-1]*, write a function *search(char pat[], char txt[])* that prints all occurrences of *pat[]* in *txt[]*. You may assume that *n > m*.

Examples:

```
Input:  txt[] = "THIS IS A TEST TEXT"
        pat[] = "TEST"
Output: Pattern found at index 10

Input:  txt[] =  "AABAACAADAABAABA"
        pat[] =  "AABA"
Output: Pattern found at index 0
        Pattern found at index 9
        Pattern found at index 12
```


Text : A A B A A C A A D A A B A A B A
Pattern : A A B A
A A B A
A A B A
A A B A A C A A D A A B A A B A
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
A A B A
Pattern Found at 0, 9 and 12

We have discussed the Naive pattern searching algorithm and the Rabin-Karp algorithm for searching patterns. The worst case complexity of both of the algorithms is O(n*m). Here, we will discuss a new algorithm for searching patterns, KMP algorithm. The time complexity of KMP algorithm is O(n) in the worst case.

KMP (Knuth Morris Pratt) Pattern Searching

The Naive pattern searching algorithm doesn't work well in cases where we see many matching characters followed by a mismatching character. Following are some examples.

```
   txt[] = "AAAAAAAAAAAAAAAAAB"
   pat[] = "AAAAB"

   txt[] = "ABABABCABABABCABABABC"
   pat[] =  "ABABAC" (not a worst case, but a bad case for Naive)
```

The KMP matching algorithm uses degenerating property (pattern having the same sub-patterns appearing more than once in the pattern) of the pattern and improves the worst case complexity to O(n). The basic idea behind KMP's algorithm is: whenever we detect a mismatch (after some matches), we already know some of the characters in the text of the next window. We take advantage of this information to avoid matching the characters that we know will anyway match. Let us consider the below example to understand this.

```
Matching Overview
txt = "AAAAABAAABA"
pat = "AAAA"

We compare first window of txt with pat
txt = "AAAAABAAABA"
pat = "AAAA"  [Initial position]
We find a match. This is same as Naive String Matching.

In the next step, we compare next window of txt with pat.
txt = "AAAAABAAABA"
pat =  "AAAA" [Pattern shifted one position]
This is where KMP does optimization over Naive. In this
second window, we only compare fourth A of pattern
with fourth character of current window of text to decide
whether current window matches or not. Since we know
first three characters will anyway match, we skipped
matching first three characters.

Need of Preprocessing?
An important question arises from the above explanation,
how to know how many characters to be skipped. To know this,
we pre-process pattern and prepare an integer array
lps[] that tells us the count of characters to be skipped.
```

Preprocessing Overview:

- KMP algorithm preprocesses pat[] and constructs an auxiliary **lps[]** of size m (same as size of pattern) which is used to skip characters while matching.
- **name lps indicates longest proper prefix which is also suffix.**. A proper prefix is prefix with whole string **not** allowed. For example, prefixes of "ABC" are "", "A", "AB" and "ABC". Proper prefixes are "", "A" and "AB". Suffixes of the string are "", "C", "BC" and "ABC".
- We search for lps in sub-patterns. More clearly we focus on sub-strings of patterns that are either prefix and suffix.
- For each sub-pattern pat[0..i] where i = 0 to m-1, lps[i] stores length of the maximum matching proper prefix which is also a suffix of the sub-pattern pat[0..i].

  ```
  lps[i] = the longest proper prefix of pat[0..i]

                which is also a suffix of pat[0..i].
  ```

  Note : lps[i] could also be defined as longest prefix which is also proper suffix. We need to use properly at one place to make sure that the whole substring is not considered.

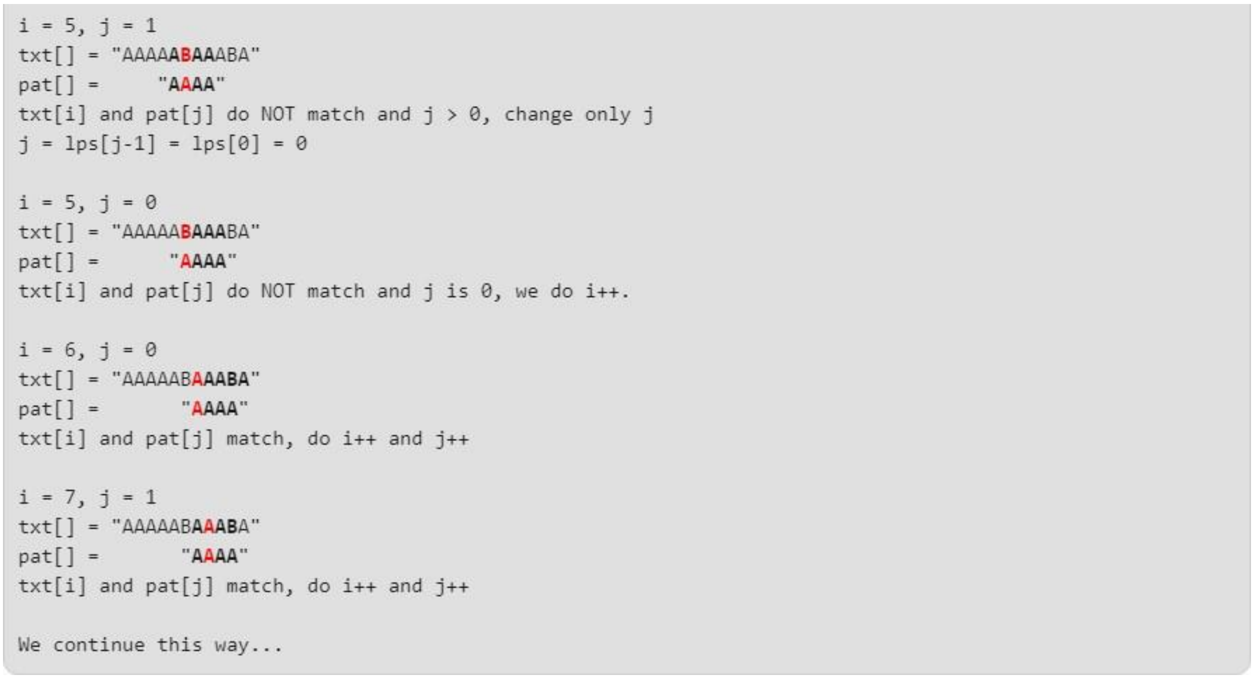
```
Examples of lps[] construction:
For the pattern “AAAA”,
lps[] is [0, 1, 2, 3]

For the pattern “ABCDE”,
lps[] is [0, 0, 0, 0, 0]

For the pattern “AABAACAABAA”,
lps[] is [0, 1, 0, 1, 2, 0, 1, 2, 3, 4, 5]

For the pattern “AAACAAAAAC”,
lps[] is [0, 1, 2, 0, 1, 2, 3, 3, 3, 4]

For the pattern “AAABAAA”,
lps[] is [0, 1, 2, 0, 1, 2, 3]
```

Searching Algorithm: Unlike Naive algorithm, where we slide the pattern by one and compare all characters at each shift, we use a value from lps[] to decide the next characters to be matched. The idea is to not match a character that we know will anyway match.

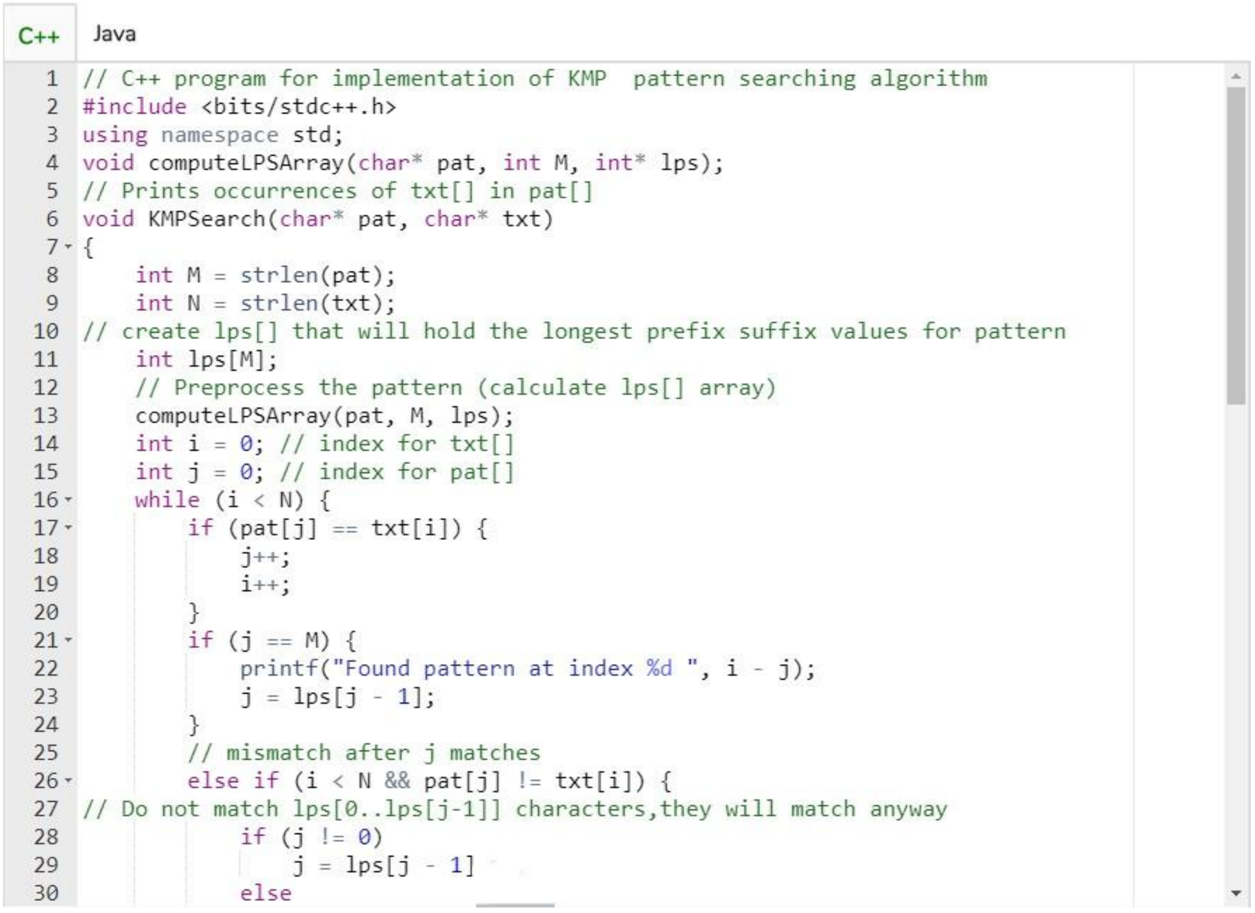
How to use lps[] to decide the next positions (or to know a number of characters to be skipped)?

- We start comparison of pat[j] with j = 0 with characters of current window of text.
- We keep matching characters txt[i] and pat[j] and keep incrementing i and j while pat[j] and txt[i] keep **matching**.
- When we see a **mismatch**
  - We know that characters pat[0..j-1] match with txt[i-j...i-1] (Note that j starts with 0 and increment it only when there is a match).
  - We also know (from above definition) that lps[j-1] is count of characters of pat[0...j-1] that are both proper prefix and suffix.
  - From above two points, we can conclude that we do not need to match these lps[j-1] characters with txt[i-j...i-1] because we know that these characters will anyway match. Let us consider above example to understand this.

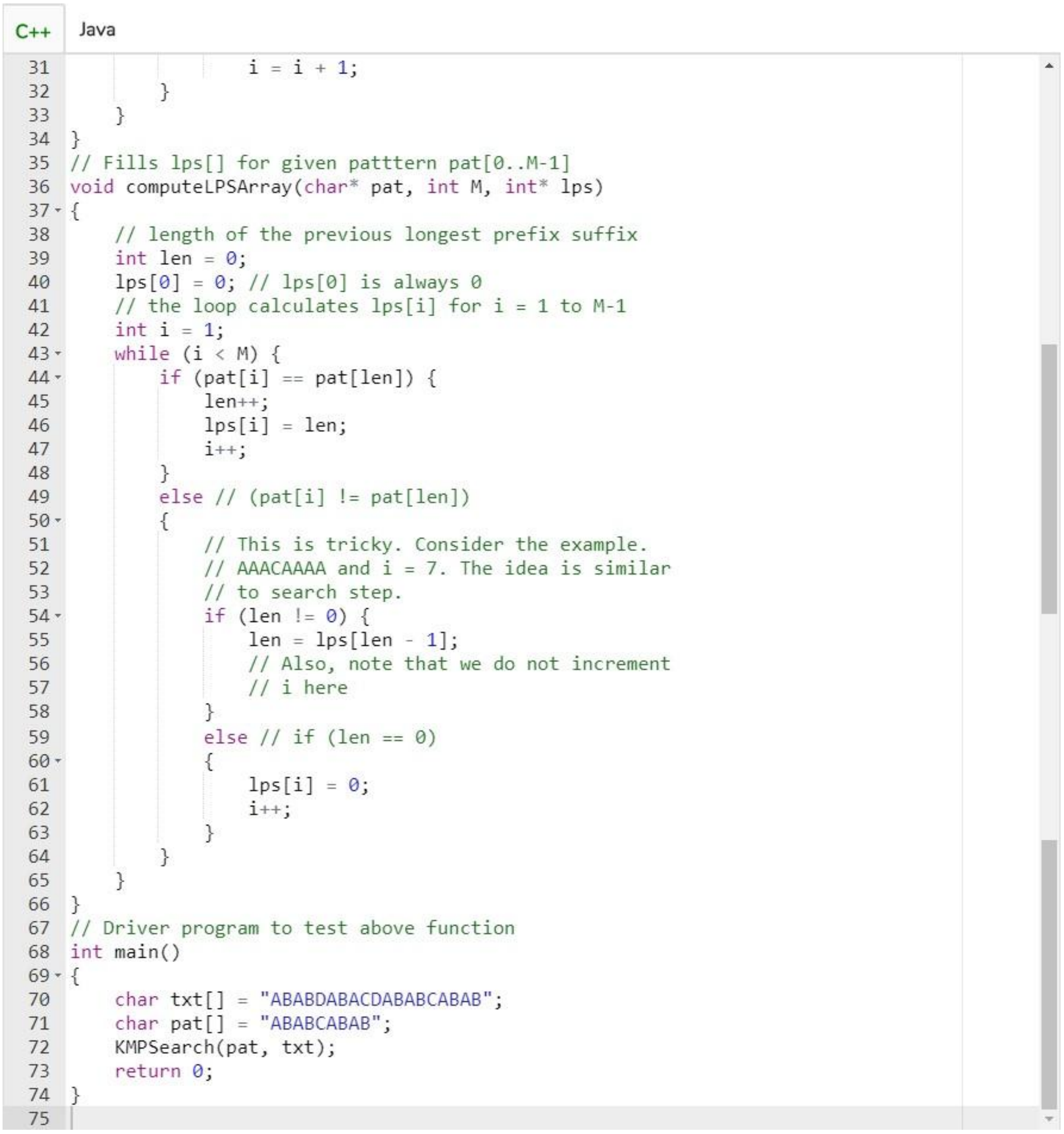
```
txt[] = "AAAAABAAABA"
pat[] = "AAAA"
lps[] = {0, 1, 2, 3}

i = 0, j = 0
txt[] = "AAAAABAAABA"
pat[] = "AAAA"
txt[i] and pat[j] match, do i++, j++

i = 1, j = 1
txt[] = "AAAAABAAABA"
pat[] = "AAAA"
txt[i] and pat[j] match, do i++, j++

i = 2, j = 2
txt[] = "AAAAABAAABA"
pat[] = "AAAA"
pat[i] and pat[j] match, do i++, j++

i = 3, j = 3
txt[] = "AAAAABAAABA"
pat[] = "AAAA"
txt[i] and pat[j] match, do i++, j++

i = 4, j = 4
Since j == M, print pattern found and reset j,
j = lps[j-1] = lps[3] = 3
Here unlike Naive algorithm, we do not match first three
characters of this window. Value of lps[j-1] (in above
step) gave us index of next character to match.
i = 4, j = 3
txt[] = "AAAAABAAABA"
pat[] =  "AAAA"
txt[i] and pat[j] match, do i++, j++

i = 5, j = 4
Since j == M, print pattern found and reset j,
j = lps[j-1] = lps[3] = 3

Again unlike Naive algorithm, we do not match first three
characters of this window. Value of lps[j-1] (in above
step) gave us index of next character to match.
i = 5, j = 3
txt[] = "AAAAABAAABA"
pat[] =   "AAAA"
txt[i] and pat[j] do NOT match and j > 0, change only j
j = lps[j-1] = lps[2] = 2

i = 5, j = 2
txt[] = "AAAAABAAABA"
pat[] =    "AAAA"
txt[i] and pat[j] do NOT match and j > 0, change only j
j = lps[j-1] = lps[1] = 1
```



```
i = 5, j = 1
txt[] = "AAAAABAAABA"
pat[] =      "AAAA"
txt[i] and pat[j] do NOT match and j > 0, change only j
j = lps[j-1] = lps[0] = 0

i = 5, j = 0
txt[] = "AAAAABAAABA"
pat[] =       "AAAA"
txt[i] and pat[j] do NOT match and j is 0, we do i++.

i = 6, j = 0
txt[] = "AAAAABAAABA"
pat[] =        "AAAA"
txt[i] and pat[j] match, do i++ and j++

i = 7, j = 1
txt[] = "AAAAABAAABA"
pat[] =        "AAAA"
txt[i] and pat[j] match, do i++ and j++

We continue this way...
```



```cpp
// C++ program for implementation of KMP  pattern searching algorithm
#include <bits/stdc++.h>
using namespace std;
void computeLPSArray(char* pat, int M, int* lps);
// Prints occurrences of txt[] in pat[]
void KMPSearch(char* pat, char* txt)
{
    int M = strlen(pat);
    int N = strlen(txt);
// create lps[] that will hold the longest prefix suffix values for pattern
    int lps[M];
    // Preprocess the pattern (calculate lps[] array)
    computeLPSArray(pat, M, lps);
    int i = 0; // index for txt[]
    int j = 0; // index for pat[]
    while (i < N) {
        if (pat[j] == txt[i]) {
            j++;
            i++;
        }
        if (j == M) {
            printf("Found pattern at index %d ", i - j);
            j = lps[j - 1];
        }
        // mismatch after j matches
        else if (i < N && pat[j] != txt[i]) {
// Do not match lps[0..lps[j-1]] characters,they will match anyway
            if (j != 0)
                j = lps[j - 1]
            else
```


C++ Java

```
                i = i + 1;
            }
        }
}
// Fills lps[] for given patttern pat[0..M-1]
void computeLPSArray(char* pat, int M, int* lps)
{
    // length of the previous longest prefix suffix
    int len = 0;
    lps[0] = 0; // lps[0] is always 0
    // the loop calculates lps[i] for i = 1 to M-1
    int i = 1;
    while (i < M) {
        if (pat[i] == pat[len]) {
            len++;
            lps[i] = len;
            i++;
        }
        else // (pat[i] != pat[len])
        {
            // This is tricky. Consider the example.
            // AAACAAAA and i = 7. The idea is similar
            // to search step.
            if (len != 0) {
                len = lps[len - 1];
                // Also, note that we do not increment
                // i here
            }
            else // if (len == 0)
            {
                lps[i] = 0;
                i++;
            }
        }
    }
}
// Driver program to test above function
int main()
{
    char txt[] = "ABABDABACDABABCABAB";
    char pat[] = "ABABCABAB";
    KMPSearch(pat, txt);
    return 0;
}

```

Output:

```
Found pattern at index 10
```

Preprocessing Algorithm: In the preprocessing part, we calculate values in lps[]. To do that, we keep track of the length of the longest prefix suffix value (we use a len variable for this purpose) for the previous index. We initialize lps[0] and len as 0. If pat[len] and pat[i] match, we increment len by 1 and assign the incremented value to lps[i]. If pat[i] and pat[len] do not match and len is not 0, we update len to lps[len-1]. See computeLPSArray () in the below code for details.

Illustration of preprocessing (or construction of lps[])

```
pat[] = "AAACAAAA"

len = 0, i  = 0.
lps[0] is always 0, we move
to i = 1

len = 0, i  = 1.
Since pat[len] and pat[i] match, do len++,
store it in lps[i] and do i++.
len = 1, lps[1] = 1, i = 2

len = 1, i  = 2.
Since pat[len] and pat[i] match, do len++,
store it in lps[i] and do i++.
len = 2, lps[2] = 2, i = 3
len = 2, i  = 3.
Since pat[len] and pat[i] do not match, and len > 0,
set len = lps[len-1] = lps[1] = 1

len = 1, i  = 3.
Since pat[len] and pat[i] do not match and len > 0,
len = lps[len-1] = lps[0] = 0

len = 0, i  = 3.
Since pat[len] and pat[i] do not match and len = 0,
Set lps[3] = 0 and i = 4.
We know that characters pat
len = 0, i  = 4.
Since pat[len] and pat[i] match, do len++,
store it in lps[i] and do i++.
len = 1, lps[4] = 1, i = 5

len = 1, i  = 5.
Since pat[len] and pat[i] match, do len++,
store it in lps[i] and do i++.
len = 2, lps[5] = 2, i = 6

len = 2, i  = 6.
Since pat[len] and pat[i] match, do len++,
store it in lps[i] and do i++.
len = 3, lps[6] = 3, i = 7
len = 3, i  = 7.
Since pat[len] and pat[i] do not match and len > 0,
set len = lps[len-1] = lps[2] = 2

len = 2, i  = 7.
Since pat[len] and pat[i] match, do len++,
store it in lps[i] and do i++.
len = 3, lps[7] = 3, i = 8

We will stop here as we have constructed the whole lps[].
```