Introduction to matrix

A **matrix** represents a collection of numbers arranged in order of rows and columns. It is necessary to enclose the elements of a matrix in parentheses or brackets.

A matrix with 9 elements is shown below:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

The above Matrix M has 3 rows and 3 columns. Each element of matrix [M] can be referred to by its row and column number. For example, $a_{23} = 6$

Order of a Matrix : The order of a matrix is defined in terms of its number of rows and columns.

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Order of a matrix = No. of rows × No. of columns

Therefore, Matrix [M] is a matrix of order 3 × 3.
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Transpose of a Matrix

The transpose $[M]^T$ of an **m x n** matrix [M] is the n x m matrix obtained by interchanging the rows and columns of [M].

Transpose of a matrix A is defined as:

if $A= [a_{ij}]$ mxn:
    then $A^T = [b_{ij}]$ nxm where $b_{ij} = a_{ji}$

For Example, transpose of matrix M, M^T will be:

$$M^T = \begin{matrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{matrix}$$

Properties of transpose of a matrix:

- $(A^T)^T = A$
- $(A+B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$

Properties of Matrix addition and multiplication:

1. A+B = B+A (Commutative)
2. (A+B)+C = A+ (B+C) (Associative)
3. AB ≠ BA (Not Commutative)
4. (AB) C = A (BC) (Associative)
5. A (B+C) = AB+AC (Distributive)

Terminologies

- **Square Matrix:** A square Matrix has as many rows as it has columns. i.e. no of rows = no of columns.
- **Symmetric matrix:** A square matrix is said to be symmetric if the transpose of original matrix is equal to its original matrix. i.e. $(A^T) = A$.
- **Skew-symmetric:** A skew-symmetric (or antisymmetric or antimetric[1]) matrix is a square matrix whose transpose equals its negative.i.e. $(A^T) = -A$.
- **Diagonal Matrix:**A diagonal matrix is a matrix in which the entries outside the main diagonal are all zero. The term usually refers to square matrices.
- **Identity Matrix:**A square matrix in which all the elements of the principal diagonal are ones and all other elements are zeros.Identity matrix is denoted as I.
- **Orthogonal Matrix:** A matrix is said to be orthogonal if $AA^T = A^TA = I$.
- **Idemponent Matrix:** A matrix is said to be idemponent if $A^2 = A$.
- **Involutary Matrix:** A matrix is said to be Involutary if $A^2 = I$.
- **Singular Matrix**: A square matrix is said to be singular matrix if its determinant is zero i.e. |A|=0
- **Nonsingular Matrix**: A square matrix is said to be non-singular matrix if its determinant is non-zero.

Note: Every Square Matrix can uniquely be expressed as the sum of a symmetrix matrix and skew symmetric matrix. A = 1/2 (AT + A) + 1/2 (A - AT).

Trace of a matrix: trace of a matrix is denoted as tr(A) which is used only for square matrix and equals the sum of the diagonal elements of the matrix. For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad tr(A) = 1+5+9 = 15$$