Factorial

Factorial: In mathematics, the factorial of a number say **N** is denoted by **N!**. The factorial of a number is calculated by finding multiplication of all integers between 1 and N(both inclusive.)

For Example, **4! = 4 * 3 * 2 * 1 = 24**.

That is,

```
N! = N*(N-1)*(N-2)*....*2*1
```

Note: As, per convention, **0! = 1**.

Sample Problem: Given a number N, the task is to count number of **trailing zeroes** in factorial of N. That is, number of zeroes at the end in the number **N!**.

For Example:

```
Input: N = 5
Output: 1
Factorial of 5 is 120 which has one trailing 0.

Input: N = 20
Output: 4
Factorial of 20 is 2432902008176640000 which has
4 trailing zeroes.
```

An efficient way to solve this problem is to observe the properties of prime factorization. Consider prime factors of **N!**. A trailing zero is always produced by the prime factors **2 and 5**. If we can count the number of 5s and 2s in prime factorization of N!, our task is done.

Consider the following examples:

- **N = 5**: There is one 5 and 3 2s in prime factors of 5! (2 * 2 * 2 * 3 * 5). So count of trailing 0s is 1.
- **N = 11**: There are two 5s and three 2s in prime factors of 11! (2 8 * 34 * 52 * 7). So count of trailing 0s is 2.

We can easily observe that the number of 2s in prime factors is always more than or equal to the number of 5s. So if we count 5s in prime factors, we are done.

Now, how to count total number of 5s in prime factors of N!? A simple way is to calculate floor(N/5). For example, 7! has one 5, 10! has two 5s. It is not done yet, there is one more thing to consider. Numbers like 25, 125, etc have more than one 5. For example if we consider 28!, we get one extra 5 and number of 0s become 6. Handling this is simple, first divide N by 5 and remove all single 5s, then divide by 25 to remove extra 5s and so on. Following is the summarized formula for counting trailing 0s.

```
Trailing 0s in N! = Count of 5s in prime factors of n!
                  = floor(n/5) + floor(n/25) + floor(n/125) + ....
```