ID: Name:

1. Show that postage of 24 cents or more can be achieved by using only 5-cent and 7-cent stamps.

By strong mathematical induction

Basis step:

n = 24, 24 = 5+5+7+7

n = 25, 25 = 5+5+5+5+5

n = 26, 26 = 5+7+7+7

n = 27, 27 = 5+5+5+5+7

n = 28, 28 = 7+7+7+7

inductive step:

假設 n<k 時命題成立，考慮 n = k 因 k-5 < k =>k-5 可用 5-cent and 7-cent 組成，此時多一張 5-cent 即可組成 k cent 所以 n=k 時成立

2. Use mathematical induction to show the following: If $t(1) = c_1$ and $t(n) = t(n-1) + c_2 n, n > 1, then$ $t(n) = \frac{n(n+1)}{2}c_2 + c_1 - c_2$.

Basis step:

n = 1 時 $t(1) = c_1 = \frac{1\times 2}{2}c_2 + c_1 - c_2$ 成立,

Inductive step:

假設 n = k 時命題成立

⇨ $t(k) = \frac{k(k+1)}{2}c_2 + c_1 - c_2$ 則 n = k + 1 時

$$t(k+1) = t(k) + c_2(k+1) = \frac{k(k+1)}{2}c_2 + c_1 - c_2 + c_2(k+1)$$

$$= (k+1)c_2\left(1 + \frac{k}{2}\right) + c_1 - c_2 = \frac{(k+1)(k+2)}{2}c_2 + c_1 - c_2$$

3. Prove by induction that $n^2 < n!$ $for\ integer$ $n \geq 4$.

Basis step:

n = 4 時 16 < 24 成立

Inductive step:

假設$n = k \geq 4$時命題成立=> $k^2 < k!$ 則 n = k+1 時

$$(k+1)^2 = k^2 + 2k + 1 < k! + 2k + 1 < k! + 2k + k = k! + 3k < k! + k \cdot k$$
$$< k! + k(k!) = (1+k)k! = (k+1)!$$

4. Prove by induction that $n^3 + 2n$ *is dividible by* 6 *for all integer* $n > 1$ *and* n *is an even number*.

假設$n = 2t, t \in Z^+$=> $n^3 + 2n = (2t)^3 + 2(2t) = 8t^3 + 4t$
對 t 做 mathematical induction
Basis step:
t = 1 時$8t^3 + 4t = 12$被 6 整除
inductive step:
假設 t = k $8k^3 + 4k$被 6 整除成立
則 t=k+1 時$8(k+1)^3 + 4(k+1) = 8(k^3 + 3k^2 + 3k + 1) + 4k + 4 =$
$(8k^3 + 4k) + 6(4k^2 + 4k + 2)$=>可被 6 整除