1. Show that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

2. Show that if a, b, and c are integers, where $a \neq 0$ and $c \neq 0$, such that $ac \mid bc$, then $a \mid b$.

3. Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \leq c \leq 12$ such that

a) $c \equiv 9a \pmod{13}$.
b) $c \equiv 11b \pmod{13}$.
c) $c \equiv a + b \pmod{13}$.
d) $c \equiv 2a + 3b \pmod{13}$.
e) $c \equiv a^2 + b^2 \pmod{13}$.
f) $c \equiv a^3 - b^3 \pmod{13}$.

4. Convert the binary expansion of each of these integers to a decimal expansion.

a) $(1\ 1111)_2$
b) $(10\ 0000\ 0001)_2$
c) $(1\ 0101\ 0101)_2$
d) $(110\ 1001\ 0001\ 0000)_2$

5. Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$
b) $(135AB)_{16}$
c) $(ABBA)_{16}$
d) $(DEFACED)_{16}$

6. Use the Euclidean algorithm to find

a) gcd(12, 18).

b) gcd(111, 201).

c) gcd(1001, 1331).

d) gcd(12345, 54321).

e) gcd(1000, 5040).

f) gcd(9888, 6060).

7. Use the extended Euclidean algorithm to express gcd(26, 91) as a linear combination of 26 and 91.

7. Find all solutions, if any, to the system of congruences $x \equiv 7 \pmod{9}$, $x \equiv 4 \pmod{12}$, and $x \equiv 16 \pmod{21}$.

8. Show that 15 is an inverse of 7 modulo 26.

9. Which memory locations are assigned by the hashing function $h(k) = k \textbf{ mod } 97$ to the records of insurance company customers with these Social Security numbers?

a) 034567981 **b)** 183211232
c) 220195744 **d)** 987255335

10. What is the original message encrypted using the RSA system with $n = 43 \cdot 59$ and $e = 13$ if the encrypted message is 0667 1947 0671? (To decrypt, first find the decryption exponent d which is the inverse of $e = 13$ modulo $42 \cdot 58$.)