1. List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if

a) $a = b$.
b) $a + b = 4$.
c) $a > b$.
d) $a \mid b$.
e) $\gcd(a, b) = 1$.
f) $\text{lcm}(a, b) = 2$.

f) $\{(1,2), (2,1), (2,2)\}$.

(a) $\{(0,0), (1,1), (2,2), (3,3)\}$

(b) $\{(1,3), (2,2), (3,1), (4,0)\}$

(c) $\{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$

(d) $\{(1,0), (1,1), (1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0)\}$.

(e) $\{(0,1), (1,0), (1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (4,1), (4,3)\}$.

2. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
c) $\{(2, 4), (4, 2)\}$
d) $\{(1, 2), (2, 3), (3, 4)\}$
e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

Each property needs to be answered. (Yes/No)

	reflexive	symmetric	anti symmetric	transitive
a	N	N	N	Y
b	Y	Y	N	Y
c	N	Y	N	N
d	N	N	Y	N
e	Y	Y	Y	Y
f	N	N	N	N

3. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

a) $x \neq y$.
b) $xy \geq 1$.
c) $x = y + 1$ or $x = y - 1$.
d) $x \equiv y \pmod{7}$.
e) x is a multiple of y.
f) x and y are both negative or both nonnegative.
g) $x = y^2$.
h) $x \geq y^2$.

	reflexive	symmetric	anti symmetric	transitive
a	N	Y	N	N
b	N	Y	N	Y
c	N	Y	N	Y
d	Y	Y	N	Y
e	Y	N	N	Y
f	Y	Y	N	Y
g	N	N	Y	N
h	N	N	Y	Y

4. Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$, and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$. Find $S \circ R$.

$$\{(1,1), (1,2), (2,1), (2,2)\}.$$

5. Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order).

a) $\{(1, 1), (1, 2), (1, 3)\}$
b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$
c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
d) $\{(1, 3), (3, 1)\}$

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

6. List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

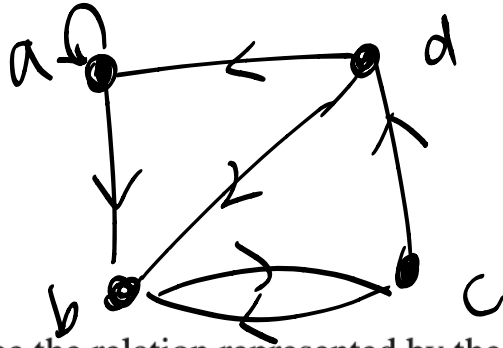b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) $(1,1), (1,3), (2,2), (3,1), (3,3)$

(b) $(1,2), (2,2), (3,2)$

(c) $(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)$

7. Draw the directed graph that represents the relation {(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)}.

8. Let R be the relation represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find the matrices that represent

a) R^2.   b) R^3.   c) R^4.

(a)
$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

9. Let R be the relation on the set $\{0, 1, 2, 3\}$ containing the ordered pairs $(0, 1)$, $(1, 1)$, $(1, 2)$, $(2, 0)$, $(2, 2)$, and $(3, 0)$. Find the

a) reflexive closure of R.   **b)** symmetric closure of R.

(a) $R = \{(0,0), (0,1), (1,1), (1,2), (2,0), (2,2), (3,0), (3,3)\}$

(b) $R = \{(0,1), (a2), (0,3), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2),$
$(3,0)\}$

10. Draw the directed graph of the reflexive closure of the relations with the directed graph shown.