- 1 Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

  a) $\neg\forall x\forall y P(x, y)$ **b)** $\neg\forall y\exists x P(x, y)$

  c) $\neg\forall y\forall x(P(x, y) \vee Q(x, y))$

  d) $\neg(\exists x\exists y\neg P(x, y) \wedge \forall x\forall y Q(x, y))$

  e) $\neg\forall x(\exists y\forall z P(x, y, z) \wedge \exists z\forall y P(x, y, z))$

- 2 Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

  a) $\exists x\forall y(xy = y)$

  b) $\forall x\forall y(((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$

  c) $\exists x\exists y((x^2 > y) \wedge (x < y))$

  d) $\forall x\forall y\exists z(x + y = z)$

- 3 For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

  a) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."

  b) "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."

  c) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."

  d) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

  e) "What is good for corporations is good for the United States." "What is good for the United States is good for you." "What is good for corporations is for you to buy lots of stuff."

  f) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents."

- 4 Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x(P(x) \wedge Q(x))$ is true.

1. $\exists x P(x) \vee \exists x Q(x)$	Premise
2. $\exists x P(x)$	Simplification from (1)
3. $P(c)$	Existential instantiation from (2)
4. $\exists x Q(x)$	Simplification from (1)
5. $Q(c)$	Existential instantiation from (4)
6. $P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7. $\exists x(P(x) \wedge Q(x))$	Existential generalization

- 5 Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable.

- 6 Use a direct proof to show that the sum of two odd integers is even.

- 7 Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

  a) a proof by contraposition.
  b) a proof by contradiction.

- 8 Show that the propositions p_1, p_2, p_3, p_4, and p_5 can be shown to be equivalent by proving that the conditional statements $p_1 \to p_4$, $p_3 \to p_1$, $p_4 \to p_2$, $p_2 \to p_5$, and $p_5 \to p_3$ are true.

- 9 Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$.

- 10 Let $S = x_1y_1 + x_2y_2 + \cdots + x_ny_n$, where $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are orderings of two different sequences of positive real numbers, each containing n elements.

  a) Show that S takes its maximum value over all orderings of the two sequences when both sequences are sorted (so that the elements in each sequence are in nondecreasing order).

  b) Show that S takes its minimum value over all orderings of the two sequences when one sequence is sorted into nondecreasing order and the other is sorted into nonincreasing order.