1. List the members of these sets.

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$

b) $\{x \mid x \text{ is a positive integer less than } 12\}$

c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$

d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

a) $\{-1, 1\}$ **b)** [1,2,3,4,5,6,7,8,9,10,11] **c)** {0, 1, 4, 9, 16, 25, 36, 49, 64 , 81} **d)** $\emptyset$

2. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Show that $A \subseteq C$.

Suppose that $x \in A. Because\ A \subseteq B, this\ implies\ that\ x\epsilon B. Because\ B \subseteq c, we\ see\ that\ x \in C. Because\ x \in A\ implies\ that\ x \in C, it\ follows\ that\ A \subseteq C.$

3. Determine whether each of these statements is true or false.

a) $0 \in \emptyset$

b) $\emptyset \in \{0\}$

c) $\{0\} \subset \emptyset$

d) $\emptyset \subset \{0\}$

e) $\{0\} \in \{0\}$

f) $\{0\} \subset \{0\}$

g) $\{\emptyset\} \subseteq \{\emptyset\}$

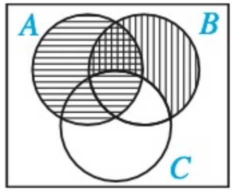
a) F **b) F c)** F **d)** T **e)** F **f)** F **g)**T

- 4 Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

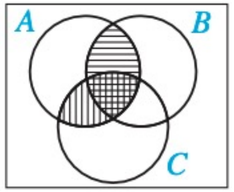
  a) $A \cap (B - C)$ **b)** $(A \cap B) \cup (A \cap C)$

  c) $(A \cap \overline{B}) \cup (A \cap \overline{C})$

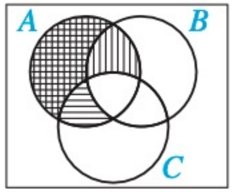
a) The double-shaded portion



b) The desired set is the entire shaded portion



c) The desired set is the entire shaded portion.



- 5 Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find

  a) $\bigcup_{i=1}^{n} A_i$. **b)** $\bigcap_{i=1}^{n} A_i$.

a) {1,2,3,...,*n*} **b)** {1}

- 6 Let A, B, and C be sets. Use the identities in Table 1 to show that $\overline{(A \cup B)} \cap \overline{(B \cup C)} \cap \overline{(A \cup C)} = \overline{A} \cap \overline{B} \cap \overline{C}$.

By De Morgan's law, the left-hand side equals(A∩B)∩(B∩C)∩(A∩C).By the commutative, associative, and idempotent laws, this simplifies to the right-hand side

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

- 7

Find these values.

a) $\lceil \frac{3}{4} \rceil$ **b)** $\lfloor \frac{7}{8} \rfloor$

c) $\lceil -\frac{3}{4} \rceil$ **d)** $\lfloor -\frac{7}{8} \rfloor$

e) $\lceil 3 \rceil$ **f)** $\lfloor -1 \rfloor$

g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$ **h)** $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

a)1 **b)**0 **c)**0 **d)**−1 **e)**3 **f)**−1 **g)**2 **h)** 1

- 8

Determine whether the function $f: \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if

a) $f(m, n) = m + n$.

b) $f(m, n) = m^2 + n^2$.

c) $f(m, n) = m$.

d) $f(m, n) = |n|$.

e) $f(m, n) = m - n$.

a) Onto **b)** Not onto **c)** Onto **d)** Not onto **e)** Onto

- 9 Find these terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.

  a) a_0 **b)** a_1 **c)** a_4 **d)** a_5

  a)3 **b)**−1 **c)**787 **d)**2639

- 10 Compute each of these double sums.

  a) $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$ **b)** $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i+3j)$

  c) $\sum_{i=1}^{3} \sum_{j=0}^{2} i$ **d)** $\sum_{i=0}^{2} \sum_{j=1}^{3} ij$

  a)21 **b)**78 **c)**18 **d)**18