1. Show that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

If $a \mid b$ and $b \mid a$, there are integers c and d such that $b = ac$ and $a = bd$. Hence, $a = acd$. Because $a \neq 0$ it follows that $cd = 1$. Thus, either $c = d = 1$ or $c = d = -1$. Hence,either $a = b$ or $a = -b$

2. Show that if a, b, and c are integers, where $a \neq 0$ and $c \neq 0$, such that $ac \mid bc$, then $a \mid b$.

Because $ac \mid bc$ there is an integer k such that $ack = bc$. Hence, $ak = b$, so $a \mid b$.

3. Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \leq c \leq 12$ such that

a) $c \equiv 9a \pmod{13}$.

b) $c \equiv 11b \pmod{13}$.

c) $c \equiv a + b \pmod{13}$.

d) $c \equiv 2a + 3b \pmod{13}$.

e) $c \equiv a^2 + b^2 \pmod{13}$.

f) $c \equiv a^3 - b^3 \pmod{13}$.

a)10 **b)**8 **c)**0 **d)**9 **e)**6 **f)**11

4. Convert the binary expansion of each of these integers to a decimal expansion.

a) $(1\ 1111)_2$ **b)** $(10\ 0000\ 0001)_2$

c) $(1\ 0101\ 0101)_2$ **d)** $(110\ 1001\ 0001\ 0000)_2$

a) 31 b) 513 c) 341 d) 26896

5. Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$ **b)** $(135AB)_{16}$

c) $(ABBA)_{16}$ **d)** $(DEFACED)_{16}$

a) 1000 0000 1110 b) 1 0011 0101 1010 1011 c) 1010 1011 1011 1010 d) 1101 1110 1111 1010 1100 1110 1101

6. Use the Euclidean algorithm to find

a) gcd(12, 18).
b) gcd(111, 201).
c) gcd(1001, 1331).
d) gcd(12345, 54321).
e) gcd(1000, 5040).
f) gcd(9888, 6060).

a) 6 **b)** 3 **c)** 11 **d)** 3 **e)** 40 **f)** 12

7. Use the extended Euclidean algorithm to express gcd(26, 91) as a linear combination of 26 and 91.

$(-3)\cdot 26+1\cdot 91=13$

8. Find all solutions, if any, to the system of congruences $x \equiv 7 \pmod 9$, $x \equiv 4 \pmod{12}$, and $x \equiv 16 \pmod{21}$.

All integers of the form $16 + 252k$, where k is an integer

9. Show that 15 is an inverse of 7 modulo 26.

$15 \cdot 7 = 105 \equiv 1 \pmod{26}$

10. Which memory locations are assigned by the hashing function $h(k) = k$ **mod** 97 to the records of insurance company customers with these Social Security numbers?

a) 034567981 **b)** 183211232
c) 220195744 **d)** 987255335

a) 91 b) 57 c) 21 d) 5

11. What is the original message encrypted using the RSA system with $n = 43 \cdot 59$ and $e = 13$ if the encrypted message is 0667 1947 0671? (To decrypt, first find the decryption exponent d which is the inverse of $e = 13$ modulo $42 \cdot 58$.)

1808 1121 0417