1. A multiple-choice test contains 10 questions. There are four possible answers for each question.

a) In how many ways can a student answer the questions on the test if the student answers every question?

b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

a) 4^{10} b) 5^{10}

2. How many 4-element RNA sequences

a) contain the base U?

b) do not contain the sequence CUG?

c) do not contain all four bases A, U, C, and G?

d) contain exactly two of the four bases A, U, C, and G?

a) 175 b) 248 c)232 d) 84

TABLE 1 Combinations and Permutations With and Without Repetition.

Type	***Repetition Allowed?***	***Formula***
r-permutations	No	$\frac{n!}{(n-r)!}$
r-combinations	No	$\frac{n!}{r!(n-r)!}$
r-permutations	Yes	n^r
r-combinations	Yes	$\frac{(n+r-1)!}{r!(n-1)!}$

3. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

Because there are six classes, but only five weekdays, the pigeonhole principle shows that at least two classes must be held on the same day.

4. Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

Let $a, a + 1, \dots, a + n - 1$ be the integers in the sequence. The integers $(a + i)$ **mod** $n, i = 0,1,2,\dots, n - 1$, are distinct, because $0 < (a+j)-(a+k) < n$ whenever $0 \leq k < j \leq n-1$. Because there are n possible values for $(a + i)$ **mod** n and there are n different integers in the set, each of these values is taken on exactly once. It follows that there is exactly one integer in the sequence that is divisible by n.

5. Find the value of each of these quantities.

a) $P(6, 3)$
b) $P(6, 5)$
c) $P(8, 1)$
d) $P(8, 5)$
e) $P(8, 8)$
f) $P(10, 9)$

a) 120 b) 720 c) 8 d) 6720 e) 40320 f) 3628800

6. How many bit strings of length 10 contain

a) exactly four 1s?
b) at most four 1s?
c) at least four 1s?
d) an equal number of 0s and 1s?

a) 210 b) 386 c) 848 d) 252

7. Find the expansion of $(x+y)^4$

a) using combinatorial reasoning, as in Example 1.

b) using the binomial theorem.

THE BINOMIAL THEOREM Let x and y be variables, and let n be a nonnegative integer. Then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j}y^j = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n.$$

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

8. Use the binomial theorem to expand $(3x^4 - 2y^3)^5$ into a sum of terms of the form cx^ay^b, where c is a real number and a and b are nonnegative integers.

$$\sum_{j=0}^{5} \binom{5}{j}(3x^4)^{5-j}(-2y^3)^j = 243x^{20} - 810x^{16}y^3 + 1080x^{12}y^6 - 720x^8y^9 + 240x^4y^{12} - 32y^{15}$$

9. In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed?

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10. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose

a) six bagels?
b) a dozen bagels?
c) two dozen bagels?
d) a dozen bagels with at least one of each kind?
e) a dozen bagels with at least three egg bagels and no more than two salty bagels?

a) 1716 b) 50,388 c) 2629575 d) 330 e)9724

No salty bagels:
$n = 7$ (7 kinds of bagels excluding the salty bagel), $r = 9$ (12 − 3 egg bagels). Therefore, there are $C(n+r-1, r) = C(7+9-1, 9) = C(15, 9) = 15!/(9! \times 6!) =$ **5005** .

One salty bagel:
$n = 7$ (7 kinds of bagels excluding the salty bagel), $r = 8$ (12−3 egg bagels −1 salty bagel). Therefore, there are $C(n+r-1, r) = C(7+8-1, 8) = C(14, 8) = 14!/(8! \times 6!) = 3,003$ ways to choose.

Two salty bagels:
$n = 7$ (7 kinds of bagels excluding the salty bagel), $r = 7$ (12−3 egg bagels −2 salty bagels). Therefore, there are $C(n+r-1, r) = C(7+7-1, 7) = C(13, 7) = 13!/(7! \times 6!) = 1,716$ ways to choose.

So, the answer to this problem is:
$C(15, 9) + C(14, 8) + C(13, 7) = 455 + 3,003 + 1,716 =$ **9724**