ID: Name:

1. Give a recursive definition of the set of positive integers that are multiples of 5.

$$5 \in S, and\ x + y \in S\ if\ x, y \in S$$

2. Let S be the set of positive integers defined by

Basis step: $5 \in S$.

Recursive step: If $n \in S$, then $3n \in S$ and $n^2 \in S$.

a) Show that if $n \in S$, then $n \equiv 5 (\mathrm{mod} 10)$. (20 points)

b) Show that there exists an integer $m \equiv 5 \pmod{10}$ that does not belong to S. (5 points)

a) *Basisstep:* $5 \equiv 5(\mathrm{mod}10)$. *Inductive step:* If $n \equiv 5(\mathrm{mod}10)$, then $3n \equiv 3 \cdot 5 = 15 \equiv 5(\mathrm{mod}10)$ and $n^2 \equiv 5^2 = 25 \equiv 5(mod\ 10)$.

b) $35 \notin S$ because 35 is not a multiple of 3 nor a perfect

3. Give a recursive algorithm for computing nx whenever n is a positive integer and x is an integer, using just addition.

procedure *mult*(*n*: positive integer, *x*: integer)

if $n = 1$ **then return** x

else return $x + mult\ (n - 1, x)$

4. Give a recursive definition of

a) the set of even integers. (10 points).

b) the set of positive integers congruent to 2 modulo 3. (10 points)

c) the set of positive integers not divisible by 5. (5 points)

a) $0 \in S$, and if $x \in S$, then $x+2 \in S$ and $x-2 \in S$.

b) $2 \in S$, and if $x \in S$, then $x+3 \in S$.

c) $1 \in S$, $2 \in S$, $3 \in S$, $4 \in S$, and if $x \in S$, then $x+5 \in S$.