1. Find the smallest relation containing the relation $\{(1, 2), (1, 4), (3, 3), (4, 1)\}$ that is

a) reflexive and transitive.
b) symmetric and transitive.
c) reflexive, symmetric, and transitive.

(a) $\{(1,1), (1,2), (1,4), (2,2), (3,3), (4,1), (4,2), (4,4)\}$

(b) $\{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (4,1), (4,2), (4,4)\}$

(c) $\{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (4,1), (4,2), (4,4)\}$

2. Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

(a) Equivalence relation

(b) Not reflexive, not transitive

(c) Equivalence relation

(d) Not transitive

(f) Not symmetric, not transitive

3. Which of these relations on the set of all functions from $\mathbf{Z}$ to $\mathbf{Z}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) $\{(f, g) \mid f(1) = g(1)\}$
b) $\{(f, g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$
c) $\{(f, g) \mid f(x) - g(x) = 1 \text{ for all } x \in \mathbf{Z}\}$
d) $\{(f, g) \mid \text{ for some } C \in \mathbf{Z}, \text{ for all } x \in \mathbf{Z}, f(x) - g(x) = C\}$
e) $\{(f, g) \mid f(0) = g(1) \text{ and } f(1) = g(0)\}$

a) Equivalence relation

b) Not transitive

c) Not reflexive, not symmetric, not transitive

d) Equivalence relation

f) Not reflexive, not transitive.

4. Which of these collections of subsets are partitions of $\{1, 2, 3, 4, 5, 6\}$?

a) $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$
b) $\{1\}, \{2, 3, 6\}, \{4\}, \{5\}$
c) $\{2, 4, 6\}, \{1, 3, 5\}$
d) $\{1, 4, 5\}, \{2, 6\}$

a) No

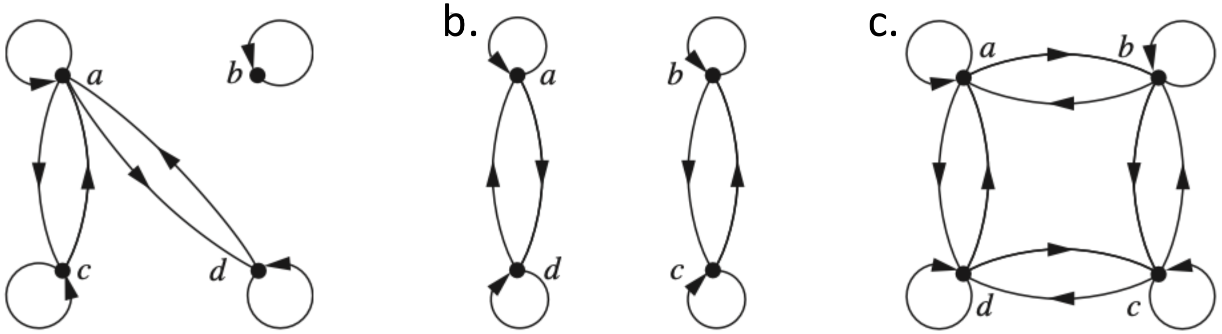
b) Yes

c) Yes

d) No

5. Determine whether the relation with the directed graph shown is an equivalence relation.

a. b. c.



a) No

b) Yes

c) No.

6. Show that the relation R on the set of all bit strings such that $s R t$ if and only if s and t contain the same number of 1s is an equivalence relation.

1. Reflexive : because a bit str s has the same number of 1s as itself.

2. Symmetric : s and t having the same number of 1s implies that t and s do.

3. Transitive : s and t having the same number of 1s, and t and u having the same number of 1s implies that s and u have the same number of 1s.

7. Which of these relations on $\{0, 1, 2, 3\}$ are partial orderings? Determine the properties of a partial ordering that the others lack.

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

b) $\{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

c) $\{(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)\}$

d) $\{(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

(a) is a partial ordering

(b) Not antisymmetric, not transitive.

(c) is a partial ordering

(d) is a partial ordering

(e) Not antisymmetric, not transitive.

8. Is (S, R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if

a) a is taller than b?

b) a is not taller than b?

c) $a = b$ or a is an ancestor of b?

d) a and b have a common friend?

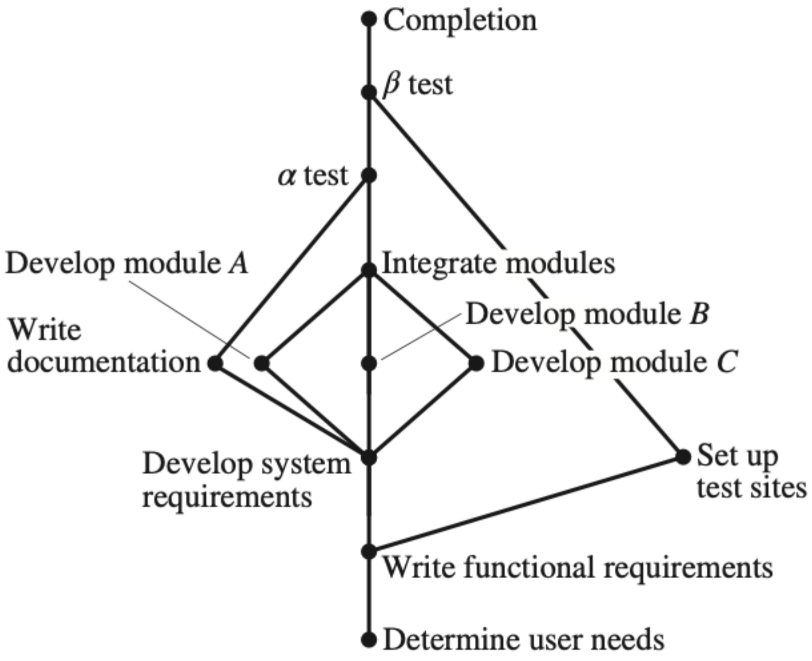
a) No

b) No

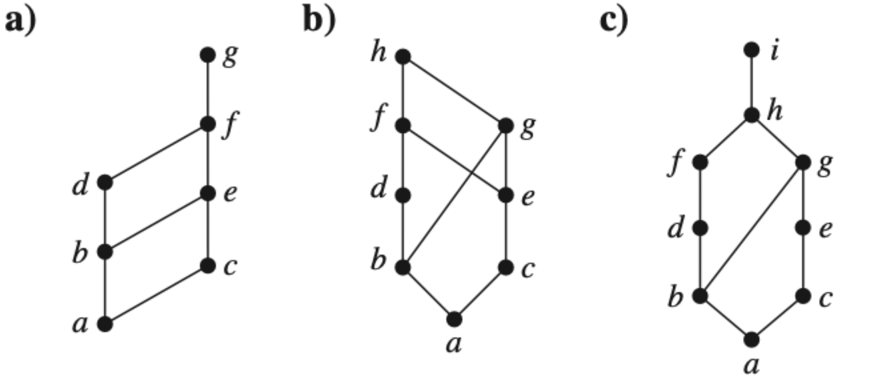
c) Yes

d) No

9. Find an ordering of the tasks of a software project if the Hasse diagram for the tasks of the project is as shown.



10. Determine whether the posets with these Hasse diagrams are lattices.



1. Determine user needs
2. Write functional requirements
3. Set up test sites.
4. Develop system requirements
5. Write documentation
6. Develop module A
7. Develop module B
8. Develop module C
9. Integrate modules
10. α test
11. β test
12. Completion

a) Yes b) No c) Yes