1. Find the generating function for the finite sequence 2, 2, 2, 2, 2, 2.

$$G(x) = \sum_{k=0}^{5} 2x^k = (2 + 2x + 2x^2 + 2x^3 + 2x^4 + 2x^5) = 2(1 + x + x^2 + x^3 + x^4 + x^5) = 2(\frac{1 - x^6}{1 - x})$$

$\frac{1 - x^{n+1}}{1 - x} = \sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$	1 if $k \leq n$; 0 otherwise

2. Use generating functions to determine the number of different ways 10 identical balloons can be given to four children if each child receives at least two balloons.

a. What is the generating function?
b. How many ways to give the 10 identical ballons to four children if each child receives at least two balloons?

a) $C_1 + C_2 + C_3 + C_4 = 10 \Rightarrow G(x) = (x^2 + x^3 + x^4)^4$

b) The coefficient of $x^{10} \Rightarrow G(x) = (x^2 + x^3 + x^4)\ (x^2 + x^3 + x^4)\ (x^2 + x^3 + x^4)\ (x^2 + x^3 + x^4)$ => 10

3. How many elements are in $A_1 \cup A_2$ if there are 12 elements in A_1, 18 elements in A_2, and

a) $A_1 \cap A_2 = \emptyset$?

b) $|A_1 \cap A_2| = 1$?

c) $|A_1 \cap A_2| = 6$?

d) $A_1 \subseteq A_2$?

a) 12+18 = 30 b) 12+18 – 1 = 29 c) 30 – 6 = 24 d) 18

4. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if

a) the sets are pairwise disjoint.

b) there are 50 common elements in each pair of sets and no elements in all three sets.

c) there are 50 common elements in each pair of sets and 25 elements in all three sets.

d) the sets are equal.

a) 300 b) 300 – 50 – 50 – 50 = 150 c) 300 – 150 + 25 = 175 d) 100

5. How many derangements of {1, 2, 3, 4, 5, 6} begin with the integers 1, 2, and 3, in some order?

起始需為123的錯排＝>僅兩種可能 312, 231
456的錯排=>僅兩種可能564, 645 因此所有可能= 2 x 2 = 4