1. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

a) $a_n = 6n$.
b) $a_n = 2n + 1$.
c) $a_n = 10^n$.
d) $a_n = 5$.

a) $a_{n+1} = a_n + 6$ for $n \geq 1$ and $a_1 = 6$
b) **b)** $a_{n+1} = a_n + 2$ for $n \geq 1$ and $a_1 = 3$
c) **c)** $a_{n+1} = 10a_n$ for $n \geq 1$ and $a_1 = 10$
d) **d)** $a_{n+1} = a_n$ for $n \geq 1$ and $a_1 = 5$

2. Give a recursive definition of

a) the set of even integers.
b) the set of positive integers congruent to 2 modulo 3.
c) the set of positive integers not divisible by 5.

a) $0 \in S$, and if $x \in S$, then $x+2 \in S$ and $x-2 \in S$.
b) $2 \in S$, and if $x \in S$, then $x+3 \in S$.
c) $1 \in S$, $2 \in S$, $3 \in S$, $4 \in S$, and if $x \in S$, then $x+5 \in S$.

3. Let S be the set of positive integers defined by

Basis step: $5 \in S$.

Recursive step: If $n \in S$, then $3n \in S$ and $n^2 \in S$.

a) Show that if $n \in S$, then $n \equiv 5 \pmod{10}$.
b) Show that there exists an integer $m \equiv 5 \pmod{10}$ that does not belong to S.

a) *Basisstep:* $5 \equiv 5(\text{mod}10)$. *Inductive step:* If $n \equiv 5(\text{mod}10)$, then $3n \equiv 3 \cdot 5 = 15 \equiv 5(\text{mod}10)$ and $n^2 \equiv 5^2 = 25 \equiv 5(mod\ 10)$.

b) $35 \notin S$ because 35 is not a multiple of 3 nor a perfect square.

4. Give a recursive algorithm for computing nx whenever n is a positive integer and x is an integer, using just addition.

procedure *mult*(*n*: positive integer, *x*: integer)
if $n = 1$ **then return** x
else return $x + mult\ (n - 1, x)$

5. Devise a recursive algorithm for computing the greatest common divisor of two nonnegative integers a and b with $a < b$ using the fact that $\gcd(a, b) = \gcd(a, b - a)$.

procedure ***gcd***(***a, b***: **nonnegative integers**) {***a*** < ***b*** **assumed to hold**}

if ***a*** **= 0 then return** ***b***

else if ***a*** **=** ***b*** **−** ***a*** **then return** ***a***

else if *a* < *b* − *a* **then return** *gcd*(*a*, *b* − *a*)

else return *gcd*(*b* − *a*, *a*)