

1. Determine which of these are linear homogeneous recurrence relations with constant coefficients. Also, find the degree of those that are. 35%

a) $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$

b) $a_n = 2na_{n-1} + a_{n-2}$

d) $a_n = a_{n-1} + 2$

f) $a_n = a_{n-2}$

c) $a_n = a_{n-1} + a_{n-4}$

e) $a_n = a_{n-1}^2 + a_{n-2}$

g) $a_n = a_{n-1} + n$

a) Degree 3 b) No. c) Degree 4 d) No e) No f) Degree 2 g) No

2. Solve these recurrence relations together with the initial conditions given. 35%

a) $a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$

b) $a_n = a_{n-1}$ for $n \geq 1$, $a_0 = 2$

c) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

d) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$

e) $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 1$

f) $a_n = 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 4$

g) $a_n = a_{n-2}/4$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

a) $a_n = 3 \cdot 2^n$ b) $a_n = 2$ c) $a_n = 3 \cdot 2^n - 2 \cdot 3^n$ d) $a_n = 6 \cdot 2^n - 2 \cdot n2^n$ e) $a_n = -1/2 \cdot n(-2)^n$ f) $a_n = 2^n - (-2)^n$ g) $a_n = (1/2)^{n+1} + (1/2)(-1/2)^n$

3. a) Determine values of the constants A and B such that $a_n = An + B$ is a solution of recurrence relation $a_n = 2a_{n-1} + n + 5$. 30%
- b) Use Theorem 5 to find all solutions of this recurrence relation.
- c) Find the solution of this recurrence relation with $a_0 = 4$.

$$a) An + B = 2(A(n - 1) + B) + n + 5$$

$$An + B = 2An - 2A + 2B + n + 5 = (2A + 1)n - 2A + 2B + 5$$

$$a) A = -1, B = -7 \quad b) a_n = \alpha 2^n - n - 7 \quad c) a_n = 11 \cdot 2^n - n - 7$$