1. Find the smallest relation containing the relation $\{(1, 2), (1, 4), (3, 3), (4, 1)\}$ that is
   a) reflexive and transitive.
   b) symmetric and transitive.
   c) reflexive, symmetric, and transitive.

2. Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.
   a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
   b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
   c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
   d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
   e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

3. Which of these relations on the set of all functions from $\mathbf{Z}$ to $\mathbf{Z}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

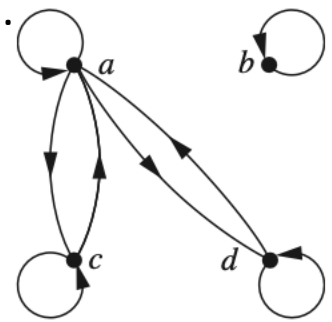
a) $\{(f, g) \mid f(1) = g(1)\}$
b) $\{(f, g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$
c) $\{(f, g) \mid f(x) - g(x) = 1 \text{ for all } x \in \mathbf{Z}\}$
d) $\{(f, g) \mid \text{ for some } C \in \mathbf{Z}, \text{ for all } x \in \mathbf{Z}, f(x) - g(x) = C\}$
e) $\{(f, g) \mid f(0) = g(1) \text{ and } f(1) = g(0)\}$

4. Which of these collections of subsets are partitions of $\{1, 2, 3, 4, 5, 6\}$?

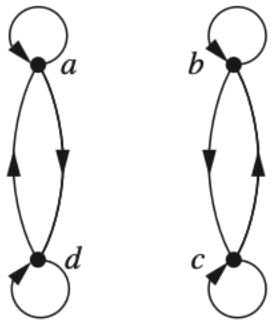
a) $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$
b) $\{1\}, \{2, 3, 6\}, \{4\}, \{5\}$
c) $\{2, 4, 6\}, \{1, 3, 5\}$
d) $\{1, 4, 5\}, \{2, 6\}$

5. Determine whether the relation with the directed graph shown is an equivalence relation.

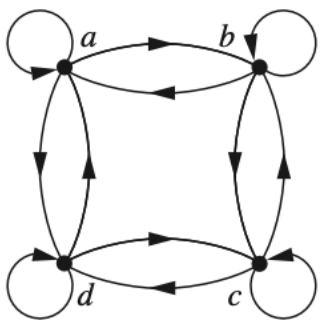
a.



b.



c.



6. Show that the relation R on the set of all bit strings such that $s\,R\,t$ if and only if s and t contain the same number of 1s is an equivalence relation.

7. Which of these relations on $\{0, 1, 2, 3\}$ are partial orderings? Determine the properties of a partial ordering that the others lack.

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

b) $\{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

c) $\{(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)\}$

d) $\{(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

8. Is (S, R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if

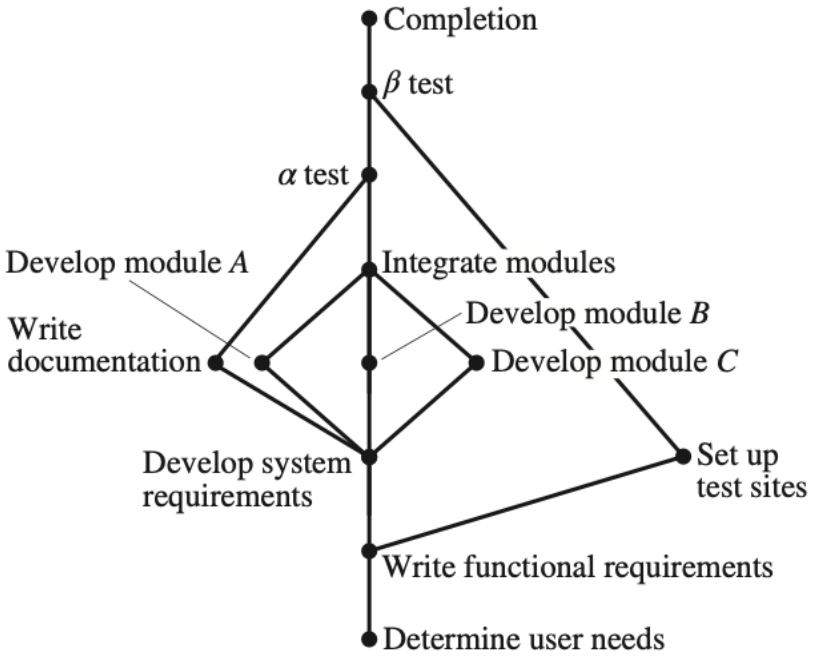
a) a is taller than b?

b) a is not taller than b?

c) $a = b$ or a is an ancestor of b?

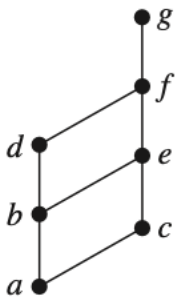
d) a and b have a common friend?

9. Find an ordering of the tasks of a software project if the Hasse diagram for the tasks of the project is as shown.

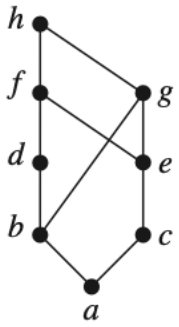


10. Determine whether the posets with these Hasse diagrams are lattices.

a)



b)



c)