1. Find the generating function for the finite sequence 2, 2, 2, 2, 2, 2.

有限

$G(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

$a_i = 2$

$\Rightarrow G(x) = 2 + 2x + 2x^2 + \cdots + 2x^5$

$= 2(1 + x + x^2 + \cdots + x^5)$

$= 2 \cdot \left(\frac{1-x^6}{1-x}\right)$ #

$\frac{\text{对称}}{|x|<1}$

2. Use generating functions to determine the number of different ways 10 identical balloons can be given to four children if each child receives at least two balloons.

至少2個

a. What is the generating function?

b. How many ways to give the 10 identical ballons to four children if each child receives at least two balloons?

a) $x_1 + x_2 + x_3 + x_4 = 10, x_i \geq 2$

$G(x) = (x^2 + x^3 + \cdots)(x^2 + x^3 + \cdots)(x^2 + x^3 + \cdots)(x^2 + x^3 + \cdots)$

$= \frac{x^2}{(1-x)} \times \frac{x^2}{(1-x)} \times \frac{x^2}{(1-x)} \times \frac{x^2}{(1-x)} = \frac{x^8}{(1-x)^4}$ #

b) 每人先拿2顆

剩下自由分 → $10 - 2\times4 = 2$

$H_{\text{選取}}^{\text{類}} = H_2^4$

$= C_2^{4+2-1}$

$= C_2^5 = 10$ #

3. How many elements are in $A_1 \cup A_2$ if there are 12 elements in A_1, 18 elements in A_2, and

a) $A_1 \cap A_2 = \emptyset$?

b) $|A_1 \cap A_2| = 1$?

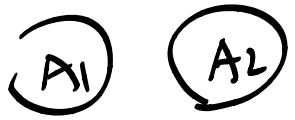
c) $|A_1 \cap A_2| = 6$?

d) $A_1 \subseteq A_2$?

$|A_1| = 12$

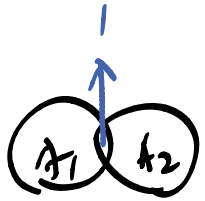
$|A_2| = 18$

a)



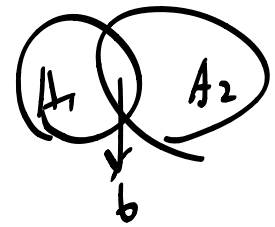
$12 + 18 = 30$ #

b)



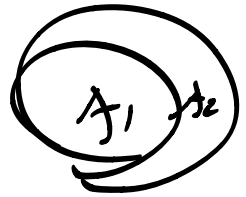
$12 + 18 - 1 = 29$ #

c)



$12 + 18 - 6 = 24$ #

d)



$A_1 \cap A_2 = A_1$

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

$= 12 + 18 - 12$

$= 18$ #

4. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if

a) the sets are pairwise disjoint.

b) there are 50 common elements in each pair of sets and no elements in all three sets.

c) there are 50 common elements in each pair of sets and 25 elements in all three sets.

d) the sets are equal.

3. How many elements are in $A_1 \cup A_2$ if there are 12 elements in A_1, 18 elements in A_2, and

a) $A_1 \cap A_2 = \emptyset$? **b)** $|A_1 \cap A_2| = 1$?

c) $|A_1 \cap A_2| = 6$? **d)** $A_1 \subseteq A_2$?

$|A_1| = 100$

$|A_2| = 100$

$|A_3| = 100$

4. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if

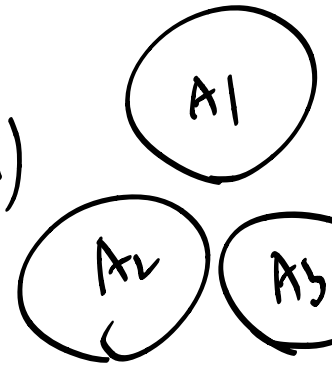
a) the sets are pairwise disjoint. 無交集

b) there are 50 common elements in each pair of sets and no elements in all three sets.

c) there are 50 common elements in each pair of sets and 25 elements in all three sets.

d) the sets are equal.

a) $|A_1 \cup A_2 \cup A_3|$



$= A_1 + A_2 + A_3 - A_1 \cap A_2 - A_2 \cap A_3 - A_3 \cap A_1 + A_1 \cap A_2 \cap A_3$

$= 100 + 100 + 100 - 0 - 0 - 0 + 0$

$= 300$

b)

$100 + 100 + 100 - 50 - 50 - 50 + 0$

$= 150$

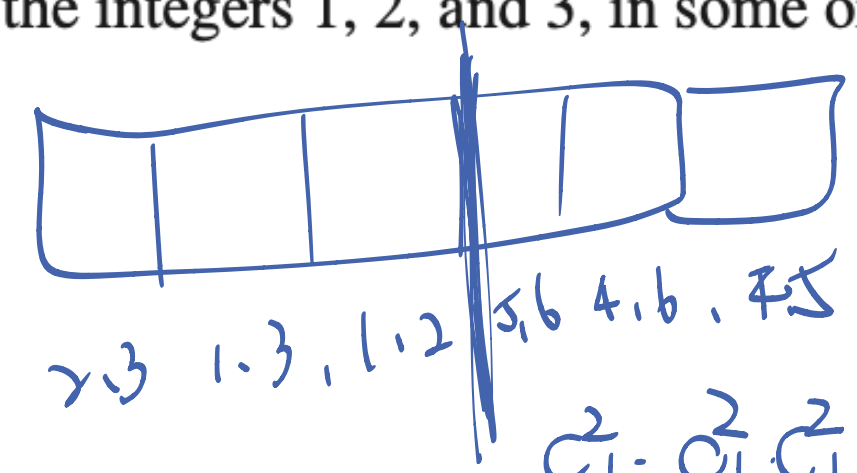
c) $100 + 100 + 100 - 50 - 50 - 50 + 25$

$= 175$

d $100 + 100 + 100 - 100 - 100 - 100 + 100$

$= 100$

5. How many derangements of {1, 2, 3, 4, 5, 6} begin with the integers 1, 2, and 3, in some order?



$C_1^2 \cdot C_1^2 \cdot C_1^2$

$C_1^2 \cdot C_1^2 \cdot C_1^2$

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