Introduction to Probability
Batch 23 (Class of 2025 Winter) Foundation Term

Individual Assignment
Total Points: 35

This is an **Individual Assignment** and has **35% weightage** in the course.

Due Date: 16th February 2025, 11:55 PM

Assignment Deliverables:

1. A **.pdf document** with only the relevant answers and explanations
    - Include tables, figures, solutions/code outputs to support your answers wherever required.
    - Do not copy questions in the report – include only the question numbers.
    - Code files rendered/exported as pdfs will not be considered for evaluation.
2. **R markdown (.rmd) / Python (.ipynb) / Excel (.xlsx) code files** used to solve the assignment
    - Include comments wherever required.
    - Solutions and outputs must be retained in the fully executed code files before submission.
3. **Assignment Submission Form** must be **attached separately** with your full name and PGID. Submissions will not be accepted or may be subject to penalty without the Assignment Submission Form being submitted.

General Instructions:

1. This is an Individual Assignment.
2. **Do NOT submit .zip files**.
3. Please note that both the report (pdf file) and the Excel (.xlsx)/code file (.rmd / .ipynb) are mandatory for evaluation.
4. Code files rendered/exported as pdfs will strictly not be considered for evaluation.
5. The Honor Code for this submission is **2N-b**. **Please look through the Honor Code restrictions carefully before attempting the assignment as there will be strong consequences for breaking them**.
6. Upload your submissions to the '**ITP Individual Assignment**' on LMS.
7. Any late submission will attract a penalty as mentioned in the course outline.
8. **Email submissions are NOT allowed**. All the submissions must be made on LMS.
9. Handwritten content will not be considered for evaluation.
10. There is no penalty for early submissions!
11. **Please adhere to the given instructions. Submissions will not be considered if the instructions are not followed**.

Assignment Details

Total Marks:35 Marks

1. **[3.5 Marks]** For the following situations describe the distribution, including parameters, of the given random variables. Give the most reasonable distribution for the situation.
    a. Ravi believes there is a 40 percent chance of rain tomorrow. Let X indicate the presence or absence of rain tomorrow.
    b. Priyanshu spends his free afternoons watching ship traffic in the harbor. Each hour about 4 large ships arrive to dock at the port. Let X be the number of large ships which arrive in the next hour.
    c. Priya and Praveen are playing a strategy game. They are equally likely to win, and play 10 matches. Let X denote the number of Priya's wins out of those matches.
    d. Neha is playing a board game where income per turn is generated by rolling a standard six-sided die and multiplying the result by 100. Let X be the income earned on a turn.
    e. Every day there is a 10% chance that Shyam will receive no mail. It's Monday. Let X be the number of times he receives mail over the next 5 days.

2. **[3.5 Marks]** In two dice rolls, let X be the outcome of the first die, and Y the outcome of the second die. Then X + Y is the sum of the two dice. Describe the following events in terms of simple outcomes of the random experiment:

    (a) $\{X + Y = 4\}$. (Example solution: $\{(1, 3), (2, 2), (3, 1)\}$.)
    (b) $\{X + Y = 9\}$.
    (c) $\{Y = 3\}$.
    (d) $\{X = Y\}$.
    (e) $\{X > 2Y\}$

3. **[3.5 Marks]** A sample space has four elements (1,2,3,4). For the potential probability functions for the sample space below, state whether they are valid or not. Provide support for your response.

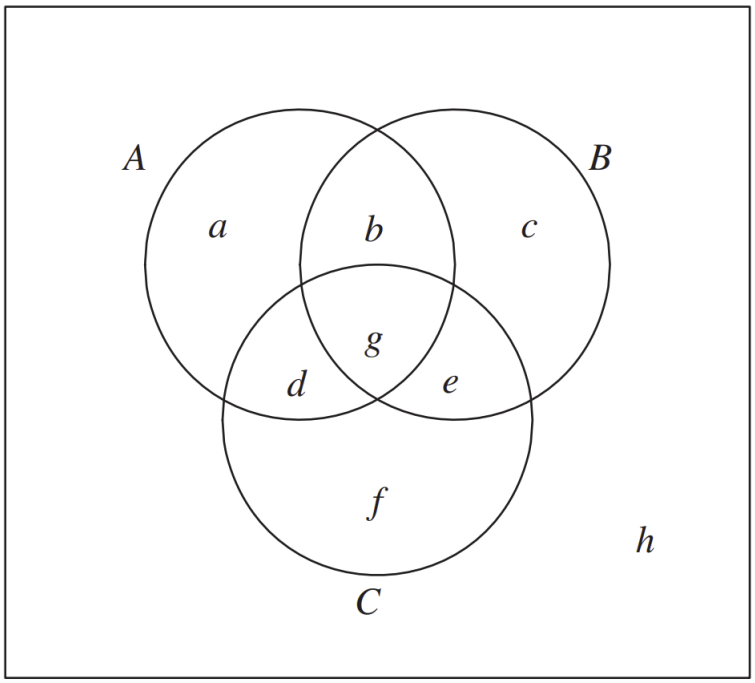
    (a) P(1) = 0.6, P(2) = 0.05, P(3) = 0.4, P(4) = 0.2.
    (b) P(1) = 0.5, P(2) = 0.2, P(3) = 0.2, P(4) = 0.1.
    (c) P(1) = 0.15, P(2) = 0.3, P(3) = 0.1, P(4) = 0.45.
    (d) P(1) = 0.3, P(2) = 0.3, P(3)=−0.2, P(4) = 0.6.

4. **[3.5 Marks]** At a large university, there are 20 graduate students in one graduate program. A professor is soliciting funding to allow between 3 and 6 of the students to attend a conference.
    - How many possible subgroups of students are there that might attend the conference?

- The funding came through for six students to attend the conference. The three most senior students hope to get to go, but the professor has decided to allocate the funding randomly. What is the probability that the three most senior students end up being able to attend the conference?

5. **A) [1.75 Marks]** Suppose A and B are mutually exclusive, with P(A) = 0.30 and P(B) = 0.60. Find the probability that (a) At least one of the two events occurs. (b) Both of the events occur. (c) Neither event occurs. (d) Exactly one of the two events occur.

   B) [1.75 Marks]



   See the assignment of probabilities to the Venn diagram in the figure. Find the following (in terms of a,b,c,…,h):

   (a) P(No events occur).

   (b) P(Exactly one event occurs).

   (c) P(Exactly two events occur).

   (d) P(Exactly three events occur).

   (e) P(At least one event occurs).

   (f) P(At least two events occur).

   (g) P(At most one event occurs).

   (h) P(At most two events occur)

6. **[3.5 Marks]** Suppose X is a random variable that takes values on {0, 0.01, 0.02, ..., 0.99, 1}. If each outcome is equally likely, find (a) $P(X \leq 0.33)$. (b) $P(0.55 \leq X \leq 0.66)$

7. **[3.5 Marks]** Every person in a group of 1000 people has a 1% chance of being infected by a virus. Assume that the process of being infected is independent from person to person. Using random variables, write expressions for the following probabilities and solve in R. (a) The probability that exactly 10 people are infected. (b) That probability that at least 16 people are infected. (c) The probability that between 12 and 14 people are infected. (d) The probability that someone is infected (Hint: Use a Binomial distribution)

8. **[3.5 Marks]** A homeowner's association has recently updated its policies and determines the number of violations of policies each home currently has. The distribution of X, the number of violations per home, is shown in the table. Use the table to address the questions below

x	0	1	2	3
$P(X = x)$	0.4	0.3	0.2	

- Determine P(X = 3) to complete the distribution.
- Two homes in the neighborhood are randomly selected. What is the probability both have no violations?
- Two homes in the neighborhood are randomly selected. What is the probability one has at most 1 violation and the other has at least 2 violations?
- Suppose a random sample of 5 homes in the association is taken. What is the probability that fewer than 3 of the sampled homes have at most 1 violation each?

9. **[3.5 Marks]** A hotel has 100 single rooms, all occupied. In each room the number of phone calls made by the guest has a Poisson distribution with rate $\lambda = 2$.
   - Find the probability that more than six calls will be made by the guest in any one room. Use any statistical software as needed (e.g. R, Python, Excel etc.) for the computation.
   - Find the probability that in at least three rooms more than six calls will be made by the guest. Use any statistical software as needed (e.g. R, Python, Excel etc.) for the computation.

10. **[3.5 Marks]** An elevator's weight capacity is 1000 pounds. Three men and three women are riding the elevator. Adult male weight is normally distributed with mean 172 pounds and standard deviation 29 pounds. Adult female weight is normally distributed with mean 143 pounds and standard deviation 29 pounds. Find the probability that the passengers' total weight exceeds the elevator's capacity.