

# **Computer Vision**

(Summer Semester 2022)

Lecture 8

**Dense Motion Estimation** 



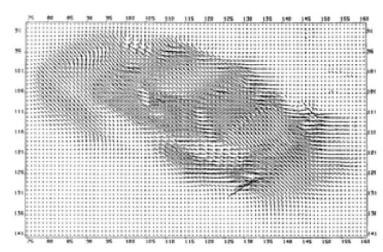
- Slides based on
  - Szeliski's book "Computer Vision: Algorithms and Applications": <a href="http://szeliski.org/Book/">http://szeliski.org/Book/</a>
  - Slides from Brown University: <a href="https://cs.brown.edu/courses/csci1430/">https://cs.brown.edu/courses/csci1430/</a>
  - Slides from Elli Angelopoulou from summer term 2014: https://www5.cs.fau.de/lectures/ss-14/computer-vision-cv/



#### **Dense Motion Estimation**

- Estimate the "flow" of objects visible in single pixels from two following frames of an animation
  - → flow coming from object motion



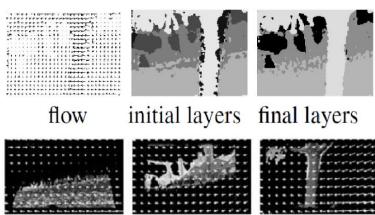




#### **Dense Motion Estimation**

- Estimate the "flow" resulting from camera movement
  - → magnitude of flow related to depth
  - → depth estimator



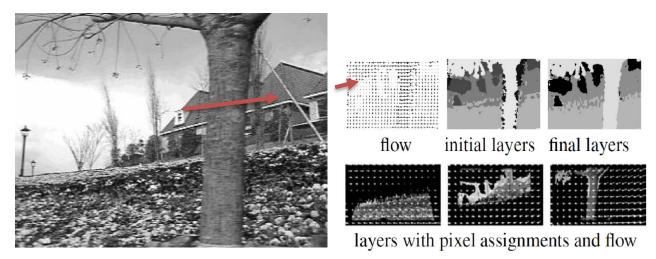


layers with pixel assignments and flow



#### **Dense Motion Estimation**

- Estimate the "flow" resulting from camera movement
  - → magnitude of flow related to depth
  - $\rightarrow$  depth estimator



"Dense": estimate flow for all pixels, not only for certain features

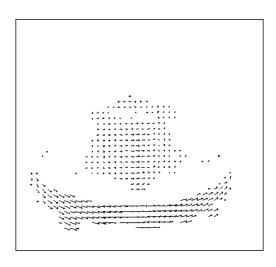


#### **Motion estimation: Optical flow**

Optical flow is the apparent motion of objects or surfaces



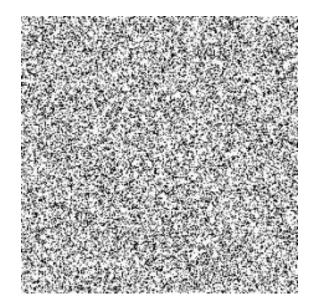




Optical flow only approximates motion flow:
 white plate has no features and thus no optical flow

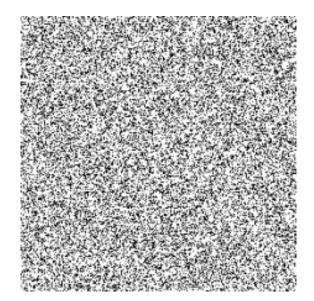


Sometimes, motion is the only cue





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• Even "impoverished" motion data can evoke a strong percept



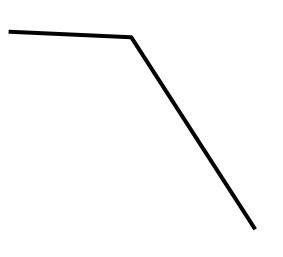


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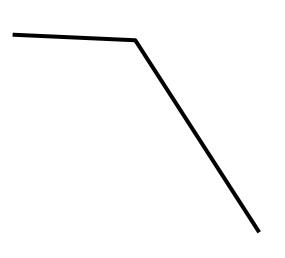


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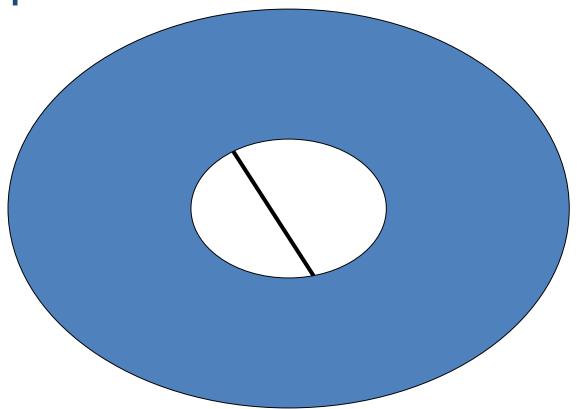




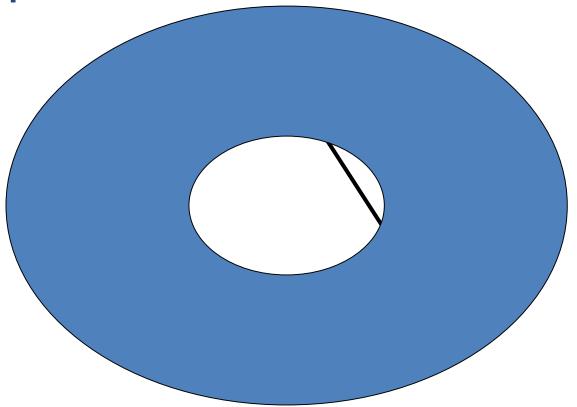




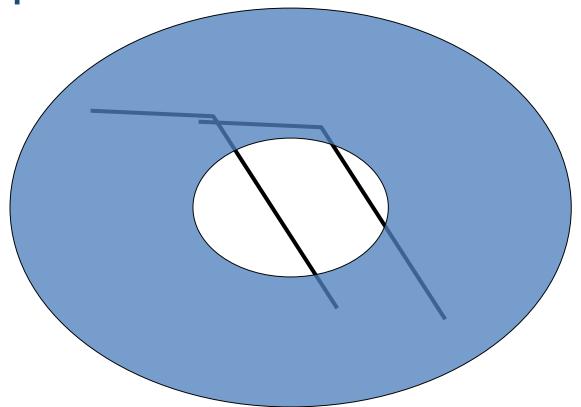






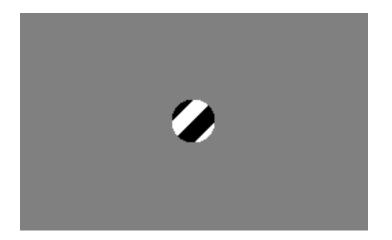








# The barber pole illusion



http://en.wikipedia.org/wiki/Barberpole\_illusion



# The barber pole illusion

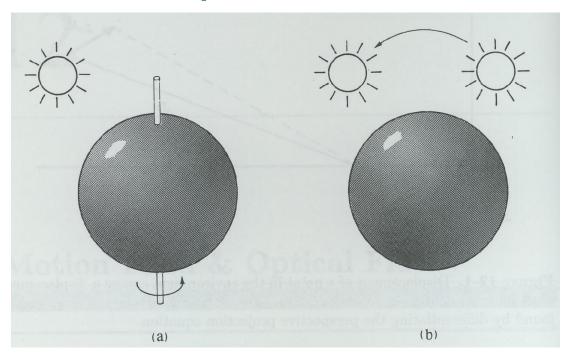




http://en.wikipedia.org/wiki/Barberpole\_illusion



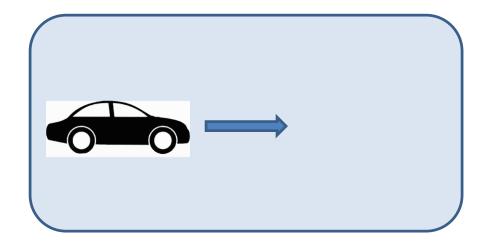
### Motion flow vs. optical flow



left: sphere rotates → motion flow, but no optical flow right: light moves → highlight moves → optical flow, but no motion flow



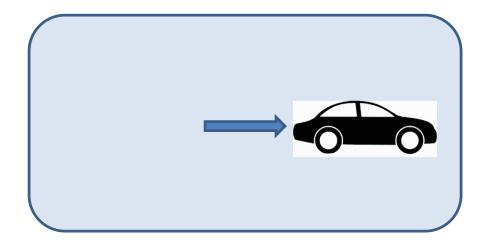
Often used assumption: translation parallel to image plane



3D translation in world → 2D translation in image space



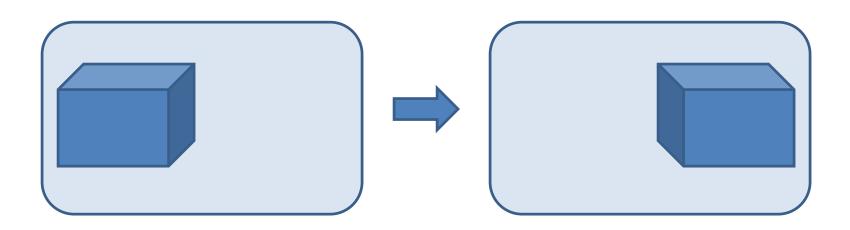
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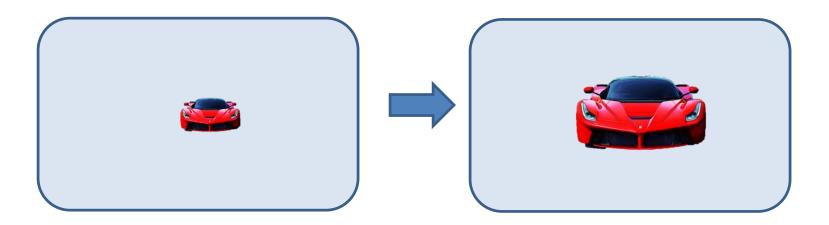
- Not really true: ignores parallax effects
- Closer objects move faster in image space than distant ones
- True for translational movement of object or camera



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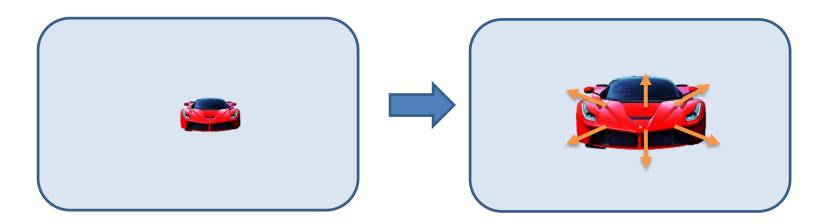
Motion towards the camera becomes a radial optical flow



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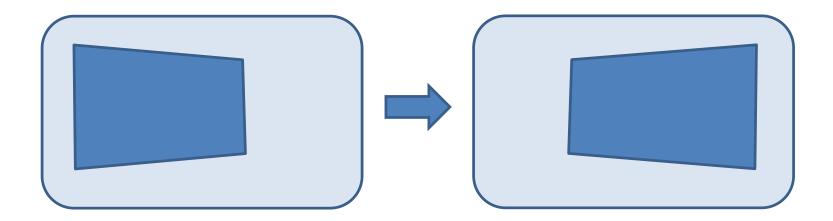


Motion towards the camera becomes a radial optical flow





■ Finally: camera rotate → additional distortion



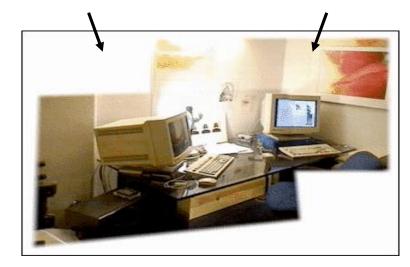


### **Image Alignment**

- Goal: Estimate a single v translation (transformation) for the entire image.
- The entire image has the same translation value so the optical flow values for every pixel is the same.
- This is typically an easier problem than general motion estimation.
- We can compute it very well with pyramid-based methods like the Lucas-Kanade one (see later)

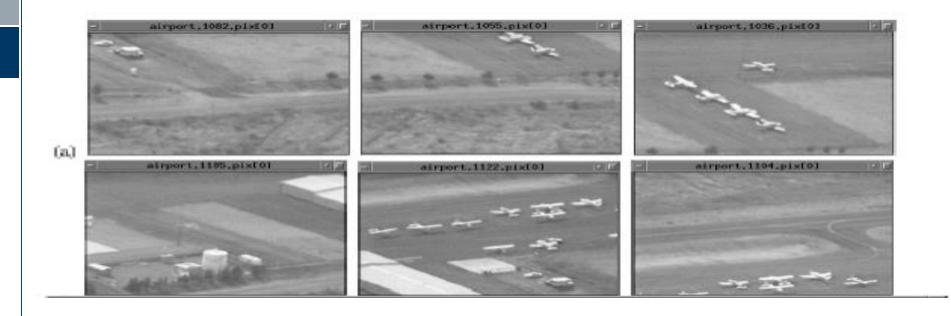








# **Mosaicing – input images**





### **Mosaicing – Final Result**



Static background messaic of an airport video clip.

(a) A few representative frames from the minute-long video dip. The video shows an airport being imaged from the air with a moving camera. The same itself is static (i.e., no moving objects). (b) The static background mosaic image which provides an extended view of the entire some imaged by the camera in the one-minute video clip.



#### **Translational Motion**

- Entire image moves, e.g. from camera shake, walking camera man etc.
- Simplified assumption: all pixels move by same amount
- More realistic:
  - translational movement of object in front of static background
  - translational movement of some objects in front of static background with translational movement
    - → assumption of many video coders
  - requires separation of background and foreground object(s)

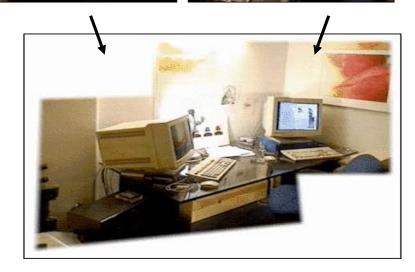


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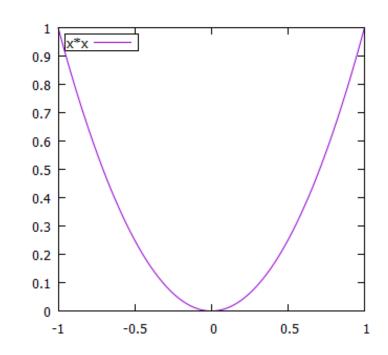
- Given two images  $I_0(x)$  and  $I_1(x)$  with x being the pixel position (x,y)
- We are looking for the displacement vector  $\mathbf{u} = (u, v)$  that minimizes the difference between  $I_1(\mathbf{x} + \mathbf{u})$  and  $I_0(\mathbf{x})$
- But how can we measure the difference between two images?
- Simple solution: sum of squared difference:

$$E_{\text{SSD}}(\boldsymbol{u}) = \sum_{i} [I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2 = \sum_{i} e_i^2,$$

- Color images:
  - extend sum over color channels
  - or use luminance only



- SSD:
  - small errors are less important
  - larger errors strongly penalized
- → tolerant to low-amplitude noise
- later: easy for minimization!
- Downside: large errors get dominant → not very robust

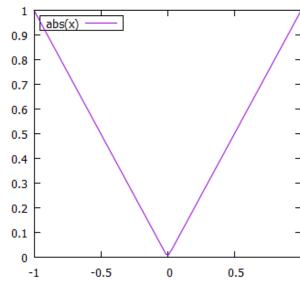




More robust: sum of absolute differences

$$E_{\text{SAD}}(\boldsymbol{u}) = \sum_{i} |I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)| = \sum_{i} |e_i|.$$

Large errors do not dominate so much



**Dense Motion Estimation** 

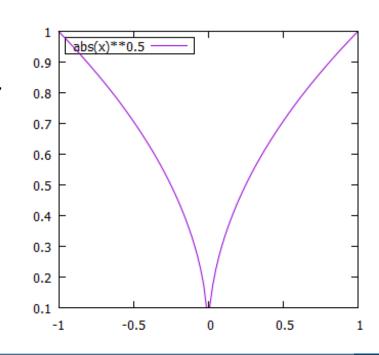
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•  $L_p$ -Norms:

$$E_L(\boldsymbol{u}) = \left(\sum_i |I_1(\boldsymbol{x}_i + \boldsymbol{u}_i) - I_o(\boldsymbol{x})|^p\right)^{\frac{z}{p}}$$

- In our case the exponent  $\frac{1}{p}$  can be removed...
- p = 2: SSD
- p = 1: SAD
- p < 1 also makes sense (right plot), but is not a norm (violates triangle inequality)





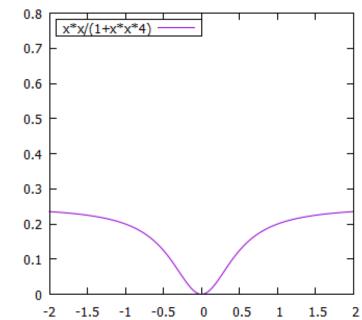
• General function: robust kernel function  $\rho$ 

$$E_{\text{SRD}}(\boldsymbol{u}) = \sum_{i} \rho(I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)) = \sum_{i} \rho(e_i).$$

(SRD = "sum of robust distances")

- $\rho(x) = x^p$ :  $L_p$ -norm
- Problem with L<sub>p</sub>-norm:
   not differentiable at zero
   → bad for optimization methods
- Alternative robust kernel functions, e.g. Geman-McClure kernel

$$\rho_{\rm GM}(x) = \frac{x^2}{1 + x^2/a^2}$$





- Up to now, we ignored the fact that  $I_1(x + u)$  can be outside the image and that we also want to consider masked images  $\rightarrow w_0(x)$  and  $w_1(x)$  are masks that are one inside the regions of interest
- $E_{WSRD}(\boldsymbol{u}) = \sum_{i} w_0(\boldsymbol{x_i} + \boldsymbol{u}) w_1(\boldsymbol{x_i}) \rho(I_1(\boldsymbol{x_i} + \boldsymbol{u}) I_0(\boldsymbol{x_i}))$  $\rightarrow$  "W" stands for "windowed"
- Only counts differences for pixels where both masks are one
- Possibly also normalize by area of counted pixels  $A(u) = \sum_i w_0(x_i + u)w_1(x_i)$ :

$$E_{NWSRD}(\boldsymbol{u}) = \frac{1}{A(\boldsymbol{u})} E_{WSRD}(\boldsymbol{u})$$



- Bias and Gain
  - sometimes, both images have not been taken with same exposure, aperture, or even camera
  - in this case, colors are not directly comparable, but we need a mapping, e.g. a linear one:

$$I_0(\mathbf{x}) \to \alpha I_0(\mathbf{x}) + \beta$$

The SSD error then becomes

$$E_{SSD}(\boldsymbol{u}) = \sum_{i} (I_1(\boldsymbol{x_i} + \boldsymbol{u}) - \alpha I_0(\boldsymbol{x}) - \beta)^2$$

We have to determine α and β such that this error is minimized
 → linear regression



#### **Error Metrics**

Oftentimes a better alternative: cross correlation

$$E_{\mathrm{CC}}(\boldsymbol{u}) = \sum_{i} I_0(\boldsymbol{x}_i) I_1(\boldsymbol{x}_i + \boldsymbol{u}).$$

or even better: normalized cross-correlation

$$E_{\text{NCC}}(\boldsymbol{u}) = \frac{\sum_{i} [I_0(\boldsymbol{x}_i) - I_0] [I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_1]}{\sqrt{\sum_{i} [I_0(\boldsymbol{x}_i) - \overline{I_0}]^2} \sqrt{\sum_{i} [I_1(\boldsymbol{x}_i + \boldsymbol{u}) - \overline{I_1}]^2}},$$

$$\overline{I_0} = \frac{1}{N} \sum_{i} I_0(\boldsymbol{x}_i) \text{ and}$$

$$\overline{I_1} = \frac{1}{N} \sum_{i} I_1(\boldsymbol{x}_i + \boldsymbol{u})$$

Also works when pictures were taken with different exposure!

with



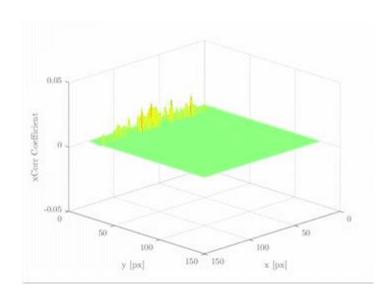
- Decide upon error metric E(u)
- Search for  $\boldsymbol{u}$  that minimizes  $E(\boldsymbol{u})$
- Full Search: examine all possible values for  $oldsymbol{u}$ , maybe in a limited radius
  - → very expensive
  - → only on discrete grid (e.g. pixels, not subpixel-wise)

For a region of 100 x 100 in each image, there are almost 200 x 200 values to search!



## **Cross-Correlation + Sliding Window**





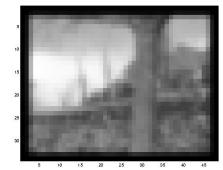
 https://en.wikipedia.org/wiki/Crosscorrelation#/media/File:Cross\_Correlation\_Animation.gif

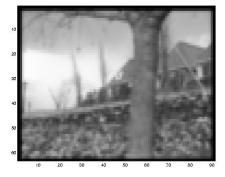


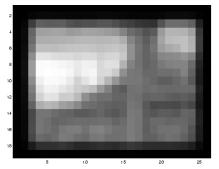
#### Use image pyramid

- starting at the coarsest resolution, we always search in a small neighborhood
- This results in an initially coarse flow resolution on a small neighborhood, and expands to a fine flow resolution on a larger neighborhood



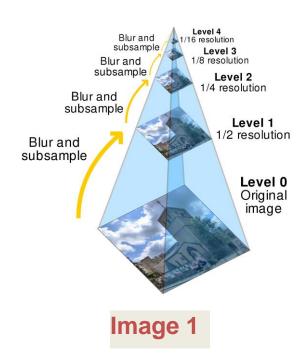


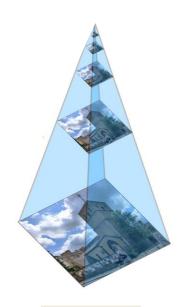






## Coarse-to-fine optical flow estimation





**Image 2** 

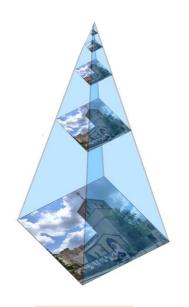
https://en.wikipedia.org/wiki/Pyramid\_(image\_processing)#/media/File:Image\_pyramid.svg

**Gaussian pyramid of Image1** 

Gaussian pyramid of Image2



## Coarse-to-fine optical flow estimation



u=1.25 pixels

u=2.5 pixels

u=5 pixels

u=10 pixels/

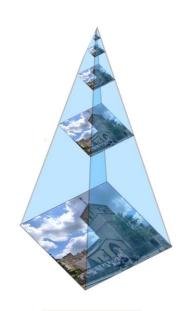


Image 2

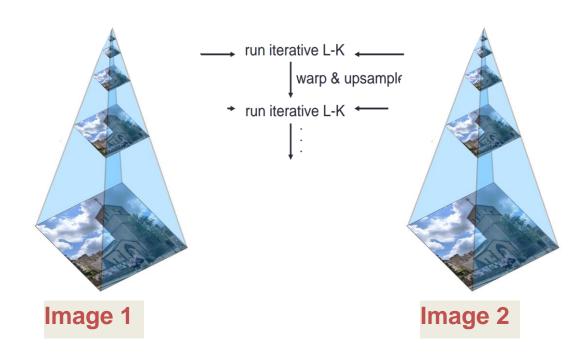
**Image 1** 

**Gaussian pyramid of Image1** 

**Gaussian pyramid of Image2** 



## Coarse-to-fine optical flow estimation



Gaussian pyramid of Image1

Gaussian pyramid of Image2



- Up to now: discrete search, usually pixel-wise
- how about subpixel values for *u*?
- Assume, we found as optimum some u
- Starting from there, we want to make small steps  $\Delta u$  to further optimize u for some pixel  $x_i$ :

$$I_1(\mathbf{x_i} + \mathbf{u} + \Delta \mathbf{u}) \approx I_1(\mathbf{x_i} + \mathbf{u}) + J_1(\mathbf{x_i} + \mathbf{u})\Delta \mathbf{u}$$

using the first order Taylor expansion of  $I_1$  with

$$J_1(x_i + u) = \nabla I_1(x_i + u) = (\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y})(x_i + u)$$



- $J_1(x)$  is the Jacobian of Image 1 in x
- i.e. the row vector of the derivatives of  $I_1$  in x and y
- use central differences or forward or backward differences
- $J_1(x)\Delta x$ : how much does the color in  $I_1$  change, if we move from x in direction  $\Delta x$ ?
- Obviously, this works well if we move by less than a pixel → we move within a linear interpolation
- For larger steps, it only works if the image is smooth



Method of Lucas-Kanade (1981):

$$E_{\text{LK-SSD}}(\boldsymbol{u} + \Delta \boldsymbol{u}) = \sum_{i} [I_1(\boldsymbol{x}_i + \boldsymbol{u} + \Delta \boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2$$

$$\approx \sum_{i} [I_1(\boldsymbol{x}_i + \boldsymbol{u}) + \boldsymbol{J}_1(\boldsymbol{x}_i + \boldsymbol{u})\Delta \boldsymbol{u} - I_0(\boldsymbol{x}_i)]^2$$

$$= \sum_{i} [\boldsymbol{J}_1(\boldsymbol{x}_i + \boldsymbol{u})\Delta \boldsymbol{u} + e_i]^2,$$

with  $e_i = I_1(\boldsymbol{x}_i + \boldsymbol{u}) - I_0(\boldsymbol{x}_i)$ 

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• set 
$$A = \begin{pmatrix} J_1(x_1 + u) \\ \vdots \\ J_1(x_n + u) \end{pmatrix} \in \mathbb{R}^{n \times 2}$$
 and  $b = \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} \in \mathbb{R}^n$ 

- b is called residuum
- these matrices and vectors are huge (one row per pixel)
- usually, we won't do this for an entire image, but for smaller patches (segmented foreground object, sub-patch  $\rightarrow$  see later)
- Then we get as error:

$$E(\Delta u) = ||A\Delta u - b||^2$$



We want to minimize this error:

$$(a-b)^2 = a^2 - 2ab + b^2$$

- $E(\Delta u) = ||A\Delta u b||^2$
- This is a typical non-linear least squares problem
- We write:

$$E(\Delta u) = \Delta u^{T} (A^{T} A) \Delta u - 2\Delta u^{T} (A^{T} b) + b^{T} b$$

and find the minimum by setting the derivative to zero:

$$2A^{T}A\Delta u - 2A^{T}b = 0$$

$$A^{T}A\Delta u = A^{T}b$$

$$2x2 \text{ matrix} \quad 2D \text{ vector}$$

• and solve this equation for  $\Delta u_{\bullet}$ 

solution similar as for the camera matrix estimation

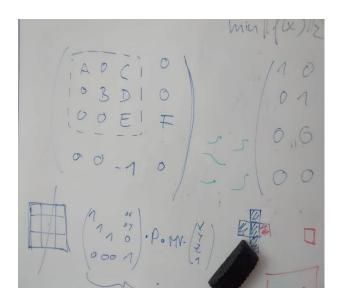


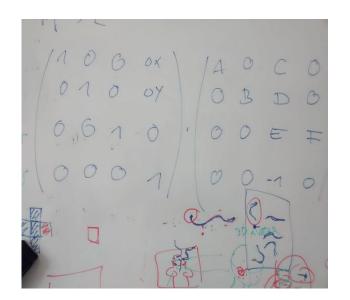
• We iterate this, and to have better stability, we only proceed a fraction  $\alpha$  in direction of  $\Delta u$ :

```
// given: starting value u
repeat
   set A = Jacobian of I1 at u
   set b = residual vector
   solve A^T*A*delta_u = A^T*b for delta_u
   u += alpha * delta_u
until convergence
```



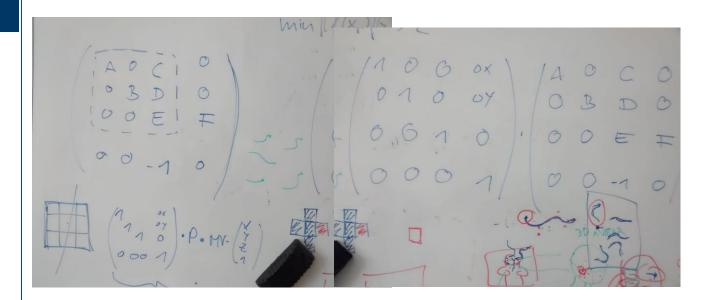
 Simple translation model maybe okay to stitch scanned documents → rotation disallowed





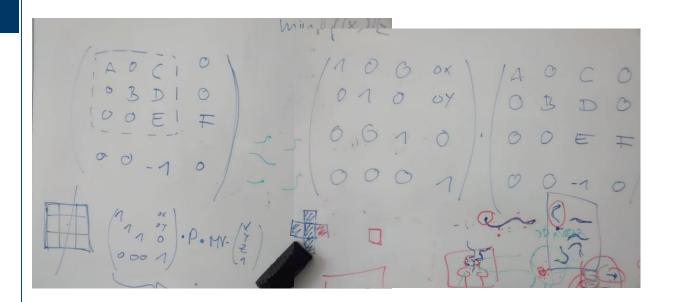


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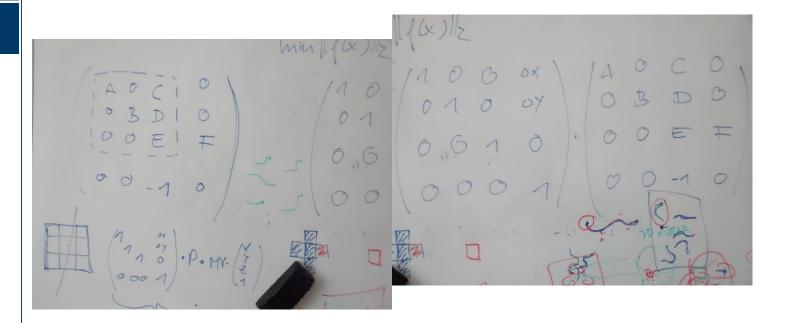


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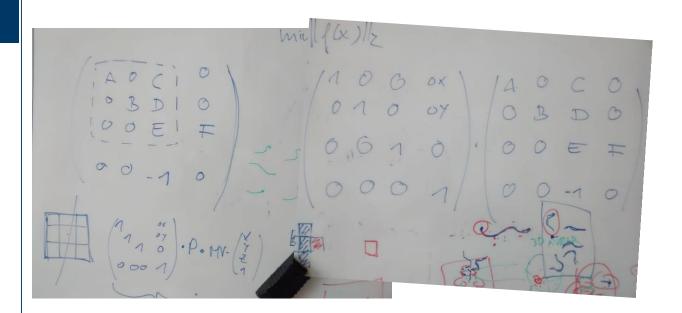


Rotation can be necessary





Rotation can be necessary



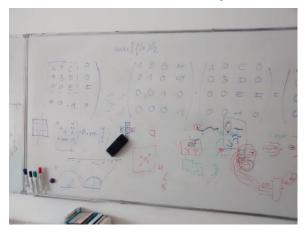


- → Search for translation and rotation
- Parameters: translation u and rotation  $\phi$
- $E_{SSD}(u, \Phi) = \sum_{i} (I_1(R(\Phi)x_i + u) I_0(x_i))^2$ where  $R(\Phi)$  is the rotation matrix for  $\Phi$
- New problem:

arg min  $E_{SSD}(u, \Phi)$ 



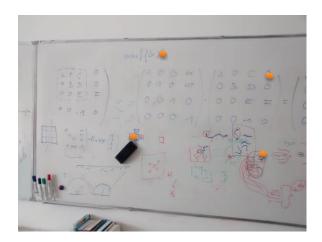
- Even more complicated: camera rotation
  - → image planes not parallel
    - Projective mapping needed: 8 degrees of freedom
    - Needed for panorama stitching!







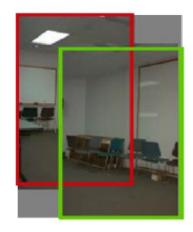
- Even more complicated: camera rotation
  - o projective mapping: defined by mapping of four image points
    - → 8 degrees of freedom



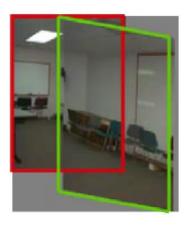




Different motion models possible, e.g.:

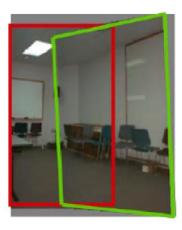


(a) translation [2 dof]



(b) affine [6 dof]





(c) perspective [8 dof] (d) 3D rotation [3+ dof]



- We describe parametric motion with a parameter vector p
- Motion is then  $x \to x'(x, p)$
- Lucas-Kanade with parametric motion:

$$egin{array}{lll} E_{ ext{LK-PM}}(p+\Delta p) & = & \sum_i [I_1(x'(x_i;p+\Delta p)) - I_0(x_i)]^2 \\ & pprox & \sum_i [I_1(x'_i) + J_1(x'_i)\Delta p - I_0(x_i)]^2 \\ & = & \sum_i [J_1(x'_i)\Delta p + e_i]^2, \end{array}$$

• where  $J_1$  is now with respect to parameter vector p:

$$J_1(x') = \frac{\partial I_1(x')}{\partial x'} \cdot \frac{\partial x'(x)}{\partial p}$$

gradient from image

gradient from motion model



Transform	Matrix	Parameters $oldsymbol{p}$	Jacobian $J$
translation	$\left[egin{array}{ccc} 1 & 0 & t_x \ 0 & 1 & t_y \end{array} ight]$	$(t_x,t_y)$	$\left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]$
Euclidean	$\left[ egin{array}{ccc} c_{ heta} & -s_{ heta} & t_x \ s_{ heta} & c_{ heta} & t_y \end{array}  ight]$	$(t_x,t_y,\theta)$	$\left[\begin{array}{ccc} 1 & 0 & -s_{\theta}x - c_{\theta}y \\ 0 & 1 & c_{\theta}x - s_{\theta}y \end{array}\right]$
similarity	$\left[ egin{array}{ccc} 1+a & -b & t_x \ b & 1+a & t_y \end{array}  ight]$	$(t_x,t_y,a,b)$	$\left[ egin{array}{cccc} 1 & 0 & x & -y \ 0 & 1 & y & x \end{array}  ight]$
affine	$\left[\begin{array}{ccc} 1 + a_{00} & a_{01} & t_x \\ a_{10} & 1 + a_{11} & t_y \end{array}\right]$	$(t_x, t_y, a_{00}, a_{01}, a_{10}, a_{11})$	$\left[\begin{array}{cccccc} 1 & 0 & x & y & 0 & 0 \\ 0 & 1 & 0 & 0 & x & y \end{array}\right]$
projective	$\left[ egin{array}{cccc} 1+h_{00} & h_{01} & h_{02} \ h_{10} & 1+h_{11} & h_{12} \ h_{20} & h_{21} & 1 \end{array}  ight]$	$(h_{00}, h_{01}, \dots, h_{21})$	





$$x' = \frac{(1+h_{00})x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + 1} \text{ and } y' = \frac{h_{10}x + (1+h_{11})y + h_{12}}{h_{20}x + h_{21}y + 1}.$$

$$J = \frac{\partial f}{\partial p} = \frac{1}{D} \begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -x'x & -x'y \\ 0 & 0 & 0 & x & y & 1 & -y'x & -y'y \end{bmatrix}$$



## **General Optical Flow**

- Optical Flow: one motion vector per pixel
- General approach:

$$E_{\mathrm{SSD-OF}}(\{u_i\}) = \sum_i [I_1(x_i + u_i) - I_0(x_i)]^2.$$
 motion of pixel i

- cannot work directly:
  - each  $u_i$  is 2D, so we have 2n unknowns (n is the number of pixels), but only n equations
- obvious solution:
  - for each pixel in  $I_0$ , find some pixel in  $I_1$  with same color  $\rightarrow u_i$
  - 1 million pixels, 256 colors → many, many solutions

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- Patch-based approach:
  - for each pixel (or a coarser grid on the image) compute translational flow on a local neighborhood to determine the pixel's flow
- Algorithm as before, but on smaller patches
- Also works on image pyramid
- runs into problems for disocclusions (newly visible region will always generate error)
  - → problematic near silhouettes
  - → see later: layer-based flow

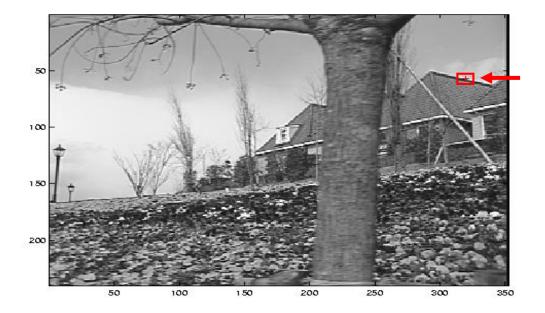


- Observation: solution not always obvious
- low textured region: optimal translation unclear



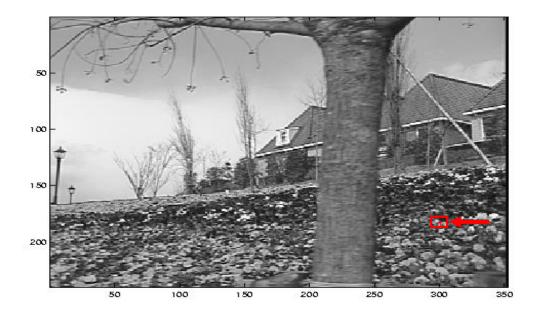


- Observation: solution not always obvious
- at edges: translation along edge leads to similar error





- Observation: solution not always obvious
- highly textured region: works best
- Repetitive structures can be tricky





SSD surfaces:

 at the three
 image locations,
 offset a small
 neighborhood
 and compute

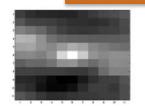
 SSD error

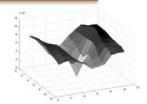






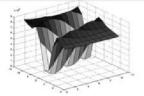
#### clear optimum



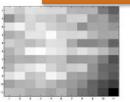


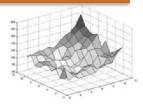
#### valley → unclear in one direction





#### no clear optimum







## **Regularization-based Optical Flow**

General optical flow underconstrained:

$$E_{SSD-OF}(\{u_i\}) = \sum_{i} [I_1(x_i + u_i) - I_0(x_i)]^2$$

- Regularization-based flow
  - **u**, should not be chosen arbitrarily
  - instead: neighboring  $u_i$  should be similar
  - add regularization term that penalizes dissimilar  $u_i$ :

$$E_{SSD-OF}(\{u_i\}) = \sum_{i} [I_1(x_i + u_i) - I_0(x_i)]^2 + \alpha^2 \|\nabla u_i\|^2$$



#### **FlowNet**

Estimate optical Flow in a data driven way

#### FlowNet: Learning Optical Flow with Convolutional Networks

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2015



### **FlowNet**

Estimate optical Flow in a data driven way

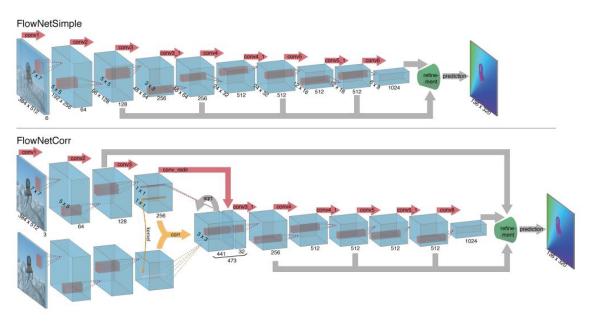
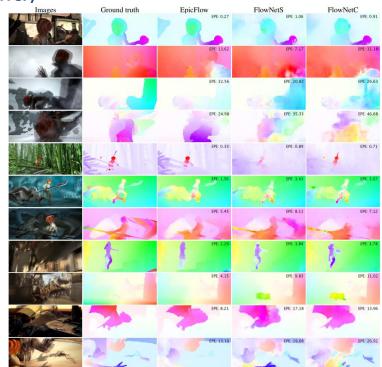


Figure 2. The two network architectures: FlowNetSimple (top) and FlowNetCorr (bottom).



### **FlowNet**

- Estimate optical Flow in a data driven way
- Synthetic training data
  - Ground truth flow
  - Real motion, not apparent motion



#### FlowNet 2.0

#### FlowNet 2.0: Evolution of Optical Flow Estimation with Deep Networks

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#### Abstract

The FlowNet demonstrated that optical flow estimation can be cast as a learning problem. However, the state of the art with regard to the quality of the flow has still been defined by traditional methods. Particularly on small displacements and real-world data, FlowNet cannot compete with variational methods. In this paper, we advance the concept of end-to-end learning of optical flow and make it



6 Dec 2016



#### FlowNet 2.0 run iterative L-K warp & upsampl run iterative L-K Image 1 Image 1 Warped Warped FlowNetS FlowNetS FlowNetC Flow Image 2 Image 2 Magnitude arge Displacemen Flow Flow Displacement Image 2 Brightnes Brightnes Brightness Error Error Error Image 1 Flow FlowNet-SD Flow Image 1 Magnitude Small Displacement Flow Image 2 Brightness

Figure 2. Schematic view of complete architecture: To compute large displacement optical flow we combine multiple FlowNets. Braces indicate concatenation of inputs. *Brightness Error* is the difference between the first image and the second image warped with the previously estimated flow. To optimally deal with small displacements, we introduce smaller strides in the beginning and convolutions between upconvolutions into the FlowNetS architecture. Finally we apply a small fusion network to provide the final estimate.

Error



74

**Example application** 



https://www.youtube.com/watch?v=iOcXdGZUvSo



# **Example application**



https://www.youtube.com/watch?v=wZcBLc4ifuQ



## Rotating Snakes Akiyoshi Kitaoka

