

# Computer Vision

## (Summer Semester 2023)

### Lecture 9

### Stereo Vision

- Slides based on
  - Szeliski's book "Computer Vision: Algorithms and Applications":  
<http://szeliski.org/Book/>
  - Slides from Brown University:  
<https://cs.brown.edu/courses/csci1430/>
  - Slides from Elli Angelopoulou from summer term 2014:  
<https://www5.cs.fau.de/lectures/ss-14/computer-vision-cv/>



**3DV 2024**  
International Conference on 3D Vision  
Davos, Switzerland  
March 18-21, 2024

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#3DV2024



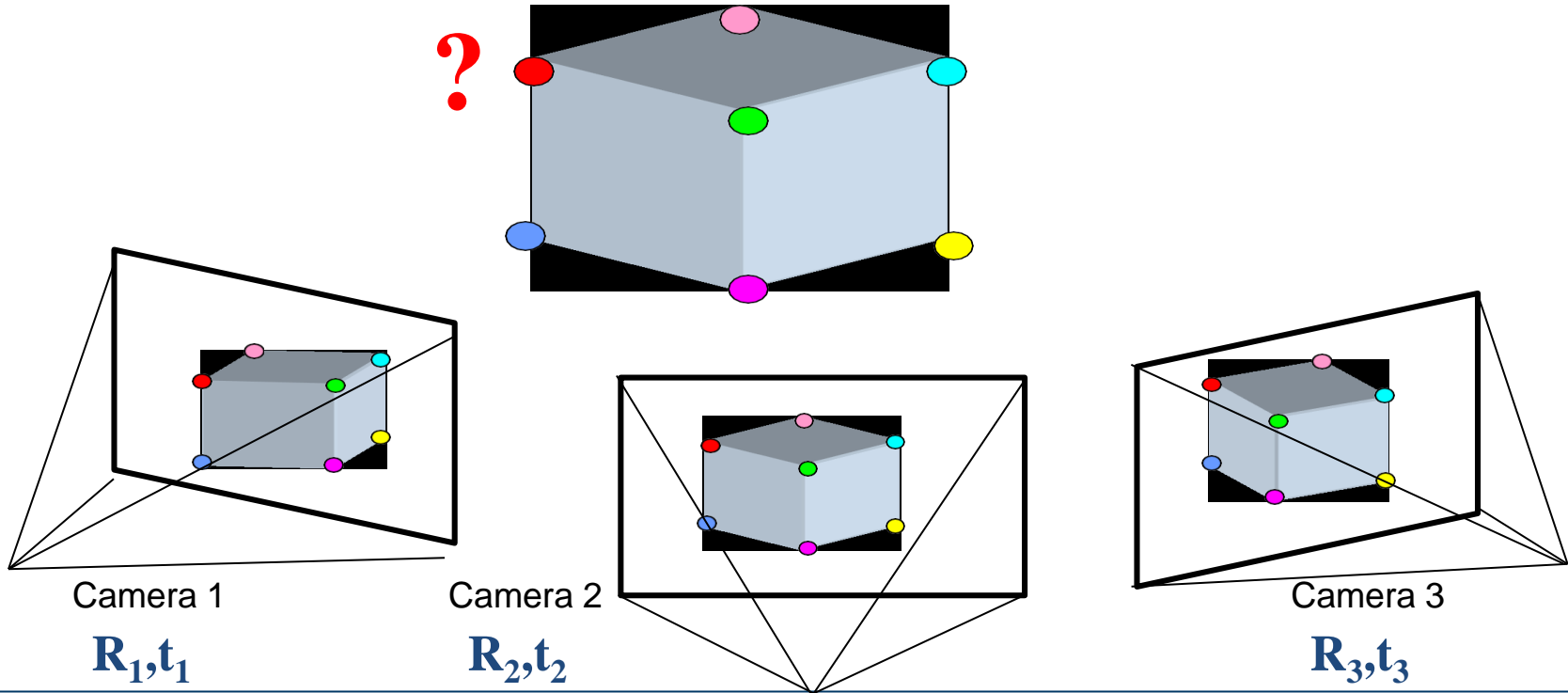
All Cool 3D Computer Vision Applications

Multiple View Geometry  
in computer vision  
SECOND EDITION  
Richard Hartley and Andrew Zisserman

5 44 189 13.5K

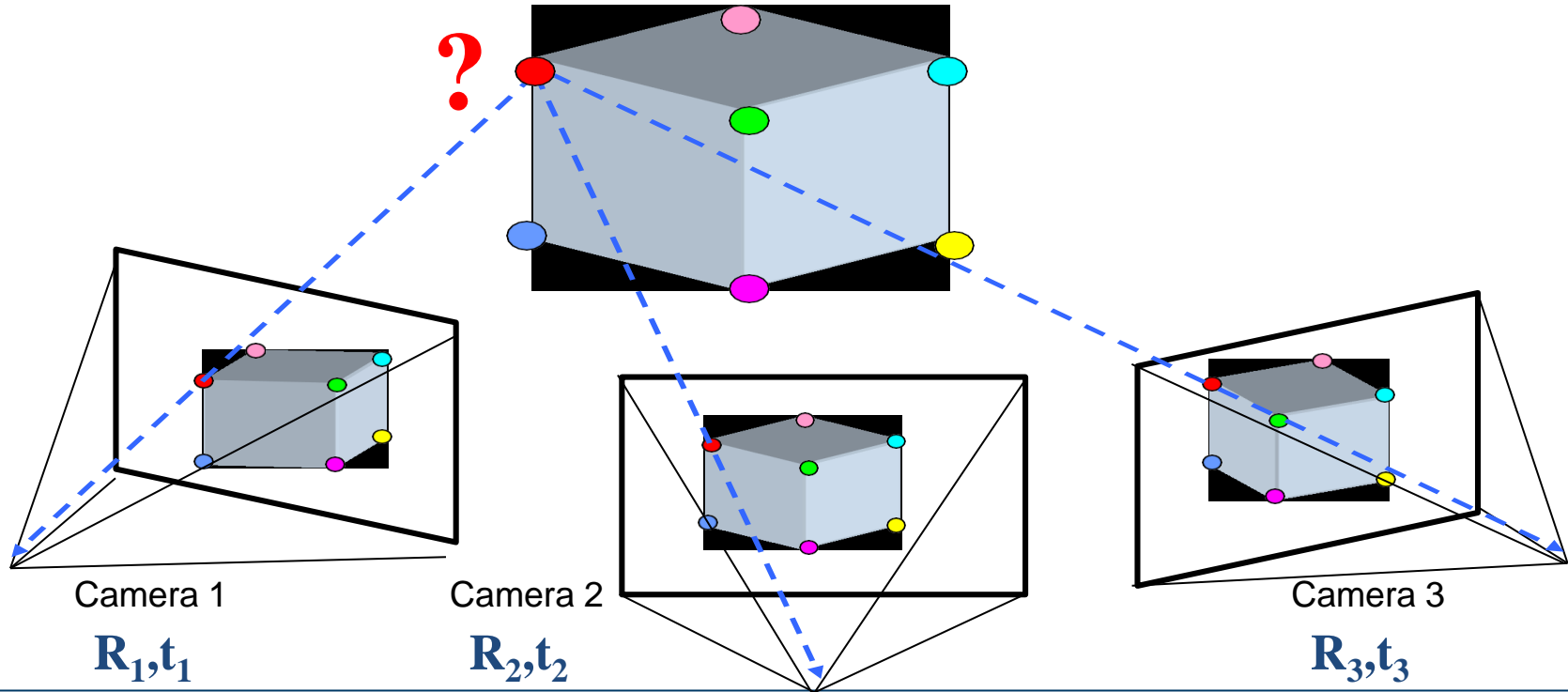
# Multi-view geometry problems

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point



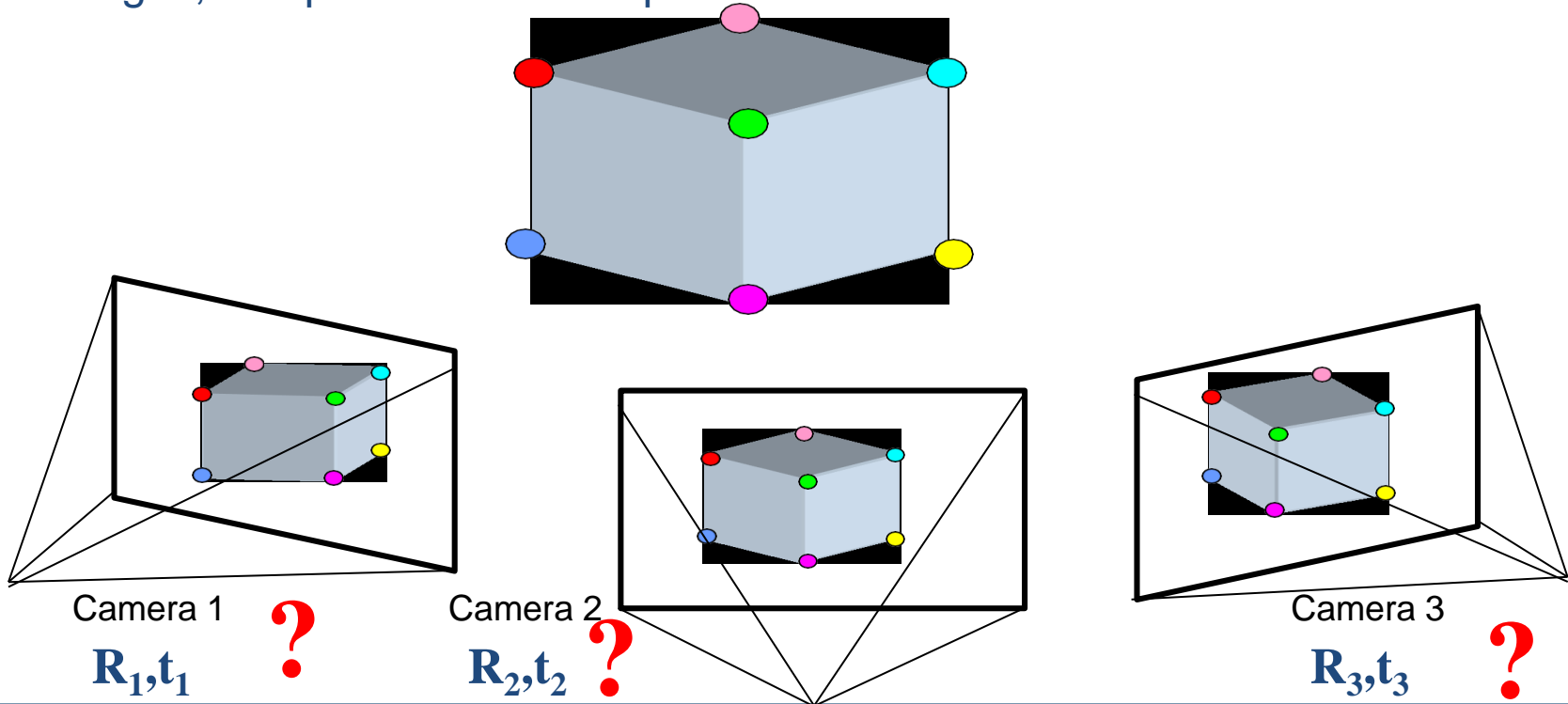
# Multi-view geometry problems

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point



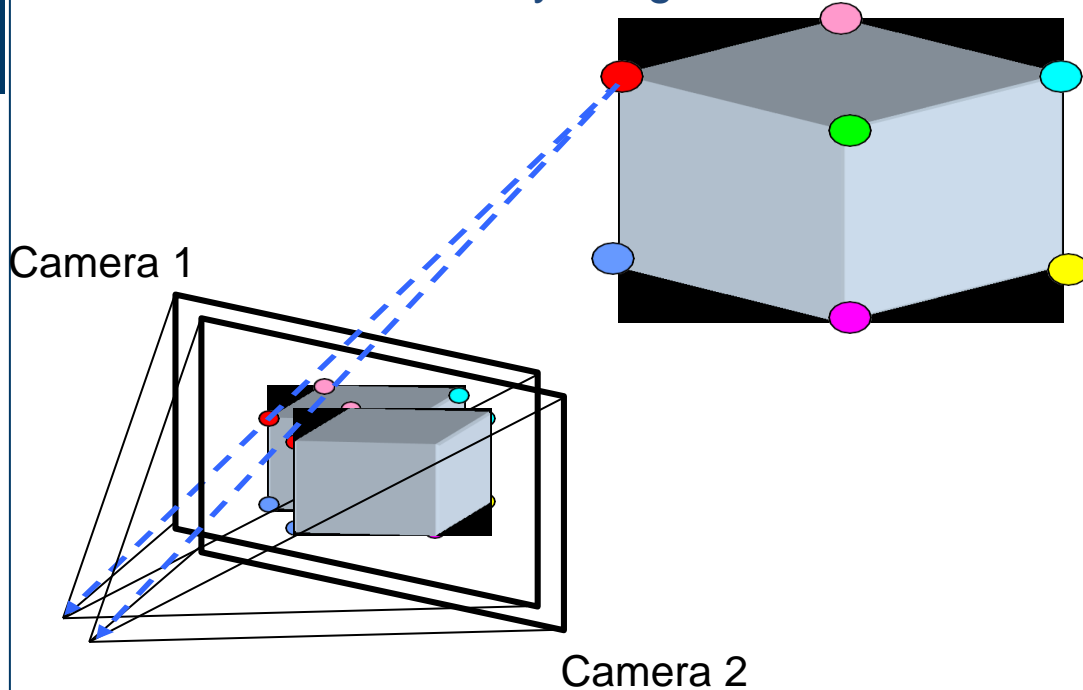
# Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters



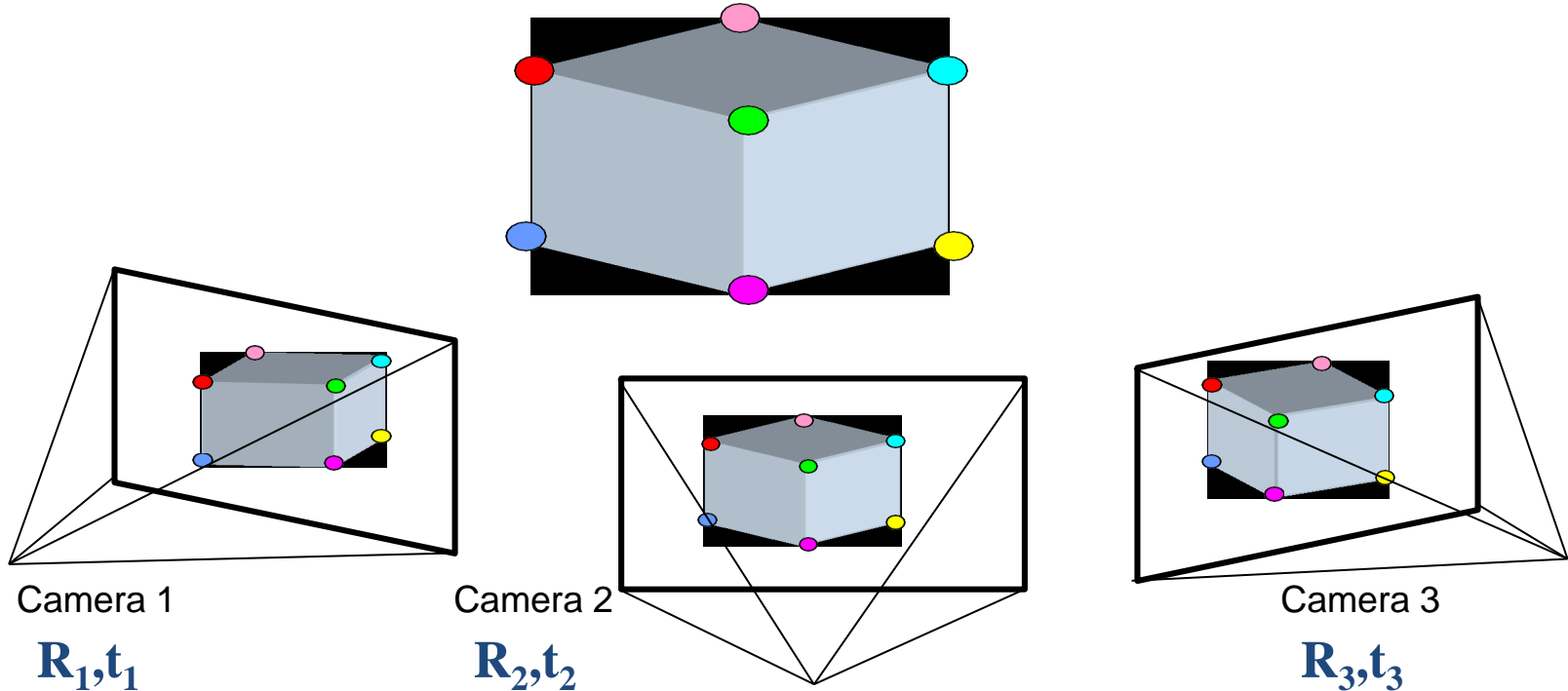
# Multi-view geometry problems

- **Optical flow:** Given two images, find the location of a world point in a second close-by image with no camera info.



# Multi-view geometry problems

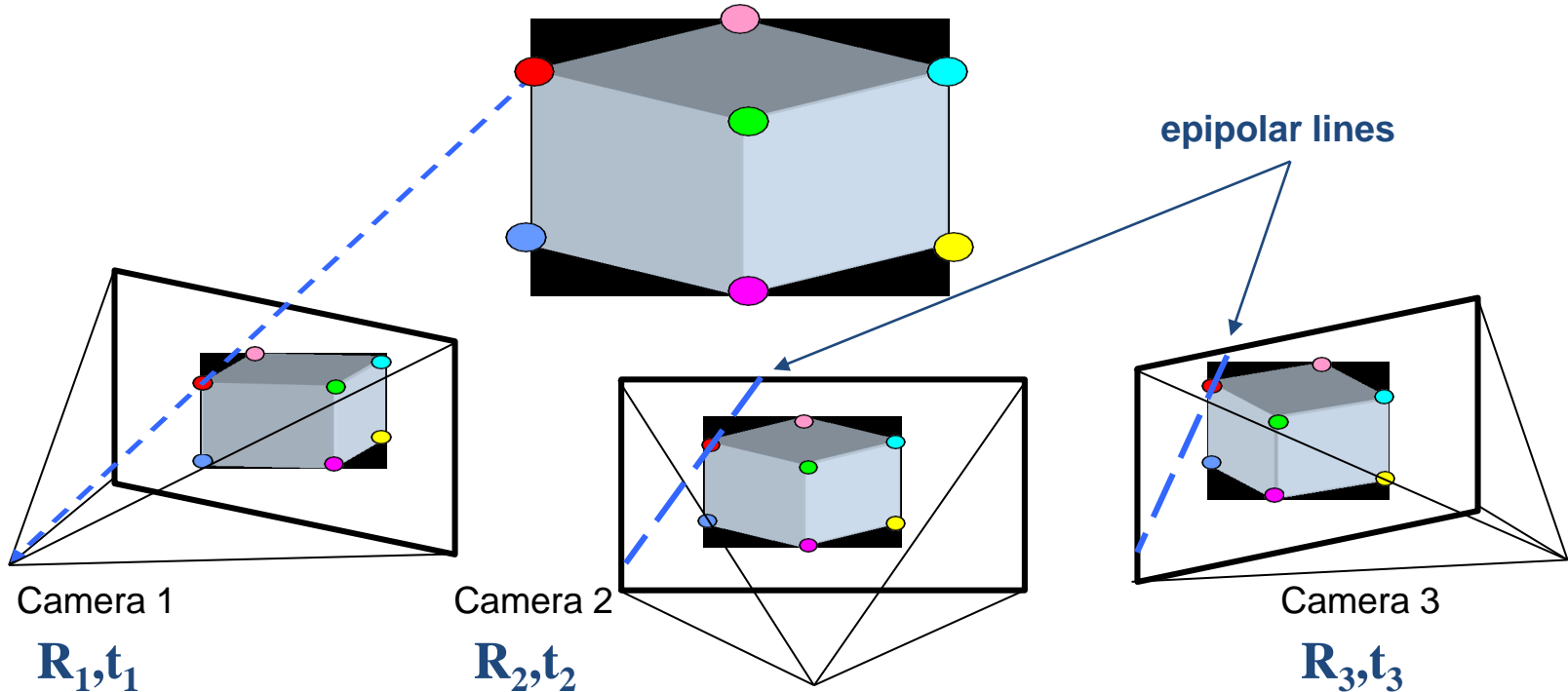
- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?





# Multi-view geometry problems

- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?



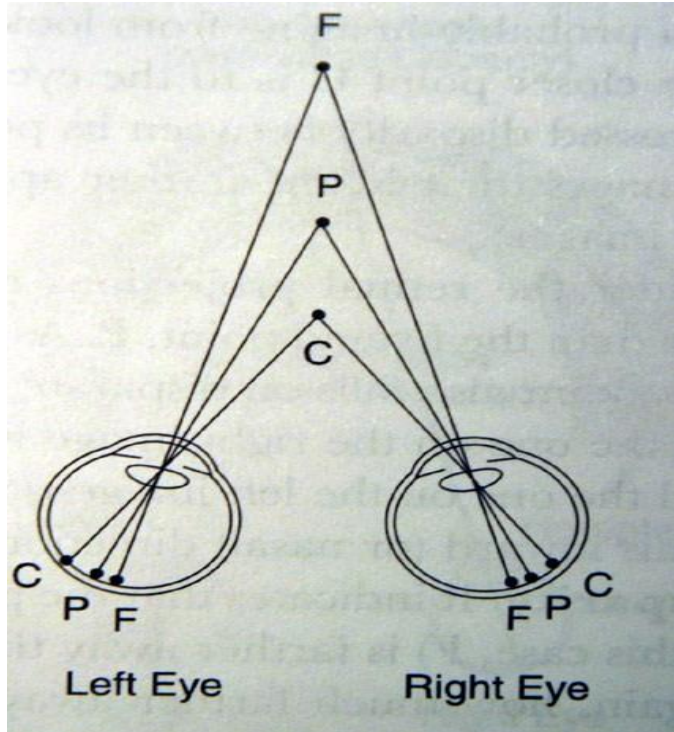
# Binocular Stereo Example



# Stereo Vision

- **Goal:** Infer information about the 3-D structure and distances of a scene from two or more images taken from different viewpoints.
- A stereo system must solve two subproblems:
  - Correspondence problem
  - Reconstruction
- *Correspondence Problem:* which pixel (point) on the left image and on the right image are projections of the same scene point.
- Once the point correspondence is established, we can compute the relative shift, the *disparity*, between the two projections.
- *Reconstruction:* The disparity data is then converted to a 3D map. In order to transform the disparity data to 3D measurements, we need some form of knowledge about the geometry of the stereo system.

# Depth from Binocular Disparity



**P** converging point

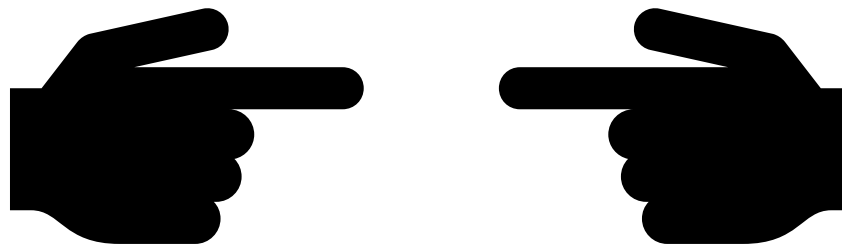
**C** object nearer  
projects to the  
outside of the **P**,  
disparity = +

**F** object farther  
projects to the  
inside of the **P**,  
disparity = -

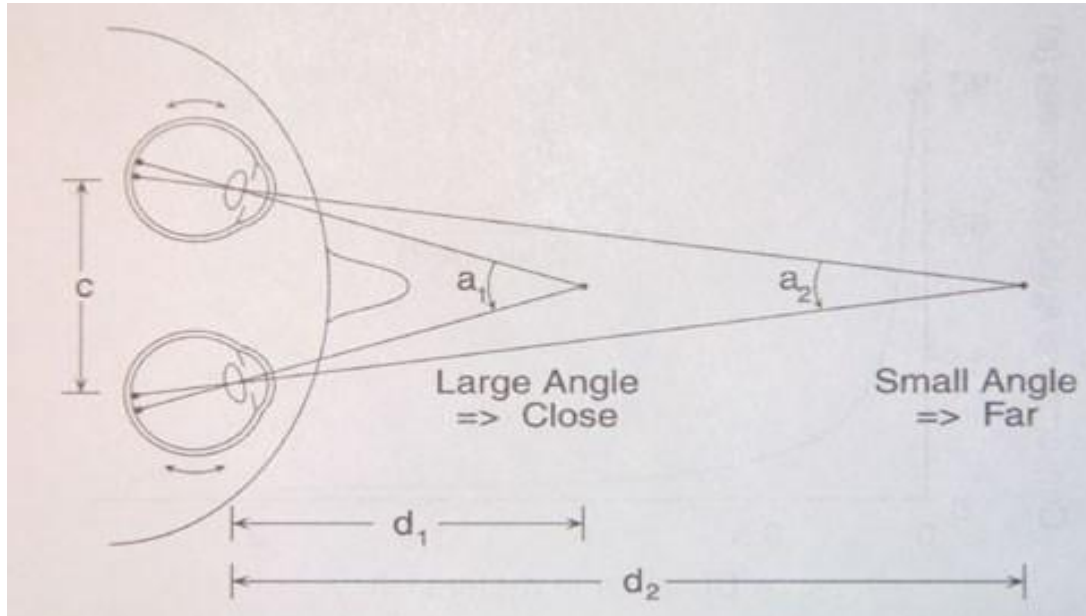
Sign and magnitude of disparity

# Disparity

**Put your hands in front of your face and look at a far object with one eye closed then the other**



# Depth from Convergence

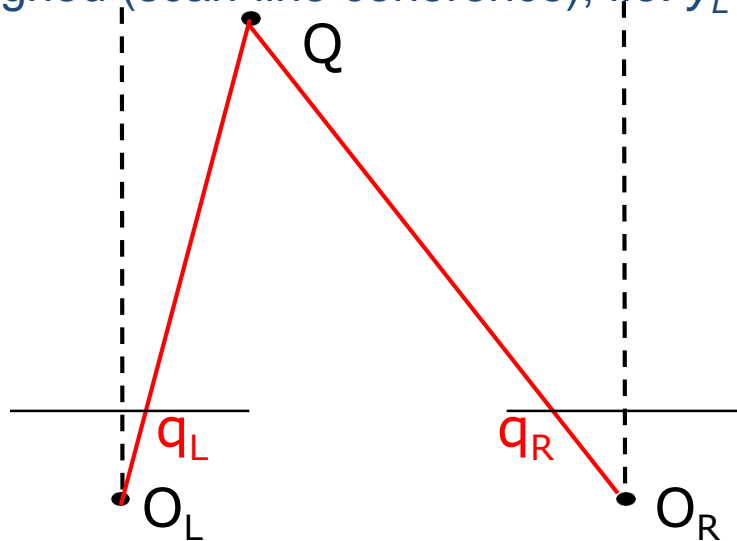


$$d = \frac{c}{2 \tan(a/2)}$$

Human performance: up to 2-2.5 meters

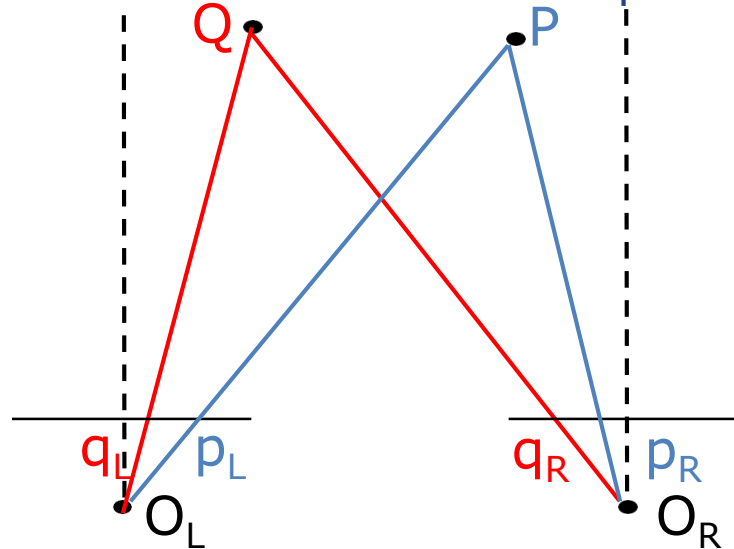
# Simple Binocular Stereo Setup

- Parallel optical axes, i.e. the fixation point (the point where the 2 optic axes intersect) is at infinity.
- Both image planes lie on the same plane.
- Their scan lines are aligned (scan-line coherence), i.e.  $y_L = y_R$ .



# Correspondence and Triangulation

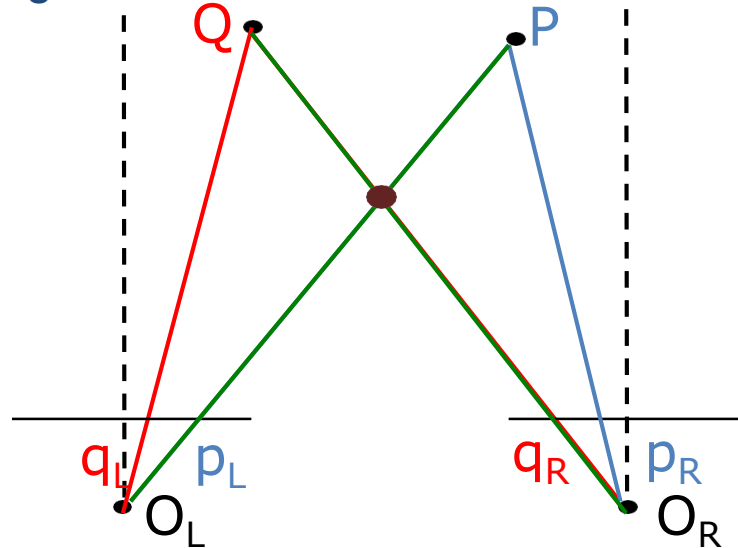
- When we correspond correctly (i.e.  $q_L$  with  $q_R$  and  $p_L$  with  $p_R$ ), the intersection of the corresponding rays gives the 3D location of scene point that generated the projections (i.e.  $Q$  and  $P$  accordingly).
- Only points lying on the same scanline can correspond





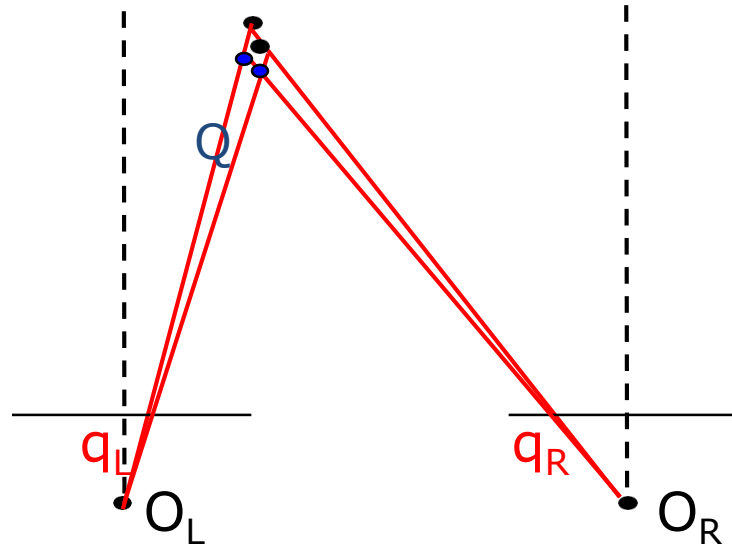
# Impact of Correspondence

- A mistake in correspondence, e.g.  $q_R$  is matched with  $p_L$ , will result in the intersection of rays that correspond to projections of distinct points (Q and P). As a result the wrong 3D location is recovered.

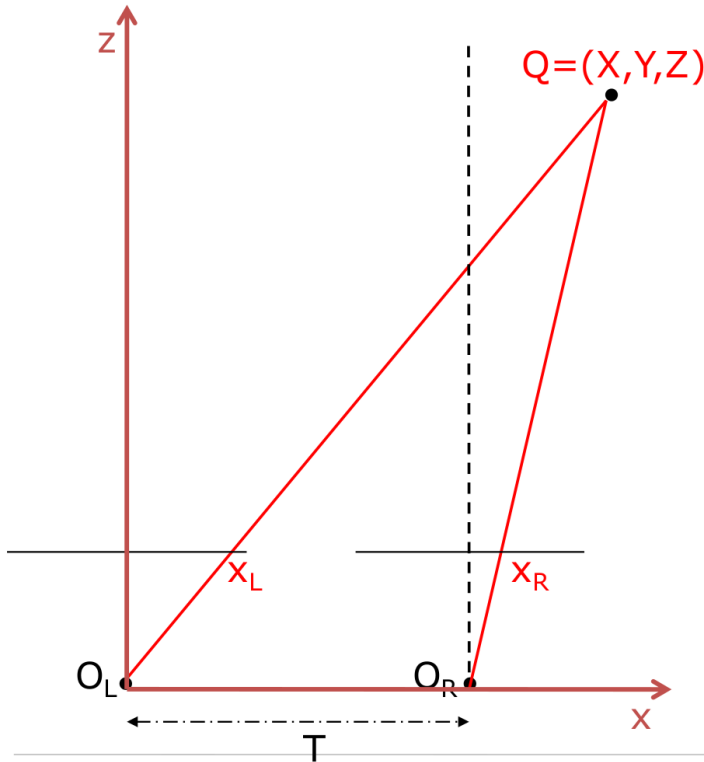


# Noise and Correspondence

- The noise in the image capture process (sensor noise, quantization, discretization) introduces inaccuracies in the projection rays that directly affect the triangulation process.

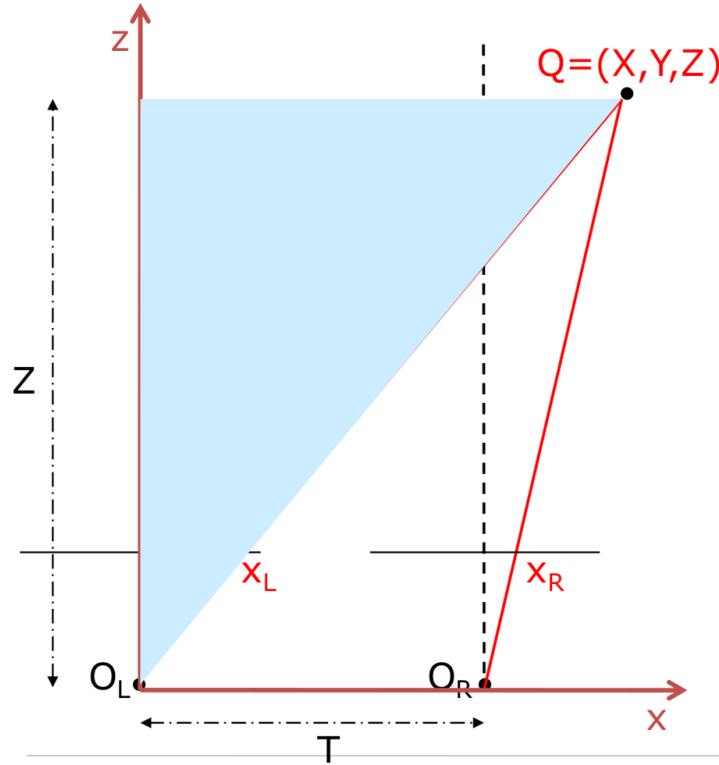


# Triangulation

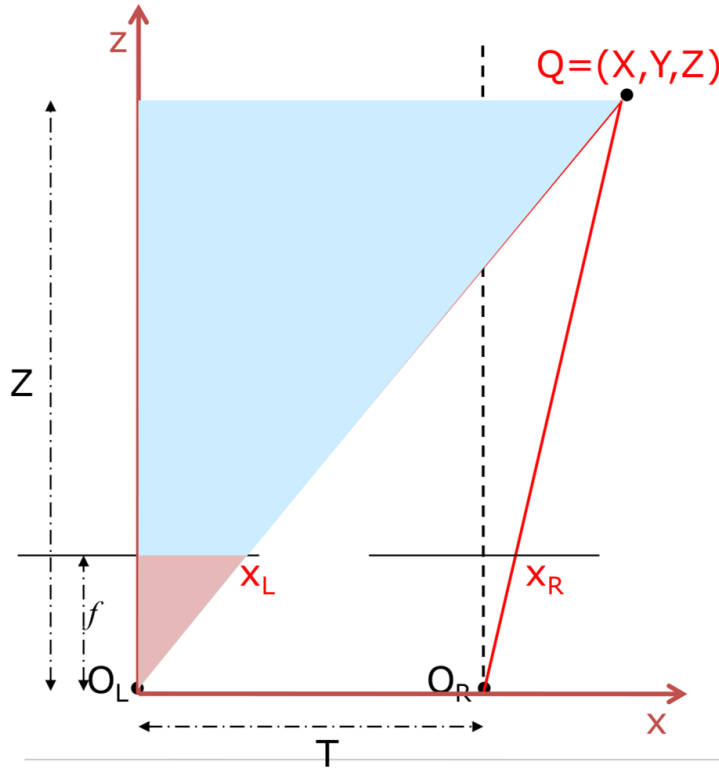


- Assume that the correspondence has been correctly established.
- Under the simple binocular setup (parallel optic axes and scan-line coherence), the only difference between the two projections  $q_L$  and  $q_R$  is in the  $x$ - component, i.e.  $x_L$  versus  $x_R$ .
- Let  $T$  be the **baseline**, i.e. the distance between the two COPs.

# Triangulation



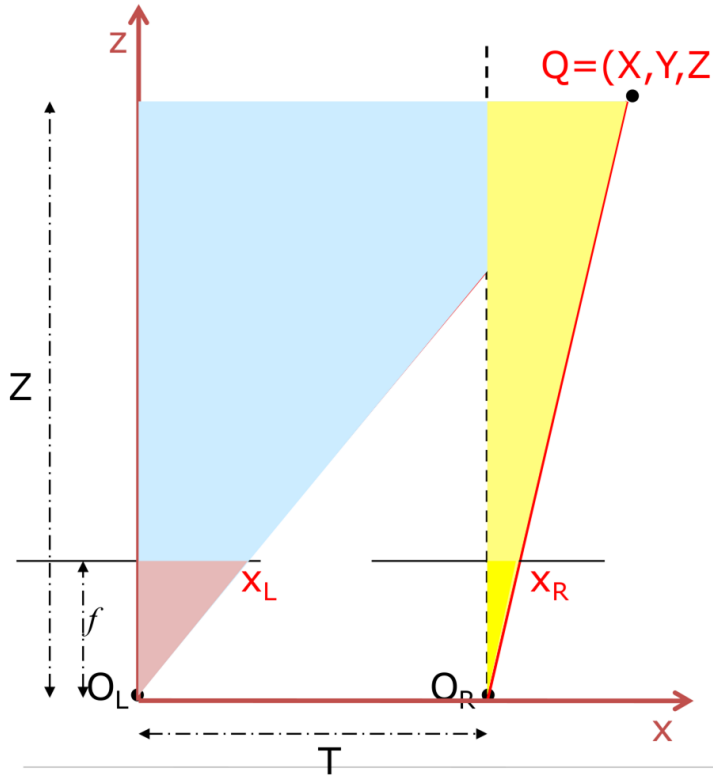
# Triangulation



From the similar triangles:

$$\frac{x_L}{f} = \frac{X}{Z} \Rightarrow X = x_L \frac{Z}{f}$$

# Triangulation



From the similar triangles:

$$\frac{x_L}{f} = \frac{X}{Z} \Rightarrow X = x_L \frac{Z}{f}$$

From the 2<sup>nd</sup> set of similar triangles:

$$\frac{x_R}{f} = \frac{X - T}{Z}$$

By replacing \$X\$ in the 2<sup>nd</sup> eq.:

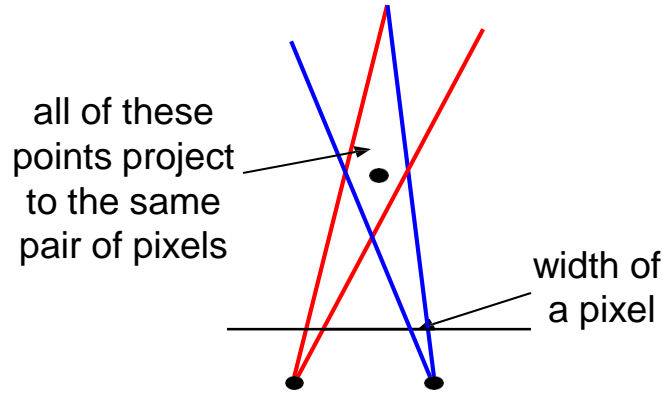
$$\frac{x_R}{f} = \frac{x_L \frac{Z}{f} - T}{Z} \Rightarrow x_R Z = x_L Z - fT$$

$$\Rightarrow Z = f \frac{T}{x_L - x_R} = f \frac{T}{d}$$

where  $d$  is the *disparity*:

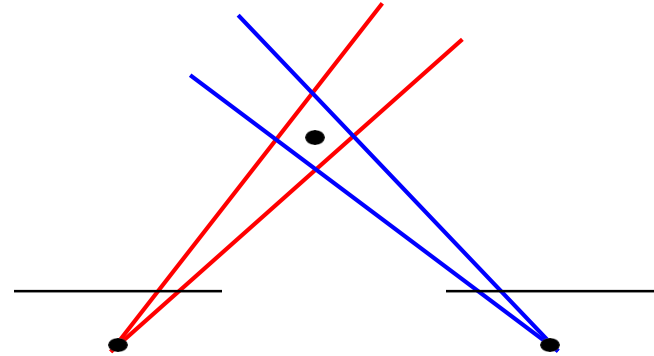
$$d = x_L - x_R$$

# Impact of Baseline



## Small Baseline

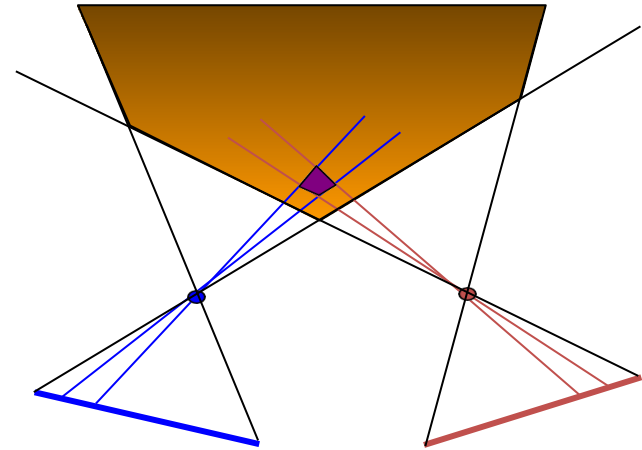
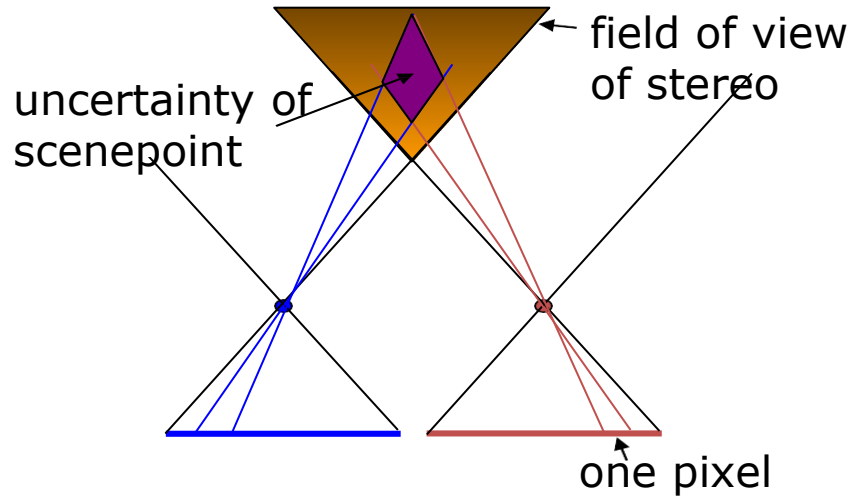
- What's the optimal baseline?
- Too small: large depth error
  - Too large: difficult search problem



## Large Baseline

- Appearance may change between the 2 viewpoints
- Decrease in the region of the scene that is mutually visible.

# Vergence



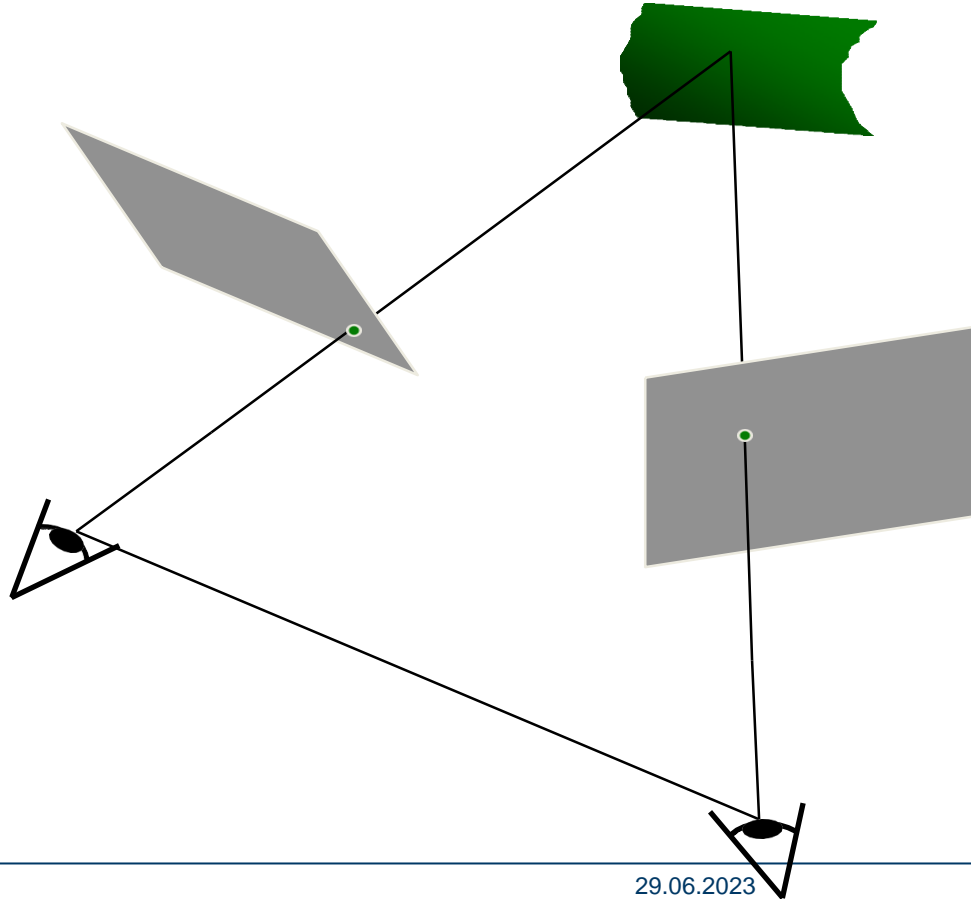
Optical axes of the two cameras need not be parallel

- Solution: Vergence (turn cameras towards each other)
  - Increases the field of view
  - Increases accuracy in the correspondence



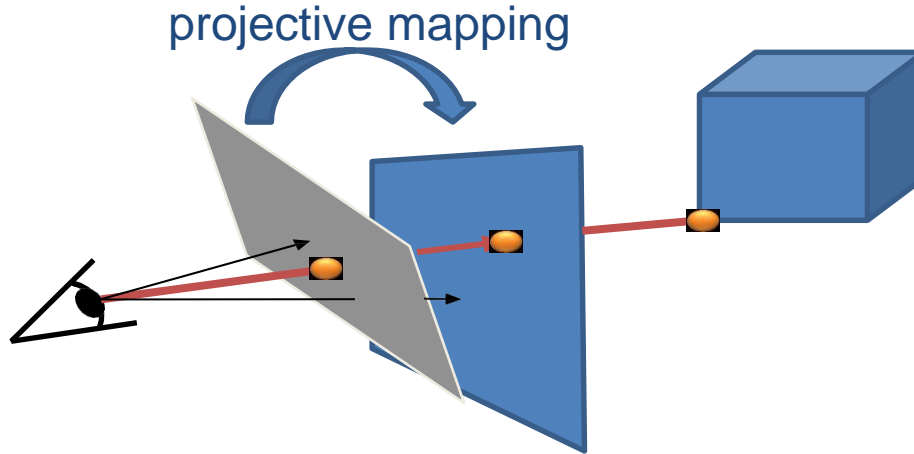
# Stereo Image Rectification

- So far we have assumed:
  - parallel optic axes
  - scan-line coherence
- Such a setup can lead to inaccuracies.
- More commonly cameras are *verged*, i.e. **the 2 optic axes intersect each other.**
- Can we use the same math?



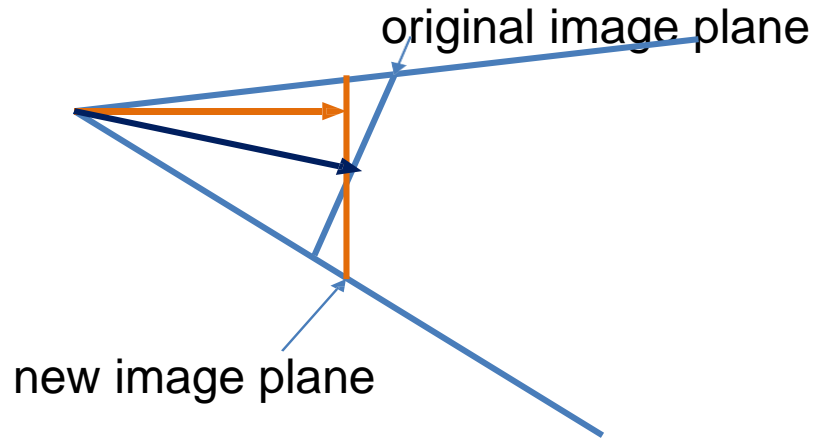
# Stereo Image Rectification

- Yes, with a simple trick: we can warp an image to a new image plane
- This warping is a projective mapping
- It assumes that the camera position does not change



# Stereo Image Rectification

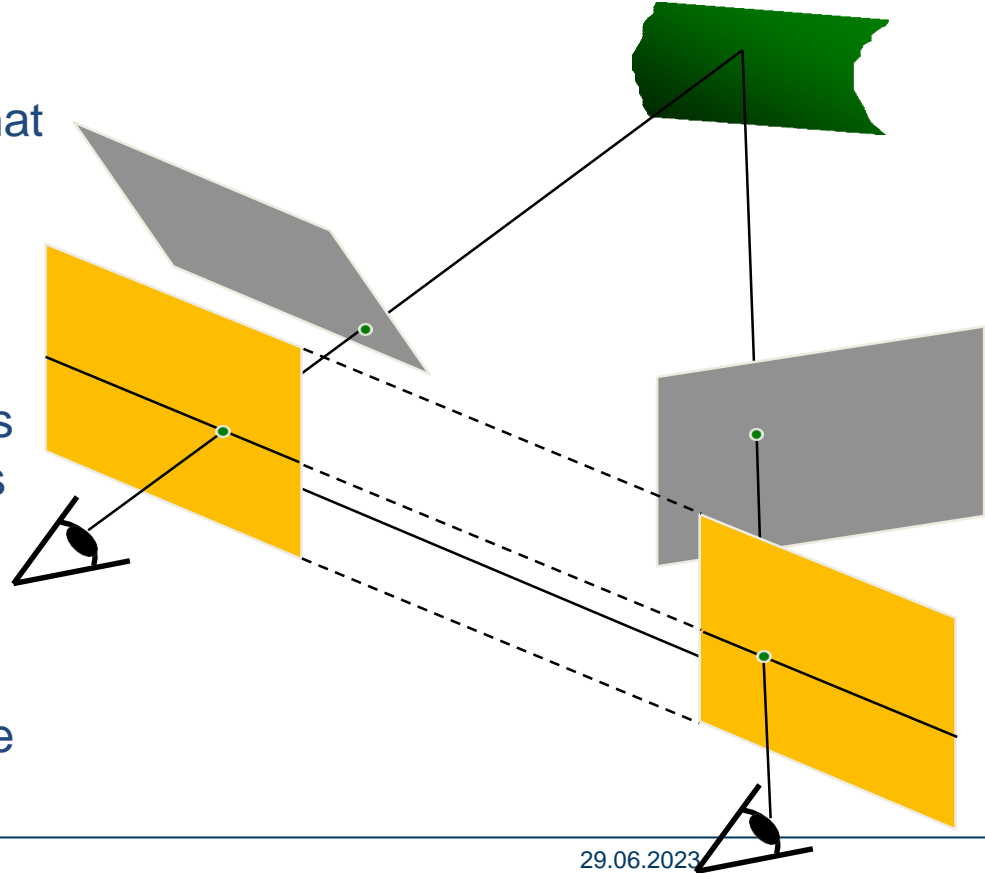
- 2D example



# Stereo Image Rectification

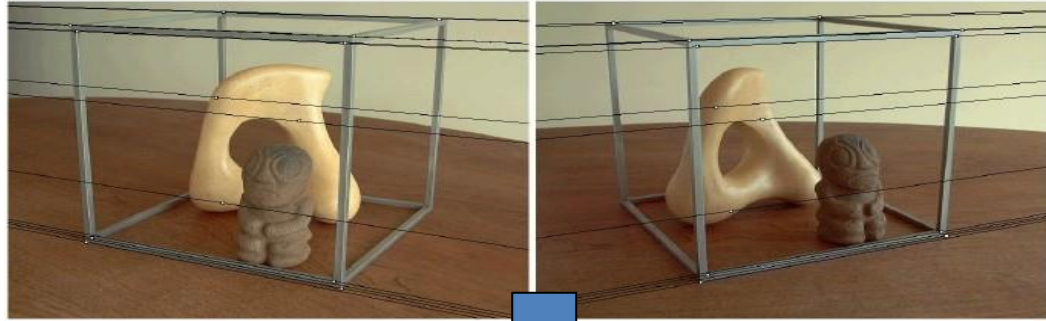
## Rectification

- Re-project the two images so that the previous simple setup is achieved
- Define a new image plane which is perpendicular on the plane spawned by the two cameras positions and the view directions
- Warp images to this plane, create two new images, which are now scan-line coherent.
- Do all the computations on these **rectified** images.



# Stereo Rectification Example

original images



after rectification:  
scan-line  
coherent



all point correspondences on the same scanline

# Correspondence Problem

- Assumptions:
    - Most scene points are visible from both viewpoints
    - Corresponding image regions look similar
  - It is a search problem: Given an element in the left image, search the right image to find the corresponding element.
  - Three underlying questions:
    - What do we match between the two images? (objects, edges, pixels, sets of pixels?)
    - What measure of similarity do we use?
    - Can we search in a systematic way?
- **we discussed the same problem already for optical flow !**

# MAGIC EYE

*A New Way of Looking at the World*



**3D Illusions by N.E. Thing Enterprises**

**SCHOLASTIC**

# Point Correspondence

1. Create an image of suitable size. Fill it with random dots. Duplicate the image.
2. Select a region in one image.
3. Shift this region horizontally by a small amount. The stereogram is complete.

To view the stereogram, focus on a point behind the image by a small amount until the two images "snap" together

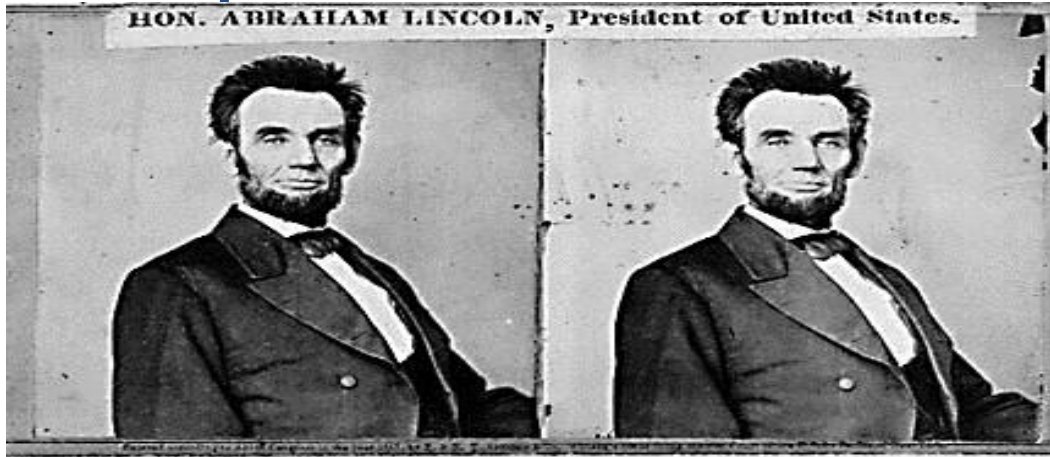


[https://en.wikipedia.org/wiki/Random\\_dot\\_stereogram](https://en.wikipedia.org/wiki/Random_dot_stereogram)

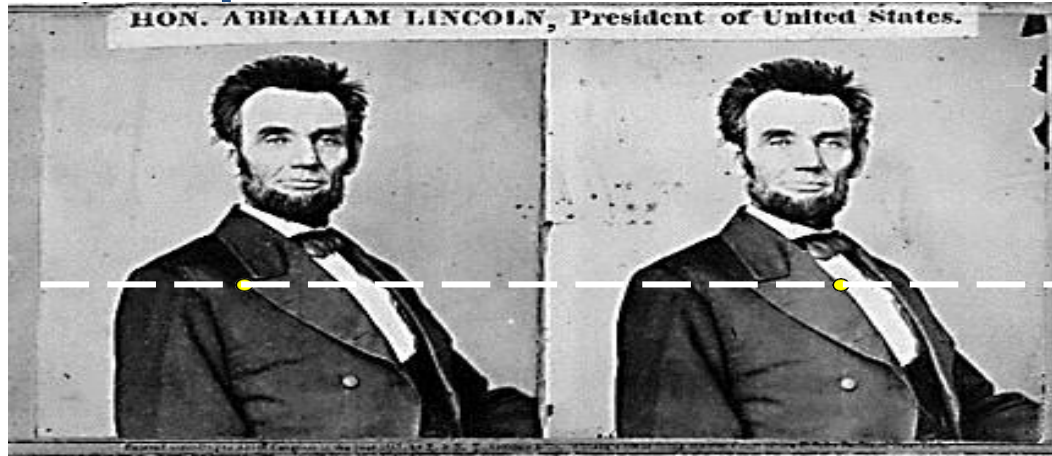
Julesz: had huge impact because it showed that recognition not needed for stereo.



# Point Correspondence in Practice



# Point Correspondence in Practice

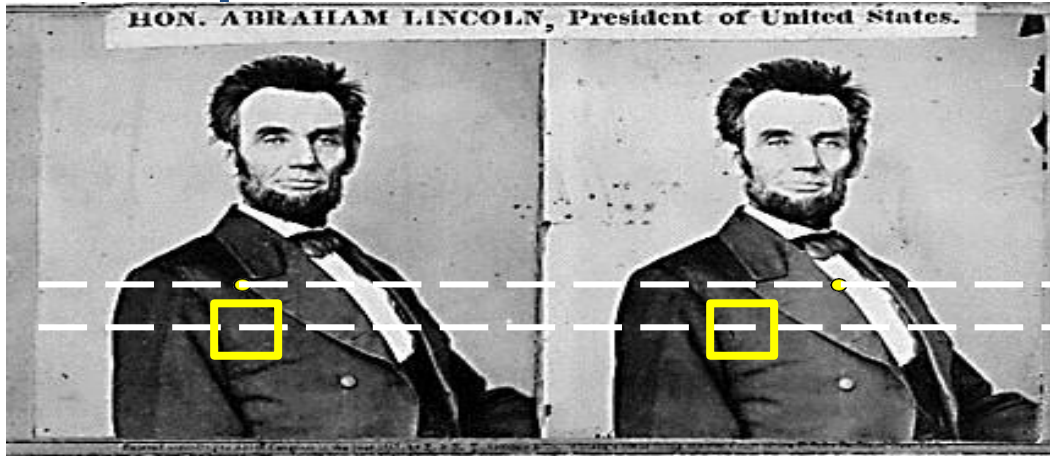


For each scan-line (special case of epipolar line)

For each pixel in the left image

- compare with every pixel on same epipolar line in right image

# Point Correspondence in Practice



For each scan-line (more properly epipolar line)

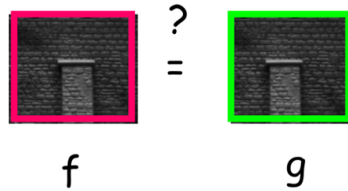
For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with closest intensity value (or more general minimum match cost).
- This will never work, so:

# Compare Regions around Points

- **Idea:** Compare intensity profiles around neighborhoods of potential points.
- Elements to be matched are now image windows of fixed size.
- The similarity measure is the correlation between windows in the two images.

# Similarity Metrics



$$SSD = \sum_{[i,j] \in R} (f(i,j) - g(i,j))^2$$

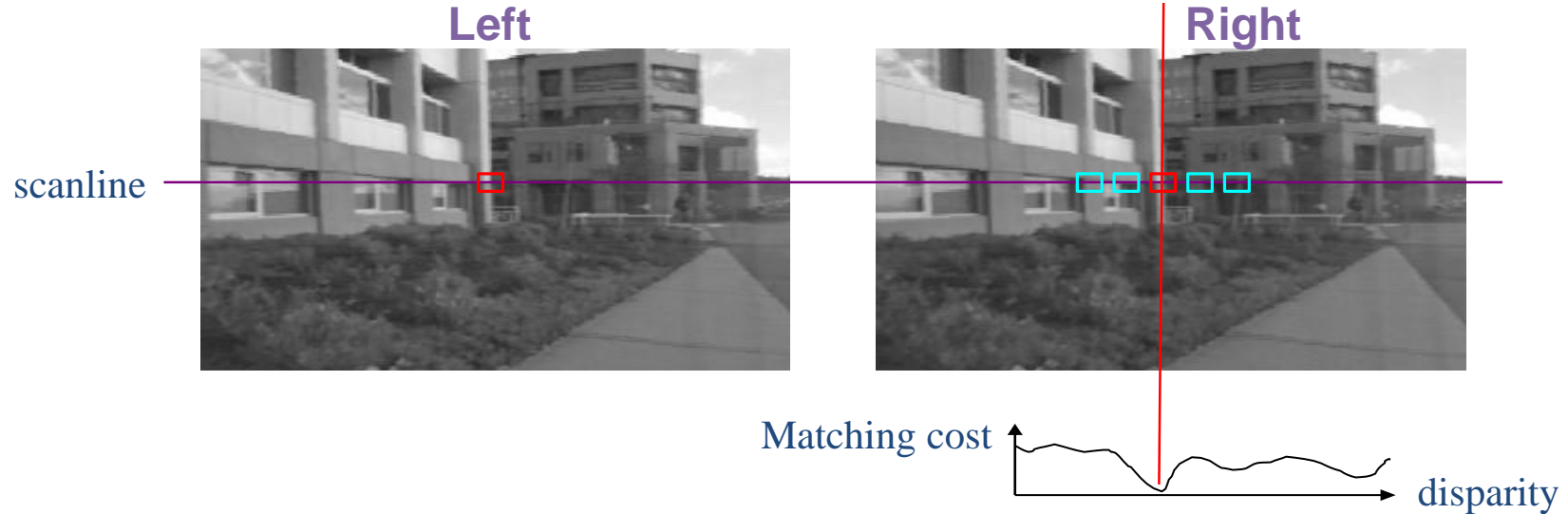
$$C_{fg} = \sum_{[i,j] \in R} f(i,j)g(i,j)$$

$$NC_{fg} = \frac{1}{n-1} \sum_{[i,j] \in R} \frac{(f(i,j) - \bar{f})(g(i,j) - \bar{g})}{\sigma_f \sigma_g}$$

Most  
popular

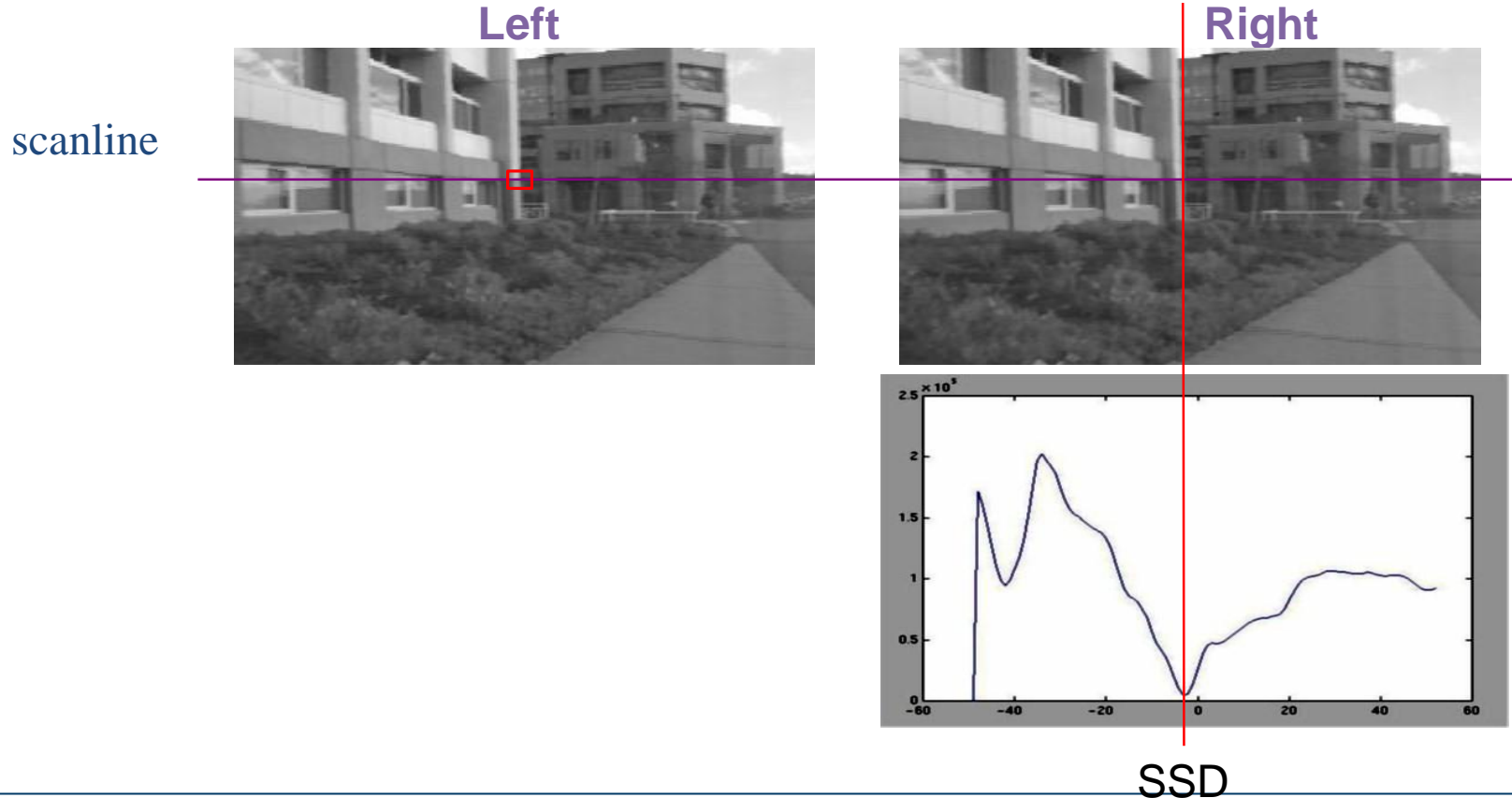
For each window, match to the closest window on the horizontal (epipolar) line

# Correspondence search with similarity constraint



- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: **SSD** or **normalized correlation**

# Correspondence search with similarity constraint

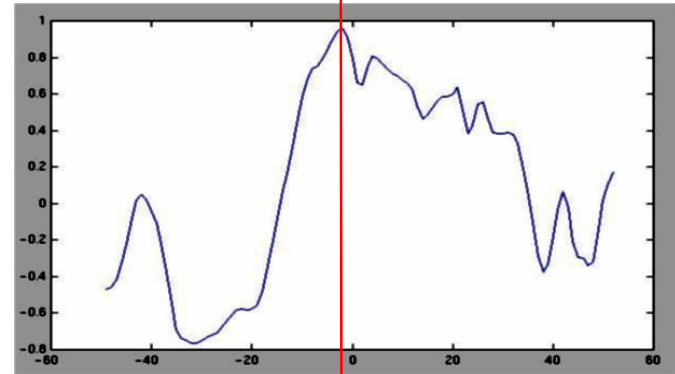


# Correspondence search with similarity constraint

Left

Right

scanline



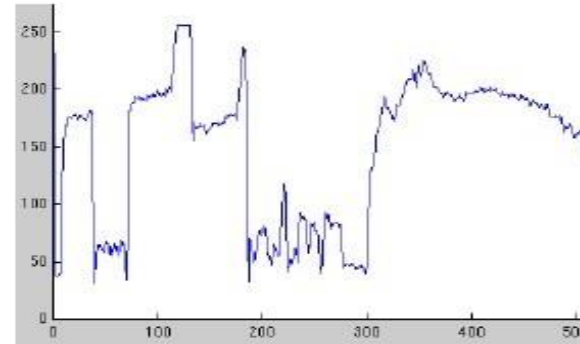
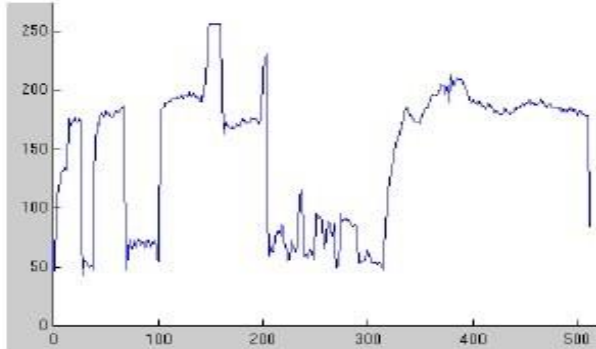
Norm. corr



# Correspondence problem



Intensity  
profiles



- Clear correspondence between intensities, but also noise and ambiguity

# Correlation-based window matching



left image band (x)

# Correlation-based window matching



left image band ( $x$ )

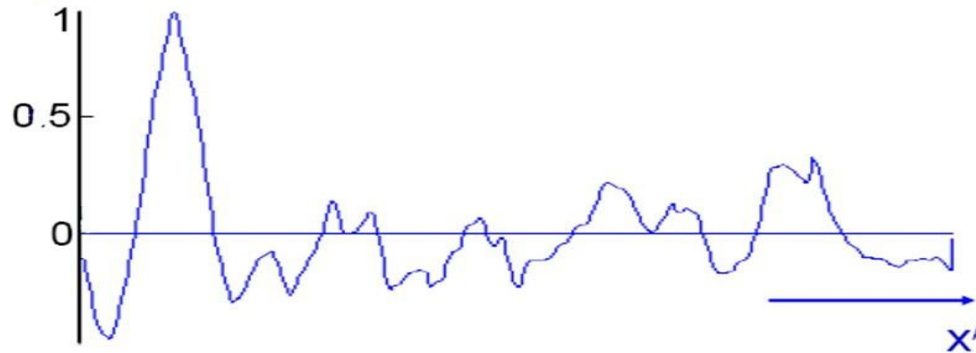
right image band ( $x'$ )

# Correlation-based window matching



left image band ( $x$ )

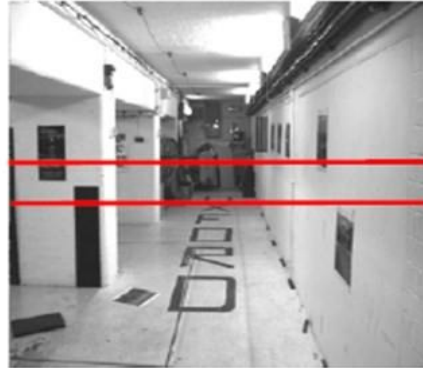
right image band ( $x'$ )



↑  
cross  
correlation

disparity =  $x' - x$

# Correlation-based window matching



target region

left image band ( $x$ )

right image band ( $x'$ )

# Correlation-based window matching



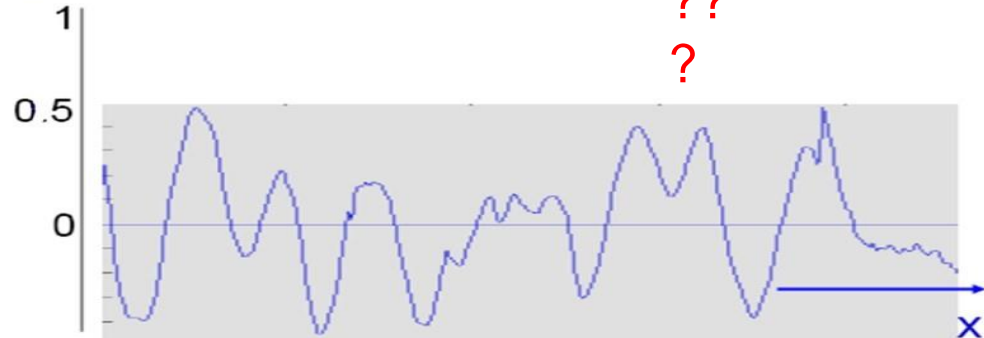
target region



left image band ( $x$ )

right image band ( $x'$ )

??  
?



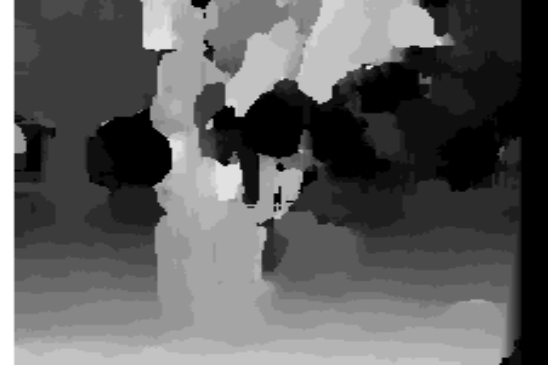
cross  
correlation

Textureless regions are non-distinct; high ambiguity for matches.

# Window Size



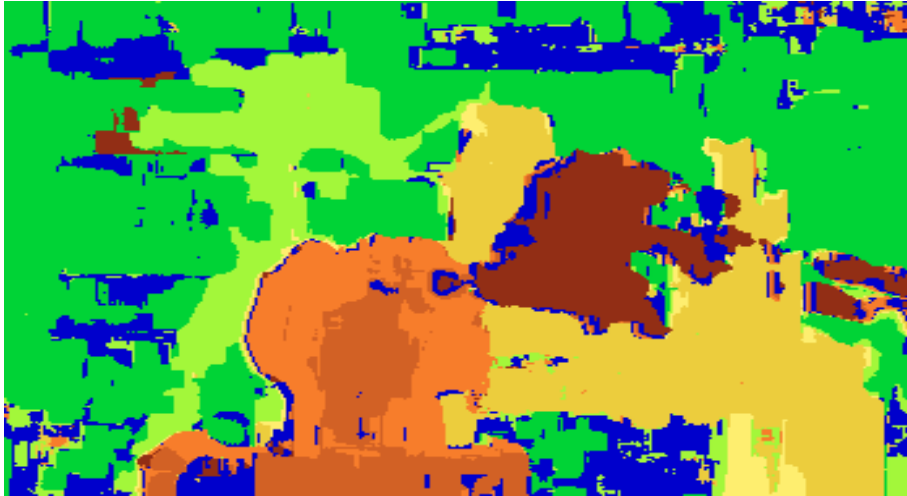
$W = 3$



$W = 20$

- Smaller window: more detail, more noise.
- Larger window: less noise, less detail
- Better results with adaptive window size

# Results with window search



Window-based matching  
(best window size)



‘Ground truth’

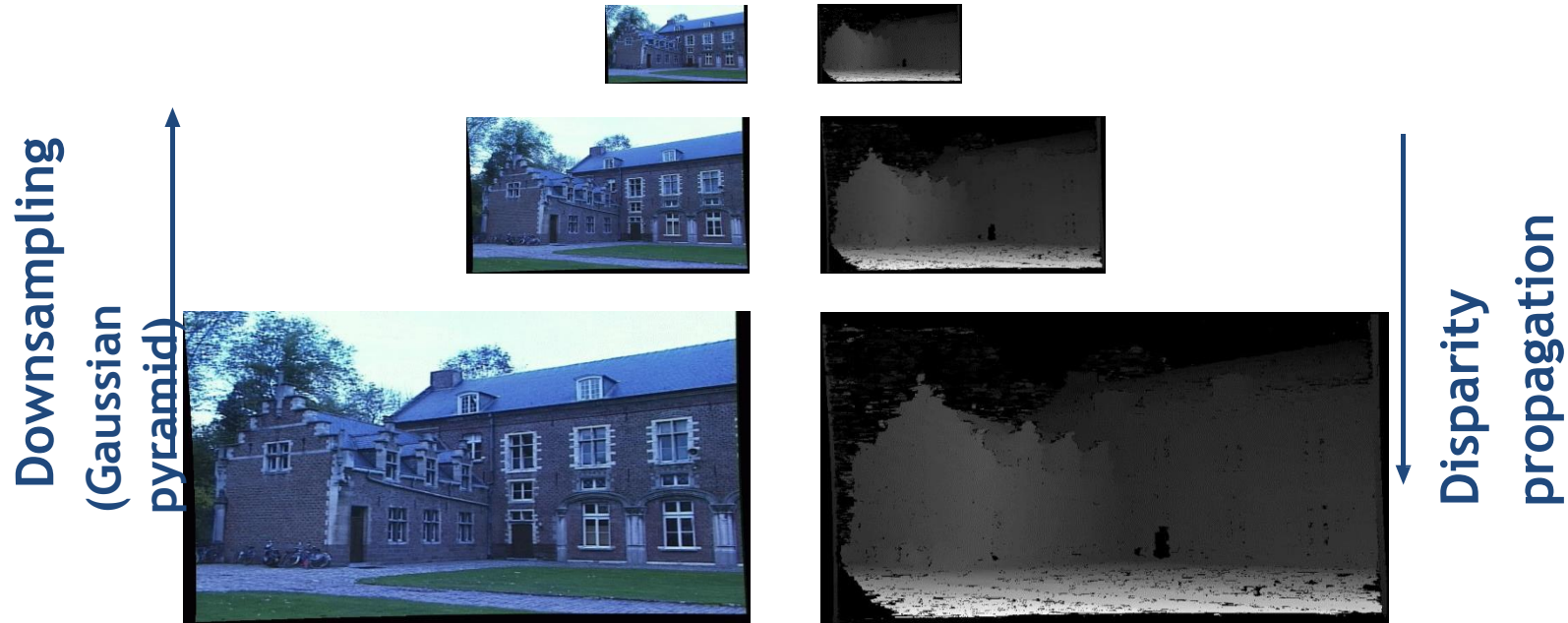


# Better solutions

- Beyond individual correspondences to estimate disparities:
- Optimize correspondence assignments jointly
  - Scanline at a time (DP)
  - Full 2D grid (graph cuts)

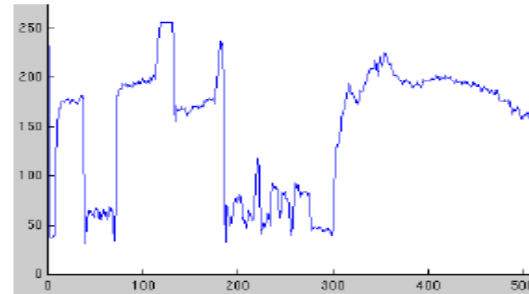
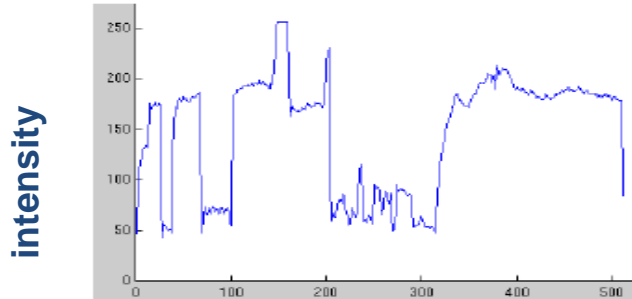
# Hierarchical Correspondence

- Allows faster computation
- Can handle large disparity ranges

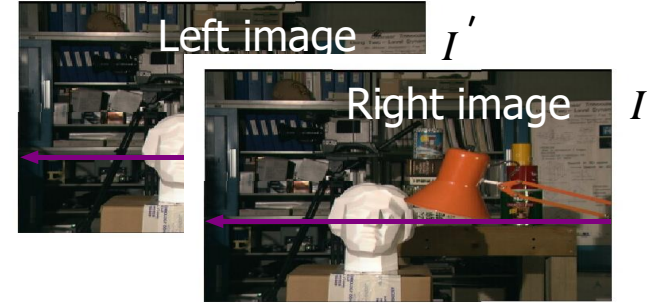
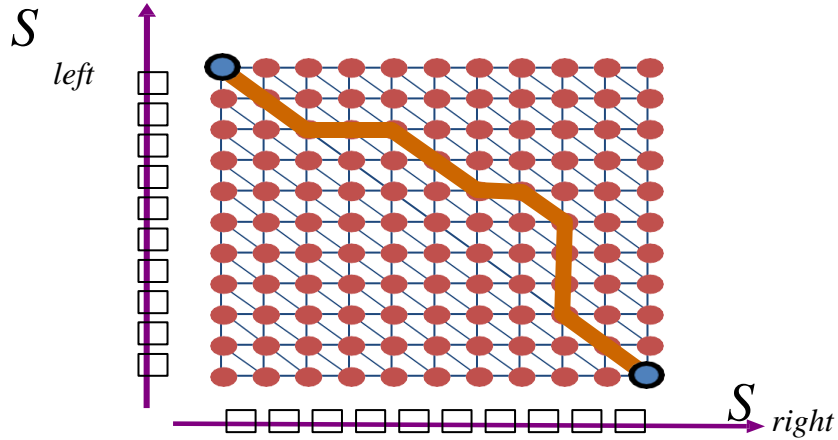


# Scanline stereo

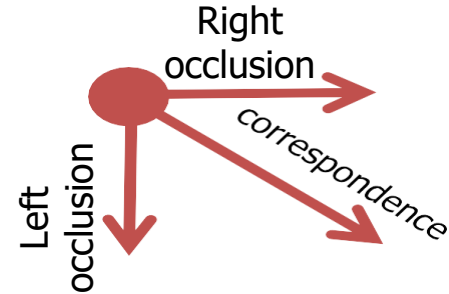
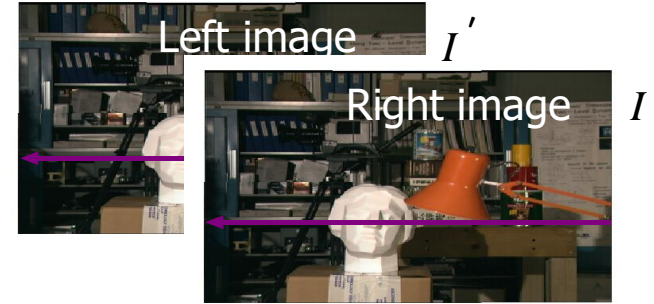
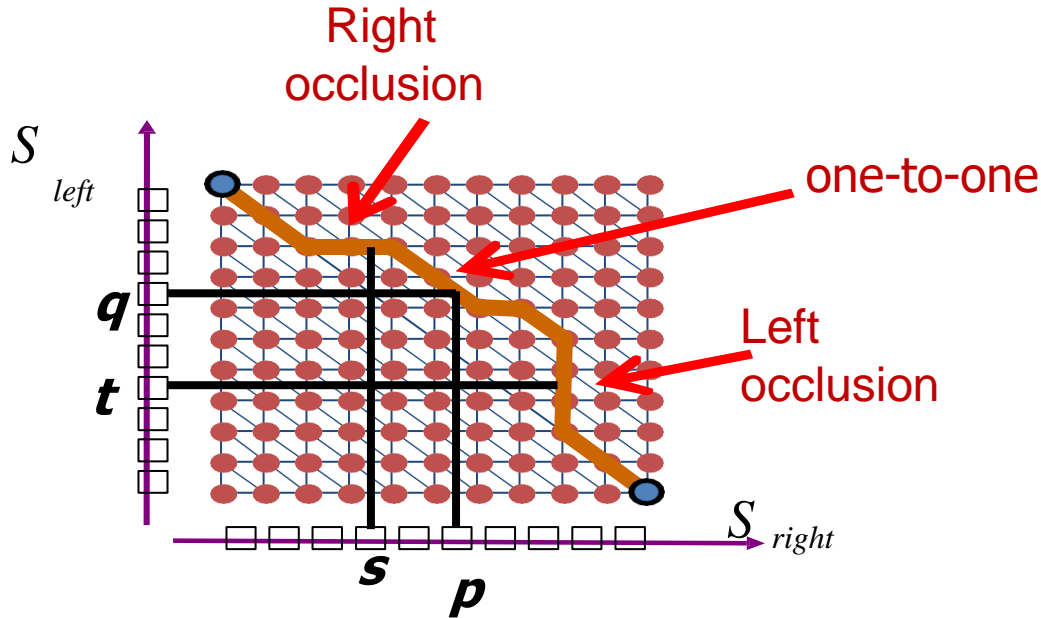
- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently



# “Shortest paths” for scan-line stereo



# “Shortest paths” for scan-line stereo

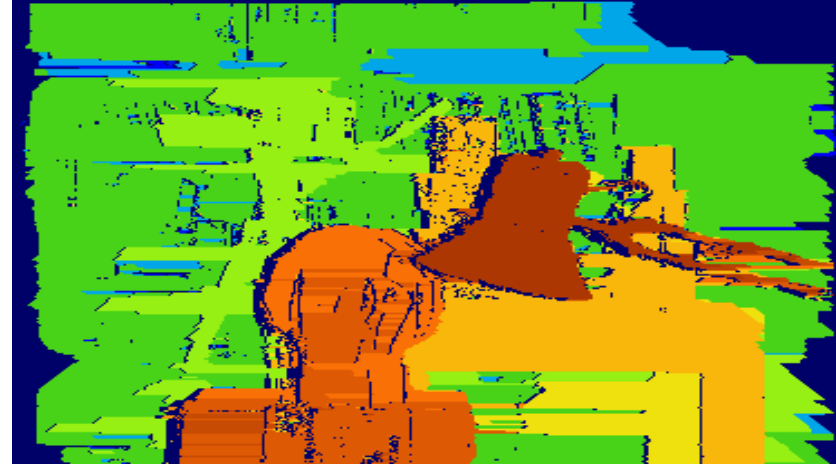


Can be implemented with dynamic programming

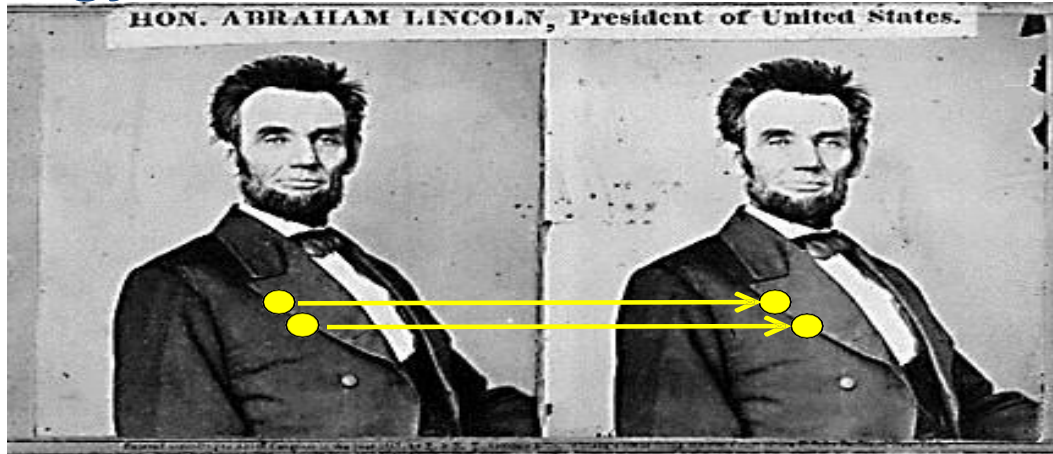
Ohta & Kanade '85, Cox et al. '96, Intille & Bobick, '01

# Coherent stereo on 2D grid

- Scanline stereo generates streaking artifacts
- Can't use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid
- E.g. look at Hirschmüller et al.: “Accurate and efficient stereo processing by semi-global matching and **mutual information**”

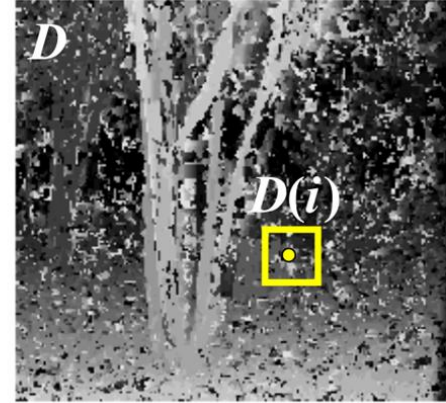
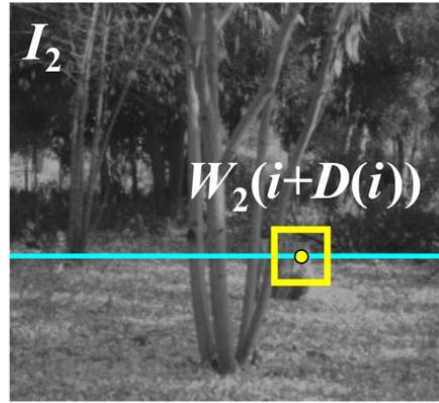
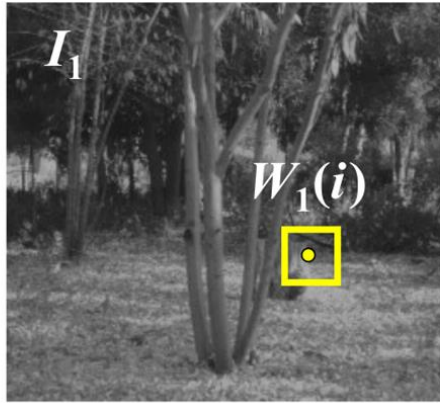


# Stereo as energy minimization



- What defines a good stereo correspondence?
  1. Match quality
    - Want each pixel to find a good match in the other image
  2. Smoothness
    - If two pixels are adjacent, they should (usually) move about the same amount

# Stereo matching as energy minimization



$$E = \alpha E_{\text{data}} (I_1, I_2, D) + \beta E_{\text{smooth}} (D)$$

$$E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2$$

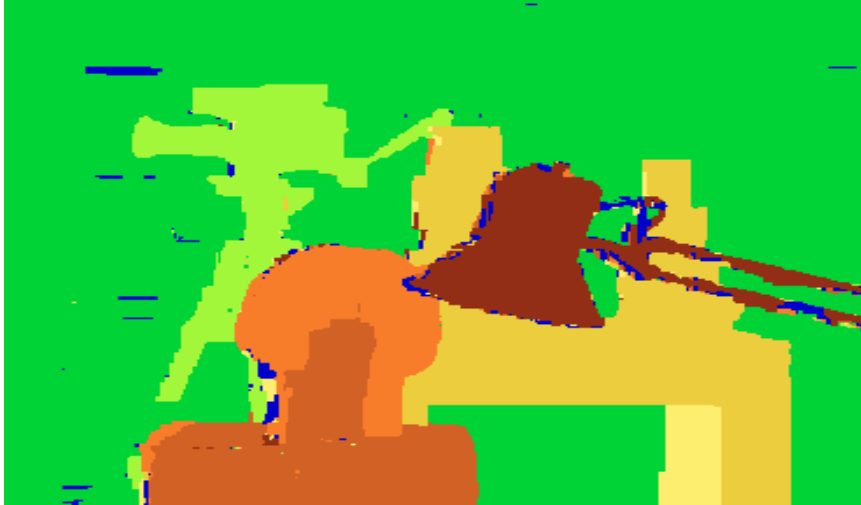
$$E_{\text{smooth}} = \sum_{\text{neighbors } i, j} \rho(D(i) - D(j))$$

Energy functions of this form can be minimized using *graph cuts*.

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001



## Better results...



Graph cut method

Boykov et al., [\*Fast Approximate Energy Minimization via Graph Cuts\*](#),  
*International Conference on Computer Vision*, September 1999.



Ground truth

# Challenges

- Low-contrast 'textureless' image regions
- Occlusions
- Violations of brightness constancy
  - Specular reflections
- Really large baselines
  - Foreshortening and appearance change
- Camera calibration errors

# Image Sources

1. The slides on image rectification are courtesy of J. Chai,  
[http://faculty.cs.tamu.edu/jchai/cpsc641\\_spring10/lectures/lecture9.ppt](http://faculty.cs.tamu.edu/jchai/cpsc641_spring10/lectures/lecture9.ppt)
2. A number of slides in this presentation have been adapted by the presentation of S. Narasimhan,  
<http://www.cs.cmu.edu/afs/cs/academic/class/15385-s06/lectures/ppts/lec-14.ppt>
3. The Lincoln image is courtesy of S. Seitz.
4. The window-matching slide is courtesy of O. Camps.
5. The example slide on hierarchical correspondence algorithms is courtesy of ETH,  
<http://www.inf.ethz.ch/personal/pomarc/courses/gcv/class07.ppt>



Suchen

ity Metrics

$f \quad ? \quad g$

$$SSD = \sum_{[i,j] \in R} (f(i,j) - g(i,j))^2$$

$$C_{fg} = \sum_{[i,j] \in R} f(i,j)g(i,j)$$

$$NC_{fg} = \frac{1}{n-1} \sum_{[i,j] \in R} \frac{(f(i,j) - \bar{f})(g(i,j) - \bar{g})}{\sigma_f \sigma_g}$$

indow, match to the closest window on the horizontal (epipolar) line in the other image.

Wiedergabe (k)

And the reason is pretty simple:  
Because it's hard.

2:35 / 8:33

FAU  
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Technische Fakultät

Empk

[ENG] Zukunftsdiagnose Science Slam: Make my points - Prof. Dr. Bernhard Egger



CRC 1483 Empkins

58 Abonnenten

Abonnieren



3



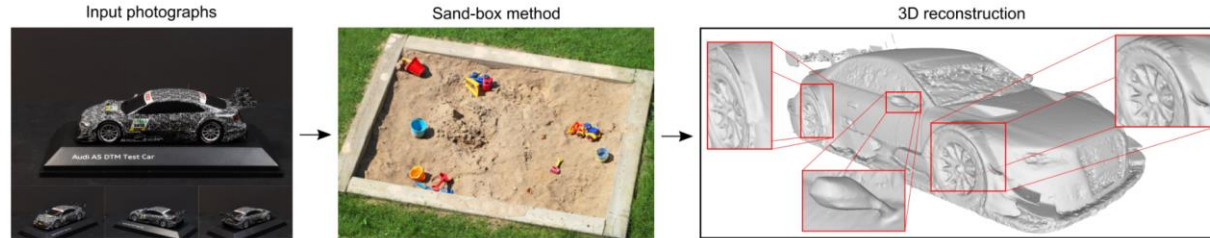
Teilen



# A Free Computer Vision Lesson for Car Manufacturers or It is Time to Retire the Erbkönig

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Oh no – we can recover very detailed 3D shape from Erbkönig photographs. How could this have happened? And even worse, the reconstruction is best in the parts where there is a pattern. Well, we guess someone didn't know about the basics of computer vision.

## Abstract

In award winning prior work [1], we identified the inability of autonomous cars to honk as the key reason that they are not broadly deployed on our streets. In this work [2], however, we suggest that the core reason is the lack of most basic computer vision knowledge of car manufacturers. To hide their most fancy new cars they put a special camouflage pattern on their so called *Erbkönig* prototypes.

