24-677 Project 2 Vrishabh Kenkre

1) Given -

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -4C_\alpha/mV_x & 4C_\alpha/m & -2C_\alpha(l_f - l_r)/mV_x \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2C_\alpha(l_f - l_r)}{I_z V_x} & \frac{2C_\alpha(l_f - l_r)}{I_z} & \frac{-2C_\alpha(l_f^2 + l_r^2)}{I_z V_x} \end{bmatrix}$$

$$\cdot \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

Also given -

$$m = 1888.6$$
$$C_\alpha = 20{,}000$$
$$l_f = 1.55$$
$$l_r = 1.39$$
$$I_z = 25854$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-42.3594}{V_x} & 42.3594 & \frac{-3.3888}{V_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.2475}{V_x} & 0.2475 & \frac{-6.7063}{V_x} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 21.1797 & 0 \\ 0 & 0 \\ 2.3981 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as it not given, we assume its identity.

$V_n = 2, 5, 8 \text{ m/s}$

Controllability = $P = [B \quad AB \quad A^2B \quad A^3B]$

Observability = $Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$

Using python (code attached below), I tested controllability & observability for all 3 values of V_n.

All of them had rank 4 — P & Q.

So the system is controllable & observable.

1.2] a) $\log(\sigma_1/\sigma_n)$ of controllability matrix approaches zero as we increase the speed. This means that the system is becoming more & more controllable.

Since min singular value is increasing, it gives higher chance of matrix being full rank.

b) As the speed increases from 1 m/s to 40 m/s, the real parts of the poles are moving towards zero. Hence, they are becoming more and more unstable.

And like after ~32 or 33 m/s, the values are greater than zero, meaning it is unstable.