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24-677

$$x_k = f(x_{k-1}, u_k) + w_k$$

$$x_k = \begin{bmatrix} X_{k-1} + \delta_t (\dot{x}_{k-1} \cos\psi - \dot{y}_{k-1} \sin t) \\ Y_{k-1} + \delta_t (\dot{x}_{k-1} \cdot \sin\psi_{k-1} + \dot{y}_{k-1} \cdot \cos\psi_{k-1}) \\ \psi_{k-1} + \delta_t \dot{\psi}_{k-1} \\ m^1_x \\ m^1_y \\ \vdots \\ m^n_x \\ m^n_y \end{bmatrix} + w_k$$

$$\Uparrow$$

$$f(x_{t-1}, u_t)$$

$$\frac{\partial f(x_{k-1}, u_t)}{\partial X_t} = F_t$$

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{x_{k-1}|k-1,\ u_k}$$

Now X_k = given = $\begin{bmatrix} x_k \\ y_k \\ \psi_k \\ m^1_x \\ m^1_y \\ \vdots \\ m^n_x \\ m^n_y \end{bmatrix}$

So $\dfrac{\partial f(x_{t-1}, u_t)}{\partial X_t} \Rightarrow$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial \psi} & \frac{\partial f_1}{\partial m^1_x} & \frac{\partial f_1}{\partial m^1_y} & \cdots \\ \vdots & & & & & \end{bmatrix}$$ ✓ Jacobian

$$
\begin{array}{c}
\begin{array}{cc} \overbrace{\hphantom{1 \quad 0 \quad -\delta_t(\dot{x}_{k-1}\sin\varphi_{k-1}+\dot{y}_{k-1}\cos\varphi_{k-1})}}^{3} & \overbrace{\hphantom{0 \; \cdots \; 0}}^{2n} \end{array} \\
\left[\begin{array}{ccc|cccc}
1 & 0 & -\delta_t\left(\dot{x}_{k-1}\sin\varphi_{k-1} + \dot{y}_{k-1}\cos\varphi_{k-1}\right) & 0 & \cdots & \cdots & \cdots \\
0 & 1 & \delta_t\left(\dot{x}_{k-1}\cos\varphi_{k-1} - \dot{y}_{k-1}\sin\varphi_{k-1}\right) & 0 & \cdots & \cdots & \cdots \\
0 & 0 & 1 & 0 & \cdots & \cdots & \cdots \\
0 & 0 & \cdots\cdots 0 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & 0 & 0 & 1 \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & & \ddots & \vdots \\
0 & 0 & \cdots\cdots 0 & 0 & \cdots & \cdots & 1
\end{array}\right]
\end{array}
\begin{array}{l} \\ \\ \\ \\ \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\} 2n \end{array}
$$

$$
\frac{\partial f(x_{k-1}, u_k)}{\partial x_t} = F_k = \begin{bmatrix} \begin{matrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{matrix} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n\times 2n} \end{bmatrix}
$$

$$
F_t \in \mathbb{R}^{(2n+3)\times(2n+3)}
$$

Now for $h(x_k) \to$

$$y_k = h(x_k) + v_k$$

$$y_k = \underbrace{\begin{bmatrix} \|m^1 - p_k\| \\ \vdots \\ \|m^n - p_k\| \\ \operatorname{atan2}(m^1_y - y_k,\ m^1_x - x_k) - \psi_t \\ \vdots \\ \operatorname{atan2}(m^n_y - y_n,\ m^n_x - x_n) - \psi_n \end{bmatrix}}_{h(x_t)} + v_t$$

$$H_t = \frac{\partial h(x_t)}{\partial x_t} \qquad H_t = \begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial \psi} & \frac{\partial h_1}{\partial m^1_x} & \cdots \\ \vdots & & & & \end{bmatrix}$$

Jacobian

$$\|m - p\| = \sqrt{(m_x - x)^2 + (m_y - y)^2}$$

So if diff^n wrt to x $\rightarrow$ $\dfrac{-(m_x^m - x)}{\sqrt{(m_x^m - x)^2 + (m_y^m - y)^2}}$

$a^m \leftarrow$

where $1 \le m \le n$

Similarly diff^n wrt y $\rightarrow$ $\dfrac{-(m_y^m - y)}{\sqrt{(m_y^m - y)^2 + (m_x^m - x)^2}}$

$b^m \leftarrow$

where $1 \le m \le n$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1 + x^2}$$

$$\frac{\partial}{\partial x} \left[\operatorname{arctan2}(m_x^i - x,\ m_y^i - y) \right)$$

$$= \frac{(m_x^m - x)^2}{(m_x^m - x)^2 + (m_y^m - y)^2} \times \left(\frac{m_y^m - y}{m_x^m - x} \right) \frac{\partial}{\partial x}$$

as $\frac{d}{dx}\left(\frac{1}{-x}\right) = \frac{1}{x^2}$

$$= \frac{m_y^m - y}{(m_x^m - x)^2 + (m_y^m - y)^2} \longrightarrow c^m$$

Similarly

$$\frac{\partial}{\partial y}\left(\operatorname{arctan2}(m'_x - x,\ m'_y - y)\right)$$

$$= \frac{(m^m_x - x)^2}{(m^m_x - x)^2 + (m^m_y - y)^2} \times \left(\frac{m^m_y - y}{m^m_x - x}\right)\frac{\partial}{\partial y}$$

$$= \frac{-(m^m_x - x)}{(m^m_x - x)^2 + (m^m_y - y)^2} \rightarrow d^m$$

Similarly, when we differentiate wrt m_x in

$$\|m - p\| \Rightarrow \frac{m^m_x - x}{\sqrt{(m^m_x - x)^2 + (m^m_y - y)^2}}$$

$$= -a^m$$

and when we differentiate wrt m^m_y -

$$-b_m = \frac{m^m_y - y}{\sqrt{(m^m_x - x)^2 + (m^m_y - y)^2}}$$

Similarly $\Rightarrow$ $-c_m =$ differentiation w.r.t m^m_x

$$= \frac{-(m^m_y - y)}{(m^m_x - x)^2 + (m^m_y - y)^2}$$

and $$-d_m = \frac{(m^m_x - x)}{(m^m_x - x)^2 + (m^m_y - y)^2}$$

So we use these a^m, b^m, c^m and d^m and substitute to make the H matrix.

$$
H = \begin{array}{c}
\\
n\left\{ \vphantom{\begin{matrix} a \\ \vdots \\ a \end{matrix}} \right. \\
n\left\{ \vphantom{\begin{matrix} a \\ \vdots \\ a \end{matrix}} \right.
\end{array}
\begin{array}{c}
\begin{array}{cc} \overbrace{\hspace{6em}}^{3} & \overbrace{\hspace{14em}}^{2n} \end{array} \\
\left[\begin{array}{ccc|ccccc}
a^1 & b^1 & 0 & -a^1 & -b^1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & 0 & 0 & \ddots & \vdots & \vdots \\
a^n & b^n & 0 & 0 & 0 & & -a^n & -b^n \\
c^1 & d^1 & -1 & -c^1 & -d^1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\
c^n & d^n & -1 & 0 & 0 & \cdots & -c^n & -d^n
\end{array}\right]
\end{array}
$$

$$H \in \mathbb{R}^{2n \times 2n+3}$$

H Matrix

Friday, 1 November 2024 1:19 PM

$$
\begin{bmatrix}
\frac{x_k - m_x^1}{\|m^1 - p_k\|} & \frac{y_k - m_y^1}{\|m^1 - p_k\|} & 0 & \frac{m_x^1 - x_k}{\|m^1 - p_k\|} & \frac{m_y^1 - y_k}{\|m^1 - p_k\|} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & & & \\
\frac{x_k - m_x^n}{\|m^n - p_k\|} & \frac{y_k - m_y^n}{\|m^n - p_k\|} & 0 & \cdots & & \cdots & \frac{m_x^n - x_k}{\|m^n - p_k\|} & \frac{m_y^n - y_k}{\|m^n - p_k\|} \\
\frac{m_y^1 - y_k}{\|m^1 - p_k\|^2} & \frac{x_k - m_x}{\|m^1 - p_k\|^2} & -1 & \frac{y_k - m_y}{\|m_y^1 - p_k\|^2} & \frac{m_x^1 - x_k}{\|m_x^1 - p_k\|^2} & \cdots & 0 & 0 \\
\vdots & & \vdots & & & & & \\
\frac{m_y^n - y_k}{\|m^n - p_k\|^2} & \frac{x_t - m_x^n}{\|m^n - p_k\|^2} & -1 & 0 & 0 & \cdots & \frac{y_k - m_y^n}{\|m^n - p_k\|^2} & \frac{x_k - m_x^n}{\|m^n - p_k\|^2}
\end{bmatrix}
$$