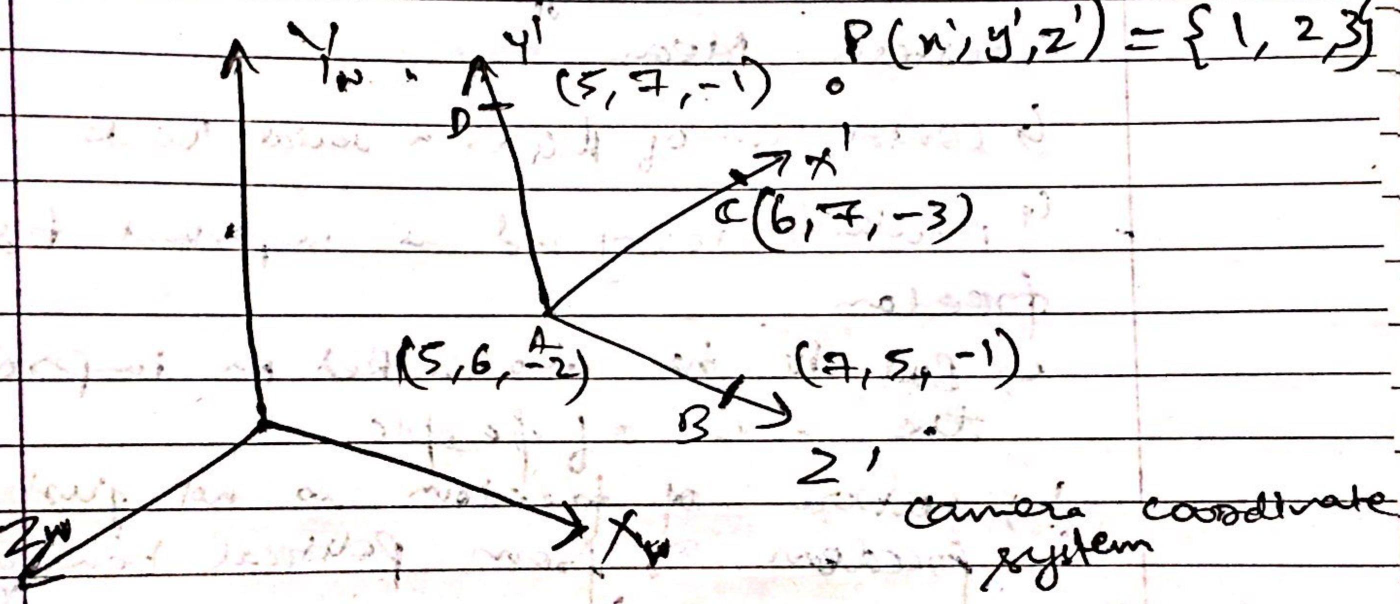
Graphics (Anand Sir)

3D Viewing

object stationary, camera moves ⇒ capturing diff views.



what is coordinate of pt P wrt window coordinates.

first check if these coordinate system are 90° to each other.

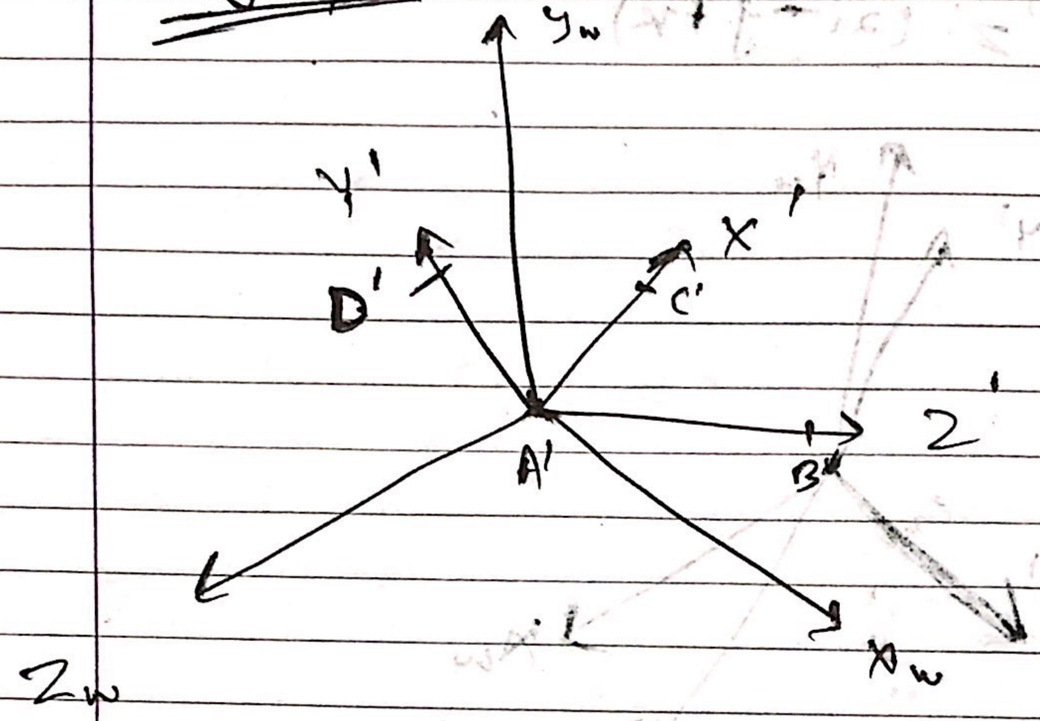
Verify new coordinate has axis vector at 90°'s.

$\overrightarrow{AB} = 2i - j + K$

$\overrightarrow{AC} = i + j - K$

$\overrightarrow{AD} = j + K$

Step 1



transformation matrix

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A'\ [0, 0, 0]$

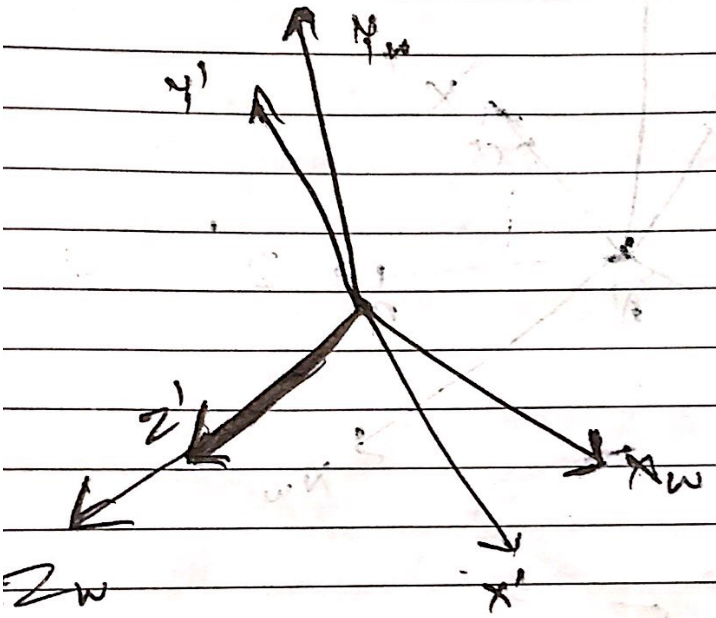
$B'\ [2, -1, 1]$

$C'\ [1, 1, -1]$

$D'\ [0, 1, 1]$

Step 2: find alignment matrix

$$A^{Z(K)}_{z'(2i-j+k)}$$



were coinciding z' with Zw and checking whether other coordinates are coinciding also.

$$A^{Z(K)}_{z'(2i-j+k)} = \begin{bmatrix} \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt3} & 0 \\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 \\ \frac{\sqrt2}{3} & -\frac{1}{\sqrt6} & \frac{1}{\sqrt6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$|z'| = \sqrt6$, $\lambda = \sqrt2$

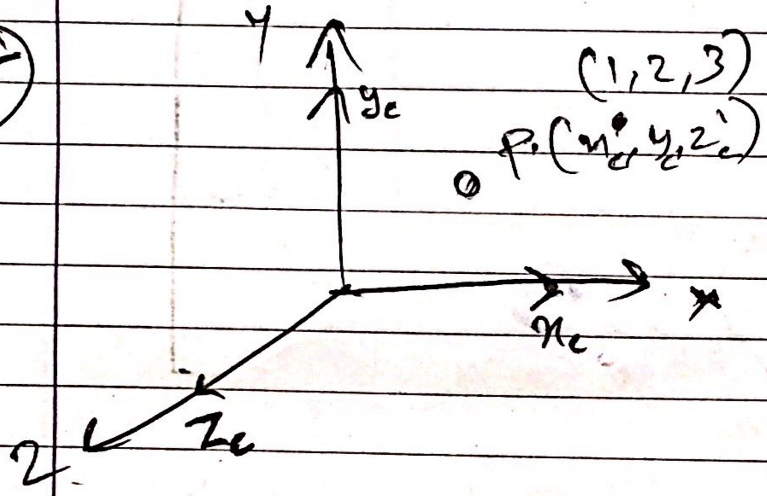
$A''\,[0, 0, 0]$

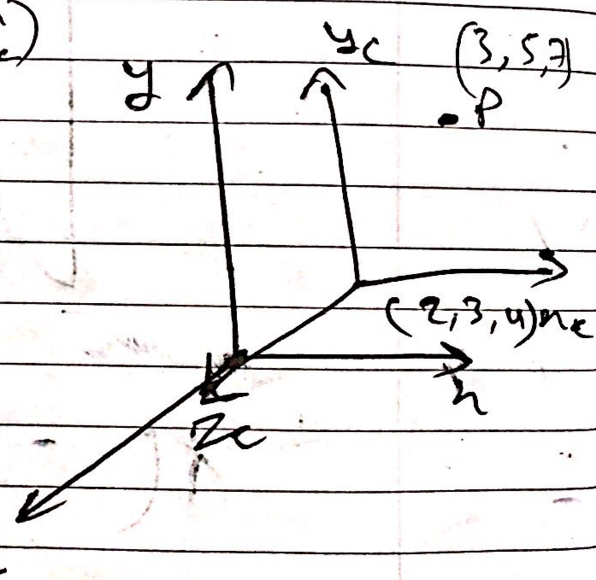
$B''\,[0, 0, \sqrt{6}]$

$C''\,[\sqrt{3}, 0, 0]$

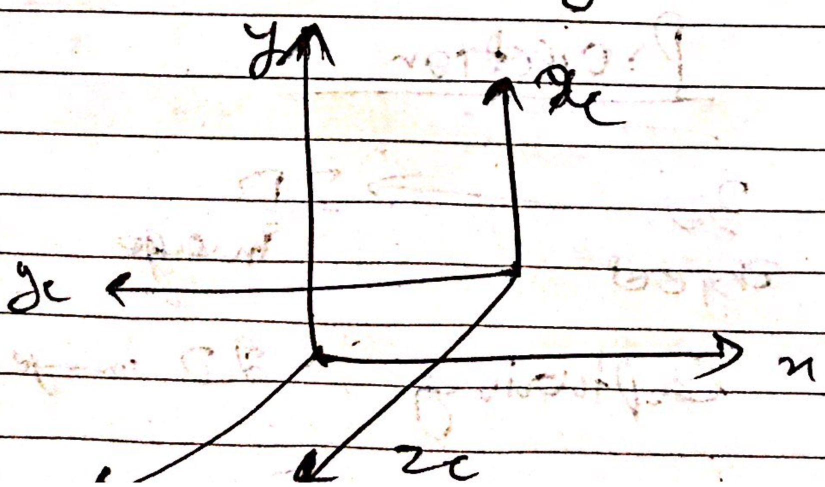
$D''\,[0, \sqrt{2}, 0]$

eg-





rotate 90° holding z



composite matrix

$$C = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\sqrt{2}}{3} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \quad \end{bmatrix}$$

$$P\left(\frac{-\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}\right)$$

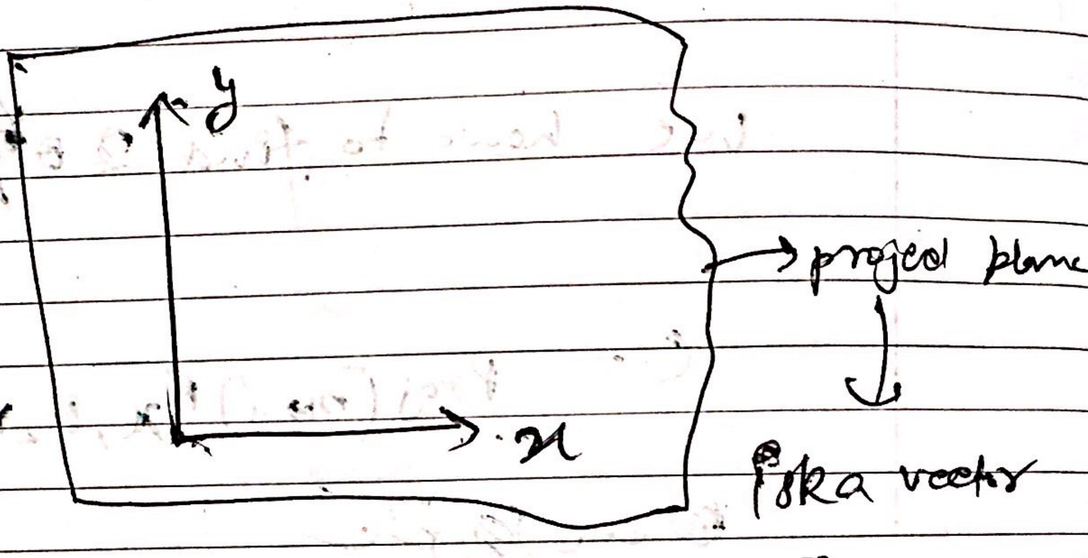
Projection

3D object $\longrightarrow$ 2D image

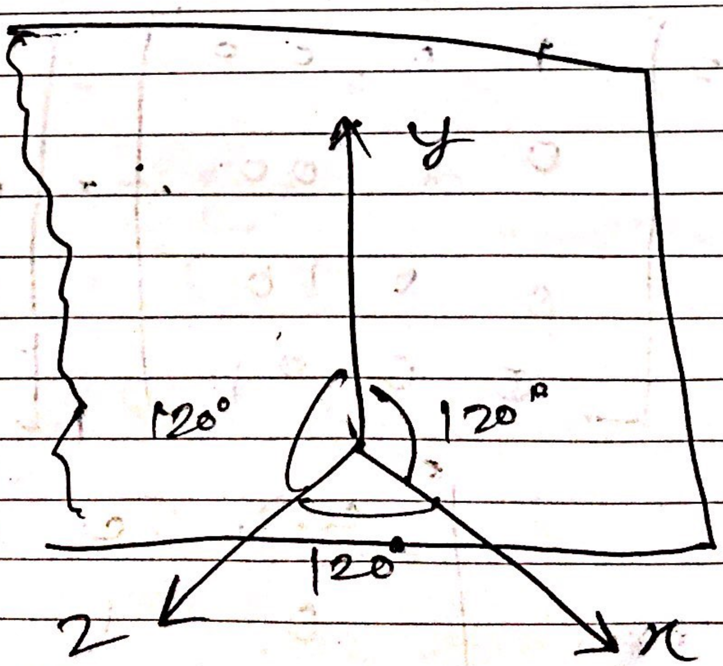
capturing : 2D image of 3D object

when projector line is parallel then it is orthographic.

isometric projection



isa vector parallel to z-axis.



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1	D	C
1	0	0
0	1	0
0	0	1
1	1	1

(a) Rotation about y axis clockwise

(b) Rotate ~~.......~~, ~~z~~

We have to find 2 θ's of rotate

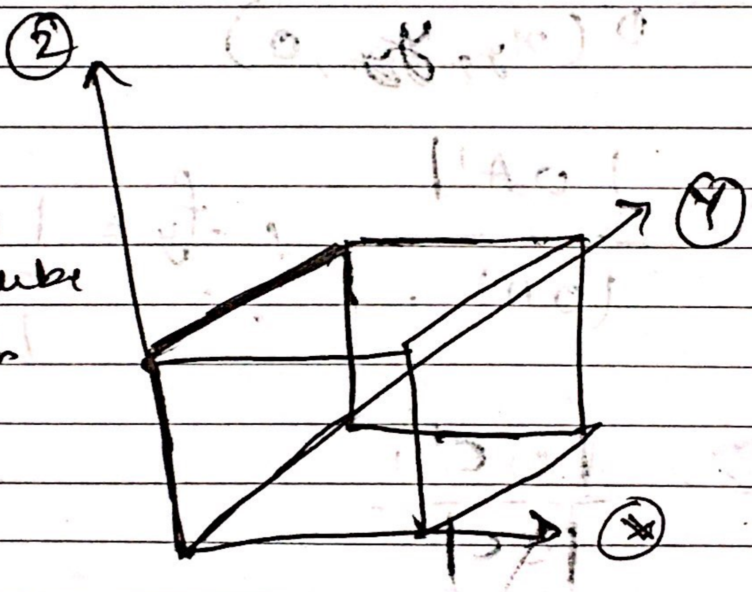
$C = \text{Proj}(x{-}y)\, R_{x,\theta_2} \times R_{y,\theta_1}$

~~ortho projection~~

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1 & -\sin\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \end{bmatrix}$$

unit cube ab taken



O [0, 0, 0]

A [1, 0, 0]

C [0, 0, 1]

D [0, 1, 0]

foreshortening factor

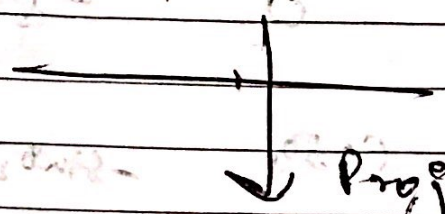
after rotation

$A(x_x, y_x, z_x)$

$C(x_z, y_z, z_z)$, $D(x_y, y_y, z_y)$

Scanned by CamScanner

A, C, D



$A'(x_x, y_x, 0)$

$C'(x_z, y_z, 0)$

$D'(x_y, y_y, 0)$

foreth. shortening vector along x-axis $\leftarrow f_x = \frac{|OA'|}{|OA|}$, $f_y = \frac{|OD'|}{|OD|}$

$$f_z = \frac{|OC'|}{|OC|}$$

eqn (1) $f_x = f_y$

$$\sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$$

$$\cos^2\theta_1 + \sin^2\theta_1 \sin^2\theta_2 = \cos^2\theta_2 \quad \text{— (1)}$$

$\lambda \; f_x : f_y : f_z$

$\underbrace{1 : 1}_{\text{eqn } 1} : 1$

eqn 2 $\quad f_y = f_z$

$$\cos^2\theta_2 = \sin^2\theta_1 + \sin^2\theta_2 \cos^2\theta_1$$

$$\theta_1 = \pm 45^\circ, \quad \theta_2 = \pm 35.26^\circ$$

this angles will gives isometric projection
(on world coordinate)

Dimetric Projection

$f_x : f_y : f_z$

$1 : 1 : K$

$1 : K : 1$

$K : 1 : 1$

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$$f_x = f_y \quad , \quad f_y = k f_z$$

$$\sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$$

$$\cos^2\theta_1 + \sin^2\theta_1 \sin^2\theta_2 = \cos^2\theta_2$$

$$\cos^2\theta_2 = k^2(\sin^2\theta_1 + \sin^2\theta_2 \cdot \cos^2\theta_1)$$

$$k^2\cos^2\theta_1 + k^2\sin^2\theta_1\sin^2\theta_2 = k^2\cos^2\theta_2$$

$$k^2\sin^2\theta_1 + k^2\sin^2\theta_2\cos^2\theta_1 = \cos^2\theta_2$$

$$k^2 + k^2\sin^2\theta_2 = (k^2+1)\cos^2\theta_2$$

$$\cancel{(k^2+1)} (\sin$$

$$k^2 - \sin^2\theta_2 + (k^2+1)\sin^2\theta_2 = (k^2+1)\cos^2\theta_2$$

$$k^2 + (k^2+1)\sin^2\theta_2 = (k^2+1)$$

value of K depends on range of $\sin\theta$ and $\cos\theta$

if $K = 1$ then isometric

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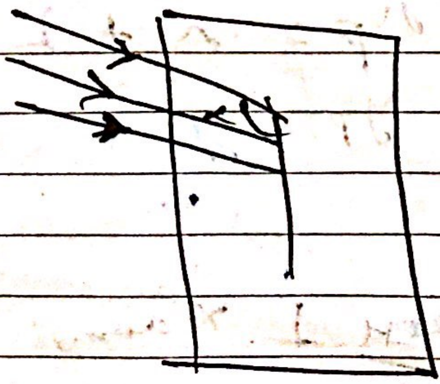
f_x, f_y & f_z

$1:1:K$	$1:K:1$	$K:1:1$
$\theta_1 = \sin^{-1}\left(\frac{K}{\sqrt{2}}\right)$	$\theta_1 = \frac{1}{\sqrt{2}}$	$(\quad)$
$\theta_2 = \sin^{-1}\sqrt{\frac{K^2}{K^2+2}}$	$\theta_2 = \cos^{-1}(\quad)$	

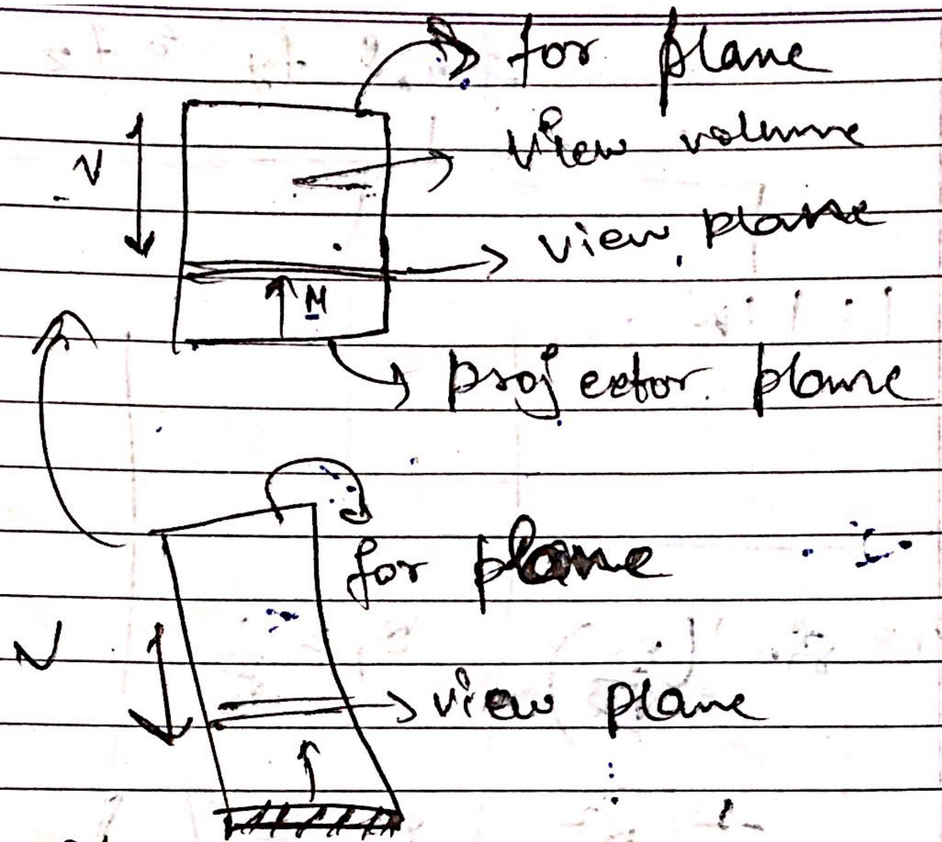
OBLIQUE Projection



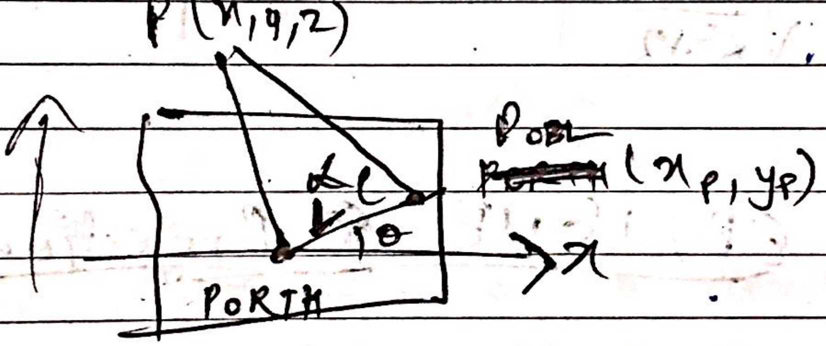
$\alpha \neq 90^\circ$

lines are not $\perp$ to the plane

→ 3D object.



$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$\left.\begin{aligned} x_p &= x + L\cos\theta \\ y_p &= y + L\sin\theta \end{aligned}\right\} \text{eq}(1)$$

$P\,P_{ORTH} \perp$ x axis,

$P\,P_{OBL}$

$\tan\alpha = \dfrac{z}{L}$

$L = \dfrac{z}{\tan\alpha}$ { Let us ~~take~~

Problems can come like

plane ko translate kiya

plane ko 1 point ke about move kia and normal banaya

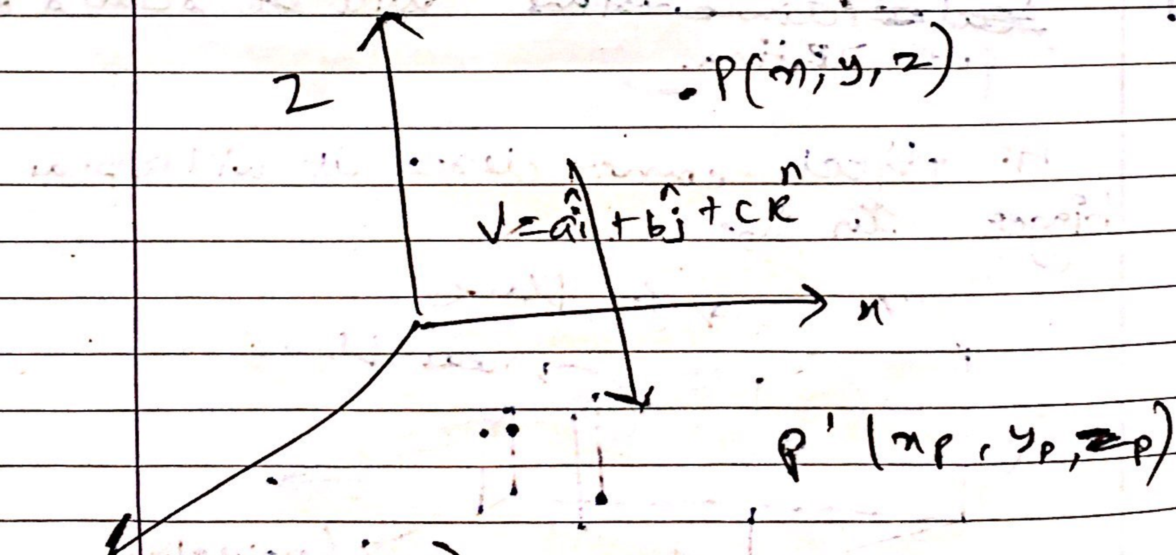
Missed

$$x_p = x + L_1\cos\theta_1 \, z$$

$$y_p = y + L_1\sin\theta_1 \cdot z$$

$$z_p = 0$$

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & L_1\cos\theta & 0 \\ 0 & 1 & L_1\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\vec{PP'} = (x_p - x)\hat{i} + (y_p - y)\hat{j} + (z_p - z)\hat{k}$$

$$\vec{PP'} \parallel \vec{V}$$

$$(x_p - x)\hat{i} + (y_p - y)\hat{j} + (z_p - z)\hat{k} = t(a\hat{i} + b\hat{j} + c\hat{k})$$

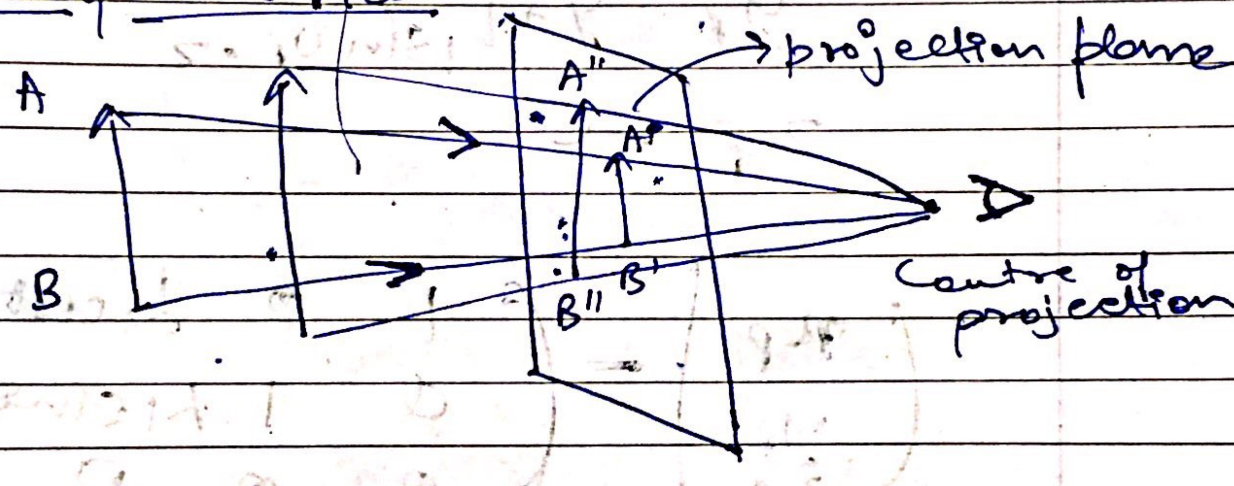
$$x_p - x = at$$

$$y_p - y = bt, \quad z_p - z = ct$$

Sociology (13/4/18).

Graphics (13/4/18)

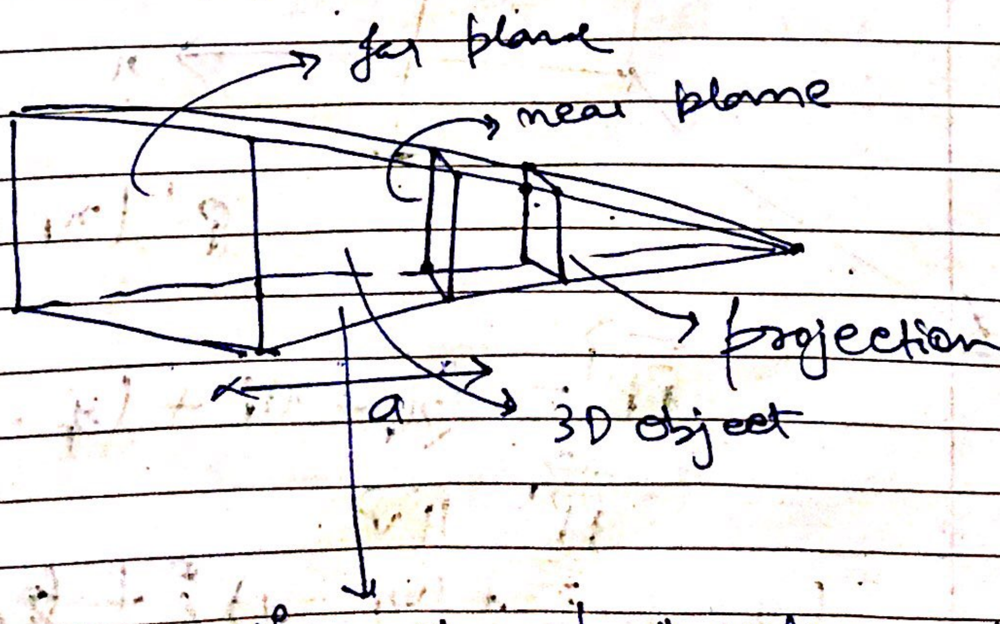
Perspective View: → converging projection lines



Size will be diminished

original ~~scaled~~ dimensions will be scaled out for retina.

As object comes closer it will appear bigger in size



View volume (~~the~~ volume where 3D object is placed)

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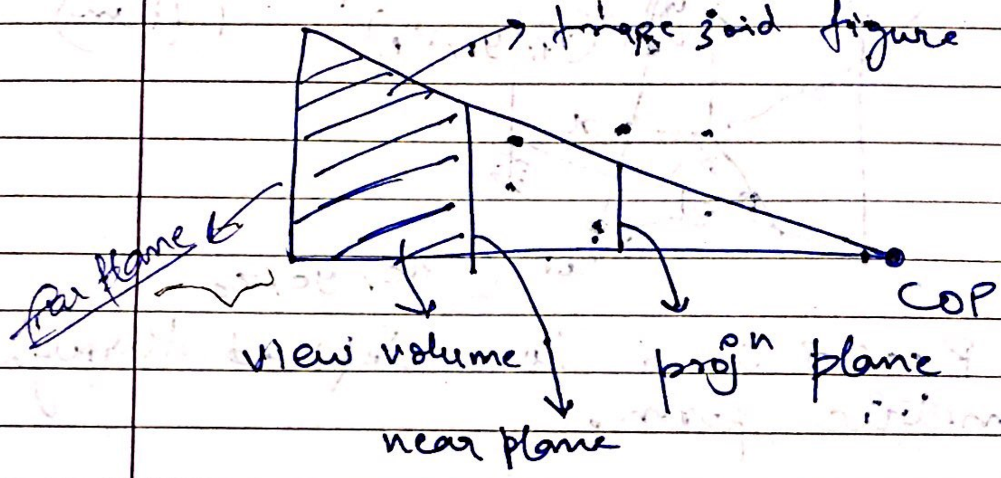
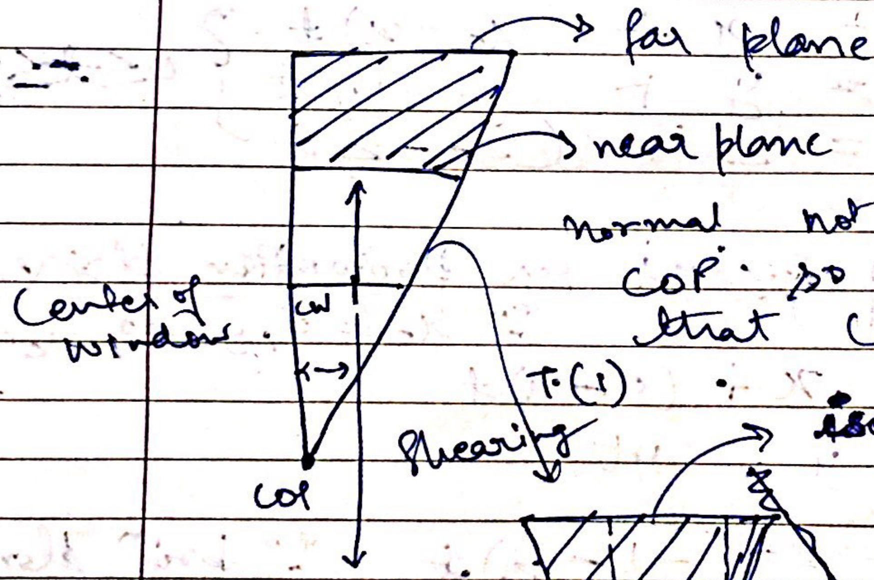
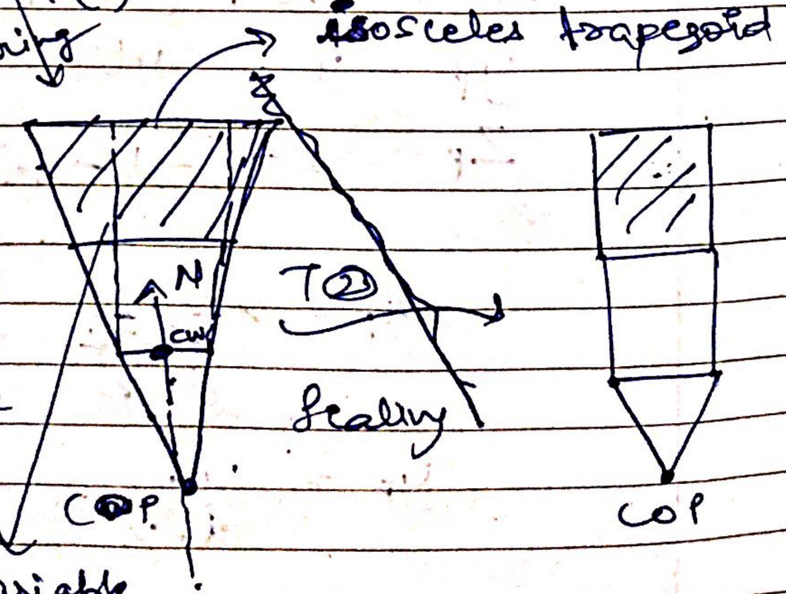
~~object~~ object should not be too far or close, so we define far and near plane.





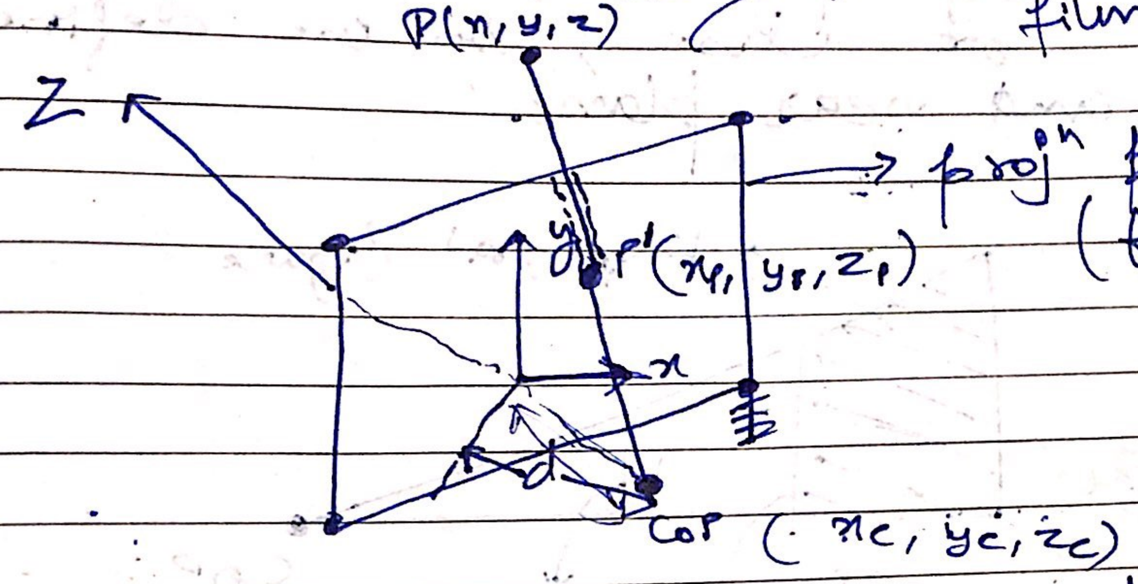
normal not aligned with COP so make it such that CW aligns with COP



Scaling karni hai taaki isosceles trapezoid converts to rectangle

We have to variable scaling here.

Case scenario - 1 → object is behind film of camera



P - CoP is a line defined through parametric form.

$$\left.\begin{aligned} x' &= x + (x_c - x)t \\ y' &= y + (y_c - y)t \\ z' &= z + (z_c - z)t \end{aligned}\right\} 0 \le t \le 1$$

<u>Point projected on projection plane</u>

$\because x_c = 0, y_c = 0, z_c = -d$

$$x_p = x + (0 - x)t$$

$$y_p = y - yt$$

$$z_p = z + (-d - z)t \quad \because \text{proj}^n \text{ plane is on } x\text{-}y \text{ plane} \therefore z_p = 0$$

$$x_p = x - \frac{xz}{d+z} = \frac{xd}{d+z}$$

$$y_p = \frac{yd}{d+z} \qquad t = \frac{z}{d+z}$$

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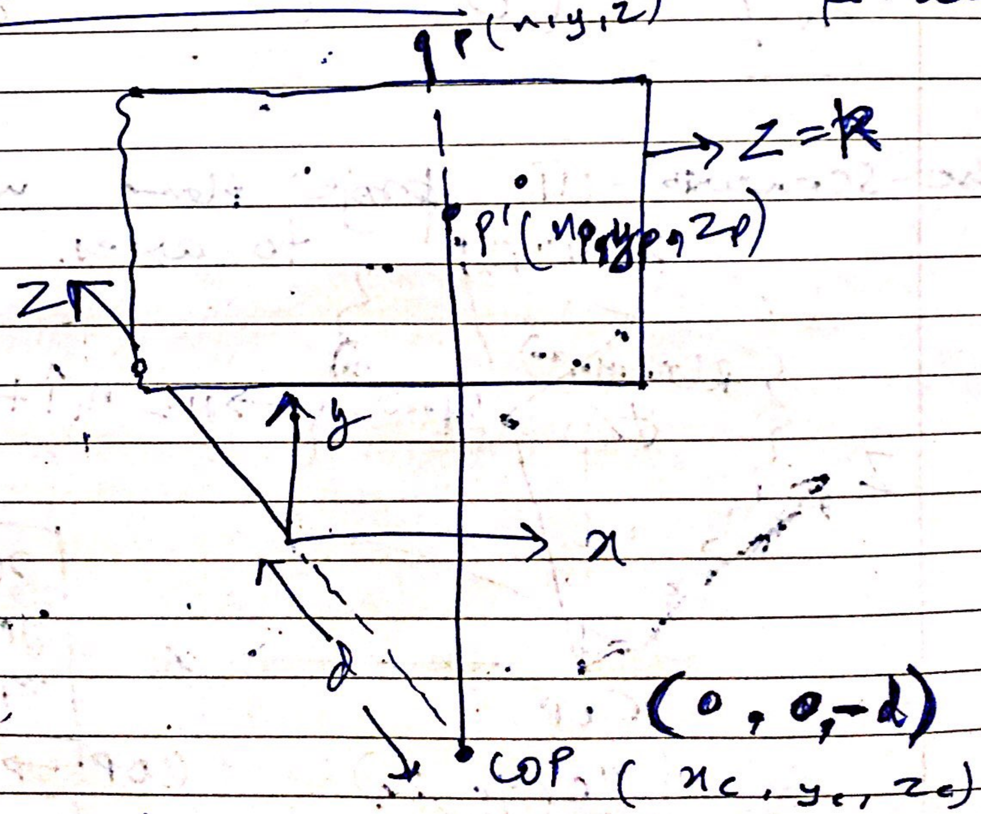
Let $h = d + z$

$x_h = x_p \cdot h$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Case scenario (2) — object stationary but we move forward.



$x_p = x + (0 - x)t \Rightarrow x_p = x - xt$

$y_p = y - yt$

$z_p = (z + k) + (-d - z)t$

$\boxed{t = \dfrac{z - k}{d + z}}$

k

$z_p = z + (-d - z)t$

$k - z = (-d - z)t$

$\boxed{x_p = x - \dfrac{x(d+k)}{d+z} = \dfrac{x(d+k)}{d+z}}$

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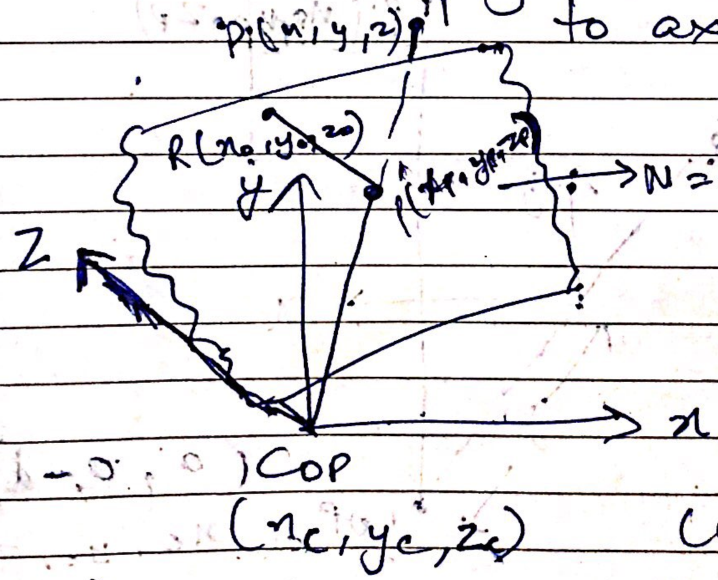
$$\boxed{y_p = \frac{y(d+k)}{d+z}}$$

$$\boxed{t = \frac{z-k}{d+z}}$$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} d+k & 0 & 0 & 0 \\ 0 & d+k & 0 & 0 \\ 0 & 0 & (k) & (kd) \\ 0 & 0 & 1 & d \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Case-Scenario-III: proje^n plane not parallel to axes.



$x_p = x - xt$
$y_p = y - yt$
$z_p = z - zt$

COP → at origin

$$\vec{V} = (x_p - x_0)\hat{i} + (y_p - y_0)\hat{j} + (z_p - z_0)\hat{k}$$

$$\vec{N} = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$$

where $\vec{V} \cdot \vec{N} = 0$

$$(x_p - x_0)n_1 + (y_p - y_0)n_2 + (z_p - z_0)n_3 = 0$$

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$$[x - xt - x_0]n_1 + [y - yt - y_0]n_2 + [z - zt - z_0]\cdot n_3 = 0$$

$$(x - x_0)n_1 + (y - y_0)n_2 + (z - z_0)n_3 - (x\cdot n_1 + y\cdot n_2 + z\cdot n_3)t = 0$$

$$\boxed{t = \frac{(x - x_0)n_1 + (y - y_0)n_2 + (z - z_0)n_3}{xn_1 + yn_2 + zn_3}}$$

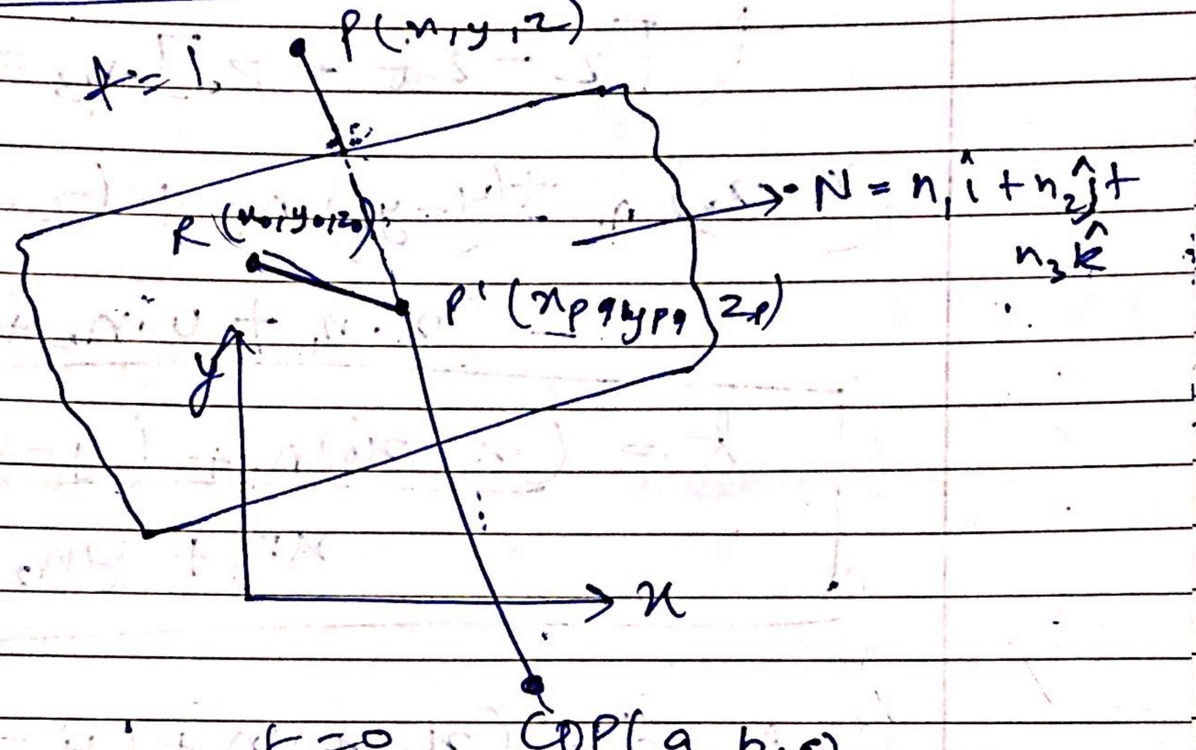
Let $d = (x - x_0)n_1 + (y - y_0)n_2 + (z - z_0)n_3$

$$x_p = x - \frac{xd}{xn_1 + yn_2 + zn_3}$$

$$y_p = y - \frac{yd}{x\cdot n_1 + yn_2 + z\cdot n_3}$$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} (h-d) & 0 & 0 & 0 \\ 0 & (h-d) & 0 & 0 \\ 0 & 0 & (h-d) & 0 \\ n_1 & n_2 & n_3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Case Scenario - IV (Generic Case)



Step ① Parametric form for P - COP

$x_1 = x' = a + (x-a)t$
$y_1 = y' = b + (y-b)t$
$z_1 = z' = c + (y-c)t$

$\vec{V} = (x_p - x_0)\hat{i} + (y_p - y_0)\hat{j} + (z_p - z_0)\hat{k}$
$\vec{n} = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$

$\vec{V} \perp \vec{n}$

$[a + (x-a)t - x_0] \cdot n_1 + [b + (y-b)t - y_0] \cdot n_2 + [c + (z-c)t - z_0] \cdot n_3 = 0$

$$t = \frac{(x_0 - a)n_1 + (y_0 - b)n_2 + (z_0 - c)n_3}{(x-a)n_1 + (y-b)n_2 + (z-c)n_3}$$

Let $d_0 = x_0 n_1 + y_0 n_2 + z_0 n_3$
$d_1 = a n_1 + b n_2 + c n_3$
$d_2 = x n_1 + y n_2 + z n_3$

$$t = \frac{d}{d_2 - d_1}$$

$$x_p = a + (x-a)t$$
$$= a + \frac{(x-a)\,d}{x n_1 + y n_2 + z n_3 - d_1}$$

$$x_p = \frac{(a n_1 + d)x + a n_2 y + a n_3 z - a d_0}{x n_1 + y n_2 + z n_3 - d_1}$$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} (a n_1 + d) & a n_2 & a n_3 & -a d_0 \\ b n_1 & (b n_2 + d) & b n_3 & -b d_0 \\ c n_1 & c n_2 & (c n_3 + d) & -c d_0 \\ n_1 & n_2 & n_3 & -d_1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Hidden Surface

Detection of hidden surfaces and eliminating it.

Six views

P T F

iso dia triametric

hidden part of cube

Tetrahedron

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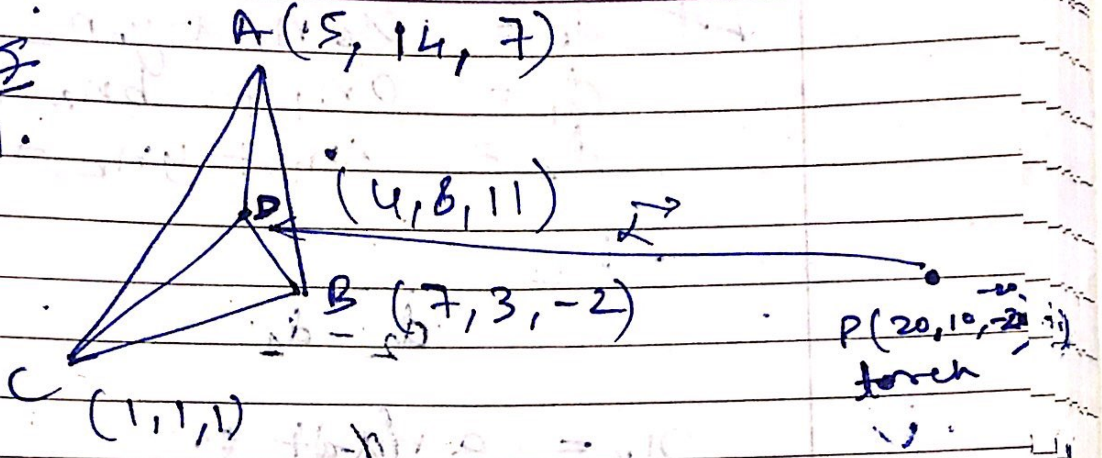
eg-

BACKFACE DETECTION METHOD

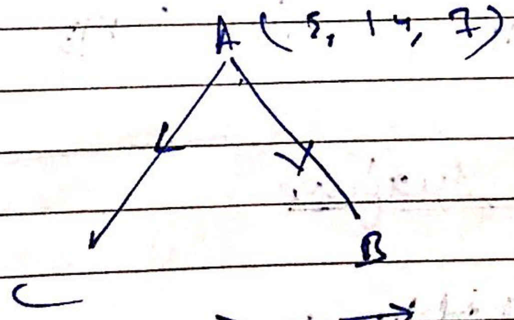


Determine which surfaces will be illuminated and which are not?

Solution step ① → List all surfaces

Surfaces ACD
ADB
BCD
ABC

Step ② Find out outward normal



$\vec{AC} = -4\hat{i} - 13\hat{j} - 6\hat{k}$

$\vec{AB} = 2\hat{i} - 11\hat{j} - 9\hat{k}$

$\vec{AC} \times \vec{AB} \Rightarrow (-)(4\hat{i} + 13\hat{j} + 6\hat{k})(2\hat{i} - 11\hat{j} - 9\hat{k})$

$\Rightarrow (-)(-44\hat{k} + 36\hat{j} - 26(\hat{k}) - 117\hat{i} + 12\hat{j} + 66(\hat{i}))$

117
66
51

$\Rightarrow (+)(-51\hat{i} + 48\hat{j} - 70\hat{k})$

drop

normal = $\boxed{(-51\hat{i} + 48\hat{j} - 70\hat{k})}$

$\boxed{ABC}$

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$$\boxed{ACD = -8\hat{i} + 2\hat{j} + \hat{k}}$$

Surface ADB

$$\vec{DB} \times \vec{DA}$$

$$(3\hat{i} - 5\hat{j} - 13k) \times (\hat{i} + 6\hat{j} - 4\hat{k})$$

$$(18\hat{k} + 12\hat{j} + 5k + 20\hat{i} - 13\hat{j} + 78\hat{i})$$

$$\Rightarrow \boxed{98\hat{i} - \hat{j} + 23k}$$

ADB

$$\boxed{BCD \Rightarrow 41\hat{i} - 69\hat{j} + 36\hat{k}}$$

(centroid)

Step 3 (mean position)

ACD $\rightarrow 10/3\,\hat{i} + \frac{23}{3}\hat{j} + \frac{19}{3}\hat{k}$

ADB $16/3\,\hat{i} + \frac{25}{3}\hat{j} + \frac{10}{3}$

BCD $\Rightarrow 4\hat{i} + 4\hat{j} + 10/3\,\hat{k}$

ABC $\Rightarrow \frac{13}{3}\hat{i} + 6\hat{j} + 2\hat{k}$

Step 4 $\boxed{L = MP - P}$

ACD $\left(\frac{10}{3} - 21\right)\hat{i} + \left(\frac{23}{3} - 10\right)\hat{j} + \left(\frac{19}{3} + 20\right)\hat{k}$

$\vec{P M_P}$

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Transport Layer → UDP, TCP.

Connection - establishment → 3/way handshake and ~~data~~ data transfer

Error control in TCP

TCP: congestion control.

Hamming ~~data~~ code, Routing Table

Sir → subnet, routing, physical devices, coloring, sections/schemes

Hardware - hub, CSMA

Graphics (NPTEL)

~~Z-Buffer Algo~~

Hidden surface Removal

world coordinate system

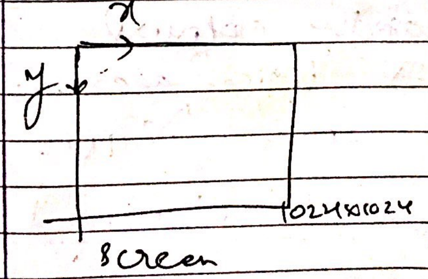
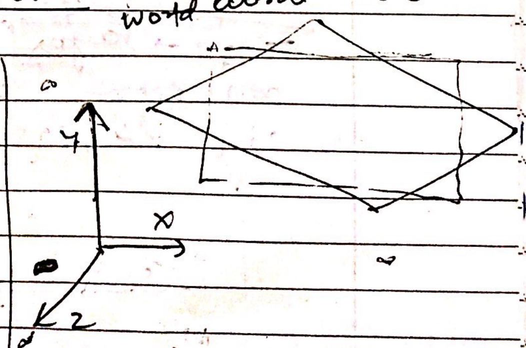


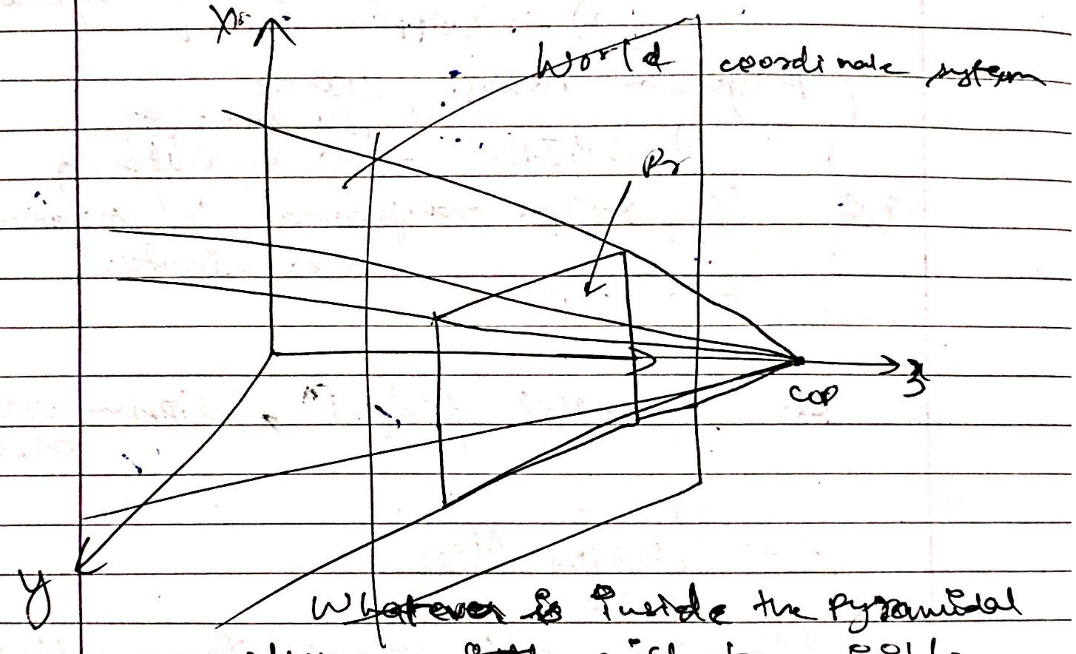
Image-space



OBJECT SPACE

A portion of world coordinate is transform onto the screen.

The hidden surface removal (HSR) Algo will work on that concerned portion.



~~Whatever~~ is inside the pyramidal volume will be visible

Object in 3D will be clipped such that only portion inside pyramid will be visible.

2. When we try to remove hidden sur then we have to specify viewing transformation

V.T — Perspective or parallel

When we know VT then we do ~~the~~ HSR

Algo in Object space:

1) → HSR for all objects
2) → clipping in 2D

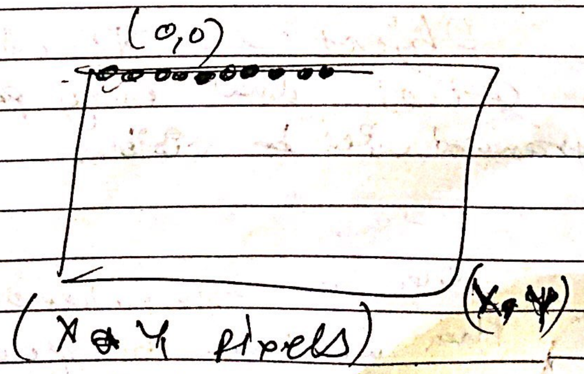
Algo in image space

1) → OBJECTS → will undergo 3D clipping
2) → then transformed to screen coordinates
→ HSR

In parallel projectⁿ, clipping volume block

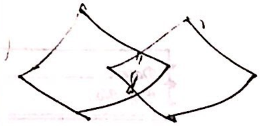
Z-Buffer Algo

Uses an extension of Frame Buffer



otal X * Y bits, each ... OBJECT SPACE

each bit will be 0 or 1



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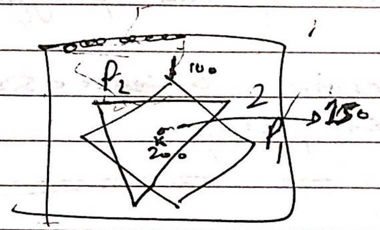
Z-Buffer: at each pixel we store depth of pixel also



Buffer main depth store kara rahe hain

Compare 150 for the value stored in Z-buffer (200) (P_1 P_2)

and decide which pixel is closer to the eye.

Use an array of $x * y$ (size)

Given: a set of polygons in image space

Algo: 1. set the frame buffer/ ~~z-buff~~ z-buff to a background value (color of bground will be initialised to all pixels)

(Z-Buffer $= Z_{min}$)

2. Scan-convert each polygon

- for each pixel, find the ~~depth of~~ $z(x,y)$ ~~that~~ ~~the~~

- if $Z(x,y) >$ Z-Buffer(x,y)

- update $z\text{-buffer}(x, y) = z(x, y)$
- update frame-buffer acc to color and intensity of that polygon

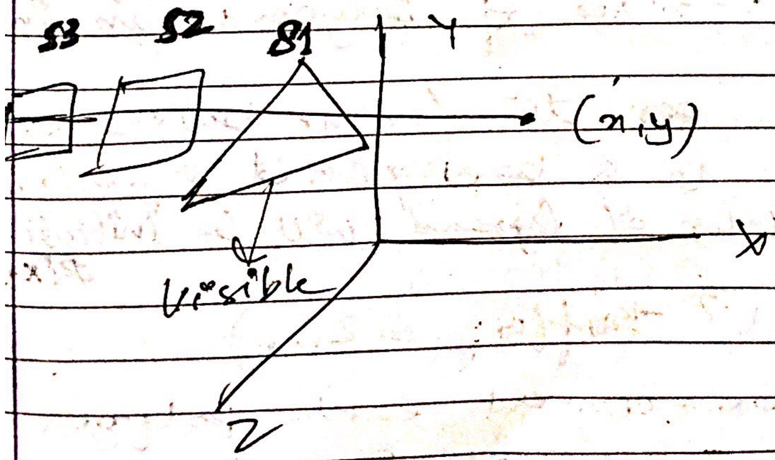
disadv: Expensive to ~~[illegible]~~ scan-convert each polygon.

Z-Buffer Method (Zeenat Hassan)

(1) based on Image space method

After projection, pixel by pixel surface ki point ki depth value kitni hai kisi proj^n plane se. (of view plane)

Jo surface paas hoga, uski intensity calculate kar lenge and that is visible surface.



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for depth value

Requests

Algorithm: buffer(x, y) $\rightarrow$ z-value

refresh(x, y) $\rightarrow$ Intensity of (x, y)

$\rightarrow$ z coordinate Normalized

$0 < z < 1$

Back-clipping plane, front-clipping plane.

Steps $\rightarrow$ ① Initialize both the buffer i.e.

buffer$(x,y) = 0$, and refresh(x,y) = I background

buffer$(x,y) = 0$

② calculate z-value for each position in the each surface and then

if $z >$ depth(x, y)

depth$(x,y) = z$ and

refresh$(x,y) =$ I surface(x,y)

$z <$ depth(x,y)

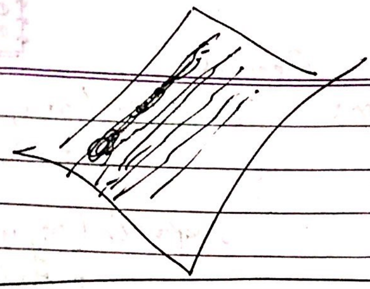
depth$(x, z) = z$

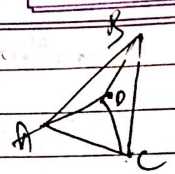
③ After processing all the surface we will get visible surface in depth(x,y) and Intensity value in refresh(x,y)

formula $Ax + By + Cz + D = 0$

Expensive to calculate $\boxed{z = \frac{-Ax - By - D}{C}}$

disadv: work for opaque surface not for transparent object.





Area Sirg → on Z-Buffer

Tetrahedron

A(1, 1, −1)

B(3, 1, −1)

C(2, 1, −3)

D(2, 2, −2)

Surfaces	Color
ACD	RED (1)
CBD	BLUE (2)
BAD	CYAN (3)
ACB	GREEN (4)

Bg → BLACK (0)

Step 1: x_{min} (1), y_{min} (1), z_{min} (−3)

x_{max} (3), y_{max} (2), z_{max} (−1)

Step 2: Plane ACD (with outward normal)

$AC \Rightarrow (1, 0, -2)$

$AD = (1, 1, -1)$

$ax + by + cz + d = 0$

$AC \times AD \Rightarrow (\hat{i} - 2\hat{k}) \times (\hat{i} + \hat{j} - \hat{k})$

$(\hat{k} + \hat{j} - 2\hat{j} + 2\hat{i})$

$(2\hat{i} - \hat{j} + \hat{k})$

$2x - y + z = d$

$2 - 1 - 1 = 0 = d$

$\boxed{2x - y + z = 0}$

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Plane CBD = $\vec{DC}$, $\vec{DB}$

$(0, -1, -1)$ $(1, -1, 1)$

$(-\hat{j} - \hat{k}) \times (\hat{i} - \hat{j} + \hat{k})$

$(\hat{k} - \hat{i} - \hat{j} + (-\hat{i}))$

$-2\hat{i} - \hat{j} + \hat{k}$

$-2x - y + z = 0$

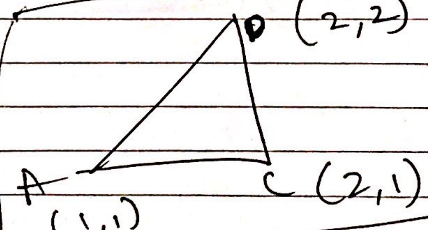
$2x + y - z = d$ D(2,2,-2)

$2(2) + (2) - (-2) = d$

$4 + 2 + 2 = d = 8$

CBD: $\boxed{2x + y - z - 8 = 0}$

Step (3)



find eqn

$AC \Rightarrow y = 1$ $(y-1) \geq 0$ (+ve) ≥ 0

$AD \Rightarrow x = y$ $(x-y) \geq 0$

$CD \Rightarrow x = 2$ $x - 2 \leq 0$ $2 - x$

$(y-1) \geq 0$ x, y

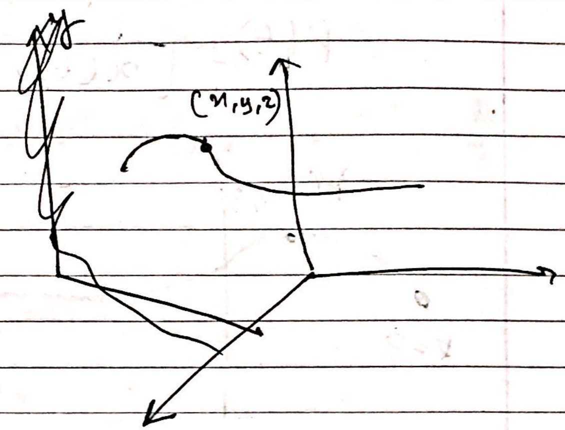
≥ 0 $2 - 1 \geq 0$ $1,$ $1 - 2 < 0$

Graphics (Anand Sir)

16/4/18 → On CamScan or Vaishnavi Message

19/4/18 →

3D Curve

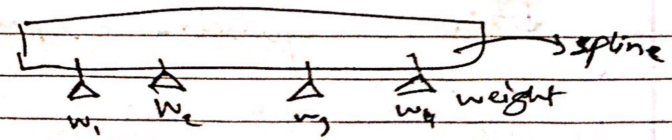


Application Domain

1. Shipping Industry
2. Car "
3. Aircraft "

they require particular shapes that are drawn people called draft person

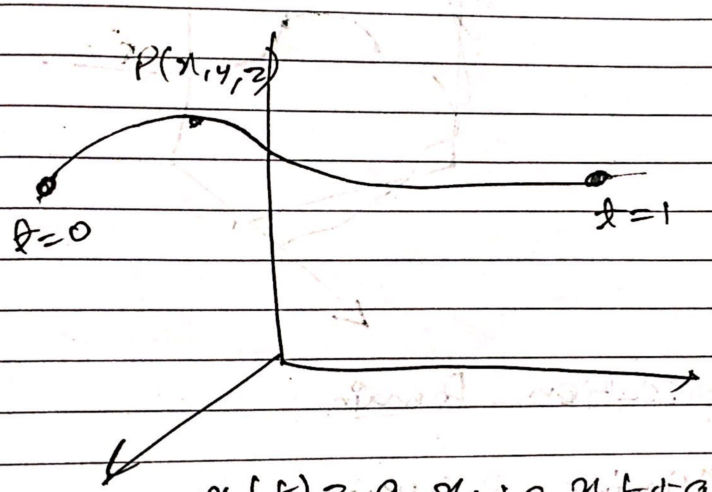
basically mathematicians



Mathematical eqn of curve is not known

Parametric form

$P(t) = [x(t), y(t), z(t)]$



$x(t) = a_0 x_0 + a_1 x_1 t + a_2 x_2 t^2 + a_3 x_3 t^3 + \dots a_n x_n t^n$

$y(t) =$ Similar

$z(t) =$ ——

$n = 1 \rightarrow$ straight line

$n = 2 \rightarrow$ quadratic curve

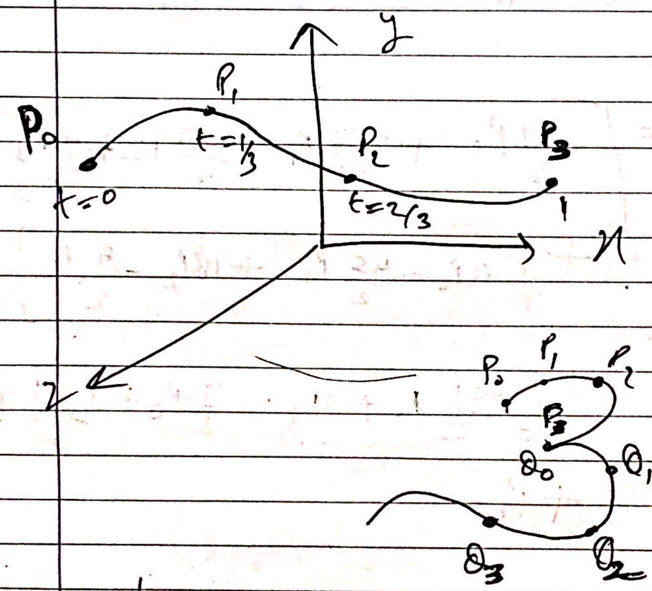
$\boxed{n = 3 \rightarrow \text{cubic curve}}$

$n = 4$ quadratic space curve eqn.

cubic curve (Parametric form)

$$x(t) = a_x t^3 + b_x t^3 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

$$\boxed{P(t) = \vec{a}\, t^3 + \vec{b}\cdot t^2 + \vec{c}\, t + \vec{d}}$$



To find $\vec{a}, \vec{b}, \vec{c}, \vec{d}$

To matrix inverse

$$P_0 = P(t)\big|_{t=0} = \vec{d}$$

$$P_1 = \frac{1}{27}\vec{a} + \frac{1}{9}\vec{b} + \frac{1}{3}\vec{c} + \vec{d}$$

$$P_2 = \frac{8}{27}\vec{a} + \frac{4}{9}\vec{b} + \frac{2}{3}\vec{c} + \vec{d}$$

$$P_3 = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

$$\vec{a} = -\frac{9}{2}P_0 + \frac{27}{2}P_1 - \frac{27}{2}P_2 + \frac{9}{2}P_3$$

$$\vec{b} = 9P_0 - \left(\frac{45}{2}\right)P_1 + 18P_2 - \frac{9}{2}P_3$$

$$\vec{c} = -\frac{11}{2}P_0 + 9P_1 - \frac{9}{2}P_2 + P_3$$

$$\vec{d} = P_0$$

$$P(t) = \left[-\frac{9}{2}P_0 + \frac{27}{2}P_1 - \frac{27}{2}P_2 + \frac{9}{2}P_3\right]t^3$$

$$+ \left[9P_0 - \frac{45}{2}P_1 + 18P_2 - \frac{9}{2}P_3\right]t^2$$

$$+ \left[-\frac{11}{2}P_0 + 9P_1 - \frac{9}{2}P_2 + P_3\right]t$$

$$+ P_0$$

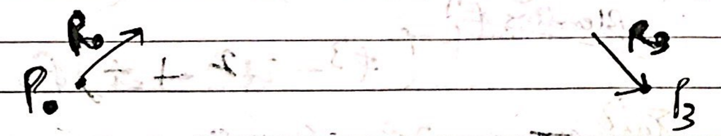
$$P(t) = \left(-\frac{9}{2}t^3 + 9t^2 - \frac{11}{2}t + 1\right)P_0 +$$

$$\left(\frac{27}{2}t^3 - \frac{45t^2}{2} + 9t\right)P_1 + \left(-\frac{27}{2}t^3 + 18t^2 - \frac{9}{2}t\right)P_2$$

$$+ \left(\frac{9}{2}t^3 - \frac{9}{2}t^2 + t\right)P_3$$

Problem A ball is thrown from pos^n P_0 with a particular speed R_0 and it is received by the wicket keeper whose position is P_3 with a speed R_3.

Ans



$$P_0 = \vec{d}$$
$(t=0)$

$$P_3 = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$
$(t=1)$

$$P'(t) = 3a^2t^2 + 2\vec{b}t + \vec{c}$$

$$R_0 = \vec{c}$$

$$R_{3\,(t=1)} = 3\vec{a} + 2\vec{b} + \vec{c}$$

Find $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ by 4 eq^n

$$\boxed{\begin{aligned}\vec{c} &= R_0 \\ \vec{d} &= P_0\end{aligned}}$$

$$\boxed{\vec{b} = 3P_3 - 3P_0 - 2R_0 - R_3}$$

$$2 \times (\vec{a} + \vec{b} + R_0 + P_0 = P_3)$$

$$3\vec{a} + 2\vec{b} + R_0 = R_3$$

$$2\vec{a} + 2\vec{b} + 2R_0 + 2P_0 = 2P_3$$

$$\vec{a} = R_0 - 2P_0 = R_3 - 2P_3$$

$$\boxed{\vec{a} = R_0 + 2P_0 + R_3 - 2P_3}$$

$$P(t) = (-2P_3 + 2P_0 + R_0 + R_3)t^3$$
$$+ (3P_3 - 3P_0 - 2R_0 - R_3)t^2$$
$$+ R_0 t$$
$$+ P_0$$

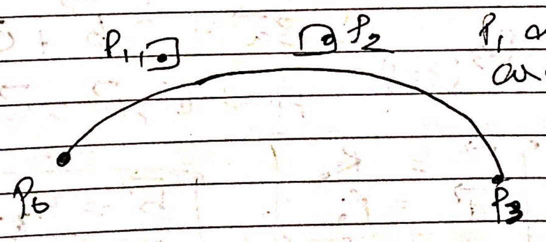
$$P(t) = \underbrace{(2t^3 - 3t^2 + 1)}_{\text{Blending fn}} P_0 + (-2t^3 + 3t^2) P_3$$
$$+ (t^3 - 2t^2 + t) R_0 + (t^3 - t^2) R_3$$

Hermite curve

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\text{Blending matrix}} \begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix}$$

Problem 2

Cubic curve



P_1 and P_2 are controls points

if we move P_1 and P_2 we get diff shapes.

P_1 and P_2 are responsible for giving special shapes

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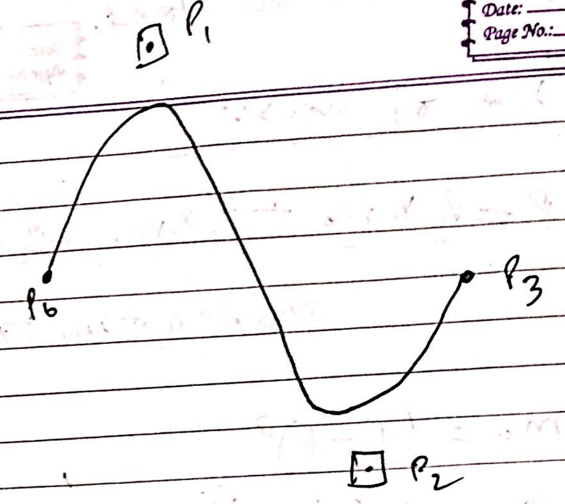
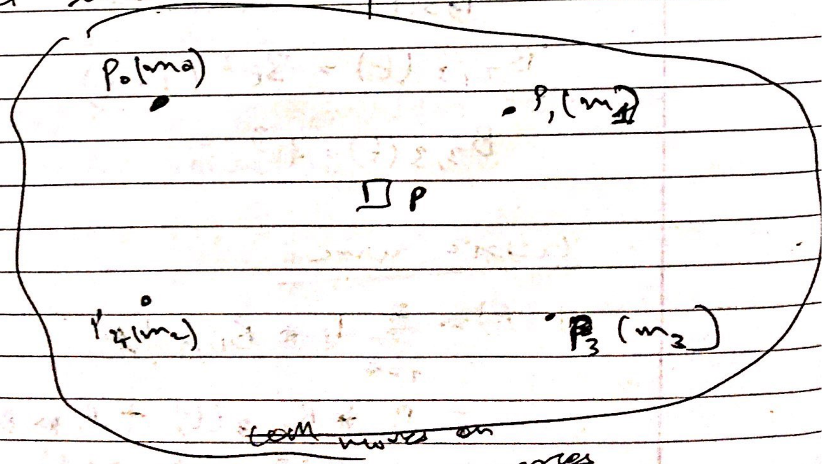


To solve this problem a sol^n was given by <u>Pierre Bezier</u>

$$\boxed{P(t) = \sum_{i=0}^{n} P_i * B_{i,n}(t) \quad 0 \le t \le 1}$$

~~Blame~~ Blending f^n

Let ~~there~~ are point masses



COM moves on changing masses

P_0 and P_3 are fixed,

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Centre of mass

$$P = \frac{P_0 \times m_0 + P_1 \times m_1 + P_2 m_2 + P_3 m_3}{m_0 + m_1 + m_2 + m_3}$$

let

$$\begin{cases} m_0 = (1-t)^3 \\ m_1 = 3t(1-t)^2 \\ m_2 = 3t^2(1-t) \\ m_4 = t^3 \end{cases}$$

for a cubic curve

$B_{0,3}(t) = (1-t)^3$

$B_{1,3}(t) = 3t(1-t)^2$

$B_{2,3}(t) = 3t^2(1-t)$

$B_{3,3}(t) = t^3$

cubic curve

$P(t) = \sum_{i=0}^{3} P_i * B_{i,3}(t)$

$= P_0 * B_{0,3}(t) + P_1 * B_{1,3}(t)$
$+ P_2 * B_{3,3}(t)$

doubt here

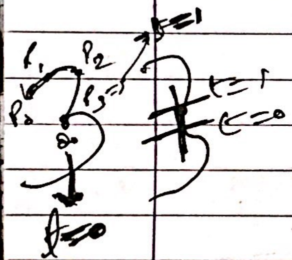
Isse baad mein Bezier Matrix banai hai.

$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) \ldots + t^3 P_3$$

$$P(t) = [-t^3 + 3t^2 - 3t + 1] P_0 + [3t^3 - 6t^2 + 3t] P_1 + [-3t^3 + 3t^2] P_2 + t^3 P_3$$

$$[t^3 \; t^2 \; t \; 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Q. Cubic bezier curve is described 1 segment by control pts

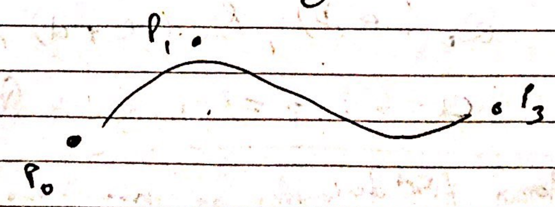


$P_0(20, 20)$
$P_1(40, 80)$
$P_2(80, 80)$
$P_3(90, 50)$

Another curve segm is given by

$Q_0(a, b)$, $Q_1(c, 20)$, $Q_2(150, 20)$
$Q_3(180, 20)$

Find a, b, c if both curve join smoothly.



$\boxed{a_0 = 90,\ b = 50}$ → continuous and smooth curve

so P_3 coincides with Q_0

$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3$$

$$P'(t)\big|_{t=0} = Q'(t)\big|_{t=1}$$

find c.

2) $P_0(2,1)$; $P_1(3,2)$, $P_2(5,0)$, $P_3(6,2)$

Bezier curve defined by ctrl pts P_0

by choosing another set of ctrl pt

$Q_0(a,b)$, $Q_1(c,d)$, $Q_2(e,f)$

$a = 6$; $b = 2$; $c = 7$; $d = 4$ $Q_3(g,h)$

* Agar first derivative se smoothness nahi ayi to second derivate se nikalo

(3) Find eqn of bezier curve which passes through the points $(0,0)$ P_0 and $(-2,1)$ P_3 and is controlled through $(7,5)$ P_1 and $(2,0)$ P_2

2 sol^{ns}

$P_1 \to (7,5)$
$P_2 \to (2,0)$
or
$P_1 (2,0)$
$P_2 (7,5)$

$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3$$

$$= (1-t)^3 (0\hat{i} + 0\hat{j}) + 3t(1-t)^2 (7\hat{i} + 5\hat{j}) + 3t^2(1-t)(2\hat{i} + 0\hat{j}) + t^3 (-2\hat{i} + \hat{j})$$

fir $\to$ sol (1)

$\hookrightarrow x(t) = 13t^3 - 36t^2 + 21t$,
$y(t) = 16t^3 - 30t^2 + 15t$

sol (2) $x(t) =$
$-17t^3 + 9t^2 + 6t$, $y(t) = -14t^3 + 15t^2$

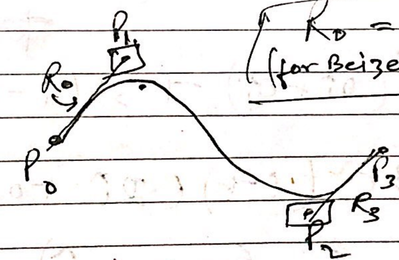
from general eqn of Bezier curve

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Q) What is transformation matrix for Hermite to Bezier and vice-versa

Hermite – Bezier
(transformation)

$R_0 = ?$ (for Beizer)



$R_0 = 3(P_1 - P_0)$
(for Beizer)

$R_0 \rightarrow$ first derivative mein $t=0$ is R_0

$R_0 \leftarrow = [P'(t)]_{t=0} \rightarrow$ of Bezier curve & its tangent

$R_3 = 3(P_3 - P_2)$
(for Bezier)

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Geometry constraint of Bezier

$$G_{Bezier} \Rightarrow \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$G_{Hermit} = \begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix} \Rightarrow M_{HBEZ} * \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Constraint matrix

$$\boxed{G_H = M_{HBEZ} * G_{Beizer}.}$$

$$G_H = \begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} * \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Basic matrix rel^n

$$\boxed{M_{BEZ} = M_H \times M_{HBEZ}}$$

$$P(t) = T \cdot M_H \cdot G_{HB} \qquad T = [t^3 \; t^2 \; t \; 1]$$

$$= T \cdot M_H \cdot (M_{HBEZ} \cdot G_B)$$

$$= T (M_H \, M_{HBEZ}) \, G_B$$

$$\boxed{P(t) = T \cdot M_{BEZ} \cdot G_B}$$

Prove that

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$$M_{Bez} = M_{H} \cdot M_{HBez}$$

$$= \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} P_0 \\ 1 \\ P_2 \\ 3 \end{bmatrix}$$

M