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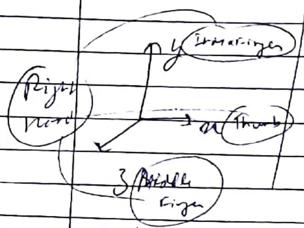
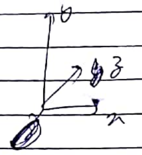
Graphics Lab

3D Transform^n → Translation
→ Scaling
→ Rotation

Right Handed Co-ordinate System | Left Handed CS
RHCS | LHCS





followed by → World, Our Eyes

Followed by → Camera

Side View	Front View	Top View
Isometric View	Dimetric View	Perspective View

CRT Screen (?).

What is homogenity in this?
→ It is an operator (addition, multiplication).

Not

Here, every transform^n follows multiplic^n
This is homogenity here.

Here,
$$\begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Augmented

Rotation

RHCS

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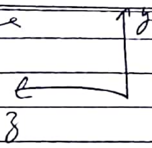
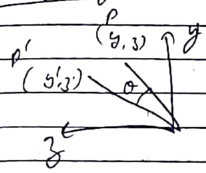
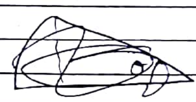
Looking from x-side

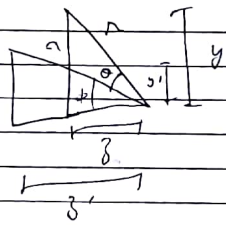


+ve rotation = anti-cw.

Rot^n from y to z ⇒ $x' = x$.

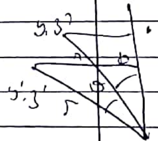






$y' = r \sin\phi$

$y = r\sin(\theta+\phi)$
$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$
$y = z'\sin\theta + y'\cos\theta$



Mine

$z' = r\cos(\theta+\phi)$

$z' = \cos\theta\, z - \sin\theta\, y'$

(confirm from here)

Sir → $y' = y\cos\theta - z\sin\theta$
$z' = y\sin\theta + z\cos\theta$

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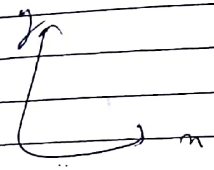
Rot^n from y →

(Self)

(Self) Rot^n from z →



$$R_{x,\theta} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & C & -S & 0 \\ 0 & S & C & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$R_{y,\theta} = \begin{matrix} C & 0 & S & 0 \\ 0 & 1 & 0 & 0 \\ -S & 0 & C & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$R_{z,\theta} = \begin{matrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

(Combination from PDFs, Books)

(C = cos, S = sin θ)

Cube

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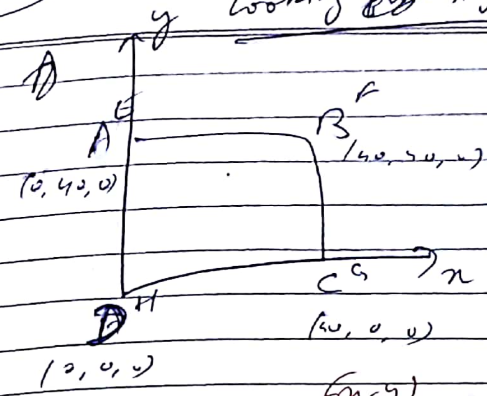
looking into x-y plane

RMCS



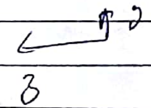
(x-y)

Front View → void drawcube_front(){
line(A_x, A_y, B_x, B_y);

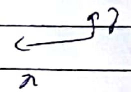
(12 edges)

(y-z)

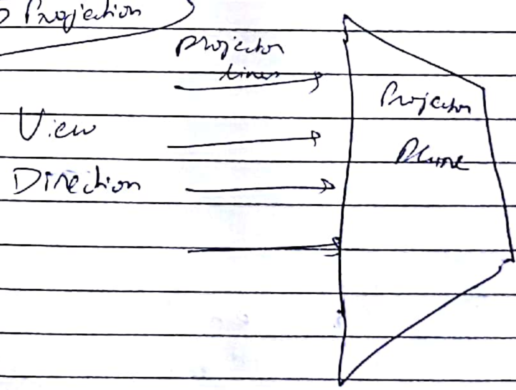
Side View → line(A_z, A_y, B_z, B_y)



Top View → line(A_x, A_z, B_x, B_z).



3D → 2D Projection



(Project Plane also makes angles with 3 axes)

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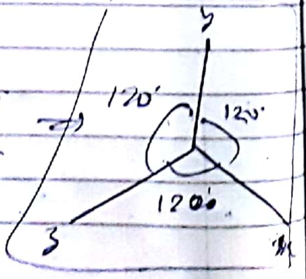
Projection lines

- Parallel
  - falling at $\alpha = 90°$ to projection plane (Orthographic Projection)
    - Isometric — Make equal angles with all co-ordinate axes
      - Imagine angle made by main diagonal of cube with its 3 edges → Screen पे [illegible]
    - Dimetric — equal angle with any 2 axes
      - xy
      - yz
      - xz
    - Trimetric — None of 3 angles with 3 axes are same.
  - $\alpha \neq 90°$ (OBLIQUE Projection)
    - Cavalier
    - Cabinet
- Non Parallel (Perspective Projection) (May be converging or diverging)



4E? Full Screen — Centre 320, 240

So, (320 + x, 240 − y)

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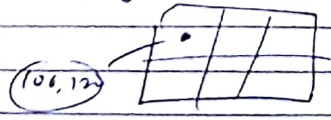
y x
480 × 640 screen

⇒ 240 × 213 Screens.

Centre → 120, 106 of first screen



Cube — 8 vertices — obj[8][4]

x [0, 40, 40, 0, 0, 40, 40, 0]

y [ ]

z [ ]

[1, 1, 1, 1, 1, 1, 1, 1] Ones

Matrix of Cube

[0, 40, 40, 0, 0, 40, 40, 0
...
1, 1, 1, 1, 1, 1, 1, 1]

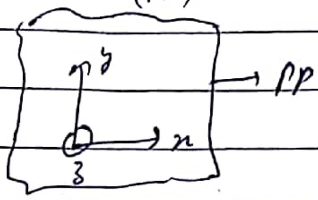
Rotx[4][4] = {1, 0, 0, 0,
0, C, S, 0,
0, S, C, 0,
0, 0, 0, 1}.

RHCS — Thumb — x
Index finger — y
Middle finger — z — Towards you.

5/4

Graphics Lab

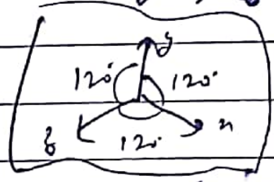
Projection Plane = Retina
(PP)



Isometric View :- PP makes equal angles with all axes.

So, Rotate about y, x.

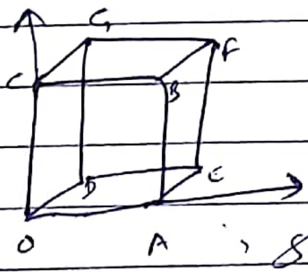
Final view —



So, $C = Proj(x-y) \times R_{x,\theta_2} \times R_{y,\theta_1}$

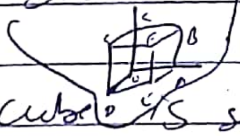
Composite Matrix

Not 120°



O	0	0	0	D	0	0	1
A	1	0	0	E	1	0	1
B	1	1	0	F	1	1	1
C	0	1	0	G	0	1	1

, should be



So, simple PP — cube is seen as a square

Isometric view — 3 faces of cube are visible.

$|OA| = |OC| = |OD| = 1.$

For CW, ACW — उपर से देखा
i.e. $\theta_x ; \theta_y$.

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After 2 rotations

$O' = (0, 0, 0)$
$A' = (x_x, y_x, z_x)$
$C' = (x_y, y_y, z_y)$
$D' = (x_z, y_z, z_z)$

$Proj(x\text{-}y) \rightarrow A' = (x_x, y_x, 0)$
$C' = (x_y, y_y, 0)$
$D' = (x_z, y_z, 0)$

~~SD Rot~~

$|OA'| = \sqrt{x_x^2 + y_x^2}$
$|OC'| = \sqrt{x_y^2 + y_y^2}$
$|OD'| = \sqrt{x_z^2 + y_z^2}$

Now, Rot^n about y — CW ⇒ -ve
Rot^n about z — ACW — +ve.

$R_{y(-\theta_1)}^{z}$

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~~$C = Proj(x-y) \times \cos\theta, 0 - \sin\theta, 0$~~

Rough

$$C = Proj(x\text{-}y) \cdot \begin{array}{c} \theta_2 \\ \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & C & -S & 0 \\ 0 & S & C & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \end{array} \times \begin{array}{c} \theta_1 \\ \begin{vmatrix} C & 0 & -S & 0 \\ 0 & 1 & 0 & 0 \\ S & 0 & C & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \end{array}$$

(Mat?)

$$= \begin{vmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ -S\theta_1 S\theta_2 & C\theta_2 & -S\theta_2 C\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \rightarrow (A)$$

Matrix ? Proj x-y

Hence, z-coordinate 2D

Reason (1) — So, $A' = \text{Column } 1 = (\cos\theta_1, -S\theta_1 S\theta_2, 0)$
$C' = (0, C\theta_2, 0)$
$D' = (-S\theta_1, -S\theta_2 C\theta_1, 0)$

Foreshortening Factor w.r.t. Edges would be Same in Isometric.

i.e. $\boxed{\dfrac{l_{x'}}{l_x} : \dfrac{l_{y'}}{l_y} : \dfrac{l_{z'}}{l_z}}$ ✓

$l_x \equiv$ Edge length.

So, $\frac{|OA'|}{|OA|} = \frac{|OC'|}{|OC|}$

$\sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$

$(\cos\theta_1)^2 + (\sin\theta_1)^2(\sin\theta_2)^2 = (\cos\theta_2)^2$ — (1)

$\frac{|OC'|}{|OC|} = \frac{|OD'|}{|OD|}$

$(\cos\theta_2)^2 = (\sin\theta_1)^2 + (\sin\theta_2^2)(\cos\theta_1^2)$ — (2)

$C\theta_1^2 - C\theta_2^2 = (\sin\theta_1 \sin\theta_2)^2$

$(C\theta_1 - C\theta_2)(C\theta_1 + C\theta_2) = \ldots$

35.26
45

$\cos^{-1}\sqrt{\frac{2}{3}}$

Find θ_1 & θ_2 from (1) & (2) & put in matrix A ⇒ Then multiply (A) by vertex matrix (4×8), to get vertices in Isometric View.

Dimetric View

Case I :- K : 1 : 1 (K=1 for Isometric).
Case II :- 1 : K : 1
Case III :- 1 : 1 : K

there, $K \frac{|OA'|}{|OA|} = \frac{|OC'|}{|OC|}$

$\Rightarrow K((\cos\theta_1)^2 + (\sin\theta_1)^2(\sin\theta_2)^2) = (\cos\theta_2)^2$ — (1)

& $\frac{|OC'|}{|OC|} = \frac{|OD'|}{|OD|}$

$\Rightarrow (\cos\theta_2)^2 = (\sin\theta_2)^2 + (\sin\theta_2)^2(\cos\theta_1)^2$.

Rough

$\cos\theta_1^2 - \cos\theta_2^2 = \sin\theta_1^2 \sin\theta_2^2$

$\cos\theta_2^2 - \sin\theta_1^2 = \sin\theta_2^2 \cos\theta_1^2$

$\frac{\cos\theta_1^2 - \cos\theta_2^2}{\sin\theta_1^2} = \frac{\cos\theta_2^2 - \sin\theta_1^2}{\cos\theta_1^2}$

$C\theta_1^4 - C\theta_2^2 C\theta_1^2 = C\theta_2^2 S\theta_1^2 - S\theta_1^4$

$C\theta_1^4 + S\theta_1^4 = C\theta_2^2$

$a^4 + b^4 = C\theta_2^2$

$1 - a^4 - b^4 = S\theta_2^2$

$$\cancel{\frac{\cos\theta_1^2}{\cos\theta_2^2} + \sin\theta_1^2(+\theta^2) = 1}$$

Isometric Solved

$$a^2 - b^2 = (1-a^2)(1-b^2)$$

$$b^2 - (1-a^2) = (1-b^2)(a^2)$$

$$a^2 - \cancel{b^2} = 1 + a^2 b^2 - a^2 \cancel{- b^2}$$

$$2a^2 = 1 + a^2 b^2 \quad — (1)$$

$$b^2 - 1 + \cancel{a^2} = \cancel{a^2} - a^2 b^2$$

$$b^2 + a^2 b^2 = 1 \quad — (2)$$

From (2) $\rightarrow$ $b^2 = \dfrac{1}{1+a^2}$

In (1) $\rightarrow$ $2a^2 = 1 + \dfrac{a^2}{1+a^2}$

$$2a^2(1+a^2) = 1 + 2a^2$$

$$\cancel{2a^2} + 2a^4 = 1 + \cancel{2a^2}$$

$$a^4 = \frac{1}{2}$$

$$a^2 = \frac{1}{\sqrt{2}}$$

$$\cos\theta_1^2 = \frac{1}{\sqrt{2}}$$

? $\rightarrow$ 45°

$$b^2 = \frac{1}{1+a^2}$$

$$= \frac{1}{1+\frac{1}{2}} \Rightarrow b^2 = \frac{2}{3}$$

So, $\cos\theta_2 = \sqrt{\frac{2}{3}}$

$$\theta_2 = \cos^{-1}\sqrt{\frac{2}{3}}$$

$\to 35.26^\circ$

For Dimetric

$$k(a^2-b^2) = 1+a^2b^2-a^2-b^2$$
$$ka^2-kb^2 = 1+a^2b^2-a^2-b^2$$

$$(k+1)a^2+(1-k)b^2 = 1+a^2b^2 \quad \text{--- (1)}$$

& $b^2+a^2b^2 = 1$ --- (2)

$$\Rightarrow b^2 = \frac{1}{1+a^2}$$

In (1) → $(k+1)a^2 + \frac{1-k}{1+a^2} = 1+\frac{a^2}{1+a^2}$

$$(1+a^2)a^2(k+1) + \cancel{1}-k = \cancel{1}+2a^2$$
$$(a^2+a^4)(k+1) = 2a^2+k$$

$$CA^2 + CA - 2A - C + 1 = 0$$

$\left(\begin{array}{l} k+1 = C \\ k = C-1 \end{array}\right)$ $CA^2 + A(C-2) - (C-1) = 0$

4 cases → ±45, ±35.26

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$$A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 - c \pm \sqrt{4 + \cancel{\;} + 5c^2 \cancel{\;} - 8c}}{2c}$$

7/4 | SE – Absent

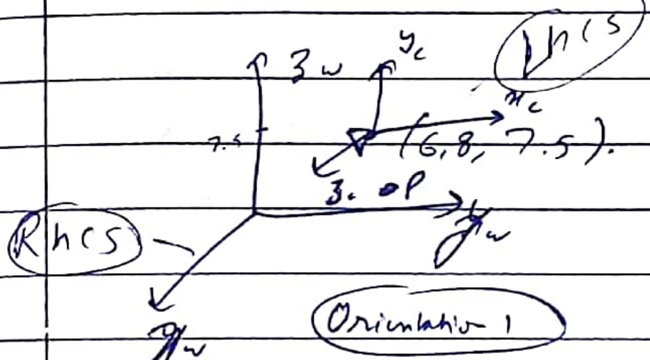
Page Replacement Algo (PRA)

V. IMP. TOPIC → (Must, in End Sem). Numerical

When there is a ~~fault~~ page fault and no eg. warning msg. b4 formatting pen drives.

2/4. **Graphics (COE-2).**

3D Transformation → View through a camera.



W = world ~~window~~

C = camera.

Z_c → Pointing to origin

y_c → Up. (not totally up)

x_c → on $Z_w = 7.5$ plane.

(Not Z_c)

$P = (1, 2, 3)$

(w.r.t. world co-ordinate system) — WCS

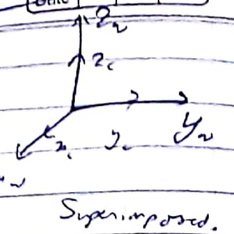
i.e. $x_w = 1$, $y_w = 2$, $z_w = 3$.

Q. Find P wrt. Camera Co-ordinates System. CCS

A. W → RHCS (Right Handed Co-ordinate System)
C → LHCS (Left do).

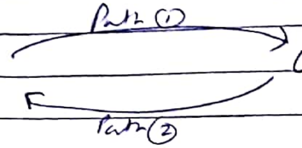
Only difference b/w LHCS & RHCS is z-axis is opposite

Goal → To orient like this —

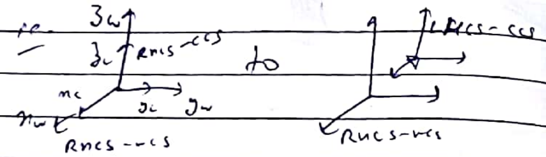


Finding Transform matrix which transforms CCS into WCS.

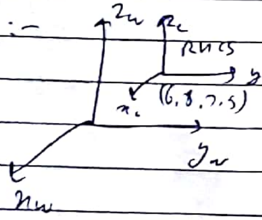
Orientation 2



Path 2 first i.e. —



Step 1:-

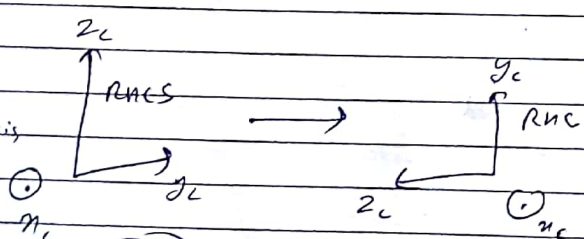


$$T = \begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step 2:-

Rot^n about x-axis by 90°

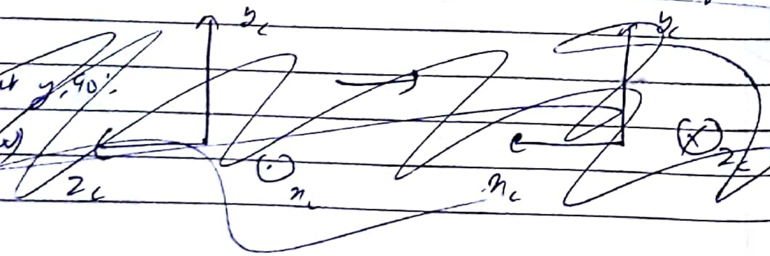
(CW or ACW?) → ACW ✓ (Look from x_c +ve point of view)



Step 3:-

Rot^n about y, 90°.

(CW or ACW)



This step is IMP

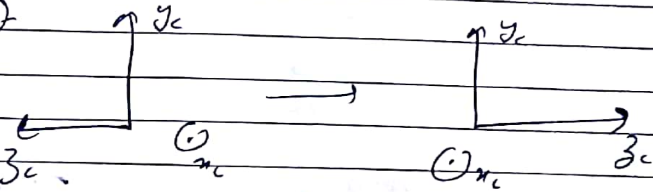
Matrix of Step 2,3 Later

Step 3: RHCS to LHCS:- Reverse z

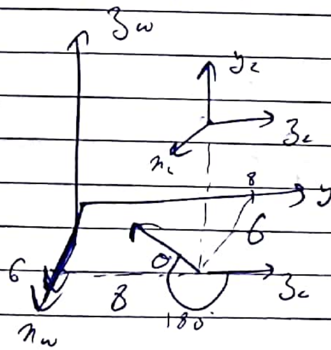
(Flip along xy plane, i.e. z axis Reverse axis $R_{z(1)}$).

(We would have just made the configuration in RHCS, but it is specified that CCS is LHCS, so RHCS to LHCS step is must.)

Step 3!



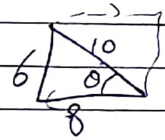
Step 4:- Point z_c towards origin.



So, Rotate CW, about y

$\phi(\text{Angle}) = -(180+\theta)$

CW → -ve



$$R_y(\theta) = \begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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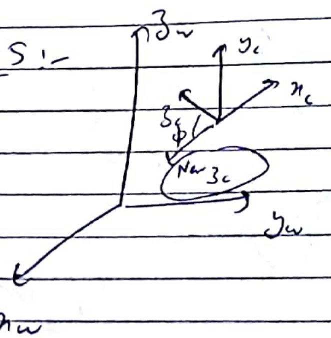
$\cos\phi = \cos(-(180+\theta))$
$= \cos(180+\theta)$
$= -\cos\theta$
$= \frac{-8}{10}$

III Quad
$\cos -ve$

$\sin\phi = \sin(-(180+\theta))$
$= -\sin(180+\theta)$
$= -(-\sin\theta)$
$= \sin\theta$
$= \frac{6}{10}$

So, $R_{y, -180+\theta} = \begin{bmatrix} -8/10 & 0 & 6/10 & 0 \\ 0 & 1 & 0 & 0 \\ -6/10 & 0 & -8/10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Step 5 :-



z_c points towards origin but it's in xy plane of WWCS.
So, bring it towards origin.

Rotate about x, by ϕ CW

So, Angle $= -\phi$.

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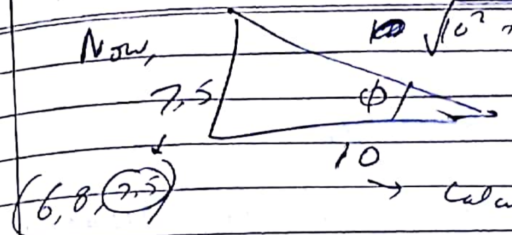
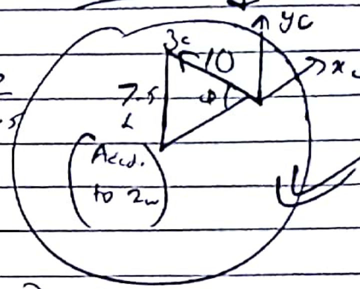
Now,



$\sqrt{10^2 + (7.5)^2} = 12.5$

→ Calculated in Step 4.



$\cos(-\phi) = \cos\phi = \frac{10}{12.5}$

$\sin(-\phi) = \frac{-7.5}{12.5}$

$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & -s & 0 \\ 0 & s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10/12.5 & 7.5/12.5 & 0 \\ 0 & -7.5/12.5 & 10/12.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Prev Step

C inverse
∵ Patch I was needed

So, $\boxed{C = R_{x,\theta_3} \times R_{y,\theta_2} \times R_{x,90}}$, & we need C^{-1}

$\times$ LHCS

Matrix of Step 2 → Simply → $R_{x,90}$ → $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Matrix of Step 3 → Simple → z reversed → $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$