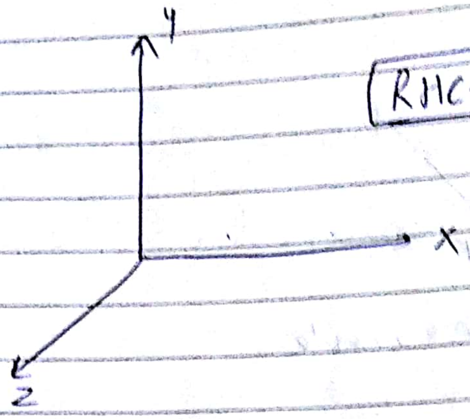
19/3/18

3D Transformation



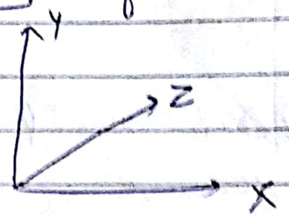
$\boxed{\text{RHCS}}$ – Right handed coordinate system

{ thumb → x axis
index finger → y axis
middle finger → z axis

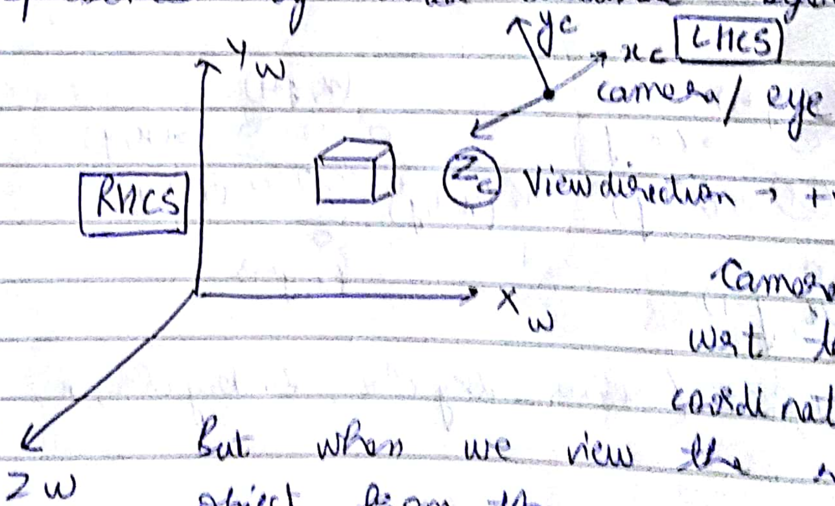
⇓

For both RHCS & LHCS

$\boxed{\text{LHCS}}$ – Left handed coordinate system



In real time, all real time objects are represented by world coordinate system in RHCS.



View direction → +ve Z axis

Camera is also wrt the world coordinate system.

But when we view the real time object from the camera, LHCS is used & we view the object in +ve Z axis

i.e.

View direction → +ve z axis

3D Transf^{m}

1. Translation → $\begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. Scaling → $\begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3. Rotation.

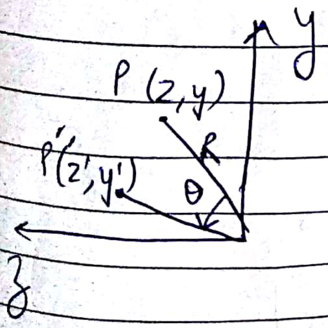
⇒ We ~~Alwys~~ always follow homogenity in terms of the operation in all the above 3 transformations.

(3) Rotation → i) about x axis
→ ii) about y axis
→ iii) about z axis

*{ Anticlockwise direc^{n} → +ve. }

(1) Rotation about x axis

see from +ve x → y → z (anticlockwise)

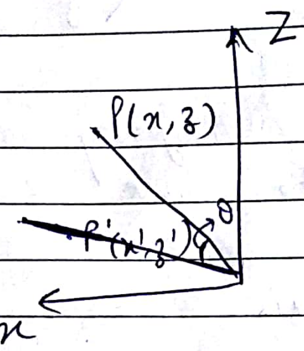


$$x' = x$$
$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$

$$R_{x,\theta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rot about x by an angle θ, anticlockwise

(ii) Rotation about y axis



$Z \rightarrow x$

$$x' = x\cos\theta + z\sin\theta$$

$$y' = y$$

$$z' = -x\sin\theta + z\cos\theta$$

$$R_{y,\theta} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(iii) Rotation about z axis

$$R_{z,\theta} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Scanned by CamScanner

Screen / Retina is 2D

→ So, the projection is taken on a plane.

Camera analogy concept

monitor/screen

Side view	Front view	Top view
Isometric view	Dimetric view	~~[illegible]~~ Perspective view

→ 6 camera pos^n for an object

→ Observing the 3D view.

{like CCTV cameras at different pos^n}

view ≡ Projection!

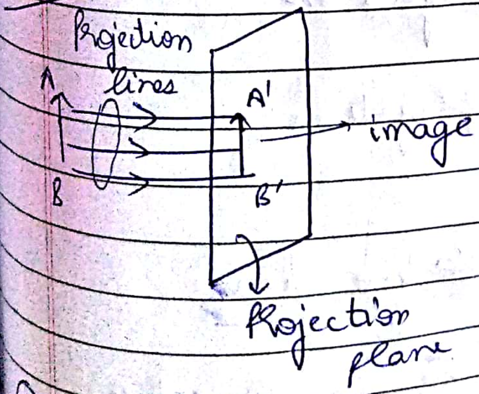
(3D → 2D)
transformation

Front view → projection on x-y plane.

Projection → ① Parallel ⟹ (Projector lines are parallel)

Projection → ② Non-parallel

①



(i) $\alpha = 90^\circ$

If $\angle AA'B' = 90^\circ$

↳ It is called as orthographic projection

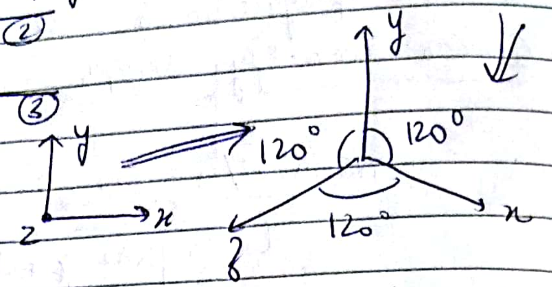
projector lines fall on plane at 90°.

(ii) $\alpha \neq 90^\circ$ Oblique projection. Projector lines are parallel only but the projection plane is tilted.

Isometric projection

① Projector lines are parallel with projection plane ⊥
projector lines make equal ∠s with all 3 coordinate axis

(i) Orthographic projec^n
(2)
(3)



We need a transformation to obtain the coordinate axis as above.

② dimetric projection

Object makes equal angle with any 2 coordinate axes.

③ Trimetric projection

unequal angles with all 3 coordinate axes.

(ii) Oblique projection

- Cabinet projection
- Cavalier projection

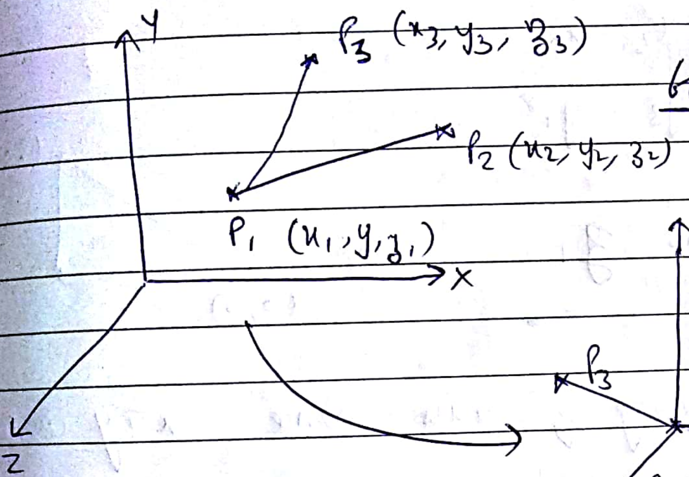
② Non-parallel → Perspective View

In real time, camera works in perspective view.

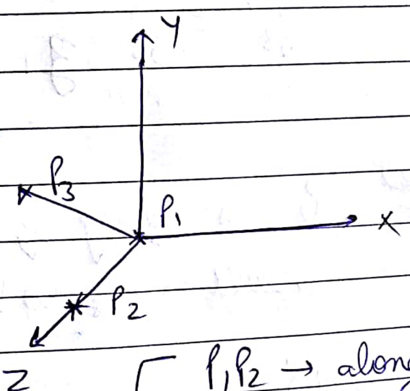
Eg: Viewing an object from far away, obj seems smaller, and large when nearby.

2 cases
- camera stationary, object moving
- object stationary, camera moving.

Exercise - 1



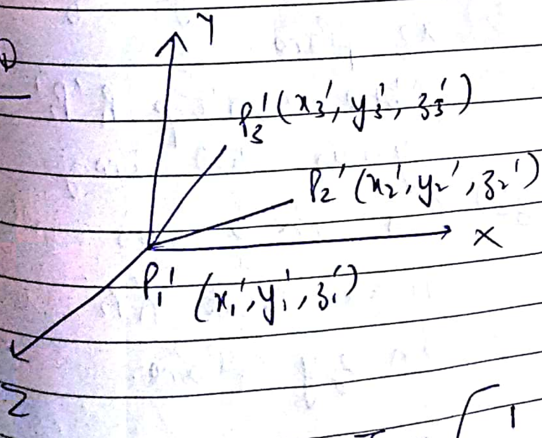
<u>Given</u> P_1, P_2, P_3 in 3 dimensional space



What kind of composite transformation is needed?

$P_1P_2 \rightarrow$ along z axis
$P_1P_3 \rightarrow$ is on y-z plane

<u>Step 1</u>

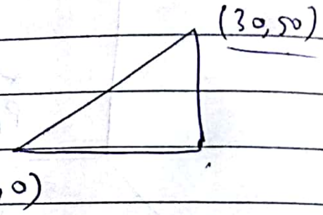


$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

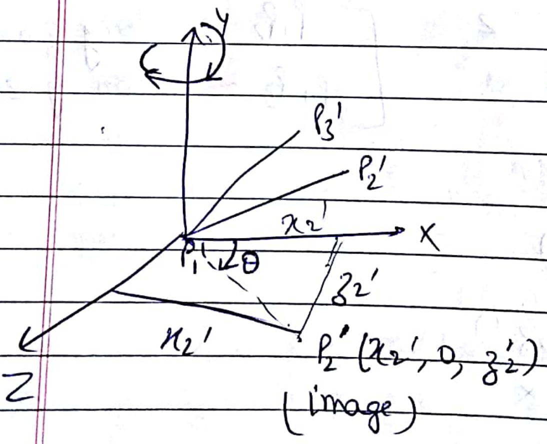
Scanned by CamScanner

$$x_2' = x_2 - x_1.$$
$$y_2' = y_2 - y_1$$
$$z_2' = z_2 - z_1$$

$$x_3' = x_3 - x_1$$
$$y_3' = y_3 - y_1$$
$$z_3' = z_3 - z_1$$



Step ② P_2 is along z axis hence x & y coordinates should be zero!



We put torch at y axis top & image of P_2 is on xz plane.
Now rotating $P_1'P_2'$ about y axis till image of P_2 lies on z axis will mean that $P_1'P_2'$ will be on zy plane.

$$R_{y,-(90-\theta)}$$

$$D_1 = \sqrt{(x_2')^2 + (z_2')^2}$$

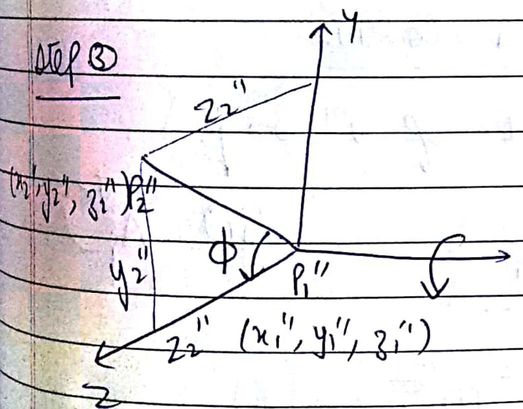
(clockwise direction rotation by an ∠ of $90-\theta$)

$$\cos(\theta - 90) = \sin\theta = \cancel{\frac{x_2'}{D_1}} \quad \frac{z_2'}{D_1}$$

$$\sin(\theta - 90) = \cos\theta = \cancel{\frac{z_2'}{D_1}} \quad \frac{-x_2'}{D_1}$$

$$R_{y,\,\theta-90} = \begin{pmatrix} \frac{z_2'}{D_1} & 0 & \frac{-x_2'}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ x_2'/D_1 & 0 & z_2'/D_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step ③



Anticlockwise rotation about x axis

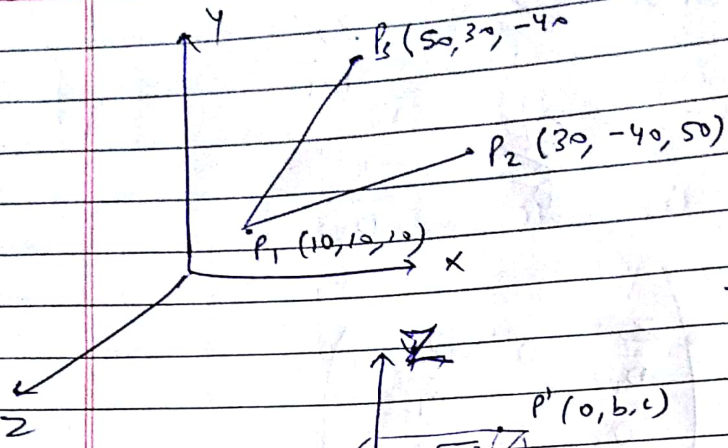
$$D_2 = \sqrt{(y_2'')^2 + (z_2'')^2}$$

$$\cos\phi = \frac{z_2''}{D_2}$$

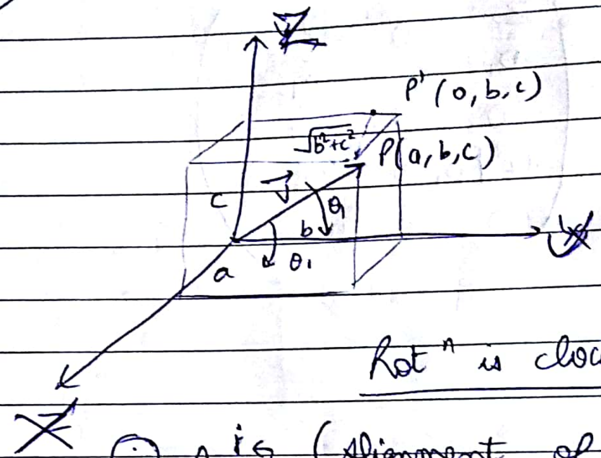
$$R_{x,\,\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_2''}{D_2} & \frac{-y_2''}{D_2} & 0 \\ 0 & \frac{y_2''}{D_2} & z_2''/D_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \sin\phi = \frac{y_2''}{D_2}$$

Step ④ x value of P_1P_3 will tell whether P_3 is on which side of yz plane & accordingly a rotation will be done to bring P_1P_3 on the y-z plane.

23/8/18



$$\vec{V} = ai + bj + ck$$



Rotⁿ is clockwise

(i) $A_{\vec{v}}^{j}$ ~~(Alignment of $\vec{v}$ on j.)~~

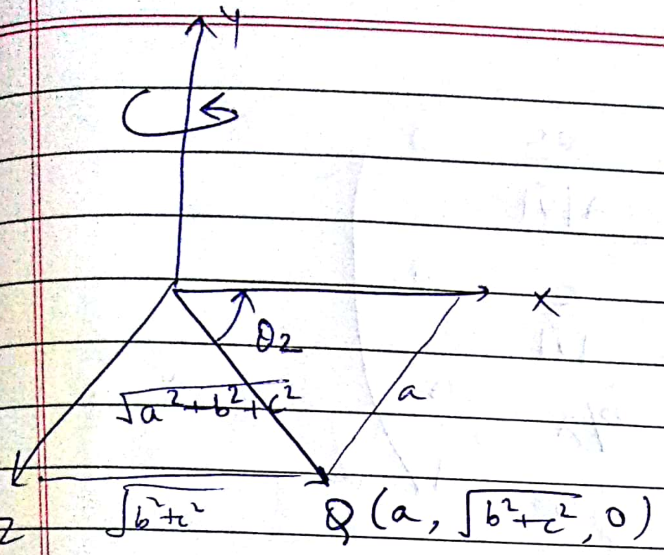
(ii) $A_{j}^{\vec{v}}$

$R_{x, -\theta_1}$

$$\cos(-\theta_1) = \frac{b}{\sqrt{b^2+c^2}}$$

$$\sin(-\theta_1) = \frac{-c}{\sqrt{b^2+c^2}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & b/\sqrt{b^2+c^2} & c/\sqrt{b^2+c^2} & 0 \\ 0 & -c/\sqrt{b^2+c^2} & b/\sqrt{b^2+c^2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b/\lambda & c/\lambda & 0 \\ 0 & -c/\lambda & b/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$R_{z,\theta_2} = \begin{bmatrix} \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & \frac{-a}{\sqrt{a^2+b^2+c^2}} & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2+c^2}} & \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos\theta_2 = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

$$|\vec{V}| = \sqrt{a^2+b^2+c^2}$$

Let $\lambda = \sqrt{b^2+c^2}$

$$\sin\theta_2 = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$R_{z,\theta_2} = \begin{bmatrix} \frac{\lambda}{|\vec{V}|} & \frac{-a}{|\vec{V}|} & 0 & 0 \\ \frac{a}{|\vec{V}|} & \frac{\lambda}{|\vec{V}|} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Composite Matrix (C) $= R_{z,\theta_2} \times R_{x,-\theta_1}$

$$= \begin{pmatrix} \frac{\lambda}{|\vec{V}|} & \frac{-a}{|\vec{V}|} & 0 & 0 \\ \frac{a}{|\vec{V}|} & \frac{\lambda}{|\vec{V}|} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b/\lambda & c/\lambda & 0 \\ 0 & -c/\lambda & b/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{\vec{v}}^{j} \circlearrowleft = C = \begin{pmatrix} \frac{\lambda}{|\vec{v}|} & \frac{-ab}{\lambda|\vec{v}|} & \frac{-ac}{\lambda|\vec{v}|} & 0 \\ \frac{a}{|\vec{v}|} & \frac{b}{|\vec{v}|} & \frac{c}{|\vec{v}|} & 0 \\ 0 & \frac{-c}{\lambda} & b/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{j}^{\vec{v}} \circlearrowleft = \left[A_{\vec{v}}^{j} \circlearrowleft\right]^{-1}$$

$$= \left[R_{z,\theta_2} \times R_{x,-\theta_1}\right]^{-1}$$

$$= (R_{x,-\theta_1})^{-1} \times (R_{z,\theta_2})^{-1}$$

$$= (R_{x,\theta_1}) \times (R_{z,-\theta_2})$$

$$\boxed{A_{\vec{v}}^{k} \circlearrowleft = R_{x,90^\circ} * A_{v}^{j} \circlearrowleft} \longleftarrow (1)$$

$P_1 (0,0,0)$ $\quad$ P_2 $(20, -50, 40)$

$$\boxed{A_{\vec{v}}^{i} = R_{z,90^\circ} * A_{v}^{j} \circlearrowleft}$$

$$\lambda = \sqrt{(50)^2+(40)^2}$$

$$= \sqrt{4100} = 64.03$$

$$|\vec{N}| = \sqrt{20^2+50^2+40^2} = \sqrt{4500}$$

$$= 67.08$$

$$C = \begin{pmatrix} \frac{64.03}{67.08} & \frac{1000}{4295.1} & \frac{-800}{4295.1} & 0 \\ \frac{20}{67.08} & \frac{-50}{67.08} & \frac{40}{67.08} & 0 \\ 0 & -40/64.03 & -50/64.03 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_{x,90^\circ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

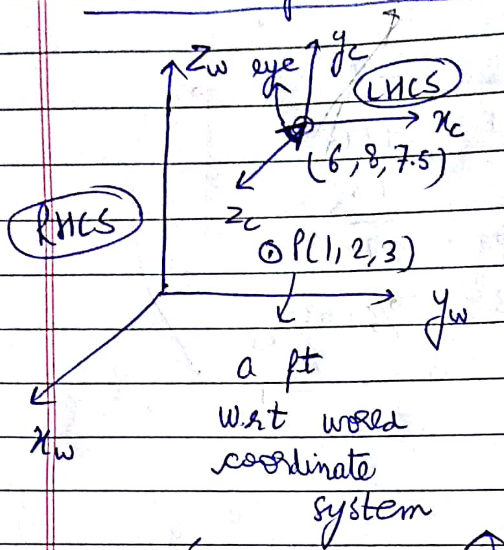
Using (1),

$R_{x,90^\circ} \times A_{\vec{V}}^{\vec{i}}$

2/4/18

3D Transformation

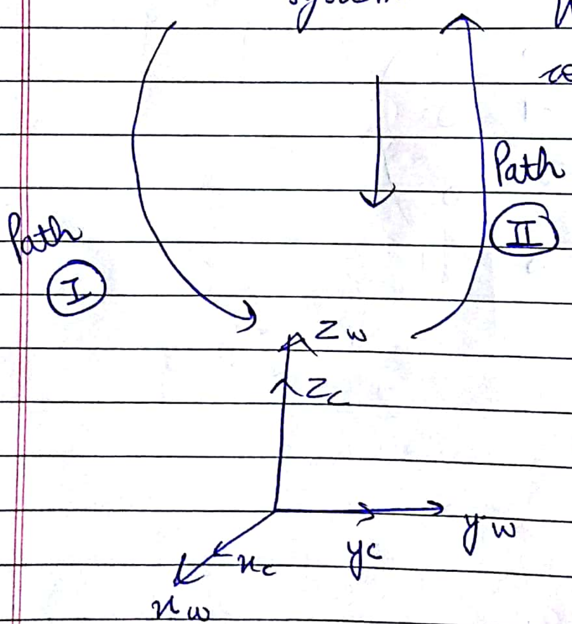
View through a camera



[Z_c is pointing to origin
y_c is up
x_c is on $z_w = 7.5$ plane

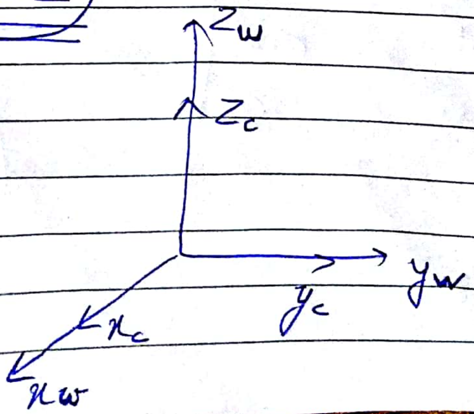
$P(1 = x_w,\ 2 = y_w,\ 3 = z_w)$

What will be P w.r.t camera coordinates?



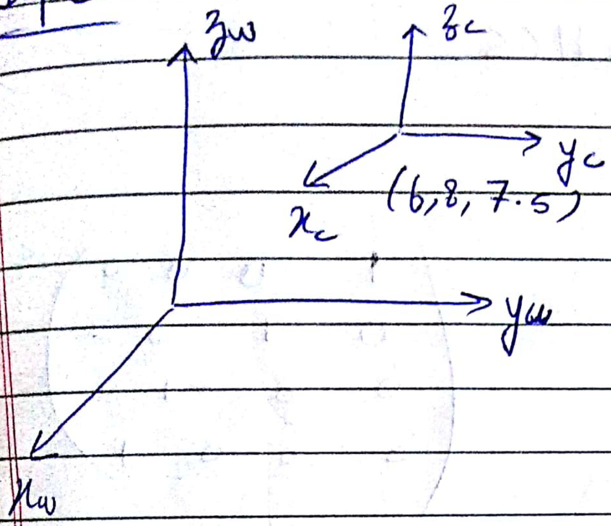
transform in such a way that the 2 axes systems coincide.

Path II



classmate
Date
Page

Step ①



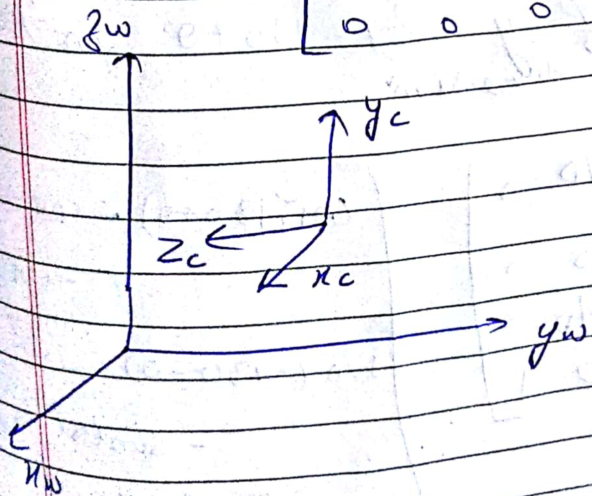
Translation

$$\begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$C = R_{x,\phi} * R_{y,\theta} * R_{x,90^\circ}$$

Step ②

$$R_{x,90^\circ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

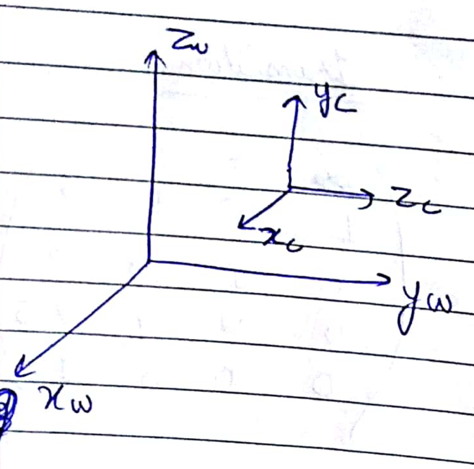


Scanned by CamScanner

classmate
Date
Page

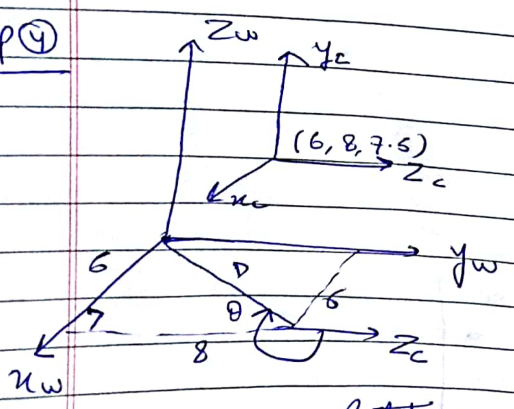
Step ③

RHCS $\xrightarrow{to}$ LHCS



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step ④



$$D = \sqrt{6^2 + 8^2} = 10$$

Rotation of y by 180+θ in the clockwise direction

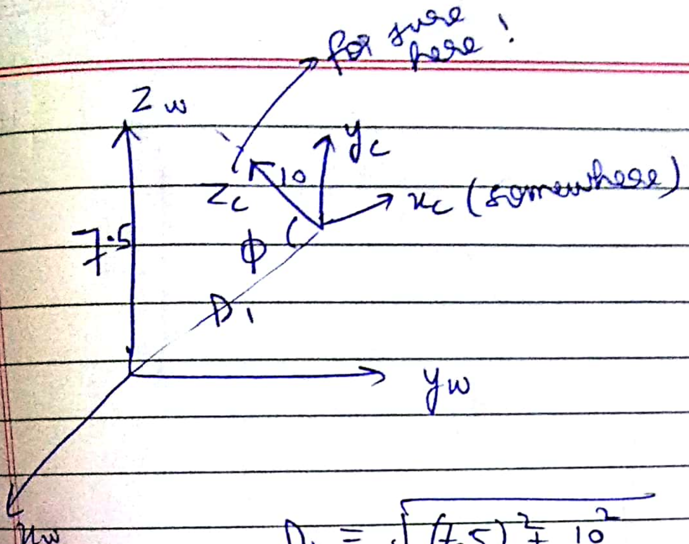
$$R_{y,-(180+\theta)} = \begin{bmatrix} -\frac{8}{D} & 0 & 6/D & 0 \\ 0 & 1 & 0 & 0 \\ -6/D & 0 & -8/D & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\sin\theta = \frac{6}{D}$

$\cos\theta = \frac{8}{D}$

$\cos(-(180+\theta)) = \cos(180+\theta)$
$= -\cos\theta$

$\sin(-180-\theta)$
$= -\sin(180+\theta)$
$= \sin\theta$



$$D_1 = \sqrt{(7.5)^2 + 10^2} = 12.5$$

Step ① Clockwise rotation about the x axis by an angle ϕ.

$$\sin\phi = \frac{7.5}{D_1}$$

$$\cos\phi = \frac{10}{D_1}$$

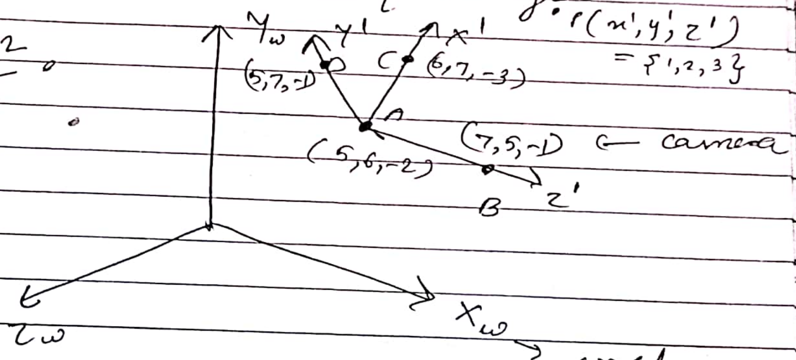
$$R_{x,-\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{10}{D_1} & 7.5/D_1 & 0 \\ 0 & -7.5/D_1 & 10/D_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Graphics

3D Viewing

* No matter which coordinate we transform, motive is to align the 2 co-ordinate system.

Ques 2.



Find ~~coor.~~ P values w.r.t world coordinate system.

① Are the coordinates 90° to each others?

$\vec{AB} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{AC} = \hat{i} + \hat{j} - \hat{k}$

$\vec{AD} = \hat{j} + \hat{k}$

$\vec{AB} \cdot \vec{AC} = 2 - 1 - 1 = 0$

$\vec{AC} \cdot \vec{AD} = 0 + 1 - 1 = 0$

$\vec{AB} \cdot \vec{AD} = 0 - 1 + 1 = 0$

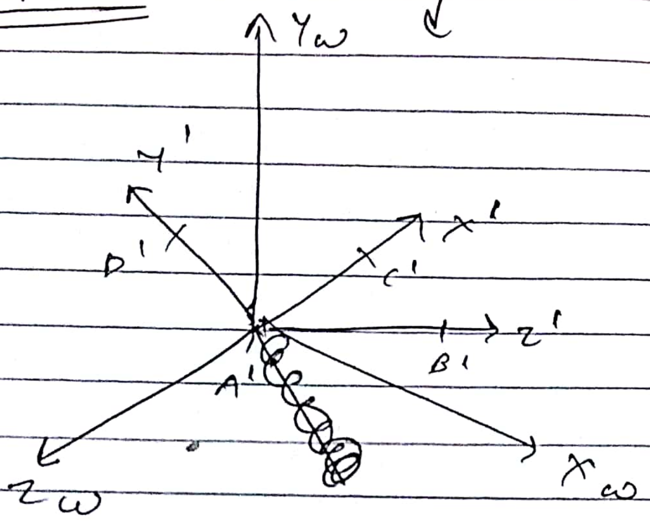
* Coordinate system is correct and at 90°

[Translation]

Date:

Page No.

Step 1



Translation to origin

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A' = [0, 0, 0]$

$B' = [2, -1, 1]$

$C' = [1, 1, -1]$

$D' = [0, 1, 1]$

$A_v' =$

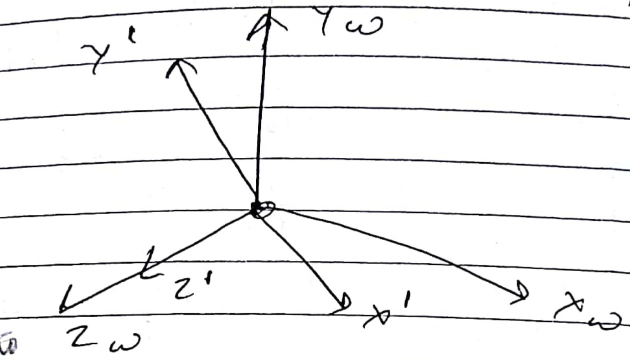
Scanned by CamScanner

To alignm to z' to z_ω

$$A_{z'}^{z(k)} = A_{z'(2i-j+k)}^{z(k)}$$

$\Uparrow$

j use the Alignment matrix.



alignment to z_ω

alignment to y axis

$$A_v^{j} = \begin{bmatrix} \frac{\lambda}{|v|} & \frac{-ab}{\lambda|v|} & \frac{-ac}{\lambda|v|} & 0 \\ \frac{a}{|v|} & \frac{b}{|v|} & \frac{c}{|v|} & 0 \\ 0 & -c/\lambda & b/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{\vec{v}}^{k} = R_{x,90} * A_v^{j}$$

$$\lambda = \sqrt{b^2+c^2} \qquad \lambda = \sqrt{2}$$

$$|\vec{v}| = \sqrt{a^2+b^2+c^2} \qquad V = \sqrt{6}$$

$$A_v^{j} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{6}} & \frac{\cancel{2}}{\cancel{2}\sqrt{3}} & \frac{-\cancel{2}}{\cancel{2}\sqrt{3}} & 0 \\ \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & -1 & -1 & \end{bmatrix}$$

$$\lambda|v| = \sqrt{12} = 2\sqrt{3}$$

Date:
Page No.

$$R_{x_3}^{90} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \frac{1}{\sqrt{3}} & +\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Coordinates after rotation

$A'' = [0, 0, 0]$

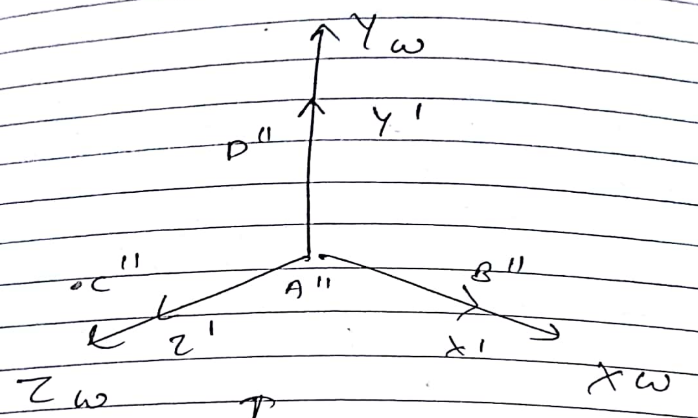
$B'' = [0, 0, \sqrt{6}]$

$C'' = [\sqrt{3}, 0, 0]$

$D'' = [0, \sqrt{2}, 0]$

Date:
Page No.

Final coordinates



In single rotation, we were able to allign the 2 coordinate system.

$$C = T_2 T_1$$

Composite matrix

$$C = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

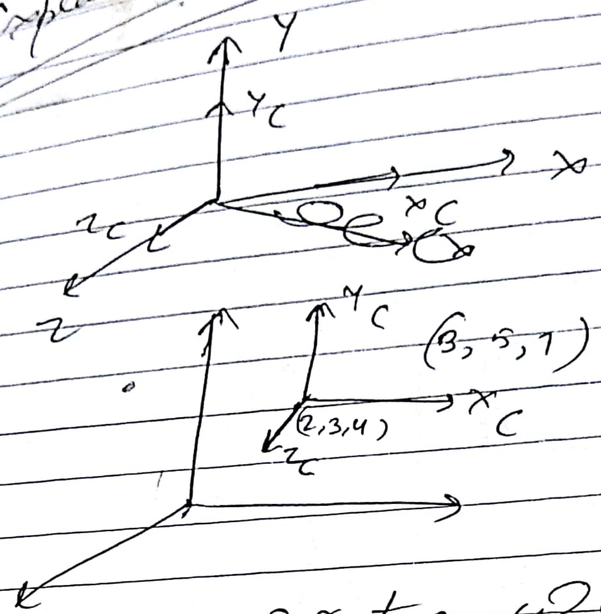
Multiply P by Ch's to

$$\begin{matrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{matrix}$$

$$\begin{array}{cccc} A & B & C & D \\ \end{array}$$
$$\begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

lation

Explanations



Objects are w.r.t. world

Final composite matrix =

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{5}{\sqrt{3}} - \frac{6}{\sqrt{3}} - \frac{2}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{6}{\sqrt{2}} - \frac{2}{\sqrt{2}} \\ \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -5\sqrt{\frac{2}{3}} + \frac{6}{\sqrt{6}} - \frac{2}{\sqrt{6}} \\ 0 & 0 & 0 & \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{13}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{4}{\sqrt{2}} \\ \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \quad \frac{10+8}{\sqrt{6}}$$

$x = \cancel{\frac{1}{\sqrt{3}} + \frac{11}{\sqrt{3}}} - \frac{3}{\sqrt{3}} - \frac{13}{\sqrt{3}} = \frac{-13}{\sqrt{3}}$

$y = \frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}} - \frac{4}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$z = \frac{\sqrt{2}}{\sqrt{3}} - \frac{2}{\sqrt{6}} - \frac{3}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$

$P = \left(\frac{-13}{\sqrt{13}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}} \right)$

$\frac{2}{\sqrt{3}}$ $\frac{2}{\sqrt{2}}$ $+\frac{2}{\sqrt{6}}$ $\frac{10+8}{\sqrt{6}}$

Projections

3D object → 2D image

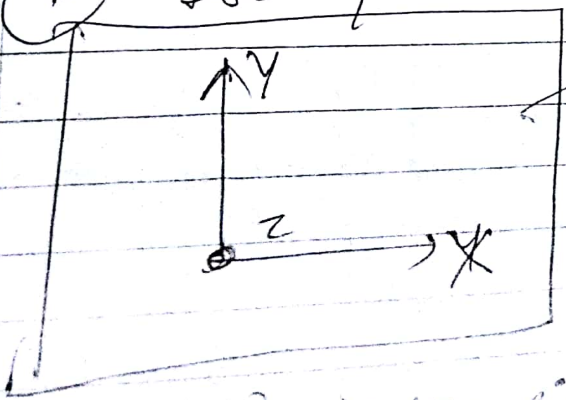
Objective

Frame on which image is formed can be static or movable

(1) When projector lines are parallel (orthographic)

Line falls at 90°

(a) Isometric projection



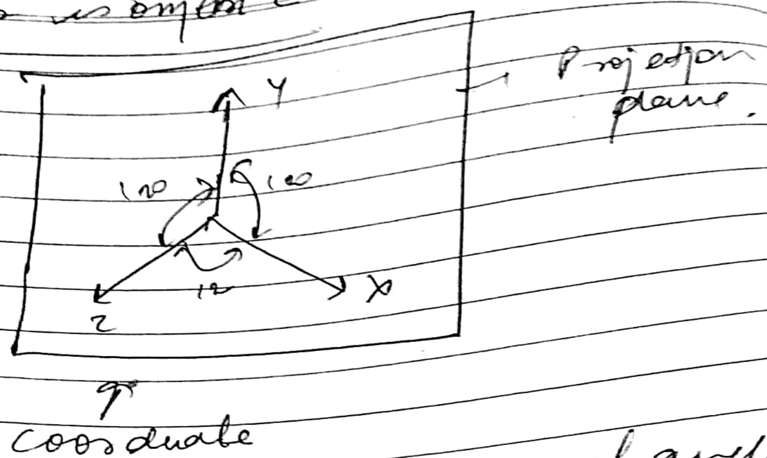
Projection plane

Z towards us.

Projector line parallel to ...

Date:
Page No.

for isometric



Projector lines falling at equal angle on coordinate axis.

How to obtain this?

$C = Proj[x\text{-}y] * R_{x,\theta_2} * R_{y,\theta_1}$

What is θ_1 and θ_2?

$C = Proj[x-y] * R_{x,\theta_2} * R_{y_1,-\theta_1}$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_1 & 0 & - \\ 0 & 1 & \\ +\sin\theta_1 & 0 & \\ 0 & 0 & \end{bmatrix}$$

Date:
Page No.

$$C = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1\sin\theta_2 & \cos\theta_2 & -\sin\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

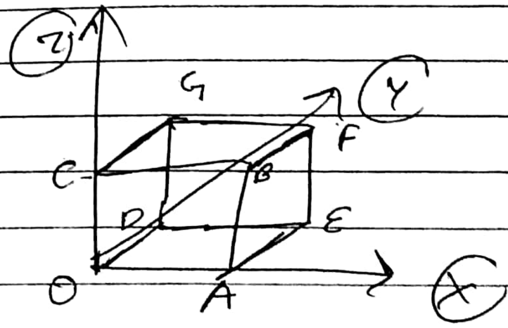
projection plane.

Equal angles = 120° need to be ensured

Image formed after rotation and before rotation, four shortening ~~vector~~ factor

l angle

The ration after should be before same.



$O = [0, 0, 0]$

$A = [1, 0, 0]$

$R_{y_1} - \theta_1$

$C = [0, 0, 1]$

$D = [0, 1, 0]$

$$\begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ +\sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Date:
Page No.

After rotation

A (x_x, y_x, z_x)

C (x_z, y_z, z_z)

D (x_y, y_y, z_y)

$\downarrow$ After Projection on (x-y plane)

A $(x_x \; y_x, 0)$

B $(x_z, y_z, 0)$

C $(x_y, y_y, 0)$

Four shortening factor

$f_x = \dfrac{|OA'|}{|OA|}$ four shorteng factor along x axis

$f_y = \dfrac{|OB'|}{|OB|}$ along y axis

$f_z = \dfrac{|OC'|}{|OC|}$ along z axis

Scanned by CamScanner

As isometric projection.

$$f_x : f_y : f_z$$
$$\underbrace{1 : 1}_{eq\ 1} : \underbrace{1}_{eq\ 2}$$

eq 1:

$$f_x = f_y$$

$$\frac{|OA'|}{|OA|} = \frac{|OB'|}{|OB|}$$

OA = OB
unit cube

$$\sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$$

$$C = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1 \sin\theta_2 & \cos\theta_2 & -\sin\theta_2 \cos\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{array}{ccc} A & B & C \\ \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Final coordinates

$$= \begin{bmatrix} \cos\theta_1 \\ -\sin\theta_1 \sin\theta_2 \end{bmatrix}$$

Scanned by CamScanner

Date:
Page No.

eq 1

$$\sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$$

$$\cos^2\theta_1 + \sin^2\theta_1 \sin^2\theta_2 = \cos^2\theta_2$$

eq 2 :

~~eee~~ $T_y = T_z$

$$\sqrt{x_y^2 + y_y^2} = \sqrt{x_z^2 + y_z^2}$$

$$\cos^2\theta_2 = \sin^2\theta_1 + \sin^2\theta_2 \cos^2\theta_1$$

$$1 + \sin^2\theta_2 (1) = 2\cos^2\theta_2$$

$$2\cos^2\theta_2 = \sin^2\theta_2 + 1$$

$\cancel{\sin^2\theta_2 +}$

$$2\cos^2\theta_2 - 1 + \cos^2\theta_2 = 1$$

$$3\cos^2\theta_2 = 2$$

$$\cos^2\theta_2 = 2/3$$

$$\cos\theta_2 = \sqrt{\frac{2}{3}}$$

$$\theta_2 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

Scanned by CamScanner

$\frac{2}{3} = \sin^2\theta_1 + \frac{1}{3}(1-\sin^2\theta_1)$

$\frac{1}{3} = \frac{2}{3}\sin^2\theta$

$\frac{1}{2} = \sin^2\theta$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta_1 = 45°$

$\theta_1 = \pm 45°$

$\theta_2 = \pm 35°26'$

Isometric can occur at only 1 combination of (θ_1, θ_2)

Put these values in C to get the final answer

Dimetric Projection

Projection lines are parallel.

Make equal angles with any 2 coordinate axes

Date:
Page No.

$f_x : f_y : f_z$

$1 : 1 : k$

$1 : k : 1$

$k : 1 : 1$

Any 3 possible cases

Case 1

$$\cos^2\theta_1 + \sin^2\theta_1 \sin^2\theta_2 = \cos^2\theta_2 \quad \text{—(1)}$$

$$\cos^2\theta_2 = k^2(\sin^2\theta_1 + \sin^2\theta_2 \cos^2\theta_1)$$

Solve these 2 equations somehow

Replace $\cos\theta_1$, $\cos\theta_2$ ...

prove for all cases

~~Correct~~

Date:

Page No.

$f_x : f_y : f_z$

$1 : 1 : k$

$\theta_1 = \sin^{-1}\left(\frac{k}{\sqrt{2}}\right)$

$\theta_2 = \sin^{-1}\left(\frac{k^2}{k^2+2}\right)$

$1 : k : 1$

$\theta_1 = \frac{1}{\sqrt{2}}$

$\theta_2 = \cos^{-1}\left(\sqrt{\frac{2k^2}{2+k^2}}\right)$

$k : 1 : 1$

$\theta_1 = \cos^{-1}\left(\sqrt{\frac{2}{2+k^2}}\right)$

$\theta_2 = \cos^{-1}\left(\sqrt{\frac{2}{2+k^2}}\right)$

Find k ranges now

② case 1

$1 > \frac{k}{\sqrt{2}} > -1$

$1 > \sqrt{\frac{k^2}{k^2+2}} > -1$

$1.414 > k > -1.414$

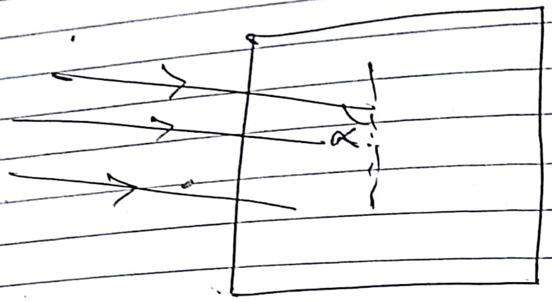
range is constant

$1 > \frac{k^2}{k^2+1} > 0$

$k^2+1 > k^2 > 0$

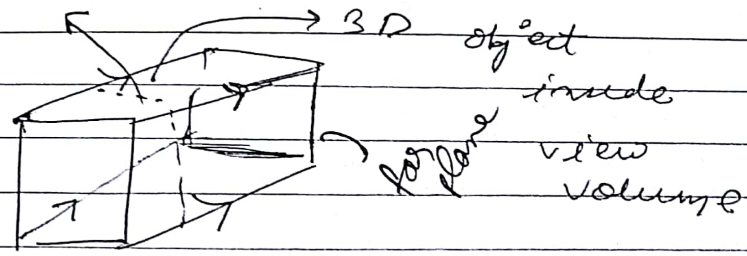
Always true

OBLIQUE Projection



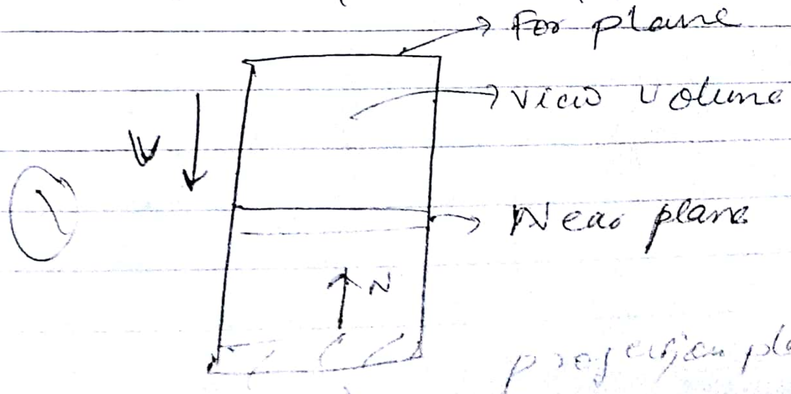
Projector lines are parallel but not falling on plane with 90.

$\alpha \neq 90^\circ$



These projector lines go far form a parallelopiped

Top View



Date:
Page No.

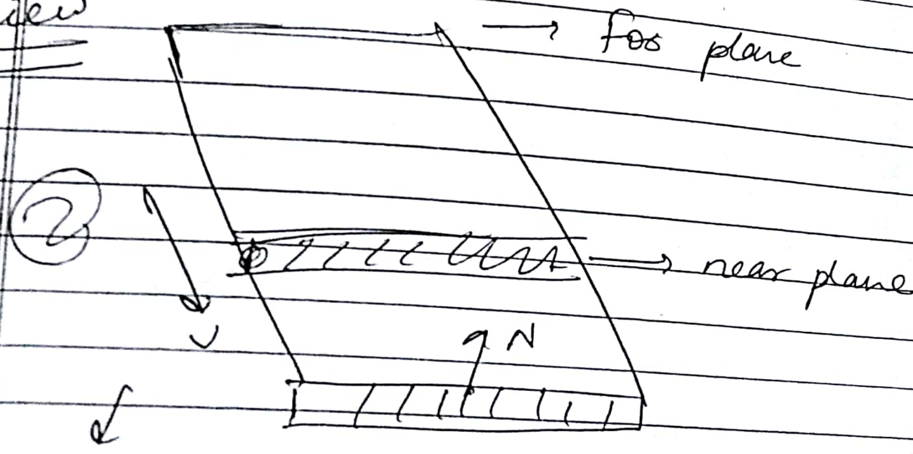
In case of orthographic projection

For a camera, near plane
No clear
far plane → No clear.

Object outside the volume is clipped.

For oblique

Top view

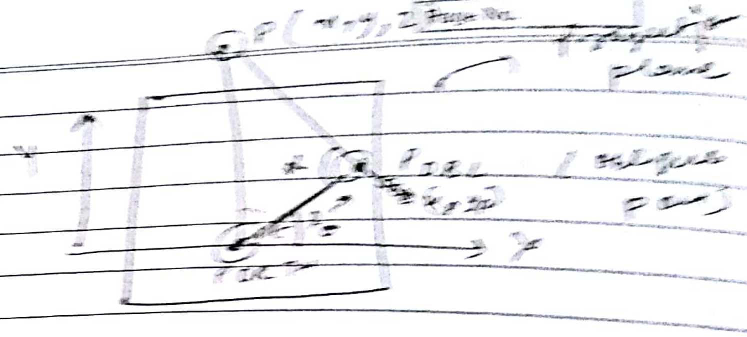


We need to transform

fig ② → fig 1

composite matrix will give the transformation

Scanned by CamScanner



$P \cdot P_{orth} \perp x$ axis

orthographic point

$P \cdot P_{obl}$ α angle

P_{obl} is (x_p, y_p)

$x_p = x + l\cos\theta$

$y_p = y + l\sin\theta$

$\tan\alpha = z/l$

$l = z/\tan\alpha$

{ let us take $\frac{1}{\tan\alpha} = l_1$ }

$l = z \times l_1$

$x_p = x + l_1 \cos\theta$

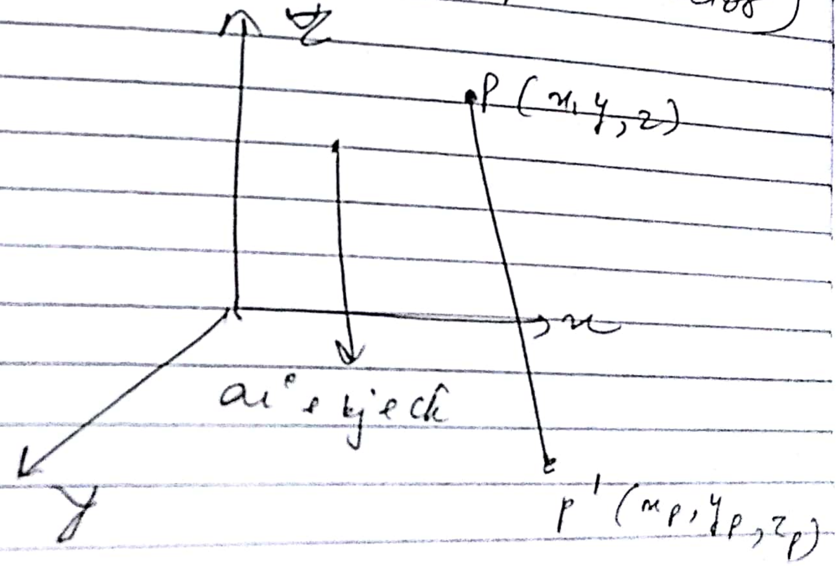
Scanned by CamScanner

$z_p = 0$

There is shearing involved in oblique project

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & l_1\cos\alpha & 0 \\ 0 & 1 & l_1\sin\alpha & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \quad \text{--- (1)}$$

wp (View direction vector)



find shearing matrix

$PP' = (x_p - x)\hat{i} + (y_p - y)\hat{j} + (z_p - z)\hat{k}$

Date:
Page No.

PP_1 and View are parallel

$\overrightarrow{PP_1} \parallel \vec{v}$

$(x_p - x)\hat{i} + (y_p - y)\hat{j} + (z_p - z)\hat{k}$

$= t\left[a\hat{i} + b\hat{j} + c\hat{k}\right]$

$x_p - x = at$

$y_p - y = bt$

$z_p - z = ct$

$\underline{z_p = 0}$ (as it projection on xy plane)

$t = -z/c$

$x_p = x + at$

$= x - \frac{az}{c}$

$y_p = y - \frac{bz}{c}$

Putting in matrix 1

$x_p = (l_1\cos\theta)(z) + x$

$\cancel{x} - \frac{az}{c} = \cancel{x} + l_1\cos\theta\,(z)$

$t = -a/c$