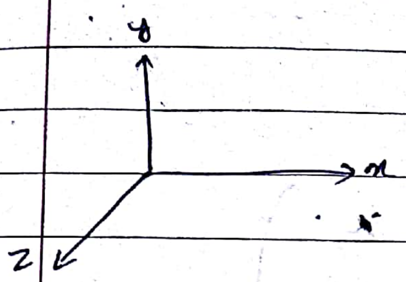
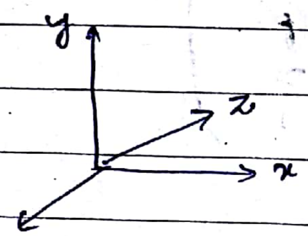
3D - Transformation



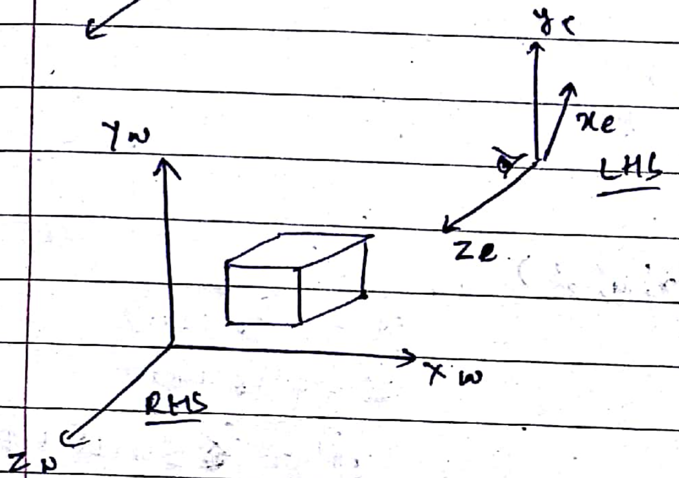
RHCS

Right handed co-ordinate system.



LHCS

Left handed co-ordinate system.



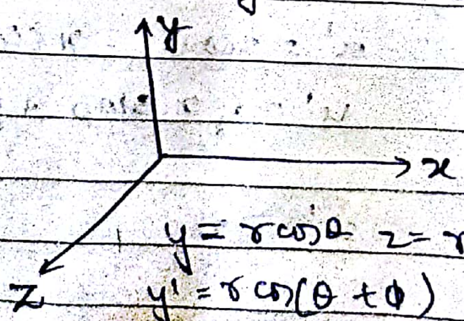
→ Translation :-

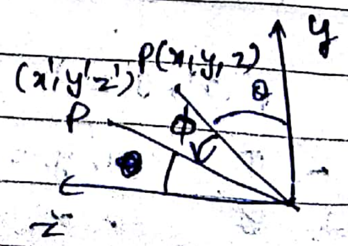
$$\begin{pmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

→ Rotation

$\boxed{A}$ About x - axis

$y \rightarrow z$ anticlockwise





$y = r\cos\theta \quad z = r\sin\theta$

$y' = r\cos(\theta + \phi)$

$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$

✓ $= y\cos\phi - z\sin\phi$

$z = r\cos(\theta+\phi)$

$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$z' = r\cos\theta$

$y' = r\sin\theta$

$z = z'\cos\phi - y'\sin\phi$

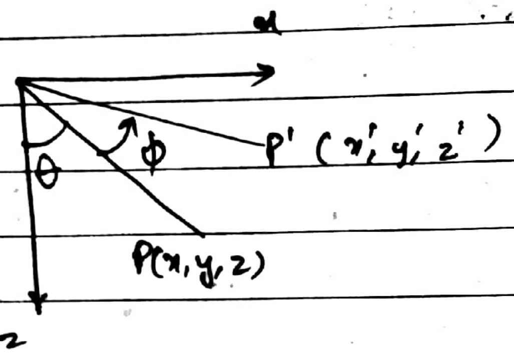
$x' = x$

$y' = y\cos\phi - z\sin\phi$

$z' = y\sin\phi + z\cos\phi$.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

B) **Rotation about y.**



$x = r\sin\theta$

$z = r\cos\theta$

$x' = r\sin(\theta+\phi)$

$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$= x\cos\phi + z\sin\phi$

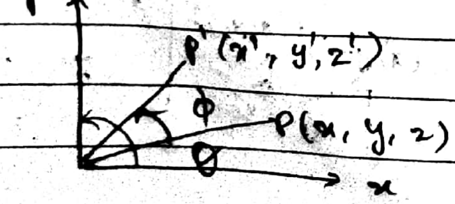
$x' = x\cos\phi + z\sin\phi$

$y' = y$

$z' = -x\sin\phi + z\cos\phi$.

$$\begin{pmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

C) **Rotation about z.**



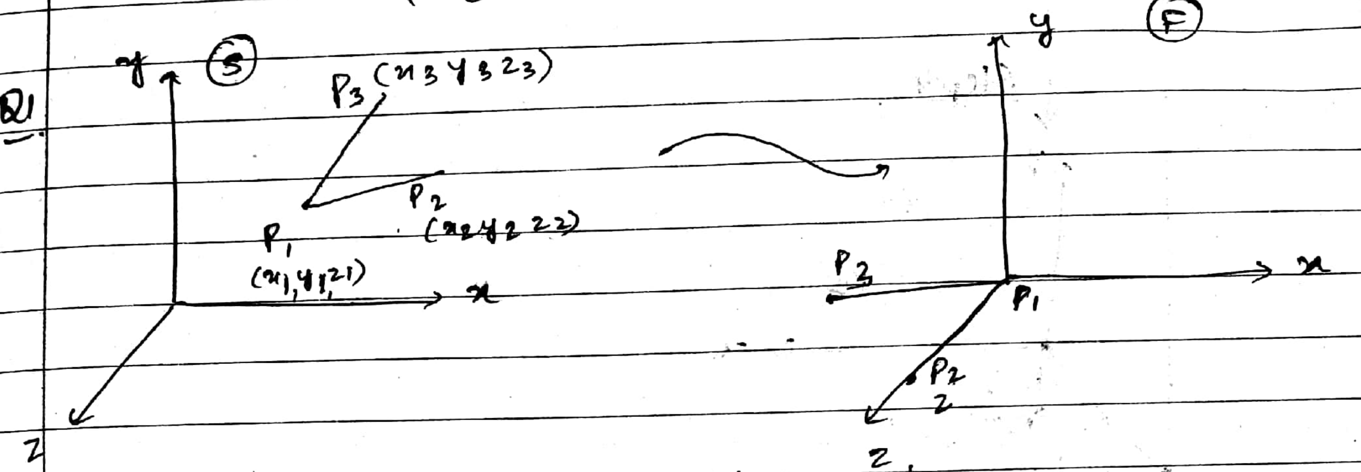
$x' = x\cos\phi - x\sin\phi$

$y' = x\sin\phi + y\cos\phi$

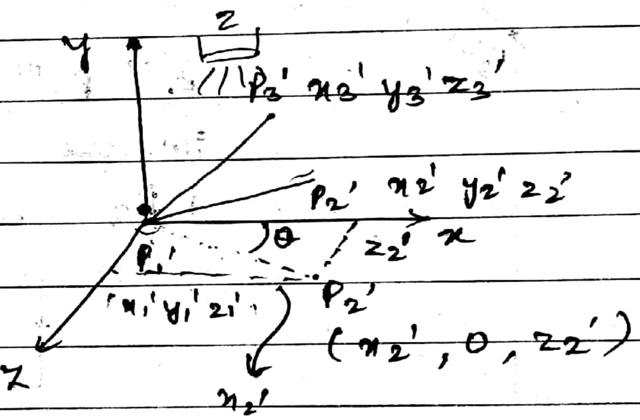
$z' = z$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q1



S1. $T = \begin{pmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$



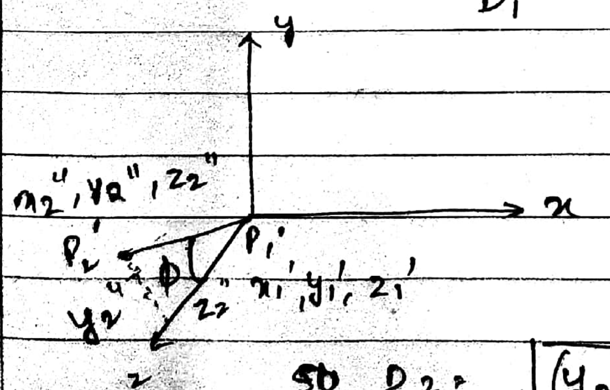
$D_1 = \sqrt{(z_2')^2 + (x_2')^2}$ Rotation is clockwise

$90 - \theta$

$\cos(\theta - 90) = \dfrac{z_2'}{D_1}$ $R_y - (90 - \theta) = \begin{pmatrix} \frac{z_2'}{D_1} & 0 & -\frac{x_2'}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_2'}{D_1} & 0 & \frac{z_2'}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\sin(\theta - 90) = \dfrac{-x_2'}{D_1}$

S2



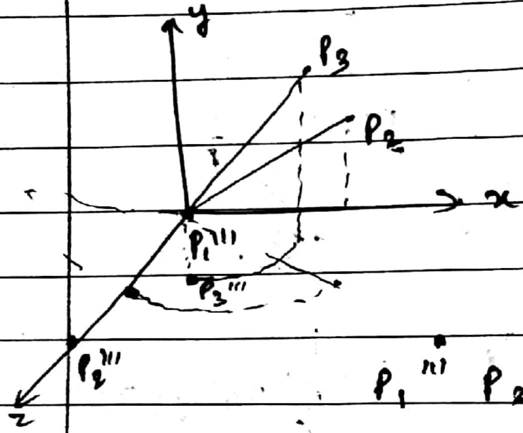
so $D_2 = \sqrt{(y_2'')^2 + (z_2'')^2}$

$\sin\phi = \dfrac{\cancel{co2}\ y_2''}{D_2}$

$\cos\phi = \dfrac{z_2''}{}$

$$\text{Rot } \phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

<u>Step 4</u>



P_1''' P_2''' are in their correct pos^n..
In the process of projection-shifting, $P_1 P_3$ might not have moved exactly
So
- YZ plane
- Infront of YZ
- Behind of YZ.

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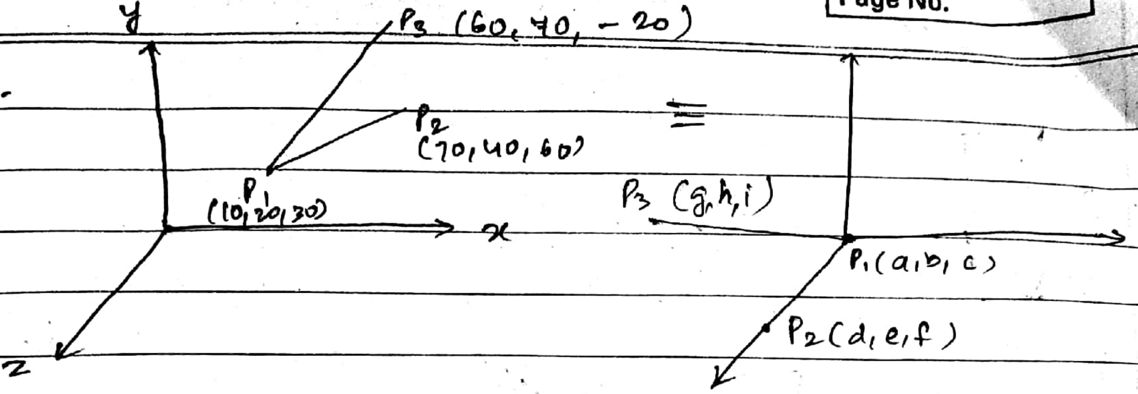
Ex2 :-



a) Find CT

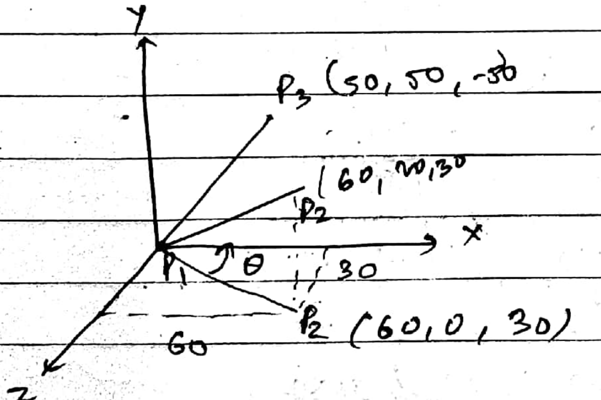
b) Determine a, b, c, d, e, f, g, h, i

S1. $T = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -30 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A$

$P_1\ (0, 0, 0)$

$P_2\ (60, 20, 30)$

$P_3\ (50, 50, -50)$



$D_1 = \sqrt{60^2 + 30^2}$

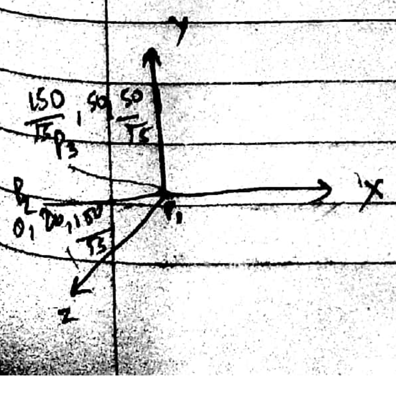
$= 30\sqrt{5}$

$\cos(\theta - 90) = \frac{\cancel{30}}{30\cancel{\sqrt{5}}} \frac{1}{\sqrt{5}}$ cos 60

$\theta = 30$

$\sin(\theta - 90) = \frac{60}{30\sqrt{5}} = -\frac{2}{\sqrt{5}}$

S2 $R_{y,\theta-90^\circ} = \begin{pmatrix} \frac{1}{\sqrt{5}} & 0 & -\frac{2}{\sqrt{5}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = B$



$P_1 = (0, 0, 0)$

$P_2 = \left(0, 20, \frac{180}{\sqrt{5}}\right)$

$P_3 = \left(\frac{150}{\sqrt{5}}, 50, \frac{50}{\sqrt{5}}\right)$

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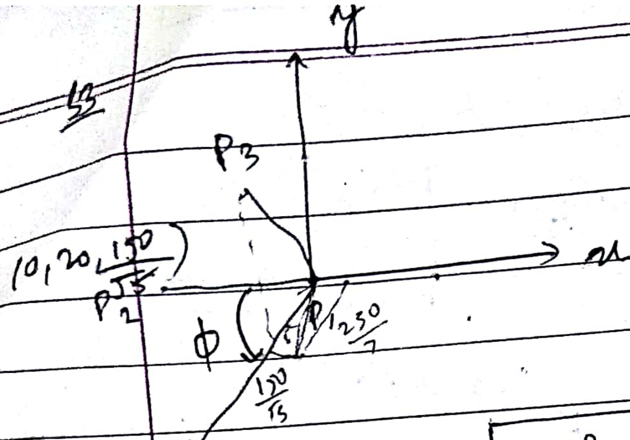

z

$$D_2 = \sqrt{20^2 + \frac{(150)^2}{5}}$$

$$= 70$$

$$\sin\phi = \frac{20}{70} . \qquad \cos\phi = \frac{15}{\sqrt{5}\times 7} .$$

$$R_{x,\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{15}{\sqrt{5}\times 7} & -\frac{2}{7} & 0 \\ 0 & \frac{2}{7} & \frac{15}{\sqrt{5}\times 7} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\frac{40}{7} \times \frac{2250}{5\times 49}$$

$P_1 = (0, 0, 0)$

$P_2 = (0, 0, 70)$ ✓

$$P_3 = \left(\frac{150}{\sqrt{5}}, \frac{750}{\sqrt{5}\times 7} \cdot \frac{-100}{7\sqrt{5}}, \right)$$

$$= \left(\frac{150}{\sqrt{5}}, \frac{650}{7\sqrt{5}}, \frac{750 + 100\sqrt{5}}{7\sqrt{5}} \right)$$

$$= \left(\frac{150}{\sqrt{5}}, \frac{650}{7\sqrt{5}}, \cancel{\frac{2\sqrt{5}}{7}} \frac{250}{7} \right)$$ ✓

54 Proj of P_3 on xz plane

$$D_3 = \sqrt{\left(\frac{150}{\sqrt{5}}\right)^2 + \left(\frac{250}{7}\right)^2}$$

$$= \sqrt{\frac{22500}{25} + \frac{62500}{49}}$$

$$= 78.895$$

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$\cos\delta = \frac{250/7}{D_3} = \frac{35.71}{78.89} = 0.45$

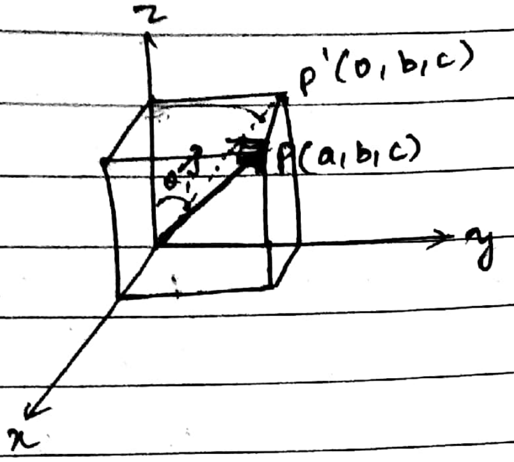
$\sin\delta = \frac{150}{\sqrt{5}\times 78.89} = \frac{150}{2.24\times 78.89} = \cancel{0.526}$ 0.852

$P_{z,\delta} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} = D$

$P_1 = (0,0,0)$
$P_2 = (0,0,70)$
$P_3 = (0, 35.42, \frac{250}{7})$

$C_T = D \times C \times B \times A$

3-D Transformation (vector)



$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$

$A_{\vec{v}}^{\hat{k}}$ = Alignment of vector $\vec{v}$ to z axis ($\hat{k}$).

S1.

Rotation about X axis.

It means vector $\vec{v}$ projection seen on Y-Z plane

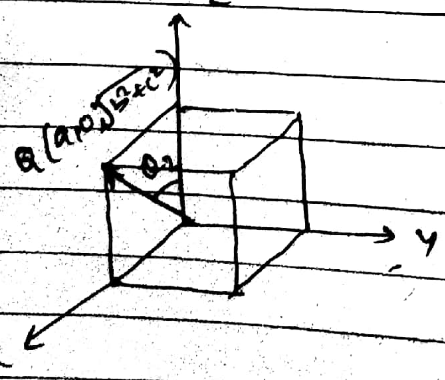
$D_1 = \sqrt{b^2 + c^2}$

$\cos\theta_1 = \frac{c}{\sqrt{b^2+c^2}}$ $\sin\theta_1 = \frac{b}{\sqrt{b^2+c^2}}$

$$R_{x,\theta_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ 1 \end{pmatrix}$$

a, 0

S2



$D_2 = \sqrt{a^2 + b^2 + c^2}$

~~$\cos\theta_1 = \frac{c}{\sqrt{b^2+c^2}}$~~

~~$\sin\theta_1 = \frac{b}{\sqrt{b}}$~~

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$$\cos\theta_2 = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\sin(-\theta_2) = -\sin\theta_2 = \frac{-a}{\sqrt{a^2+b^2+c^2}}$$

If a vector is in 3D, we need 2 rotations to align it with a co-ordinate axis.

$$R_{y,-\theta_2} = \begin{bmatrix} \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{+a}{\sqrt{a^2+b^2+c^2}} & 0 & \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

let $\lambda = \sqrt{b^2+c^2}$

& $|v| = \sqrt{a^2+b^2+c^2}$

$$R_{y,-\theta_2} = \begin{pmatrix} \frac{\lambda}{|v|} & 0 & -\frac{a}{|v|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{|v|} & 0 & \frac{\lambda}{|v|} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$C_T = R_{y,-\theta_2} \times R_{x,\theta_1}$

$$\begin{pmatrix} \frac{\lambda}{|V|} & 0 & \frac{-a}{|V|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{|V|} & 0 & \frac{\lambda}{|V|} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\lambda} & -\frac{b}{\lambda} & 0 \\ 0 & \frac{b}{\lambda} & \frac{c}{\lambda} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\lambda}{|V|} & \frac{-ab}{\lambda|V|} & \frac{-ac}{\lambda|V|} & 0 \\ 0 & \frac{c}{\lambda} & \frac{-b}{\lambda} & 0 \\ \frac{a}{|V|} & \frac{b}{|V|} & \frac{c}{|V|} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q. Align the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ with the vector $\hat{k}$.

Ans $\lambda = \sqrt{2}$
$v = \sqrt{3}$

$$C_T = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

✓ Align z axis ($\hat{k}$) to a vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$

$C_T^{-1} = \left[R_{y,-\theta_2} \times R_{x,\theta_1}\right]^{-1}$

$= \left[R_{x,\theta_1}\right]^{-1} \times \left[R_{y,-\theta_2}\right]^{-1}$

$= R_{x,-\theta_1} \times R_{y,\theta_2}$

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$$C_T^{-1} = \begin{pmatrix} \frac{\lambda}{|v|} & 0 & \frac{a}{|v|} & 0 \\ \frac{-ab}{\lambda|v|} & \frac{c}{\lambda} & \frac{b}{|v|} & 0 \\ \frac{-ac}{\lambda|v|} & -\frac{b}{\lambda} & \frac{c}{|v|} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q. Find a transf. which aligns $\vec{v} = \hat{i}+\hat{j}+\hat{k}$ with the vector $\vec{N} = 2\hat{i} - \hat{j} - \hat{k}$.

$T = A_K^{\vec{N}} \times A_{\vec{v}}^{\hat{k}}$ $\qquad \vec{v} = \lambda = \sqrt{2} \qquad v = \sqrt{3}$

Ans $\quad A_{\vec{v}}^{\hat{k}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$A_K^{\vec{N}} :- \begin{pmatrix} \sqrt{\frac{2}{6}} & 0 & \frac{-2}{\sqrt{6}} & 0 \\ \frac{+2}{\sqrt{12}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & 0 \\ \frac{2}{\sqrt{12}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\qquad v = \sqrt{6} \qquad \lambda = \sqrt{2}$

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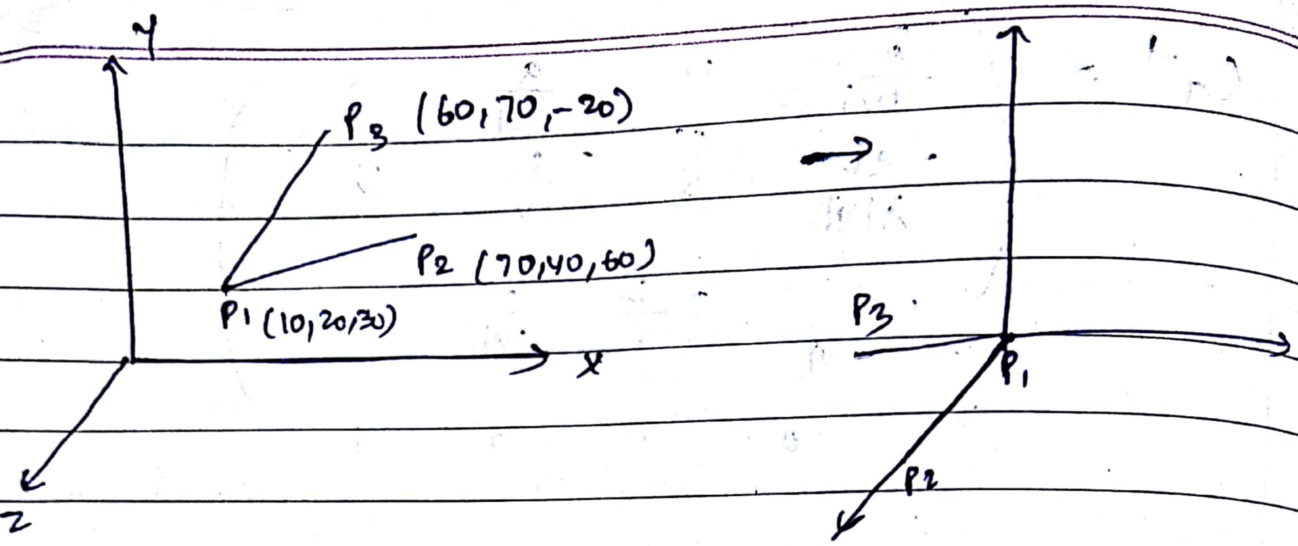
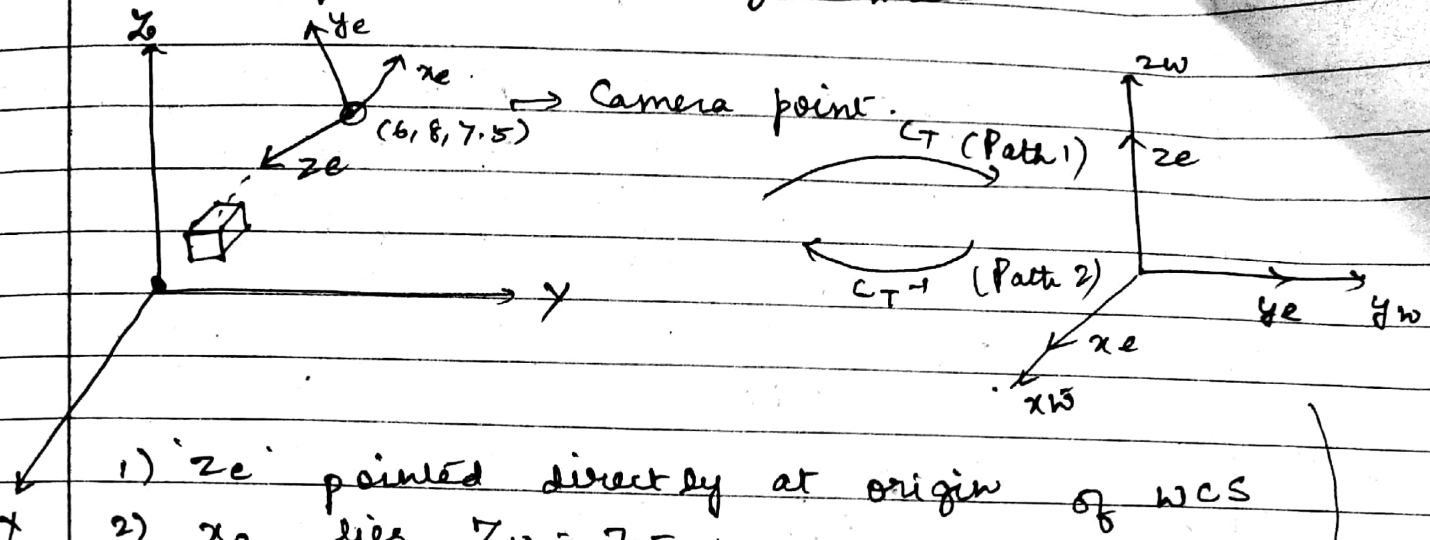
Q3



3D Transformation. Through camera

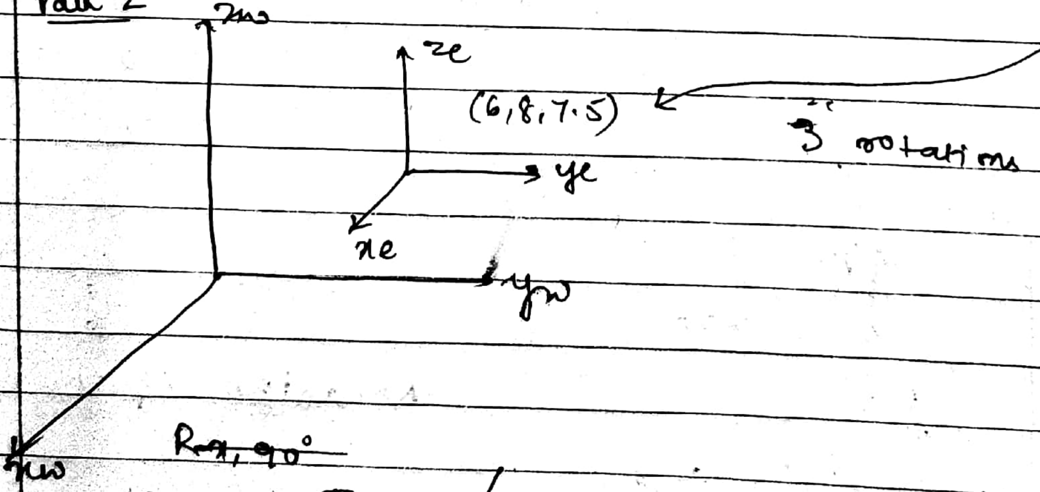


1) 'ze' pointed directly at origin of WCS
2) x_e lies $Z_w = 7.5$ plane
3) ye is up.

Q. An object is described wrt W.C.S. What will be the co-ordinates of the object with respect to CCS (camera co-ordinate system)

→ Either find CT or CT^{-1} as seen in fig above.

Path 2



~~$R_{x}, 90°$~~

S1 $T = \begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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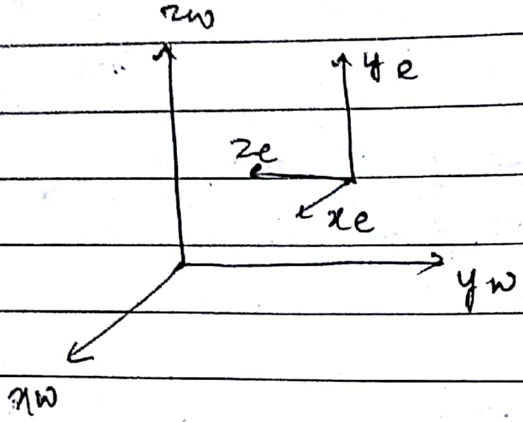
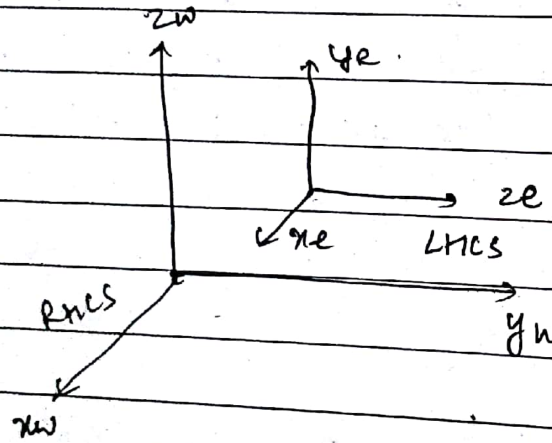
S2 Rotation about x_e by 90° (anticlockwise)

$$R_{x_e, 90^\circ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



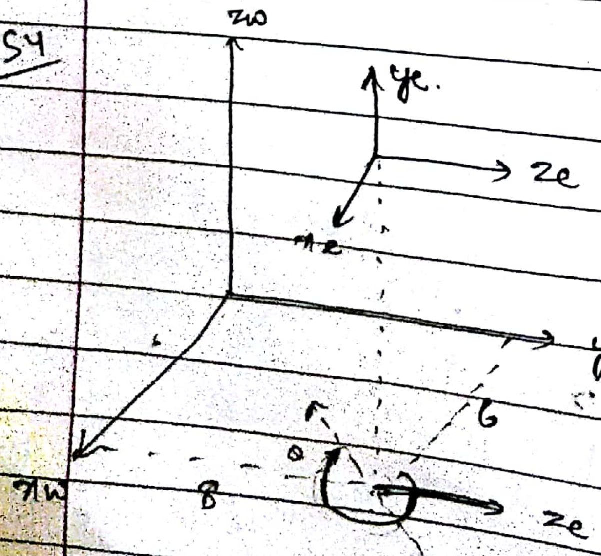
S3



$R_{x_e, 90^\circ} \times (-x)$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

S4



$\frac{8}{10}$

Rotation of y axis

$\cos(180 + \theta) = -\cos\theta$

$\cos\theta = -\frac{8}{10}$

$\sin\theta = \frac{6}{10}$

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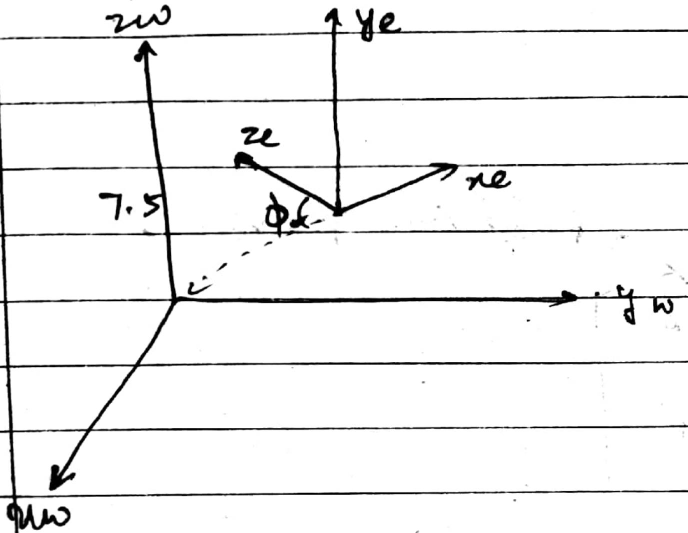
$$R_y(-(180+\theta)) = \begin{pmatrix} -8/10 & 0 & 6/10 & 0 \\ 0 & 1 & 0 & 0 \\ -6/10 & 0 & -8/10 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$D = \sqrt{10^2 + (7.5)^2}$$
$$= 12$$
$$\cos(-\phi) = \frac{10}{12.5}$$
$$\sin(-\phi) = \frac{-7.5}{12.5}$$

$$R_{xe,-\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{10}{12.5} & \frac{7.5}{12.5} & 0 \\ 0 & \frac{-7.5}{12.5} & \frac{10}{12.5} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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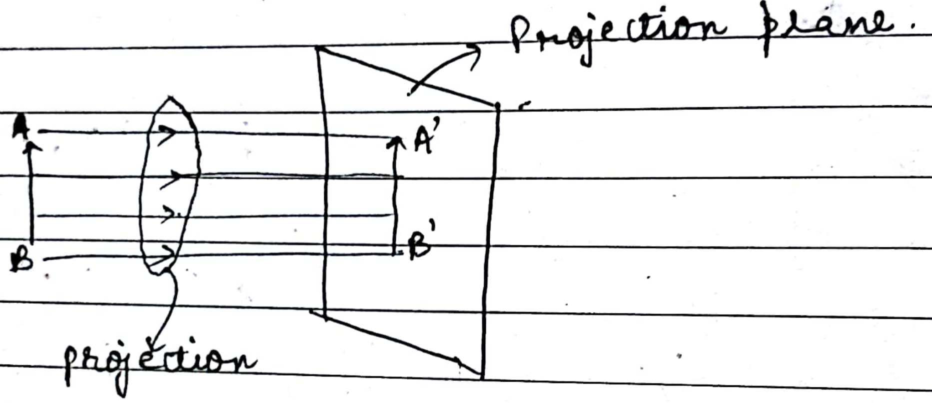
Projection

3D Transformed → 2D

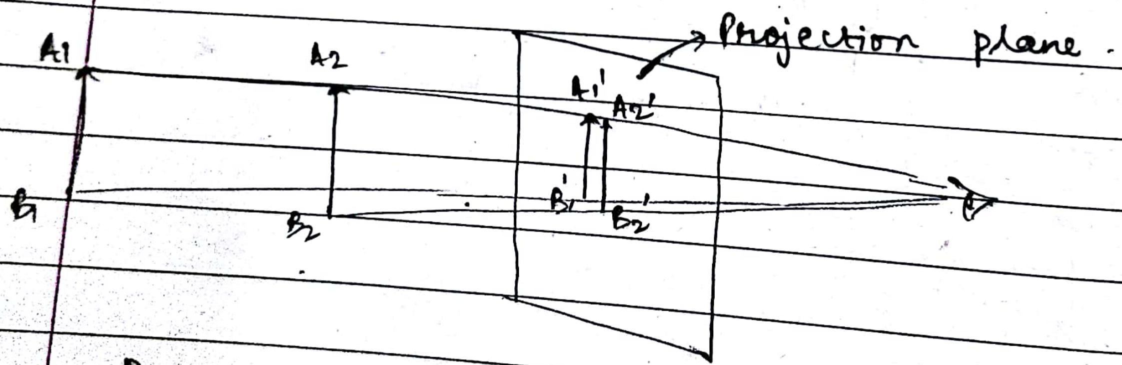
Object

(Manipulation)

Parallel Projection



Perspective Projection



1) Projection plane is stationary.
2) If object is moved towards projection plane, size of image increases.

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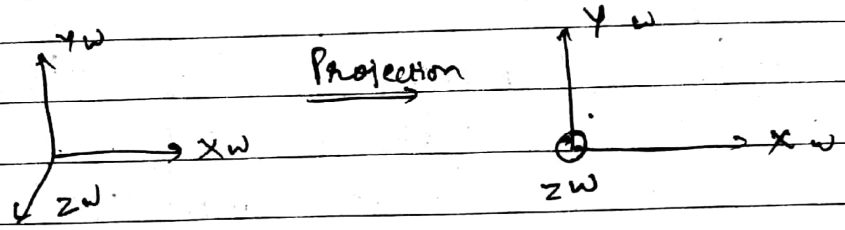
Parallel Projection :-

1) Projector lines falling on projection plane at $90°$
(orthographic projection)

2) Projector lines falling on projection plane at an angle $\alpha \neq 90°$
(Oblique projection).

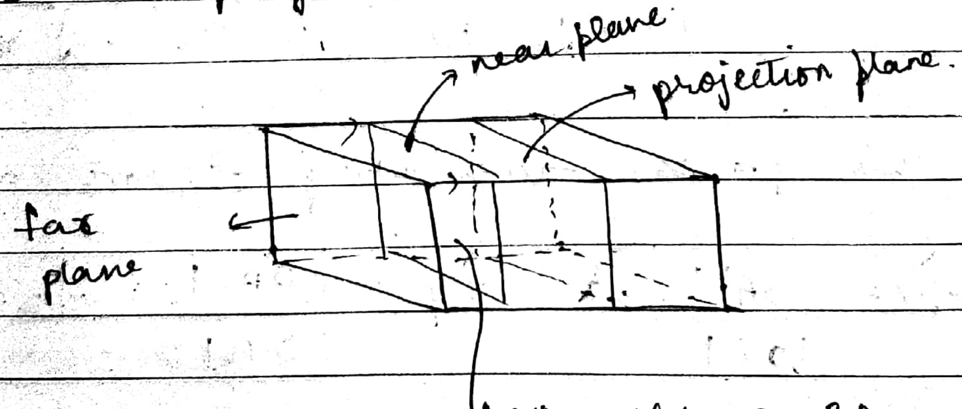


Orthographic Projection

- Isometric projection - Projector lines make an equal angle with all three co-ordinate axis
- Diametric projection - Projector lines make equal angle with any 2 co-ordinate ~~axis~~.
- Trimetric projection - Unequal angle.

Oblique Projection

- Cavalier projection
- Cabinet projection.



View Volume 3D
(Object description is not view volume)

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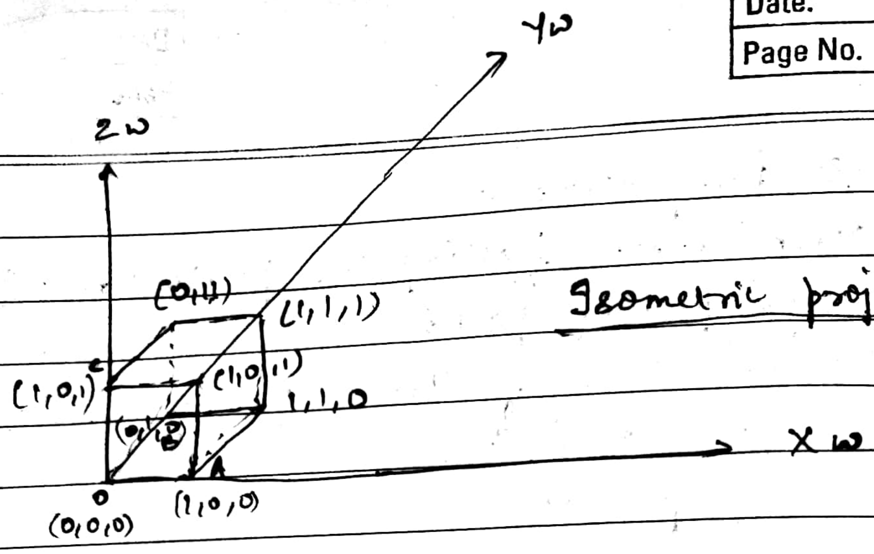
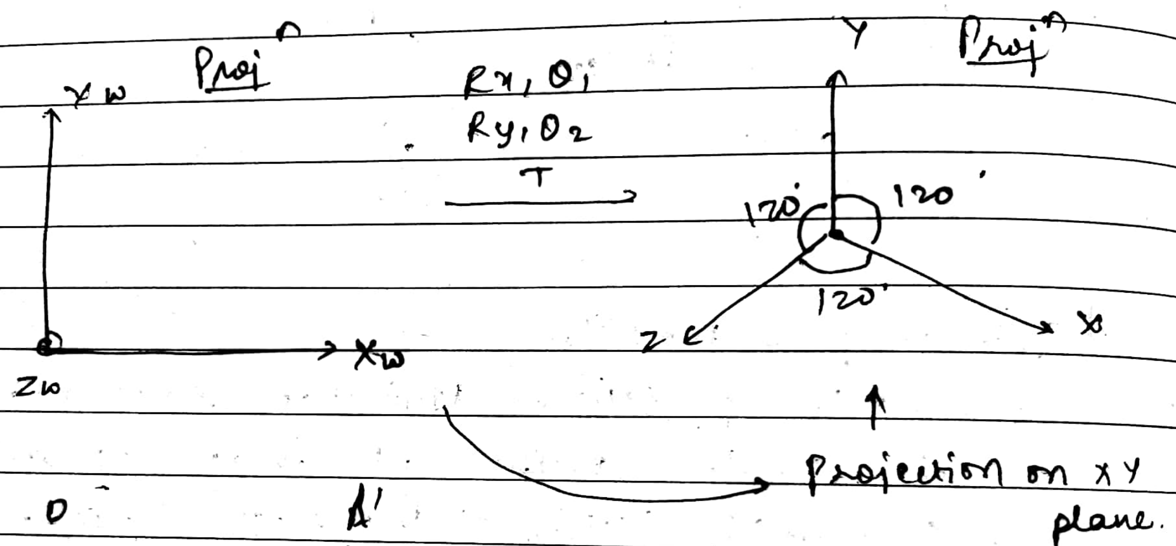


Geometric projection.

$|OA| = 1$

$|OB| = 1$

$|OC| = 1$



O
(0,0,0)

A'
$(x_x, y_x, 0)$

$OA' = \sqrt{x_x^2 + y_x^2}$

B'
$(x_y, y_y, 0)$

$OB' = \sqrt{x_y^2 + y_y^2}$

C'
$(x_z, y_z, 0)$

$OC' = \sqrt{x_z^2 + y_z^2}$

→ In this process, foreshortening factors are equal.

$$f_x = \frac{|OA'|}{|OA|} = f_y = \frac{|OB'|}{|OB|} = f_z = \frac{|OC'|}{|OC|}$$

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Rotation about X, $-\theta_1^\circ$

$$R_{x,-\theta_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 & 0 \\ 0 & -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rotation about Y, $-\theta_2^\circ$

$$R_{y,-\theta_2} = \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = R_{y,-\theta_2} \times R_{x,-\theta_1}$$

$$= \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 & 0 \\ 0 & -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_2 & \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 & 0 \\ \sin\theta_2 & -\cos\theta_2\sin\theta_1 & \cos\theta_1\cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$Proj_{x-y} \times T$

Proj on XY plane make Z 0

$$= \begin{pmatrix} \cos\theta_2 & \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{array}{ccc} A & B & C \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\theta_2 & \sin\theta_1 \sin\theta_2 & -\sin\theta_2 \cos\theta_1 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & 0 & 0 \\ \cancel{\sin\theta_1} \; 0 & & \end{pmatrix}$$

Case 1

$f_x = f_y$

$$\Rightarrow \sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$$

$$\Rightarrow \cos^2\theta_2 = \sqrt{\sin^2\theta_1 * \sin^2\theta_2 + \cos^2\theta_1}$$

$$= \sin^2\theta_1 * \sin^2\theta_2 + \cos^2\theta_1$$

Case 2 $\quad f_y = f_z$.

$$\sin^2\theta_1 \sin^2\theta_2 + \cos^2\theta_1 = \sin^2\theta_2 \cos^2\theta_1 + \sin^2\theta_1$$

$$\sin^2\theta_1 \sin^2\theta_2 = \cos^2\theta_1 - \cos^2\theta_2$$

$$\sin^2\theta_1 \sin^2\theta_2 = \sin^2\theta_1 - \cos^2\theta_1$$

$$2\cos^2\theta_1 = -\sin^2\theta_1 - \cos^2\theta_2.$$

$$2 - 2\sin^2\theta_1 = -\sin^2\theta_1 - \cos^2\theta_2$$

$$2 + \cos^2\theta_2 = \sin^2\theta_1 \checkmark$$

$$(2 + \cos^2\theta_2)(1 - \cos^2\theta_2) = (1 - \sin^2\theta_1) - \cos^2\theta_2$$

$$= (-1 - \cos^2\theta_2 - \cos^2\theta_2)$$

$$2 - \cancel{2\cos^2\theta_2} + \cos^2\theta_2 - \cos^4\theta_2 = -1 - 2\cos^2\cancel{\theta_2}.$$

$$\cos^4\theta_2 - \cos^2\theta_2 - 3 = 0$$

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$x^2 - x - 3 = 0$

$\cancel{x^2} - \cancel{2x}$

$$x = \frac{1 \pm \sqrt{\cancel{9}\,1 + 12}}{2}$$

$$\cos^2\theta_2 = \frac{1 \pm \sqrt{13}}{2}$$

$$\boxed{\begin{aligned} \theta_1 &= \pm 45^\circ \\ \theta_2 &= \pm 35.26^\circ \end{aligned}}$$

Dimetric Projection

I: $f_x = f_y \neq f_z$

II $f_x = f_z \neq f_y$

III $f_y = f_z \neq f_x$

I. $f_x = f_y \neq f_z$.

$f_x = f_y$ $\quad \sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$

$\Rightarrow \cos^2\theta_2 = \sin^2\theta_2 \sin^2\theta_1 + \cos^2\theta_1$

$\boxed{\theta_1 = \pm \sin^{-1}(\tan\theta_2)}$

$f_y = f_z$

$\sqrt{x_y^2 + y_y^2} = f_z^2$ — (2)

$\sin^2\theta_1 \sin^2\theta_2 + \cos^2\theta_1 = f_z^2$

$\therefore \boxed{\theta_2 = \pm \cos^{-1}(f_z^2)}$

II. $f_x = f_z \neq f_y$.

$f_x = f_z$

$\cancel{\sqrt{x_x^2 + x_y^2}}$ $\quad \sqrt{x_x^2 + y_x^2} = \sqrt{x_z^2 + y_z^2}$

$\cos^2\theta_2 = \sin^2\theta_2 \cos^2\theta_1 + \sin^2\theta_1$

$\cancel{\theta_1 = \pm \sin^{-1}(\tan\theta_2)}$

$\cancel{f_x = f_y = f_z}$

$\cos^2\theta_2 = \cancel{\sin^2\theta_2 \sin^2\theta_1 + \cos^2\theta_1}$

$\cancel{\theta_1 = \pm \sin^{-1}(+}$

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$f_x \neq f_y$

$\cos^2\theta_2 \neq f_y^2$

$\boxed{\theta_2 = \cos^{-1} f_y}$

III $f_y = f_z \neq f_x$

$f_y = f_z$

$\sin^2\theta_1 \sin^2\theta_2 + \cos^2\theta_1 = \sin^2\theta_2 \cos^2\theta_1 + \sin^2\theta_1$

$\sin^2\theta_2(\sin^2\theta_1 - \cos^2\theta_1) = (\sin^2\theta_1 - \cos^2\theta_1)$

$\theta_2 = \pm 90^\circ$

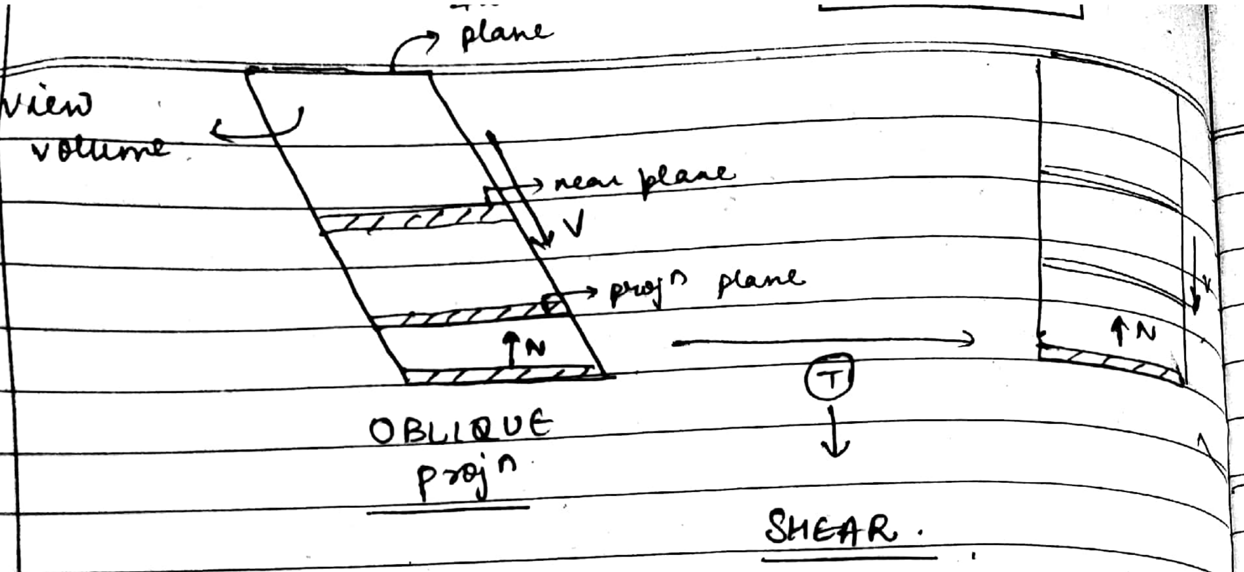
$f_z = f_x$

$\sin^2\theta_2 \cos^2\theta_1 + \sin^2\theta_1 = \cancel{\cos^2\theta_2}\ f_x$

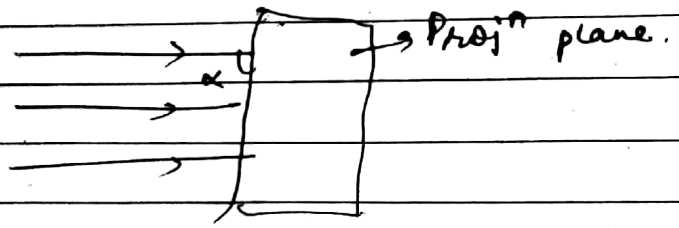
$\cos^2\theta_1 + \sin^2\theta_1 = f_x$

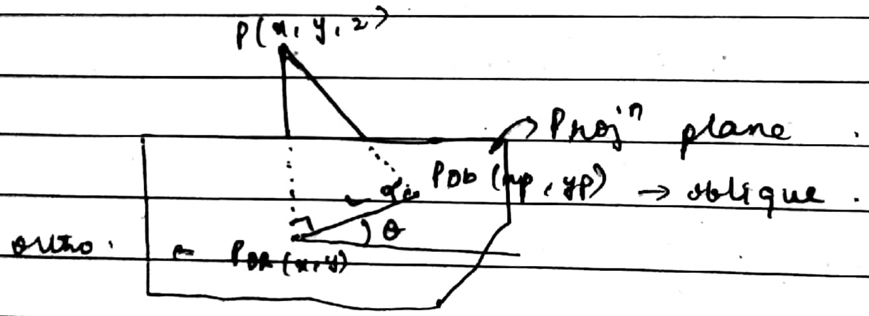
$f_y = f_x$

$\sin^2\theta_1 \sin^2\theta_2 + \cos^2\theta_1 = f_x$



Oblique Projection





L is a distance b/w P_{OR} & P_{OB}.

Δ P_{OR} P_{OB} P $\perp$ Triangle at P_{OR}

$x_p = x + L\cos\theta$

$y_p = y + L\sin\theta$

$\tan\alpha = \dfrac{z}{L}$

$L = \dfrac{z}{\tan\alpha} \approx L_1 \cdot z \quad \left(L_1 = \dfrac{1}{\tan\alpha} \right)$

Amazing ♡

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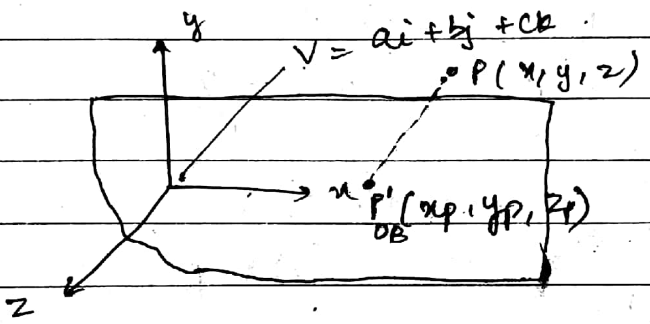
$$x_p = x + (L_1 \cdot \cos\theta) \cdot z$$
$$y_p = y + (L_1 \sin\theta)\, z$$

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & L_1\cos\theta & 0 \\ 0 & 1 & L_1\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

P1



$$x_p - x = k * a$$
$$y_p - y = k * b$$
$$\boxed{z_p - z = k * c}$$
$$\because z_p = 0$$
$$\boxed{k = \frac{-z}{c}}$$

$$x_p - x = -\frac{za}{c}$$

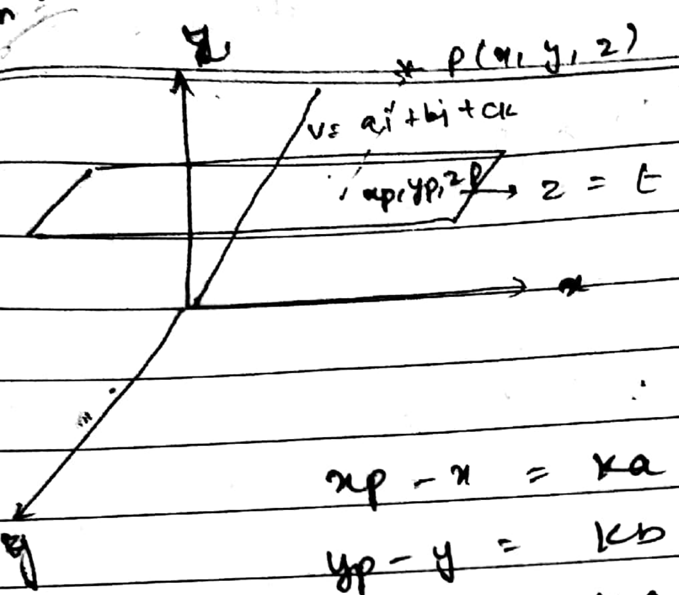
$$y_p - y = \frac{-zb}{c}$$

$$x_p = x - \frac{az}{c}$$

$$y_p = y - \frac{bz}{c}$$

2 Ang Find. Intense ✓ Even ✓

P2



$$x_p - x = ka$$

$$y_p - y = kb$$

$$z_p - z = kc$$

$$t - z = kc$$

$$k = \frac{t - z}{c}$$

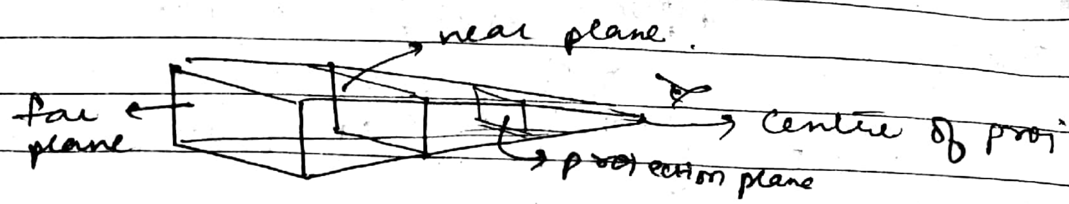
$$x_p - x = (t - z)a$$

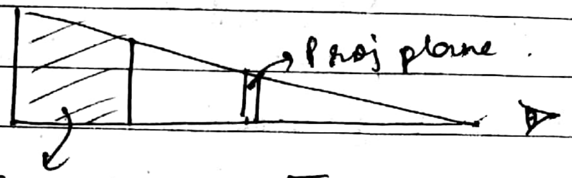
$$y_p - y = \frac{(t - z)b}{c}$$

$$z_p - z = (t - z)$$

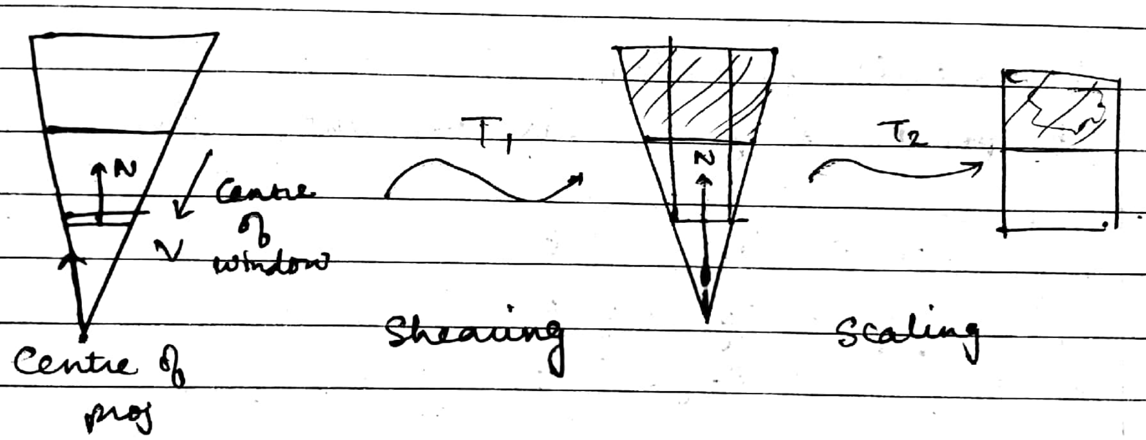
$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -a/c & at/c \\ 0 & 1 & -b/c & bt/c \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Non-parallel Projection





Trapezium = $\xrightarrow[2D]{T}$ Rectangle



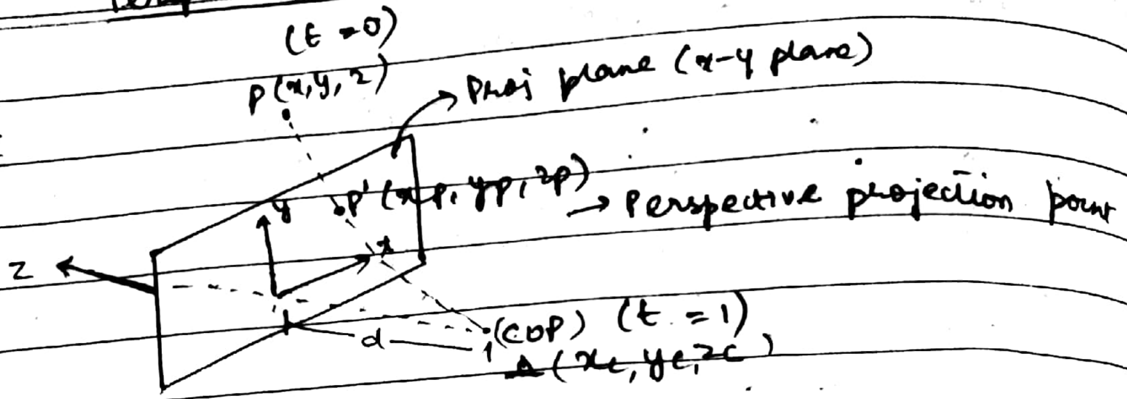
Normal to C. of. proj $\longrightarrow$ on same st line

2 Normal to C. of proj are not in st line

$C = T_2 * T_1$

Perspective Projection

Case I



Projector lines are not parallel.

Consider P - COP as a line.

Parametric form of eq^n:

A point x' y' z' is on the line.

$$x' = x + (x_c - x)t$$
$$y' = y + (y_c - y)t$$
$$z' = z + (z_c - z)t$$

$$x_c = 0, \; y_c = 0 \quad z_c = -d$$

$$z' = z + (-d - z)t$$

when $z' = z_p = 0$ (As it is on x-y plane)

$$\boxed{t = \frac{z}{d+z}}$$

$$x_p = x + \frac{(-x)z}{d+z} = \frac{dx}{d+z}$$

$$y_p = y + \frac{(-y)z}{d+z} = \frac{dy}{d+z}$$

$zh = kh$ $\quad z_p = k$

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let $h = d + z$

$x_h = h x_p \qquad y_h = h y_p \qquad z_h = h z_p$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$x_h = dx$

$h x_p = dx$

$x_p = \dfrac{dx}{h}$

Case 2 : The projection plane is ∥ to x, y & $z = k$.

$z' = z + (z_c - z)\, t$

$x_c = 0, \quad y_c = 0, \quad z_c = -d$

$z' = z + (-d - z)\, t$

when $z' = z_p = k$.

$t = \left(\dfrac{z - k}{d - z} \right)$

$dx \; \cancel{dz} \quad \cancel{xz} + kx$

$x_p = x + \dfrac{(-x)(z - k)}{d - z} = \dfrac{(k + d)\, x}{d - z}$

$y_p = y + \dfrac{(-y)(z - k)}{d - z}$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} k+d & 0 & 0 & 0 \\ 0 & d+k & 0 & 0 \\ 0 & 0 & k & kd \\ 0 & 0 & 1 & d \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$z_h = kz + kd$

$z_h = k(d + z) = kh = h z_p$

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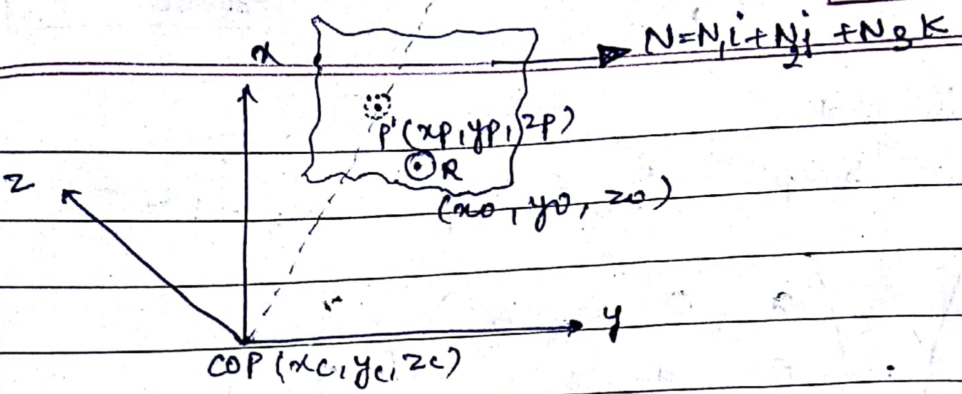
Case 3



For a perspective proj^n point:

$$x_p = xt$$
$$y_p = yt$$
$$z_p = zt$$

$$\vec{V} = (x_p - x_0) i + (y_p - y_0) j + (z_p - z_0) k$$

$$\vec{V} \cdot \vec{N} = (x_p - x_0) N_1 + (y_p - y_0) N_2 + (z_p - z_0) N_3$$

$$\Rightarrow \quad 0 = (xt - x_0) N_1 + (yt - y_0) N_2 + (zt - z_0) N_3$$

$$\Rightarrow \quad t = \frac{N_1 x_0 + N_2 \cdot y_0 + N_3 \cdot z_0}{N_1 x + N_2 y + N_3 z}$$

$$x_p = x \left(\frac{N_1 \cdot x_0 + N_2 \cdot y_0 + N_3 \cdot z_0}{N_1 \cdot x + N_2 \cdot y + N_3 \cdot z} \right)$$

Let $d_0 = N_1 \cdot x_0 + N_2 \cdot y_0 + N_3 \cdot z_0$.

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} d_0 & 0 & 0 & 0 \\ 0 & d_0 & 0 & 0 \\ 0 & 0 & d_0 & 0 \\ N_1 & N_2 & N_3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

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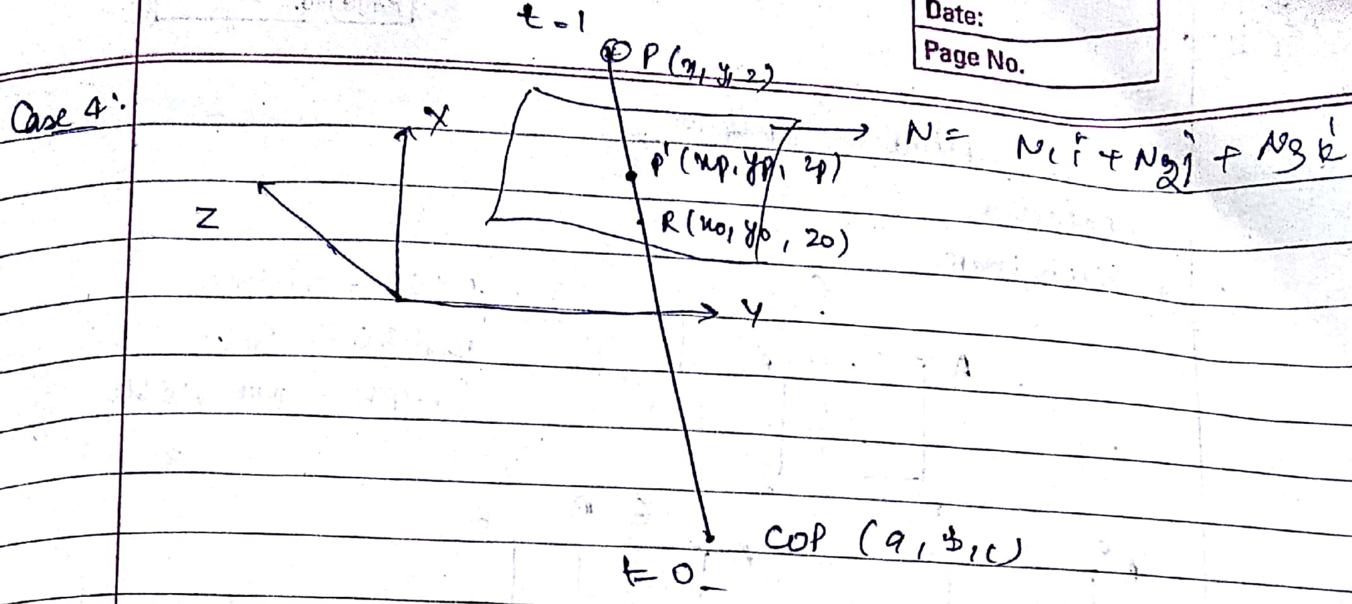
Case 4:



~~$x_p = x + (x - x) t$~~

$x_p = x + (x - a) t$

$y_p = y + (y - a) t$

$z_p = z + (z - a) t$.

$\vec{V} = (x_p - x_0) i + (y_p - y_0) j + (z_p - z_0) k$

$\vec{V} \cdot \vec{N} = 0$

~~$0 = (x_p - x_0) N_1 + (y_p - y_0)$~~

$0 = (x_p - x_0) N_1 + (y_p - y_0) N_2 + (z_p - z_0) N_3$

$$T = \begin{pmatrix} an_1 + d & an_2 & an_3 & -ad_0 \\ bn_1 & bn_2 + d & bn_3 & -bd_0 \\ cn_1 & cn_2 & cn_3 + d & -cd_0 \\ n_1 & n_2 & n_3 & -d_1 \end{pmatrix}$$

$d_0 = x_0 n_1 + y_0 n_2 + z_0 n_3$

$d_1 = an_1 + bn_2 + cn_3$

$$\boxed{d = d_0 - d_1}$$

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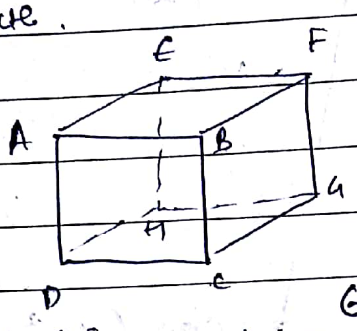
Hidden Surface

→ Detection

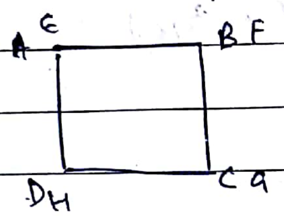
→ Eliminate.

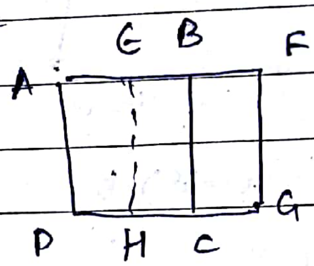


ABCD – visible

BFGC – visible

EFGH – not visible.





Method I

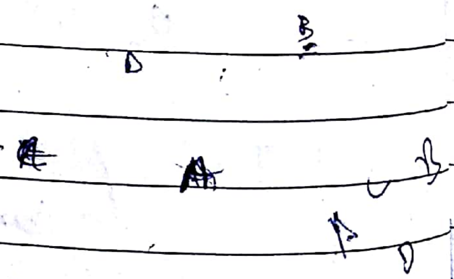
– Finding an outward normal

Tag – surface being back or front.

Q: A solid tetrahedron is given by position vector A(1,1,1), B(3,1,1), C(2,1,3). ~~for~~ D(2,2,2). Point source of light P(2,3,4)

Soln :-

1) A C D
2) C B D
3) B A D
4) A C B



Normal outward

$N_{ACD} = AC \times AD = -2\hat{i} + \hat{j} + k$

$N_{CBD} = CB \times CD = 2\hat{i} + \hat{j} + k$

$N_{BAD} = 2\hat{j} - 2K$

$N_{ACB} = AB \times AC = -4\hat{j}$ ~~(P 2, 3, 4)~~

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Mean Pos^n of Each surface

$M_{ACD} = \left(\frac{5}{3}, \frac{4}{3}, 2 \right)$

$M_{CBD} = \left(\frac{-7}{3}, \frac{4}{3}, 2 \right)$

$M_{BAD} = \left(2, \frac{4}{3}, \frac{4}{3} \right)$

$M_{ACB} = \left(2, 1, \frac{5}{3} \right)$

(-ve)

$L_{\overrightarrow{PM}_{ACD}} = \frac{1}{3}\hat{i} + \frac{5}{3}\hat{j} + 2k$

$L_{\overrightarrow{PM}_{CBD}} = -\frac{1}{3}\hat{i} + \frac{5}{3}\hat{j} + 2k$

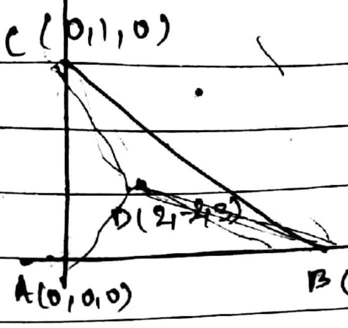
$L_{\overrightarrow{PM}_{BAD}} = \frac{5}{3}\hat{i} + \frac{8}{3}k$

$L_{\overrightarrow{P}_{ACB}} = 2\hat{j} + \frac{7}{3}k$

$N_{ACD} \cdot L_{\overrightarrow{PM}_{ACD}} = -\frac{2}{3} + \frac{5}{3} + 2$

$= +ve$. $\therefore$ visible

Q2.



i j k i j k

i) ~~ACD~~

$N_{ACD} = AC \times \cancel{AD}$

$= -\hat{j} \times (-2\hat{i} + 2\hat{j} - 3\hat{k})$

$= -2\hat{k} + 3\hat{i}$

ii) $N_{ABC} = AB \times AC$ ✓

$= \hat{i} \times \hat{k} = \hat{k}$

iii) $CDB = CB \times CD = (+\hat{i} - \hat{j}) \times (2\hat{i} - 3\hat{j} + 3\hat{k})$

iv) $ADB = AP \times AB$

$M_{ABC} = \cancel{(2, 1, \tfrac{5}{3})} = (\tfrac{1}{3}, \tfrac{1}{3}, 0)$

$\vec{L}_{PM_{ABC}} = \cancel{-1\hat{i}} - \cancel{3}\hat{j} - \frac{10}{3}\hat{k}$

$\hat{N}$

-ve

$\vec{r}_{PM_{ABC}} = -\frac{8}{3}\hat{i} - \frac{11}{3}\hat{j} - 5\hat{k}$

-ve

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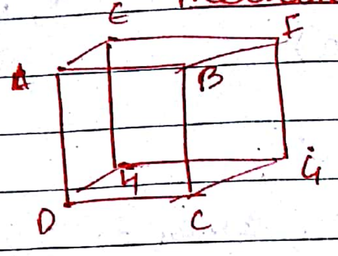
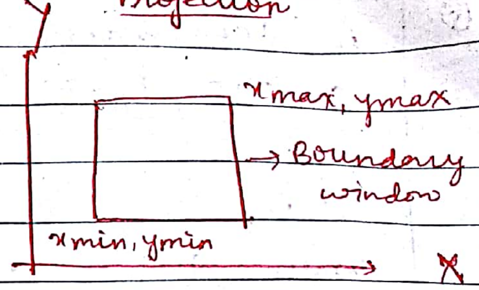
Method 2



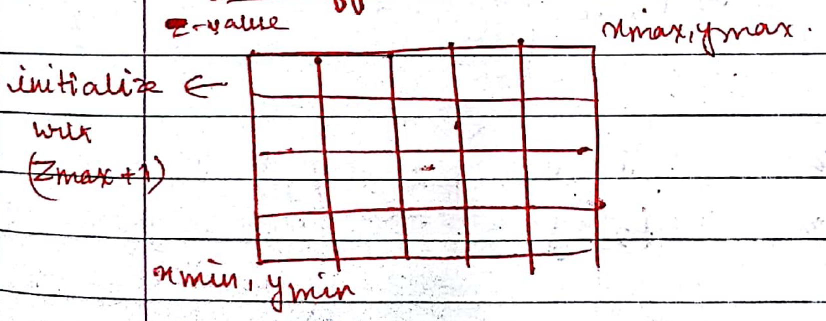
Projection



LIST (3D co.ord)

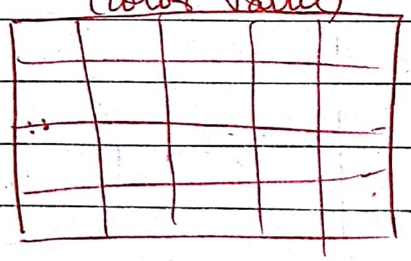
A
B
C
D } xmin, ymin
E } xmax, ymax
F } → filter ①
G
H

Z-buffer



Color Buffer

(color value)



filter 3 → zmin, zmax

Surfaces   filter 2

- ABCD
- BCFG
- BFEA
- AEHD
- CDHG
- EFGH

∀ surface taken one by one, it is mapped into Z-buffer, color buffer.

→ $ax + by + cz = d$

$$z = (d - ax + by) \times \frac{1}{c}$$

Q. A tetrahedron is A (1, 1, −1); B (3, 1, −1)
C (2, 1, −3) D (2, 2, −2)

Ans S1.

$x_{min} = 1$ $x_{max} = 3$ $z_{min} = -3$

$y_{min} = 1$ $y_{max} = 2$ $z_{max} = -1$

Surface	$ax + by + cz =$	
ACD	$2x - y + z = 0$	$z_{ACD} = -2x + y$
CBD	$-2x - y + z = -8$	$z_{CBD} = 2x + y - 8$
BAD	$-2y - 2z = 0$	$z_{BAD} = -y$
ACB	$y = 1$	X

Eq of plane

$$\begin{pmatrix} y_1 - y_2 & z_1 - z_2 \\ y_2 - y_3 & z_2 - z_3 \end{pmatrix} x + \begin{pmatrix} z_1 - z_2 & x_1 - x_2 \\ z_2 - z_3 & x_2 - x_3 \end{pmatrix} y$$

$$+ \begin{pmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{pmatrix} z = - \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} = 0$$

1, 1, −1
2, 1, −3
2, 2, −2

ACD

$$\begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix} x + \begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix} y + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} z$$

$$+2x + -y + z = 0 \quad - \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & -3 \\ 2 & 2 & -2 \end{pmatrix}$$

$$\boxed{2x - y + z = 0}$$

$1(-2 + 6) \quad +4$

$-1(-4 + 6) \quad -2 \quad = 0$

$1(4 - 2) \quad -2$

ACB

CBD

2, 1, −3

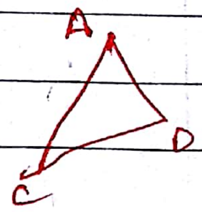
3, 1, −1

2, 2, −2

~~+~~ surface one by one

ACB when projected on x − y plane

A(1,1) C(2,1) D(2,2)



inside/outside test

1) Eqn of projn line AC $y = 1$

$f(x,y) = y - 1$

$D(2,2) > 0$

2) Eqn of ~~D~~C $x = 2$

$f(x,y) = x - 2$ ⇒ $f(x,y) = 2 - x$

$A(1,1) < 0$ $A(1,1) > 0$.

3) Eqn of proj line AD $x = y$

$f(x,y) = \cancel{y - x}\ x - y$

$C(2,1) > 0$

x y	Surface	z	Inside test
1 1	ACD CBD BAD	−1 −5, −1	T
1 2			
2 1			
2 2			
3 1			
3 2			

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x	y	Surface	z	Inside Test	Selection
1	1	A CD	-1	T	
		CBD	-5		CBD (Blue)
		BAD	-1		
1	2	A CD	0	F	
		CBD	-4	T	
		B AD	2	T	

Graphics Contd. later

Graphics contd.

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Hidden Surface (Painter Algorithm)

- Based on the concept that painter follows in painting on the canvas.

- Distant objects / surfaces are painted 1st.
- Closest " " " last.

Surface / plane

$S = \{ S_1, S_2, \ldots S_n \}$

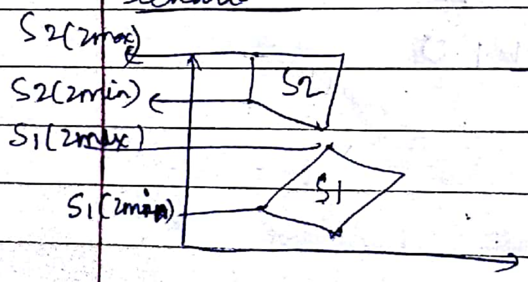
$\boxed{\text{Sort}}$

$S = \{ \bigcirc \}$

sorted acc to depth (along z axis)

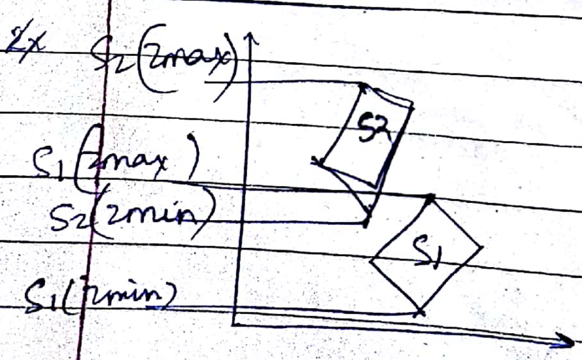
Surface at a larger z value.

Scenario 1



$\boxed{\text{no - depth overlap}}$

Scenario 2



Scanned by CamScanner

Date:
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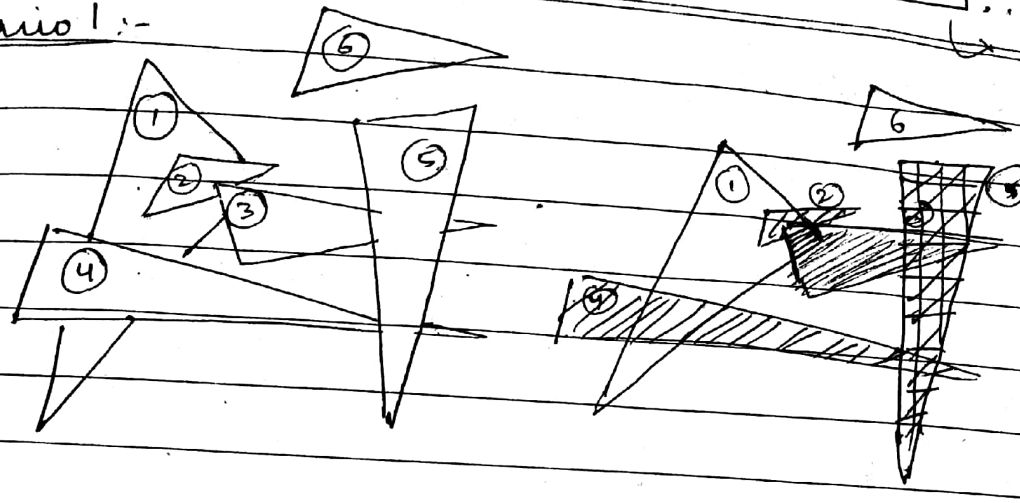
Scenario 1 :-



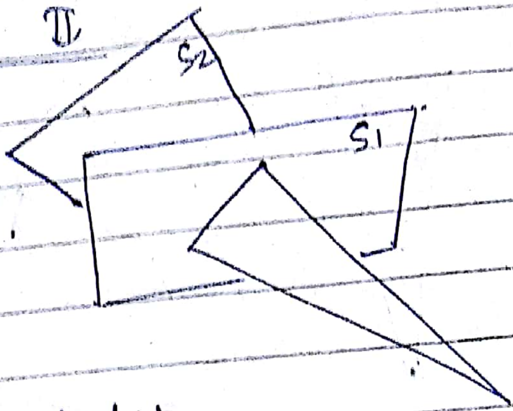
1. Surfaces are stored in an appropriate Data Struc.
2. Surfaces are numbered / tagged.
3. Sort the surfaces in decreasing order
   [no - depth overlap]

Consⁿ of table for Scenario 1

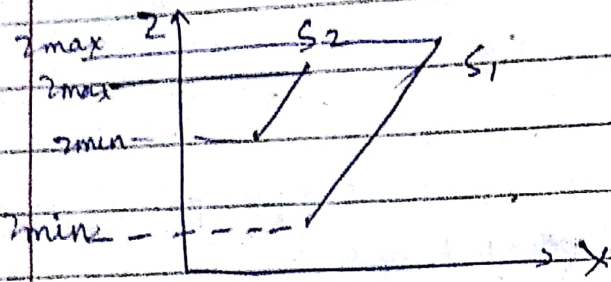
Surface	Behind Counter	List of surf. in front
1	0	2, 3, 4
2	1	3
3	2	5
4	1	5
5	2	—
6	0	—

Surface	List	BC			
1	2,3,4	0 →	-1 →	-1	-1
2	3	1 →	0 →	-1	-1
3	5	2 →	1 →	0	-1
4	5	1 →	0 →	-1	-1
5	—	2 →	2 →	1 →	0
6	—	0 →	-1 →	-1	-1

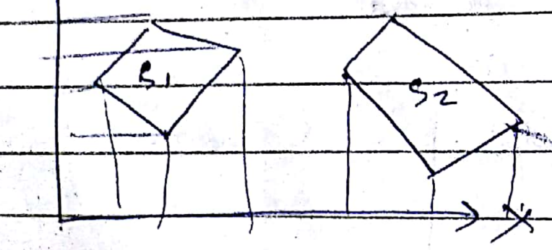
Scenario II



Depth - overlap

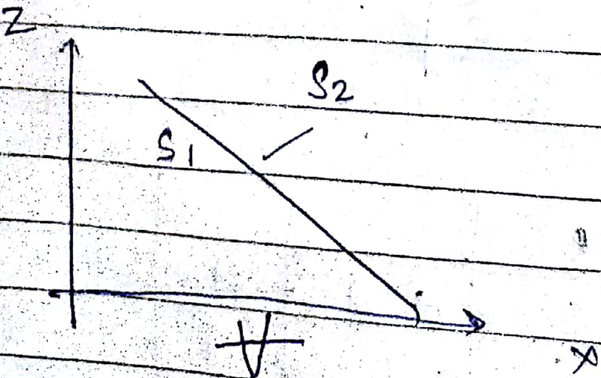


no x overlap



$\{S_1, S_2\} \longrightarrow$ Z overlap exist

no x - overlap exist



$\{S_2, S_1\}$ → paint

↳ followed.

$\{S_1, S_2\}$

~~Reorder~~

$\{S_2, S_1\}$

There is a z overlap ✓

There is a x overlap ✓

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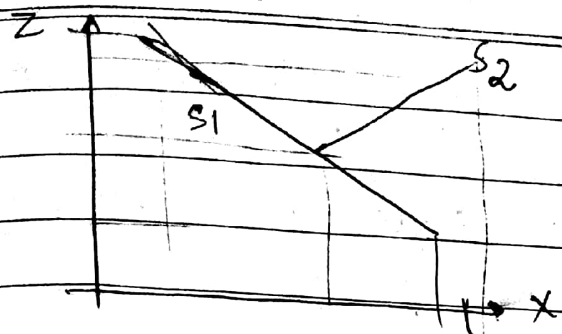
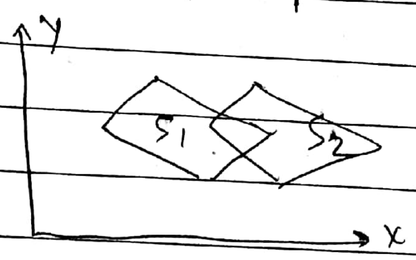


Z overlap

X overlap partial

{S1, S2}

Proj^n on x-y plane



2-6

0.5-4

Q. Consider 3 polygon Q_1 (1,1,1), (4,5,2), (5,(2,5)

Q_2 (2,2,0.5), (3,3,1.75) & (6,1,0.5)

Q3 (0.5, 2, 0.25) (2,5,1.8) & (4,4,0.8)

Surfaces are ordered acc. to the largest z - value on each surface.

Ans. Input: {Q1, Q2, Q3}

Polygon: {Q_1, Q_3, Q_2}

Surfaces with the greatest depth, call it S, is then compared to other surfaces in ~~and~~ the list to determine if there is any overlap in depth. If no depth overlap occurs, S is rendered (scan-converted)

Polygon: {Q_1, Q_3, Q_2}

↓

S

	Depth overlap
S vs Q3	Y
vs Q_2	Y

If a depth overlap occurs, following tests are done.

Test 1. The bounding rectangles in XY plane for ^check the 2 surfaces do not overlap.

4, 5 → check X extent overlap

Q_3. → check Y extent overlap

0.5, 2

	T/F
X extent overlap	T
Y extent overlap	T

Test 2 Surface S is on the outside of the overlapping surface relative to the view plane.

S plane eq^n: Q_1 (1,1,1) (4,5,2) (5,2,5)

$$\begin{pmatrix} -4 & -1 \\ 3 & -3 \end{pmatrix} x + \begin{pmatrix} -1 & -9 \\ -3 & -1 \end{pmatrix} y + \begin{pmatrix} -3 & -4 \\ -1 & 3 \end{pmatrix} z$$

1 − 9   −9 − 4

12 + 3 x

$\boxed{15x - 8y - 13z = -6}$

$$- \begin{pmatrix} 1 & 1 & 1 \\ 4 & 5 & 2 \\ 5 & 2 & 5 \end{pmatrix}$$

Q_2 ↑ 2, 2, 0.5

30 − 16 − 6.5 + 6

+ve

1(21) − 1(20 − 10) + 1(8 − 25)

21 − 10 − 17

Q_3

Test 3 Overlapping surface is on the inside of surface S relative to view plane.

Test 4 Proj^n of the 2 surfaces on to the view plane, do not overlap.

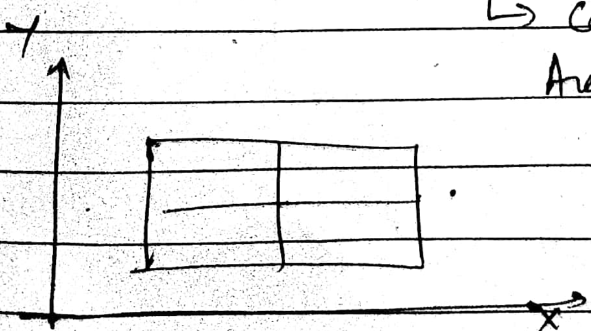
disjoint — overlapping

overlapping: Only with bounding rectangle — with surface & Q2.

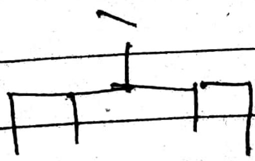
Test 1, 2, 3, 4
if fail (all)
S ← Q3

Problems definition for hidden surfaces:-
Given a set of 3D surfaces to be projected on to a 2D screen, obtain the nearest surfaces corresponding to any point on the screen.

Area Subdivision method
↳ category of Image space,
Area Object Coherence



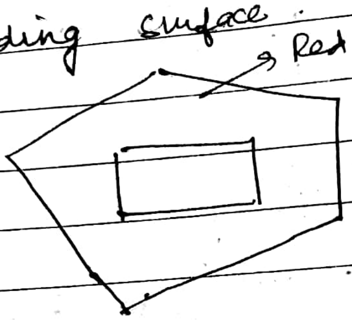
data str. — Tree



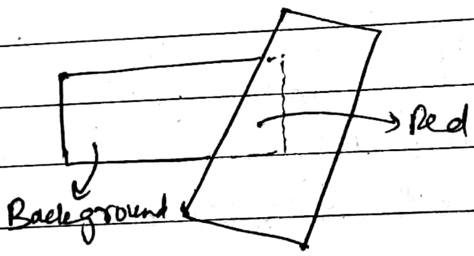
The procedure to determine whether we should subdivide an area into smaller rectangles ~~or~~ not.

1. We classify each of the surface, according to their relation with the area.

a) Surrounding surface.

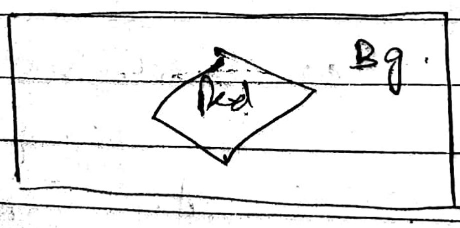


b) Overlapping surface

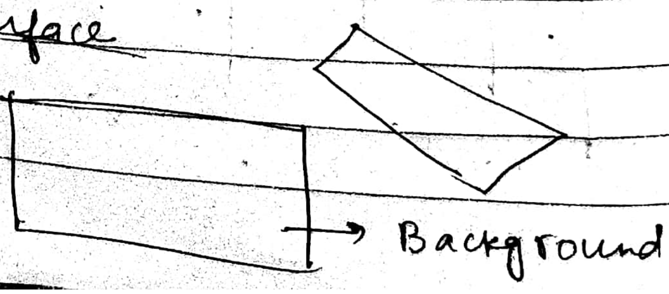


- Whenever there is an overlapping surface, apply Sutherland Hodgman or Weiler Algo.

c) Inside surface



d) Outside Surface



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Check the result from ①. If any of the following condⁿ is true, no subdivision of area is needed.

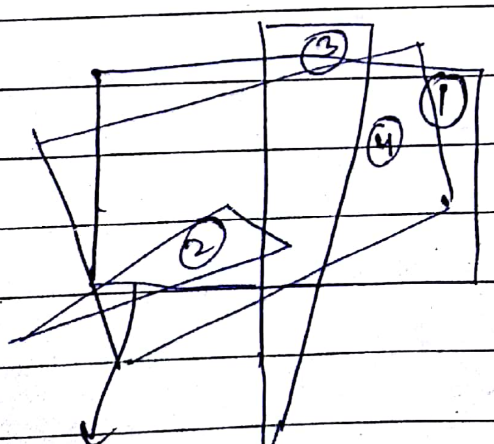
Condition

① All surfaces are outside the area.
→ Display background color on the area.

② Only 1 surface is inside or overlapping or surrounding surfaces into area.
→ Fill the area with background color followed by surface color.

③ A surrounding surface all other surface within the area boundaries. - Surfaces to be colored.

④ More than one polygon intersect contained in & surrounding the area. But, the surrounding polygon is in front of other polygon.
- Render the surface.



One above the other

Why should I render ②?