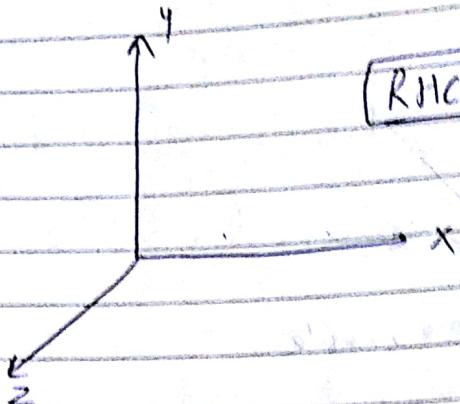


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3D Transformation

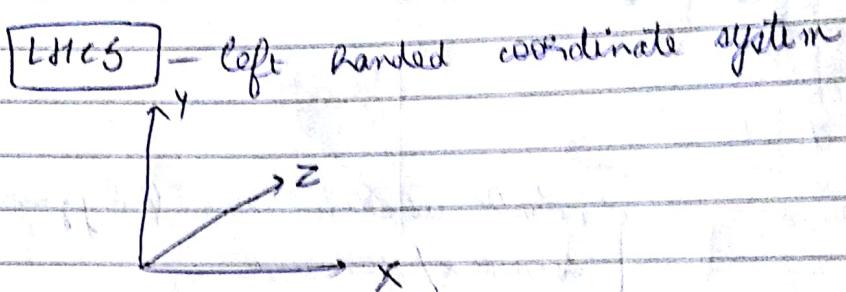
(Page)



RHCS

Right handed coordinate system

{ thumb \rightarrow x axis
index finger \rightarrow y axis
middle finger \rightarrow z axis

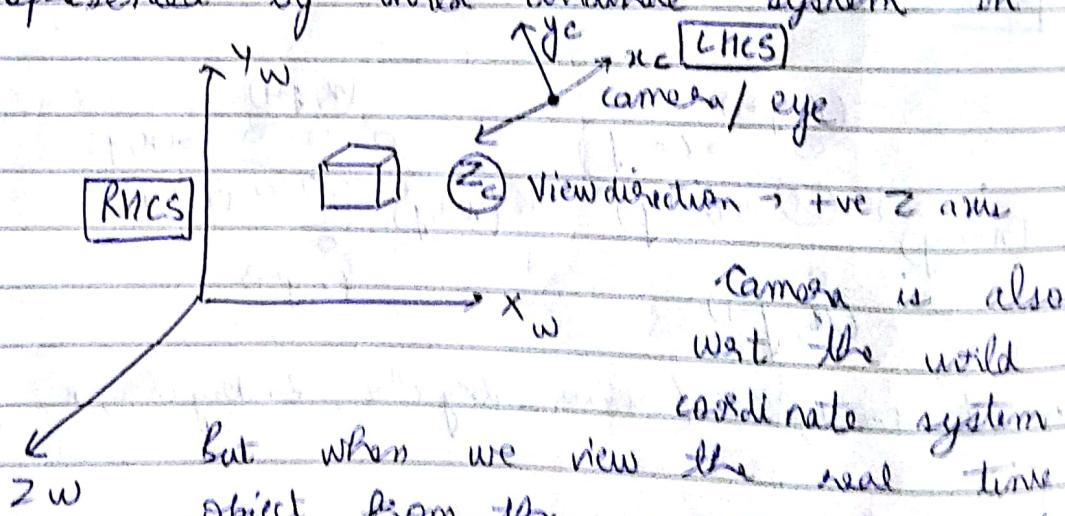


LHCS

Left handed coordinate system

For both
RHCS & LHCS

In real-time, all real-time objects are represented by world coordinate system in RHCS.



But when we view the real-time object from the camera, CSCS is used if we view the object in z axis i.e.

View direction $\rightarrow +ve z$ axis

2nd

face

front

back

left

right

up

down

left

right

up

3D Transf^m

(1)

Translation \rightarrow

$$\begin{pmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(2)

Scaling \rightarrow

$$\begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(3)

Rotation.

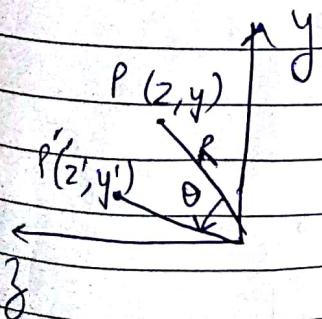
\Rightarrow We ~~also~~ always follow homogeneity in terms of the operation in all the above 3 transformations

- (i) Rotation \rightarrow
- i) about x axis
 - ii) about y axis
 - iii) about z axis

* anticlockwise direction \rightarrow +ve. }

i) Rotation about z axis

see from +ve z \rightarrow $y \rightarrow z$ (anticlockwise)



$$x' = x \cos \theta - y \sin \theta$$

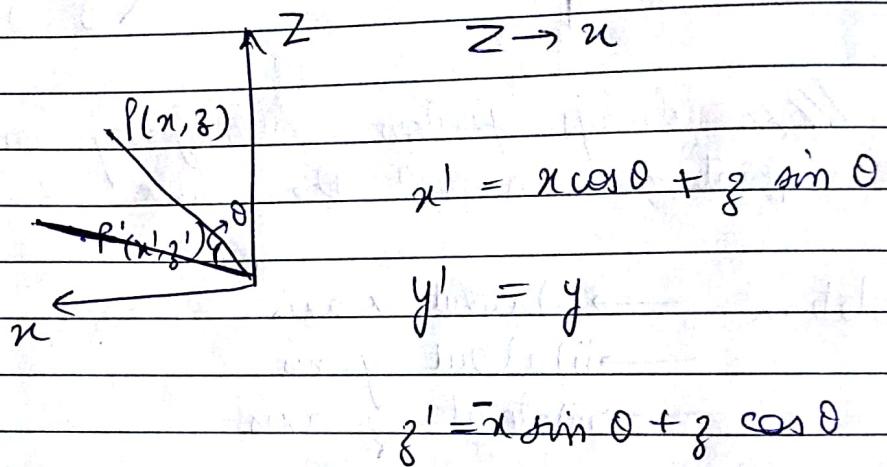
$$y' = y \cos \theta + x \sin \theta$$

$$z' = z$$

$$R_{x, \theta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rot about x by an angle θ , anticlockwise

ii) Rotation about y axis



$$R_{y, \theta} = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

iii) Rotation about z axis

$$R_{z, \theta} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Screen / Retina is 2D
 So, the projection is taken on a plane.
Camera analogy concept

Monitor / screen

side view	front view	Top view	→ 6 camera pos^n
Frontal view	Dimetric view	Isometric view	for an object.

→ Observing the 3D view.

view = projection!

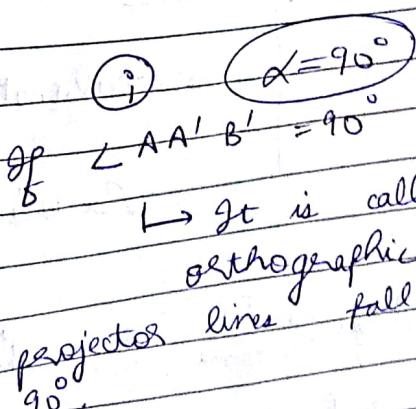
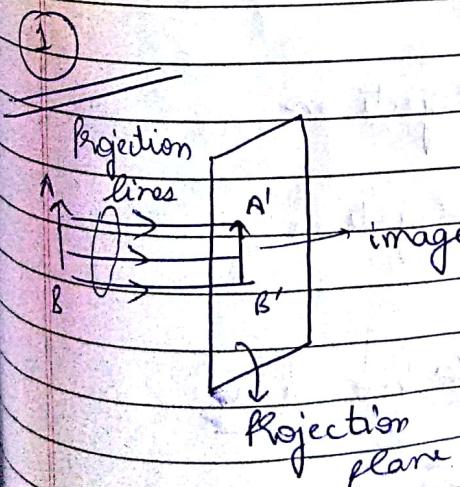
{ like CCTV cameras at different pos^n }

(3D → 2D)

Transformation

Front view → projection on xy plane.

Projection → Parallel → (Projector lines are parallel)
 (1) Non-parallel



It is called as orthographic projection
 projector lines fall on plane at 90°.

(ii) $\alpha \neq 90^\circ$ Oblique projection.
 Projector lines are parallel only but the projection plane is tilted.

(1)

Obligographic projection

Isometric projection

Projector lines are parallel with projection plane
Projector lines make equal angles with all 3 coordinate axes

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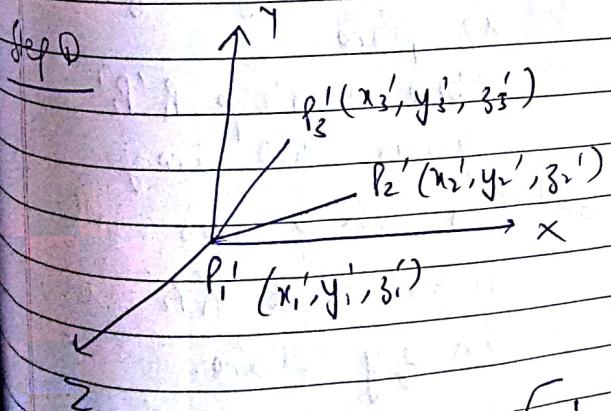
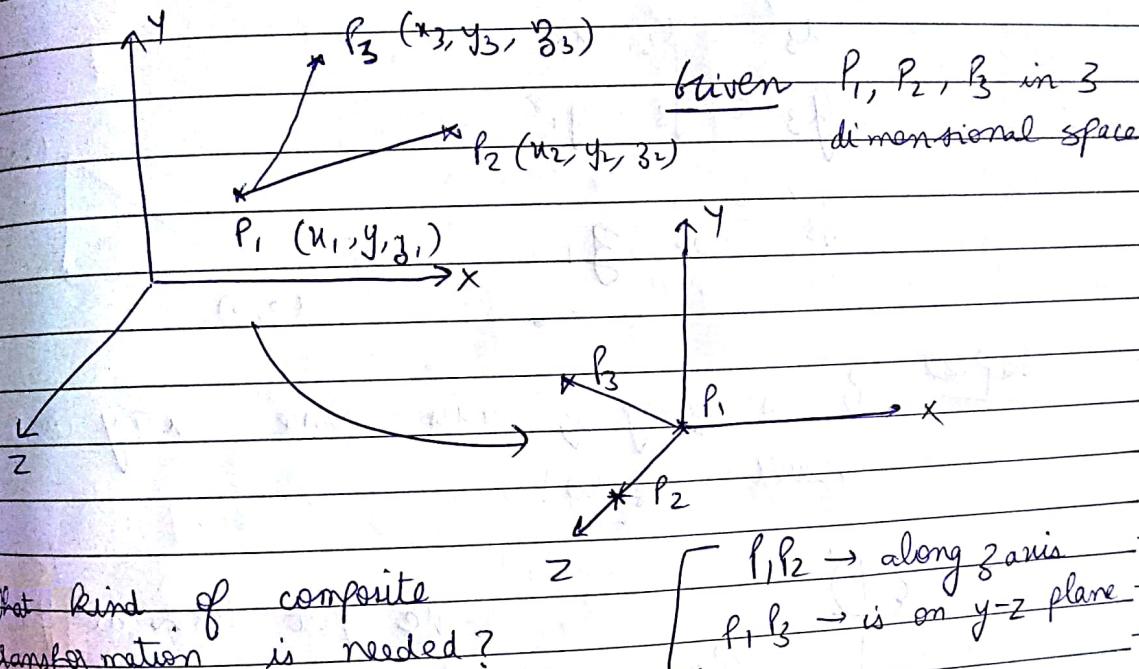
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- 2 cases
- camera stationary, object moving
 - object stationary, camera moving.

Exercise - 1



$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Obj seems

$$x_2' = x_2 - x_1.$$

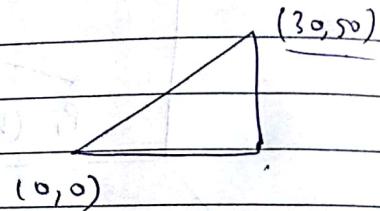
$$y_2' = y_2 - y_1$$

$$z_2' = z_2 - z_1$$

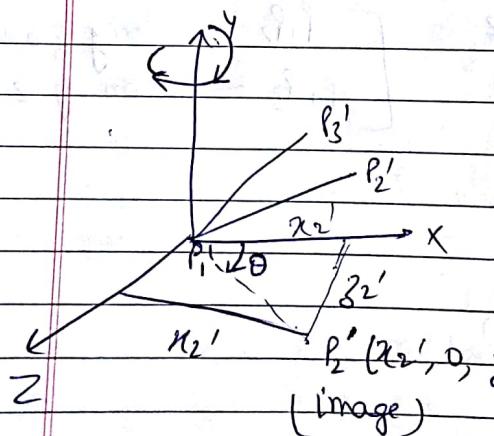
$$x_3' = x_3 - x_1$$

$$y_3' = y_3 - y_1$$

$$z_3' = z_3 - z_1$$



Step 2 P_2 is along z axis hence x & y coordinates should be zero!



We put torch at y axis top & image of P_2 is on x_3 plane.

Now rotating $P_1 P_2'$ about y axis till image of P_2 lies on z axis will mean that $P_1 P_2'$ will be on xy plane.

$$R_y, -(90-0)$$

$$D_1 = \sqrt{(x_2')^2 + (z_2')^2}$$

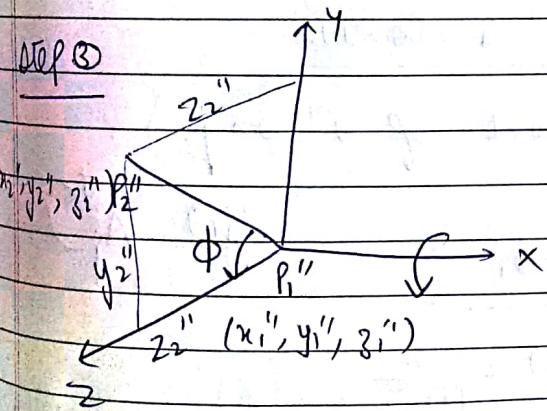
(clockwise direction
rotation by an \angle of
 $90-0$)

Step ④

$$\sin(\theta - 90^\circ) = \sin \theta = \frac{x_2'}{D_1} = \frac{z_2'}{D_1}$$

$$\sin(\theta - 90^\circ) = \cos \theta = \frac{y_2'}{D_1} = \frac{-x_2'}{D_1}$$

$$R_y, \theta - 90^\circ = \begin{pmatrix} \frac{z_2'}{D_1} & 0 & \frac{-x_2'}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_2'}{D_1} & 0 & \frac{z_2'}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Anticlockwise rotation
about x axis

$$D_2 = \sqrt{(y_2'')^2 + (z_2'')^2}$$

$$\cos \phi = \frac{z_2''}{D_2}$$

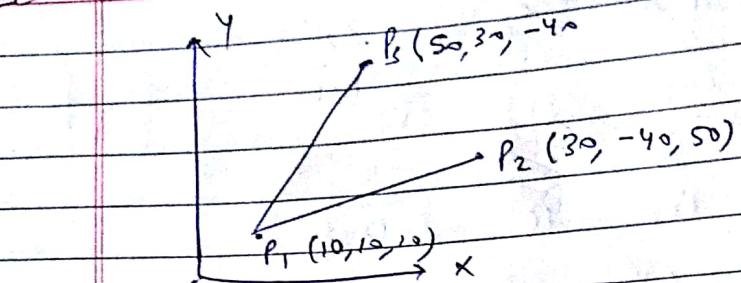
$$\sin \phi = \frac{y_2''}{D_2}$$

$$R_x, \phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_2''}{D_2} & \frac{-y_2''}{D_2} & 0 \\ 0 & \frac{y_2''}{D_2} & \frac{z_2''}{D_2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

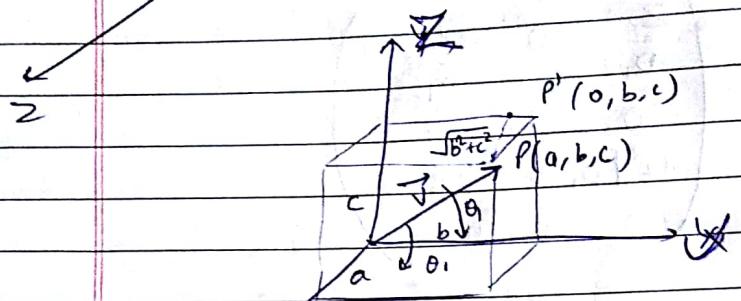
Step ④

The x value of P₁, P₂ will tell whether P₂ is on which side of Y₃ plane & accordingly a rotation will be done to bring P₂ on the Y-Z plane.

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$$\vec{J} = ai + bj + ck$$



Rot^n is clockwise

i) $A_{\vec{v} \rightarrow \vec{v}}$ (Alignment of \vec{v} on \vec{j})

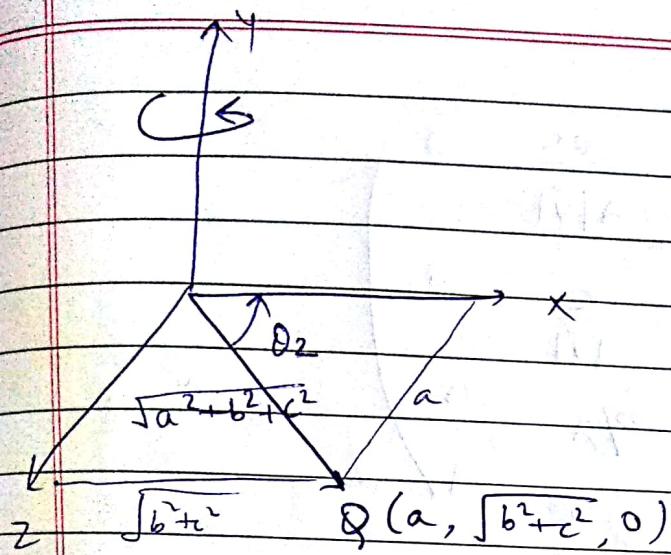
$$\underline{R_{21}, -\theta_1}$$

ii) $A_{\vec{j} \rightarrow \vec{v}}$

$$\cos(-\theta_1) = \frac{b}{\sqrt{b^2 + c^2}}$$

$$\sin(-\theta_1) = \frac{-c}{\sqrt{b^2 + c^2}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & b/\sqrt{b^2+c^2} & c/\sqrt{b^2+c^2} & 0 \\ 0 & c/\sqrt{b^2+c^2} & b/\sqrt{b^2+c^2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & b/2 & c/2 & 0 \\ 0 & -c/2 & b/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$R_{z, \theta_2} = \begin{bmatrix} \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & -\frac{a}{\sqrt{a^2+b^2+c^2}} & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2+c^2}} & \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos \theta_2 = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

$$|\vec{v}| = \sqrt{a^2+b^2+c^2}$$

$$\text{let } \lambda = \sqrt{b^2+c^2}$$

$$\sin \theta_2 = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$R_{z, \theta_2} = \begin{bmatrix} \frac{\lambda}{|\vec{v}|} & -\frac{a}{|\vec{v}|} & 0 & 0 \\ \frac{a}{|\vec{v}|} & \frac{\lambda}{|\vec{v}|} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Composite} = R_{z, \theta_2} \times R_{x, \theta_1}$$

Matrix
(c)

$$= \begin{pmatrix} \frac{\lambda}{|\vec{v}|} & -\frac{a}{|\vec{v}|} & 0 & 0 \\ \frac{a}{|\vec{v}|} & \frac{\lambda}{|\vec{v}|} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b/\lambda & c/\lambda & 0 \\ 0 & -c/\lambda & b/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_v^{ij} = C = \begin{pmatrix} \frac{a}{|v|} & \frac{-ab}{|v|} & \frac{-ac}{|v|} & 0 \\ 0 & \frac{a}{|v|} & \frac{b}{|v|} & \frac{c}{|v|} \\ 0 & 0 & \frac{b/a}{1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_v^{ij} = [A_v^{ij}]^{-1}$$

$$= [R_{z, \theta_2} \times R_{x, -\theta_1}]^{-1}$$

$$= (R_{x, -\theta_1})^{-1} \times (R_{z, \theta_2})^{-1}$$

$$= (R_{x, \theta_1}) \times (R_z, -\theta_2)$$

$$\boxed{A_v^{kj} = R_{x, 90^\circ} * A_v^{ij}} \quad \text{--- (1)}$$

$$P_1(0, 0, 0)$$

$$P_2(20, -50, 40)$$

$$\boxed{A_v^{ij} = R_{z, 90^\circ} * A_v^{ij}}$$

$$\lambda = \sqrt{(50)^2 + (40)^2}$$

$$= \sqrt{4100} = 64.03$$

$$|\vec{N}| = \sqrt{20^2 + 50^2 + 40^2} = \sqrt{4500}$$

$$= 67.08$$

$$C = \begin{pmatrix} \frac{64.03}{67.08} & \frac{100}{67.08} & \frac{-800}{67.08} & 0 \\ 0 & \frac{-50}{67.08} & \frac{40}{67.08} & 0 \\ 0 & -\frac{40}{64.03} & -\frac{50}{64.03} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_{n,90^\circ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

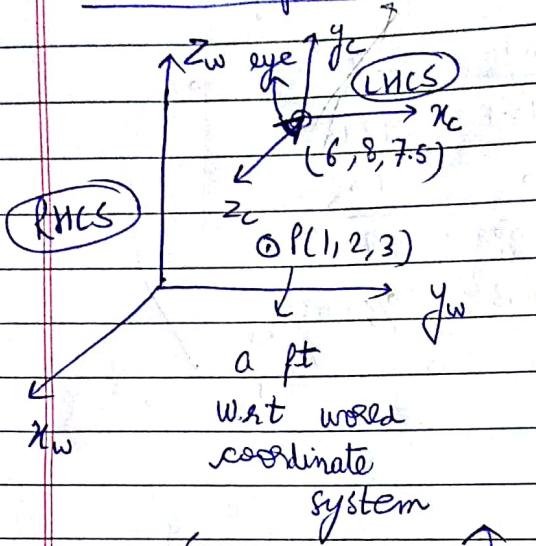
Using ①,

$$R_{n,90^\circ} \times A \xrightarrow{iS}$$

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3D Transformation

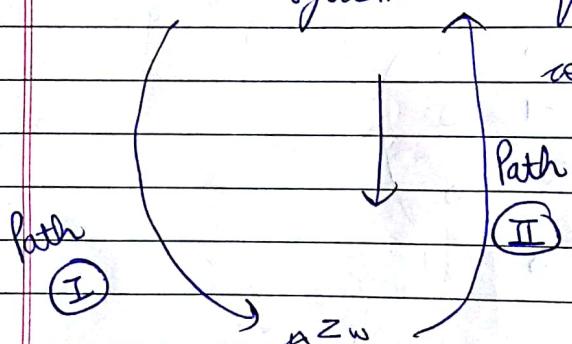
View through a camera



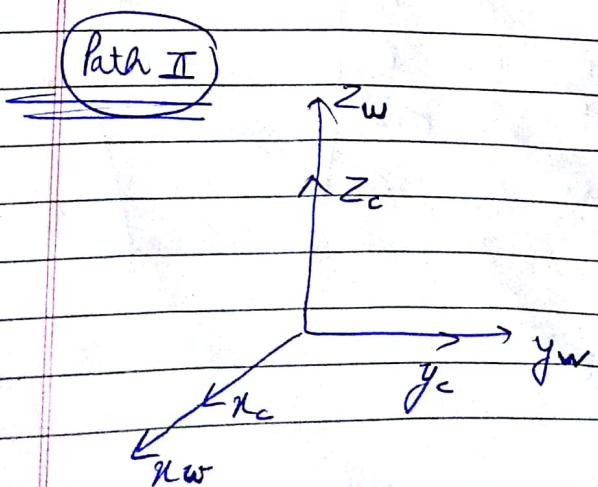
x_c is pointing to origin
 y_c is up
 x_c is on $z_w = 7.5$ plane

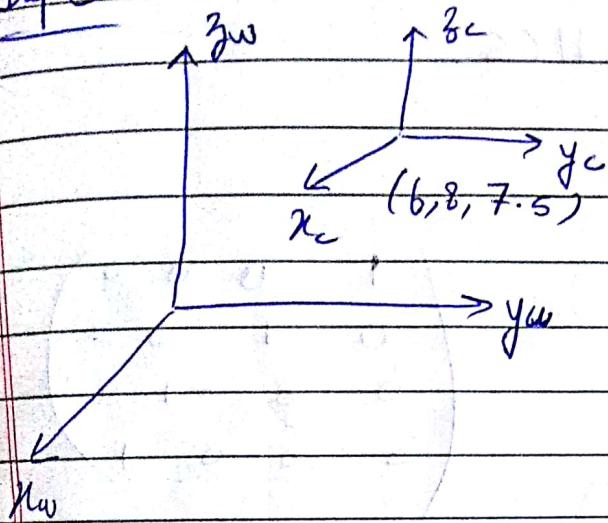
$$P(1 = x_w, 2 = y_w, 3 = z_w)$$

What will be P w.r.t camera coordinates?



transform in such a way
that the z axes systems
coincide.



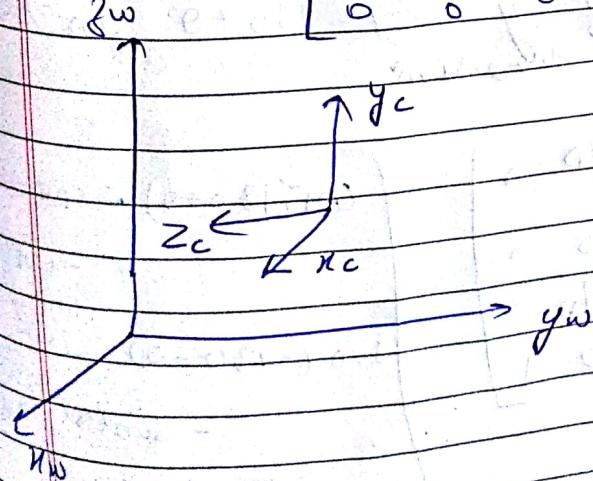
Step ①Translation

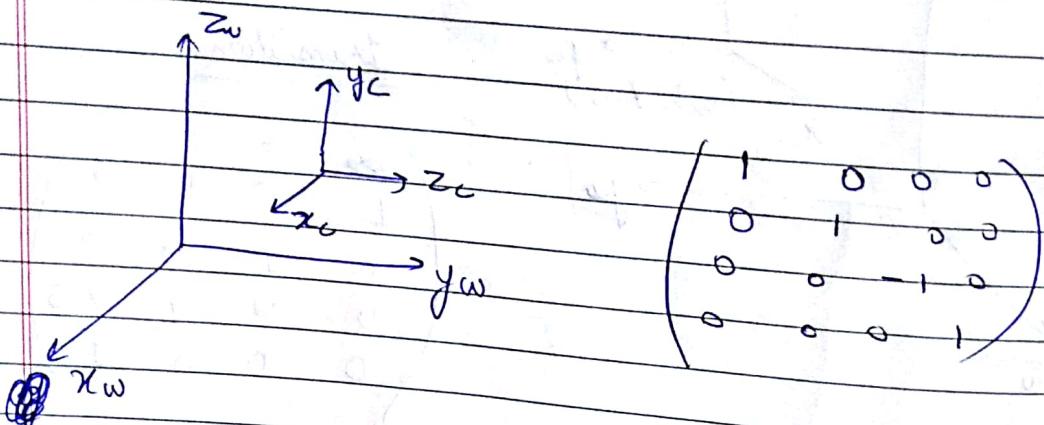
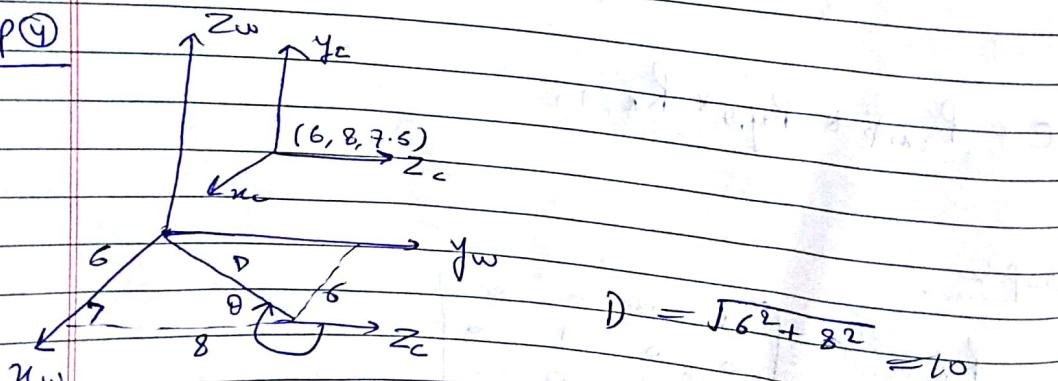
$$\begin{pmatrix} \text{Translation} \\ 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e = R_{w,\phi} * R_{y,0} * R_{x,90^\circ}$$

Step ②

$$R_{x,90^\circ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Step ②F WCS $\xrightarrow{\text{to}}$ L WCSStep ④

Rotation of y by $180 + \theta$ in the clockwise direction

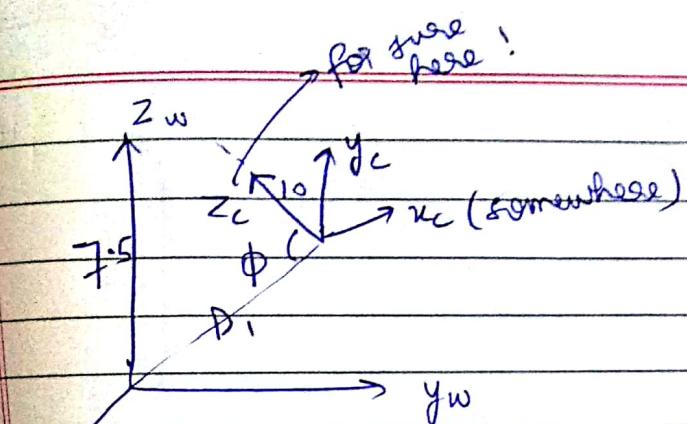
$$R_{y, -(180+\theta)} = \begin{bmatrix} -\frac{8}{D} & 0 & 6/D & 0 \\ D & 0 & 1 & 0 \\ 0 & -6/D & 0 & -8/D \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(-(180+\theta)) = \cos(180+\theta) = -\cos\theta$$

$$\sin(-(180+\theta)) = -\sin(180+\theta) = \sin\theta$$

$$\sin\theta = \frac{6}{D}$$

$$\cos\theta = \frac{8}{D}$$



$$D_1 = \sqrt{(7.5)^2 + 10^2} = 12.5$$

Step 0 Clockwise rotation about the z axis by an angle ϕ .

$$\sin \phi = \frac{7.5}{D_1}$$

$$\cos \phi = \frac{10}{D_1}$$

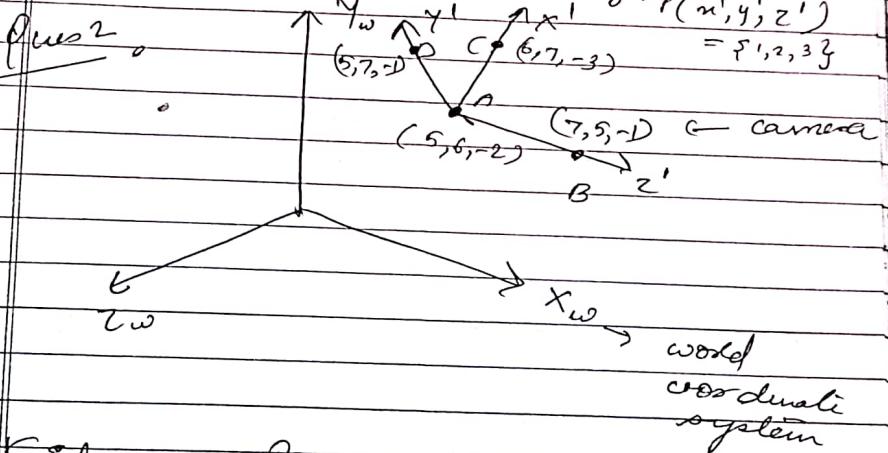
$$h_{x, -\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{10}{D_1} & \frac{7.5}{D_1} & 0 \\ 0 & -\frac{7.5}{D_1} & \frac{10}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Graphics

3D Viewing

Date: _____
Page No. _____

No matter which coordinate we transform, motive is to align the 2 co-ordinate system.



Find ~~cos~~? values w.r.t world coordinate system.

Q) Are the coordinates 90° to each other?

$$\vec{AB} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{AC} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{AD} = \hat{j} + \hat{k}$$

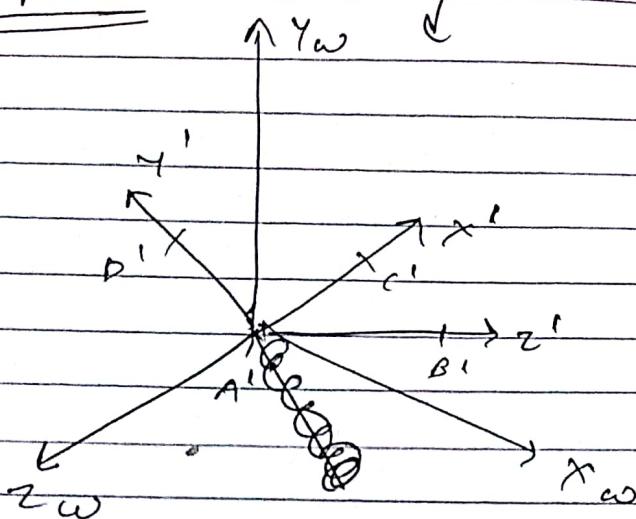
$$\vec{AB} \cdot \vec{AC} = 2 - 1 - 1 = 0$$

$$\vec{AB} \cdot \vec{AD} = 0 - 1 - 1 = 0$$

Coordinate system is correct
and at 90°

Date _____
Page No. _____

Step 1



Translation to origin

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A' = [0, 0, 0]$$

$$B' = [-2, -1, 1]$$

$$C' = [1, 1, -1]$$

$$D' = [0, 1, 1]$$

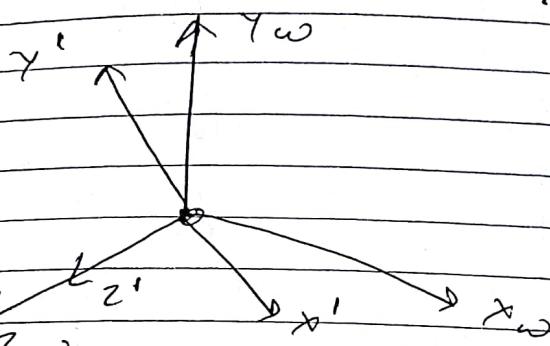
A_v =

$\text{to align } z^i \text{ to } z_w$

$$z(k) \quad z(k)$$

$$A_{z^i} = A^i(z^i - j + k)$$

\uparrow
 $\text{use the alignment matrix.}$



$\text{alignment to } z_w$

$$A_v^i = \begin{bmatrix} \frac{a}{|v|} & \frac{-ab}{|v|} & \frac{-ac}{|v|} & 0 \\ 0 & \frac{b}{|v|} & \frac{c}{|v|} & 0 \\ 0 & 0 & \frac{-c}{|v|} & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_v^k \in \mathbb{R}_{n,90} \times A_v^j$$

$$\lambda = \sqrt{b^2 e_c^2} \quad \lambda = \sqrt{2}$$

$$|\vec{v}| = \sqrt{a^2 e_b^2 e_c^2} = \sqrt{6}$$

$$A_v^j = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{6}} & \frac{2}{\sqrt{12}} & \frac{-2}{\sqrt{12}} & 0 \\ \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix} \quad \lambda |v| = \sqrt{12} = 2\sqrt{3}$$

$$R_{n_3}^{90^\circ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$\tau = \frac{1}{2} \left[\begin{array}{cccc} \frac{1}{\sqrt{3}} & +\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \left[\begin{array}{cccc} 0 & 2 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 9 & 1 & 9 \end{array} \right]$$

coordinates after solution

$$A'' = [0, 0, 0]$$

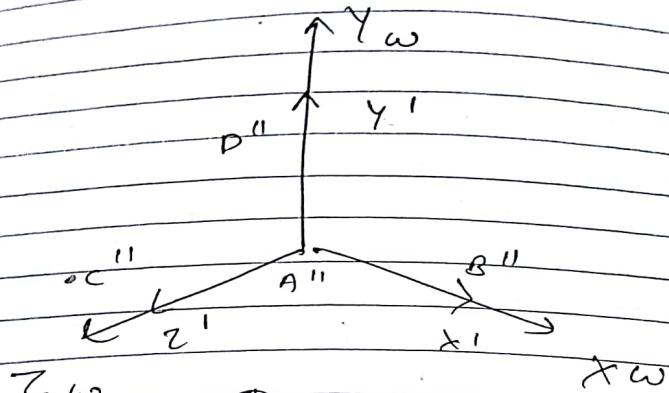
$$B'' = [0, 0, \sqrt{6}]$$

$$C'' = [\sqrt{3}, 0, 0]$$

$$D'' = [0, \sqrt{2}, 0]$$

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Final coordinates



in single notation, we

were able to align the
2 coordinate system.

$$C = T_2 T_1$$

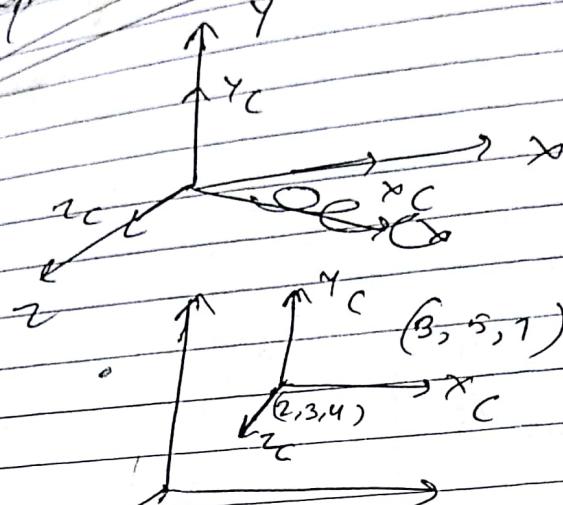
Composite matrix

lation

$$C = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\sqrt{2}}{3} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiply P by this to

Explanations



Objects are what's world

Final composite matrix =

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{5}{\sqrt{3}} & -\frac{6}{\sqrt{3}} & -2 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{6}{\sqrt{2}} & \frac{2}{\sqrt{2}} & 0 \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{5\sqrt{2}}{\sqrt{6}} & \frac{6}{\sqrt{6}} & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{13}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{4}{\sqrt{2}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \frac{10+8}{\sqrt{6}}$$

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$$x = \frac{1}{f_3} e_{33} - \frac{3}{f_3} \frac{13}{f_3} = \frac{13}{f_3}$$

$$y = \frac{2}{f_2} e_{32} - \frac{4}{f_2} \frac{13}{f_3} = \frac{1}{f_2}$$

$$z = \frac{5}{f_2} \frac{13}{f_3} - \frac{2}{f_2} - \frac{3}{f_2} \frac{13}{f_3} = \frac{1}{f_2}$$

$$P = \left(\frac{-13}{f_1 f_3}, \frac{1}{f_2}, \frac{1}{f_2} \right)$$

Projections

3D object \rightarrow 2D image

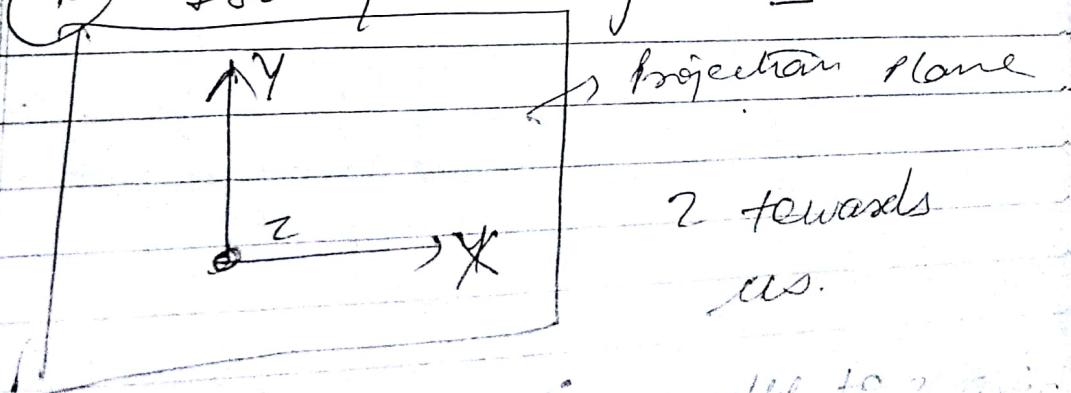
Objective

Frame on which image is formed can be static or moveable

When projector lines are parallel
(Orthographic)

line falls at 90°

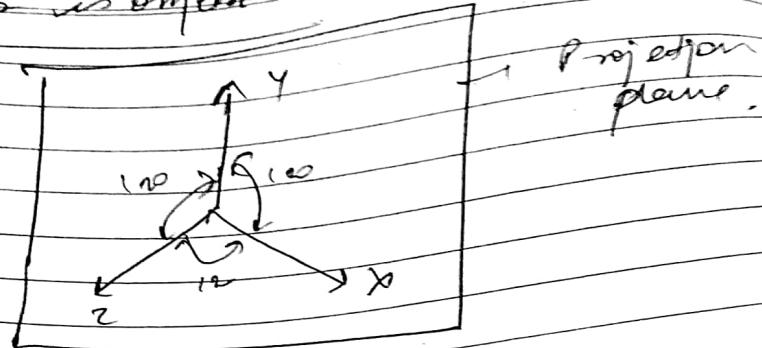
Q) Isometric projection



Projector lines parallel to?

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for isometric



Coordinate

projector lines falling at equal angle
on coordinate axis.

How to obtain this?

$$C = \text{Proj}[x-y] \times R_{x, \theta_2} \times R_{y, \theta_1}$$

What is θ_1 and θ_2 ?

$$C = \text{Proj}[x-y] \times R_{x, \theta_2} \times R_{y, -\theta_1}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_2 & -\sin \theta_2 & 0 \\ 0 & \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos \theta_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ques.

in error
done.

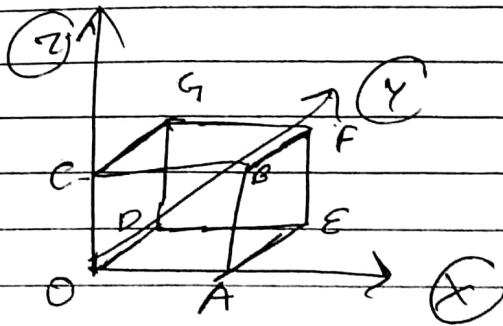
$$C = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

equal angles = 120° need to be erased

angle

Image formed after rotation and before rotation, four shortening vector factors

The ration after should be before same.



$R_y = 0$

$$\Theta = [0, 0, 0]$$

$$AC = [1, 0, 0]$$

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad C = [0, 0, 1] \quad D = [0, 1, 0]$$

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After rotation

$$A (x_n, y_n, z_n)$$

$$C (x_z, y_z, z_z)$$

$$D (x_y, y_y, z_y)$$

↓
After
Projection on
(n-y Plane)

$$A (m_n, y_n, 0)$$

$$B (m_z, y_z, 0)$$

$$C (m_y, y_y, 0)$$

Four shortening factors

$$f_x = \frac{|OA'|}{|OA|} \quad \begin{matrix} \text{four shortening} \\ \text{factors} \\ \text{along x axis} \end{matrix}$$

$$f_y = \frac{|OB'|}{|OB|} \quad \text{along y axis}$$

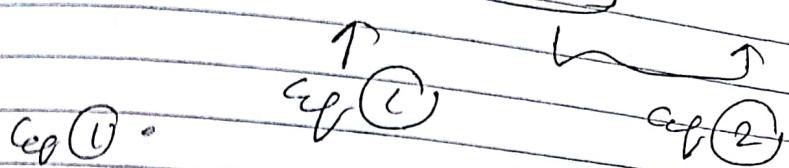
$$f_z = \frac{|OC'|}{|OC|} \quad \text{along z axis}$$

As isometric projection

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$$F_n = F_y + f_z$$

$$I = \text{ } I$$



$$F_n = F_y$$

$$\frac{|O_n'|}{|O_n|} = \frac{|O_B'|}{|O_B|}$$

$$OA = OB$$

unit cube

$$J_{n_x^2 - e_y x^2} = J_{y^2 e_y y^2}$$

$$C = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$* \begin{bmatrix} A & B & C \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Final coordinates

$$= \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \\ \cos\theta_2 \end{bmatrix}$$

c. $\frac{1}{15}$
d. $\frac{8}{15}$

Answer: Option D
Explanation:

$$\text{At } A \text{ day's work} = \frac{1}{15}$$

$$\text{At } B \text{ day's work} = \frac{1}{20}$$

$$(\text{A} + \text{B}) \text{ day's work} = \left(\frac{1}{15} + \frac{1}{20} \right) = \frac{7}{60}$$

$$(\text{A} + \text{B}) \text{ 4 day's work} = \left(\frac{7}{60} \times 4 \right) = \frac{7}{15}$$



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Eq 1

$$\sqrt{\sin^2 \theta_1 + \cos^2 \theta_1} = \sqrt{\sin^2 \theta_2 + \cos^2 \theta_2}$$

$$\cos^2 \theta_1 + \sin^2 \theta_1, \sin^2 \theta_2 + \cos^2 \theta_2$$

Eq 2

$$\sin \theta_1 - \cos \theta_1 = \sin \theta_2 - \cos \theta_2$$

$$\sqrt{\sin^2 \theta_1 + \cos^2 \theta_1} = \sqrt{\sin^2 \theta_2 + \cos^2 \theta_2}$$

$$\cos^2 \theta_1 + \sin^2 \theta_1, \sin^2 \theta_2 + \cos^2 \theta_2$$

$$1 + \sin^2 \theta_2 (1) = 2 \cos^2 \theta_2$$

$$2 \cos^2 \theta_2 = \sin^2 \theta_2 + 1$$

$$\sin^2 \theta_2 =$$

$$2 \cos^2 \theta_2 + 1 - \cos^2 \theta_2 = 1$$

$$3 \cos^2 \theta_2 = 1$$

$$\cos^2 \theta_2 = \frac{1}{3}$$

$$\cos \theta_2 = \sqrt{\frac{1}{3}}$$

$$\theta_2 = \cos^{-1} \left(\sqrt{\frac{1}{3}} \right)$$

$$\frac{2}{3} = \sin 2\alpha_1 + \frac{1}{3} (\cos 2\alpha_1)$$

$$\frac{1}{2} = \frac{1}{3} \sin 2\alpha_1$$

$$\frac{1}{2} = \sin 2\alpha_1$$

$$\sin 2\alpha_1 = \frac{1}{2}$$

$$\alpha_1 = 45^\circ$$

$$\alpha_1 = 345^\circ$$

$$\alpha_2 = 135^\circ$$

Isometric can occur at
only 1 combination
of (α_1, α_2)

Put these values in C to
get the final answer.

Diametric Projection

Projector lines are parallel.

Make equal angles with
any 3 - coordinate axes

at 60°

at 30°

at 90°

at 15°

at 45°

at 75°

at 105°

at 135°

at 150°

at 180°

at 210°

at 240°

at 270°

at 300°

at 330°

at 360°

at 390°

at 420°

at 450°

at 480°

at 510°

at 540°

at 570°

at 600°

at 630°

at 660°

at 690°

at 720°

at 750°

at 780°

at 810°

at 840°

at 870°

at 900°

at 930°

at 960°

at 990°

at 1020°

at 1050°

at 1080°

at 1110°

at 1140°

at 1170°

at 1200°

at 1230°

at 1260°

at 1300°

at 1330°

at 1360°

at 1390°

at 1420°

at 1450°

at 1480°

at 1510°

at 1540°

at 1570°

at 1600°

at 1630°

at 1660°

at 1690°

at 1720°

at 1750°

at 1780°

at 1810°

at 1840°

at 1870°

at 1900°

at 1930°

at 1960°

at 1990°

at 2020°

at 2050°

at 2080°

at 2110°

at 2140°

at 2170°

at 2200°

at 2230°

at 2260°

at 2290°

at 2320°

at 2350°

at 2380°

at 2410°

at 2440°

at 2470°

at 2500°

at 2530°

at 2560°

at 2590°

at 2620°

at 2650°

at 2680°

at 2710°

at 2740°

at 2770°

at 2800°

at 2830°

at 2860°

at 2890°

at 2920°

at 2950°

at 2980°

at 3010°

at 3040°

at 3070°

at 3100°

at 3130°

at 3160°

at 3190°

at 3220°

at 3250°

at 3280°

at 3310°

at 3340°

at 3370°

at 3400°

at 3430°

at 3460°

at 3490°

at 3520°

at 3550°

at 3580°

at 3610°

at 3640°

at 3670°

at 3700°

at 3730°

at 3760°

at 3790°

at 3820°

at 3850°

at 3880°

at 3910°

at 3940°

at 3970°

at 4000°

at 4030°

at 4060°

at 4090°

at 4120°

at 4150°

at 4180°

at 4210°

at 4240°

at 4270°

at 4300°

at 4330°

at 4360°

at 4390°

at 4420°

at 4450°

at 4480°

at 4510°

at 4540°

at 4570°

at 4600°

at 4630°

at 4660°

at 4690°

at 4720°

at 4750°

at 4780°

at 4810°

at 4840°

at 4870°

at 4900°

at 4930°

at 4960°

at 4990°

at 5020°

at 5050°

at 5080°

at 5110°

at 5140°

at 5170°

at 5200°

at 5230°

at 5260°

at 5290°

at 5320°

at 5350°

at 5380°

at 5410°

at 5440°

at 5470°

at 5500°

at 5530°

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at 5620°

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at 6100°

at 6130°

at 6160°

at 6190°

at 6220°

at 6250°

at 6280°

at 6310°

at 6340°

at 6370°

at 6400°

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at 6670°

at 6700°

at 6730°

at 6760°

at 6790°

at 6820°

at 6850°

at 6880°

at 6910°

at 6940°

at 6970°

at 7000°

at 7030°

at 7060°

at 7090°

at 7120°

at 7150°

at 7180°

at 7210°

at 7240°

at 7270°

C. 15
D. 8
Answer: Open D

Explanation:

$$\text{As 1 day's work} = \frac{1}{15}$$

$$\text{B's 1 day's work} = \frac{1}{30}$$

$$(\text{A} + \text{B})\text{'s 1 day's work} = \left(\frac{1}{15} + \frac{1}{30}\right) = \frac{1}{10}$$

$$(\text{A} + \text{B})\text{'s 4 days work} = \left(\frac{1}{10} \times 4\right) = \frac{2}{5}$$

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$\sin \theta_1 \cos \theta_2 + \sin \theta_2 \cos \theta_1$

look

• look

P

kilo

Any 3 possible cases

~~1000~~

case 1

$$\cos^2 \theta_1 + \sin^2 \theta_1, \sin^2 \theta_2,$$

$$= \cos^2 \theta_2 \quad \text{--- (1)}$$

$$\cos^2 \theta_2 = k^2 (\sin^2 \theta_1 + \sin^2 \theta_2 \cos^2 \theta_1) \quad \text{--- (2)}$$

Because

$$\cos^2 \theta_2 = k^2 \sin^2 \theta_1 + k^2 \sin^2 \theta_2 (1 - \sin^2 \theta_1)$$

$$\cos^2 \theta_2 = k^2 \sin^2 \theta_1 + k^2 \sin^2 \theta_2 - k^2 \sin^2 \theta_1 \sin^2 \theta_2$$

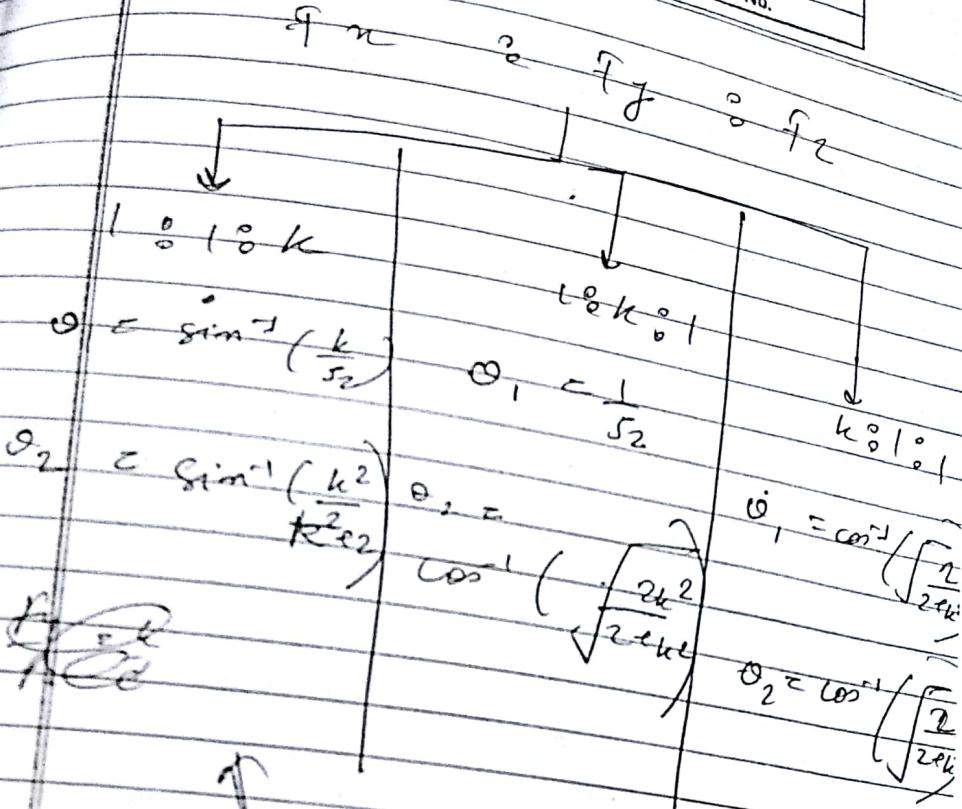
Solve these 2 equations
simultaneously

Replace

~~(cos theta_1, sin theta_1, cos theta_2,
sin theta_2)~~

solve for all cases

for θ_1 & θ_2



→ ② Find k & angles now

$$\theta_1 \rightarrow \frac{k}{\sqrt{2}k} > -1$$

$$1 > \frac{k^2}{k^2 + 1} - 1$$

$$1.0414 > k > -1.0414$$

$$1 > \frac{k^2}{k^2 + 1} > 0$$

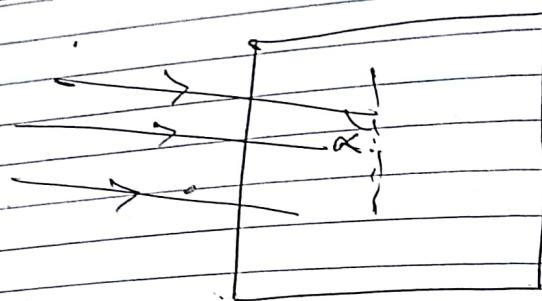
rough $k^2 + 1 > k^2 > 0$

↑ Always true

3rd part
(or 3rd part)

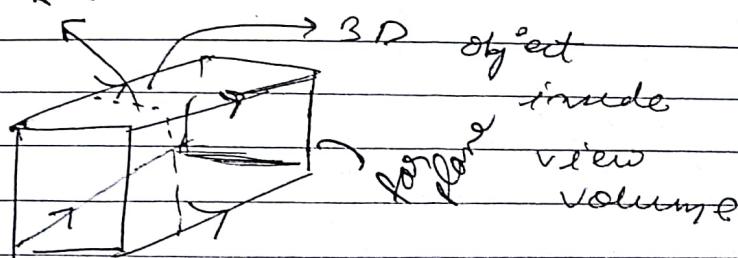
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OBLIQUE PROJECTION



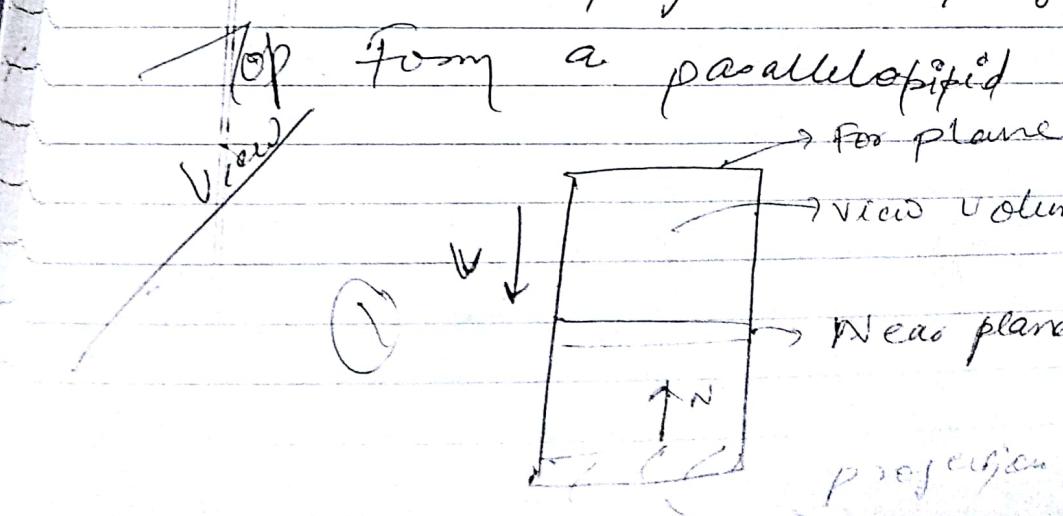
Projectors lines are parallel but not falling on plane with 90° .

$\text{proj. line} \neq 90^\circ$



These projectors lines go far

Top from a parallelopiped



projection plane

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In case of orthographic projection

For a camera, near plane

For plane \rightarrow no dear

Object outside the volume is

clipped.

For oblique

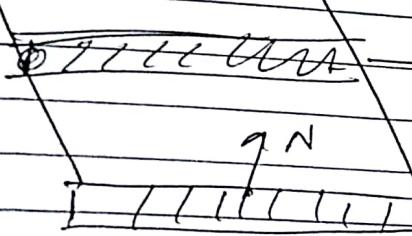
Top

View

(2)

For plane

(2)

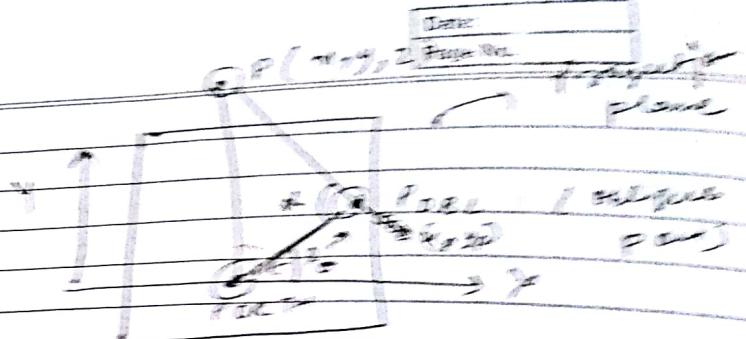


near plane

We need to transform

Fig (2) \rightarrow Fig 1

Composite matrix will give
the transformation



P. P_{max} & X_{min}

at the static
point

P. P_{min} & X_{max}

$$\text{load} \in (\alpha_1, \gamma_1)$$

$$x_p = a - b \cos \theta$$

$$y_p = y + b \sin \theta$$

$$\tan \theta = y/x$$

$$\theta = y/\tan x$$

$$\left\{ \text{let us take } \frac{1}{\tan x} = b/y \right.$$

$$\theta = y/b$$

$$y_p = a - b \cos \theta$$

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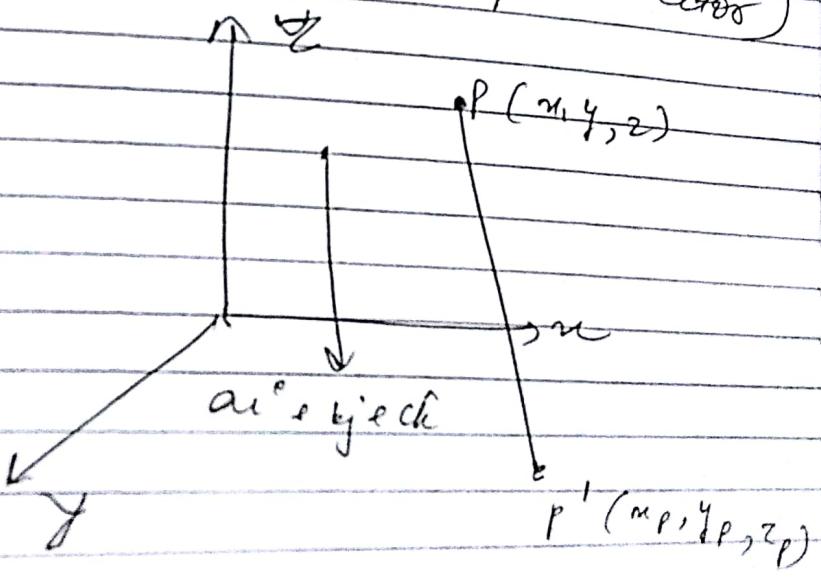
$$z_p = 0$$

There is shearing involved
in oblique project

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & h_{1000} & 0 \\ 0 & 1 & h_{1000} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

— (1)

\vec{v}_p (view direction vector)



Find Shearing matrix

$$PP' = \epsilon (x_p - x) i + (y_p - y) j + \epsilon (z_p - z) k$$

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F_P , and view are parallel
 $\vec{P_P} \parallel \vec{v}$

$$(x_p - x) \hat{i} + (y_p - y) \hat{j} + (z_p - z) \hat{k}$$

$$= -t [a \hat{i} + b \hat{j} + c \hat{k}]$$

$$x_p - x = at$$

$$y_p - y = bt$$

$$\textcircled{z_p - z = ct}$$

$z_p = 0$ (as its projection
on XY plane)

$$t = -z/c$$

$$x_p = x + at$$

$$= x - \frac{az}{c}$$

$$y_p = y - \frac{bz}{c}$$

~~x_p~~ putting in matrix 1

$$x_p = (a \cos \theta) (z) + x$$

$$\frac{x}{z} = a^2 \Rightarrow x^2 + a^2 \cos^2(\theta z) = y^2$$

$$t = -a/c$$