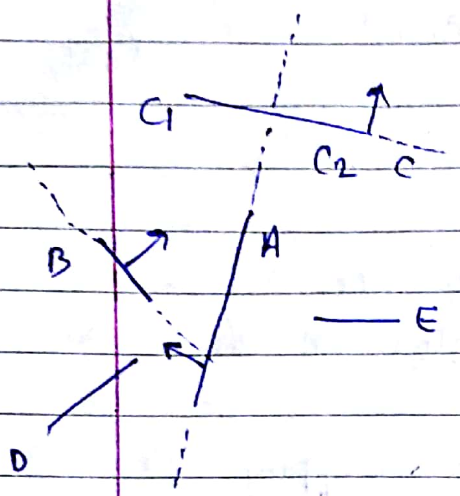
B.S.P (Binary Space Partition)



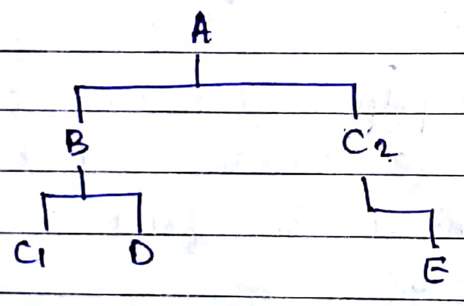
Surfaces

A
B
C
D
E

- Declaring a normal outward.
- Space in 2D is divided into 2 parts:

Front (Normal outward dir.)

Back (Away from normal)



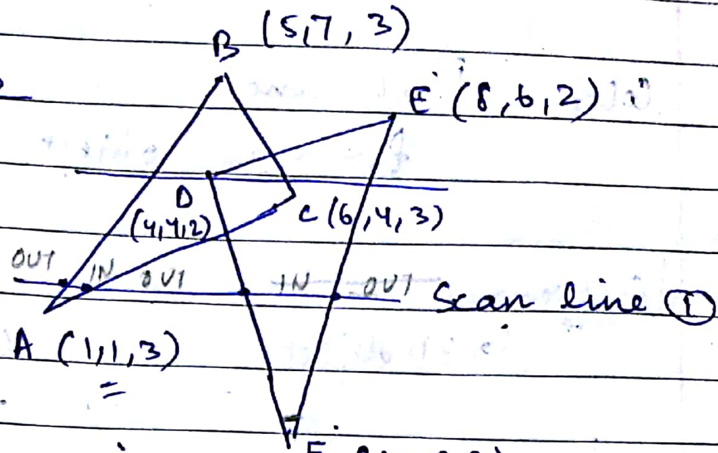
→ clockwise normal dir.

→ Back face Culling ✓
→ Z Buffer
→ Painter – No overlapp ✓ (List)
Overlapp ✓ (Reorder 4 tests)
→ Area Subdivision ✓ (But need questions)
→ Scan line
→ BSP ✓
→ Quad Tree Oct Tree

Hidden Surface

SCAN LINE METHOD

Extension of polygon fill.



- Horizontal ~~filling~~ scanning
- Vertical Scanning

Data Structure

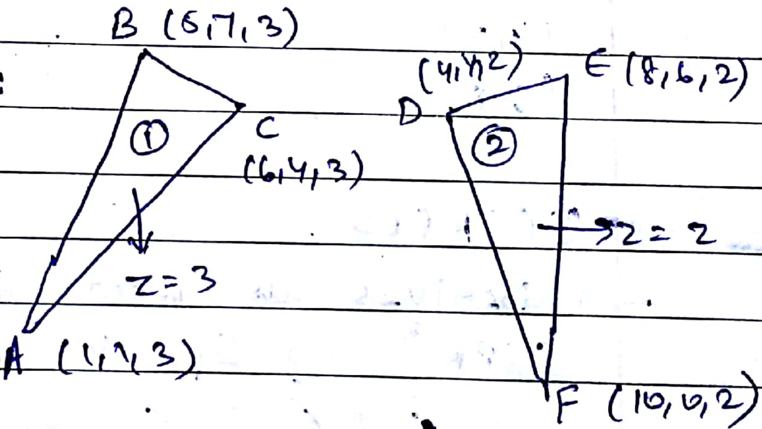
Edge

y_{max}	$x_{y_{min}}$	$1/m$	P_{ID}	→

Polygon

P_{ID}	Plane Eq	Shading info	In out	→

- Global edge table :



y_{max} 7

6

5

4 — | 7 | 6 | -1/3 | 1 | → | 6 | 4 | 2 | 2 |

3

2

1 — | 4 | 1 | 3/3 | 1 | → | 7 | 1 | 2/3 | 1 | → ∧

0 — | 4 | 10 | -3/2 | 2 | → | 6 | 10 | -1/3 | 2 | → ∧

y_{min}

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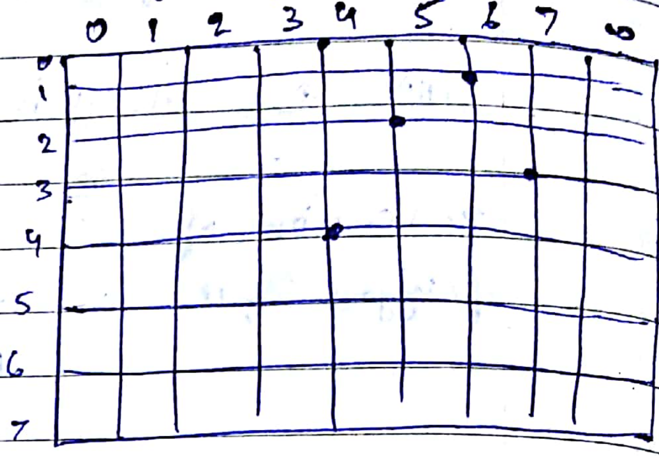
Quad Tree (2-D) (Image Space)



Color | Coherence |
from an object

| Octree |

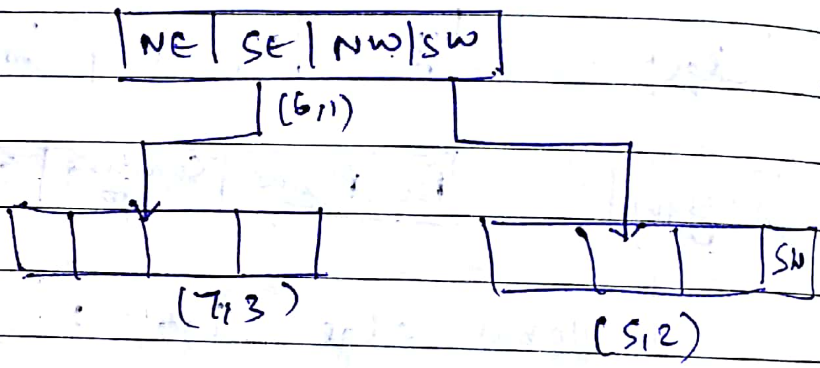
↳ 3D object

NW	NE
SW	SE

A tree with 4 children.

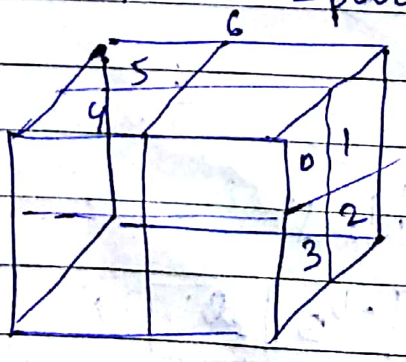
NE	SE	NW	SW

(6,1)
(5,2)
(7,3)
(4,5)
(6,8).



OCTREES.

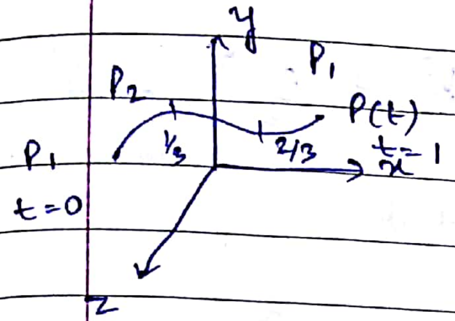
→ Derives its word from octal + Tree
(SPR) → Spartial Partioning Representation



VOXEL
↳ Smallest unit in 3D

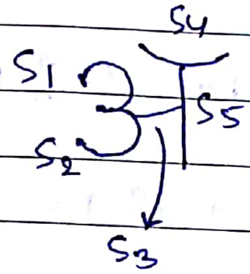
0	1	2	3	4	5	6	7

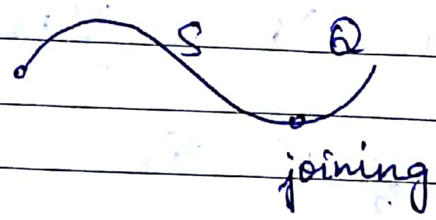
3D Curves



Segment of a curve

- font design & development
- automobile industry.





Smooth

$$P(t) = a_{x0} + a_{x1} t + a_{x2} t^2 + \cdots + a_{xn} t^n$$

- $n = 1$ line
- $n = 2$ quadratic
- $n = 3$ cubic curve.

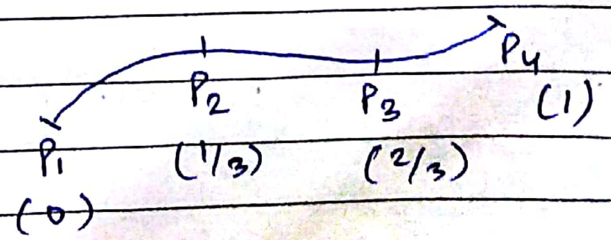
Cubic

$$P(t) = [x(t), y(t), z(t)]$$

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

$$P(t) = \vec{a} t^3 + \vec{b} t^2 + \vec{c} t + \vec{d} \quad \text{— (1)}$$

Cubic curve → 4 points need.



$P_1(t=0) = \vec{d}$

$P_2(t = 1/3)$

$$= \frac{\vec{a}}{27} + \frac{\vec{b}}{9} + \frac{\vec{c}}{3} + \vec{d}.$$

Cubic curve → 4 points need

$$P_3(t = 2/3) = \frac{8}{27}\vec{a} + \frac{4}{9}\vec{b} + \frac{2}{3}\vec{c} + \vec{d}$$

$$P_4(t = 1) = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

Solving

$$\vec{a} = \left(-\frac{9}{2}\right)P_1 + \frac{27}{2}P_2 - \frac{27}{2}P_3 + \frac{9}{2}P_4$$

$$\vec{b} = \cancel{-\frac{11}{2}P_1}\ 9P_1 - \frac{45}{2}P_2 + 18P_3 - \frac{9}{2}P_4$$

$$\vec{c} = -\frac{11}{2}P_1 + 9P_2 - \frac{9}{2}P_3 + P_4$$

$$\vec{d} = P_1$$

Put in ① blending funcⁿ

$$P(t) = P_1\left(-\frac{9}{2}t^3 + 9t^2 - \frac{11}{2}t + 1\right) +$$

$$P_2\left(\frac{27}{2}t^3 - \frac{45}{2}t^2 + 9t\right) + P_3\left(-\frac{27}{2}t^3 + 18t^2 - \frac{9}{2}t\right)$$

$$+ P_4\left(\frac{9}{2}t^3 - \frac{9}{2}t^2 + t\right)$$

for t = 0 to 1 step = 0.0001

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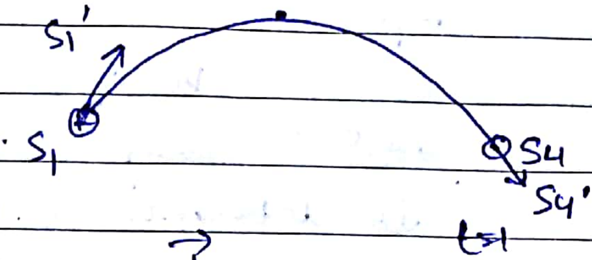
HERMITE CURVE

Q. Design a curve segment for a stone thrown at a particular dir & magnitude.



t=0 $S_1 = \vec{d}$ $\qquad S_4 = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

$S_1' = \vec{c}$ $\qquad S_4' = 3\vec{a} + 2\vec{b} + \vec{c}$

$S_4 = \vec{a} + b + S_1 + S_1'$ $\qquad S_4' = 3a + 2b + S_1'$

$b = S_4 - S_1 - S_1' - a$ $\qquad S_4' = 3a + 2S_4 - 2S_1 - 2S_1' - 2a + S_1'$

$S_4' - 2S_4 + S_1' + 2S_1 = a$

$b = S_4 + \cancel{S_1} - \cancel{S_1'} - S_4' + 2S_4 - S_1' - S_1$

- - - - (calc) :)

$\vec{a} = -2S_4 + S_4' + 2S_1 + S_1'$

$\vec{b} = 3S_4 - 3S_1 - S_4' + 2S_1'$

$\vec{c} = S_1'$

$\vec{d} = S_1$

$P(t) = S_1 \underbrace{(2t^3 - 3t^2 + 1)}_{\text{B.F.1}} + S_1' \underbrace{(t^3 - 2t^2 + t)}_{\text{B.F.2}}$
$+ S_4(-2t^3 + 3t^2) + S_4'(t^3 - t^2)$

Blending func^n

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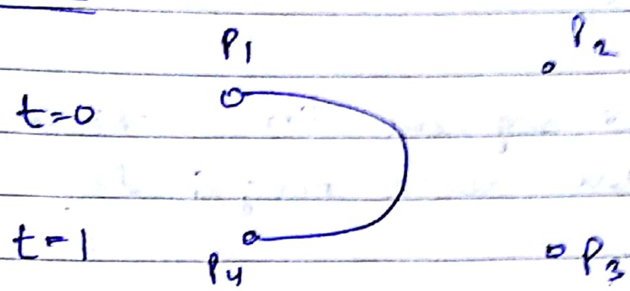
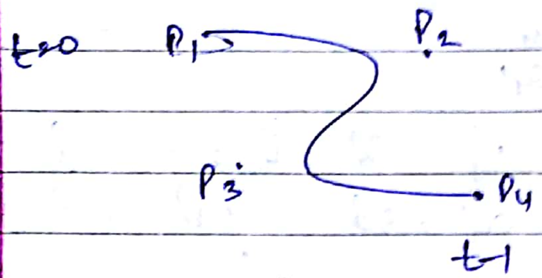
Beizer

Q-



• Intermediate points are not touching the curve.



Design

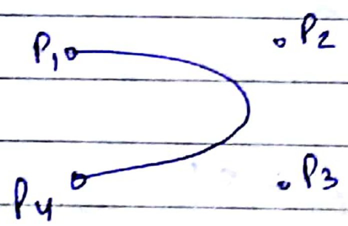
Note: Shape of curve is determined by intermediate points (acts as MAGNET)

4 points → 3 degree

$$P(t) = \vec{a}t^3 + \vec{b}t^2 + \vec{c}t + \vec{d}$$

Magnet Action (HW)

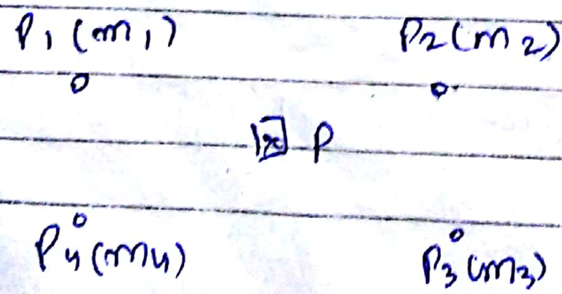
Cubic curve segment



P_2, P_3 → magnet action.

→ End points P_1, P_4 touching the curve.
→ P_2, P_3 are controlling point for shape of curve

Pierre Bezier → Renault Car.



Point masses
Location is static

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Centre of mass:-

$$P = \frac{m_1 P_1 + m_2 P_2 + m_3 P_3 + m_4 P_4}{m_1 + m_2 + m_3 + m_4}$$

in case of dynamic masses.
Now, think that at instead of being fixed, each mass is computed with a function.

$m_1 = (1-t)^3$ $\quad$ $t=0$
$m_2 = 3t(1-t)^2$ $\quad$ Same
$m_3 = 3t^2(1-t)$
$m_4 = t^3$ $\quad$ $t=1$

$$P(t) = \sum_{i=0}^{n} P_i \cdot B_{i,n}(t) \quad \rightarrow \text{Blending func}^n.$$

$$0 \le t \le 1$$

$$B_{in}(t) = {}^nC_i \cdot t^i \cdot (1-t)^{n-i}$$

$[0,3] \leftarrow$ $n=3 \rightarrow$ cubic curve

$$P(t) = \sum_{i=0}^{3} P_i \, B_{i,3}(t) \qquad 0 \le t \le 4$$

$$B_{i,3}(t) = {}^3C_i \cdot t^i (1-t)^{3-i}$$

$B_{0,3} = (1-t)^3 = 1 - t^3 - 3t + 3t^2$
$B_{1,3} = 3t(1-t)^2 = 3t + 3t^3 - 6t$
$B_{2,3} = 3t^2(1-t) = 3t^2 - 3t^3$
$B_{3,3} = t^3$

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$$P(t) = \underbrace{[t^3 \quad t^2 \quad t \quad 1]}_{T} \underbrace{\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{M_{Bez}} \underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}}_{G_{Bez}}$$

Beizer Matrix

How to connect Hermite with Beizer ?

$T \to [t^3 \quad t^2 \quad t \quad 1]$

$m \to$ basis matrix

$$M_H = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad G_H = \begin{bmatrix} B_1 \\ B_4 \\ B_1' \\ B_4' \end{bmatrix}$$

Beizer

$$P(t) = P_1(1-t)^3 + P_2\, 3t\,(1-t)^2 + P_3\, 3t^2(1-t) + P_4 t^3$$

$$P'(t) = -3P_1(1-t)^2 + [-6t(1-t) + 3(1-t)^2]P_2 + 6P_3 t(1-t) - 3P_3 t^2 + 3P_4 t^2$$

at $P = 0$,

$$P'(0) = 3(P_2 - P_1)$$

Hermite — Beizer

$$P_1' = P'(0) = 3(P_2 - P_1)$$

$$P_4' = P'(1) = 3(P_4 - P_3)$$

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$$G_H = \begin{bmatrix} P_1 \\ P_4 \\ P_1' \\ P_4' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}}_{G_{BEZ}}$$

2 points 2 derivatives
4 constraints

$$\boxed{G_H = M_{HBez} \cdot G_{BEZ}}$$

4 geometric points
⇓
4 constraints

$$P(t) = T \cdot M_H \cdot G_H$$

$$G_H = M_{HB} \cdot G_B$$

$$P(t) = T \cdot M_H (M_{HB} \cdot G_B)$$

$$= T \cdot (M_H \cdot M_{HB}) \cdot G_B$$

$$= T \cdot M_B \cdot G_B$$

$$M \cdot M_{HB} = \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{pmatrix}$$

$$= \begin{bmatrix} -1 & 1 & -3 & 3 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = M_B.$$

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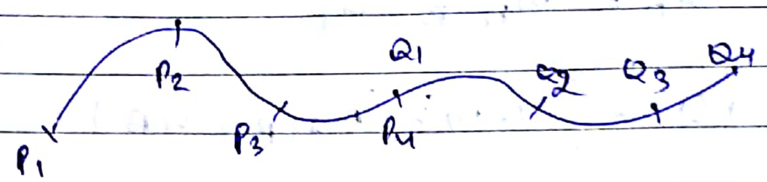
Q. Cubic Beizer curve segment described by the control points
P_1 (20, 20) P_2 (40, 80)
P_3 (80, 80) P_4 (90, 50)

another segment is described by -
Q_1 (a, b) Q_2 (c, 20)
Q_3 (150, 20) Q_4 (180, 20)

Determine a, b, c s.t. curve segments join smoothly :-



Since they join,
$P_4 = Q_1$ $a = 90, b = 50$

for smoothness, first derivatives are equal.

$$P(t) = P_1(1-t)^3 + P_2(3t(1-t)^2) + P_3\, 3t^2(1-t) + P_4 t^3$$

$$P'(1) = Q'(0)$$

$$3(P_4 - P_3) = 3(Q_2 - Q_1)$$

$$90,50 - 80,80 = c,20 - a,b$$

$$10,-30 = c-90, -30$$

$$c - 90 = 10 \qquad \boxed{c = 100}$$

Q_2 (c, d)

$$d - b = -30$$
$$d = 50 - 30 \qquad \boxed{d = 20} \rightarrow \text{given}$$

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If $Q_3(e,f)$ second derivatives equate then $P''(1) = Q''(0)$.