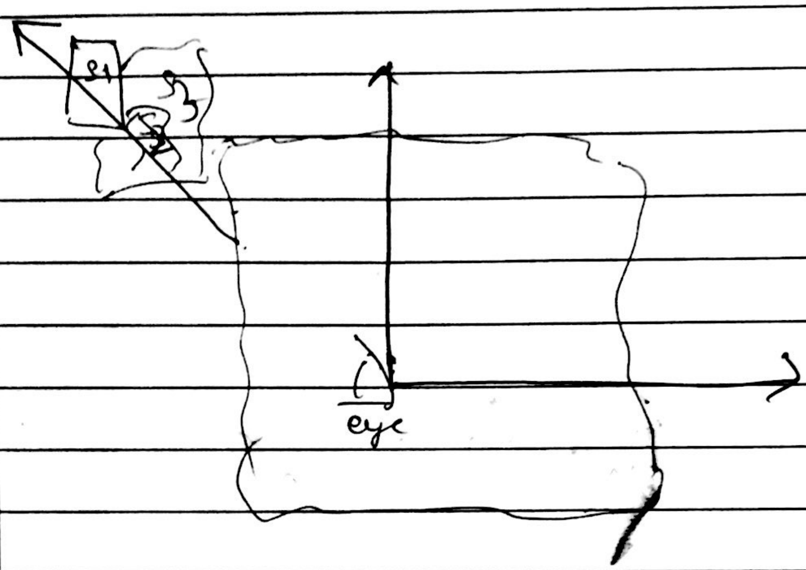
Hidden surface

1 Backface detection 2) Z-Buffer

3D object Image (2D)



$S = \{S_1, S_2, \ldots, S_n\}$

$S_1 = (x, y, z_1)$

$S_2 = (x, y, z_2)$

$S_3 = (x, y, z_3)$

View that having minimum z value.

Step 1:

find $\{x_{min}, y_{min}, z_{min}\}$

find $\{x_{max}, y_{max}, z_{max}\}$

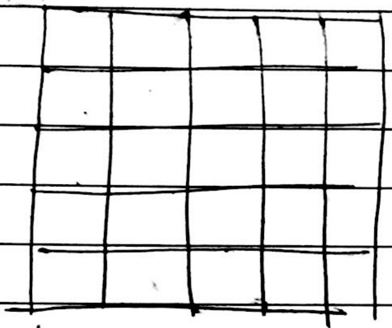
from all surfaces.

Step2 :

2 D buffer (Z-value) — same structure at M1

$\boxed{M_1}$ Z buffer

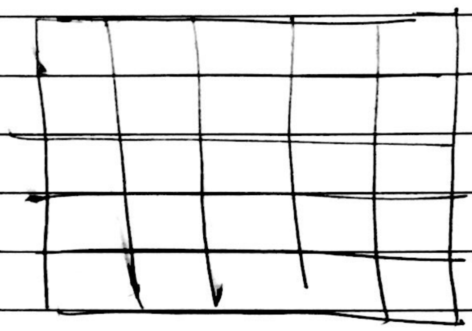
(1,1)



home PS

Every location initialize $Z_{max}+1$.

$\boxed{M_2}$ Color buffer



Every position with background color

$$ax + by + cz + d = 0$$

$$Z_{(x,y)} = -\frac{1}{c}(ax + by + d)$$

$$Z_{(x+1,y)} = Z_{(x,y)} - \frac{a}{c}$$

For each surface calculate Z_{min} for each surface (x, y) value.

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Example

Tetrahedron

A(1, 1, -1)

B(3, 1, -1)

C(2, 1, -3)

D(2, 2, -2)

Surfaces

Surface	Color
ACD	RED (1)
CBD	BLUE (2)
BAD	CYAN (3)
ACB	GREEN (4)

Background BLACK (0)

Step 1

$x_{min} = 1$, $y_{min} = 1$, $z_{min} = -3$

$x_{max} = 3$, $y_{max} = 2$, $z_{max} = -1$

Plane ACD $ax + by + cz + d = 0$

$AC = (1, 0, -2)$

$AD = (1, 1, -1)$

$$\begin{matrix} i & j & k \\ 1 & 0 & -2 \\ 1 & 1 & -1 \end{matrix}$$

Normal $= i(2) - j(-1+2) + k(1)$

$2i - j + k$

$2x - y + z = 0$

Plane CBD

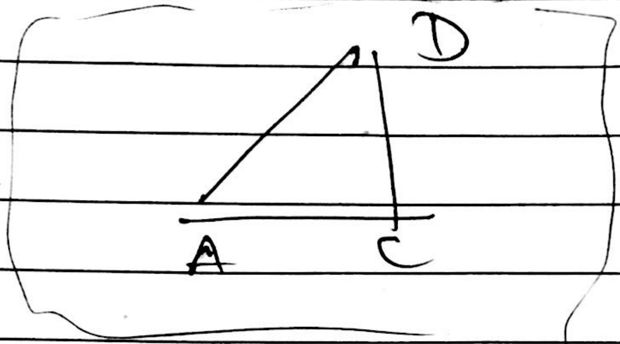
$-2x - 5y + z + 12 = 0$

$z_{CBD} = 2x - y - 8$

Step 3 (Image)

ACD (No 2 words)

A(1,1), C(2,1), D(2,2)



find equation

inside test :

line eq AC $y = 1$

line eq CD $x = 2$

line eq of AD $y = x$

f(x,y) :

for D (2,2)

$f(x,y) = y - 1 = 2 - 1 \geq 0$

A(1,1)

$f(x,y) = 2 - x \geq 0$

C(2,1)

$f(x,y) : x - y \geq 0$

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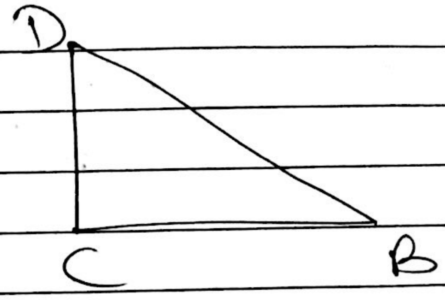
CBD (image)

C (2,1)
B (3,1)
D (2,2)



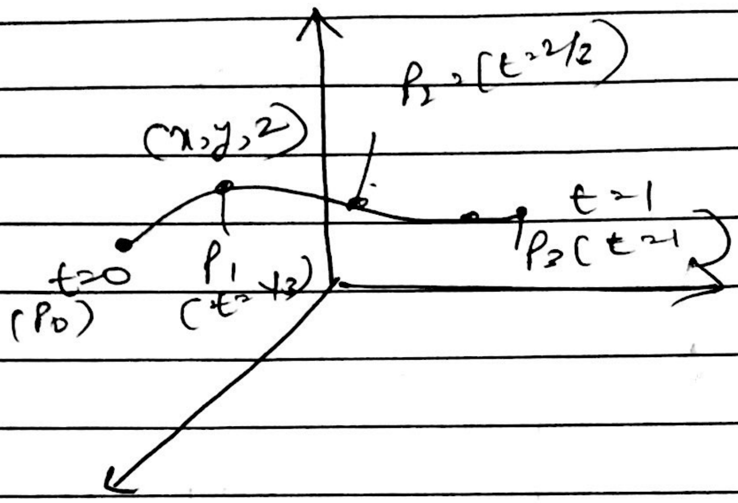
line eq. CB = $y = 1$ $\quad f(x,y)\ y-1 \geq 0$
line eq. BD $x + y = 4$ $\quad f(x,y)\ 4-x-y \geq 0$
line eq DC $x = 2$ $\quad f(x,y)\ x-2 \geq 0$

Must pass inside test

X	Y	Z	inside test	
1	1	$Z_{ACD} = -1$ $Z_{CBD} = 5$	T F	ACD (RED)
1	2	$Z_{ACD} = 0$ $Z_{CBD} = -4$	F F	Background color
2	1			
2	2			
3	1			
3	2			

when T both and Z also same choose any one.

2 D Curves.



Parametric form for curve segment

$$P(t) = [x(t), y(t), z(t)] \quad 0 \le t \le 1$$

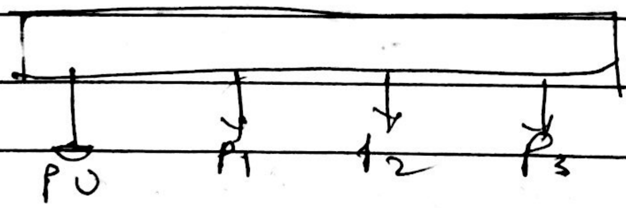
$$x(t) = a_0 x_0 + a_1 x_1 t + a_2 x_2 t^2 + \dots + a_n x_n t^n.$$

$n = 1$ = line

$n = 2$ = quadratic curve

$n = 3$ = cubic curve

$n = 4$ ~~quadratic~~ quadratic space curve.



for $n = 3$

P points are control points.

position, tangent vector,
reversed position, reversed tangent vector

$x(t) = a_x t^3 + b_x t^2 + c_x t + d_z$
$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$
$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$

$P(t) = \vec{a} t^3 + \vec{b} t^2 + \vec{c} t + \vec{d}$

$P_0 = \vec{t}$

$P_1 = \frac{\vec{a}}{27} + \frac{\vec{b}}{9} + \frac{\vec{c}}{3} + \vec{d}$

$P_2 = \frac{8\vec{a}}{27} + \frac{4\vec{b}}{9} + \frac{2}{3}\vec{c} + \vec{d}$

$P_3 = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

$$\begin{bmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \\ \vec{d} \end{bmatrix} = \begin{bmatrix} -9/2 & 27/2 & -\frac{27}{2} & \frac{9}{2} \\ 9 & -45/2 & 15 & -9/2 \\ -11/2 & 9 & -9/2 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$P(t) = \left(-\frac{9}{2}P_0 + \frac{27}{2}P_1 - \frac{27}{2}P_2 + \frac{9}{2}P_3\right) t^3$$
$$+ \left(9P_0 - \frac{45}{2}P_1 + 15P_2 - 9/2\,P_3\right) t^2 +$$
$$\left(-11/2\,P_0 + 9P_1 - 9/2\,P_2 - P_3\right) t +$$
$$+ \vec{d}$$

P_0, P_1, P_2, P_3 are given coordinates in x, y, z

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for $t = 0$ to 1 step 0.0001

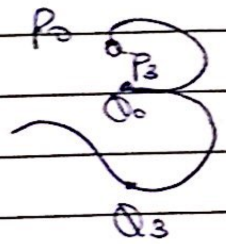
{ $P(t) = \left[-\frac{9}{2} t^3 + 9t^2 - \frac{11}{2} t + 1 \right] P_0$

$+ [\quad] P_1$

$+ [\quad] P_2$

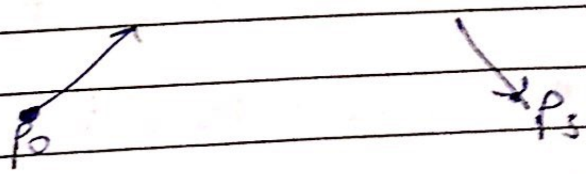
$+ [\quad] P_3$ }

Putpixel on x, y plane



Problem 1: A stone is thrown from a point P_0 with direction and magnitude

Stone is received at another point P_3.
It has location, direction and magnitude



what is parametric eq. of curve?

Hermite .

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$P_0 = (t=0) = \vec{d}$
$P_3 \quad (t=1) = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

$P'(t) = 3\vec{a}t^2 + 2\vec{b}t + \vec{C}$

direction start $P_0'(t=0) = \vec{C}$

direction end $P_3'(t=1) = 3\vec{a} + 2\vec{b} + \vec{C}$

$$\begin{array}{c} \vec{a} \\ \vec{b} \\ \vec{C} \\ \vec{d} \end{array} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_3 \\ P_0' \\ P_3' \end{bmatrix}$$

$$\begin{array}{c} P_0 \\ P_3 \\ P_0' \\ P_3' \end{array} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} \; \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for $t = 0$ to 1 . step $= 0.0001$

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$P = \vec{a}\, t^3 + \vec{b}\, t^2 + \vec{c}\, t + \vec{d}$

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