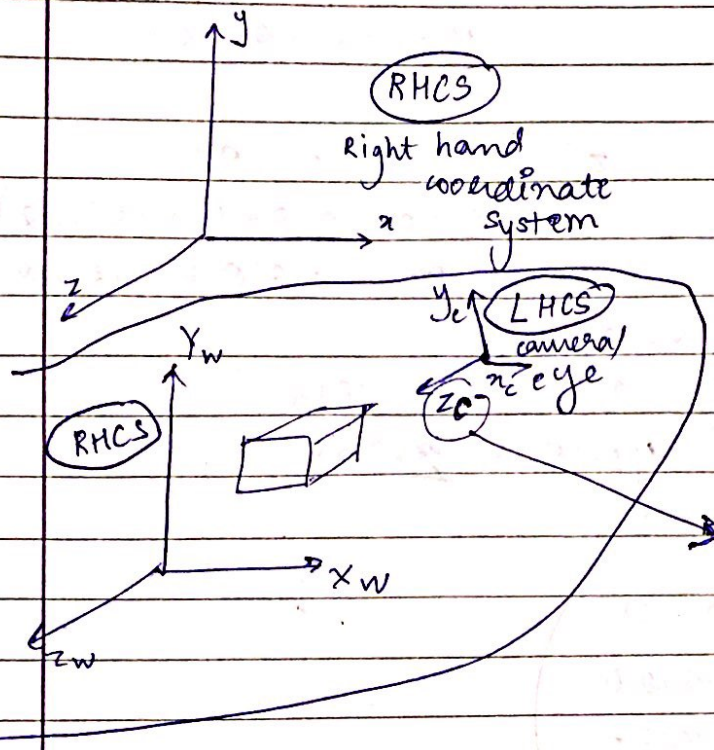
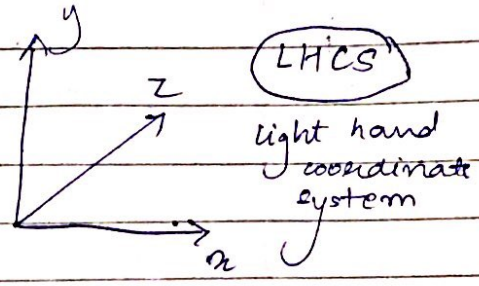
Thumb = x
first = y
middle finger = z

Page No.
Date

# 3 D TRANSFORMATION



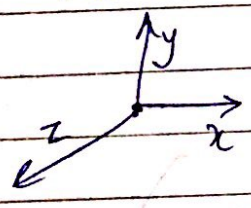




For camera, view direction +ve z axis

3D Transformation
- Translation $\begin{pmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- Scaling $\begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- Rotation
  - about x axis
  - about y axis
  - about z axis

anticlockwise rotation is +ve rotation

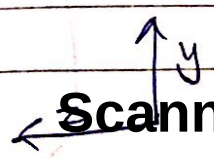
Rotation about x axis



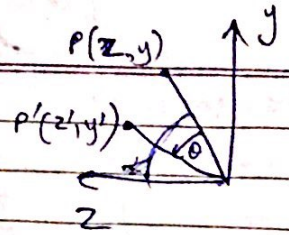

anticlockwise $y \to z$

~~We~~ Normally, we consider $\uparrow y$, $\rightarrow x$ in 2D because rotation about z axis

So here




Scanned by CamScanner




$z = R\cos(\cancel{\theta+}\,\alpha)$

$y = R\sin(\cancel{\theta+}\,\alpha)$

3D view

$z' = R\cos(\alpha-\theta)$

$= R(\cos\alpha\cos\theta + \sin\alpha\sin\theta)$

$= z\cos\theta + y\sin\theta$

$y' = R\sin(\alpha-\theta)$

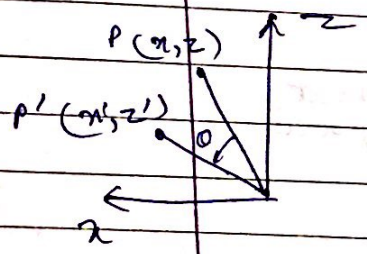
$= R(\sin\alpha\cos\theta - \sin\theta\cos\alpha)$

$= y\cos\theta - z\sin\theta$

$x' = x$

$$R_{x,\theta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

## Rotation about Y axis



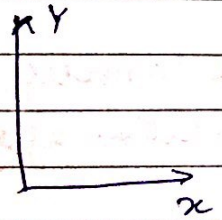

$y' = y$

$x' = x\cos\theta + z\sin\theta$

$z' = z\cos\theta - x\sin\theta$

$$R_{y,\theta} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Rotation about z axis




$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Scanned by CamScanner

monitor divided into 6 camera

| Page No. | | | |
|---|---|---|---|
| Date | | | |

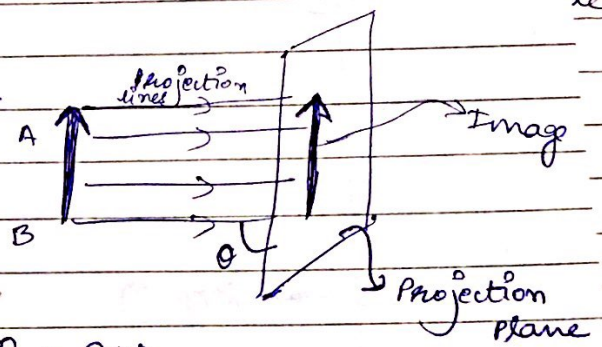
3D view

| | | |
|---|---|---|
| side view | Front view | Top view |
| Isometric view | Dimetric view | Perspective view |

View ≡ Projection since (3D → 2D)

Projection — Parallel
— Non Parallel (PERSPECTIVE view)

PARALLEL




ISOMETRIC PROJECTION

Projection lines are parallel

with projection plane it makes ⊥ angle

Projector lines make equal angles with all 3 coord axis

DIMETRIC PROJECTION

TRIMETRIC PROJECTION

→ when θ = 90° then Orthographic Projection

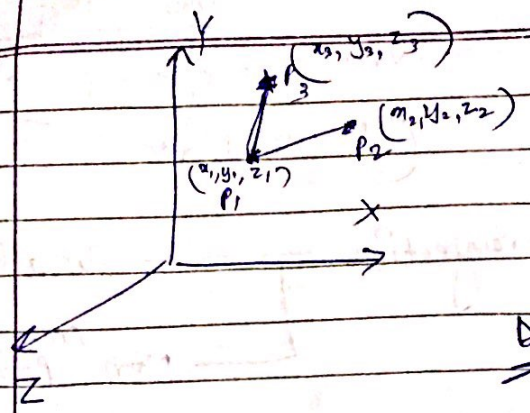
→ when θ != 90° then Oblique Projection (projection lines are still ||)

→ CABINET PROJECTION

→ CAVALIER PROJECTION

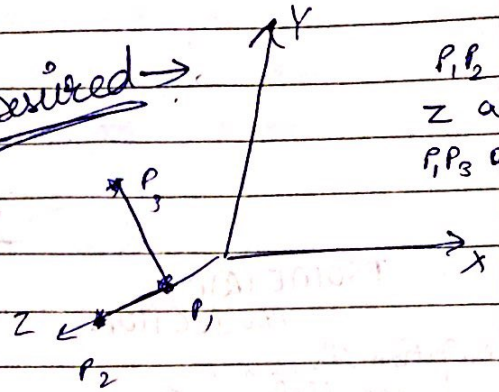
Scanned by CamScanner

Ex ①



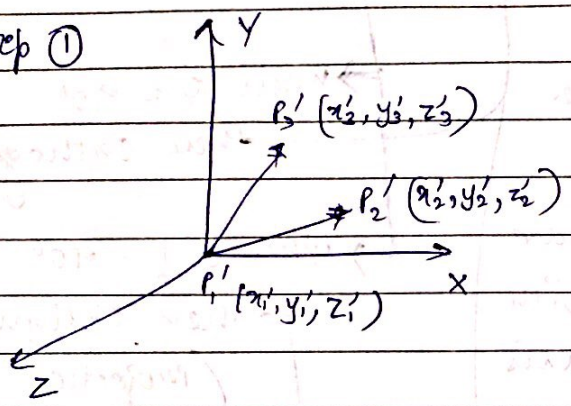

Given → $P_1$, $P_2$ & $P_3$ in 3-D space

Desired →




$P_1P_2$ along Z axis
$P_1P_3$ on Y-Z plane

Ans → Step ①

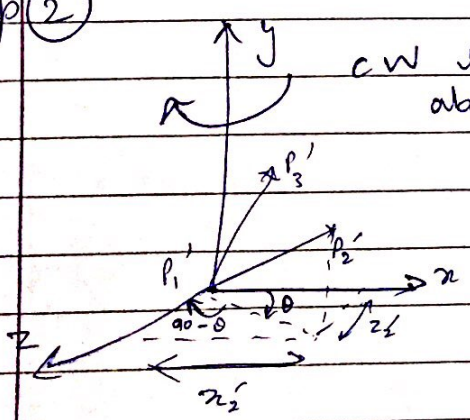



$x_2' = x_2 - x_1$
$y_2' = y_2 - y_1$
$z_2' = y_3 - y_1$

$x_3' = x_3 - x_1$
$y_3' = y_2 - y_1$
$z_3' = z_3 - z_1$

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step ②




CW rotation about y

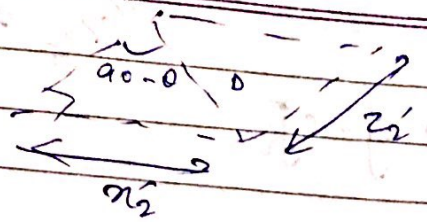
$R_{y,-(90-\theta)}$

↑ as clockwise rotation

Scanned by CamScanner

We can also have done rotation about z & then y






$$D_1 = \sqrt{x_2'^2 + z_2'^2}$$

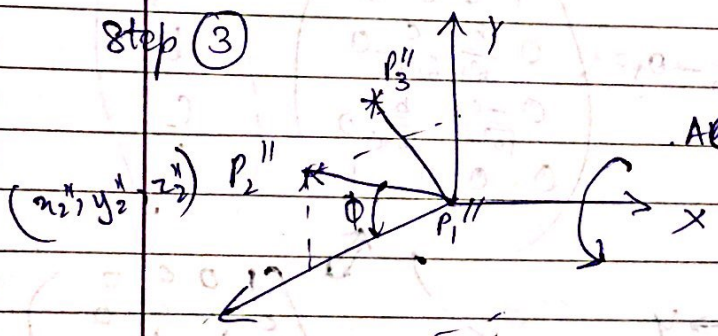
$$\cos(90-\theta) = \frac{z_2'}{D_1}$$

$$\sin(90-\theta) = \frac{x_2'}{D_1}$$

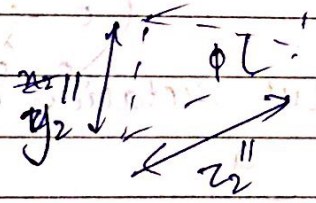
$$90-\theta = \alpha$$

$$R_{y,-\alpha} = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{z_2'}{D_1} & 0 & \frac{-x_2'}{D_1} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_2'}{D_1} & 0 & \frac{z_2'}{D_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step (3)




ACW rotation about x

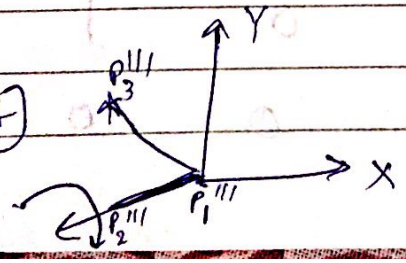



$$D_2 = \sqrt{(z_2'')^2 + (y_2'')^2}$$

$$\cos\phi = \frac{z_2''}{D_2} \qquad \sin\phi = \frac{y_2''}{D_2}$$

$$R_{x,\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_2''}{D_2} & \frac{-y_2''}{D_2} & 0 \\ 0 & \frac{y_2''}{D_2} & \frac{z_2''}{D_2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step (4)



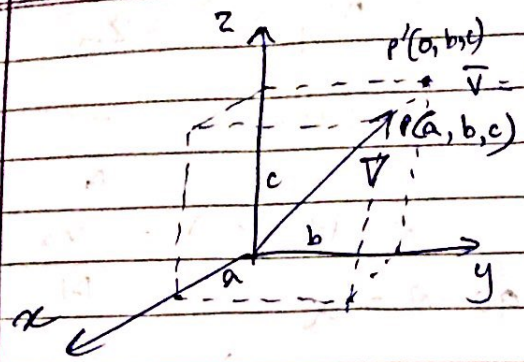

Now rotate about z axis to allign $P_1'''$ to YZ plane

Scanned by CamScanner

$\sqrt{40^2+20^2+50^2}$ 1066.54 1075.84

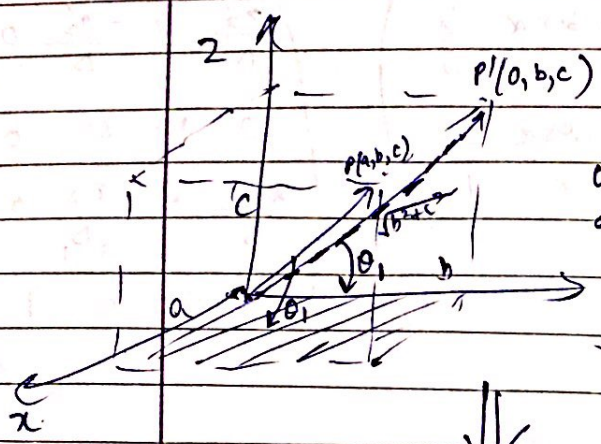
Page No. | Date

11.138 4.47, 6.66, 3.27, 8.2, 2.8




$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$

$A_{\vec{v}}^{\hat{j}}$ Align $\vec{v}$ to $\hat{j}$ i.e. y axis



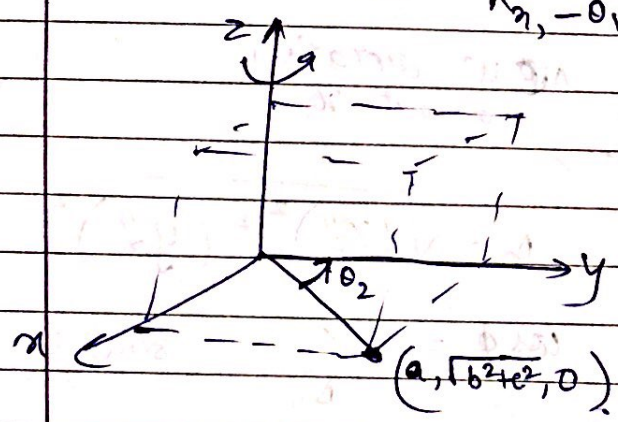

clockwise about $x$ $R_{x,-\theta_1}$

$\cos(-\theta_1) = \frac{b}{\sqrt{b^2+c^2}}$

$\sin(-\theta_1) = \frac{-c}{\sqrt{b^2+c^2}}$

$$R_{x,-\theta_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{-c}{\sqrt{b^2+c^2}} & \frac{b}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{b}{\lambda} & \frac{c}{\lambda} & 0 \\ 0 & \frac{-c}{\lambda} & \frac{b}{\lambda} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$




ACW rotation about z

$\cos\theta_2 = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$ $\sin\theta_2 = \frac{a}{\sqrt{b^2+c^2}}$

$$R_{z,\theta_2} = \begin{pmatrix} \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & \frac{-a}{\sqrt{b^2+c^2}} & 0 & 0 \\ \frac{a}{\sqrt{b^2+c^2}} & \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Scanned by CamScanner

$(AB)^{-1}AB = I$

$B^{-1}\cancel{A^{-1}}\cancel{A}B$

Page No.
Date

$|\vec{v}| = \sqrt{a^2+b^2+c^2}$ let $\lambda = \sqrt{b^2+c^2}$

$$R_{z,\theta_2} = \begin{pmatrix} \frac{\lambda}{|\vec{v}|} & \frac{-a}{|\vec{v}|} & 0 & 0 \\ \frac{a}{|\vec{v}|} & \frac{\lambda}{|\vec{v}|} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

C matrix composite $= R_{z,\theta_2} * R_{x,-\theta_1}$

$$\begin{pmatrix} \frac{\lambda}{|\vec{v}|} & \frac{-a}{|\vec{v}|} & 0 & 0 \\ \frac{a}{|\vec{v}|} & \frac{\lambda}{|\vec{v}|} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b/\lambda & c/\lambda & 0 \\ 0 & -c/\lambda & b/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \cancel{\begin{pmatrix} \frac{\lambda}{|\vec{v}|} & \frac{-ab}{(\vec{v})\lambda} + \frac{ac}{(\vec{v})\lambda} \end{pmatrix}}$$

$$A^{\vec{v}}_{\vec{j}} = \begin{pmatrix} \frac{\lambda}{|\vec{v}|} & \frac{-ab}{|\vec{v}|\lambda} & \frac{-ac}{|\vec{v}|.\lambda} & 0 \\ \frac{a}{|\vec{v}|} & \frac{b\cancel{\lambda}}{\cancel{\lambda}(\vec{v})} & \frac{c}{\cancel{\lambda}}\frac{\cancel{\lambda}}{|\vec{v}|} & 0 \\ 0 & \frac{-c}{\lambda} & \frac{b}{\lambda} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{\vec{v}}_{\vec{j}} = \left[A^{\vec{j}}_{\vec{v}}\right]^{-1} = \left[R_{z,\theta_2} * R_{x,-\theta_1}\right]^{-1}$$

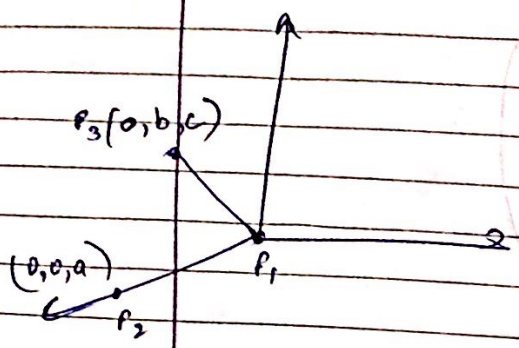
$$= \left[R_{x,-\theta_1}\right]^{-1} * \left[R_{z,\theta_2}\right]^{-1}$$

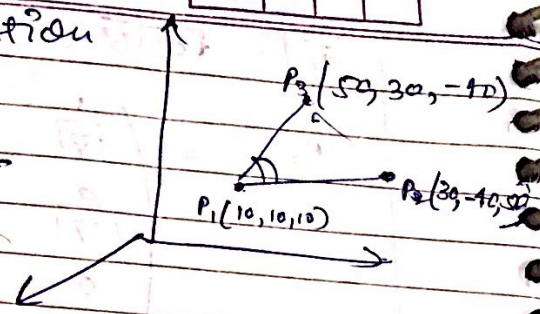
$$= R_{x,\theta_1} * R_{z,-\theta_2}$$

$$\boxed{A^{k}_{\vec{v}} = R_{x,90} * A^{j}_{v}} \qquad \boxed{A^{i}_{\vec{v}} = R_{z,-90} * A^{j}_{v}}$$

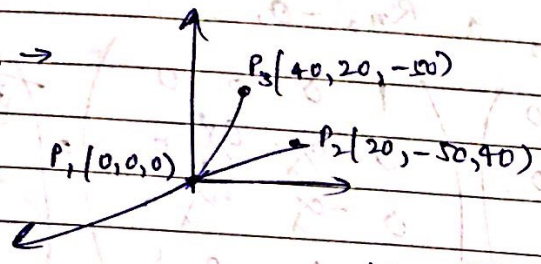
Scanned by CamScanner

like in the assignment question




Ans →




$$\vec{V} = 20\hat{i} - 50\hat{j} + 40k$$

$$|\vec{V}| = \sqrt{20^2 + 50^2 + 40^2}$$

$$= 67.08$$

$$\lambda = \sqrt{b^2 + c^2} = \sqrt{50^2 + 40^2} = 64.03$$

$$A_{\vec{V}}^{\hat{j}} = \begin{pmatrix} \frac{64.03}{67.08} & \frac{-20 \times -50}{67.08 \times 64.03} & \frac{-20 \times 40}{67.08 \times 64.03} & 0 \\ \frac{20}{67.08} & \frac{-50}{67.08} & \frac{40}{67.08} & 0 \\ 0 & \frac{-40}{64.03} & \frac{-50}{64.03} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.95 & 0.0233 & 0.186 & 0 \\ 0.3 & -0.75 & 0.6 & 0 \\ 0 & -0.625 & -0.78 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{\vec{V}}^{\hat{k}} = R_{x,90} \times A_{\vec{V}}^{\hat{j}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

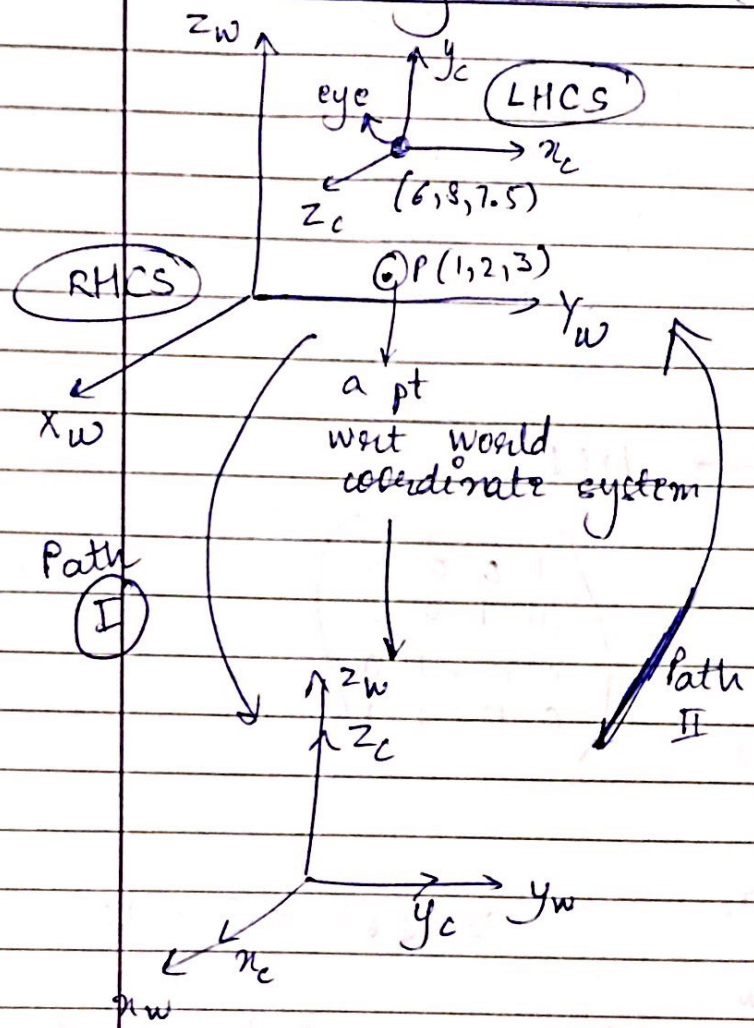
Scanned by CamScanner

# This is Test & CHECK



## 3D Transformation

View through a camera



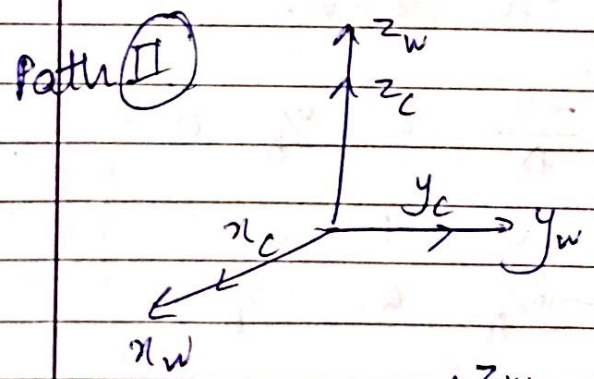

$z_c$ is pointing to origin
$y_c$ is up
$x_c$ is on $z_w = 7.5$ plane

P ( 1 = $x_w$, 2 = $y_w$, 3 = $z_w$ )

what will be P wrt the camera coordinates?

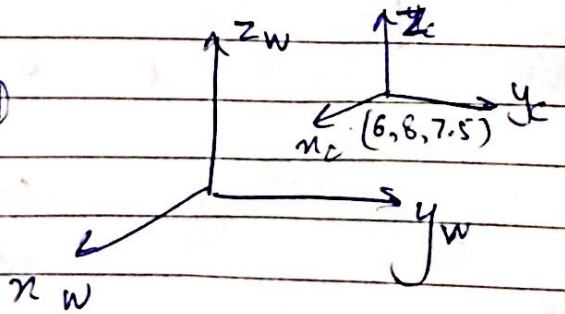
Path (I)
{ transform ~~world~~ camera coordinate system to the ~~camera~~ world coordinate and multiply point with the transformation matrix's inverse }

Path (II)




move the ~~cam~~ world system to camera

step (1)




Translation

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

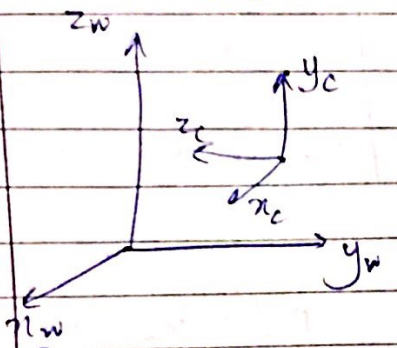
Scanned by CamScanner

$$C = R_{x\phi} + R_{y,\theta} + R_{x,90^\circ}$$

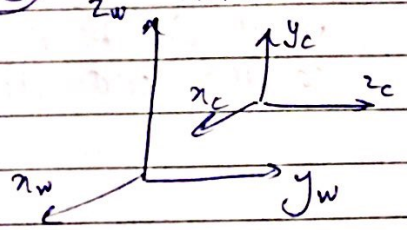
Page No.
Date

step (2)

$R_{x,90^\circ}$ : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

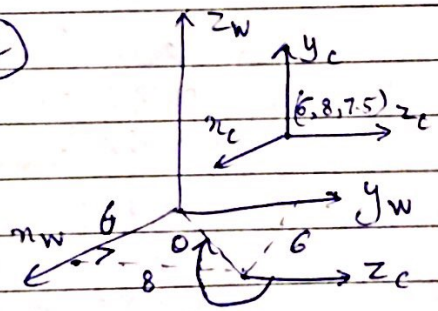



step (3) RHCS $\xrightarrow{to}$ LHCS




$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

step (4)




$D = \sqrt{6^2 + 8^2} = 10$

Rotation of y by $180 + \theta$ in the clockwise direction

$$R_{y,-(180+\theta)} = \begin{pmatrix} \frac{-8}{D} & 0 & 6/D & 0 \\ 0 & 1 & 0 & 0 \\ -6/D & 0 & -8/D & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\sin\theta = \frac{6}{D}$

$\cos\theta = \frac{8}{D}$

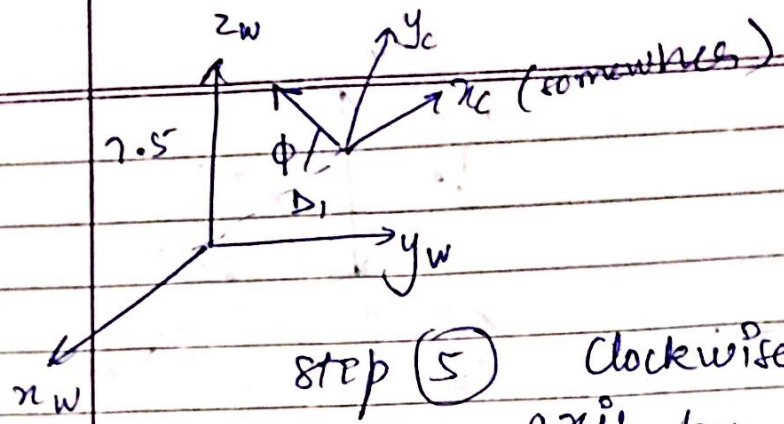
$\cos(180+\theta) = -\cos\theta$

$\cos(180+\theta) = -\cos\theta$

$\sin(180+\theta) = -(-\sin\theta)$

$= \sin\theta$

Scanned by CamScanner




$D_1 = \sqrt{(7.5)^2 + 10^2} = 12.5$

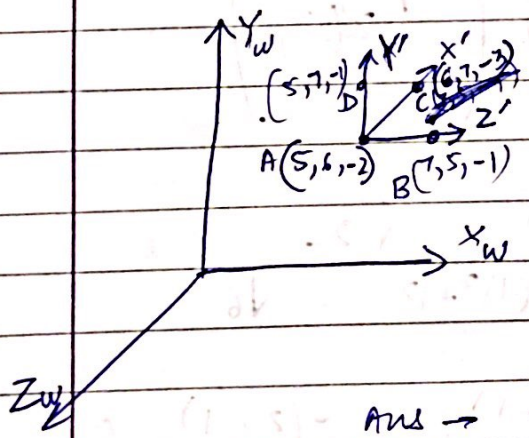
Step (5) Clockwise rotation about the $x$ axis by an angle $\phi$

$\sin\phi = 7.5/D_1$

$\cos\phi = 10/D_1$

$$R_{x,-\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 10/D_1 & 7.5/D_1 & 0 \\ 0 & -7.5/D_1 & 10/D_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## 3D Viewing




$P(x', y', z') = \{1, 2, 3\}$

Find P wrt world coordinate system?

Ans → $AB = 2i - j + k$

$AC = i + j - k$

$AD = j + k$

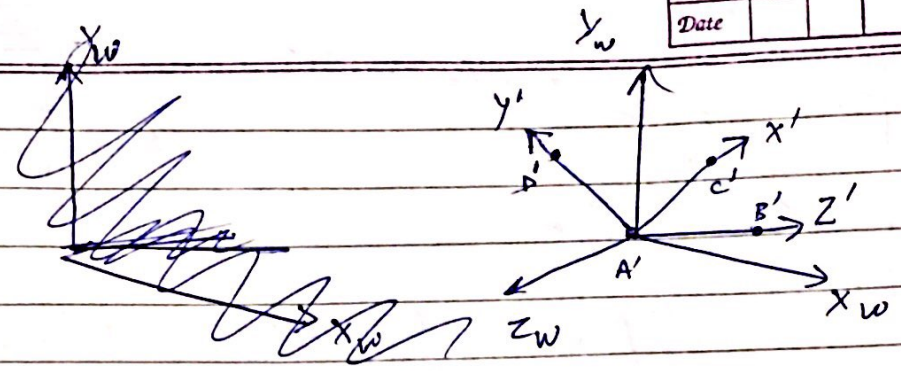
$\overline{AB} \cdot \overline{AC} = 2 - 1 - 1 = 0$ $\qquad$ $\overline{AC} \cdot \overline{AD}$ $1 - 1 = 0$

$\overline{AB} \cdot \overline{AD} = -1 + 1 = 0$

Hence, ~~they are~~ it is proper coordinate system

Scanned by CamScanner

Step ①




$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A' = [0, 0, 0]$

$B'' = [2, -1, 1]$

$C' = [1, 1, -1]$

$D' = [0, 1, 1]$

$$A^{Z(K)}_{Z'(2i - j + k)} = R_{x,90} * A^{j}_{Z'(2i - j + k)}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A^{j}_{Z'(2i - j + k)} \end{pmatrix}$$

$$A^{j}_{Z'(2i - j + k)}$$

$$\lambda = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$|\bar{v}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{6}} & \frac{-2(-1)}{\sqrt{6}\sqrt{2}} & \frac{-(2)(1)}{\sqrt{6}\sqrt{2}} & 0 \\ \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Scanned by CamScanner

| Page No. | |
|---|---|
| Date | |

$$= \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{z}_{z'} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\frac{4\sqrt{2}}{\sqrt{6}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}$

$A'' = [0, 0, 0]$ $\quad$ $B'' = [0, 0, \sqrt{6}]$

$C'' = [\sqrt{3}, 0, 0]$ $\quad$ $D'' = [0, \sqrt{2}, 0]$

As we see, $A''$, $B''$, $C''$, $D''$ values are such that the camera coordinate system aligns with the world coordinate system

Composite Matrix $\equiv$ $R_{x,90} * A^{j}_{z'} * T$

$$CM \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$ gives coordinates in world coordinate system.

Read PTO

$$\cancel{CM \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \sqrt{2/3} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{matrix} 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} & -5 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & -6 \\ \sqrt{2/3} & & & \end{matrix}}$$

Scanned by CamScanner

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \sqrt{2/3} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{-13}{\sqrt{3}} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & -2\sqrt{2} \\ \sqrt{2/3} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{2/3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Take $(M^{-1})$ now

$P(1,2,3) =$ in world ~~sy~~ coordinate system

$\cancel{\left(\frac{-13}{\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}\right)}$ This and then multiply

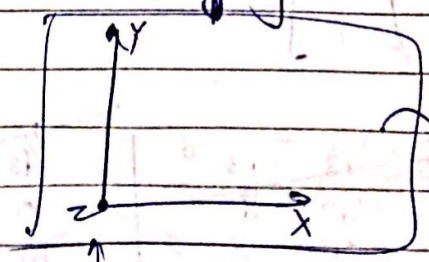
$1/\sqrt{2}$ $1/\sqrt{2}$ $-2/\sqrt{2}$

## Projection

3D object $\longrightarrow$ 2D object

Project lines are $\parallel$

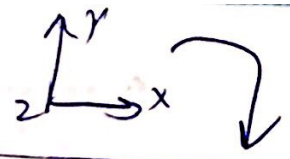
Isometric projection

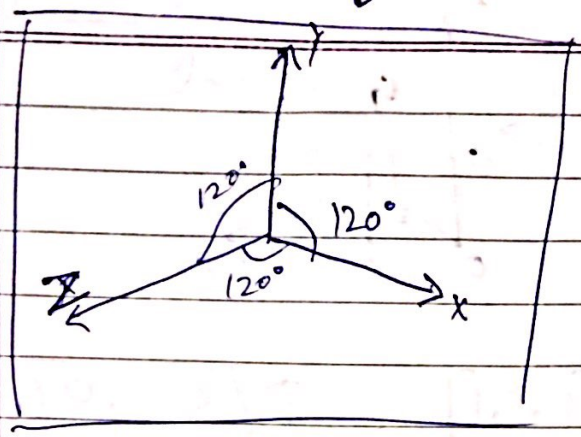



projection plane

Z is $\perp$ to page so not visible on projection

Scanned by CamScanner




projector lines are falling at equal angle on projector axis

$$C = P_{rog}[x,y] * R_{x,\theta_2} * R_{y,\theta_1}$$

$R_{y,\theta_1} \equiv R_{y,-\theta_1}$ as clockwise Rotation

$R_{x,\theta_2} \equiv R_{x,\theta_2}$ as anticlockwise rotation

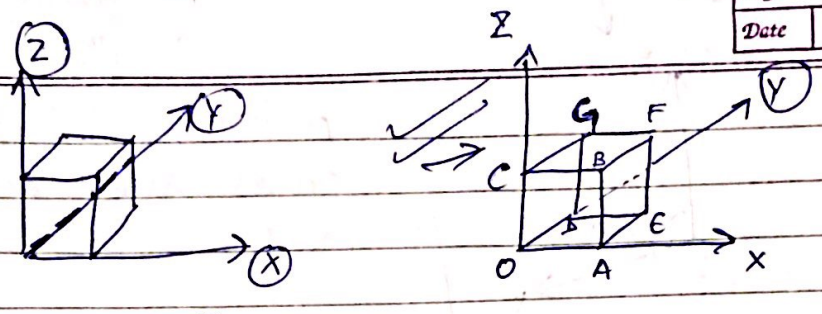
$$C = P_{roj}[x,y] * R_{x,\theta_2} * R_{y,-\theta_1}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1\sin\theta_2 & \cos\theta_2 & -\sin\theta_2\cos\theta_1 & 0 \\ \sin\theta_1\cos\theta_2 & \sin\theta_2 & \cos\theta_1\cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1\sin\theta_2 & \cos\theta_2 & -\sin\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scanned by CamScanner




O [0, 0, 0]
A [1, 0, 0]  C [0, 0, 1]  D [0, 1, 0]

A $[x_x, y_x, z_x]$  C $[x_z, y_z, z_z]$  D $[x_y, y_y, z_y]$

↓ proj (x-y)

A' $[x_x, y_x, 0]$  C $[x_z, y_z, 0]$  D $[x_y, y_y, 0]$

Foreshortening vector along x-axis $f_x = \frac{|OA'|}{|OA|}$

y-axis $f_y = \frac{|OD'|}{|OD|}$

z-axis $f_z = \frac{|OC'|}{|OC|}$

In isometric, $f_x : f_y : f_z$
$1 : 1 : 1$

Eq$^n$ (1) ⇒ $f_x = f_y$  $\frac{|OA'|}{|OA|} = \frac{|OD'|}{|OD|}$

$\sqrt{x_x^2 + y_x^2} = \sqrt{x_y^2 + y_y^2}$

multiplying the coordinates with C from before

Scanned by CamScanner

| Page No. | |
|---|---|
| Date | |

$x_x = \cos\theta_1$ $\qquad y_x = -\sin\theta_1 \sin\theta_2$

$x_y = 0$ $\qquad y_y = \cos\theta_2$

$$(\cos\theta_1)^2 + (\sin\theta_1 \sin\theta_2)^2 = (\cos\theta_2)^2$$

Eq$^n$ (2) $\quad f_y = f_z \qquad \cos^2\theta_2 = (\sin\theta_1)^2 + (\sin\theta_2 \cos\theta_1)^2$

(2) − (1)

$$\cos^2\theta_1 - \cos^2\theta_2 = \sin^2\theta_2 (\cos^2\theta_1 - \sin^2\theta_1)$$

(2) + (1)

$$\cos^2\theta_1 + \sin^2\theta_1 + \sin^2\theta_2 (\cos^2\theta_1 + \sin^2\theta_1) = 2\cos^2\theta_2$$

$2\cos^2\theta - 1$

$$1 + \sin^2\theta_2 = 2\cos^2\theta_2$$

$$1 + \sin^2\theta_2 = 2(1 - \sin^2\theta_2)$$

$$3\sin^2\theta_2 = 2 -$$

$$\sin^2\theta_2 = \frac{1}{3} \Rightarrow \theta_2 = \pm 35.26$$

$$3 - 3\sin^2\theta_1 + \frac{\sin^2\theta_1}{3} = \frac{2}{3}$$

$$1 = 2\sin^2\theta_1 \qquad \sin^2\theta_1 = 1/2$$

$$\Rightarrow \theta_1 = \pm 45^\circ$$

Scanned by CamScanner

# DIMETRIC PROJECTION

here only ~~2 are~~ parallel with 2 axis

$f_x : f_y : f_z$

$1:1:k$ $\quad$ $1:k:1$ $\quad$ $k:1:1$

$1:1:k$

Eq$^n$ (1) = $f_x = f_y$ $\quad$ $\cos^2\theta_1 + \sin^2\theta_1 \sin^2\theta_2 = \cos^2\theta_2$

Eq$^n$ (2) = $k f_y = f_z$ $\quad$ $k\cos^2\theta_2 = \sin^2\theta_1 + \sin^2\theta_2 \cos^2\theta_1$

$\cancel{k\sin^2\theta_1 + k\cos^2\theta_1 + k\sin^2\theta_2(\sin^2\theta_1 + \cos^2\theta_1) = (k+1)\cos^2\theta_2}$

$k^2 \left|\frac{ON}{OB}\right|^2 = 1 \quad )^2$

$1 + \sin^2\theta_2 = (k^2+1)\cos^2\theta_2$ $\quad$ $\cancel{1 + k\sin^2\theta_2 = (k+1)\cos^2\theta_2(1 - \sin^2\theta_2)}$

$\cancel{1 + k + k\sin^2\theta_2 = k+1 - (k+1)\sin^2\theta_2}$

$\cancel{2 = (k+2}$ $\quad$ $\cancel{(2k+1)\sin^2\theta_2 = 1}$

$(k^2+2)\sin^2\theta_2 = k^2+1-1$ $\quad$ $\cancel{\sin^2\theta_2 = \frac{1}{2k+1}}$

$\sin\theta_2 = \pm\sqrt{\frac{k^2}{k^2+2}}$ $\quad$ $\cancel{\sin\theta_2 = \sqrt{\frac{1}{2k+1}}}$

$\cancel{\cos^2\theta_2 = 1 - \frac{1}{2k+1} = \frac{2k}{2k+1}}$

$\theta_2 = \sin^{-1}\left(\pm\sqrt{\frac{k^2}{k^2+2}}\right)$

$\cancel{\theta_2 = \cos^{-1}\sqrt{\frac{2k}{2k+1}}}$

$\cancel{\frac{k^2}{2}}$

Scanned by CamScanner

$$\cos^2\theta_1 + \frac{K^2}{K^2+2}\left(1-\cos^2\theta_1\right) = 1 - \frac{K^2}{K^2+2}$$

$$\frac{2}{K^2+2}\cos^2\theta_1 = \frac{K^2+2-2K^2}{K^2+2} = \frac{2-K^2}{K^2+2}$$

$$\cos^2\theta_1 = \frac{2-K^2}{2} = 1-\frac{K^2}{2}$$

$$\sin^2\theta_1 = \frac{K^2}{2}$$

$$\theta_1 = \sin^{-1}\left(\frac{+\sqrt{K}}{2}\right)$$

$1:1:K$ $\quad$ $1:K:1$ $\quad$ $K:1:1$

$\theta_1 \rightarrow \sin^{-1}\left(\pm\sqrt{\frac{K}{2}}\right)$ $\quad$ $\cos^{-1}\sqrt{\frac{2}{2+K^2}}$

$\theta_2 \rightarrow \sin^{-1}\left(+\sqrt{\frac{K^2}{K^2+2}}\right)$ $\quad$ $\cos^{-1}\left(\sqrt{\frac{2K^2}{K^2+2}}\right)$ $\quad$ $\cos^{-1}\left(\sqrt{\frac{2}{2+K^2}}\right)$

$$K^2\cos^2\theta_1 + K^2\sin^2\theta_1\sin^2\theta_2 = \cos^2\theta_2$$

$$\cos^2\theta_2 = \sin^2\theta_1 + \sin^2\theta_2\cos^2\theta_1$$

$$K^2 + K^2\sin^2\theta_2 = (K^2+1)\cos^2\theta_2$$

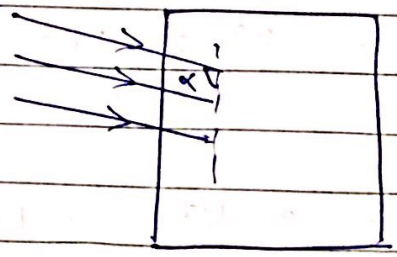
$$K^2+1 - (K^2+1)\sin^2\theta_2$$

$$(2K^2+1)\sin^2\theta_2 = 1$$

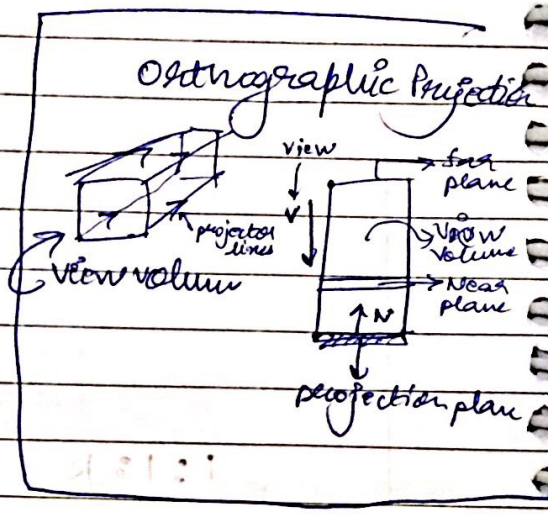
$$\sin^2\theta_2 = \frac{1}{2K^2+1} \qquad \cos^2\theta_2 = \frac{2K^2}{2K^2+1}$$

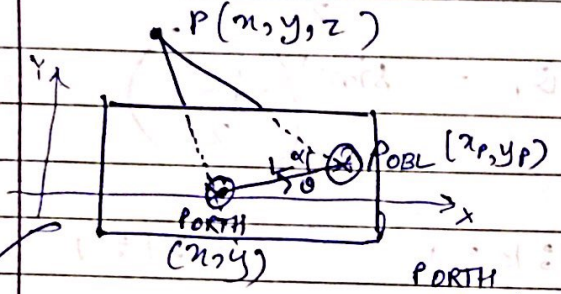
Scanned by CamScanner

# OBLIQUE PROJECTION




$\alpha \neq 90°$

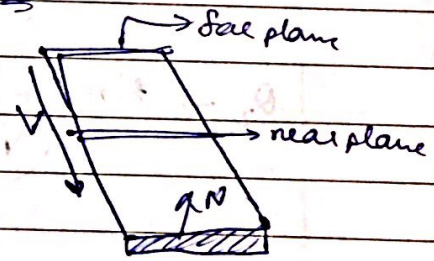



$P_{ORTH}$ = P orthographic

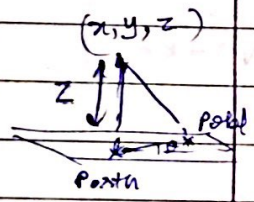
$P_{OBL}$ = P oblique

Oblique




$PP_{ORTH} \perp X_{axis}$

{as orthographic}




$PP_{OBL}$ & $X_{axis}$ have α angle

$$x_p = x + L\cos\theta$$
$$y_p = y + L\sin\theta$$
$$z_p = 0$$

$$\tan\alpha = \frac{Z}{L} \qquad L = \frac{Z}{\tan\alpha}$$

let us take $\frac{1}{\tan\alpha} = L_1$

$$L = L_1 * Z$$

$$x_p = x + L_1 Z\cos\theta$$
$$y_p = y + L_1 Z\sin\theta$$
$$z_p = 0$$

Scanned by CamScanner

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L\cos\theta & 0 \\ 0 & 1 & L\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

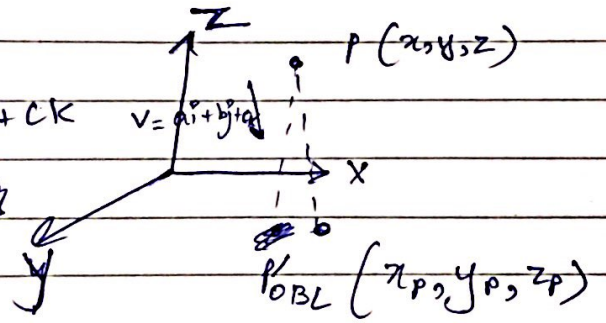
↳ This is the matrix what we get in shearing

Hence, ~~not~~ Oblique projection is basically shearing of Orthographic Projection.

Example ( View direction)

$V = ai + bj + ck$

view dir {oblique}




$z_p = 0$

$$\overrightarrow{PP'} = (x_p - x)i + (y_p - y)j + (z_p - z)k$$

$$\overrightarrow{PP'} \parallel \vec{V}$$

$$\Rightarrow (x_p - x)i + (y_p - y)j + (z_p - z)k = t(ai + bj + ck)$$

$x_p - x = at$ $\qquad x_p = x + at = x - \frac{a}{c}z$

$y_p - y = bt$ $\qquad y_p = y + bt = y - \frac{b}{c}z$

$z_p - z = ct$ $\qquad z_p = 0$ $\qquad t = -z/c$

Both are same

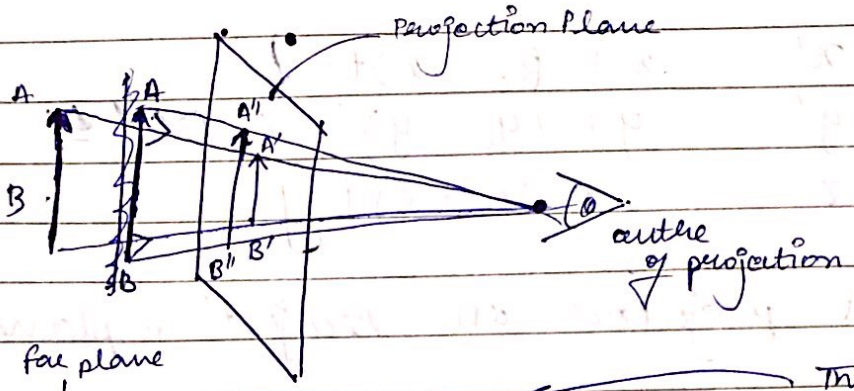
$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -a/c & 0 \\ 0 & 1 & -b/c & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scanned by CamScanner

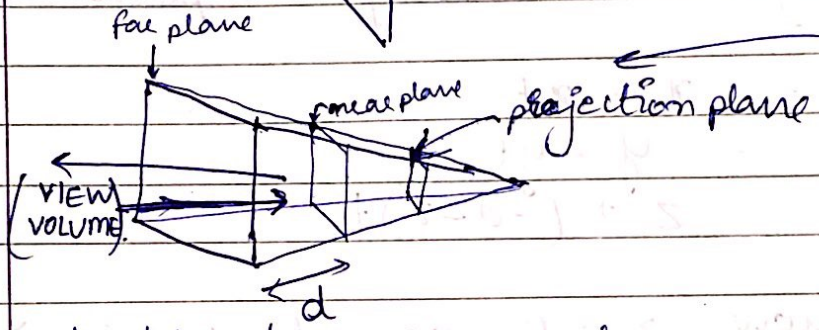
axiometric projection

Page No.
Date

# PERSPECTIVE VIEW

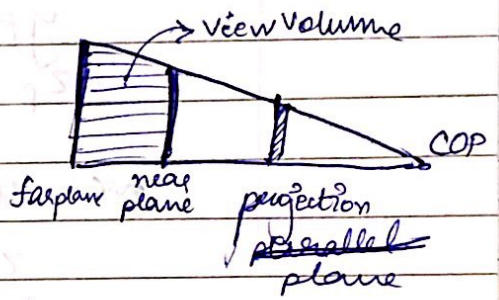



When AB is brought near to projn plane
$A''B'' > A'B'$

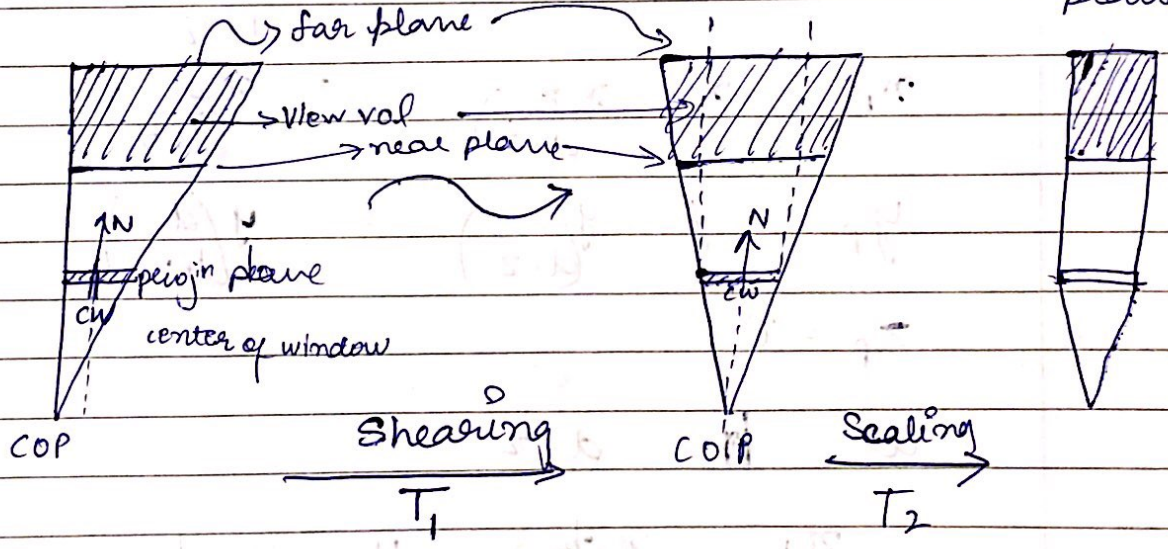



3D object is here

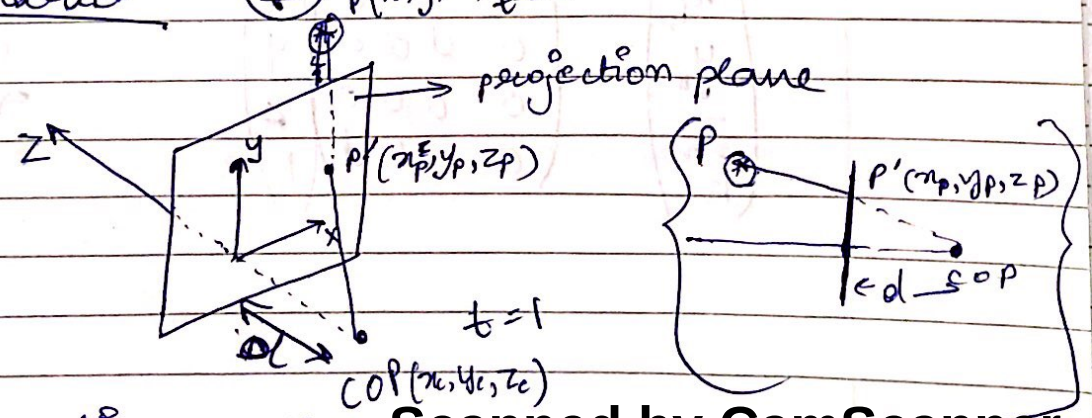
This when viewed from side




d depends on camera lens




## Case Scenario — I. $P(x,y,z)$ $t=0$




P COP is a line defined through parametric eqn

Scanned by CamScanner

Page No.
Date

Any point on this line P COP $(x', y', z')$

$$\left.\begin{aligned} x' &= x + (x_c - x)t \\ y' &= y + (y_c - y)t \\ z' &= z + (z_c - z)t \end{aligned}\right\} \; 0 \le t \le 1$$

Point projected on projection plane

$$x_p = x - xt$$
$$y_p = y - yt$$
$$z_p = z + (-d - z)t$$

we know $z_p = 0$ as plane is on XY plane

$$z = (d+z)t \qquad t = \frac{z}{d+z}$$

$$x_p = x - x\left(\frac{z}{d+z}\right) = x\left(\frac{d}{d+z}\right)$$

$$y_p = y - y\left(\frac{z}{d+z}\right) = y\left(\frac{d}{d+z}\right)$$

$$z_p = 0$$

let $h = d + z$

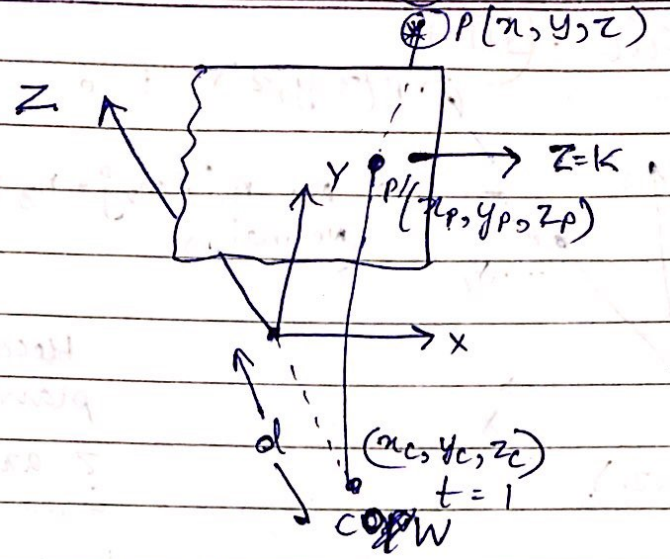
$$x_h = x_p \cdot h \qquad y_h = y_p \cdot h \qquad z_h = z_p \cdot h = 0$$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$\{h = z + d\}$

Scanned by CamScanner

Case Scenario - II




we have now just shifted the projection plane to $Z = K$

$x_c = y_c = 0 \qquad z_c = d$

$$x_p = x + (x_c - x)t$$
$$y_p = y + (y_c - y)t$$
$$z_p = z + (z_c - z)t$$

$$K = z + (-d - z)t$$
$$t = \cancel{z - dt - zt}$$
$$t = \frac{k - z}{-(d+z)} = \frac{z-k}{d+z}$$

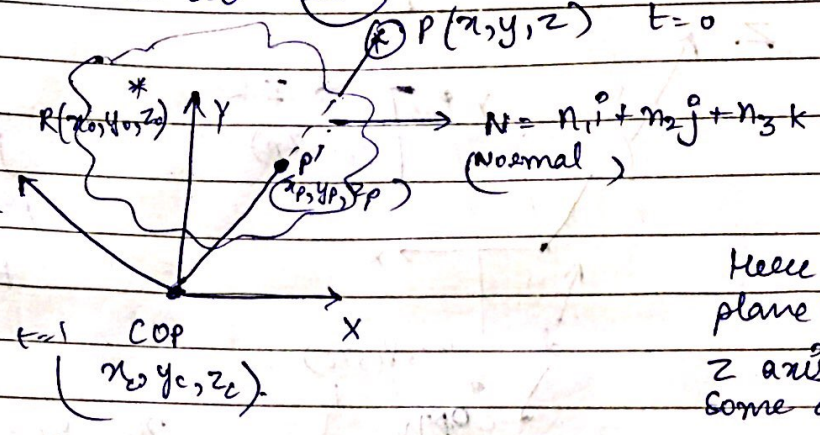
$$x_p = x - x\left(\frac{z-k}{d+z}\right)$$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} d+k & 0 & 0 & 0 \\ 0 & d+k & 0 & 0 \\ 0 & 0 & k & dk \\ 0 & 0 & 1 & d \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Check this once

Scanned by CamScanner

Case Scenario - III




Here the projection plane is not ⊥ to z axis but at some angle
Hence Normal is given

$(x', y', z')$ any point on $\overline{P\,COP}$ line

$(x_p, y_p, z_p)$ is particular point on proj$^n$ plane

$$x' = x + (x_c - x)t$$
$$y' = y + (y_c - y)t$$
$$z' = z + (z_c - z)t$$

$x_c = y_c = z_c = 0$ as COP is origin

$$\left.\begin{aligned} x' &= x - xt \\ y' &= y - yt \\ z' &= z - zt \end{aligned}\right\} (1)$$

$RP' \perp N$ Hence their dot product will be 0

$$\bar{v} = (x_p - x_0)i + (y_p - y_0)j + (z_p - z_0)k$$

$$\bar{v} \cdot N = 0 \quad \cancel{(x - xt)(x_0)} \quad (x - xt - x_0)n_1 + (y - yt - y_0)n_2 + (z - zt - z_0)n_3 = 0$$

$$\Rightarrow (x - x_0)n_1 + (y - y_0)n_2 + (z - z_0)n_3 - t(xn_1 + yn_2 + zn_3) = 0$$

$$t = \frac{(x - x_0)n_1 + (y - y_0)n_2 + (z - z_0)n_3}{xn_1 + yn_2 + zn_3}$$

Scanned by CamScanner

{ In case (3) ~~then~~ d is not distance b/w COP & origin plane as COP is origin }

| Page No. | |
|---|---|
| Date | |

let $d = (x-x_0)n_1 + (y-y_0)n_2 + (z-z_0)n_3$

~~d is COP~~

$$x_p = x - x\frac{d}{xn_1+yn_2+zn_3}$$

$$y_p = y - y\frac{d}{xn_1+yn_2+zn_3}$$

$$z_p = z - z\frac{d}{xn_1+yn_2+zn_3}$$

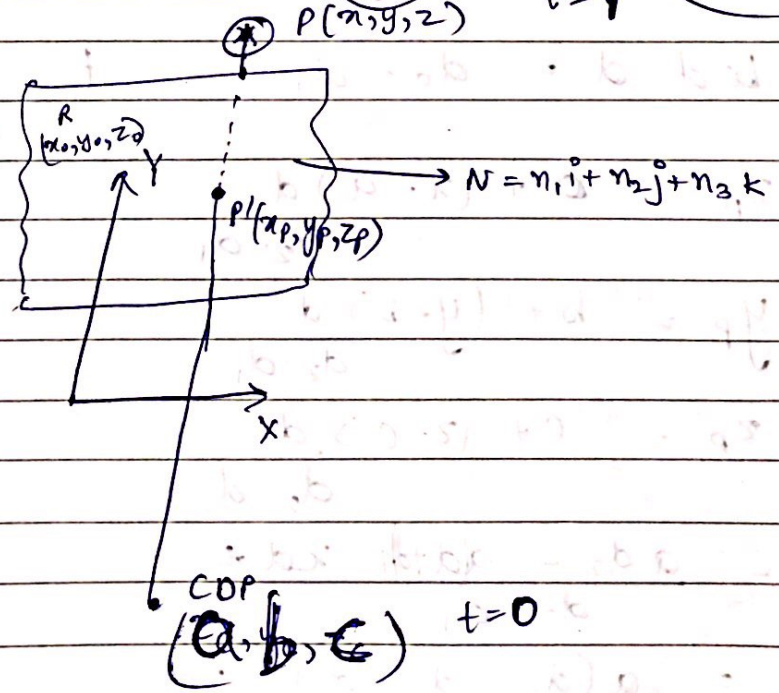
let $h = xn_1+yn_2+zn_3$

$z_p = z - z\frac{d}{h}$ $\quad z_p h = zh -$

$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} h-d & 0 & 0 & 0 \\ 0 & h-d & 0 & 0 \\ 0 & 0 & h-d & 0 \\ n_1 & n_2 & n_3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

{ h-d is constant }

Case Scenario — IV (GENERIC)




$$\cancel{x' = x + (x_c - x)t}$$
$$\cancel{y' = y + (y_c - y)t}$$
$$\cancel{z' = z + (z_c - z)t}$$

$$x' = a + (x-a)t$$
$$y' = b + (y-b)t$$
$$z' = c + (z-c)t$$

Scanned by CamScanner

Page No.
Date

$$\vec{v} = (x_p - x_0)i + (y_p - y_0)j + (z_p - z_0)k$$
$$\vec{n} = n_1 i + n_2 j + n_3 k$$

$$\vec{v} \perp \vec{n} \qquad \vec{v} \cdot \vec{N} = 0$$

$$(a + (x-a)t - x_0)n_1 + (b + (y-b)t - y_0)n_2$$
$$+ (c + (z-c)t - z_0)n_3 = 0$$

$$(a-x_0)n_1 + (b-y_0)n_2 + (c-z_0)n_3 + \left((x-a)n_1 + (y-b)n_2 + (z-c)n_3\right)t = 0$$

$$t = \frac{(x_0-a)n_1 + (y_0-b)n_2 + (z_0-c)n_3}{n_1(x-a) + n_2(y-b) + n_3(z-c)}$$

Let $d_0 = x_0 n_1 + y_0 n_2 + z_0 n_3$
$d_1 = a n_1 + b n_2 + c n_3$
$d_2 = x n_1 + y n_2 + z n_3$

let numerator be d $\quad d = d_0 - d_1 \quad t = \dfrac{d}{d_2 - d_1}$

$$x_p = a + (x-a)\frac{d}{d_2-d_1}$$

$$y_p = b + (y-b)\frac{d}{d_2-d_1}$$

$$z_p = c + (z-c)\frac{d}{d_2-d_1}$$

$$x_p = \frac{a d_2 - a(d_1+d) + xd}{d_2-d_1}$$

$$= \frac{a(d_2 - d_1 - d) + xd}{d_2 - d_1}$$

Sir →
$$\boxed{x_p = \frac{(an_1+d)x + an_2 y + an_3 z - ad_0}{xn_1 + yn_2 + zn_3 - d_1}}$$

Scanned by CamScanner

Page No.
Date

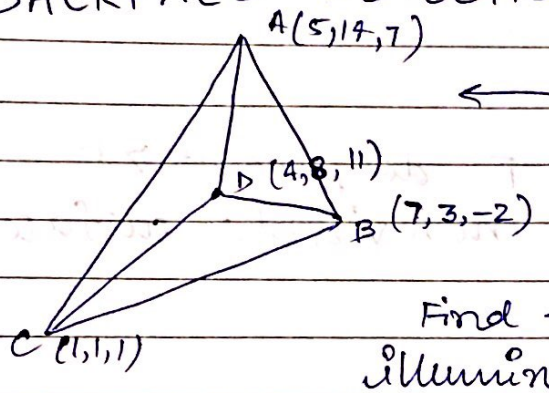
$$\begin{pmatrix} x_h \\ y_h \\ z_h \\ h \end{pmatrix} = \begin{pmatrix} (an_1+d) & an_2 & an_3 & -ad_0 \\ bn_1 & (bn_2+d) & bn_3 & -bd_0 \\ cn_1 & cn_2 & (cn_3+d) & -cd_0 \\ n_1 & n_2 & n_3 & -d_1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

## HIDDEN SURFACE

(Detection & then Elimination)

## BACKFACE DETECTION METHOD

Eg →




← Tetrahedron

point light source

P (20,10,−20)

Find those surfaces which are illuminated and those which are not illuminated

Ans →

| Step ① | Step ② |
|---|---|
| ACD | $-8i+2j+k$ |
| ADB | $98i-j+23k$ |
| BCD | $41i-69j+36k$ |
| ABC | $-51i+48j-70k$ |

find outward normal {outward normal that goes out of the solid}

**ACD**

$AC = -4i-13j-6k$ $\quad AD = -i-6j+4k$

~~ABC = AB × AC  2i − 11j~~ ~~ACD~~

$$\begin{matrix} i & j & k \\ -4 & -13 & -6 \\ -1 & -6 & 4 \end{matrix} \begin{matrix} -4 \\ -1 \end{matrix}$$

$$(24-13)k + (-52-36)i + (6+16)j$$

$$22j - 88i + 11k$$

~~(2i − 8j + k)~~

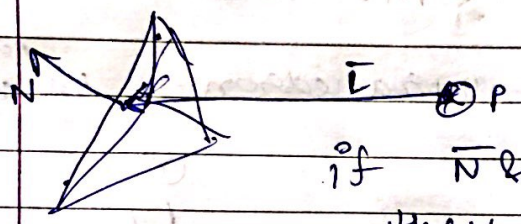
N of ACD $(-8i+2j+k)$

Scanned by CamScanner

| Step ① | Outward Normal Step ② | (Mean position) Step ③ | $\bar{L} = mp - p$ Step 4 |
|---|---|---|---|
| ACD | | $10/3\,i + 23/3\,j + 19/3\,k$ | $\left(\frac{10}{3}-20\right)i + \left(\frac{23}{3}-10\right)j + \left(\frac{19}{3}+20\right)k$ |
| ADB | | $16/3\,i + 25/3\,j + 16/3\,k$ | $\left(\frac{16}{3}-20\right)i + \left(\frac{25}{3}-10\right)j + \left(\frac{16}{3}+20\right)k$ |
| BCD | | $4i + 4j + 10/3\,k$ | $(4-20)i + (4-10)j + \left(\frac{10}{3}+20\right)k$ |
| ABC | | $13/3\,i + 6j + 2k$ | $\left(\frac{13}{3}-20\right)i + (6-10)j + (2+20)k$ |

$\bar{L} \cdot \bar{N} > 0$ non-visible

$< 0$ visible



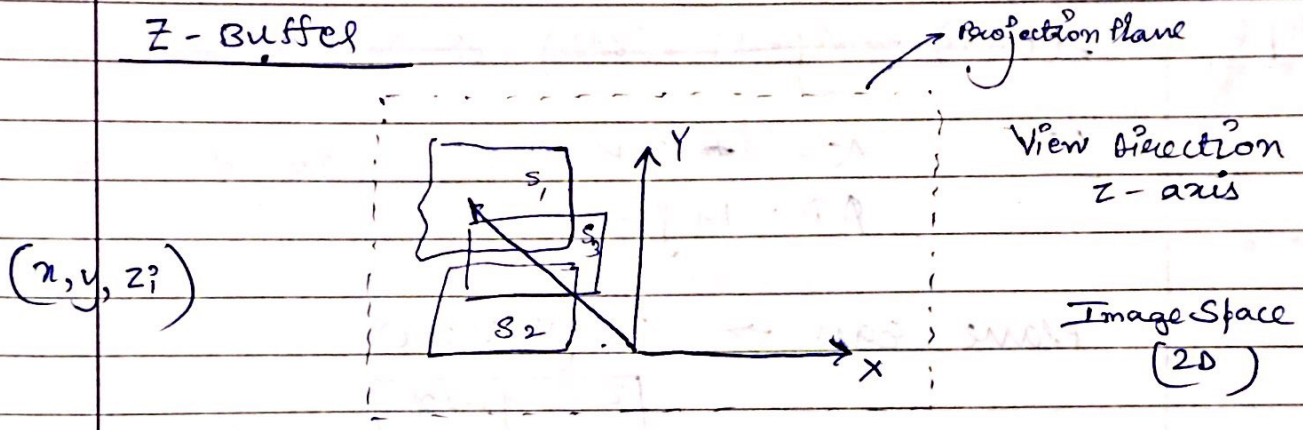

if $\bar{N}$ & $\bar{L}$ dot product $> 0$
then non visible surface

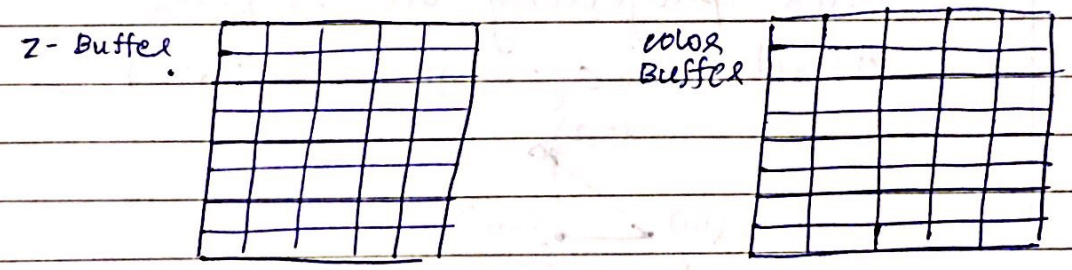
CG

| Page No. | |
|---|---|
| Date | |

# HIDDEN SURFACE

## Z - Buffer




$S: \{S_1, S_2, S_3 \ldots S_n\}$

$\downarrow$

$\{x_{min}, x_{max}, y_{min}, y_{max}, z_{min}, z_{max}\}$

We will initialise z-Buffer with $z_{max} + 1$
we initialise color buffer with Bkgd color

Eg → A tetrahedron is given by position vector
A (1,1,-1), B (3,1,-1), C (2,1,-3), D (2,2,-2)

surface, S →
ACD (Blue [1])
CBD (Green [2])
BAD (cyan [3])
ACB (Red [4])

Bkgndcolor (Black [0])

Scanned by CamScanner

Page No.
Date

Step ① $x_{min} = 1$, $x_{max} = 3$, $y_{min} = 1$, $y_{max} = 2$, $z_{min} = -3$, $z_{max} = -1$

Step ② ~~$(x-1)+(y-1)+(z+1) = (3-2)+(2-1)+-2$~~

ACD $AC = i - 2K$ $\qquad 2x - y + z = 0$

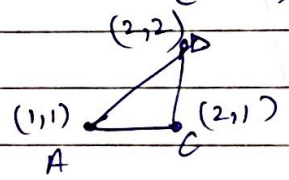
$AD = 1 + j - K$

plane eqn → $2x - y + z = 0$

$\boxed{z = y - 2x}$

After projecting on the plane

$A(1,1)$ $\quad C(2,1)$ $\quad D(2,2)$




$\boxed{\text{inside test}}$

AC $\frac{y-1}{x-1} = 0$ $\qquad f(x,y) = y-1$

$y - 1 = 0$ for D $f(x,y) > 0$

AD ⇒ $y = x$ $\qquad f(x,y) = x - y$

for C $f(x,y) > 0$

CD ⇒ $x = 2$ $\qquad f(x,y) = 2 - x$

for A $f(x,y) > 0$

(we have chosen the eqns as such $f(x,y) > 0$)

~~For~~ ~~since~~ from step ①

(we put the points in the line eqns to check whether point is inside or on the surface)

for ACD $z = y - 2x$

| $x$ | $y$ | $z$ ACD | Inside Test |
|---|---|---|---|
| 1 | 1 | -1 | T |
| 1 | 2 | 0 | F |
| 2 | 1 | -3 | T |
| 2 | 2 | -2 | |
| 3 | 1 | -5 | |
| 3 | 2 | -4 | |

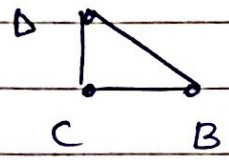
Scanned by CamScanner

| Page No. | |
|---|---|
| Date | |

CBD plane $\rightarrow$ $2x + y - z = 8$

$-z = 8 - 2x - y$ $\quad z = y + 2x - 8$

B (3,1) C(2,1) D(2,2)




CD $\Rightarrow x = 2$ $\quad x - 2 \geq 0$

CB $\Rightarrow y = 1$ $\quad y - 1 \geq 0$

BD $\Rightarrow x + y = 4$ $\quad 4 - x - y \geq 0$

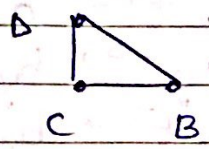
| $x$ | $y$ | $z$ | Inside Test |
|---|---|---|---|
| 1 | 1 | ACD = -1 | T |
| | | BCD = -5 | F |
| 1 | 2 | ACD = 0 | F |
| | | BCD = -4 | |
| 2 | 1 | ACD = -3 | T |
| | | BCD = -3 | |
| 2 | 2 | ACD = -2 | |
| | | BCD = -2 | |
| 3 | 1 | ACD = -5 | |
| | | BCD = -1 | |
| 3 | 2 | ACD = -4 | |
| | | BCD = 0 | |

Scanned by CamScanner

CBD plane $\rightarrow$ $2x + y - z = 8$

$-z = 8 - 2x - y$ $\quad z = y + 2x - 8$

B (3,1) C(2,1) D(2,2)



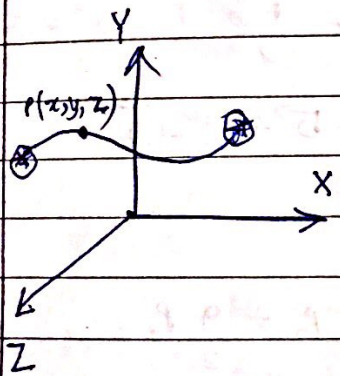

$CD \Rightarrow x = 2$ $\quad x - 2 \geq 0$

$CB \Rightarrow y = 1$ $\quad y - 1 \geq 0$

$BD \Rightarrow x + y = 4$ $\quad 4 - x - y \geq 0$

| x | y | z | Inside Test |
|---|---|---|---|
| 1 | 1 | ACD = -1 | T |
| | | BCD = -5 | F |
| 1 | 2 | ACD = 0 | F |
| | | BCD = -4 | |
| 2 | 1 | ACD = -3 | T |
| | | BCD = -3 | |
| 2 | 2 | ACD = -2 | |
| | | BCD = -2 | |
| 3 | 1 | ACD = -5 | |
| | | BCD = -1 | |
| 3 | 2 | ACD = -4 | |
| | | BCD = 0 | |

## 3D CURVE

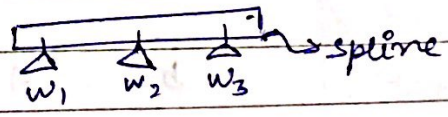



Mathematical eqn of curve is not known

Application Domain

Draft person (mathematician) $\rightarrow$ Shipping Industry

$\rightarrow$ Car Industry




### Parametric form

$$P(t) = [x(t), y(t), z(t)] \quad \{0 \leq t_0 \leq 1\}$$

$$x(t) = a_0 x_0 + a_1 x_1 t + a_2 x_2 t^2 \ldots\ldots a_n x_n t^n$$

similarly, $y(t)$ and $z(t)$

When $n = 1$ straightline

$n = 2$ Quadratic curve

$n = 3$ cubic curve

$n = 4$ Quadratic space curve

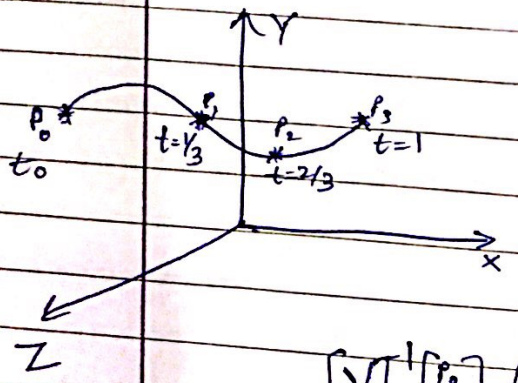
$\vdots$

For $n=3$ cubic space we need atleast 4 points to uniquely identify

cubic curve (Parametric form)

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$
$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

$$P(t) = \vec{a} t^3 + \vec{b} t^2 + \vec{c} t + \vec{d}$$




$$\begin{cases} P_0 = P(t) \text{ at } t=0 = \vec{d} \\ P_1 = P(t) \text{ at } t=1/3 \\ \quad \frac{\vec{a}}{27} + \frac{\vec{b}}{9} + \frac{\vec{c}}{3} + \vec{d} \\ P_2 = P(t) \text{ at } t = 2/3 \\ \quad \frac{8\vec{a}}{27} + \frac{4\vec{b}}{9} + \frac{2\vec{c}}{3} + \vec{d} \\ P_3 = \vec{a} + \vec{b} + \vec{c} + \vec{d} \end{cases}$$

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} & & & \\ & X & & \\ & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$[X]^{-1} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\vec{a} = -\frac{9}{2} P_0 + \frac{27}{2} P_1 - \frac{27}{2} P_2 + \frac{9}{2} P_3$$

$$\vec{b} = 9 P_0 - \frac{45}{2} P_1 + 18 P_2 - \frac{9}{2} P_3$$

$$\vec{c} = -\frac{11}{2} P_0 + 9 P_1 - \frac{9}{2} P_2 + P_3$$

$$\vec{d} = P_0$$

$$P(t) = \left(-\frac{9}{2} P_0 + \frac{27}{2} P_1 - \frac{27}{2} P_2 + \frac{9}{2} P_3\right) t^3 + \left(9P_0 - \frac{45}{2} P_1 + 18 P_2 - \frac{9}{2} P_3\right) t^2$$
$$+ \left(-\frac{11}{2} P_0 + 9 P_1 - \frac{9}{2} P_2 + P_3\right) t + P_0$$

Scanned by CamScanner

$$= \left[\frac{-9t^3+9t^2-11t+1}{2}\right]P_0 + \cancel{(\quad)P_1 + (\quad)}\left[\frac{27t^3-45t^2+9t}{2}\right]P_1$$

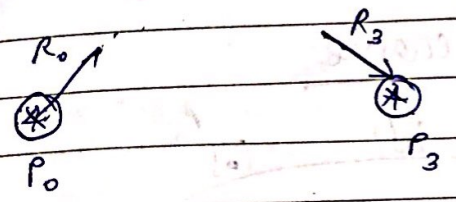
$$+ \left(\frac{-27}{2}t^3 + 18t^2 - \frac{9t}{2}\right)P_2 + \left[\frac{9}{2}t^3 - \frac{9t^2}{2} + t\right]P_3$$

Now, we vary $t=0$ to $t=1$ with step 0.0001

Problem

A ball is thrown from a position $P_0$ with a particular speed $R_0$ and it is received by the wicket keeper whose position is $P_3$ with a speed $R_3$.

Ans →




Hermite

$P_0 = \vec{d}$ $\{t=0\}$ $\qquad$ $P_3 = \bar{a}+\bar{b}+\bar{c}+\bar{d}$ $\{t=1\}$

$$\frac{d\,P(t)}{dt} = 3\vec{a}t^2 + 2\vec{b}t + \vec{c} = v(t)$$

$$R_0 = \vec{c} \equiv v(0)$$

$$R_3 = 3a+2b+c \equiv v(1)$$

$$\begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Find inv of

$$\vec{a} = -2P_3 + 2P_0 + R_0 + R_3$$

$$\vec{b} = 3P_3 - 3P_0 - 2R_0 - R_3$$

$$\vec{c} = R_0$$

$$\vec{d} = P_0$$

Scanned by CamScanner

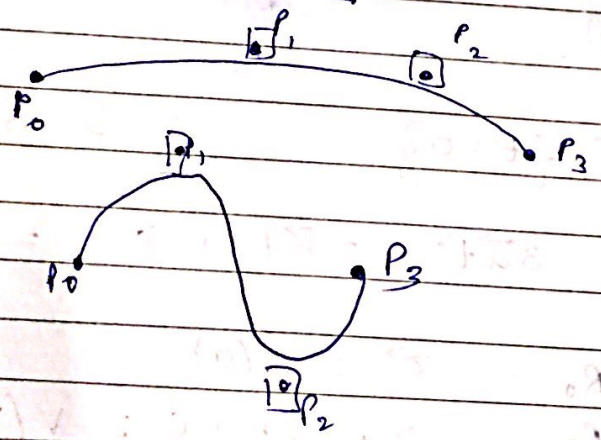
$$P(t) = (-2P_3 + 2P_0 + R_0 + R_3)t^3 + (3P_3 - 3P_0 - 2R_0 - R_3)t^2 + R_0 t + P_0$$

$$P(t) = (-2t^3 + 3t^2)P_3 + (2t^3 - 3t^2 + 1)P_0 + (t^3 - 2t^2 + t)R_0 + (t^3 - t^2)R_3$$

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\text{Blending Matrix}} \begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix}$$

Problem 2

Cubic curve




$P_1, P_2 \rightarrow$ control Point

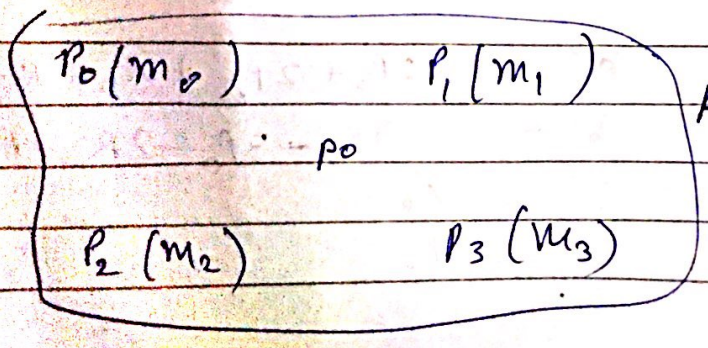
Not lying on curve but modify shape of curve

Peitree Bezen

$$P(t) = \sum_{i=0}^{n} P_i \cdot B_{i,n}(t)$$

$$0 \le t \le 1$$

Let these are point masses




centre of mass $P$

$$P = \frac{P_0 * m_0 + P_1 * m_1 + P_2 * m_2 + P_3 * m_3}{m_0 + m_1 + m_2 + m_3}$$

Scanned by CamScanner

$m_0 = (1-t)^3$

$m_1 = 3t(1-t)^2$

$m_2 = 3t^3(1-t)$

$m_3 = t^3$

At $t=0$

$m_1 = m_2 = m_3 = 0$

$t=1$

$m_0 = m_1 = m_2 = 0$

$n=3$ cubic space curve

$B_{0,3}(t) = (1-t)^3$ $\quad B_{1,3}(t) = 3t(1-t)^2$

$B_{2,3}(t) = 3t^2(1-t)$ $\quad B_{3,3}(t) = t^3$

cubic curve $\quad P(t) = \sum_{i=0}^{3} P_i * B_{i,3}(t)$

$= P_0 * B_{0,3}(t) + P_1 * B_{1,3}(t) + P_2 * B_{2,3}(t) + P_3 * B_{3,3}(t)$

$P(t) = (1-t)^3 * P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$

$= [1 - t^3 + 3t^2 - 3t]P_0 + [3t^3 - 6t^2 + 3t]P_1 + [-3t^3 + 3t^2]P_2 + t^3 P_3$

Q. <u>Cubic Bezier curve problem</u>

$P_0(20,20)$, $P_1(40,80)$, $P_2(80,80)$, $P_3(90,50)$

Another curve segment

$Q_0(a,b)$, $Q_1(6,20)$, $Q_2(150,20)$, $Q_3(180,20)$

Both the curve join ~~so~~ smoothly

Ans $\rightarrow$ since both curves join smoothly,

$Q_0 = P_3$ so $a = 90$ $b = 50$

smooth means tangent same

Scanned by CamScanner

$C = 100$

Page No.
Date

$$P'(t) = \cancel{3(1-t)^2} - 3(1-t)^2 P_0 + 3(1-t)^2 P_1 - 6t(1-t) P_1 + 6t(1-t) P_2 - 3t^2 P_2 + 3t^2 P_3$$

$P'(t)_{t=1} = Q'(t)_{t=0}$

Q. $P_0 = (2,1)$ $P_1 = (3,2)$ $P_2 (5,0)$ $P_3 (6,2)$

By properly choosing another set of control point $Q_1 (a,b)$, $Q_2 (c,d)$, $Q_3 (e,f)$, $Q_4 (g,h)$ so that curve is smooth.

Ans → cubic curve is smooth till 2nd degree

So $P''(t)_{t=1} = Q''(t)_{t=0}$

$a = 6$ $b = 2$

$\begin{cases} c = 7 \; d = -1 \\ e = 7 \; f = 10 \end{cases}$

$$P''(t) = 6(1-t)P_0 - 6(1-t)P_1 - 6(1-t)P_1 + 6tP_1 + 6(1-t)P_2$$

Q. Find eqn of Beizer curve which passes through $(0,0)$ & $(-2,1)$ and is controlled through $(7,5)$ & $(2,0)$.

Ans → $P_1$ $P_2$ or $P_1$ $P_2$

$(7,5)$ $(2,0)$ $(2,0)$ $(7,5)$

So two solutions possible

Scanned by CamScanner

Page No.
Date

Sol$^n$ 1) $x(t) = 13t^3 - 36t^2 + 21t$
$y(t) = 16t^3 - 30t^2 + 15t$

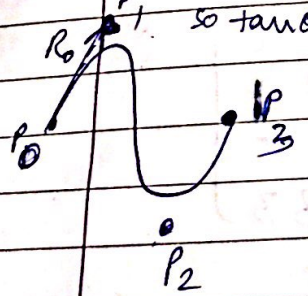
Sol$^n$ 2) $x(t) = -17t^3 + 9t^2 + 6t$
$y(t) = -14t^3 + 15t^2$

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

↳ this came from $P(t) = \left[-t^3+3t^2-3t+1\right]P_0 + \left[3t^3-6t^2+3t\right]P_1 + \left[-3t^3+3t^2\right]P_2 + \left[t^3\right]P_3$

## Hermite - Bezier {Transformation}

we assume $P_0 P_1$ & $\parallel R_0$ so tangent same.




$R_0$ (for Bezier)

$P'(t)$ at $t=0$ $= R_0$

$R_0 = -3P_0 + 3P_1 = 3(P_1 - P_0)$

$R_3 = 3(P_3 - P_2)$ $\{t=1\}$

Geometric constraint

$G_{Bezier} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$ $G_{Hermite} = \begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix}$

Scanned by CamScanner

$$G_H = M_{HB} * G_B$$ where M is a transformation matrix

$$\begin{bmatrix} P_0 \\ P_3 \\ R_0 \\ R_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Basis matrix of Bezier

$$M_{Bez} = M_H * M_{HB}$$

$$\begin{bmatrix} -1 & 3 & -3 & 1 \\ \text{work} & & & \\ \text{as} & & & \\ \text{done earlier} & & & \end{bmatrix}$$

$$P(t) = T \cdot M_H \cdot G_H$$

$$= T \cdot M_H \cdot (M_{HB}\, G_B)$$

$$= T \cdot (M_H \cdot M_{HB}) \cdot G_B$$

$$= T \cdot M_B \cdot G_B$$

verify that ~~Basis~~

$$M_B = M_H * M_{HB}$$

$$= \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

which is same as before so verified

Scanned by CamScanner

B-spline was proposed to avoid these disadvantages

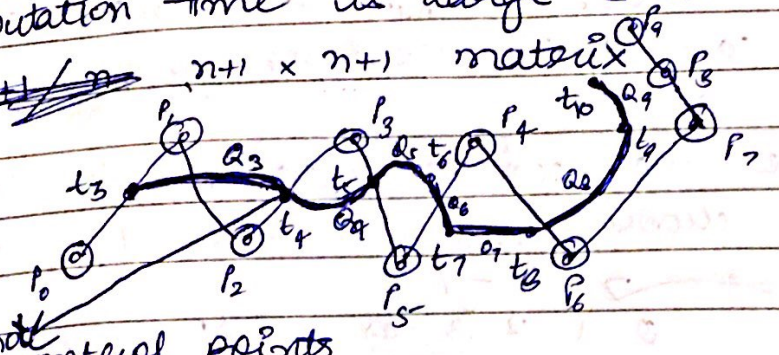
Page No.
Date

## B - spline

spline has two disadvantages

1) Polynomial coefficients are dependent on ~~#~~ all the control points, moving any point affect the entire curve.

2) computation time is large since it involves inversion of ~~n+1/n~~ $n+1 \times n+1$ matrix




In Bezier $n=3$, $n$ is deg of eqn since Bezier is cubic $n=3$

P → control points (knot)

Q → segments of curve  t → points on curve

Explaination 1) → change in control point has global effect. But only local control points should affect curve.

(if n+1 control points, n-2 curve segments can be addressed)

if there are n+1 control points $(P_0, \ldots P_n)$

$Q_3$  $\underbrace{P_0\ P_1\ P_2\ P_3}$ these should affect $Q_3$ segment

movement of corresponding control points should only effect that segment only.

$Q_4$  $P_1\ P_2\ P_3\ P_4$

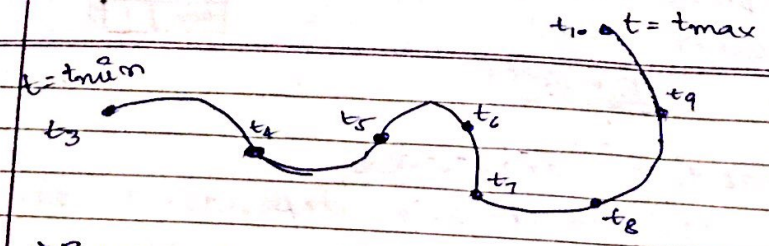
$Q_5$  $P_2\ P_3\ P_4\ P_5$

$Q_6$  $P_3\ P_4\ P_5\ P_6$

⋮

(if last curve segment $P_n\ P_0\ P_1\ P_2$ then closed curve)

Scanned by CamScanner




If we choose $t_{min} = 0$ & $t_{max} = 1$, we can get floating points when we sample
so we choose $t_{min}$ and $t_{max}$, such that we can avoid floating points

Since we need to address for 4 ~~at~~ {cubic curve}
we choose $t_{min} = 0$ $t_{max} = 7$

$P(t)$

| 0 | 1 | 2 | 3 | 4 | | $P_0'$ |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | | $P_1$ |
| 2 | 3 | 4 | 5 | 6 | | $P_2'$ |
| 3 | 4 | 5 | 6 | 7 | | $P_3$ |

$$\boxed{P(t) = \sum_{i=0}^{n} P_i * B_{i,d}(t)}$$

$t$ (knot)

$t = \{t_0, t_1, t_2 \ldots t_n\}$ set of knots

$t_{min} \le t \le t_{max}$

$2 \le d \le n+1$

degree $\Rightarrow d-1$
for cubic degree $= 3$
$d - 1 = 3$
$d = 4$

recursion $f^n$ {since $d = 4$, $4 \le n+1$, $3 \le n$}

## Cox - De - Boor Recursive $f_n$

$$B_{i,d}(t) = \left\{\frac{t - t_i}{t_{i+d-1} - t_i}\right\} B_{i,d-1}(t) + \left\{\frac{t_{i+d} - t}{t_{i+d} - t_{i+1}}\right\} B_{i+1,d-1}(t)$$

Scanned by CamScanner

where $B_{i,1}(t) = 1$ if $t_i \le t < t_{i+1}$

else

$B_{i,1}(t) = 0$

$$P(t) = P_0 * B_{0,4}(t) + P_1 * B_{1,4}(t) + P_2 * B_{2,4}(t) + P_3 * B_{3,4}(t)$$

sampling is for $n+d=7$

$$t: \{\underset{0}{t_0}, \underset{1}{t_1}, t_2, t_3, t_4, t_6, \underset{7}{t_7}\}$$

$$B_{0,4}(t) = \left\{\frac{t-0}{3-0}\right\} B_{0,3}(t) + \left(\frac{4-t}{4-1}\right) B_{1,3}(t)$$

$t_{0+4-1} = t_3$

$\frac{t_4 - t}{t_4 - t_1}$

Do from BK / Net

$$P(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{matrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{matrix}$$

Q. 4 points $P_0(a,b)$, $P_1(20,50)$, $P_2(40,40)$, $P_3(70,c)$ are available for drawing B-spline curve such that curve starts from the $(21,43)$ and terminates with slope $(-1/2)$

Ans → ~~first point → $\begin{bmatrix} 1/6 & 4/6 & 1/6 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$~~

~~$(a,b) = P_0 = \frac{a}{6} + \frac{20 \times 4}{6} + \frac{40}{6}$~~

$a = -120$

Scanned by CamScanner

$\cancel{\frac{a}{6} + \frac{1}{3}(20) + \frac{1}{6}(40) = 0}$

$\cancel{a = -120}$

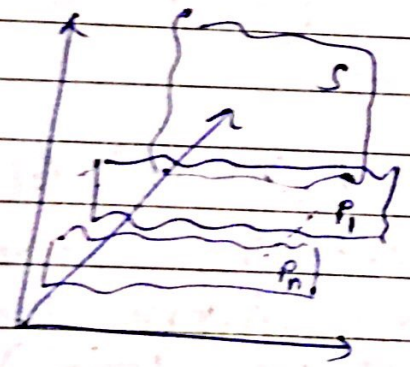
$a + 80 + 40$

$\frac{a + 120}{6} =$

## Hidden Surface

Painter Algo / Depth Sorting / Priority Algo
→ Newell & Sancha

If a curve follows standard practice done by any painter
→ distant object is painted first
(sorting of polygon in order of decreasing DEPTH)
→ nearer objects are painted, accordingly




Each surface has vertices $(x_i, y_i, z_i)$

Set : $\{S, P_1, \ldots P_n\}$

Sort them acc to their depth {z value}

When sorting we look into

z-extent ($z_{max}$ (of this surface) $z_{min}$ (of this surface))

y-extend ($y_{max}$ $y_{min}$)

x-extent ($x_{max}$ $x_{min}$)

} Bounding Rectangle (for an image)

Scanned by CamScanner

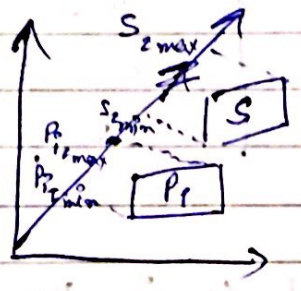
(b/w two surfaces)

No overlap along z axis

similarly for others

| | Overlapping | | |
|---|---|---|---|
| $Z_{extent}$ | ✗ | ✗ | ✗ |
| $Y_{extent}$ | ✗ | ✗ | ✓ |
| $X_{extent}$ | ✗ | ✓ | ✗ |
| | paint them according to depth | paint them acc to depth | paint them acc to depth |

Here no z overlap




$S_{z_{min}} > P_{z_{max}}$ no overlap along z axis

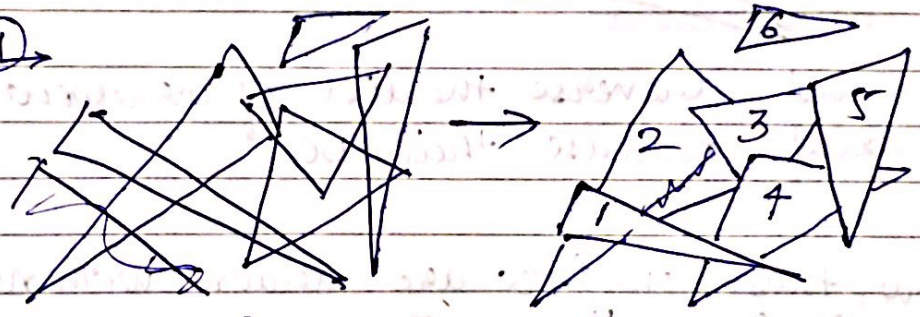
But here is x overlap no z overlap

Hence, we first paint S and then $P_i$

(Cases, y may be overlapped)

~~Sim~~

Eg. 1




Given there is no z overlapping

Solution → STEP1 Creating a table

| surface # | Counter (Behind) | List (surfaces in Front) |
|---|---|---|
| 1 | 2 | – {nothing in front} |
| 2 | 0 | 3, 4, 1 |
| 3 | 1 | 4, 5 – |
| 4 | 2 | 1, 5 – |
| 5 | 2 | – |
| 6 | 0 | – |

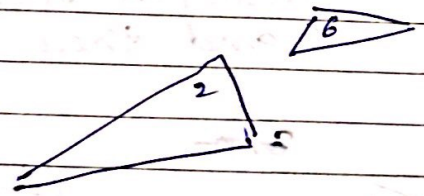
{we have not considered 3 behind 1}

Scanned by CamScanner

Step (2) Processing the constructed Table

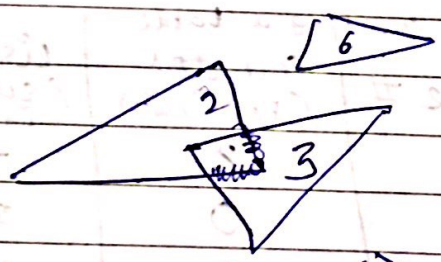
| Surface # | List (surfaces in front) | behind counter BC | Pass (1) | Pass (2) |
|---|---|---|---|---|
| 1 | – | 2 | 1 | 0 |
| 2 | ~~3~~, ~~4~~, ~~1~~ | 0 ✓ | | |
| 3 | ~~4~~, ~~5~~ | 1 | 0 ✓ | |
| 4 | ~~1~~, ~~5~~ | 2 | 1 | 0 ✓ |
| 5 | – | 2 | | 1 → 0 ✓ |
| 6 | – | 0 ✓ | | |

During Pass (1), look at 0 in BC and draw it

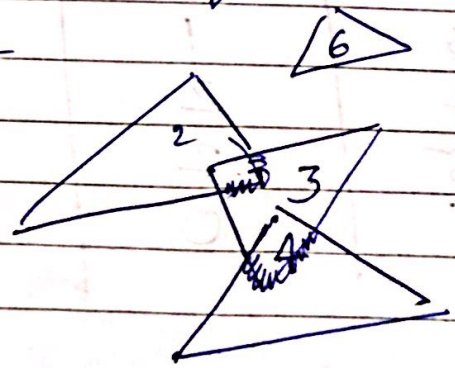



and traverse the list of corresponding surfaces and decrease their BC

Now, those surfaces are drawn with new BC as 0. Here, 3 so

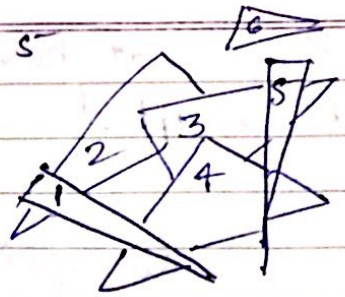



Now, 4




Scanned by CamScanner

Now, 1 and 5

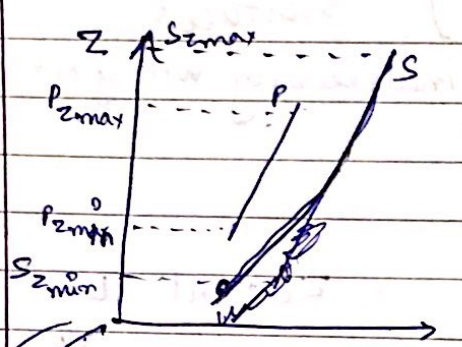



Now

$z_{extent}$

Overlap ✓

Now, all the cases will have depth overlap (z-overlap)

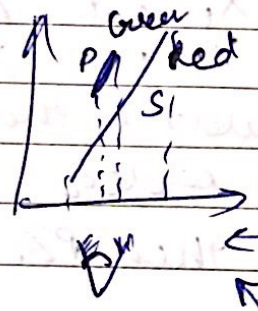



here x, y, z all overlap

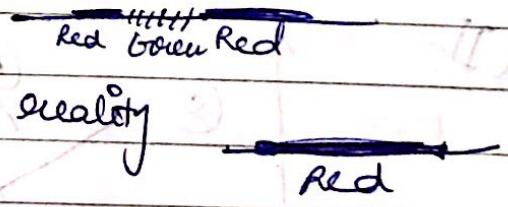
Set → S, P
{sorted according to $z_{max}$}

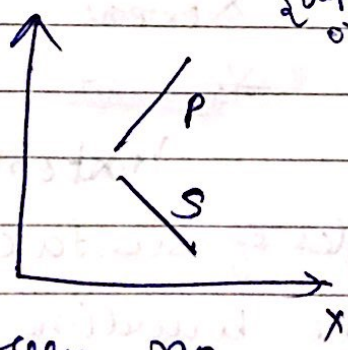
So acc to this we draw S first & then P

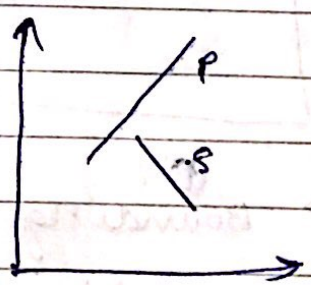
So we will see P from this viewpoint but in reality P is not visible




These both same z overlap but in reality {depth overlap}



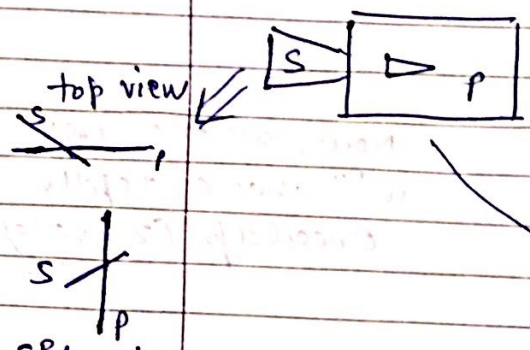




Here no z overlap {depth overlap}

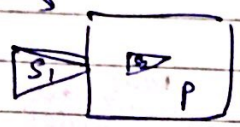
Scanned by CamScanner

whether the surfaces that have been sorted are needed to be swapped



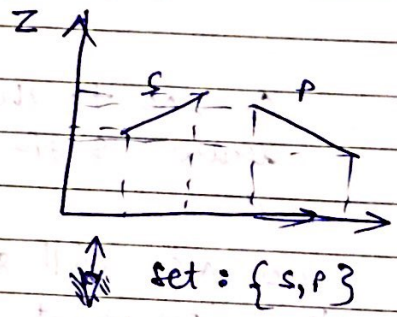

Query ① surfaces sorted should remain in same order

Query ② surfaces if passing / crossing each other, needs to break into two sub surfaces




$S_1, S_2, P$ individually will yield correct result

Case I




Zextent ✓

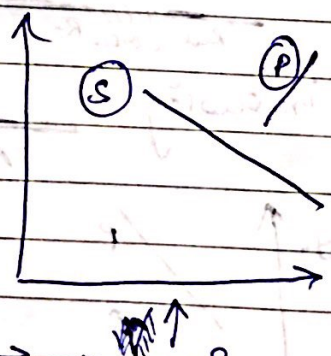
Xextent ✗

Yextent ✗

(should n't change any order)

i.e. S is painted first and then P&. ....

Case II




Zextent ✓

X extent ✓

~~Yextent~~

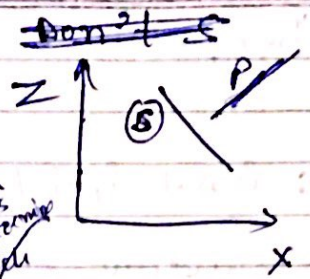
Yextent ✓ or ✗

→ Bounding rectangles of surface (P) is completely inside the bounding rectangle of (S) then

Don't Scan P

(This will happen when Y is overlapping)

case III



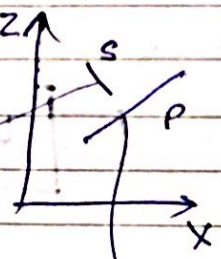

zextent ✓
Xextent ✓
Yextent ✓ or X

Partially obscured Problem

we perform diff tests to determine which case

then we proceed for Test (2)

Test (2)




S is outside of surface P wrt view direction of plane i.e. XY plane

plane eqn of S

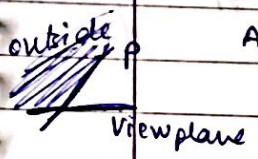
$Ax + By + Cz = D$

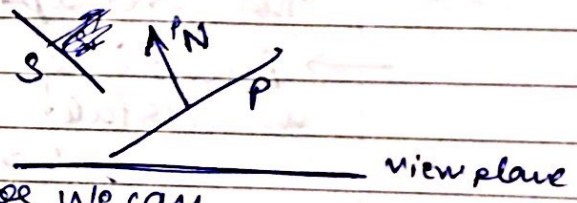
Plane eqn of P

Take all the vertices of S and put it in plane eqn of surface P

$A_p x' + B_p y' + C_p z' + D_p > 0$

then outside

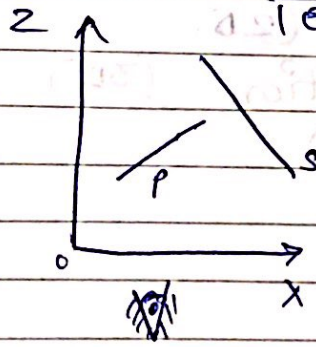



or We can use outward Normal of P wrt view plane

We use $P_N$ to determine if S is outside if any vector on surface S is within 90° of $P_N$

If this test (2) fails then Test (3)

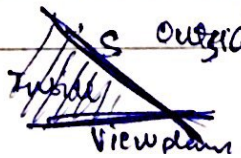



surface P is inside of S

vertices of P is put in plane eqn of surface S

if $A_s x + B_s y + C_s z + D < 0$

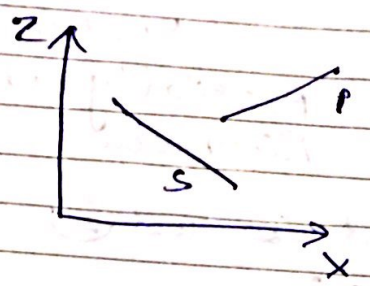
then P is inside S




Scanned by CamScanner

So, we scan convert S and then paint P

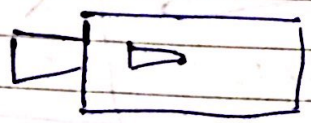
Next Scenario




→ S is not inside P

→ Bounding rectangle of P and S overlap

→ P is not inside of S



→ Only bounding rectangle overlaps & surface not overlaps

→ Bounding rectangle overlaps as well as surface overlaps

↳ we go for polygon clipping by Weiler Atherton

$\{S, P\} \longrightarrow \{S, P_1, P_2\}$

→ Area subdivision (2D)

→ Binary Space Partition (3D)

→ Octree (3D)

Do from book

Scanned by CamScanner

Page No.
Date

Q. $S_1$: $T_{11}$ (0,7,5), $T_{12}$ (6,7,5), $T_{13}$ (0,1,5)

$S_2$: $T_{21}$ (0,6,7), $T_{22}$ (0,1,2), $T_{23}$ (5,1,7)

Apply Painters Algo.

Ans → Here $x, y, z$ all 3 are overlapping

Set: ($S_2$, $S_1$) bcs (7 > 5)

Bounding Rectangle → $S_1$ → 0–6, 1–7, 5–5

$S_2$ → 0–5, 1–6, 2–7

~~Since both do not completely overlap we proceed with next tests~~

Eqn of $S_1$ → $z = 5$

Eqn of $S_2$ → $x + y - z + 1 = 0$

Bounding Rectangles are overlapping and surfaces are overlapping

Now, apply Weiler Atherton on ($S_1$, $S_2$)

Scanned by CamScanner