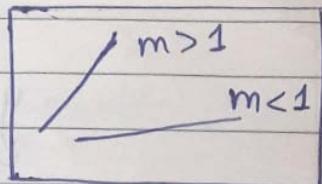


02/01/2018

Computer Graphics

2D on screen



→ Drawn L to R

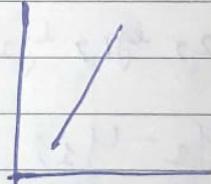
#include <graphics.h>

Primitives: Line, Circle

LINE:

$$y = mx + c$$

for $m > 1$



y changes more as compared to x.

$$x = y/m - c/m$$

$$x_i = \frac{y_i}{m} - \frac{c}{m}$$

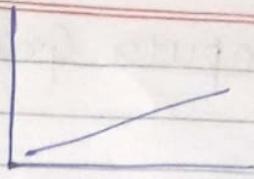
$$x_{i+1} = \frac{y_{i+1}}{m} - \frac{c}{m} \quad (y_{i+1} = y_i + 1)$$

$$x_{i+1} = \frac{y_i}{m} - \frac{c}{m} + \frac{1}{m}$$

$$\boxed{x_{i+1} = x_i + \frac{1}{m}}$$

$$[y_{i+1} = y_i + 1]$$

for $m < 1$



Changes in x are more than those in y

$$y = mx + c$$

$$y_i = mx_i + c$$

$$y_{i+1} = mx_{i+1} + c$$

$$(x_{i+1} = x_i + 1)$$

$$y_{i+1} = mx_i + c + m$$

$$\boxed{y_{i+1} = y_i + m} \quad [x_{i+1} = x_i + 1]$$

Algorithm to draw a line:

mline(x_1, x_2, y_1, y_2) {

$$dy = y_2 - y_1;$$

$$dx = x_2 - x_1;$$

$$m = dy/dx$$

if ($\text{abs}(dy) < \text{abs}(dx)$) {

$$x = x_1, y = y_1;$$

display(x, y);

while ($x < x_2$) {

$$x++;$$

$$y += m;$$

display(x, y);

}

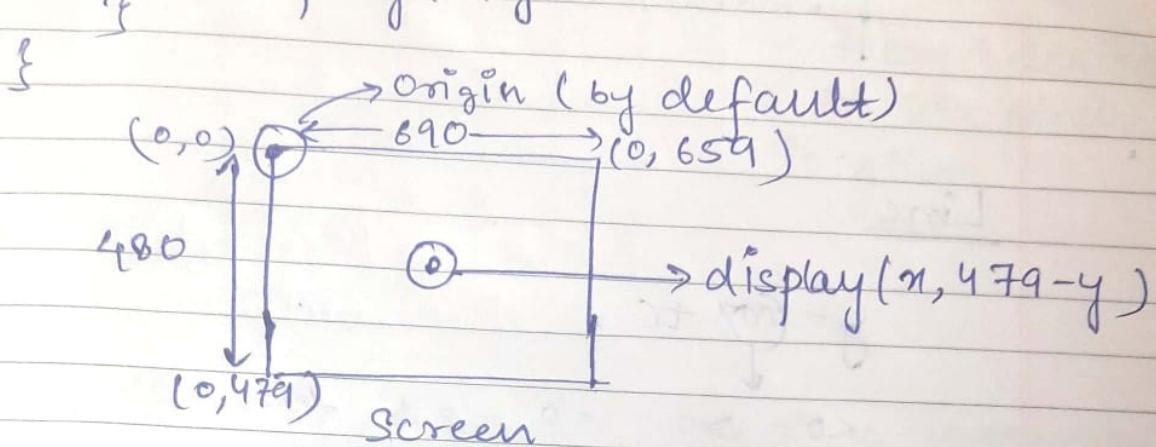
if ($\text{abs}(dy) \geq \text{abs}(dx)$) {

$$x = x_1, y = y_1;$$

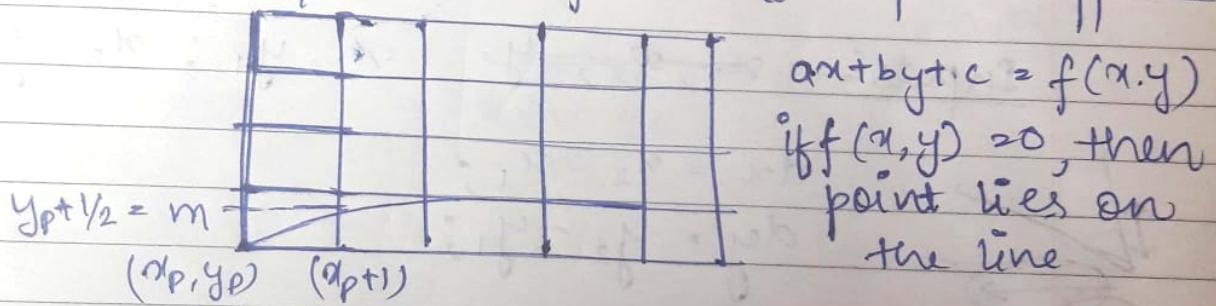
display(x, y);

while ($y < y_2$) {

$y++;$
 $x^2 = 1/m;$
 $\text{display}(x, y)$



Another approach of line ~~Midpoint Approach~~



$f(x_p, y_p) = ax_p + by_p + c$

$f(m) = a(x_p + 1) + b(y_p + 1/2) + c$

deciding value \leftarrow Case 1: if $f(m) = 0$, then lies on the line

Case 2: $f(m) > 0$

Case 3: $f(m) < 0$

04/01/2018

Page _____
my companion

Primitive Drawing
→ mathematical equation

Line

$$y = mx + c$$

\downarrow
 $< 0 \quad > 0 \quad = 0 \quad = 1 \quad = \infty$

Smallest unit: Pixel

Inputs: x_1, y_1, x_2, y_2

$$dx = x_2 - x_1;$$

$$dy = y_2 - y_1;$$

$$x = x_1, y = y_1;$$

putpixel(x, y, WHITE);

while($x < x_2$)

$m < 0$ { floating pt. & floating pt. computation takes more time.

$y + = m;$

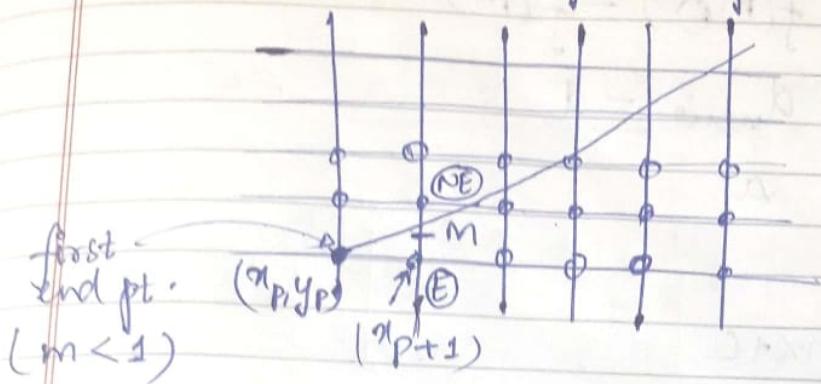
$x + +,$

putpixel(x, y, WHITE);

MidPoint Approach:

To avoid floating pt. computation

Screen is in the form of a grid



Every intersection is a pixel location

$\therefore m < 1, dy < dx \Rightarrow x$ is incremented
& accordingly y is computed (major movement
in x).

$$f(x, y) = ax + by + c$$

↓ ↓ ↓
 < 0 = 0 > 0

$$f(M) = f(x_p + 1, y_p + 1/2)$$

$$d_{\text{decision}} = f(M) = a(x_p + 1) + b(y_p + 1/2) + c$$

↓
 keeps on
 updating depending on value of $f(M)$

if $d < 0$
selection is (E)

$$f(M_E) = f(x_p + 2, y_p + 1/2)$$

$$d_{\text{new}} = a(x_p + 2) + b(y_p + 1/2) + c$$

$$\Delta E = f(M_{\text{new}}) - f(M)$$

$$\Delta E = d_{\text{new}} - d$$
$$= a$$

$$y = mx + c$$

$$\frac{dy}{dx}$$

Replace m with $\frac{dy}{dx}$.

$$dx \cdot y = dy \cdot x + dx \cdot c$$

$$f(x, y) = (\underbrace{dy}_{(a)})x - dx \cdot y + dx \cdot c = 0$$

$$\Rightarrow \Delta E = a = dy$$

else if $d > 0$
Selection is NE

$$f(M_{\text{NE}}) = f(x_{p+2}, y_p + 3/2)$$

$$d_{\text{new}} = f(a(x_{p+2}), b(y_p + 3/2) + c)$$

$$\Delta E = d_{\text{new}} - d$$

$$\Delta_{\text{NE}} = f(M_{\text{NE}}) - f(M)$$

$$\Delta NE = va + b$$

$$\Rightarrow \underline{a+b} = dy - dx$$

$$d = f(x_p + 1, y_p + 1/2)$$

$$= ax_p + a + by_p + b/2 + c$$

$$= \underbrace{ax_p + by_p + c}_{\downarrow} + a + b/2$$

This pt. is on line ; so it will be 0.

$$d_{in} = a + b/2.$$

To remove $1/2$ from $b/2$, multiply it with a scalar qty, that is, 2.

$$f(x, y) = 2(ax + by + c)$$

$$d_{in} = 2a + b = \underline{2dy - dx}$$

$$\text{if } d < 0, \Delta E = 2dy = 2a.$$

$$\text{else } \Delta NE = 2(dy - dx) \\ = 2(a + b)$$

$$\underline{d = f(x_p + 1, y_p + 1/2)}$$

Changes in previous code:

```
d = 2*(dy-dx); d = dy - dx;  
while (x <= x2)  
{  
    if d < 0  
        d+ = 2*dy;  
    else  
        {d+ = 2*(dy-dx);  
         y++;}  
    x++;  
    putpixel(x, y, WHITE);  
}
```

Example:

$$x_1 = 10$$

$$y_1 = 10$$

$$x_2 = 20$$

$$y_2 = 18$$

$$m < 1$$

$$dy = 8$$

$$dx = 10$$

$$x = 10 \quad (x_1)$$

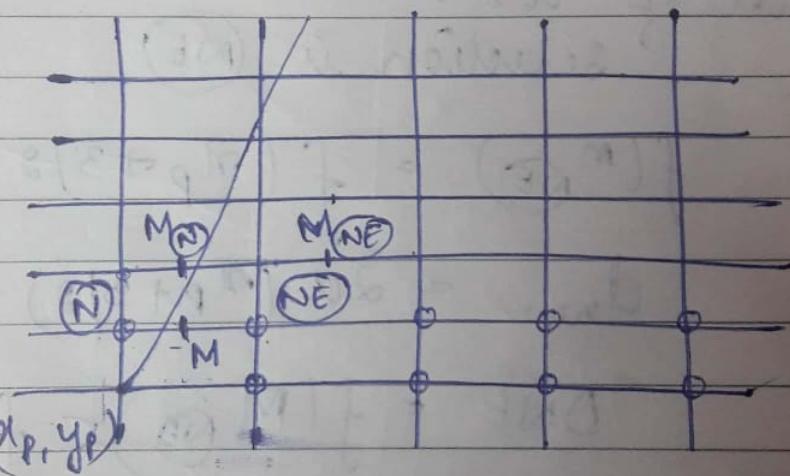
$$y = 10 \quad (y_1)$$

Check Table

x _i	y _i	d
10	10	6
11	11	2
12	12	-2
13	13	14

{ complete the rest of the table}

For $m > 1$:



At (x_p, y_p) :

$$f(x_p, y_p) = 2(ax_p + by_p + c)$$

$$d = f(M) = f(x_p + 1/2 + y_p + 1)$$

$$d = 2a(x_p + 1/2) + 2b(y_p + 1) + 2c$$

if $d < 0$

selection is (N)

$$f(M_N) = f(x_p + \frac{1}{2}, y_p + 2)$$

$$d_{new} = 2a(x_p + \frac{1}{2}) + 2b(y_p + 2) + 2c$$

$$\Delta N = f(M_N) - f(M)$$

$$\Delta N = d_{new} - d$$

$$\Delta N = 2b = -2dx.$$

else if $d \geq 0$

selection is (NE)

$$f(M_{NE}) = f(x_p + 3/2, y_p + 2)$$

$$d_{new} = 2a(x_p + 3/2) + 2b(y_p + 2) + 2c$$

$$\Delta NE = f(M_{NE}) - f(M)$$

$$\Delta NE = d_{new} - d$$

$$\Delta NE = 2a + 2b = 2(dy - dx)$$

$$d = f(x_p + 1/2, y_p + 1)$$

$$d = ax_p + a/2 + by_p + b + c$$

$$d = \underbrace{ax_p + by_p + c}_{=0} + a/2 + b$$

$$d = a/2 + b$$

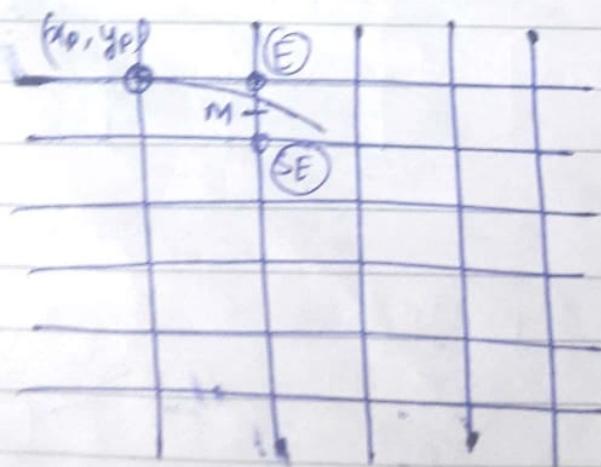
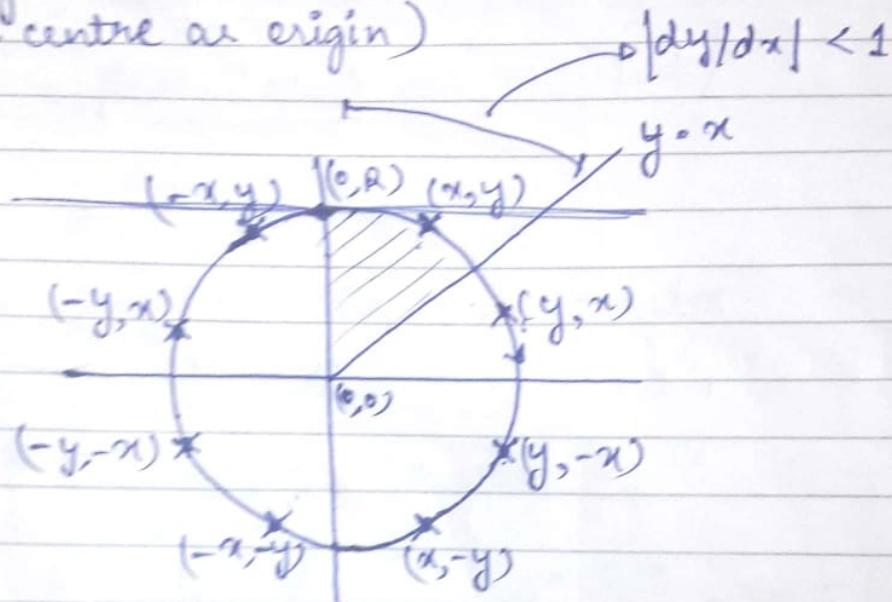
$$f(x,y) = 2(ax+by+c)$$

$$d_{in} = a + 2b = dy - 2dx$$

Scan ConversionPrimitive Circle:

$$x^2 + y^2 = R^2$$

(with centre at origin)

At (x_p, y_p)

$$f(x_p, y_p) = x_p^2 + y_p^2 - R^2$$

$$d = f(M) = f(x_p + 1, y_p - \frac{1}{2}) \\ = (x_p + 1)^2 + (y_p - \frac{1}{2})^2 - R^2$$

1st differential:

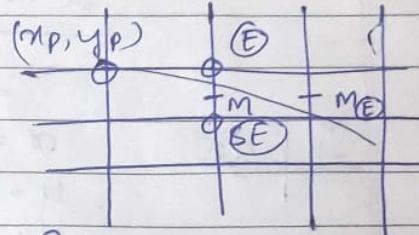
if $d < 0$

selection is (E) (assumed)

$$d_{\text{new}} = f(M_E)$$

$$= f(x_p + 2, y_p - \frac{1}{2})$$

$$= (x_p + 2)^2 + (y_p - \frac{1}{2})^2 - R^2$$



$$\Delta E = d_{\text{new}} - d = \boxed{3 + 2x_p} \quad (2 \text{ degree circle reduced to degree 1})$$

else

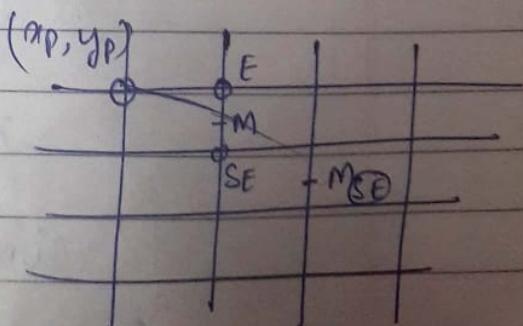
Selection is (SE)

$$d_{\text{new}} = f(M_{\text{SE}})$$

$$= f(x_p + 2, y_p - \frac{3}{2})$$

$$= (x_p + 2)^2 + (y_p - \frac{3}{2})^2 - R^2$$

$$\Delta SE = d_{\text{new}} - d = \boxed{2x_p - 2y_p + 5} = \boxed{2(x_p - y_p) + 5}$$



Initialization:

$$d = (x_p^2 + y_p^2 - R^2) + 2x_p + 1 - y_p + \frac{1}{4}$$

$$d = 2x_p - y_p + 5/4$$

$$\boxed{d = 5/4 - R}$$

(Starting from (0, R))

$$\boxed{d \approx 1 - R}$$

Inputs:

$$x = 0;$$

$$y = R;$$

$$d = 1 - R;$$

Circle Symmetry (x, y); → 8 times putpixel

while ($x \leq y$)

{ if $d < 0$

$$d+ = 3 + 2x;$$

else

$$\{ d+ = 2(x-y) + 5;$$

$$y--;$$

}

$x++;$ (becoz of major movement in x)

circleSymmetry (x, y);

}

$$R = 8$$

x	y	d
0	8	-7
1	8	-4
2	8	-1
3	7	-6
4	7	3
5	6	2
6	5	5

* Rather for going for 1st diff., we go for 2nd diff. bcoz time taken for multiplication is more than arithmetic addition so arithmetic computation time can be reduced.

Prerequisites for 2nd differential:

At (x_p, y_p) , 1st differential are:

$$\Delta E = 2x_p + 3$$

$$\Delta S E = 2(x_p - y_p) + 5$$

If $d < 0$
Selection is ①

At ① : (x_{p+1}, y_p)

$$\Delta E_{\text{new}} = 2x_p + 5$$

$$\Delta SE_{\text{new}} = 2(x_p - y_p) + 7$$

$$\boxed{\frac{\Delta^2 E}{(\Delta E_{\text{new}} - \Delta E)} = 2}$$

for the selection of E

$$\boxed{\frac{\Delta^2 SE}{(\Delta SE_{\text{new}} - \Delta SE)} = 2}$$

else selection is (SE)

$$\text{At } (SE) : (x_p + 1, y_p - 1)$$

~~$$\Delta E_{\text{new}} = 2x_p + 5$$~~

$$\Delta SE_{\text{new}} = 2(x_p - y_p) + 9$$

$$\boxed{\frac{\Delta^2 E}{(\Delta E_{\text{new}} - \Delta E)} = 2}$$

$$\boxed{\frac{\Delta^2 SE}{(\Delta SE_{\text{new}} - \Delta SE)} = 4}$$

Now, we have to initialise ΔE & ΔSE .

$$d = 1 - R$$

$$\Delta E = 3$$

$$\Delta SE = 5 - 2R$$

} Starting from R, R

B Changes in previous code:

$$x = 0;$$

$$y = R;$$

$$\Delta E = 3;$$

$$\Delta SE = 5 - 2R;$$

circle symmetry (x, y):

while ($x \leq y$) \rightarrow for 1st octant only

{ if $d < 0$

$$d+ = \Delta E;$$

$$\Delta E+ = 2;$$

$$\Delta SE+ = 2;$$

}

else

{

$$d+ = \Delta SE;$$

$$\Delta E+ = 2;$$

$$\Delta SE+ = 4; \quad y--;$$

}

$x++;$

circle symmetry (x, y):

$$R = 8$$

x	y	d	deltaE	deltaSE
0	6	-7	3	-11
1	8	-4	5	-9
2	8	1	7	-7
3	7	-6	9	-3
4	7	3	11	-1.
5	6	2	13	3
6	5	5	15	7

$$\text{grad}(E) = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j \quad \text{If } \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$$

Date _____
my companion _____

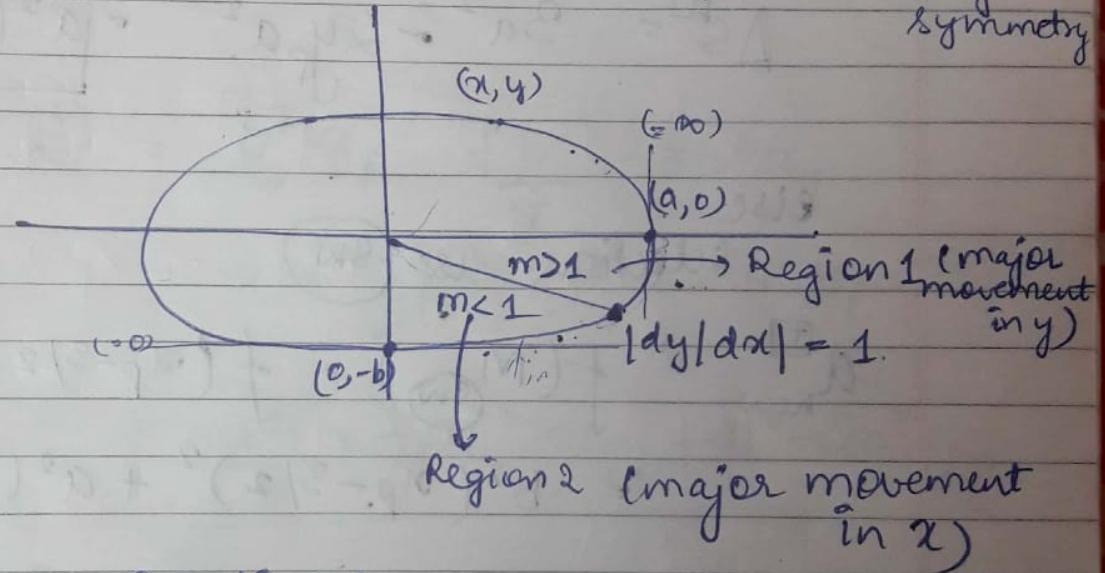
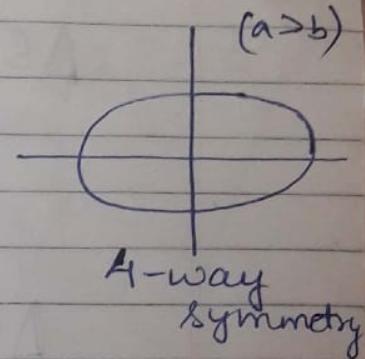
11/01/2018

Scan Conversion

Ellipse:

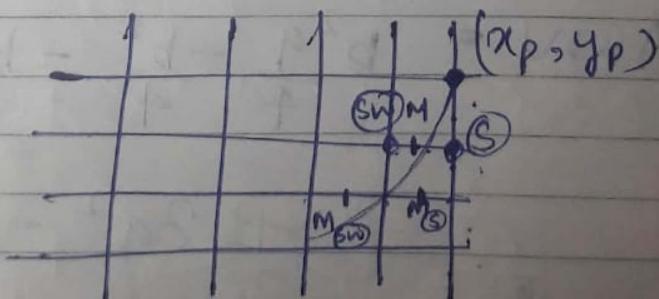
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f(x, y) = b^2x^2 + a^2y^2 - a^2b^2$$



Ist Derivative:

Region 1:



$$d = f(M) = f(x_p - 1/2, y_p - 1) = b^2(x_p - 1/2)^2 + a^2(y_p - 1)^2 - a^2b^2$$

If $d < 0$

Selection is S

$$d_{\text{new}} = f(M_S) = f(x_p - 1/2, y_p - 2)$$

$$d_{\text{new}}^{\boxed{I}} = a^2(x_p - 1/2)^2 + b^2(y_p - 2)^2 - a^2b^2$$

$$\Delta S = d_{\text{new}}^{\boxed{I}} - d_{\boxed{I}}$$

$$\Delta S = a^2 y_p^2 + 4a^2 - 4y_p a^2 - a^2 y_p^2 \\ - a^2 + 2y_p a^2$$

$$\Delta S^{\boxed{I}} = 3a^2 - 2y_p a^2 - \boxed{a^2(3 - 2y_p)}$$

else

selection is (S_W)

$$d_{\text{new}}^{\boxed{I}} = f(M_{\textcircled{SW}}) = f(x_p - 3/2, y_p - 2) \\ = b^2(x_p - 3/2)^2 + a^2(y_p - 2)^2 - a^2b^2$$

$$\Delta S_W = d_{\text{new}}^{\boxed{I}} - d_{\boxed{I}}$$

$$\Delta S_W = b^2 \frac{9}{4} - b^2 = b^2 3x_p + b^2 x_p$$

$$+ 3a^2 - 2y_p a^2$$

$$\Delta S_W^{\boxed{I}} = b^2 - 2b^2 x_p + 3a^2 - 2y_p a^2 \\ = \boxed{a^2(3 - 2y_p) + b^2(2 - 2x_p)}$$

Starting from $(a, 0)$ (clockwise) my companion

\boxed{I}

$$d_{in}^{[I]} = b^2(x_p - \frac{1}{2})^2 + a^2(y_p - 1)^2 - a^2b^2$$

$$d_{in}^{[I]} = b^2(x - \frac{1}{2})^2 + a^2 - a^2b^2$$

$$d_{in}^{[I]} = a^2 \cdot b^2 + \frac{b^2}{4} - ab^2 + a^2 - a^2b^2$$

$$d_{in}^{[I]} =$$

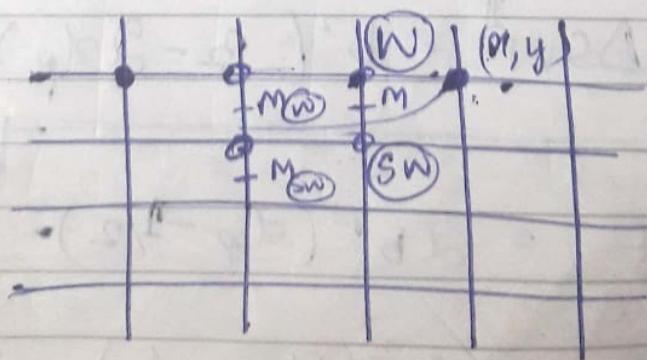
$$d_{in}^{[I]} = b^2 x_p^2 + \frac{b^2}{4} - b^2 x_p$$

$$+ a^2 y_p^2 + a^2 - 2a^2 y_p - a^2 b^2$$

$$d_{in}^{[I]} = \frac{b^2}{4} - b^2 x_p + a^2 - 2a^2 y_p$$

$$d_{in}^{[I]} = \frac{b^2}{4} - b^2 a + a^2$$

Region II:



$$d^{\text{II}} = f(M) = f(x_p - 1, y_p - 1/2)$$

$$d^{\text{III}} = b^2(x_p - 1)^2 + a^2(y_p - 1/2)^2 - a^2 b^2$$

Selection is \textcircled{N}

$$d_{\text{new}}^{\text{II}} = f(M_{\textcircled{N}}) = f(x_p - 2, y_p - 1/2)$$

$$d_{\text{new}}^{\text{II}} = b^2(x_p - 2)^2 + a^2(y_p - 1/2)^2 - a^2 b^2$$

$$\Delta W^{\text{II}} = d_{\text{new}}^{\text{II}} - d^{\text{III}} = 3b^2 - 2b^2 x_p$$

$$\boxed{\Delta W^{\text{II}} = 2b^2(3 - 2x_p)}$$

Selection is \textcircled{SN}

$$d_{\text{new}}^{\text{II}} = f(M_{\textcircled{SN}}) = f(x_p - 2, y_p - 3/2)$$

$$d_{\text{new}}^{\text{II}} = b^2(x_p - 2)^2 + a^2(y_p - 3/2)^2 - a^2 b^2$$

$$\Delta SW^{\text{II}} = d_{\text{new}}^{\text{II}} - d^{\text{III}}$$

$$\boxed{\Delta SW^{\text{II}} = b^2(3 - 2x_p) + 2a^2(1 - y_p)}$$

$$\frac{\partial f}{\partial x} = 2b^2(x_p - 1/2)$$

$$\frac{\partial f}{\partial y} = a^2(y_p - 1)$$

II Derivative :

Prerequisites \rightarrow 1st derivative

Region I :

At (x_p, y_p) : 1st differential are:

$$\Delta S = a^2(3 - 2y_p)$$

$$\Delta SW = a^2(3 - 2y_p) + b^2(2 - 2x_p)$$

Selection is (S) :

At (S) : $(x_p, y_p - 1)$

$$\Delta S_{\text{new}} = a^2(3 - 2(y_p - 1))$$

$$\Delta S_{\text{new}} = a^2(3 - 2y_p + 2)$$

$$\Delta S_{\text{new}} = a^2(5 - 2y_p)$$

$$\Delta SW_{\text{new}} = a^2(3 - 2(y_p - 1)) \\ + b^2(2 - 2x_p)$$

$$\Delta SW_{\text{new}} = a^2(5 - 2y_p) + b^2(2 - 2x_p)$$

$$\Delta^2 S_{\text{①}} = \Delta S_{\text{new}} - \Delta S = 2a^2$$

$$\Delta^2 SW_{\text{①}} = \Delta SW_{\text{new}} - \Delta SW = 2a^2$$

Selection is SW.

At SW: $(x_p - 1, y_p - 1)$

$$\Delta S_{\text{new}} = a^2(3 - 2(y_p - 1))$$

$$\Delta S_{\text{new}} = a^2(5 - 2y_p)$$

$$\Delta S_{\text{SW new}} = a^2(3 - 2(y_p - 1)) +$$

$$b^2(2 - 2(x_p - 1))$$

$$\Delta S_{\text{SW new}} = a^2(5 - 2y_p) + b^2(4 - 2x_p)$$

$$\Delta^2 S_{\text{(I)}}^{\text{SW}} = \Delta S_{\text{new}} - \Delta S = 2a^2$$

$$\Delta^2 S_{\text{(I)}}^{\text{SW}} = \Delta S_{\text{SW new}} - \Delta S_{\text{new}} = \frac{2a^2}{+ 2b^2}$$

Region II:At (x_p, y_p) : 1st differential are:

$$\Delta W = b^2(3 - 2x_p)$$

$$\Delta SW = b^2(3 - 2x_p) + 2a^2(1 - y_p)$$

Selection is \textcircled{W} :At \textcircled{W} : $(x_p - 1, y_p)$

$$\Delta W_{\text{new}} = b^2(3 - 2(x_p - 1))$$

$$\Delta W_{\text{new}} = b^2(5 - 2x_p)$$

$$\Delta SW_{\text{new}} = b^2(3 - 2(x_p - 1)) + 2a^2(1 - y_p)$$

$$\Delta SW_{\text{new}} = b^2(5 - 2x_p) + 2a^2(1 - y_p)$$

$$\Delta^2 W_{\text{II}}^{\textcircled{W}} = \cancel{\Delta W_{\text{new}}} - \Delta W$$

$$\Delta^2 W_{\text{II}}^{\textcircled{W}} = 2b^2$$

$$\boxed{\Delta^2 SW_{\text{II}}^{\textcircled{W}} = \Delta SW_{\text{new}} - \Delta SW = 2b^2}$$

Selection is \textcircled{SW} :At \textcircled{SW} : $(x_p - 1, y_p - 1)$

$$\Delta W_{\text{new}} = b^2(3 - 2(x_p - 1)) = b^2(5 - 2x_p)$$

$$\Delta SW_{\text{new}} = b^2(3 - 2(x_p - 1)) + 2a^2(2 - y_p + 1)$$

$$\Delta SW_{\text{new}} = b^2(5 - 2x_p) + 2a^2(2 - y_p)$$

$$\Delta^2 W_{\text{II}}^{\text{(sw)}} = \Delta W_{\text{new}} - \Delta W = 2b^2$$

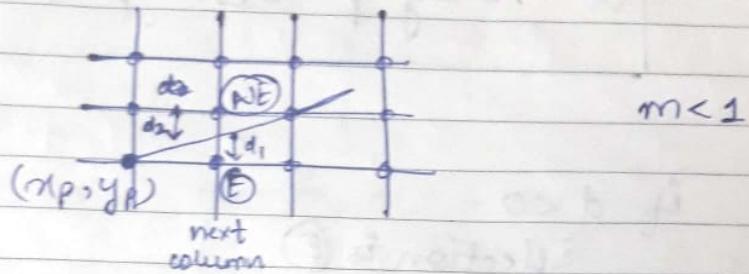
$$\Delta^2 SW_{\text{II}}^{\text{(sw)}} = \Delta SW_{\text{new}} - \Delta SW = 2b^2 + 2a^2$$

- Difference in measurable qty. for a primitive should either be +ve or -ve

Bresenham's Approach

- Find a measurable qty
- Difference in measurable qty for decision making.

LINE:



$$\text{At } (x_p, y_p): \quad y_p = mx_p + c$$

- For every primitive, there is a unique mathematical equation & ^{unique} measurable quantity.
- Identify a measurable qty. for every primitive
- Here, measurable qty is a distance.

$$\text{At } (x_p + 1): \quad y_p = m(x_p + 1) + c$$

$$\begin{aligned} d_1 &= y - y_p && \rightarrow ① \\ d_2 &= (y_p + 1) - y && \rightarrow ② \end{aligned}$$

$$d = d_1 - d_2 = 2y - y_p - y_p - 1$$

$$\boxed{d = 2[m(x_p + 1) + c] - 2y_p - 1}$$

↳ floating pt. computation
(because of m)

$$\underbrace{d \cdot \Delta x}_{\text{d}} = 2[\Delta y(x_p + 1) + c \cdot \Delta x] - 2y_p \Delta x - \Delta x$$

$$d = 2\Delta y x_p + 2\Delta y - 2y_p \Delta x - \Delta x + 2c \Delta x$$

$$d = 2\Delta y x_p - 2\Delta x y_p + [2\Delta y - \Delta x - 2c \cdot \Delta x] \text{ constant}$$

$$\boxed{d = 2\Delta y x_p - 2\Delta x y_p + C} \quad \rightarrow (3)$$

if $d < 0$
 Selection is E

$$d_{\text{new}} = 2\Delta y (x_p + 1) - 2\Delta x y_p + C$$

$$\boxed{\Delta E = d_{\text{new}} - d = 2\Delta y}$$

else
 Selection is NE

$$d_{\text{new}} = 2\Delta y (x_p + 1) - 2\Delta x (y_p + 1) + C$$

$$\boxed{\Delta NE = d_{\text{new}} - d = 2(\Delta y - \Delta x)}.$$

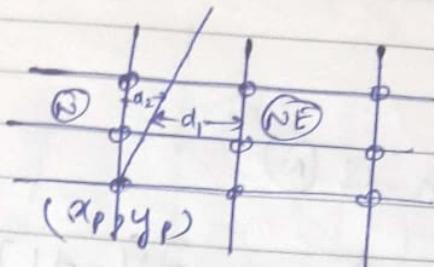
For d_{in} :

$$d = 2[m(x_p + 1) + c] - 2y_p - 1$$

$$d = 2\Delta x [m x_p - y_p^0 + c] + (2\Delta y - \Delta x)$$

$$\boxed{d_{in} = 2\Delta y - \Delta x}$$

For $m > 1$:



$$d_1 = -(y_p + 1) - c$$

$$d_2 = x_p - x - c$$

$$\text{At } (y_p+1) \rightarrow x = \frac{1}{m}[(y_p+1) - c] \quad y = mx + c$$

$$\frac{y_p - c}{m} = x_p \quad (\text{At } x_p, y_p)$$

$$d = d_1 - d_2 = y_p + 1 - x - x + x_p$$

$$d = 2x_p - 2x + 1$$

$$d = 2x_p - 2\left[\frac{1}{m}(y_p + 1) - c\right] + 1$$

$$d \cdot \Delta y = 2x_p \Delta y - 2 \cdot [(y_p + 1) \Delta x - c \Delta x]$$

$$d = 2x_p \Delta y - 2y_p \Delta x + [2c \Delta x - 2(y_p + 1) \Delta x + \Delta y]$$

$$[d = 2x_p \Delta y - 2y_p \Delta x + c]$$

if $d < 0$

selection is **NE**

$$d_{\text{new}} = 2(y_p + 1) \Delta y - 2(y_p + 1) \Delta x + c$$

$$\Delta NE = d_{new} - d = 2(\Delta y - \Delta x)$$

else

selection is N

$$d_{new} = 2x_p \Delta y - 2(y_p + 1) \Delta x + c$$

$$\Delta N = d_{new} - d = -2 \Delta x$$

for d_{in} : $\Delta y - 2 \Delta x$

$$d \Delta y = 2x_p \Delta y - 2y_p \Delta x - 2 \Delta x + 2c + \Delta N + \Delta y$$

$$d = 2[x_p - y_p]$$

calculate d_{in} .

$$\begin{aligned} d_i &= 2x_p + 1 - 2x \\ &= 2x_p + 1 - 2\left[\frac{1}{m}(y_p + 1) - c\right] \\ &= 1 + 2\left[x_p - \frac{1}{m}y_p + c - \frac{1}{m}\right] \end{aligned}$$

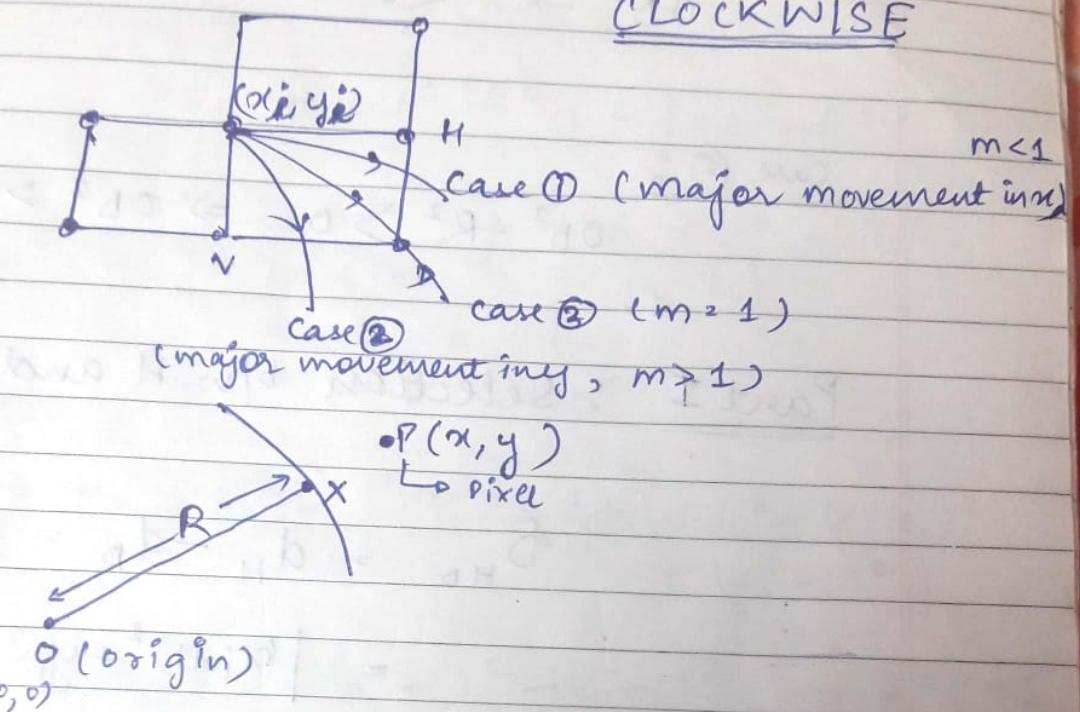
$$= 1 - \frac{2}{m} = \frac{1 - 2 \Delta x}{\Delta y}$$

$$d_{in} = \Delta y - 2 \Delta x$$

16/03/2018

CIRCLE: $(x^2 + y^2 = R^2)$

CLOCKWISE



$$d_p = \sqrt{OP^2 - OX^2}$$

$$d_p = \sqrt{(x^2 + y^2) - R^2}$$

Measurable qty for a circle.

$$d_H = \sqrt{(x_i + 1)^2 + y_i^2 - R^2}$$

$$d_D = \sqrt{(x_i + 1)^2 + (y_i - 1)^2 - R^2}$$

$$d_V = \sqrt{x_i^2 + (y_i - 1)^2 - R^2}$$

Case ①:

$$OD^2 - R^2 < 0 \Rightarrow OD^2 < R^2$$

Case ②:

$$OD^2 - R^2 > 0 \Rightarrow OD^2 > R^2$$

Part I : Selection b/w H and D

$$\begin{aligned}\delta_{HD} &= d_H - d_D = |OH^2 - R^2| - |OD^2 - R^2| \\ &= |(x_i + 1)^2 + y_i^2 - R^2| - \\ &\quad |(x_i + 1)^2 + (y_i - 1)^2 - R^2|\end{aligned}$$

$$\therefore OD^2 < R^2$$

$$OH^2 > R^2$$

$$\delta_{HD} = (x_i + 1)^2 + y_i^2 - R^2 - [R^2 - (x_i + 1)^2 - (y_i - 1)^2]$$

$$\delta_{HD} = 2 \underbrace{[(x_i + 1)^2 + (y_i - 1)^2 - R^2]}_{\Delta D_i} + 2y_i - 1$$

$$\delta_{HD} = \Delta D_i + 2y_i - 1$$

Part II: Selection b/w V & D

my companion

$$\delta_{VD} = d_V - d_D$$

$$= |OV^2 - R^2| - |OD^2 - R^2|$$

$$\therefore OD^2 > R^2 \text{ and } OV^2 < R^2$$

$$\delta_{VD} = 2R^2 - OV^2 - OD^2$$

$$= 2R^2 - [(x_i^2 + (y_i - 1)^2 + (x_i + 1)^2 + (y_i - 1)^2)]$$

$$= 2[R^2 - (x_i + 1)^2 + (y_i - 1)^2] + 2x_i + 1$$

(1) Pixel H is chosen

$$x_{i+1} \rightarrow x_i + 1$$

$$y_{i+1} \rightarrow y_i$$

$$\Delta D_i^o = (x_i + 1)^2 + (y_i - 1)^2 - R^2 \quad \textcircled{1}$$

$$\Delta D_{i+1}^o = (x_i + 2)^2 + (y_i - 1)^2 - R^2 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1}$$

$$\boxed{\Delta D_{i+1}^o = \Delta D_i^o + 2x_i + 3}$$

(2) Pixel V is chosen

$$\begin{aligned}x_{i+1}^{\circ} &\rightarrow x_i^{\circ} \\y_{i+1}^{\circ} &\rightarrow y_i^{\circ} - 1\end{aligned}$$

$$\Delta D_{i+1} = (x_i^{\circ} + 1)^2 + (y_i^{\circ} - 2)^2 - R^2 \quad \text{---(3)}$$

$$(3) - (1)$$

$$\boxed{\Delta D_{i+1} = \Delta D_i - 2y_i + 3}$$

(3) Pixel D is chosen

$$\begin{aligned}x_{i+1}^{\circ} &\rightarrow x_i^{\circ} + 1 \\y_{i+1}^{\circ} &\rightarrow y_i^{\circ} - 1\end{aligned}$$

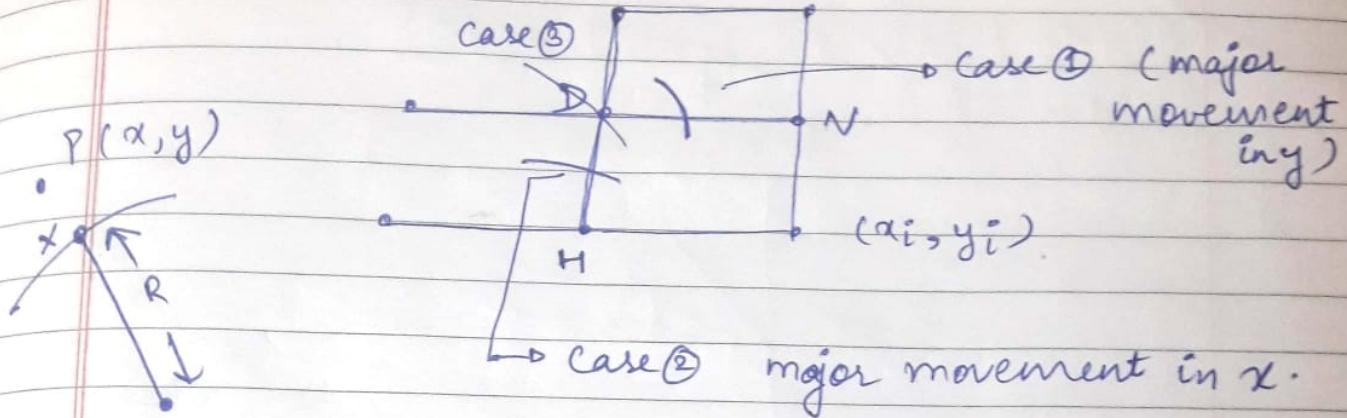
$$\Delta D_{i+1} = (x_i^{\circ} + 2)^2 + (y_i^{\circ} - 2)^2 - R^2$$

$$\boxed{\Delta D_{i+1} = \Delta D_i + 2(x_i^{\circ} - y_i^{\circ}) + 6}$$

$$\Delta D = 2(1 - R)$$

(Starting from
0, R)

ANTI CLOCKWISE: (Starting from R, 0)



$$d_P = |O_P^2 - O_X^2|$$

$$d_P = |\alpha^2 + y^2 - R^2|$$

$$d_V = |x_i^2 + (y_i + 1)^2 - R^2|$$

$$d_D = |(x_i - 1)^2 + (y_i + 1)^2 - R^2|$$

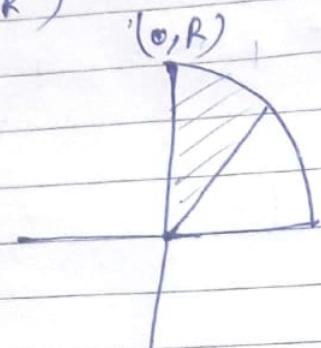
$$d_H = |(x_i - 1)^2 + y_i^2 - R^2|$$

↓
complete

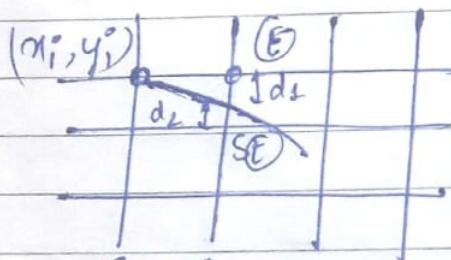
18/01/2018

Bresenham's Approach

Circle (with 1st quadrant):
 $(x^2 + y^2 = R^2)$



1st octant: major movement in x.



1st derivative:

for column position $(x_i + 1)$:

$$d_1 = y_p^2 - y^2 \quad (\text{w.r.t } \theta)$$

$$d_2 = y^2 - (y_{p+1})^2 \quad (\text{w.r.t. } \theta)$$

$$y^2 = R^2 - (x_p + 1)^2$$

$D = d_1 - d_2 \rightarrow$ Taking diff. in measurable qty.

$$D = y_p^2 - y^2 - y^2 + (y_p - 1)^2$$

$$D = y_p^2 - 2y^2 + y_p^2 + 1 - 2y_p$$

$$\boxed{D = \frac{2y_p^2 - 2y^2 - 2y_p + 1}{2y_p^2 - 2(R^2 - (x_p + 1)^2) - 2y_p + 1}}$$

If $D < 0$
Selection is (E)

$$x_{p+1} \rightarrow x_p + 1$$

$$y_{p+1} \rightarrow y_p$$

$$D_{\text{new}} = 2y_p^2 - 2(R^2 - (x_p + 2)^2) \div 2y_p + 1$$

$$D_{\text{new}} = 2y_p^2 - 2R^2$$

$$\Delta E = D_{\text{new}} - D = 2(R^2 - (x_p + 2)^2) \\ - 2(R^2 - (x_p + 1)^2)$$

$$\Delta E = 2[R^2 - (x_p + 2)^2 - R^2 + (x_p + 1)^2]$$

$$\Delta E = 2[-3 - 2x_p] = -2[3 + 2x_p]$$

$$\boxed{\Delta E = 2[2x_p + 3]}$$

else if $\Delta \geq 0$
Selection is (SE)

$$x_{p+1} \rightarrow x_p + 1$$

$$y_{p+1} \rightarrow y_p - 1$$

$$\begin{aligned} D_{\text{new}} = & 2(y_p - 1)^2 - 2(R^2 - x_p + 2)^2 \\ & - 2(y_p - 1) + 1 \end{aligned}$$

$$\Delta SE = D_{\text{new}} - D$$

$$\boxed{\Delta SE = 2x_p^2 - 2(2x_p + 3) - 4(y_p - 1)}$$

For D_{in}

$$D_{(0,R)} = 2R^2 - 2(R^2 - 1) - 2R + 1$$

$$\boxed{D_{in} = 2R - 3 - 2R}$$

2nd Derivative:

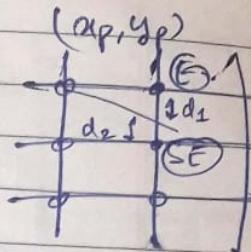
$$\Delta E = 2(2x_p + 3)$$

$$\Delta SE = 2(2x_p + 3) - 4(y_p - 1)$$

(P, R)

$$\rightarrow \Delta E^i = 6$$

$$\rightarrow \Delta SE^i = 10 - \text{[redacted]} \quad 10 - 4R$$



If $D < 0$
selection is \textcircled{E}

$$x_{p+1} \rightarrow x_p + 1$$

$$y_{p+1} \rightarrow y_p$$

$$\Delta E_{\text{new}} = 2(2x_p + 5)$$

$$\Delta SE_{\text{new}} = 2(2x_p + 5) - 4(y_p - 1)$$

$$\Delta^2 E_{\textcircled{E}} = \Delta E_{\text{new}} - \Delta E$$

$$\boxed{\Delta^2 E_{\textcircled{E}} = 4}$$

$$\boxed{\Delta^2 SE_{\textcircled{E}} = \Delta SE_{\text{new}} - \Delta SE = 4}$$

if $D >= 0$

selection is (SE)

$$x_{p+1} \rightarrow x_p + 1$$

$$y_{p-1} \rightarrow y_p - 1$$

$$\Delta E_{\text{new}} = 2(2x_p + 5)$$

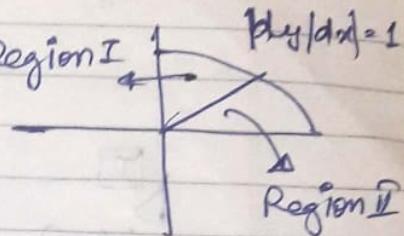
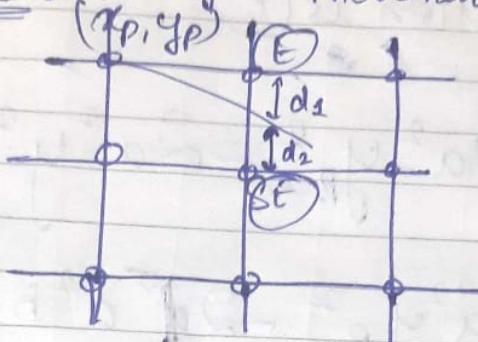
$$\Delta S_{\text{E}}_{\text{new}} = 2(2x_p + 5) - 4(y_p - 2)$$

$$\boxed{\Delta^2 E_{\text{(SE)}} = \Delta E_{\text{new}} - \Delta E = 4}$$

$$\boxed{\Delta^2 S_{\text{E}}_{\text{(SF)}} = \Delta S_{\text{E}}_{\text{new}} - \Delta S_{\text{E}} = 8}$$

Ellipses: $(b^2x^2 + a^2y^2 = a^2b^2)$

Region I: (major movement in x)



$$d_1 = a^2 y_p^2 - a^2 y^2$$

$$d_2 = a^2 y^2 - a^2 (y_p - 1)^2$$

$$a^2 y^2 = a^2 b^2 - b^2 (x_p + 1)^2$$

$$D^{\boxed{I}} = d_1 - d_2$$

$$D^{\boxed{I}} = -2a^2 y^2 + 2a^2 y_p^2 - a^2 + 2y_p a^2$$

$$D^{\boxed{I}} = 2a^2 y_p^2 + 2y_p a^2 - a^2 \cancel{- 2a^2 y^2}$$

$$- 2(a^2 b^2 - b^2 (x_p + 1)^2)$$

$$D^{\boxed{II}} = a^2 (2y_p^2 + 2y_p)$$

$$D^{\boxed{II}} = 2a^2 y_p^2 - 2a^2 y^2 + 2y_p a^2 - a^2$$

if $D < 0$

selection is \textcircled{E}

$$D_{\text{new}}^{\text{I}} = 2a^2 y_p^2 - 2a^2 y^2 + 2y_p a^2 - a^2$$

$$D_{\text{new}}^{\text{II}} = 2a^2 y_p^2 - 2a^2 b^2 - b^2 (x_p+2)^2 \\ + 2y_p a^2 - a^2$$

$$\Delta E^{\text{I}} = D_{\text{new}}^{\text{II}} - D_{\text{new}}^{\text{I}}$$

$$\Delta E^{\text{I}} = -2[a^2 b^2 - b^2 (x_p+2)^2] \\ + 2[a^2 b^2 - b^2 (x_p+1)^2]$$

$$\Delta E^{\text{I}} = 2b^2 (x_p+2)^2 - 2b^2 (x_p+1)^2$$

$$\Delta E^{\text{II}} = 2b^2 (x_p^2 + 4 + 4x_p - x_p^2 - 2x_p - 1) \\ \boxed{\Delta E^{\text{II}} = 2b^2 (3 + 2x_p)}$$

else if $D \geq 0$

Selection is \textcircled{SE}

$$D_{\text{new}}^{\square} = 2a^2y_p^2 - 2a^2y^2 + 2y_p a^2 - a^2$$

$$D_{\text{new}}^{\square} = 2a^2(y_p^2) - 2[a^2b^2 - b^2(x_p+2)^2]$$

$$+ 2y_p a^2 - a^2$$

$$\Delta SE^{\square} = D_{\text{new}}^{\square} - D^{\square}$$

$$\Delta SE^{\square} = \frac{4a^2y_p^2}{-2[a^2b^2 - b^2(x_p+2)^2]}$$

$$+ 2[a^2b^2 - b^2(x_p+1)^2]$$

$$\Delta SE^{\square} = 2b^2[(x_p+2)^2 - (x_p+1)^2] + 4a^2y_p^2$$

$$\boxed{\Delta SE^{\square} = 2b^2[3 + 2x_p] + 4a^2[1 - y_p]} \quad \cancel{+ 4a^2y_p^2}$$

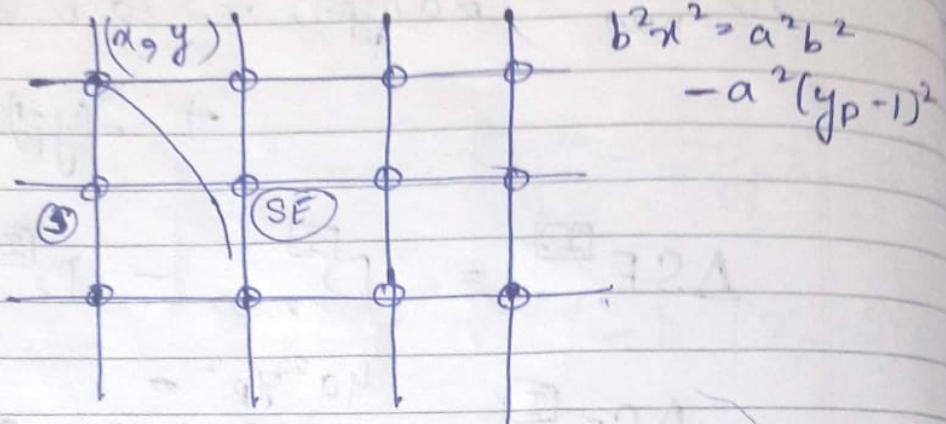
For D_{in}^{\square} :

Starting from (0, b):

$$D_{\text{in}}^{\square} = 2a^2b^2 + 2ba^2 - a^2 - 2(a^2b^2 - b^2)$$

$$\Rightarrow \boxed{D_{\text{in}}^{\square} = 2ba^2 - a^2 + 2b^2}$$

Region II: (major movement in y)



$$b^2 x^2 = a^2 b^2$$

$$-a^2 (y_p - 1)^2$$

note

$$d_1 = -b^2 x_p^2 + b^2 x^2$$

$$d_2 = -b^2 x^2 + b^2 (x_p + 1)^2$$

$$D^{(II)} = d_1 - d_2$$

$$D^{(IV)} = -b^2 x_p^2 - b^2 (y_p + 1)^2 + 2b^2 x^2$$

$$D^{(III)} = 2b^2 x^2 + b^2 [(x_p + 1)^2 - x_p^2]$$

$$D^{(II)} = 2[a^2 b^2 - a^2 (y_p - 1)^2] - b^2 [(x_p + 1)^2 + (x_p)^2]$$

if $D < 0$, selection $\overset{is}{\wedge} S$

$$x_{p+1} \rightarrow x_p$$

$$y_{p+1} \rightarrow y_{p-1}$$

$$D_{new}^{(I)} = 2[a^2 b^2 - a^2 (y_p - \delta)^2] - b^2 [(x_p + 1)^2 + (x_p)^2]$$

$$\Delta S^{(II)} = D_{new}^{(I)} - D^{(II)}$$

$$\Delta S^{\text{II}} = 2a^2(2y_p - 3)$$

if $D > 0$, selection is SE

$$x_{p+1} \rightarrow x_p + 1$$

~~$y_{p+1} \rightarrow y_p - 1$~~

$$D_{\text{new}}^{\text{II}} = 2[a^2 b^2 - a^2(y_p - 2)^2] - b^2 [(x_{p+2})^2 + (x_{p+1})^2]$$

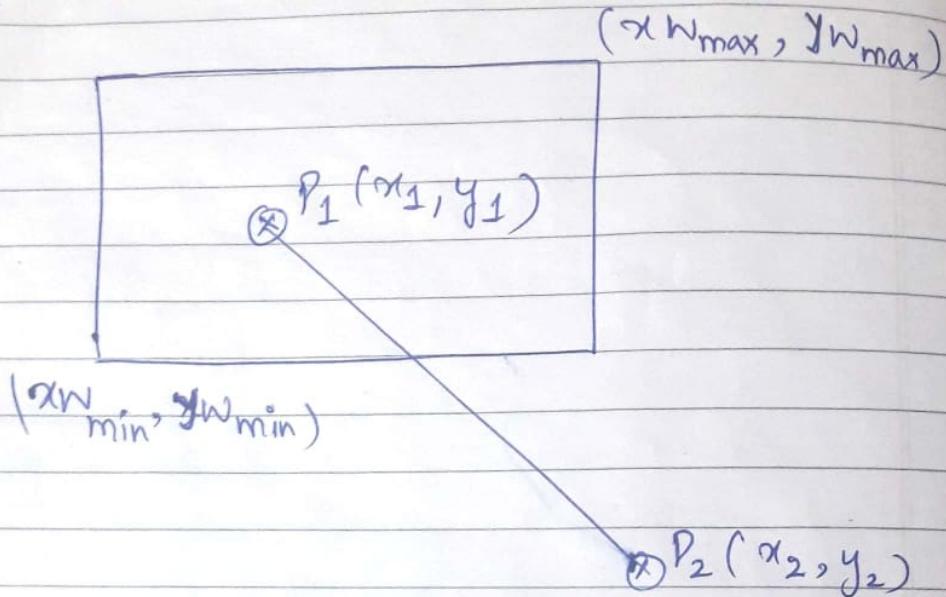
$$\Delta S_E^{\text{II}} = D_{\text{new}}^{\text{II}} - D^{\text{III}}$$

$$\Delta S_E^{\text{II}} = 2a^2(2y_p - 3) - b^2(2(2x_{p+2}))$$

$$\Delta S_E^{\text{II}} = 2a^2(2y_p - 3) - 4b^2(x_p + 1)$$

CLIPPING

Rectangular window,



$$\min \leq x \leq \max$$

$$\min \leq y \leq \max$$

{ int inside ()

Accordingly

return (0) or (1)

}

// depending on whether it lies inside or outside.

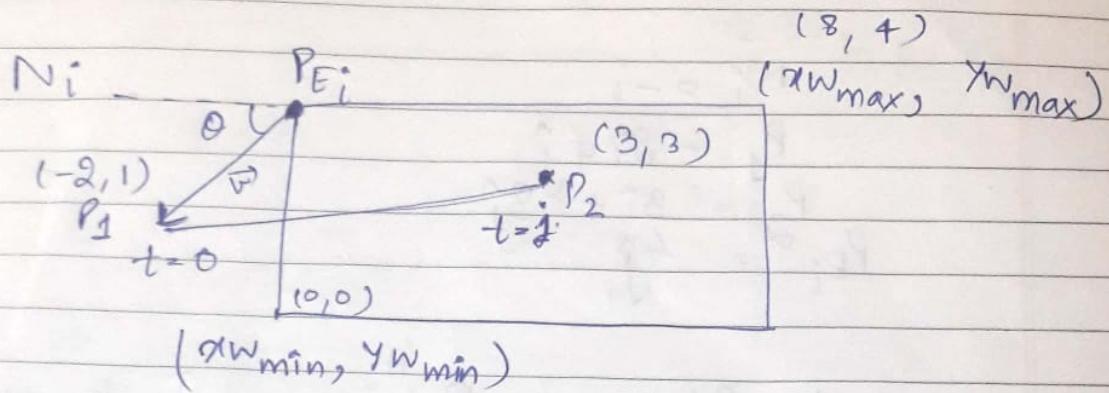
0 0 (Both pts. out)

0 1

1 0

1 1 Both pts. in (no clipping reqd.)
line retained.

Cyrus - Beck Method:



$$\text{Parametric eqn: } P(t) = P_1 + (P_2 - P_1)t \quad \text{--- (1)}$$

Drop a vector from an edge & let it swing like a pendulum.

$$\vec{V} = [P(t) - P(E_i)]$$

$\vec{V} \cdot \vec{N}_i$ $\begin{cases} < 0 & (\text{pt. is inside, } \theta > 90^\circ) \\ = 0 & (\text{pt. is on edge}) \\ > 0 & (\text{pt. is outside, } \theta < 90^\circ) \end{cases}$

$$\vec{V} \cdot \vec{N}_i = 0 \quad (\text{At intersection})$$

$$[P(t) - P(E_i)] \cdot \vec{N}_i = 0$$

$$[P_1 + (P_2 - P_1)t - P_{Ei}] \cdot \vec{N}_i = 0$$

$$N_i \cdot (P_1 - P_{Ei}) + \vec{N}_i \cdot (P_2 - P_1)t = 0$$

$$\boxed{t = \frac{N_i \cdot (P_1 - P_{Ei})}{\vec{N}_i \cdot (P_2 - P_1)}}$$

left Edge :

(B)
my companion

$$N_1^o = -\hat{i}$$

$$P_1 = 2\hat{i} + \hat{j}$$

$$P_{Ei}^2 = \frac{3}{4}\hat{i} + \frac{3}{4}\hat{j}$$

$t = 2/5 \rightarrow$ Put it in egn ① to get the intersection pt

$- N_1^o \cdot (P_2 - P_1)$

$\begin{cases} > 0 & \rightarrow \text{is potentially exiting} \\ < 0 & \rightarrow \text{is potentially entering} \end{cases}$

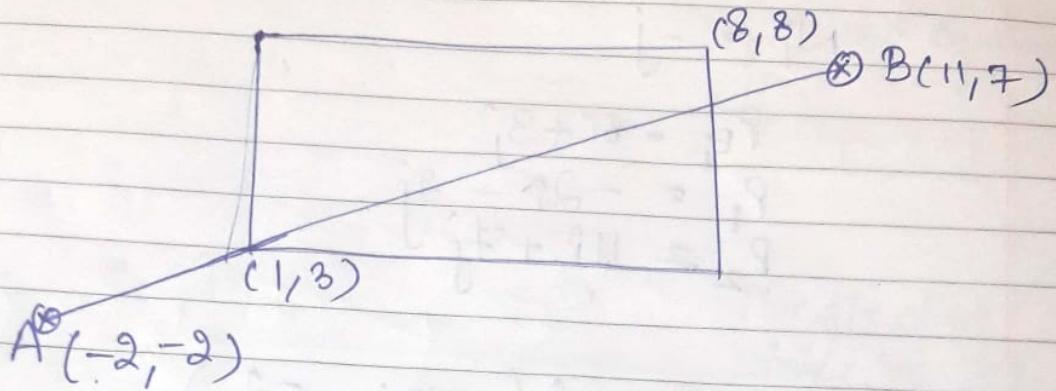
If

$$0 \leq t \leq 1$$

then only we have to find the intersection pt

Otherwise, the edge & the line ~~do not~~ do not intersect. So, there is no need to find the intersection pt.

Example 1:



Left Edge:

~~top~~

$$N_L = -\hat{i}$$

$$P_{E_L} = \vec{AB}(1, 8) = \hat{i} + 8\hat{j}$$

$$P_1 = -2\hat{i} - 2\hat{j}$$

$$P_2 = 11\hat{i} + 7\hat{j}$$

$$t_L = \frac{-\hat{i} \cdot (-2\hat{i} - 2\hat{j}) - \hat{i} \cdot 8\hat{j}}{\hat{i} \cdot (11\hat{i} + 7\hat{j} + 2\hat{i} + 2\hat{j})}$$

$$t_L = \frac{-\hat{i}(-3\hat{i} - 10\hat{j})}{\hat{i}(13\hat{i} + 9\hat{j})}$$

$$t_L = \frac{3 \cancel{-} 0}{13 + 0} = \boxed{\frac{3}{13}}$$

Bottom Edge:

my companion

$$N_i = -j$$

$$P_{E_i} = 8i + 3j$$

$$P_1 = -2i - 2j$$

$$P_2 = 11i + 7j$$

$$t_B = \frac{-j \cdot (-10i - 5j)}{j \cdot (13i + 9j)}$$

$$\boxed{t_B = 5/9}$$

Right Edge:

$$N_i = +i$$

$$P_{E_i} = 8j + 8i, P_1 = -2i - 2j, P_2 = 11i + 7j$$

$$t_R = \frac{i \cdot (-10i - 10j)}{-j \cdot (13i + 9j)}$$

$$t_R = \frac{-10}{-13} = \boxed{\frac{10}{13}}$$

Up Edge:

$$N_i = +j$$

$$P_{E_i} = 8i + 8j, P_1 = -2i - 2j, P_2 = 11i + 7j$$

$$t_T = \frac{j \cdot (-10i - 10j)}{-j \cdot (13i + 9j)} = \frac{-10}{-9} = \boxed{10/9}$$

$$t_L = \frac{3}{13}$$

Denominator values

> 0

$$t_B = \frac{5}{9}$$

> 0

$$t_R = \frac{10}{13}$$

< 0

$$t_T = \frac{10}{9}$$

< 0

< 0

} Max of those two

Neglected
since $t \geq 1$.

$$\max(t_L, t_B) = t_B$$

clipped line: line (t_B, t_R)

Potentially entering $\rightarrow \max(D, > 0)$

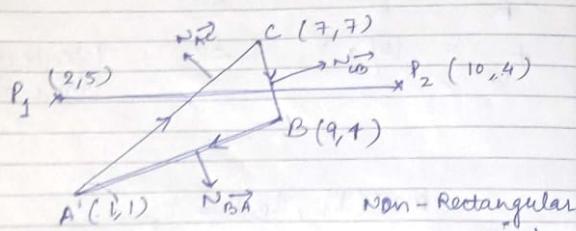
Potentially exiting $\rightarrow \min(D, < 0)$

23/01/2018

Line Clipping

Cyrus - Beck :

Example 3 :



This window is convex
as all the angles < 180°

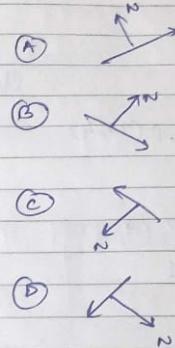
Can be convex
or concave

We can traverse the window either in Q_1 or Q_3 dir.
When \rightarrow RHS is always towards the window, LHS \rightarrow outside.

Edges	Normal (N _i)	P _{Ei}	P ₁ - P _{Ei}	N _i · (P ₁ - P _{Ei})	(P ₂ - P ₁) · N _i + t	
\overrightarrow{AC} (6 \hat{i} + 6 \hat{j})	-6 \hat{i} + 6 \hat{j}	$\hat{i} + \hat{j}$	$\hat{i} + 4\hat{j}$	18	-54	$\frac{1}{3}(P_{E\text{ent}})(P_{\text{Exit}})$
\overrightarrow{CB} (2 \hat{i} - 3 \hat{j})	3 \hat{i} + 2 \hat{j}	$7\hat{i} + 7\hat{j}$	-5 \hat{i} - 2 \hat{j}	-19	22	$\frac{19}{22}(P_{E\text{ent}})(P_{\text{Exit}})$
\overrightarrow{BA} (-8 \hat{i} - 3 \hat{j})	3 \hat{i} - 8 \hat{j}	9 $\hat{i} + 4\hat{j}$	-7 $\hat{i} + \hat{j}$	-29	92	$\frac{29}{32}(P_{E\text{ent}})(P_{\text{Exit}})$

$$P_2 - P_1 = 8\hat{i} - \hat{j}$$

Edge	Normal
(A)	- +
(B)	+ +
(C)	--
(D)	+ -

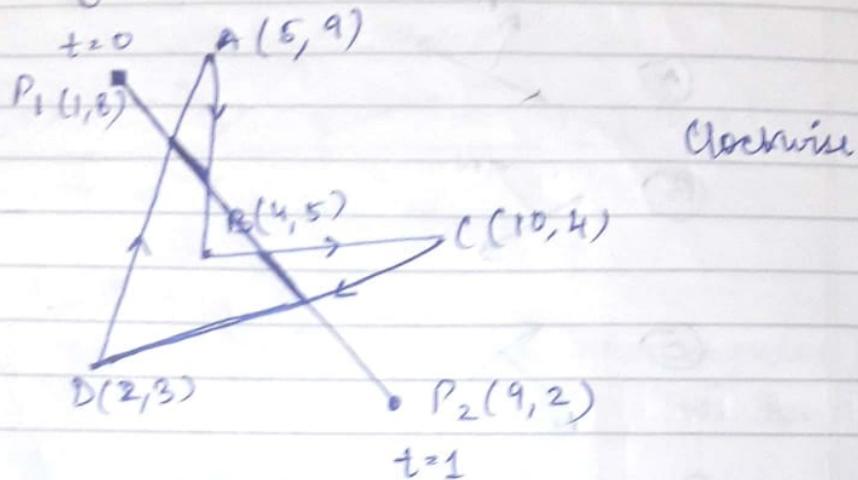


Increasing
order

25/01/2018

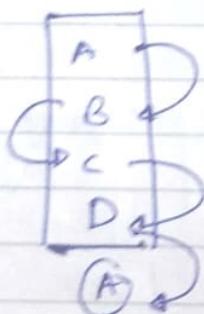
Line Clipping

non-Rectangular window:



Concave window

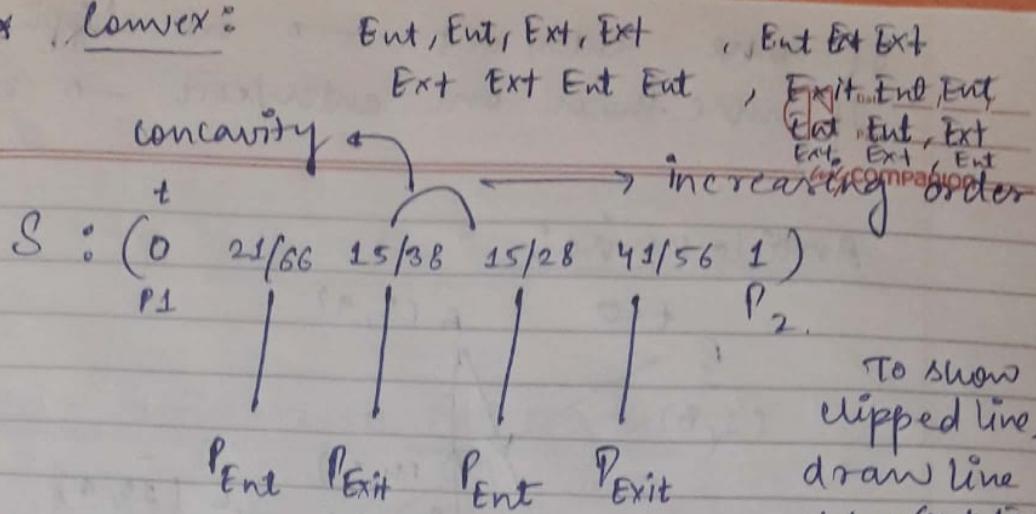
arr[]



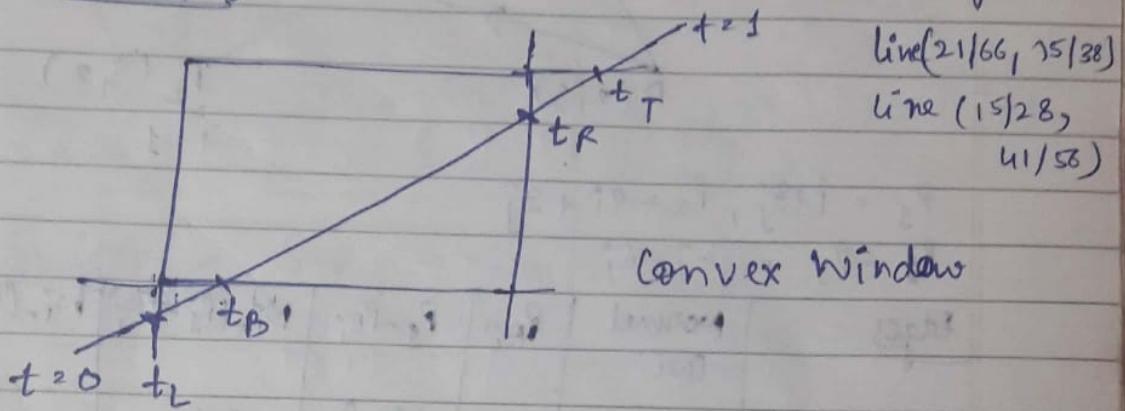
$$\begin{aligned}P_1 &= i + B \hat{j} \\P_2 &= 9\hat{i} + 2\hat{j} \\P_2 - P_1 &= 8\hat{i} - 6\hat{j}\end{aligned}$$

Edge	Normal ($N_i\hat{i}$)	P_{Ei}	$P_1 - P_{Ei}$	$N_i \cdot (P_1 - P_{Ei})$	$N_i \cdot (P_2 - P_1)$	t
$\vec{AB} = -\hat{i} - 4\hat{j}$	$4\hat{i} - \hat{j}$	$8\hat{i} + 9\hat{j}$	$-4\hat{i} - \hat{j}$	-15	88 (P_{Exit})	$15/38$
$\vec{BC} = 6\hat{i} - \hat{j}$	$\hat{i} + 6\hat{j}$	$4\hat{i} + 5\hat{j}$	$2\hat{i} + 3\hat{j}$	15	-28 (P_{End})	$15/28$
$\vec{CD} = -8\hat{i} - \hat{j}$	$\hat{i} - 8\hat{j}$	$10\hat{i} + 4\hat{j}$	$-9\hat{i} + 4\hat{j}$	-41	56 (P_{Exit})	$41/53$
$\vec{DA} = 3\hat{i} + 6\hat{j}$	$-6\hat{i} + 3\hat{j}$	$2\hat{i} + 3\hat{j}$	$\hat{i} + 5\hat{j}$	21	-66 (P_{End})	$21/66$

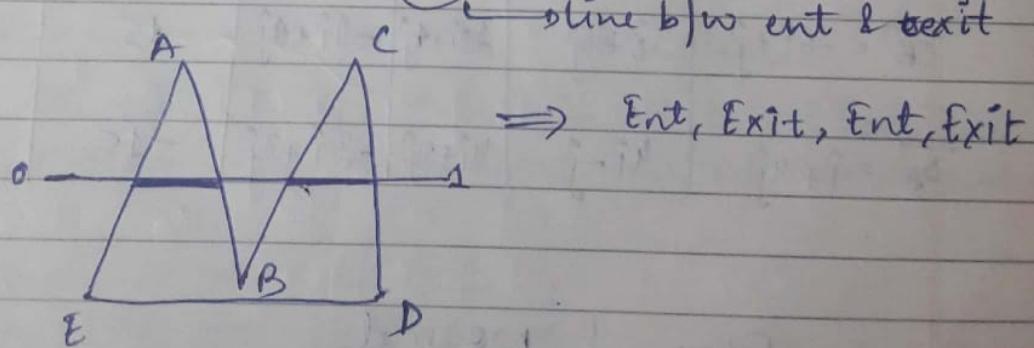
\Rightarrow Convex:



For Convex:

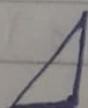


$$S : (0, P_{\text{Ent}}, P_{\text{Exit}}, P_{\text{Ent}}, P_{\text{Exit}}, 1)$$



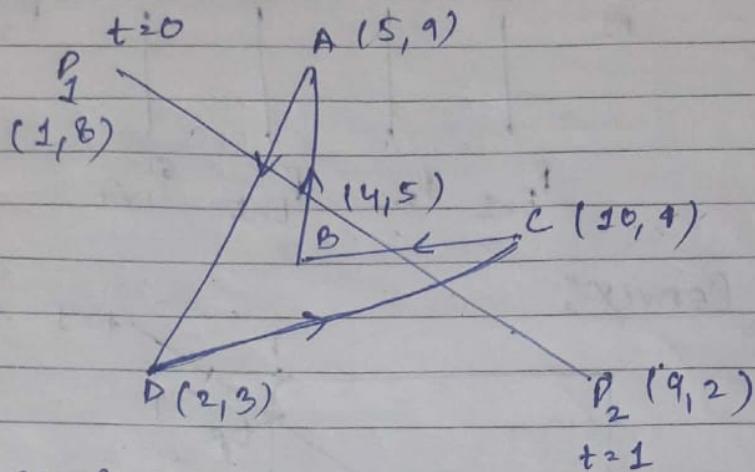
If such pairs (Ent, Exit) are more than 1 b/w $t=0$ & $t=1$. \Rightarrow then clipping in concave part.

$\Rightarrow t < 0$ or $t > 1$



If one enter/exit \rightarrow one clipping
 If more than one enter/exit \rightarrow more than 1 clipping

Anticlockwise.



$$P_1 = i + 8j, P_2 = 9i + 2j$$

$$P_2 - P_1 = 8j - 6i$$

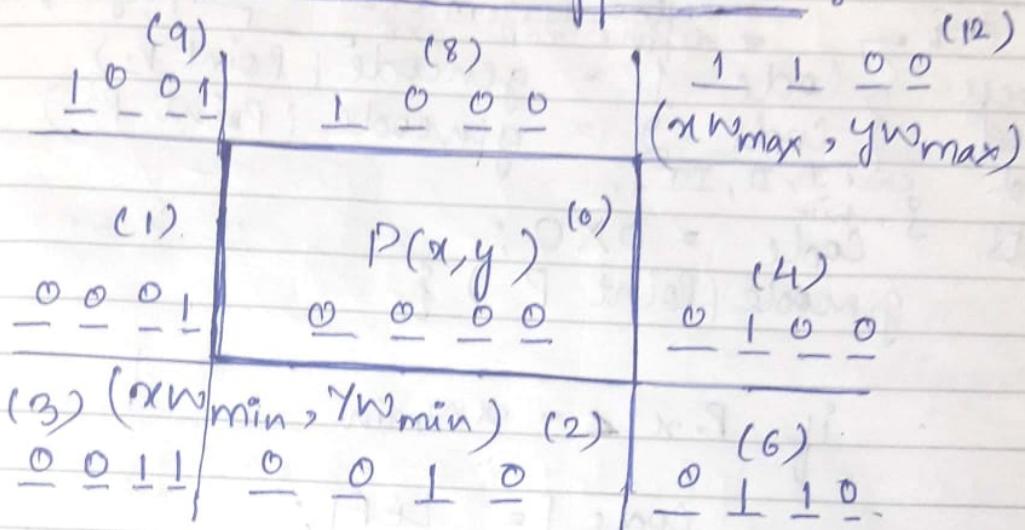
Edges	Normal (N _i)	P _{Ei} :	P ₁ - P _{Ei}	N _i .(P ₁ - P _{Ei})	N _i .(P ₂ - P ₁)	t
$\overrightarrow{AD} = -3i - 6j$	$-6i + 3j$	$5i + 9j$	$-4i - j$	21	$-66(P_{\text{Ent}})$	$21/66$
$\overrightarrow{DC} = 8i + j$	$i - 8j$	$2i + 3j$	$-i + 5j$	-41	$56(P_{\text{Exit}})$	$41/56$
$\overrightarrow{CB} = -6i + j$	$i + 6j$	$10i + 4j$	$-9i + 4j$	15	$-28(P_{\text{Ent}})$	$15/28$
$\overrightarrow{BA} = i + 4j$	$4i - j$	$4i + 5j$	$-3i + 3j$	-15	$38(P_{\text{Exit}})$	$15/38$

Edge	Normal
++	+-
+-	--
-+	++
--	-+

$$S: (0 \downarrow \frac{21}{66} \downarrow \frac{15}{38} \downarrow \frac{15}{28} \downarrow \frac{41}{56})$$

Line Clipping

Cohen - Sutherland Approach :



$$x_{w\min} \leq x \leq x_{w\max}$$

$$y_{w\min} \leq y \leq y_{w\max}$$

A rectangular window has 9 regions.

Point can be anywhere in these 9 regions.
Each region has a unique region code (address).

Left side \rightarrow LSB 1.

Bottom side \rightarrow 2nd bit 1

Right side \rightarrow 3rd bit 1.

Upper side \rightarrow MSB 1

Input: $P_1(x_1, y_1), P_2(x_2, y_2)$

Method:

do {

#define LEFT 0x1
 #define RIGHT 0x4
 #define BOTTOM 0x2
 #define TOP 0x8

oflag = 0; done = 0;

Address assigned to end points }
 case I, II, III;
 while {
 Code = 0x0;
 gencode (Point P) {

if $P.x \leq x_{W\min}$

Code1 = LEFT;

if $P.x \geq x_{W\max}$

Code1 = RIGHT;

if $P.y \leq y_{W\min}$

Code1 = BOTTOM;

if $P.y \geq y_{W\max}$

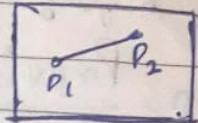
Code1 = TOP;

}

Case I:

$$\text{Code}(P_1) = 0000$$

$$\text{Code}(P_2) = 0000$$



AND

OR



$$\begin{cases} P_1 \text{ AND } P_2 = 0 \\ P_1 \text{ OR } P_2 = 0 \end{cases} \left\{ \begin{array}{l} \text{True} \\ \text{False} \end{array} \right.$$

True \Rightarrow Line is inside, no clipping

Case II:

$$\text{Code}(P_3) = 0001$$

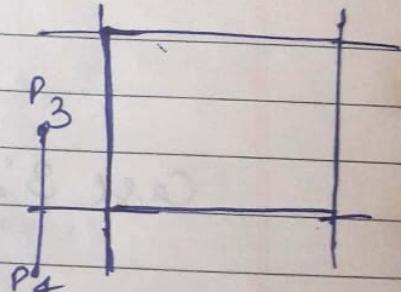
$$\text{Code}(P_4) = 0011$$

$$P_1 \text{ AND } P_2 \neq 0$$

AND



NON-ZERO



\Rightarrow Line is outside

Case III:

$$\text{Code}(P_5) = 0001$$

$$\text{Code}(P_6) = 0010$$

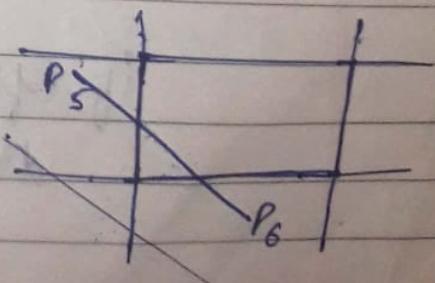


AND $\rightarrow 0$

OR $\rightarrow \text{NON-ZERO}$

$$P_1 \text{ AND } P_2 = 0$$

$$P_1 \text{ OR } P_2 \neq 0$$



For Case III:

First clip. Then

- * Keep checking iteratively whether after clipping line falls under case I or Case II

flag = 0; done = 0;

do { Previous code }

case 1: if (code1 & code2 == 0) &&

(code1 | code2 == 0)

{ /* line is visible */

flag = 1; done = 1;

cxit();

}

case 2: if (code1 & code2 != 0)

{ /* line is not visible */

} cxit(); done = 1;

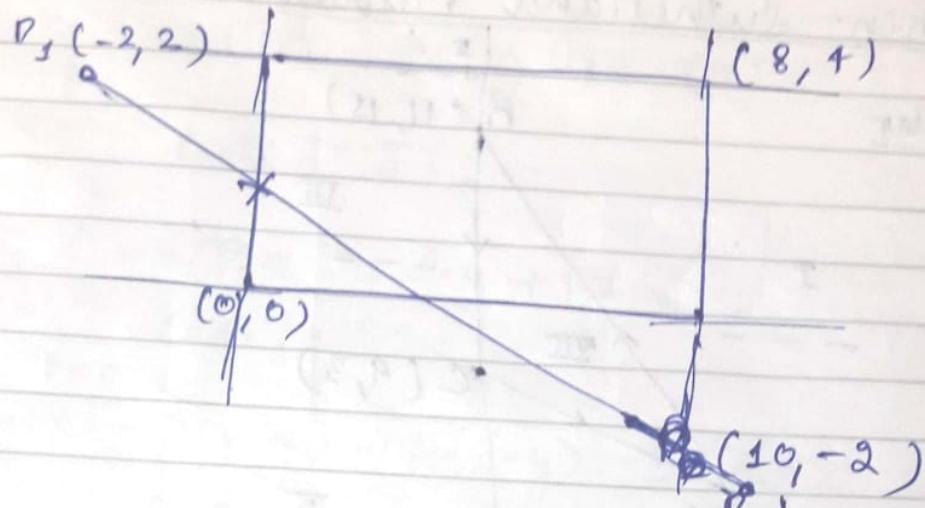
Case 3: if (code1 & code2 == 0) && (code1 | code2 != 0)

find intersection();

} while (done != 1)

if (flag) → // line is inside
drawline(P₁, P₂)

EXAMPLE 1



$$\begin{aligned} \text{Code } (P_1) &= 0001 \\ \text{Code } (P_2) &= 0110 \end{aligned}$$

clipped
at left edge

case (3 LEFT = 0x1;	flag	done
0	0	0

$$\begin{aligned} \text{if } (\text{LEFT} \& \text{code } P_1 == 1) \\ x_I &= x_W \min; \\ y_I &= y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} * (x_I - x_W \min) \end{aligned}$$

$$P_1 \leftarrow (x_I, y_I)$$

$$\text{code } (P_1) = 00000$$

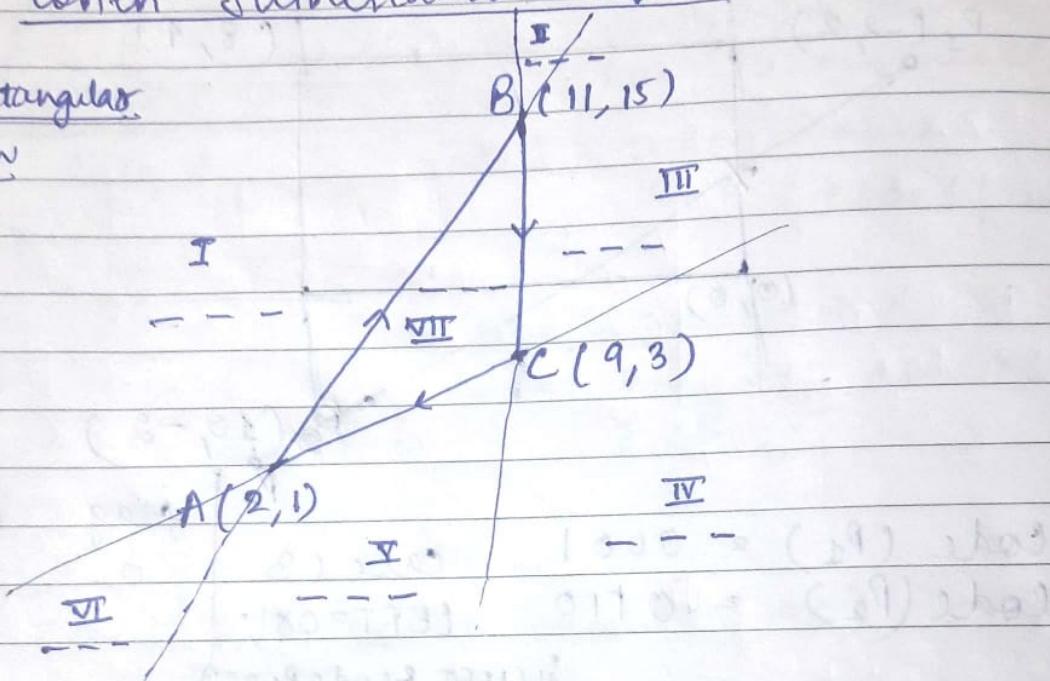
$$\text{code } (P_2) = 0110$$

30/01/2018

Line Clipping

Cohen-Sutherland Method:

Non-rectangular window



Inside Test

Equation

$$AB \\ f(x,y) = 14x - 9y - 19$$

$$f(x,y) = c(9,3)$$

$$> 0$$

$$BC \\ f(x,y) = 6x + y + 51$$

$$f(x,y) = A(2,1)$$

$$\text{---} > 0$$

$$CA \\ f(x,y) = 2x + 7y - 3$$

$$f(x,y) = B(11,15)$$

$$> 0$$

$> 0 \rightarrow$ Point is inside the window.

```
#define I1 1  
#define I2 2  
#define I3 4
```

```
int code = 0;
```

```
int gencode (Point P)
```

```
{
```

```
if ( 14(P.x) - 9(P.y) - 19 <= 0 )
```

```
code |= I1;
```

```
if ( -6(P.x) + P.y + 51 <= 0 )
```

```
code |= I2;
```

```
if ( -2(P.x) + 7(P.y) - 3 <= 0 )
```

```
code |= I3;
```

```
}
```



Date _____ / _____ / _____

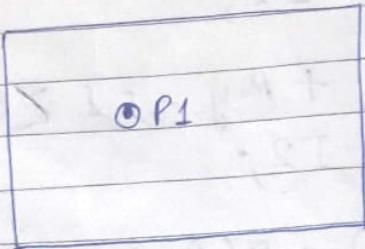
Page _____

mycompanion

01/02/2018

Line Clipping

Nicholl - Lee - Nicholl :



Computation time faster than other algorithms but code length is large

Step 1:

Locate one of the end point of a line.

Part I :

End point is inside the rectangular window.

Part II :

End point is on the edge side.
(Left, Right, Top, Bottom)

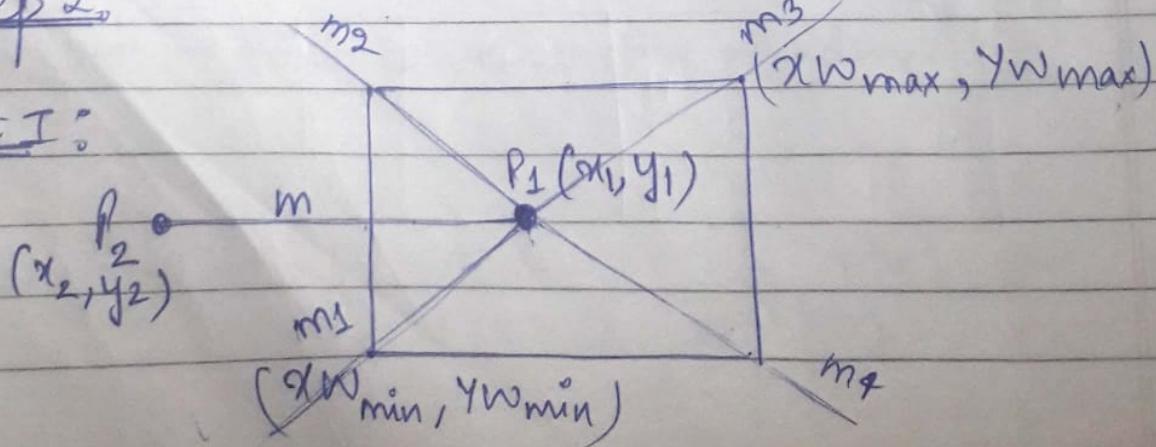
Part III :

End point is on the corner.

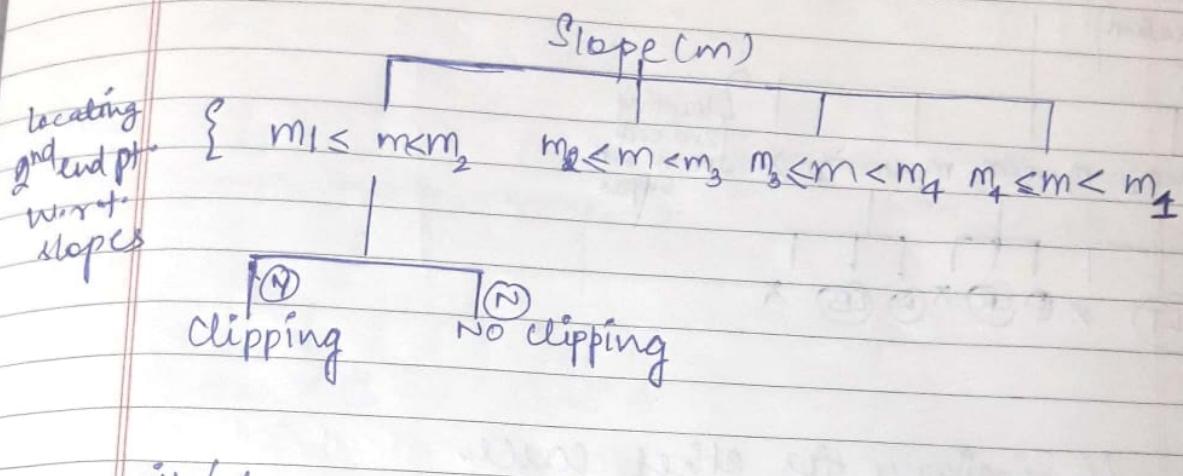
(left top, left bottom, right top, right bottom)

Step 2:

Part I :



If line $(P_1 - P_2)$ is rotated, then its slope m' will lie between any of the slopes m_1, m_2, m_3, m_4 .



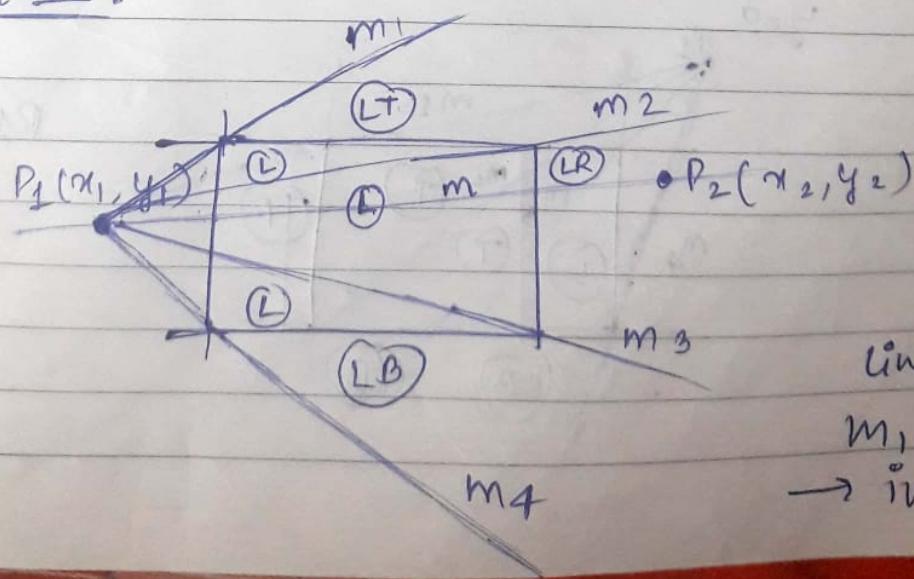
if $\{ (m > m_2) \& (m < m_1) \& (P_1 \cdot x_1 > P_2 \cdot x_2) \}$
 if $(P_2 \cdot x_2 < x_{w\min})$
 clipping

~~No clip~~

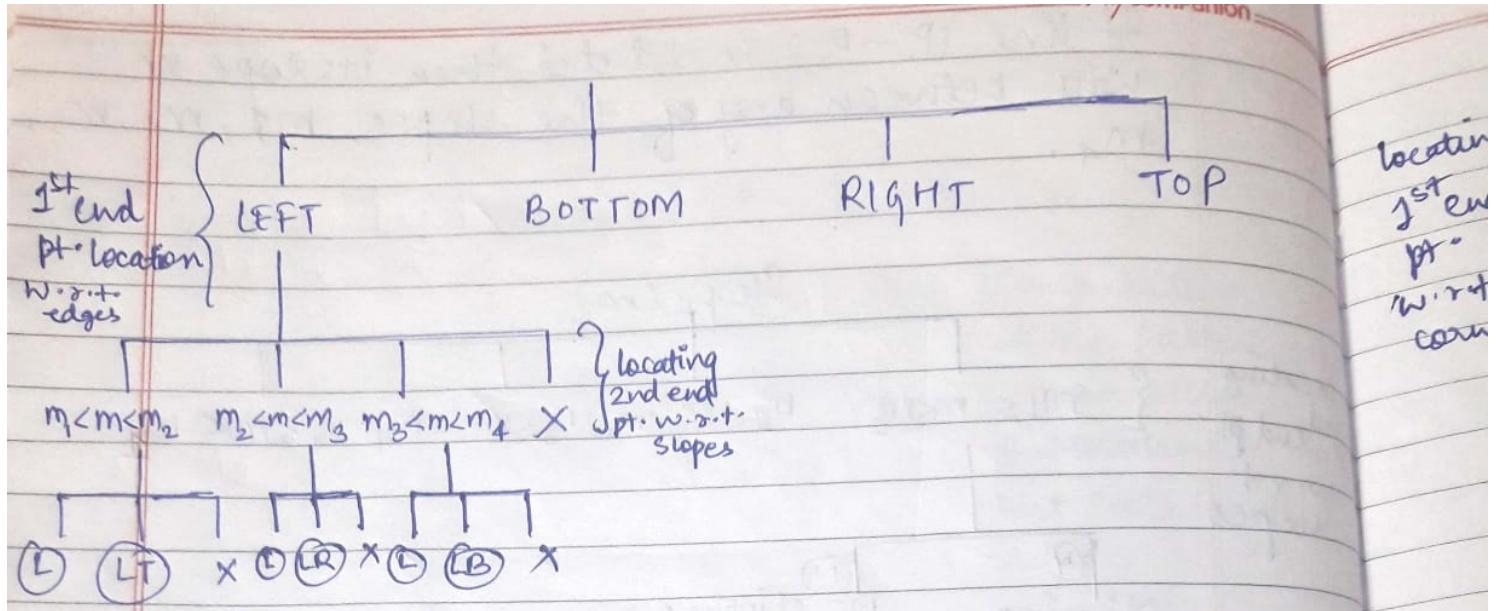
?

Similarly for other cases.

Part II:

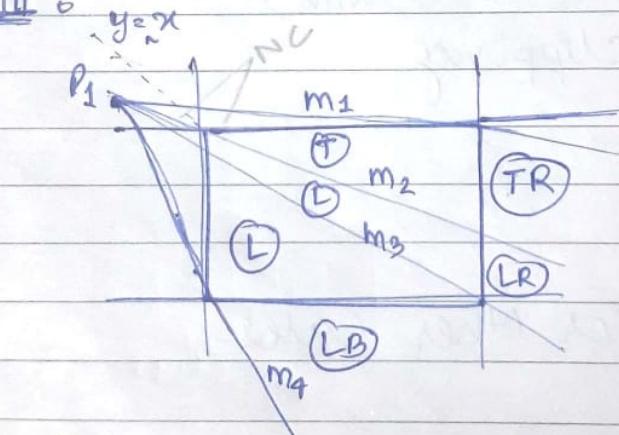


line b/w
 m_1 & m_4
 → invisible

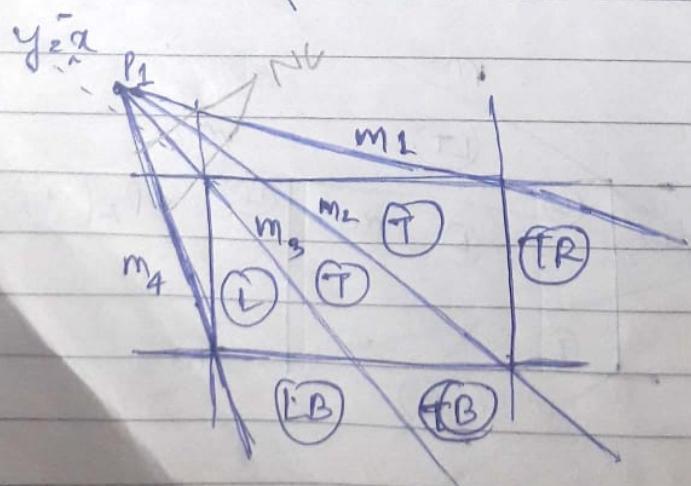


// Similarly for other cases.

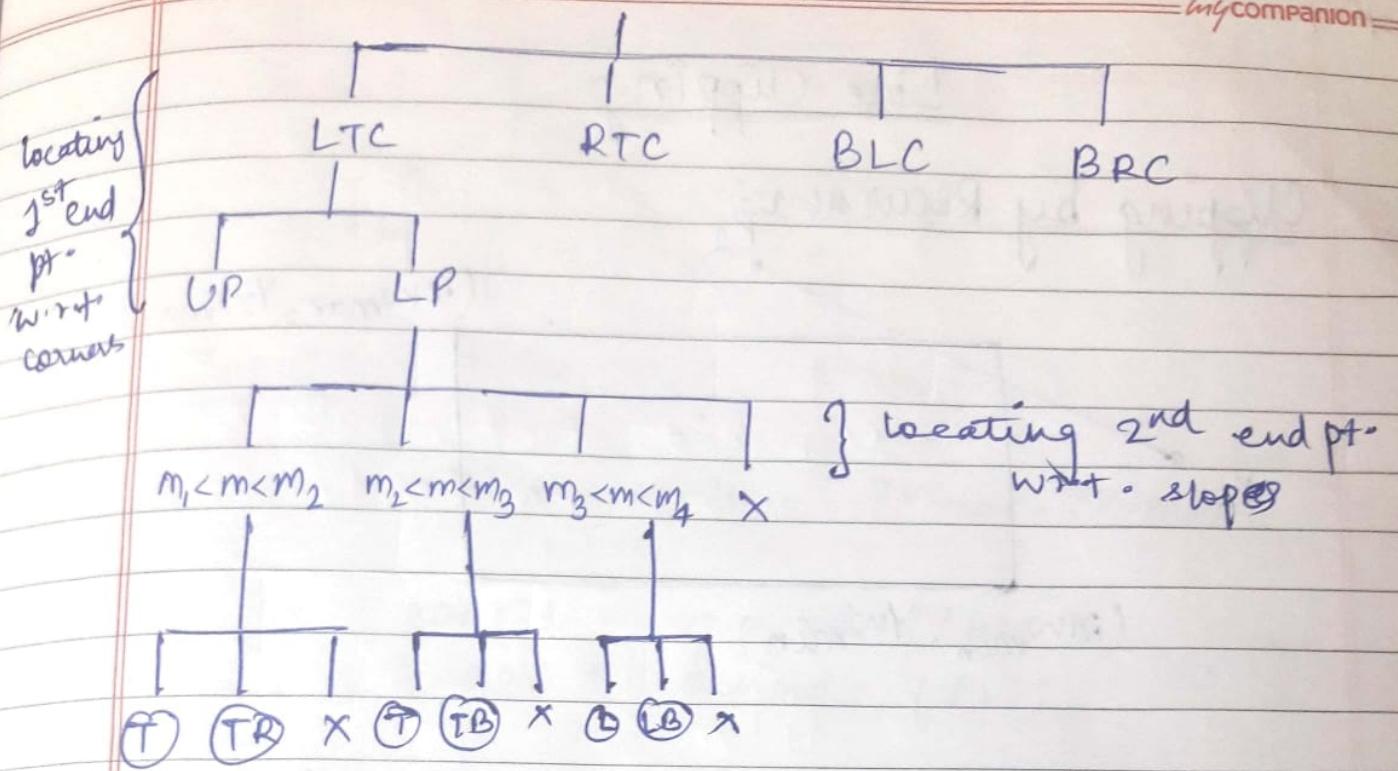
Part III



P1 in lower part of $y = x$



P1 above $y = x$



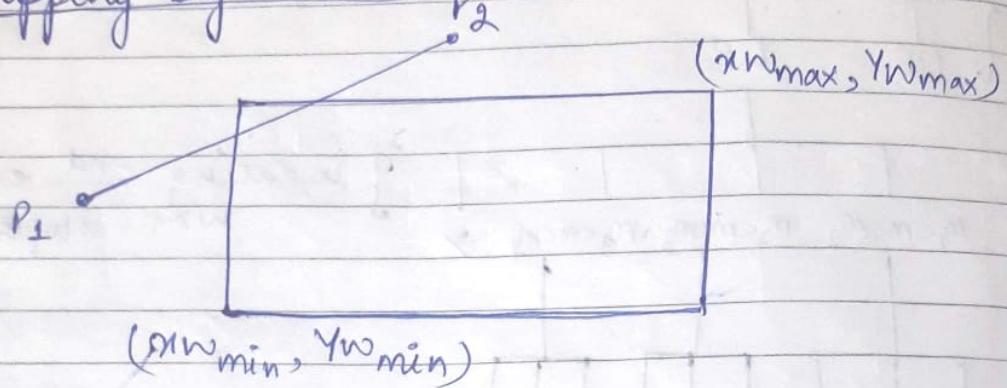
// Similarly for other cases.

05/02/2018

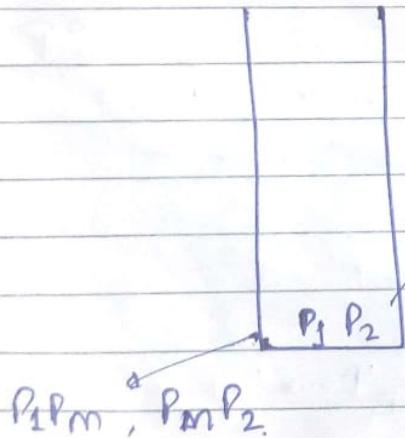
my companion

Line Clipping

Clipping by Recursion; (Mid Point Subdivision)

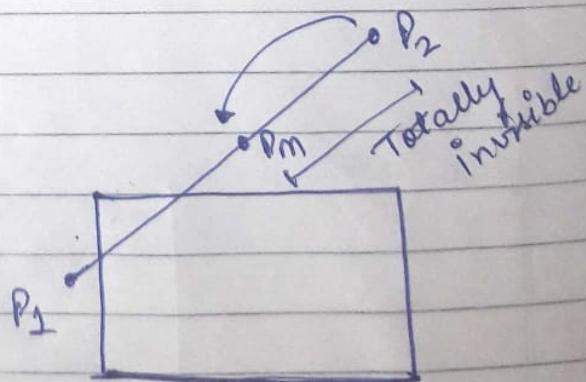
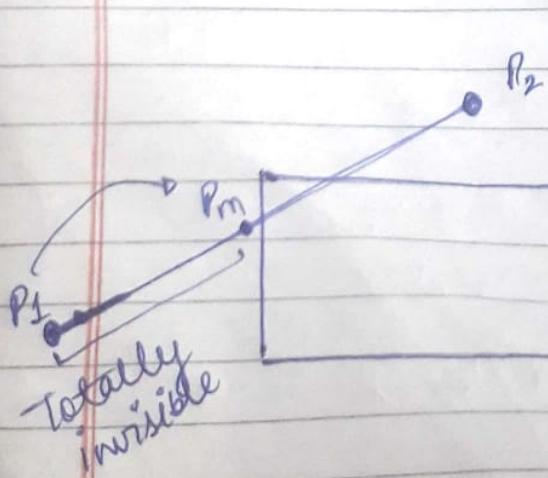


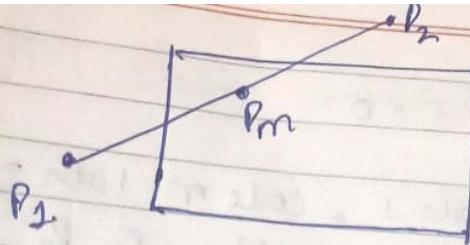
mid point of a line can be easily calculated.



Cohen Sutherland:

- totally visible
- totally invisible
- partially visible





Both are
visible

Input: $P_1(x_1, y_1)$, $P_2(x_2, y_2)$

$x_{W\min}, x_{Y\min}, x_{W\max}, y_{W\max}$.

code 1 = gencode(P_1);
code 2 = gencode(P_2);

Step ①

if code1 & code2 both = 0
then draw line P_1, P_2

Step ②

if code 1 & code 2 $\neq 0$
/* line invisible */

Step ③ if $P_m = (P_1 + P_2)/2$;

code m = gencode(P_m);

Step ④

if code m $\neq 0$

then

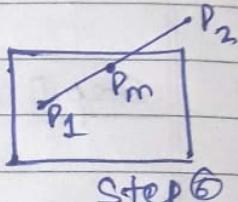
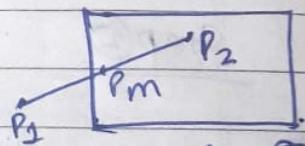
if code 1 & code m $\neq 0$ then
 $P_1 \leftarrow P_m$; goto step①

else if code 2 & code m $\neq 0$ then

$P_2 \leftarrow P_m$, goto step①

Step ⑤if code $m = 0$ if code 1, code m both = 0
then consider $P_1 P_m P_2$

else if code

else if code 2 ; code m both = 0
then consider $P_1 P_m$.Step ⑥consider $P_1 P_m$ do {
 $P_{m_1} = (P_1 + P_m)/2;$
code $m_1 = \text{gencode}(P_{m_1});$
if code $m_1 < 0$ then $P_1 = P_{m_1}$, else
 $P_m = P_{m_1},$ Step ⑥while {
 $P_{m_1} \cdot x <> x_{W_{\min}} \&$
 $P_{m_1} \cdot x <> x_{W_{\max}} \&$ Step ⑦($P_{m_1} \cdot y <> y_{W_{\min}} \&$
 $P_{m_1} \cdot y <> y_{W_{\max}}$) $P_1 \leftarrow P_{m_1}$ Step ⑦consider $P_m P_2$

do {

 $P_{m_2} = (P_2 + P_m)/2 ; \text{code } m_2 = \text{gencode}(P_{m_2})$ if code $m_2 < 0$ then $P_2 = P_{m_2}$, else $P_m = P_{m_2}$

{}

while (same condition as step⑥)

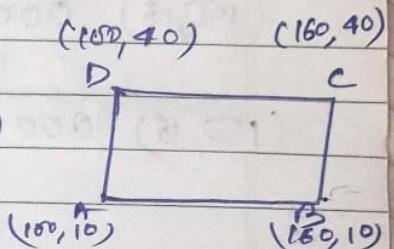
$$P_2 \leftarrow P_m$$

EXAMPLE :

$$P_1(120, 5), P_2(180, 30)$$

Window ABCD

$$A(100, 10), B(160, 10), C(160, 40), D(100, 40)$$



P_1	Code 1	P_2	Code 2	P_m	Code m.	Remarks
-------	--------	-------	--------	-------	---------	---------

(120, 5) 0010 (180, 30) 0100 150, 18 0000 Consider P_1, P_m
& consider $P_m P_2$.

consider P_1, P_m

(120, 5) 0010 (150, 18) 0000 (135, 12) 0000

(120, 5) 0010 (135, 12) 0000 (128, 9) 0010

(128, 9) 0010 (135, 12) 0000 (132, 11) 0000

(128, 9) 0010 (132, 11) 0000 (130, 10) 0001

(128, 9) 0010 (130, 10) $\underline{P_m}$

Consider P_m P₂

(135,12) 0000 150

P₁ code 1 P₂ code 2 P_m Codem Remarks.

(150,18) 0000 (180,30) 0100 (165,24) 0100

(150,18) 0000 (165,24) 0100

Polygon Clipping

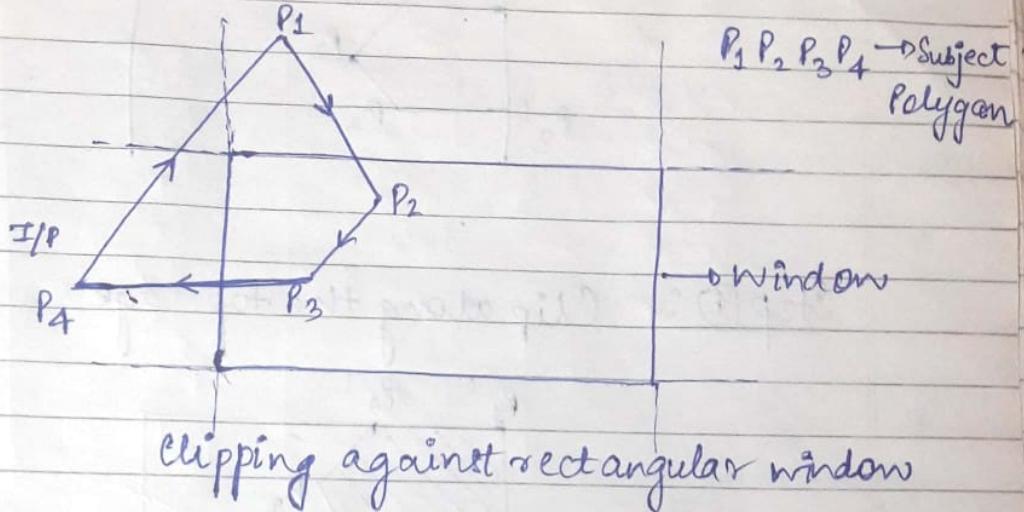
Window
Rectangular
Non-Rectangular
Polygon
Convex
Concave.

06/02/2018

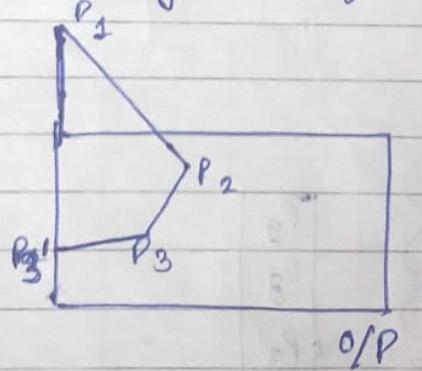
my companion

CLIPPING

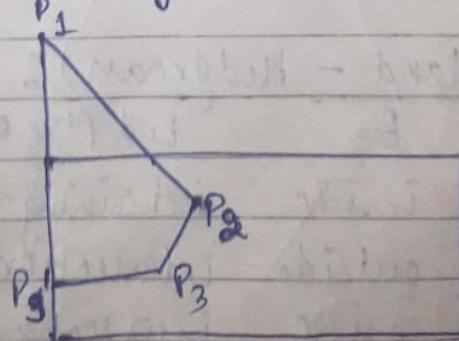
Polygon Clipping



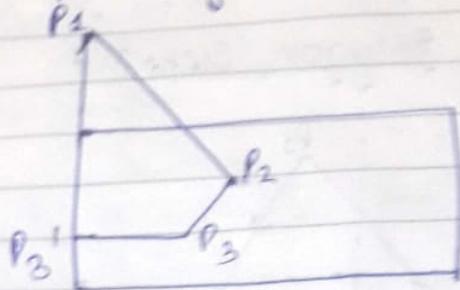
Step ① : Clip along the left edge



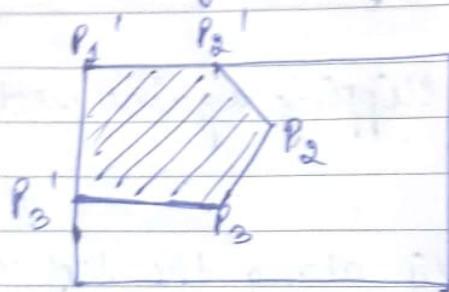
Step ② : Clip along the bottom edge



Step ③: Clip along the right edge



Step ④: Clip along the top edge

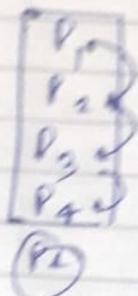


Rules (Sutherland - Hodgman):

E_1	E_2	End pts. of edge ($E_1 \& E_2$)
inside	inside	Retaining the 2 nd End Pt. (E_2)
inside	outside	Intersection pt. is computed & stored
outside	inside	intersection pt & 2 nd end pt to be stored
outside	outside	NIL

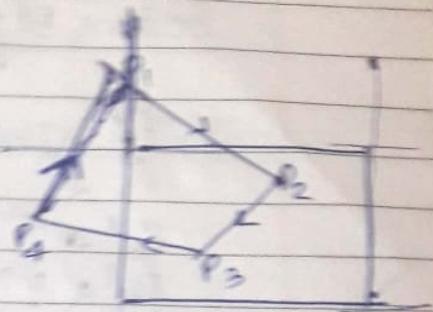
EXAMPLE:

Not
left
edge

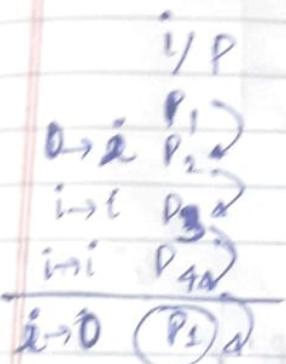


i/P

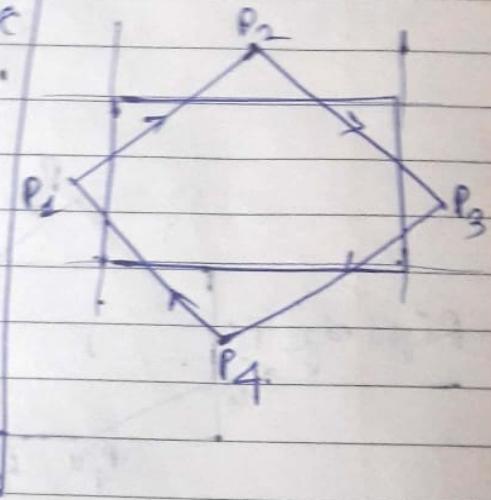
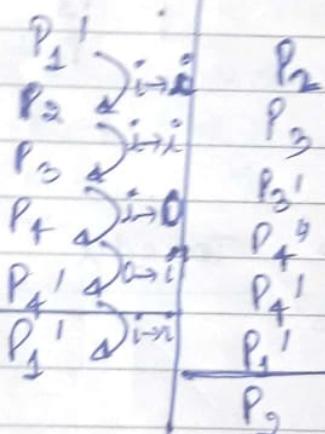
After Processing



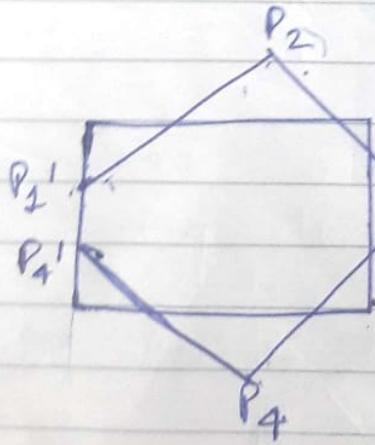
EXAMPLE:



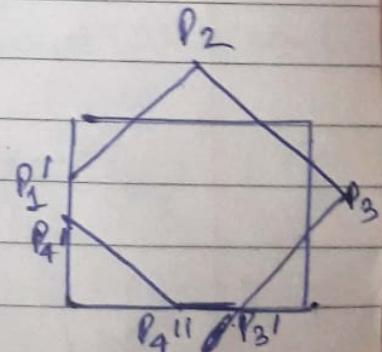
left edge | Bottom edge

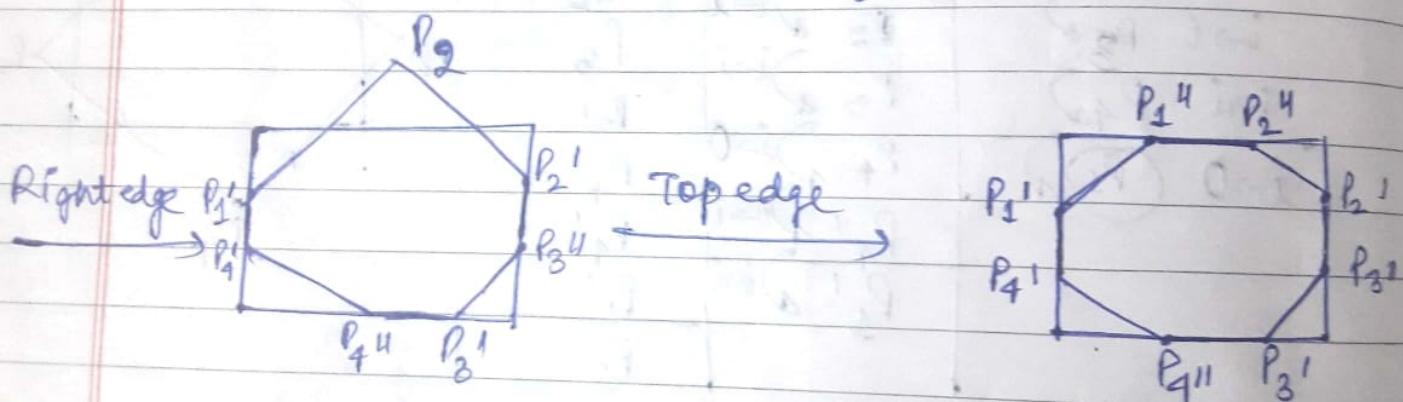
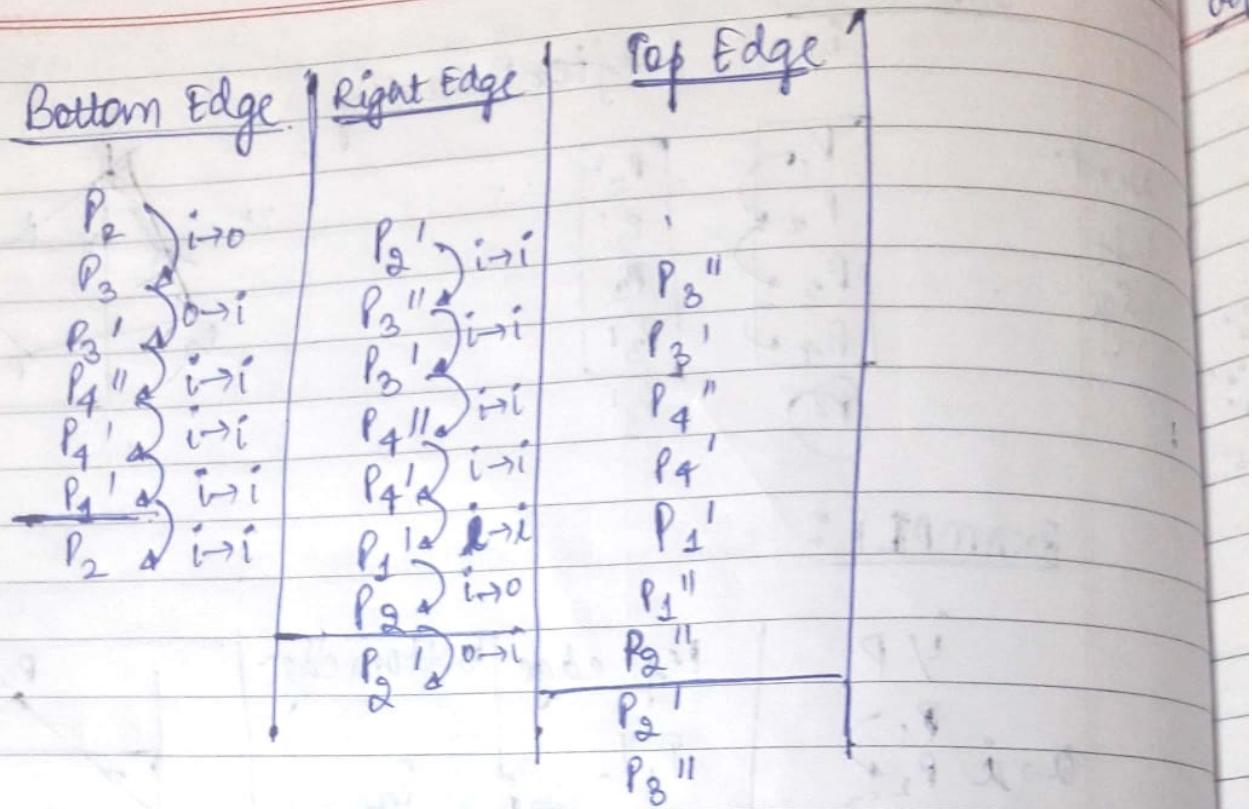


After
clipping
at
left edge



Bottom
edge

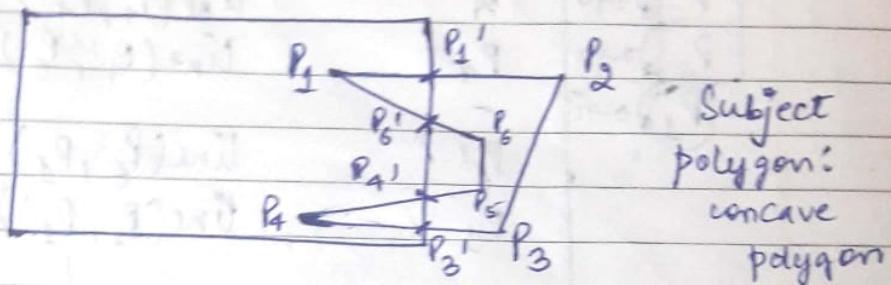




Polygon Clipping

Sutherland-Hodgman Method

EXAMPLE 1 :



Input : (a) window coordinates
 (b) Subject Polygon coordinates

Method:

Rules

- (a) $i \rightarrow o$ Store intersection point (I)
- (b) $o \rightarrow i$ Store intersection point (I), second vertex (P_2')
- (c) $i \rightarrow i$ Store second vertex (P_2')
- (d) $o \rightarrow o$ Nothing is stored

Output: Set of Clipped polygons.

i/p | Right edge

P_1'	i/o	P_3'
P_2'	0>0	-
P_3'	0>0	-
P_4'	0>i	P_3'
P_5'	i>0	P_4'
P_6'	0>0	P_4'
(P_1')	0>i	-

line($P_3' P_3'$) x (Not reqd.)

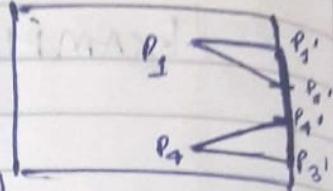
line($P_3' P_4'$)

line($P_4' P_4'$)

line($P_4' P_6'$) x (Not reqd.)

line($P_6' P_6'$)

line($P_1' P_1'$)



We should remove (P_1', P_3') & (P_4', P_6') .

In place of these two, (P_1', P_6') & (P_4', P_3') should be there.

Step 1:

	i/o	traversed
P_1'	1	01
P_3'	0	01
P_4'	0	01
P_4'	1	01
P_6'	0	01
P_6'	0	01
P_1'	0	02

c(if i/o ①)

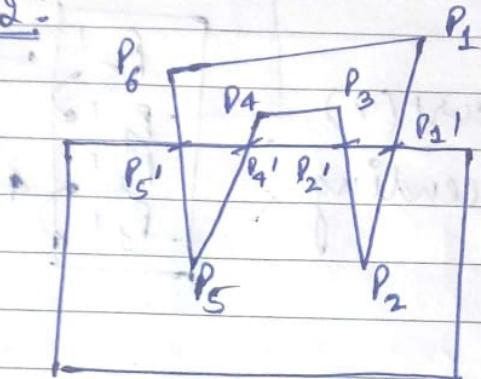
Step 2:

P_1'	sort(y) descending	P_1'
P_3'		P_6'
P_4'		P_4'
P_6'		P_3'

c(if i/o ①)

(% coordinate

of each intersection pt. is same, so sort them acc. to
y coordinate).

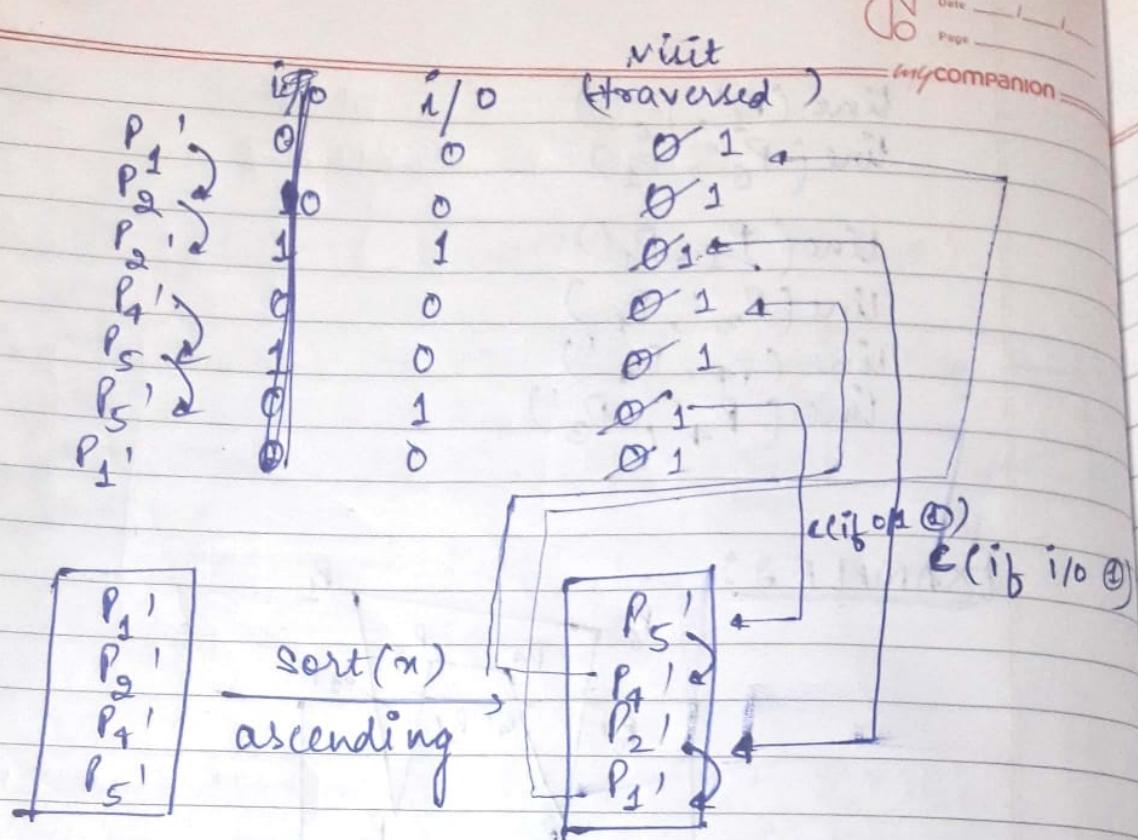
line (P_1', P_6') line (P_6', P_1) line (P_3, P_1') line (P_2', P_4) line (P_4, P_4') line (P_4', P_3') EXAMPLE 2:

IP

Top edge

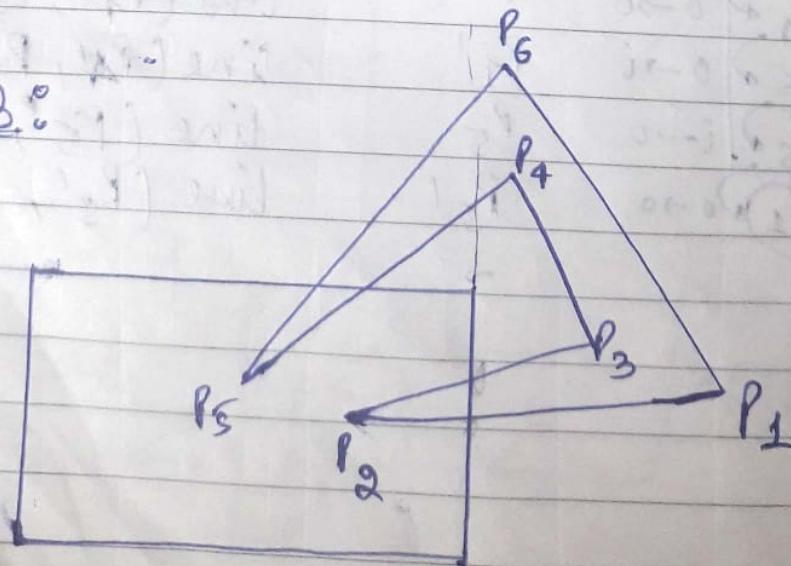
P_1	$0 \rightarrow i$	P_1'
P_2	$i \rightarrow o$	P_2'
P_3	$i \rightarrow o$	P_3'
P_4	$o \rightarrow o$	-
P_5	$o \rightarrow i$	P_5'
P_6	$i \rightarrow o$	P_6'
P_1	$o \rightarrow o$	-

line (P_1', P_2) line (P_2, P_2') line (P_2', P_1') × (Not Reqd.)line (P_4', P_5) line (P_4', P_5') line (P_5', P_5') (Not Reqd.)line (P_5', P_1') (Not Reqd.)



line (P_1', P_2')
 line (P_2', P_2')
 line (P_2', P_1')
 line (P_4', P_5')
 line (P_5', P_5')
 line (P_5', P_4')

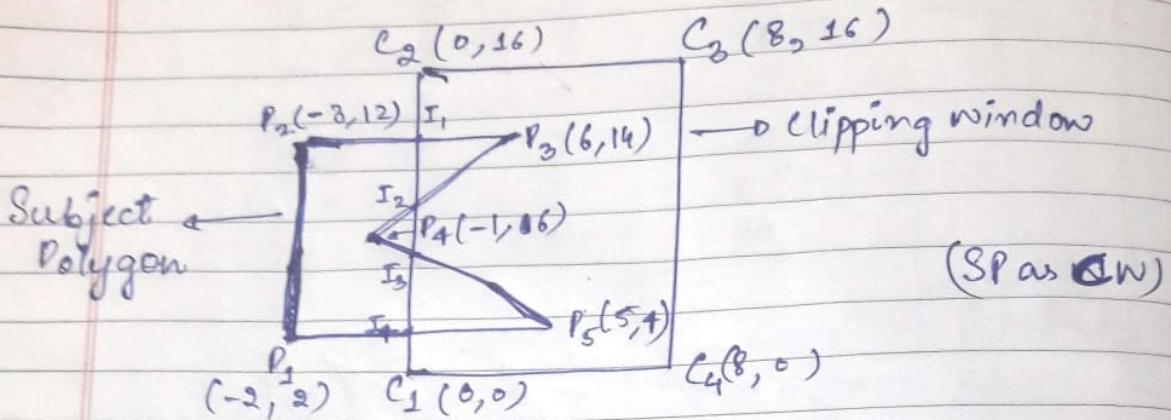
Example 3.



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Polygon Clipping WEILER - ATHERTON



- Walk over the polygon.
- Illumination starts on entry \rightarrow traversal is on clipping window
- whenever you exit, take a right turn (traversing in clockwise direction).

$$t = \frac{N \cdot (C_1 - P_E)}{-N \cdot (C_2 - C_1)}$$

Edge (Clipping window)

For edge $C_1 C_2$:

Edge		(1)	(0)	t	Intersection Pt.
Edge of SP	Normal	Entry/Exit			
$P_1 P_2$	$-i + 10j$	$-10i - j$	Exit	$-9/8$	\times
$P_2 P_3$	$i + 2j$	$-2i + 9j$	Entry	$19/24$	I_1
$P_3 P_4$	$-7i - 8j$	$+9i - 7j$	Exit	$20/512$	I_2
$P_4 P_5$	$6i - 2j$	$2i + 6j$	Entry	$2/32$	I_3
$P_5 P_1$	$-7i - 2j$	$2i - 7j$	Exit	$9/56$	I_4

Step ① :

SP $P_1 \ P_2 \ P_3 \ P_4 \ P_5$

CW $C_1 \ C_2 \ C_3 \ C_4$

Step ② :

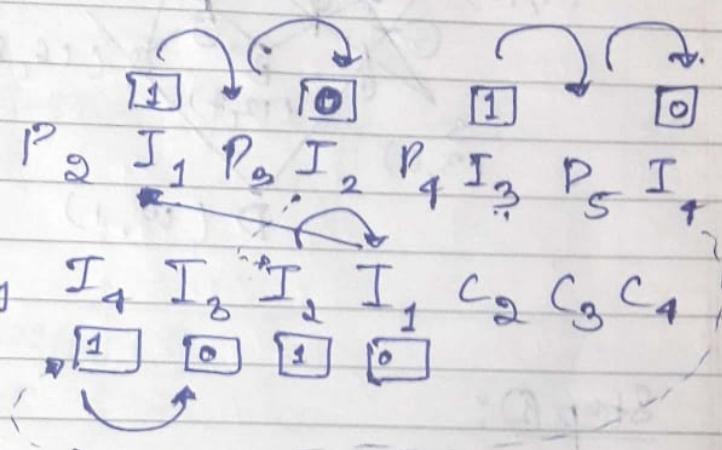
- ① Evolve
- ② Intersection pt.
- ③ Entry/Exit
- Determine

Step ③ :

SP
(updated)

: $P_1 \ P_2 \ I_1 \ P_3 \ I_2 \ P_4 \ I_3 \ P_5 \ I_4$

CW
(acc-to-t)
values



Step ④ :

line (I_1, P_3)]	Entry list $\{I_1, I_3\}$
line (P_3, I_2)]	clipped polygon ₁
line (I_2, I_1)]	
line (I_3, P_5)]	clipped polygon ₂
line (P_5, I_4)]	
line (I_4, I_1)]	

Start traversing, whenever a match with entry list found, start illuminating.

* Note : Right turn \rightarrow switching from SP to CW

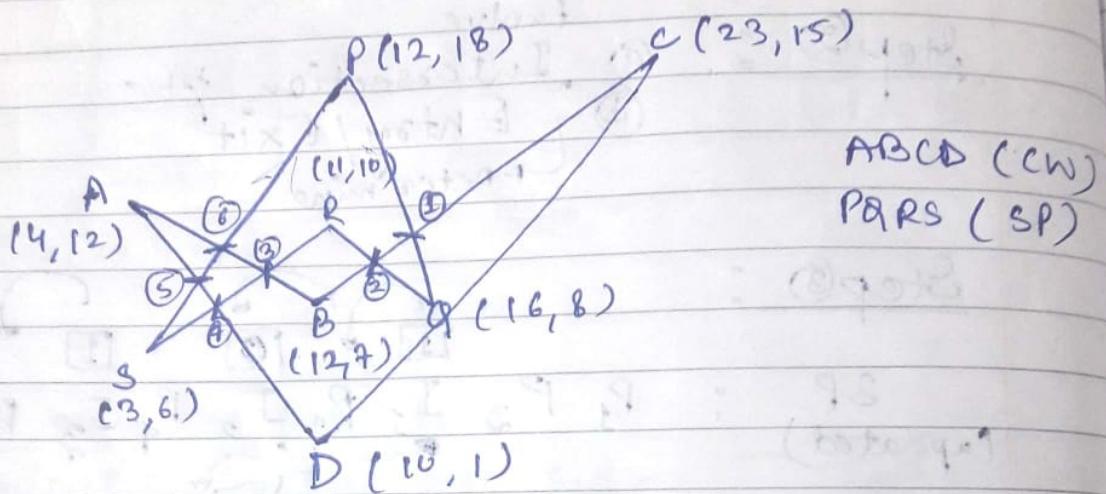
15/02/2018

Polygon Clipping

my companion

Weiler Atherton

Non-Rectangular Clipping Window



Step ①:

SP : P Q R S

CW : A B C D

Step ② :

intersection point computation

SP : Edge 1 (PA) $4\hat{i} - 10\hat{j}$
(clockwise traversal).

$$Q-P = 4\hat{i} - 10\hat{j}$$

$$t = \frac{N_i(P - E_i)}{-N_i(Q - P)}$$

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Edge (cw)	Normal	$P - E_i$	Denominator	t
AB (8i - 5j)	$5i + 8j$	$8i + 6j$	60	$\frac{22}{15} > 1$ X
BC (11i + 8j)	$-8i + 11j$	$11j$	142	$\frac{121}{142} < 1$ ✓ (o → i)
CD (-13i - 14j)	$14i - 13j$	$25i + 22j$ $-11i + 3j$	-186	$\frac{193}{186} > 1$ X
DA (-6i + 11j)	$-11i - 6j$	$18i + 7j$ $2i + 17j$	-16	$\frac{194}{16} > 1$ X

SP	P	$\frac{193}{186}$	Q	
	-1	0	-1	

SP: Edge 2 (RS) : $-8i - 4j$

$$R - S = -8i - 4j$$

Edge (cw)	Normal (n)	$R - E_i$	Denominator	t
AB (8i - 5j)	$5i + 8j$	$7i - 2j$	$\frac{1}{2}$ (End) (o → i)	$0.263 (19/72)$ ③
BC (11i + 8j)	$-8i + 11j$	$-i + 3j$	-20 (i → o)	$-\frac{4}{20} < 1$ X
CD (-13i - 14j)	$14i - 13j$	$-12i - 5j$	60 (o → i)	$-\frac{103}{60} > 1$ X
DA (-6i + 11j)	$-11i - 6j$	$i + 9j$	$-\frac{112}{112} (i \rightarrow o)$	$\frac{65}{112} (0.58)$ ④

SP : to be clipped

CW : against which to be clipped

in SP $\leftarrow \begin{cases} i \rightarrow o(1) \\ o \rightarrow i(0) \end{cases}$

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SP
Tagging
Traversal

P	Q	R	S	T	U	V	W	X	Y	Z
-1	0	-1	1	-1	0	1	-1	0	1	1
0	0	1	0	0	1	0	0	0	1	0

CW

A	B	C	D	E	F	G	H	I	J	K
-1	0	1	-1	0	1	-1	-1	0	1	1
0	1	0	0	0	0	0	0	0	1	0

O/P:

line (P, Q)
line (Q, R)
line (R, S)
line (S, T)
line (T, U)
line (U, V)
line (V, W)

□, □ ---

control transfers

- * Search for Tagging value 0 in SP. (Before that, make traversal value 1)
- * -1 in SP \rightarrow continue to next Pt.
- * 1 in SP \rightarrow Switch control to CW list,

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Filling Polygon Flipping

Polygon →

Non-horizontal (no horizontal edges)

Non-vertical (No vertical edges)

(case II)

Scan line in filling the polygon.

T.

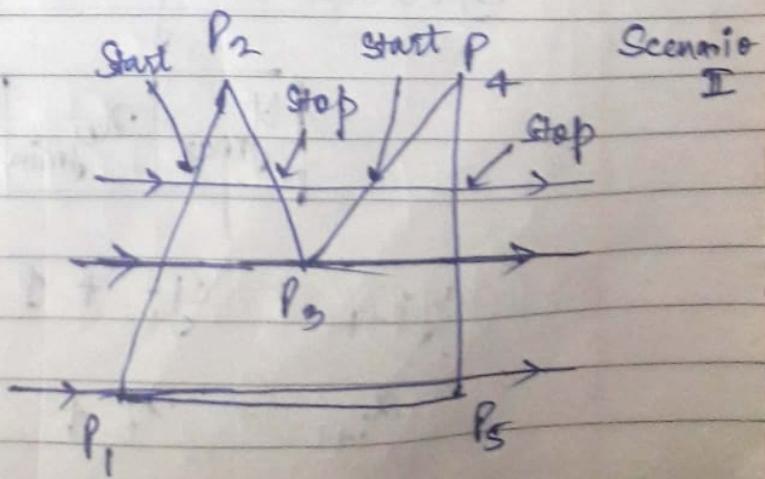
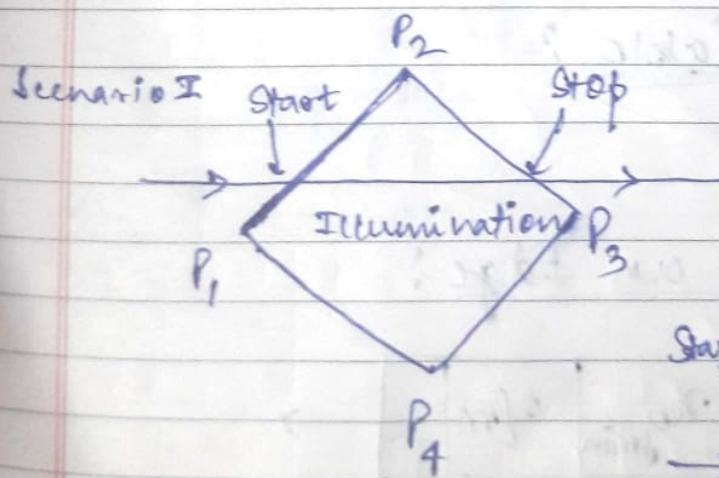
→ Horizontal Scanning

→ Vertical Scanning

① Horizontal Scanning: (non-horizontal edges)

Start scanning & (either $L \rightarrow R$ or $R \rightarrow L$)

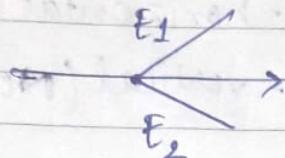
Illumination starts when an edge is encountered & stops when another edge is encountered.



- Vertices are formed from 2 consecutive edges
- when a scan line passes through a vertex

Single encounter @

N.o.t.o.
2edges



$$E_1 \cdot y_{\min} = E_2 \cdot y_{\max}$$

(either side of
scan line)

Double encounter

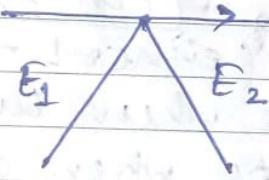
N.o.t.o.
2edges

(a)



$$E_1 \cdot y_{\min} = E_2 \cdot y_{\max}$$

(c)



$$E_1 \cdot y_{\max} = E_2 \cdot y_{\max}$$

Global Edge Table :

Structure of an Edge :

y_{\max}	$x_{y_{\min}}$	$1/m$
------------	----------------	-------

$$y_{i+1} = y_i + 1$$

(B → T)
(L → R)

$$x_{i+1} = x_i + 1/m$$

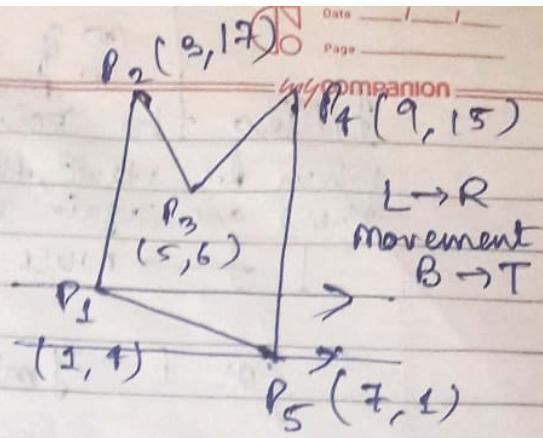
$P_1 P_5$

1	4	-2	1
---	---	----	---

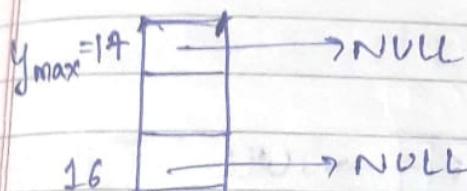
$T \rightarrow B$ (and $R \rightarrow L$):

$$y_{i+1} = y_i - 1$$

$$\alpha_{i+1} = x_i - 1/m$$

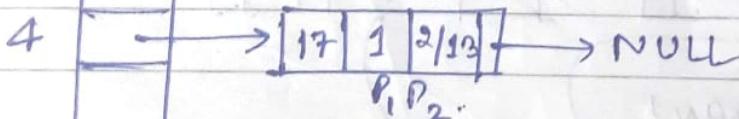
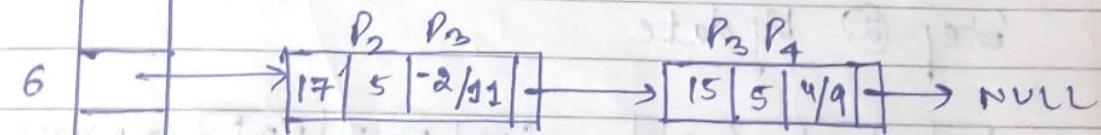


$$[y_{\min} \mid \alpha_y \max] \pm 1/m$$

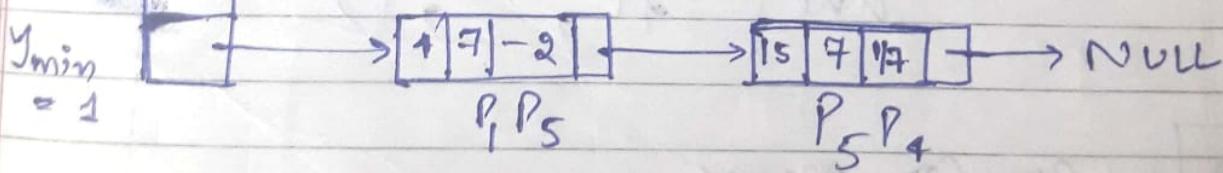


← Array of linked lists.

← Global edge Table



2 → NULL

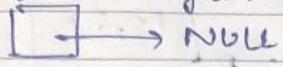


Global Edge Table

- Steps:
- ① Merge
 - ② Update (if the nodes are previous)
 - ③ if $y_i = y_{\max}$ of any node
draw $x_{i+1} = \frac{x_1 + x_m}{2}$
DELETE THAT my companion NODE

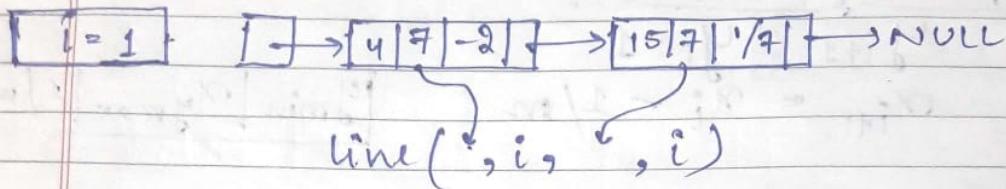
Filling the polygon (Scan line):

when we start, there are no edges & when we end.
there are again no edges.



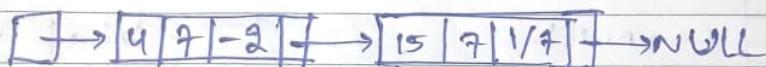
(Start)

for $i = y_{\min}$ to y_{\max} .

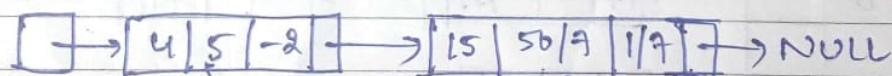


$i=2$

Step ① Merge



Step ② Update



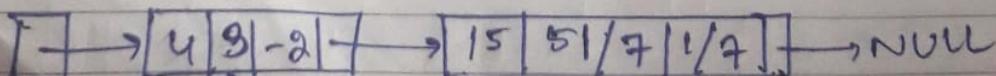
Step ③ draw

line(, i, , i)

$i=3$

Step ① Merge

Step ② Update



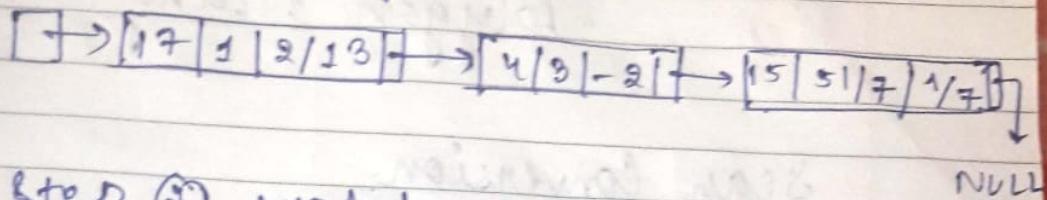
Step ③ draw

line(, i, , i)

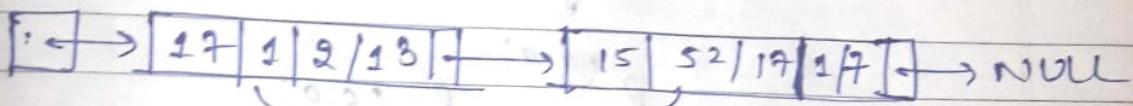
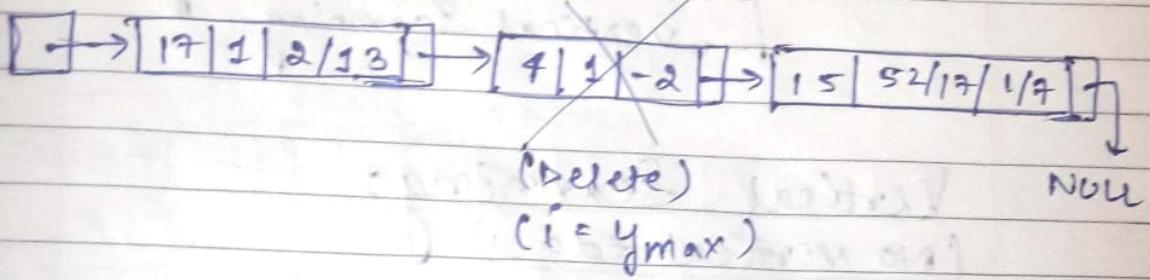
i = 4

Step ① Merge

(sorted acc.
using x)
(L \leftarrow R)



Step ② update



Step ③ draw line(\nearrow , i \nearrow , i \searrow)

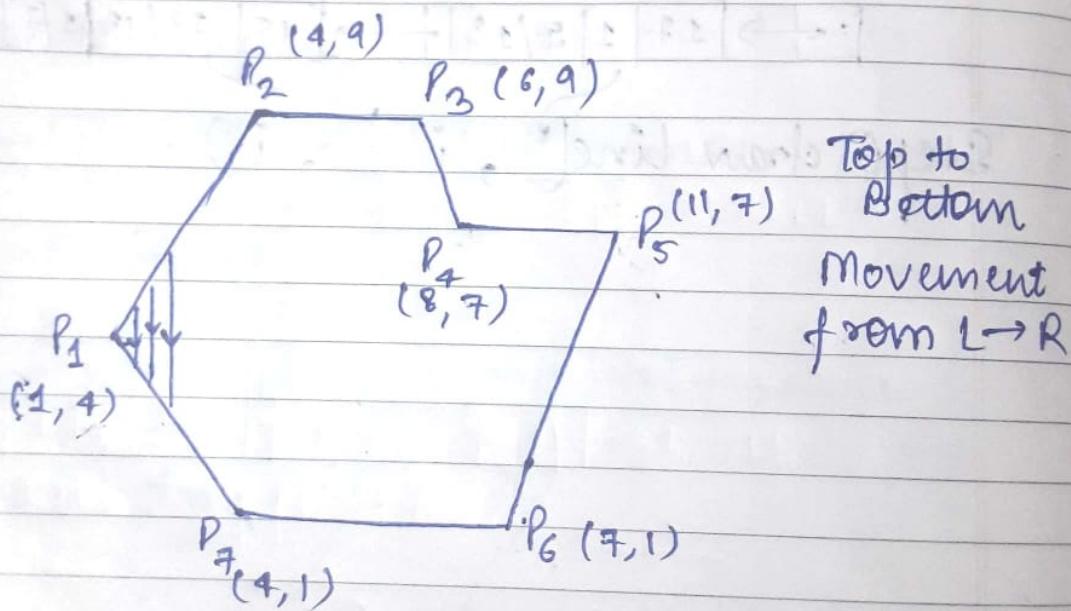
20/02/2018

Polygon Filling

Scan conversion

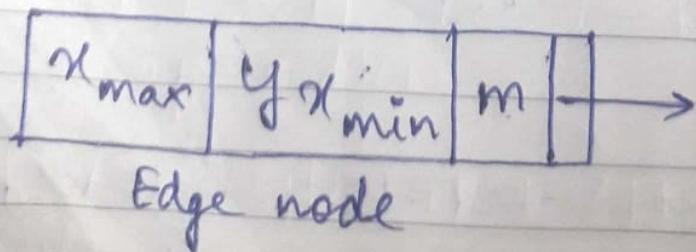
- Horizontal Scanning
- vertical Scanning

Vertical Scanning: (no vertical edges)

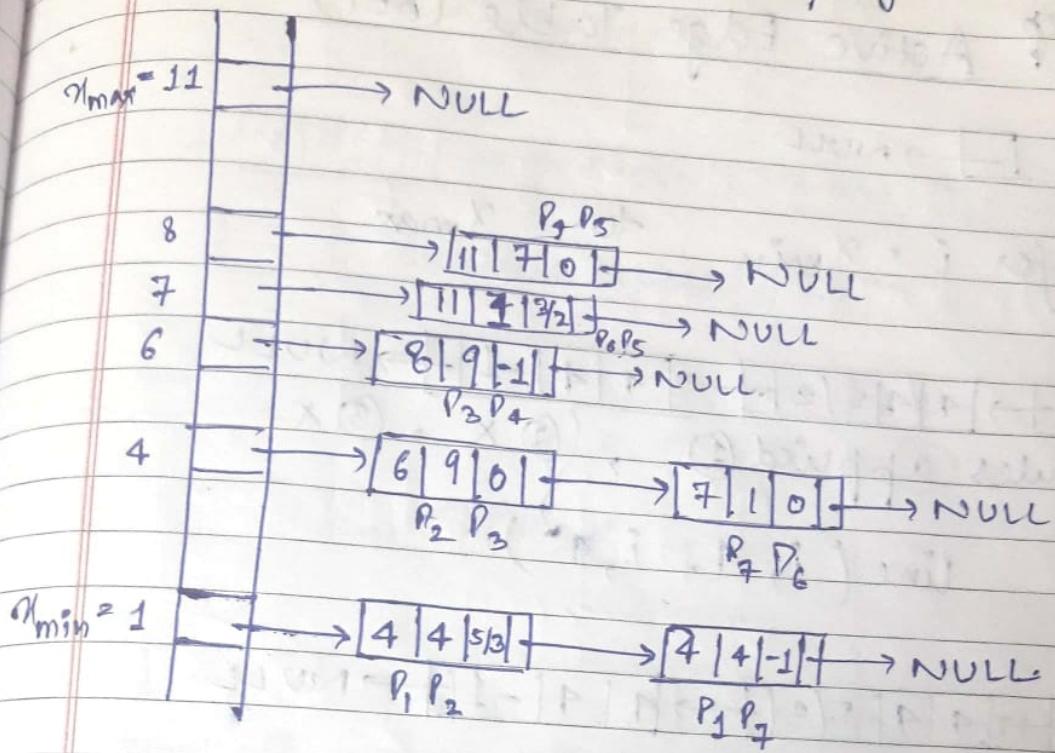


Step ①

Edge-Structure



Step ②: Construction of global edge table



Step ③: Filling of polygon
(Active Edge Table)

Rules (for processing Active Edge Table):

- ① Merge (Active edge list at an instant with global edge table).
- ② Update (only those edges which are old ones).
 $x_{i+1} \rightarrow x_i + 1 ; y_{i+1} = y_i \pm m$
- ③ Delete (only that edge whose first member element matches at that instant).
- ④ Draw the scan line (vertical).

~~Step~~ Active Edge Table (AET),

$\boxed{-} \rightarrow \text{NULL}$

for $i = x_{\min}$ to x_{\max}

$i=1$ $\boxed{-} \rightarrow \boxed{4|4|5/3} \rightarrow \boxed{4|4|-1} \rightarrow \text{NULL}$
Rules applied ①, ② X, ③ X
line(i, 4, i, 4)

$i=2$ $\boxed{-} \rightarrow \boxed{4|4|5/3} \rightarrow \boxed{4|4|-1} \rightarrow \text{NULL}$
Rules applied ①

$\boxed{-} \rightarrow \boxed{4|4/3|5/3} \xrightarrow{(t+m)} \boxed{4|3|-1} \rightarrow \text{NULL}$
Rule applied ③

(③ X)
line(i, 17/3, i, 3)

$i=3$ $\boxed{-} \rightarrow \boxed{4|17/3|5/3} \rightarrow \boxed{4|3|-1} \rightarrow \text{NULL}$
Rule applied ①

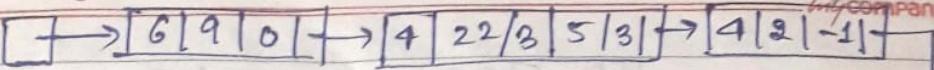
$\boxed{-} \rightarrow \boxed{4|22/3|5/3} \rightarrow \boxed{4|21|-1} \rightarrow \text{NULL}$
Rule applied ②

(③ X)

line(0, 22/3, i, 2)

(sorted according to y) ($T \rightarrow B$)

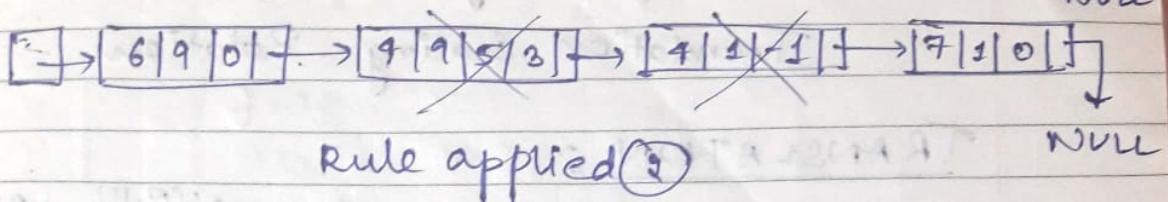
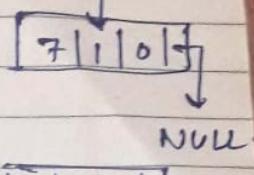
$i=4$



Rule applied ①

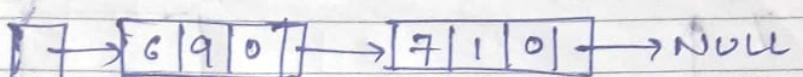
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Rule applied ②

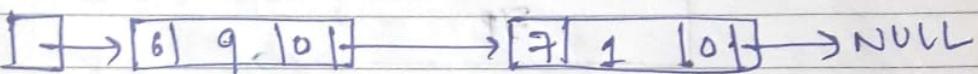
NULL



Rule applied ③

line($i, q, l, 1$)

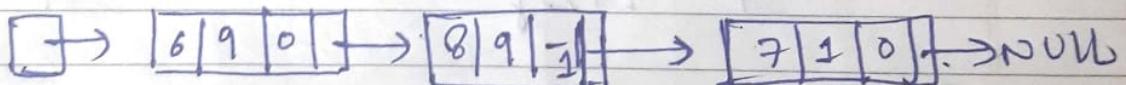
$i=5$



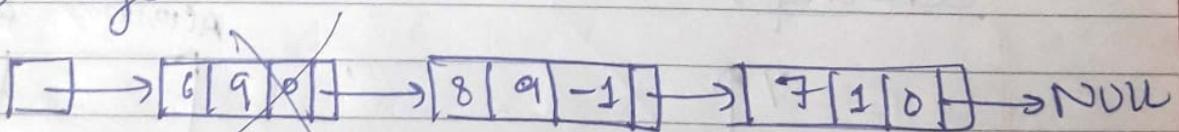
① merge ② update ③ X

line($i, q, i, 1$)

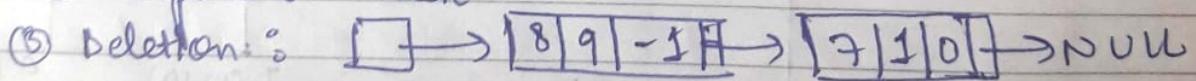
$i=6$



① merge



② update Delete

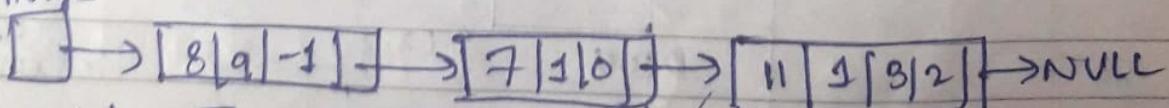


③ Deletion :

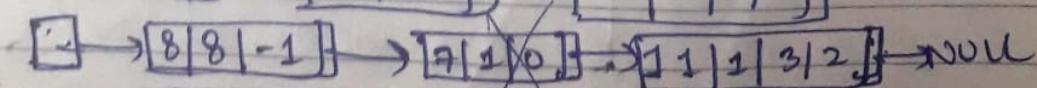
line($i, q, i, 1$)

$i=7$

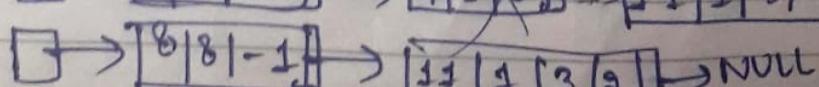
① merge



② update



③ Delete



line($i, q, i, 1$)

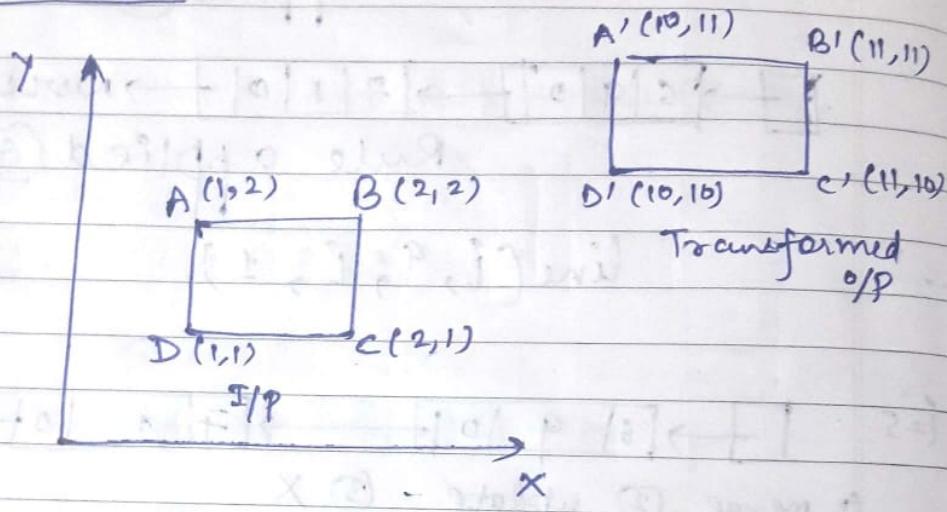
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2D TRANSFORMATION

Translation Scaling Rotation.

TRANSLATION :

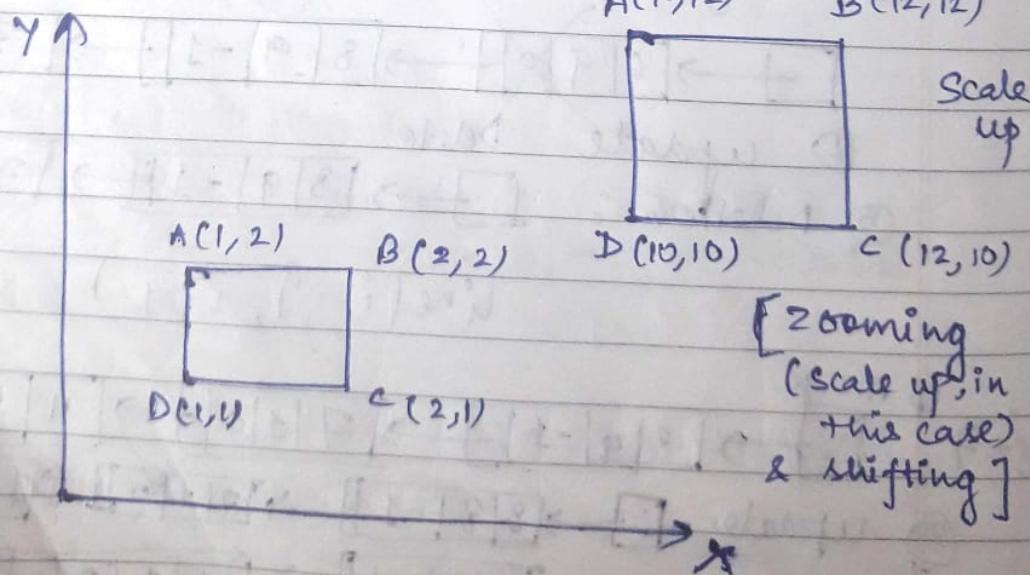


$$x' = x + \Delta x$$

$$y' = y + \Delta y$$

\pm : Transformation operator

SCALING :



S_x : Scaling factor in x -direction

S_y : Scaling factor in y -direction

$$x' = (x-1)S_x + \Delta x + 1$$

$$y' = (y-1)S_y + \Delta y + 1$$

Transformation operator: $\begin{pmatrix} & \\ & \end{pmatrix}$, *

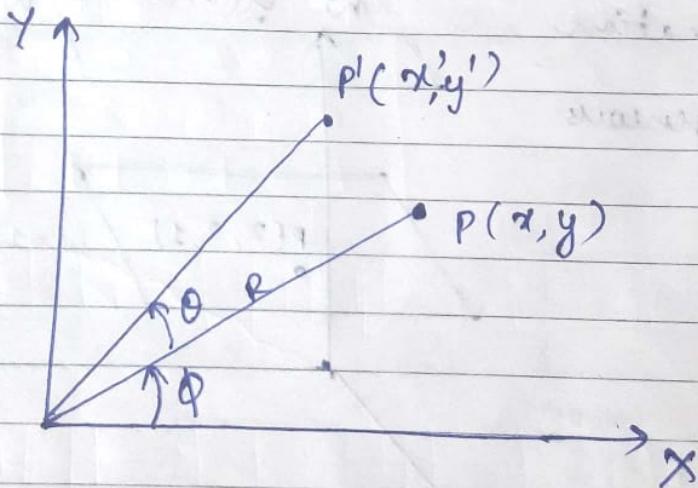
↓ L. Scaling

Translation

ROTATION:

(can be about any pt.)

$$\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \end{aligned}$$



$$x' = R\cos(\theta + \phi)$$

$$y' = R\sin(\theta + \phi)$$

$$x' = R\cos\theta \cos\phi - R\sin\theta \sin\phi$$

$$y' = R\sin\theta \cos\phi + R\cos\theta \sin\phi$$

$$\Rightarrow \boxed{x' = x\cos\phi - y\sin\phi}$$

$$y' = R \sin \theta \cos \phi + R \cos \theta \sin \phi$$

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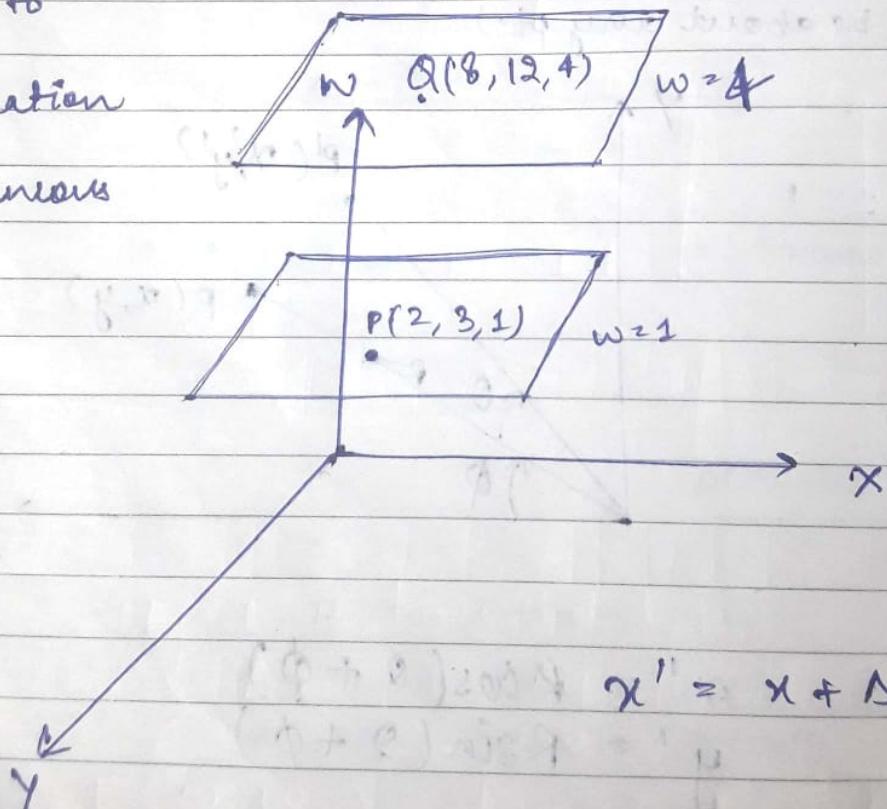
$$\boxed{y' = x \sin \theta + y \cos \theta}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Transformation operator : *

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

We need to
make the
transformation
operator
homogeneous



$$x' = x + \Delta x$$

Instead of taking 2-D, represent the pt. in
a homogeneous coordinate system.

$$\hookrightarrow [x/w, y/w, z]$$

Translation:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

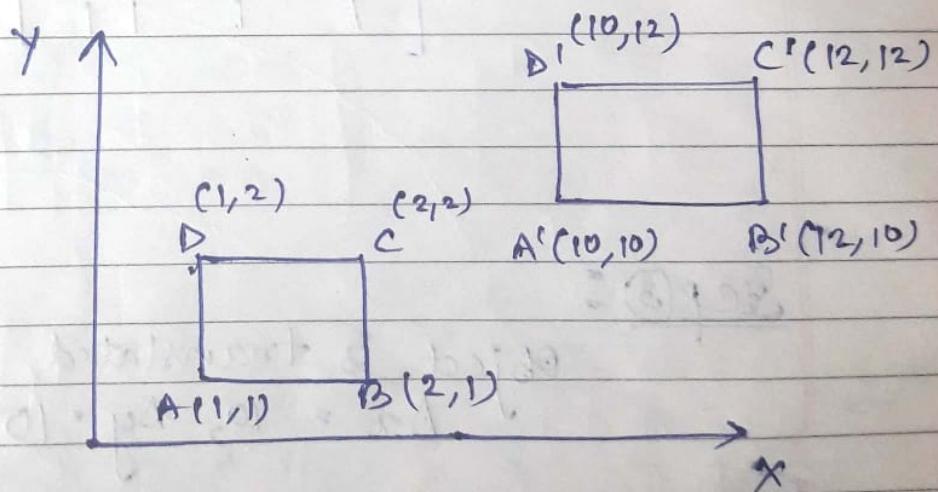
Every pt. in 2D can be taken as a pt. in 3D with $w=1$ in a plane $w=1$.

Scaling: $\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

Rotation: $\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

Homogenous transformation operator: *

EXAMPLE 1:



Step ①:

Object is brought to origin

bcz scaling is w.r.t the origin

T_1
(Translation matrix)

$\rightarrow A \text{ to } (0,0)$

$$= \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

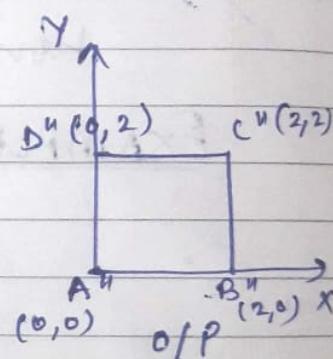
$$T_1 = \begin{bmatrix} A' & B' & C' & D' \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Step ②:

Object to be scaled up
 $S_x = 2, S_y = 2$

Scaling matrix $\rightarrow \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{bmatrix} A' & B' & C' & D' \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} A'' & B'' & C'' & D'' \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



Step ③:

Object is translated

$$\Delta x = 10, \Delta y = 10$$

$$\begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Translation Matrix (T_2)

$$\Rightarrow \begin{bmatrix} A''' & B''' & C''' & D''' \\ 10 & 12 & 12 & 10 \\ 10 & 10 & 12 & 12 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

T_2

S

T_1



$$C = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rightarrow T_1 \times S$

$$C = \begin{pmatrix} 2 & 0 & 8 \\ 0 & 2 & 8 \\ 0 & 0 & 1 \end{pmatrix}$$

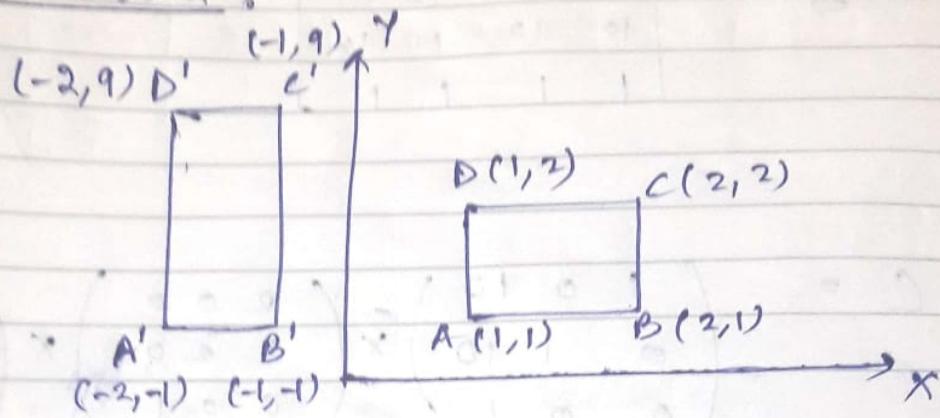
(composite matrix)

$$(C) \rightarrow \begin{pmatrix} 2 & 0 & 8 \\ 0 & 2 & 8 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} A & B & C & D \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix} \leftarrow (IP)$$

$$O/P = \begin{bmatrix} A''' & B''' & C''' & D''' \\ 10 & 12 & 12 & 10 \\ 10 & 10 & 12 & 12 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\Rightarrow [O/P \cdot 2 \quad C * IP]$

EXAMPLE 2 :



$$T_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

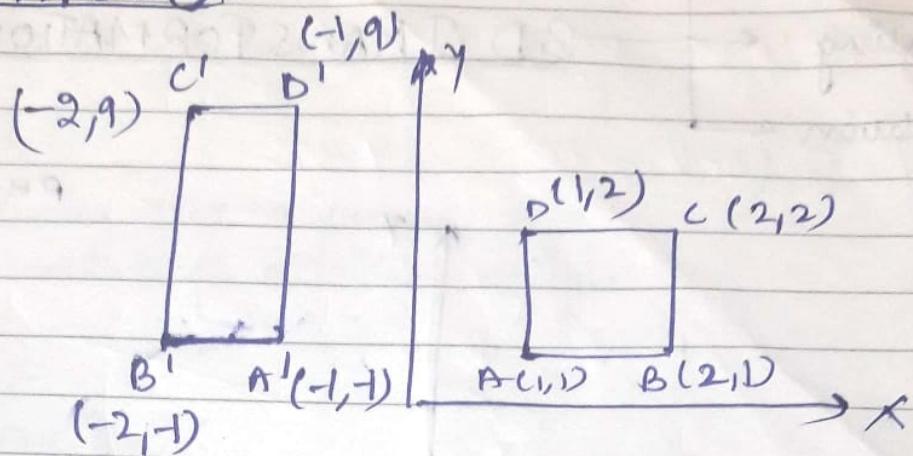
$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = T_2 * S * T_1$$

$$C = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

EXAMPLE 3:



Reflection about Y-Axis

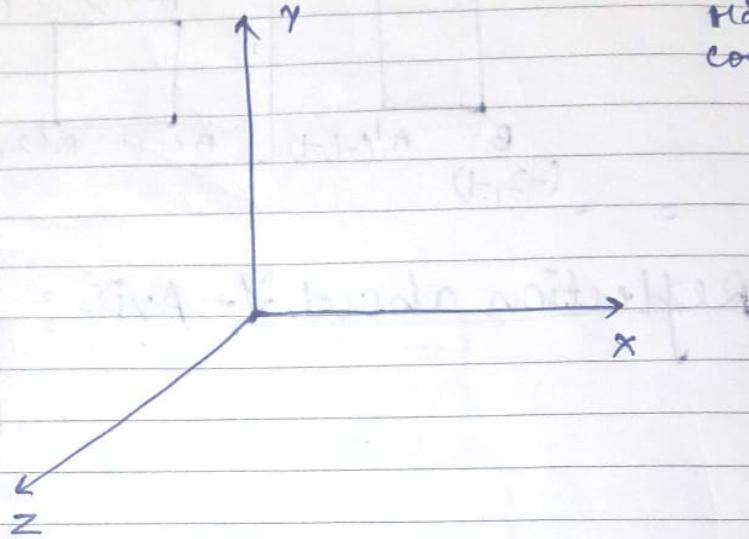
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Rotation
Scaling
Translation

3D TRANSFORMATION



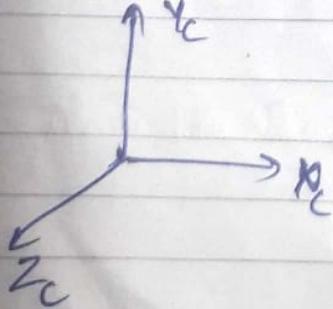
RHCS (Right Handed Coordinate System)

Camera Analogy:

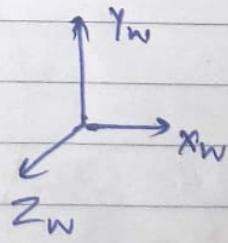
Camera



(Camera Coord. system)

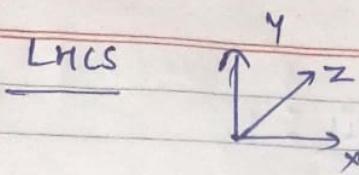


object



stated as RHCS
world
coordinate
system
(for the
Object)

Object is referred
w.r.t. world
coordinate system



- z is along the viewing dir^n
- eye is always at the origin

Translation :

$$T = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling :

$$S = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation :

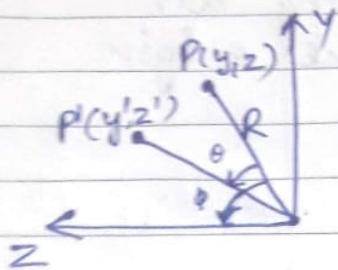
- Rotation about x-axis
- Rotation about y-axis
- Rotation about z-axis

RD

Any rotation in anticlockwise dir^n is considered as +ve.

Rotation about X-axis:

Anticlockwise: $Y \rightarrow Z$



$$y' = R \cos \theta, z' = R$$

$$x' = x$$

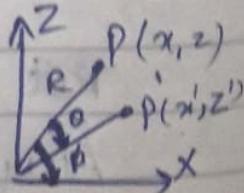
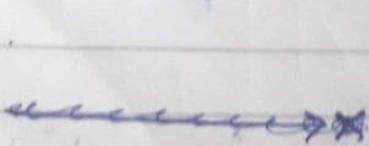
$$y' = y \cos \theta - z \sin \theta$$

$$z' = z \cos \theta + y \sin \theta$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation about Y-axis:

Anticlockwise rotation: $Z \rightarrow X$



$$y' = y$$

$$x = R \cos \phi, y = R \sin \phi$$

$$z = R \cos \theta$$

$$x' = x \cos \theta - z \sin \theta$$

$$y' = x \sin \theta + z \cos \theta$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

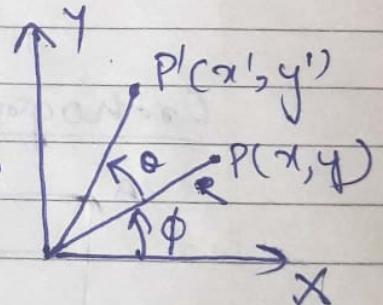
Rotation about Z-axis:

Anticlockwise rotation: $x \rightarrow y$

$$x' = R \cos(\theta + \phi)$$

$$x' = R \cos \theta \cos \phi - R \sin \theta \sin \phi$$

$$\Rightarrow x' = x \cos \theta - y \sin \theta$$



$$y' = R \sin(\theta + \phi)$$

$$y' = R \sin \theta \cos \phi + R \cos \theta \sin \phi$$

$$\Rightarrow y' = x \sin \theta + y \cos \theta$$

$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Projection -

→ Parallel

→ Non → Parallel

Isometric
Dimetric
Trimetric
Front

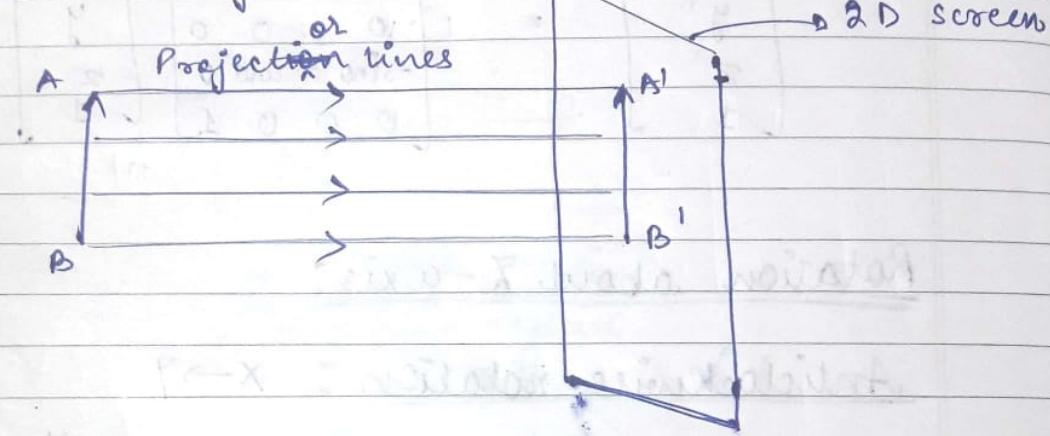
Orthographic

Obllique

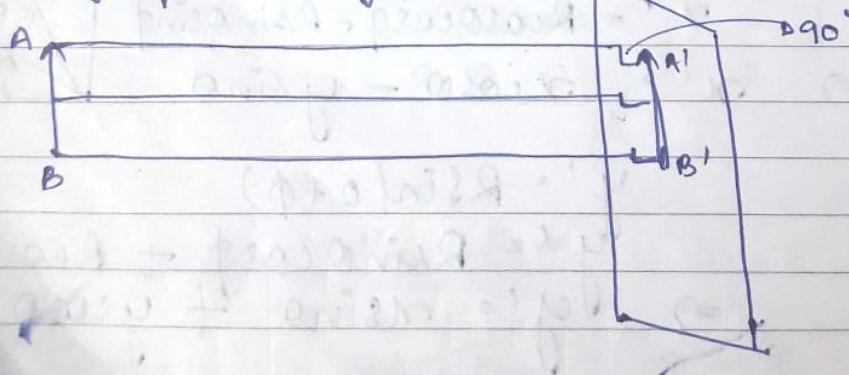
Cabinet

Cavalier

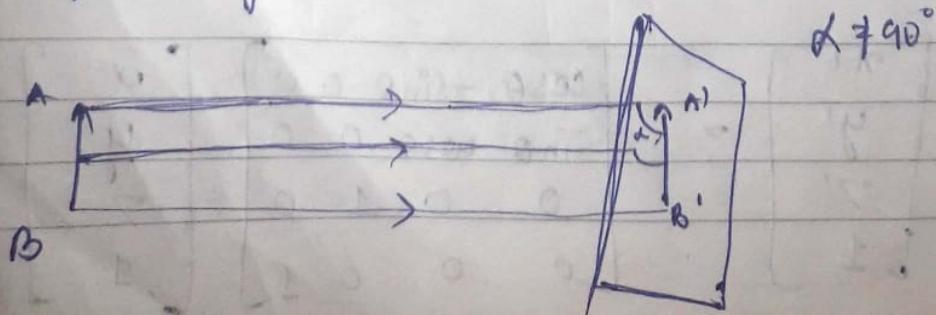
Parallel Projection:



Orthographic Projection:



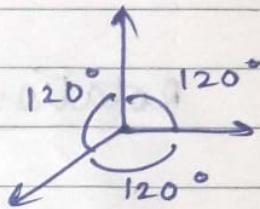
Obllique Projection:



Isometric Projection:

Parallel projection lines falling ^{or} on the screen at an angle of 45° . These lines make equal angle with all the 3 coordinate axis.

If these coordinate axis are projected on a plane, then these axis will make 120° angle with each other.



Dimetric Projection:

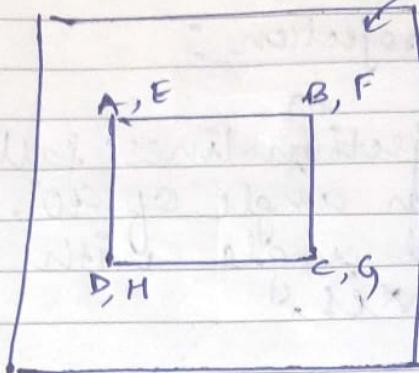
Orthogonal Projection + Projector lines make equal angles only with 2-axis.

Trimetric Projection:

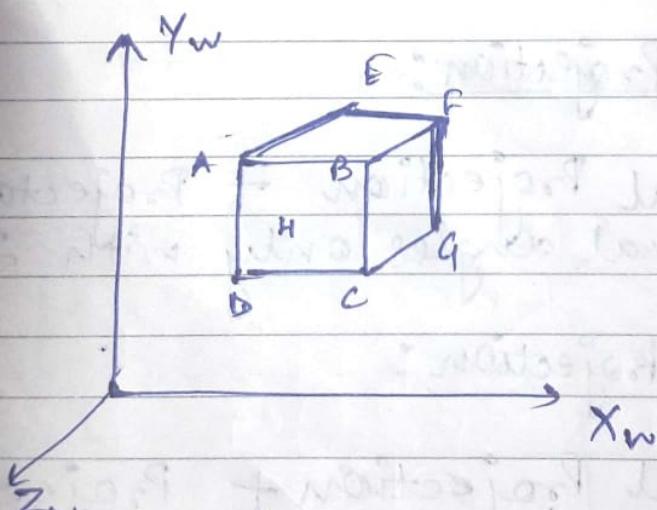
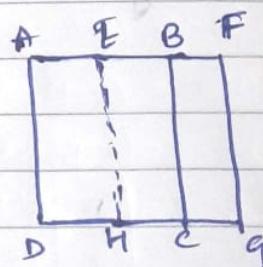
Orthogonal Projection + Projector lines don't make equal angles with any axis.

15/03/2020

Projections plane



Cube

rotation about y-axis :

Projection (x-y)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A & B & C & D & \dots & H \end{bmatrix}$$

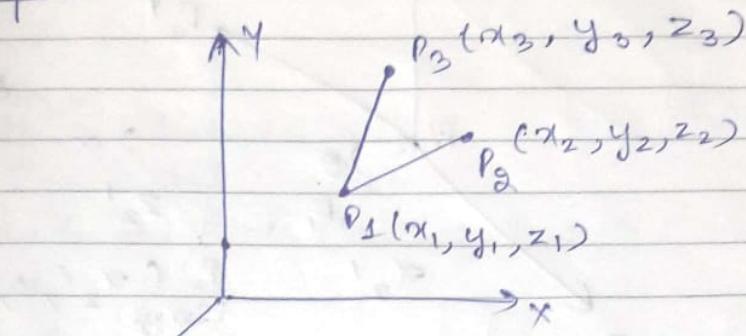
15/03/2018

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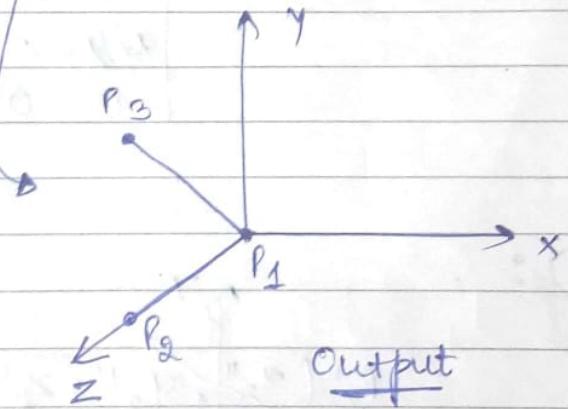
3D TRANSFORMATION

Example:



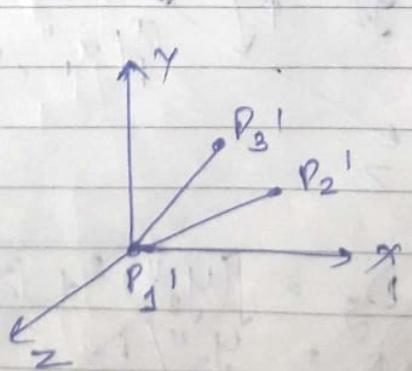
Input

C (composite matrix, to be derived)



P_1, P_2 along
z-axis
& P_1, P_3
along is
on y-z
plane.

Step ①



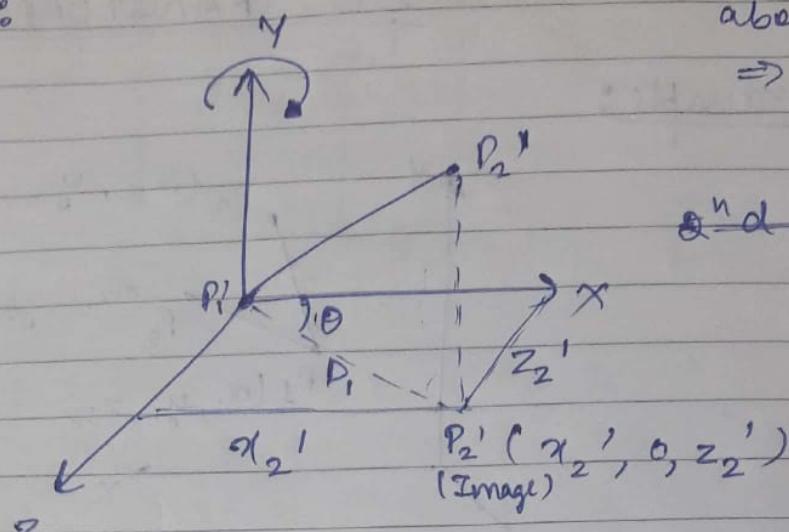
$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_1' = (0, 0, 0)$$

$$P_2' = (x_2 - x_1, y_2 - y_1, z_2 - z_1) \Rightarrow (x_2', y_2', z_2') \\ P_3' = (x_3 - x_1, y_3 - y_1, z_3 - z_1) \Rightarrow (x_3', y_3', z_3')$$

(in clockwise direction)

Step (2) :

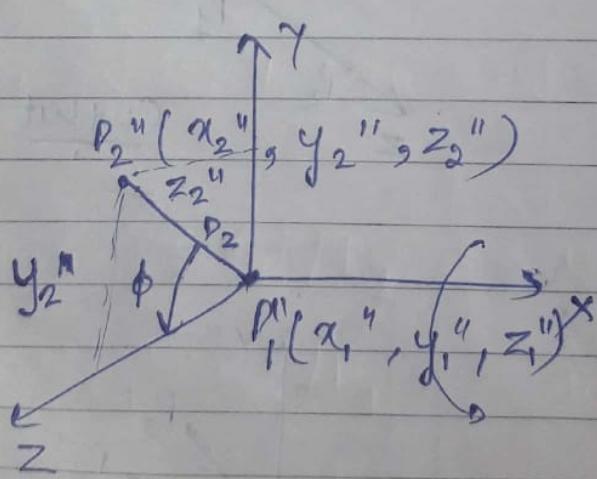


1st Rotation:
about y-axis
 $\Rightarrow P_1, P_2'$ on
y² plane
2nd Rota

$$D_1 = \sqrt{(z_2')^2 + (x_2')^2}, \sin(\theta - 90^\circ) = \frac{\cos\theta}{D_1} = \frac{x_2'}{D_1}, \cos(\theta - 90^\circ) = \sin\theta = \frac{z_2'}{D_1}$$

$$R_{y, -(90-\theta)} = \begin{bmatrix} z_2'/D_1 & 0 & -x_2'/D_1 & 0 \\ 0 & 1 & 0 & 0 \\ x_2'/D_1 & 0 & z_2'/D_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step (3) :



2nd Rotation:
Rotation about
x-axis
 \downarrow
 $P_1'' P_2''$
along
z-axis
(Anticlockwise)

$$D_2 = \sqrt{(y_2'')^2 + (z_2'')^2}$$

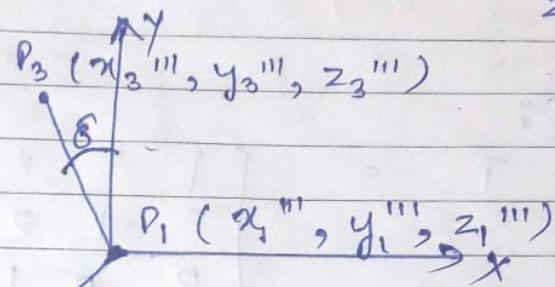
$$\sin\phi = \frac{y_2''}{D_2}$$

$$\cos\phi = \frac{z_2''}{D_2}$$

$$R_{x, \phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & z_2''/D_2 & -y_2''/D_2 & 0 \\ 0 & y_2''/D_2 & z_2''/D_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step ④ :

$P_1 P_2$ now on Z-axis but
 $P_1 P_3$ can now be on
 left or right of YZ plane
 (δ for P_3 can be +ve
 also, -ve also)

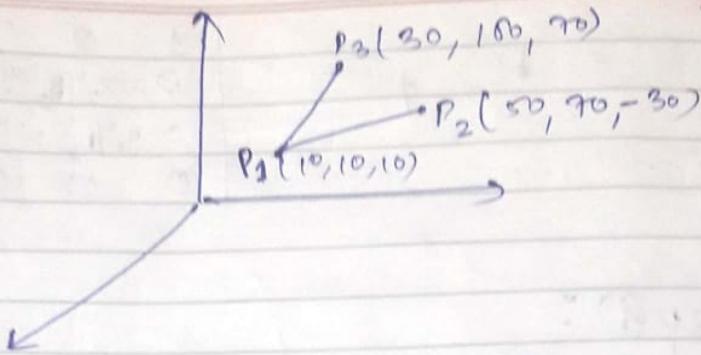


Say δ angle of rotation
 about Z-axis to bring P_3 in YZ
 plane. If already aligned, 0.

Rotation about
 Z-axis
 (anticlockwise)
 Hold
 Z-axis
 & rotate
 about it.

$$C = R_{z, \delta} * R_{x, \phi} * R_{y, -\theta} * T$$

Example:



Step①:

$$T = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_1' = (0, 0, 0)$$

$$P_2' = (40, 60, -40)$$

$$P_3' = (20, 90, 60)$$

Step②:

$$D_1 = \sqrt{1600 + 1600} = \sqrt{3200} = 10\sqrt{32} \\ = 40\sqrt{2}$$

$$\sin(0-90^\circ) = -\cos\theta = -z_2'/D_1 = -40/40\sqrt{2} = \underline{\underline{-\frac{1}{\sqrt{2}}}}$$

$$\cos(0-90^\circ) = \sin\theta = z_2'/D_1 = -40/40\sqrt{2} = \underline{\underline{-\frac{1}{\sqrt{2}}}}$$

By, $\theta = \begin{bmatrix} \sqrt{2} & 0 & -\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{2} & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$R_y, -\theta = \begin{bmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 40 & 20 \\ 0 & 60 & 90 \\ 0 & -40 & 60 \\ 1 & 1 & 1 \end{bmatrix}$$

↓

$$\Rightarrow \begin{bmatrix} 0 & 0 & -80/\sqrt{2} \\ 0 & 60 & 90 \\ 0 & 80/\sqrt{2} & -40/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$P_1''(0, 0, 0)$$

$$P_2''(0, 60, 80/\sqrt{2}) \Rightarrow P_2''(0, 60, 40\sqrt{2})$$

$$P_3''(-80/\sqrt{2}, 90, -40/\sqrt{2})$$

$$\Rightarrow (-40\sqrt{2}, 90, -20\sqrt{2})$$

Step ③:

$$D_2 = \sqrt{3600 + 8100} = \sqrt{11700} \\ = 10\sqrt{117}$$

$$D_2 = \sqrt{3600 + 3200} = \sqrt{6800} \\ = 10\sqrt{68} \\ = 20\sqrt{17}$$

$$\begin{aligned}\sin \phi &= 60 / 20\sqrt{17} = \frac{3\sqrt{17}}{\sqrt{17}} \\ \cos \phi &= 40\sqrt{2} / 20\sqrt{17} = \frac{2\sqrt{2}}{\sqrt{17}}\end{aligned}$$

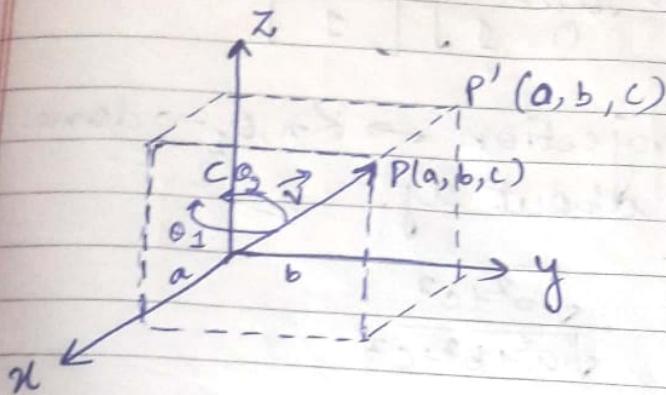
$R_{\alpha, \phi}$

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3D TRANSFORMATION (vector)



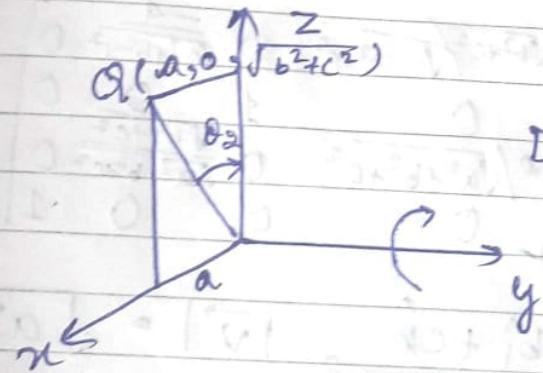
A_v^k : composite transformation to align the vector \vec{v} along k vector

$$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$$

OP' → Image of OP on yz plane

Projection in yz plane if rotation about x .

if rotation about x -axis anticlockwise



$$D = \sqrt{a^2 + b^2 + c^2}$$

$$\cos \theta_1 = c / \sqrt{b^2 + c^2}$$

$$\sin \theta_1 = b / \sqrt{b^2 + c^2}$$

$$R_{x, \theta_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 & 0 \\ 0 & \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/\sqrt{b^2+c^2} & -b/\sqrt{b^2+c^2} & 0 \\ 0 & b/\sqrt{b^2+c^2} & c/\sqrt{b^2+c^2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{-b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & \frac{-b}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix}$$

First part of projection $\rightarrow R_{x, \theta_1} \rightarrow$ done
Now, rotation about y .

$$\cos(-\theta_2) = \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\sin(-\theta_2) = \frac{-a}{\sqrt{a^2+b^2+c^2}}$$

$$R_{y, -\theta_2} = \begin{bmatrix} \sqrt{b^2+c^2}/\sqrt{a^2+b^2+c^2} & 0 & \frac{-a}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ a/\sqrt{a^2+b^2+c^2} & 0 & \sqrt{b^2+c^2}/\sqrt{a^2+b^2+c^2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}, |\vec{v}| = \sqrt{a^2+b^2+c^2}$$

Let $\lambda = \sqrt{b^2+c^2}$

$$\Rightarrow \begin{bmatrix} \lambda/|\vec{v}| & 0 & -a/|\vec{v}| & 0 \\ 0 & 1 & 0 & 0 \\ a/|\vec{v}| & 0 & \lambda/|\vec{v}| & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow R_{y, -\theta_2}$$

$$R_{x, \theta_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a/\lambda & -b/\lambda & 0 \\ 0 & b/\lambda & a/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

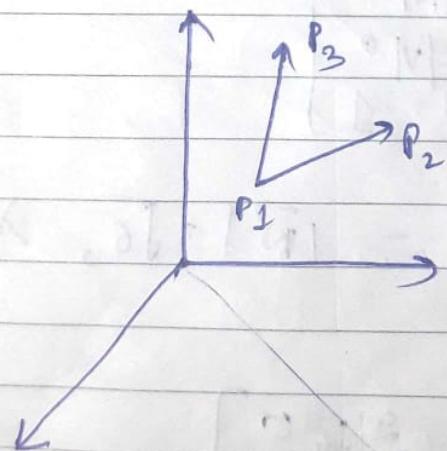
$$A_{\vec{v}}^{\vec{k}} = \begin{bmatrix} \lambda/v_1 & 0 & -a/v_1 & 0 \\ 0 & 1 & 0 & 0 \\ a/v_1 & 0 & \lambda/v_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/\lambda & b/\lambda & 0 \\ 0 & b/\lambda & c/\lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{\vec{v}}^{\vec{k}} = \begin{bmatrix} \lambda/v_1 & -ab/\lambda/v_1 & -ac/\lambda/v_1 & 0 \\ 0 & c/\lambda & -b/\lambda & 0 \\ a/v_1 & b/v_1 & c/v_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 1 Align vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ with the vector \vec{k} .

$$|\vec{v}| = \sqrt{3}, \quad \lambda = \sqrt{2}$$

$$A_{\vec{v}}^{\vec{k}} = \begin{bmatrix} \sqrt{2}/3 & 0 & -1/\sqrt{6} & -1/\sqrt{6} & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Now, we can directly use the transformation matrix to align $P_1 P_2$ along z -axis

Alignment of \vec{k} to vector \vec{v} : Reverse of previous

$$\begin{aligned}
 A_K^{\vec{v}} &= [A_V^K]^{-1} \\
 &= [R_{y, -\theta_2} \times R_{x, \theta_1}]^{-1} \\
 &= [R_{x, \theta_1}]^{-1} \times [R_{y, -\theta_2}]^{-1} \\
 &= R_{x, -\theta_1} \circ R_{y, \theta_2}
 \end{aligned}$$

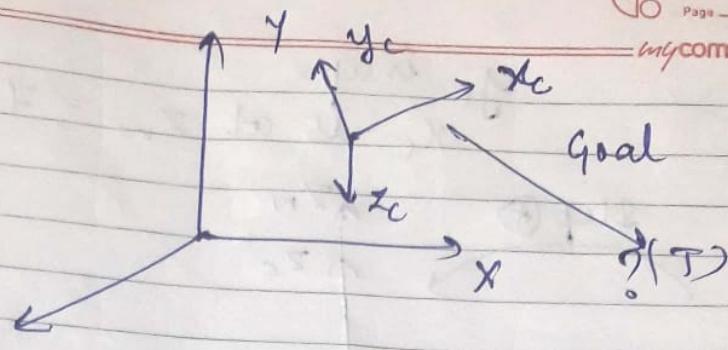
Question 2 Align vector $\vec{v} = i + j + k$ with the vector $\vec{N} = 2i - j - k$

$$A_V^{\vec{N}} = A_K^{\vec{N}} \times A_V^{\vec{k}}$$

$$\begin{bmatrix}
 \lambda/v | & 0 & a/v | & 0 \\
 -ab/\lambda/v | & c/\lambda & b/v | & 0 \\
 -ac/\lambda/v | & -b/\lambda & c/v | & 0 \\
 0 & 0 & 0 & -1
 \end{bmatrix} \leftarrow A_K^{\vec{v}}$$

$$\vec{N} = 2i - j - k, |\vec{N}| = \sqrt{6}, \lambda_N = \sqrt{2}$$

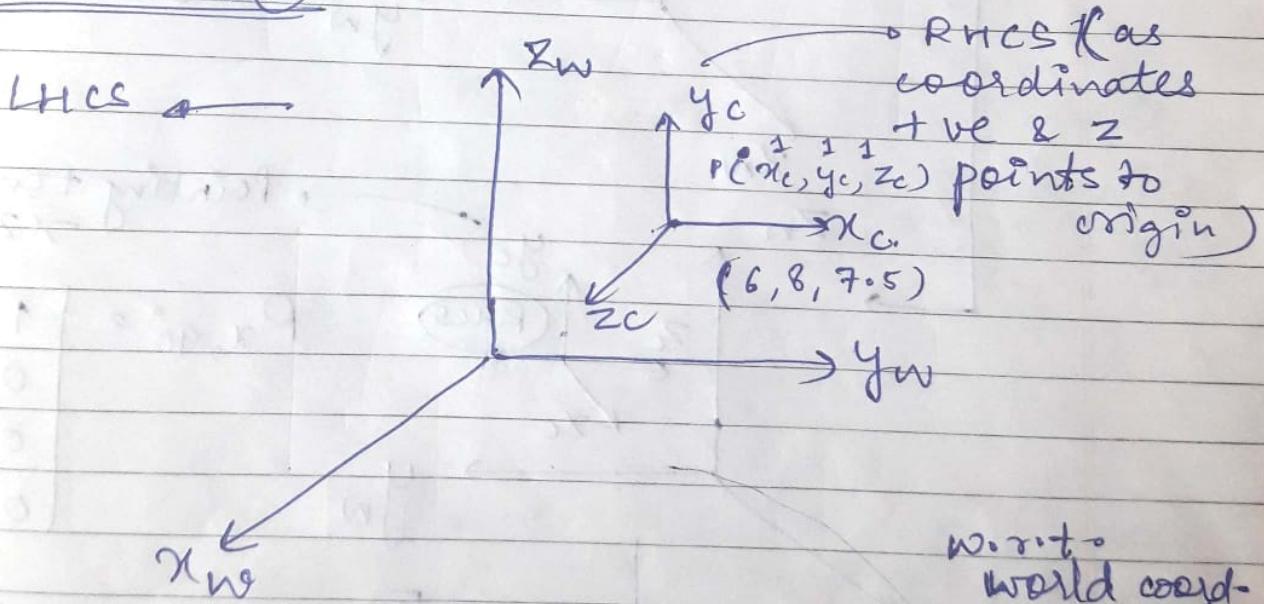
$$A_K^{\vec{N}} = \begin{bmatrix} 1/\sqrt{3} & 0 & 2/\sqrt{6} & 0 \\ 2/\sqrt{12} & -1/\sqrt{2} & -1/\sqrt{6} & 0 \\ 2/\sqrt{12} & 1/\sqrt{2} & -1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Goal ~~not~~ \rightarrow Transformation matrix to convert coordinate axis to x_c, y_c, z_c

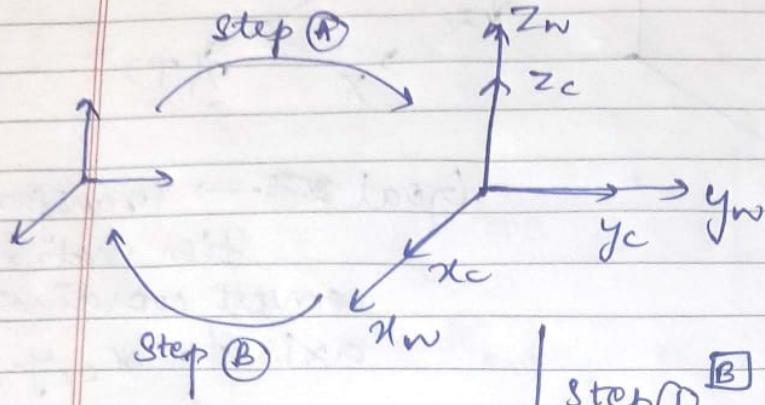
To see how an object looks in different coordinate axes.

EXERCISE (2):



A camera is positioned at $(6, 8, 7.5)$ w.r.t. world coordinate system. Point P is $(3, 1, 1)$ w.r.t. camera coordinate system. What will be point P w.r.t. world coordinate system. Given view direction is towards origin of wcs.

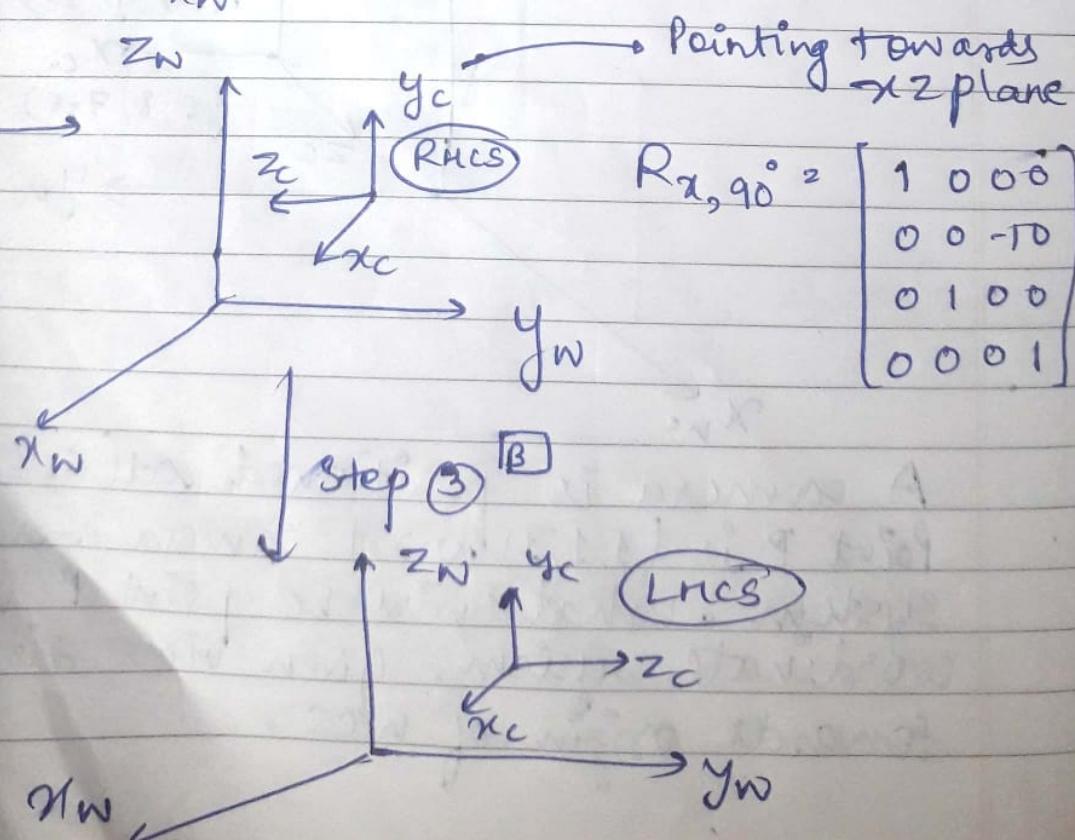
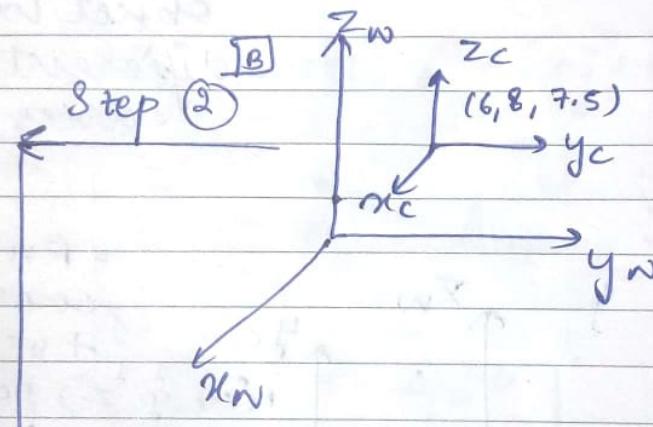
y_c is up
 x_c is at $z_w = 7.5$



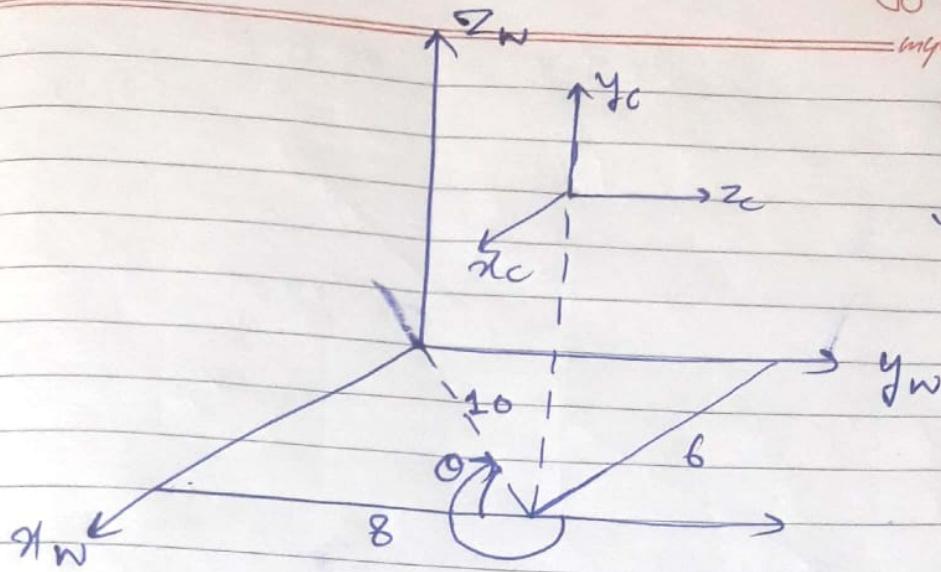
Step A & Step B
 (B) are inverse.
 Finding step B
 can give for step A

- (1) Translation
- (2) Rotate along x_c so that pts. towards -y
- (3) Rotate to point z_c towards z axis (edge)
- (4) Rotate to point z_c towards origin

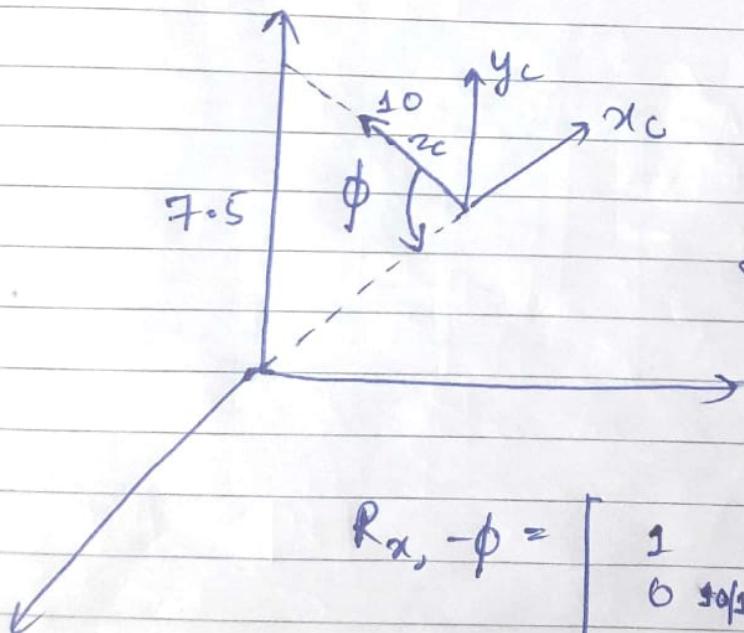
$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation
is
clockwise



$$R_y, -(180 + \theta) = \begin{bmatrix} -8/10 & 0 & 6/10 & 0 \\ 0 & 1 & 0 & 0 \\ -6/10 & 0 & -8/10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation is
clockwise

$$\sqrt{(7.5)^2 + (10)^2} = 12.5$$

$$R_x, -\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10/12.5 & 7.5/12.5 & 0 \\ 0 & 7.5/12.5 & 10/12.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

New
viewing
dir^n is
towards
origin

$$\cos(-\phi) = 10/12.5$$

$$\sin(-\phi) = -7.5/12.5$$

Camera \rightarrow Close images : blurred
There is a ^{too} far images also blurred
volume for clear vision.

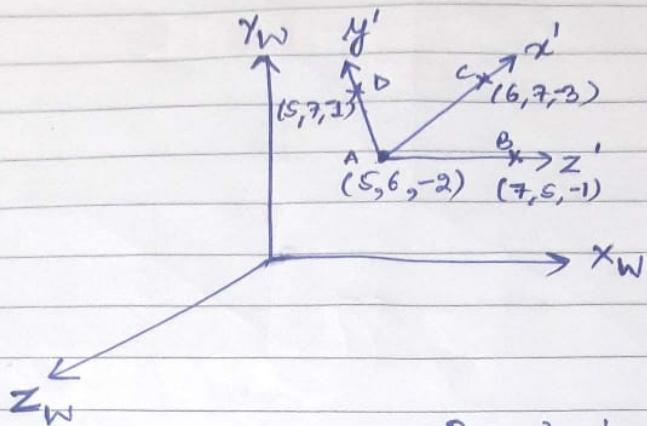
02/04/2018.

Answer: $\frac{-13}{\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}$
Date _____
Page _____

my companion

3D TRANSFORMATION

(Assignment)



$$P(x', y', z') = \{1, 2, 3\}$$

$$P_w \rightarrow ?$$

Step ①:

(Translation)

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordinates

after

translation:

$$A' \rightarrow (0, 0, 0)$$

$$B' \rightarrow (2, -1, 1)$$

$$C' \rightarrow (1, 1, -1)$$

$$D' \rightarrow (0, 1, 1)$$

$$P' \rightarrow (-4, -4, 5)$$

Step ②: Align z' axis

$$\vec{z}' = 2\hat{i} - \hat{j} + \hat{k}, |z'| = \sqrt{6}, \lambda = \sqrt{2}$$

$$A_{\vec{z}'} = \begin{bmatrix} \lambda/|z'| & -ab/\lambda|z'| & -ac/\lambda|z'| & 0 \\ 0 & c/\lambda & -b/\lambda & 0 \\ a/|z'| & b/|z'| & c/|z'| & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{Z'} = \begin{bmatrix} \sqrt{2}/\sqrt{6} & 2/\sqrt{12} & -2/\sqrt{12} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

New Values \Rightarrow $\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} A' & B' & C' & D' \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0 & 0 & \sqrt{3} \cdot 0 \\ 0 & 0 & 0 \cdot \sqrt{2} \\ 0 & \sqrt{6} & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

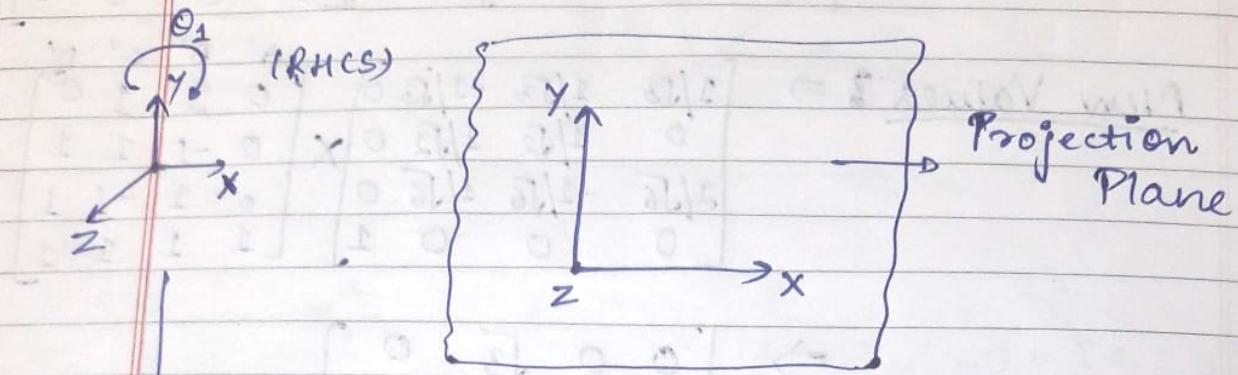
(All the axes are aligned)

Final coordinates of P:

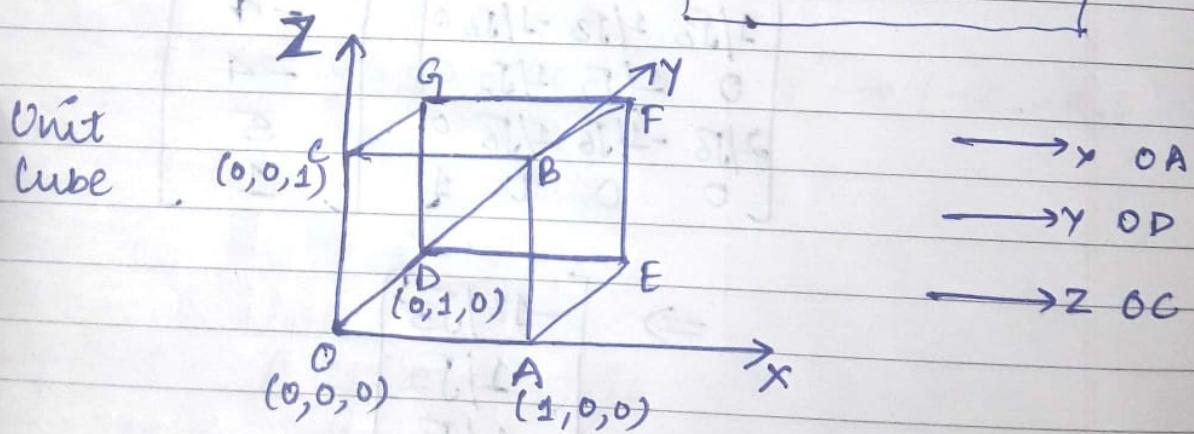
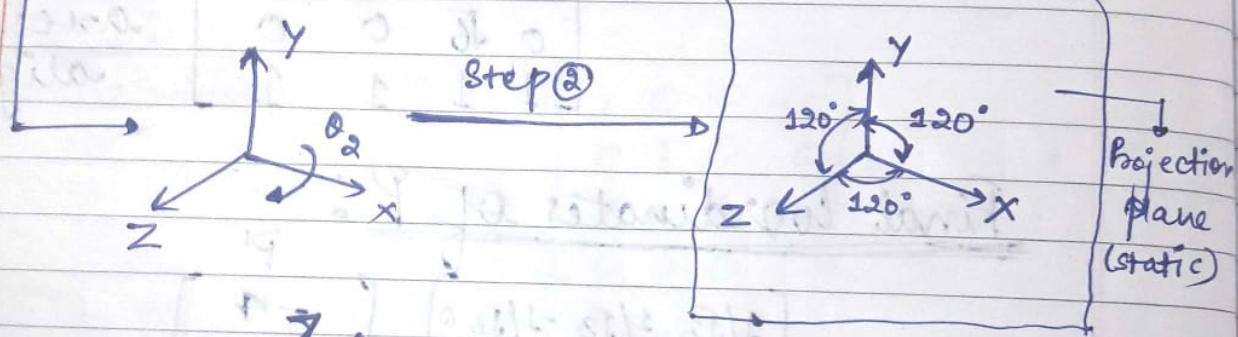
$$\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P' \\ -4 \\ -4 \\ 5 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -18/\sqrt{3} \\ 1/\sqrt{2} \\ 1/\sqrt{6} \\ 1 \end{bmatrix}$$

$$P_W \rightarrow (-18/\sqrt{3}, 1/\sqrt{2}, 1/\sqrt{6})$$

Isometric View

Step①



\rightarrow x OA
 \rightarrow y OD
 \rightarrow z OC

After 2 rotations:

$$A' = (x_x, y_x, 0)$$

$$D' = (x_y, y_y, 0)$$

$$C' = (x_z, y_z, 0)$$

$$f_x \ |OA| = 1$$

$$f_x' \ |OA'| = \sqrt{x_x^2 + y_x^2}$$

For shortening factor along x-axis = $\frac{f_x'}{f_x}$

Property for Isometric projection:

$$\boxed{\frac{f_x'}{f_x} = \frac{f_y'}{f_y} = \frac{f_z'}{f_z}}$$

$$R_{y, -\theta_1} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{x, \theta_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \text{Proj}_{x-y} * R_{x, \theta_2} * R_{y, -\theta_1}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1 \cos\theta_2 & \cos\theta_2 & -\sin\theta_2 \cos\theta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A' = (\cos\theta_1, -\sin\theta_1 \sin\theta_2, 0)$$

$$D' = (0, \cos\theta_2, 0)$$

$$C' = (-\sin\theta_1, -\sin\theta_2 \cos\theta_1)$$

Eqn ①:

$$\frac{fx'}{fx} = \frac{fy'}{fy}$$

$$fy' = |OD'| = \sqrt{\cos^2 \theta_2}$$

$$fy = |OD| = 1$$

$$\cos^2 \theta_1 + \sin^2 \theta_1 \sin^2 \theta_2 = \cos^2 \theta_2$$

Eqn ②:

$$\frac{fy'}{fy} = \frac{fz'}{fz}$$

$$\cos^2 \theta_2 = \sin^2 \theta_1 + \sin^2 \theta_2 \cos^2 \theta_1.$$

Add ① & ②:

$$1 + \sin^2 \theta_2 \cos^2 \theta_1 + \sin^2 \theta_1 \sin^2 \theta_2 = 1 + 2 \cos^2 \theta_2$$

$$1 + \sin^2 \theta_2 (\cos^2 \theta_1 + \sin^2 \theta_1) = 1 + 2 \cos^2 \theta_2$$

$$1 + \sin^2 \theta_2 = 2 \cos^2 \theta_2$$

$$8\sin^2 \theta_2 = -1 + 2\cos^2 \theta_2$$

$$\begin{aligned} 1 - \cos^2 \theta_2 &= -1 \\ 2 &= \cos^2 \theta_2 \\ \cos \theta_2 &= \end{aligned}$$

$$1 - \cos^2 \theta_2 = 2\cos^2 \theta_2 - 1$$

$$2 = 3\cos^2 \theta_2$$

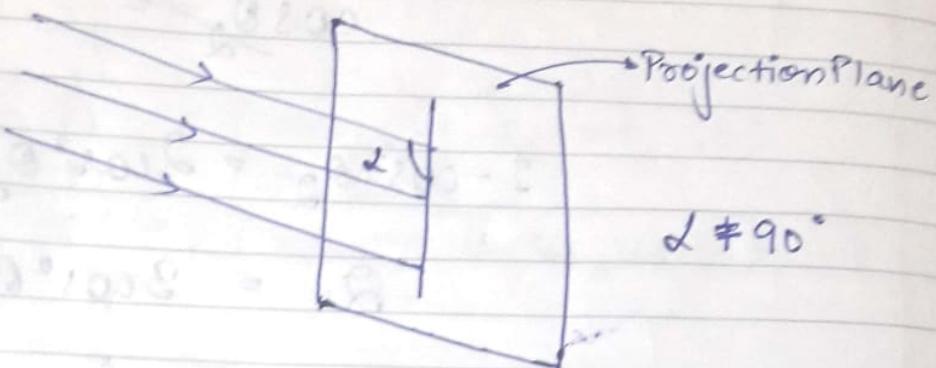
$$\cos \theta_2 = \pm \sqrt{\frac{2}{3}}$$

$$\boxed{\theta_2 = \pm 35.26}$$

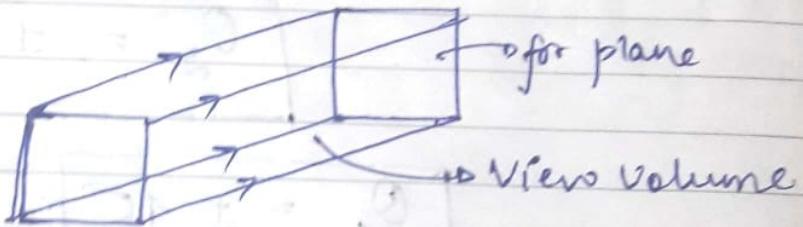
$$\boxed{\theta_1 = \pm 45^\circ}$$

OBLIQUE PROJECTION

Parallel
Projector
lines



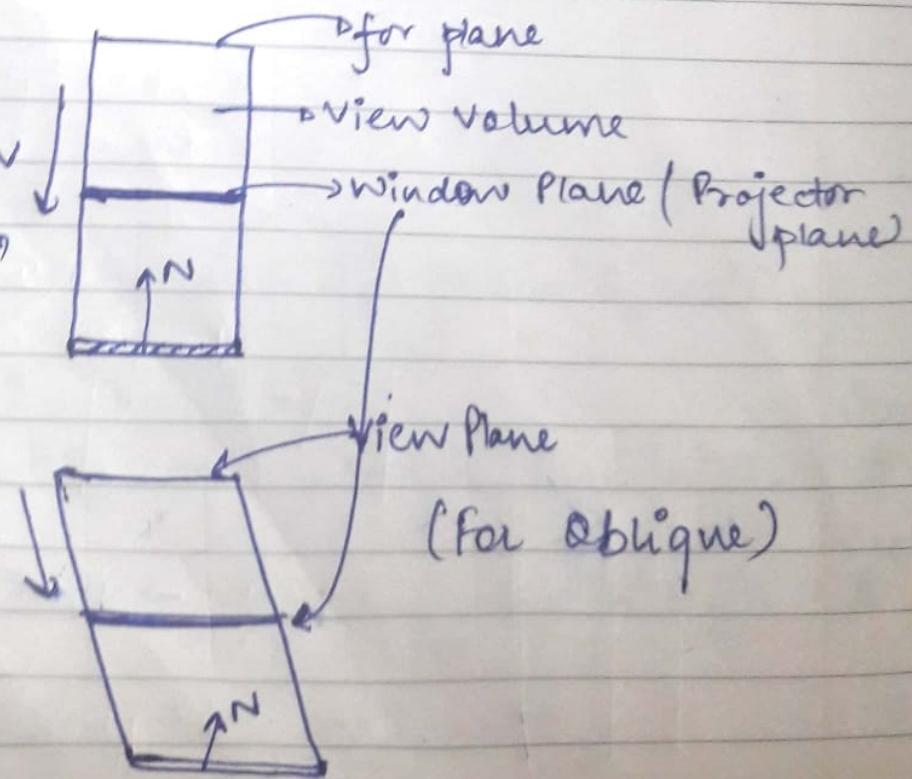
In 3D.:



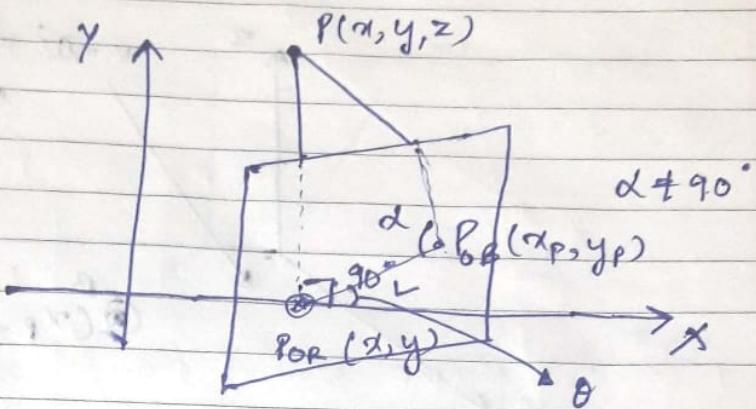
V: Viewing dirn

composite
transformation
is a sharing
operation

Projector
lines.



$$C = \text{Proj}(x-y) \times \vec{A_v}$$



$$x_p = x + L \cos \theta$$

$$y_p = y + L \sin \theta$$

$$\boxed{\tan \alpha = \frac{z}{L}}$$

$$L = \frac{z}{\tan \alpha}$$

$$L = z L_1 \quad (\text{let } L_1 = 1 / \tan \alpha)$$

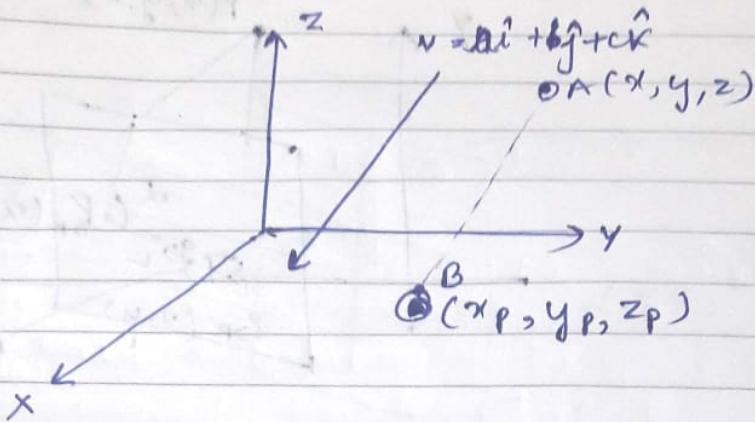
$$x_p = x + L_1 \cos \theta z$$

$$y_p = y + L_1 \sin \theta z$$

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & L_1 \cos \theta & 0 \\ 0 & 1 & L_1 \sin \theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

EXAMPLE 1:

Derive the equation of



$$\overrightarrow{AB} = (x_p - x)\hat{i} + (y_p - y)\hat{j} + (z_p - z)\hat{k}$$

$$\therefore \overrightarrow{AB} \parallel \vec{v}$$

$$(x_p - x)\hat{i} + (y_p - y)\hat{j} + (z_p - z)\hat{k} = t(a\hat{i} + b\hat{j} + c\hat{k})$$

$$\begin{aligned} \Rightarrow (x_p - x) &= at \\ (y_p - y) &= bt \\ (z_p - z) &= ct \end{aligned}$$

$$z_p \neq 0$$

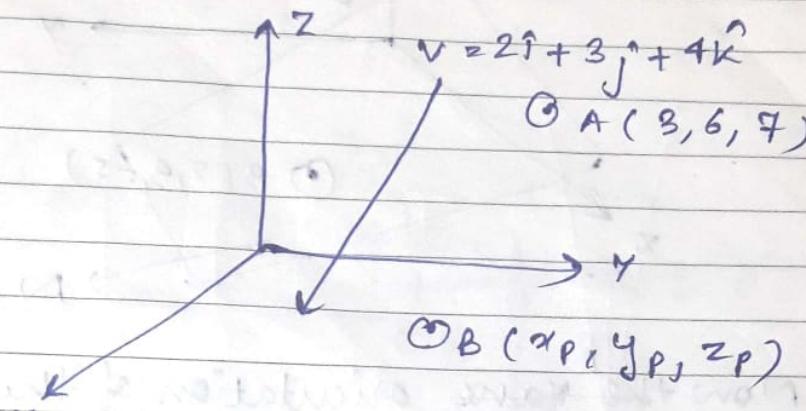
$$\Rightarrow -z = ct$$

$$\Rightarrow \boxed{t = -z/c}$$

$$x_p = x - \left(\frac{a}{c}\right)z$$

$$y_p = y - \left(\frac{b}{c}\right)z$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -a/c & 0 \\ 0 & 1 & -b/c & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

EXAMPLE 2:

Find B.

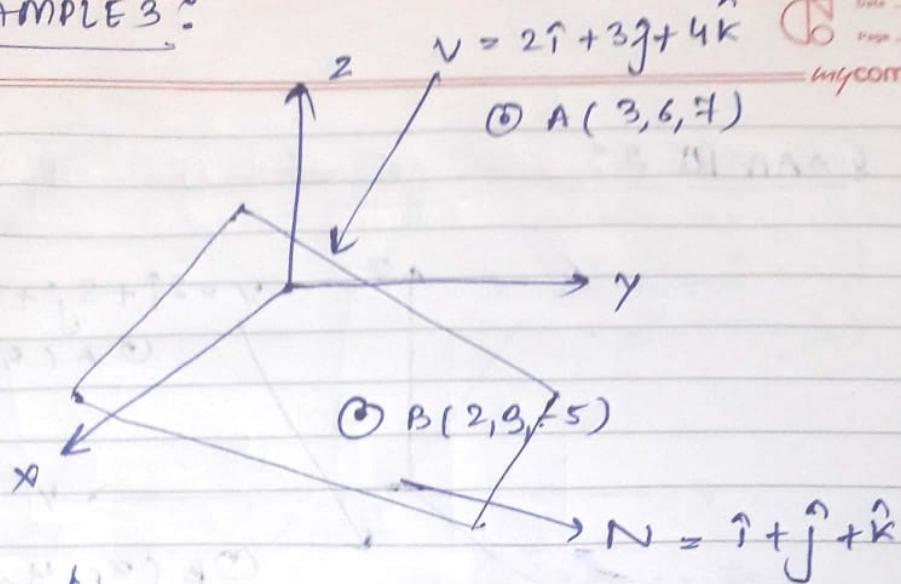
$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 7 \\ 1 \end{bmatrix}$$

$$(x_p, y_p, z_p) \rightarrow (-0.5, 0.75, 0)$$

$$B \rightarrow (-\frac{1}{2}, \frac{3}{4}, 0)$$

Now if $z = -30$, what is B?

EXAMPLE 3:



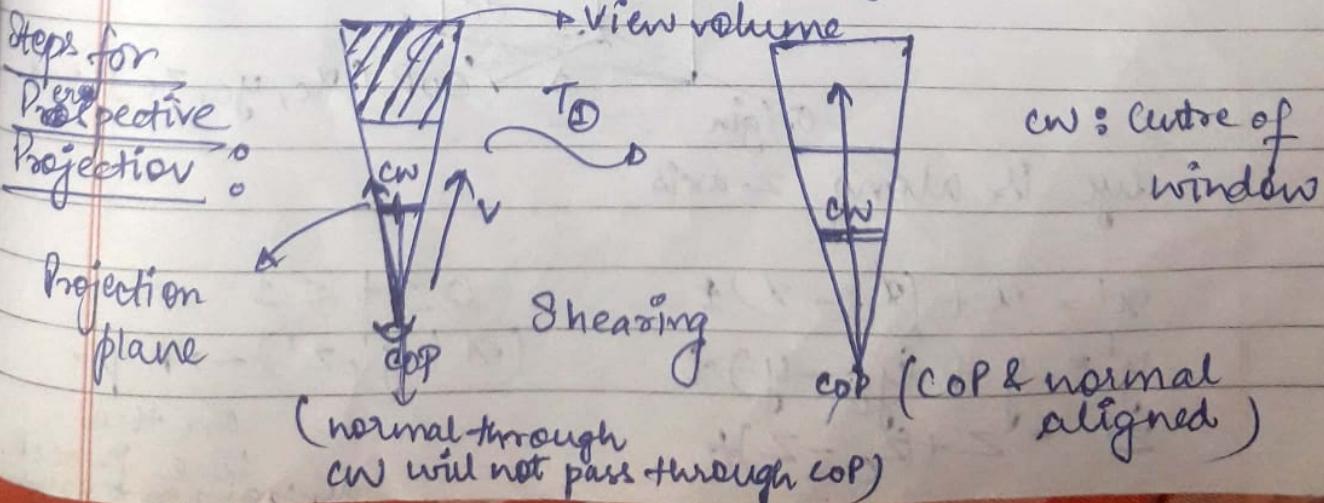
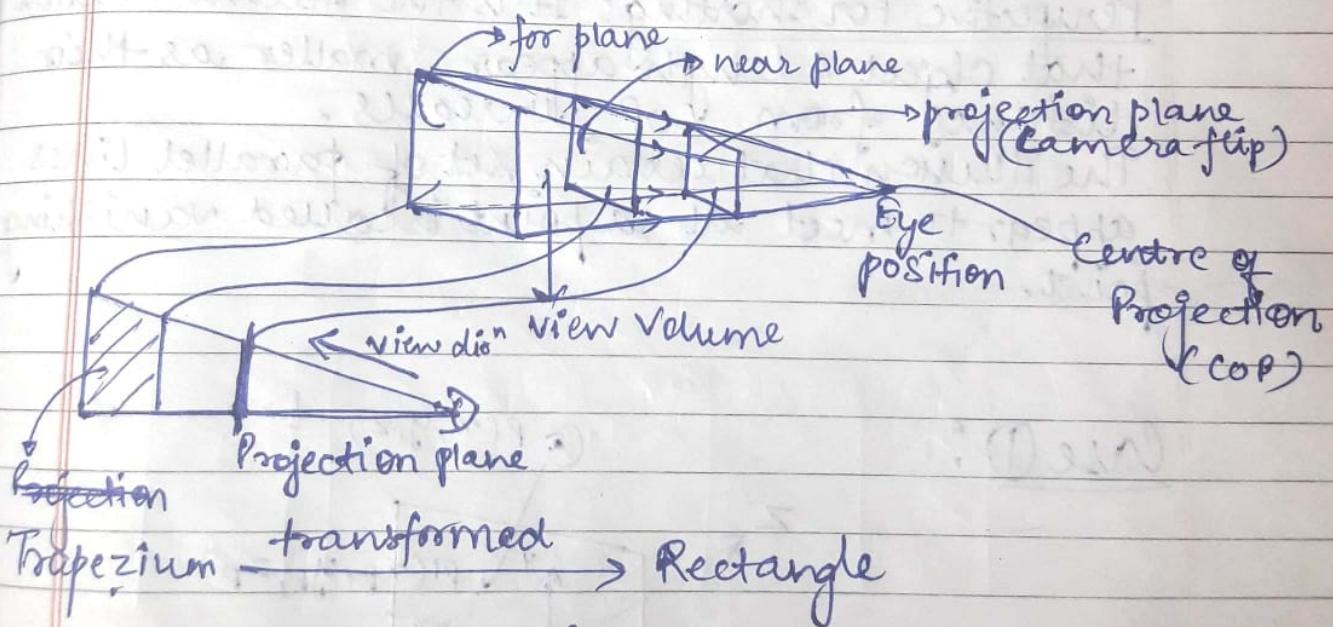
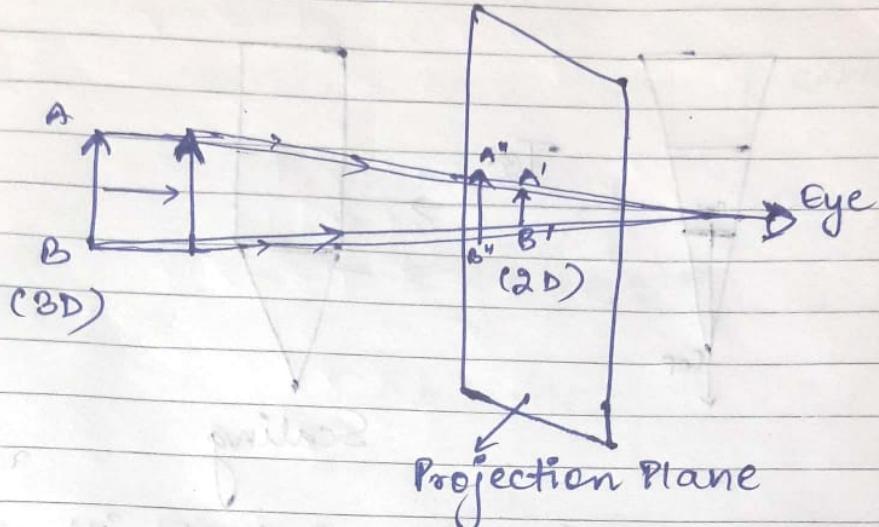
Now the plane orientation of the projection plane is changed. What will be B ?

$$\begin{aligned} & ((x_p - 3)\hat{i} + (y_p - 6)\hat{j} + (z_p - 7)\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) \\ &= (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) \end{aligned}$$

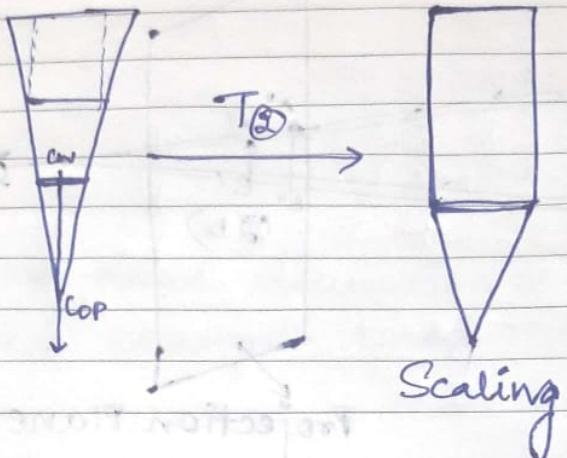
19/07/2018

Perspective Projection

(non-parallel projector lines, converging to a point)



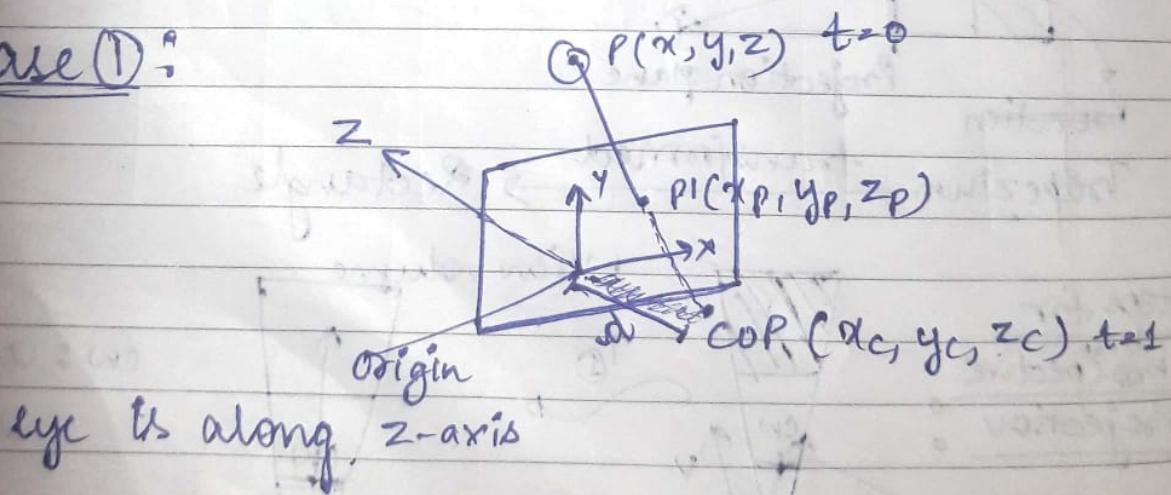
After this transformation, the trapezium becomes an isosceles trapezium.



Perspective For Shortening: It is the illusion that object & length appear smaller as their distance from COP increases.

The illusion that certain set of parallel lines appears to meet at a point is called vanishing point.

Case ①:



$$x' = x + (x_c - x)t$$

$$y' = y + (y_c - y)t$$

$$z' = z + (z_c - z)t$$

$$0 \leq t \leq 1$$

$$(x', y', z') \rightarrow \text{A pt}$$

on the
line $P \rightarrow \text{COP}$.
depending on t value

For a projection point at (x_p, y_p, z_p)

$$\therefore z_p = 0$$

$$x_c = 0, y_c = 0, z_c = -d \quad (\text{camera is on } z\text{-axis})$$

$$x_p = x + (-x)t$$

$$y_p = y + (-y)t$$

$$z_p = z + (-d+z)t$$

$$\boxed{t = z/(d+z)}$$

$$x_p = x - \frac{xz}{d+z} = \frac{xd}{d+z}$$

$$y_p = y - \frac{yz}{d+z} = \frac{yd}{d+z}$$

Let $h = d+z$ (homogenous factor)

$$\therefore x_h = x_p * h$$

$$y_h = y_p * h$$

$$x_h = xd$$

$$y_h = yd$$

$$h = d+z$$

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 1 & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Case (2): Projection plane is moved near to the object at a distance k from the origin, that is, projection plane is at $Z = k$

$$x_p = x + (-x)t$$

$$y_p = y + (-y)t$$

$$z_p = k = z - t(d - z)t$$

$$t = \frac{(z-k)}{d+z}$$

$$x_p = x - \frac{x(z-k)}{d+z} = \frac{x(d+k)}{d+z}$$

$$y_p = y - \frac{y(z-k)}{d+z} = \frac{y(d+k)}{d+z}$$

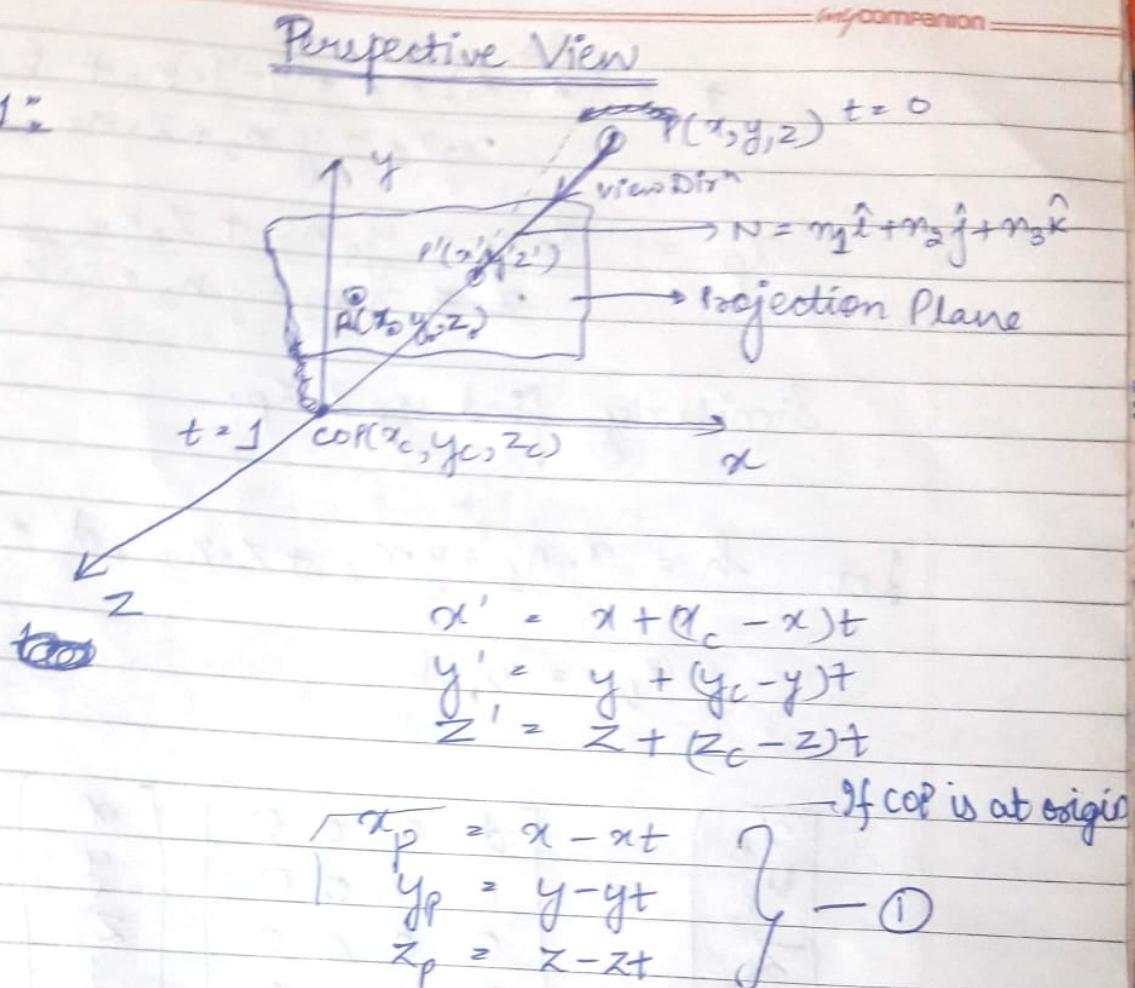
$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ 1_h \end{bmatrix} = \begin{bmatrix} d+k & 0 & 0 & 0 \\ 0 & d+k & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

12/09/18

IN Date _____
Page _____
my companion

Type 1:

COP at origin



$$x' = x + (x_c - x)t$$

$$y' = y + (y_c - y)t$$

$$z' = z + (z_c - z)t$$

if COP is at origin

$$\begin{cases} x_p = x - xt \\ y_p = y - yt \\ z_p = z - zt \end{cases} \quad \text{--- (1)}$$

$$\overrightarrow{P'R} = (x_p - z_o)\hat{i} + (y_p - y_o)\hat{j} + (z_p - z_o)\hat{k}$$

$$\therefore \overrightarrow{P'R} \perp \overrightarrow{N}$$

$$(x_p - x_o)n_1 + (y_p - y_o)n_2 + (z_p - z_o)n_3 = 0$$

$$(x - xt - x_o)n_1 + (y - yt - y_o)n_2 + (z - zt - z_o)n_3 = 0$$

$$-[x \cdot n_1 + y \cdot n_2 + z \cdot n_3]t + [(x - x_o)n_1 + (y - y_o)n_2 + (z - z_o)n_3] = 0$$

$$t = \frac{(x - x_0)n_1 + (y - y_0)n_2 + (z - z_0)n_3}{x \cdot n_1 + y \cdot n_2 + z \cdot n_3}$$

$$x_p = \frac{x_0n_1 + y_0n_2 + z_0n_3}{x \cdot n_1 + y \cdot n_2 + z \cdot n_3} \times x$$

Similarly find y_p & z_p .

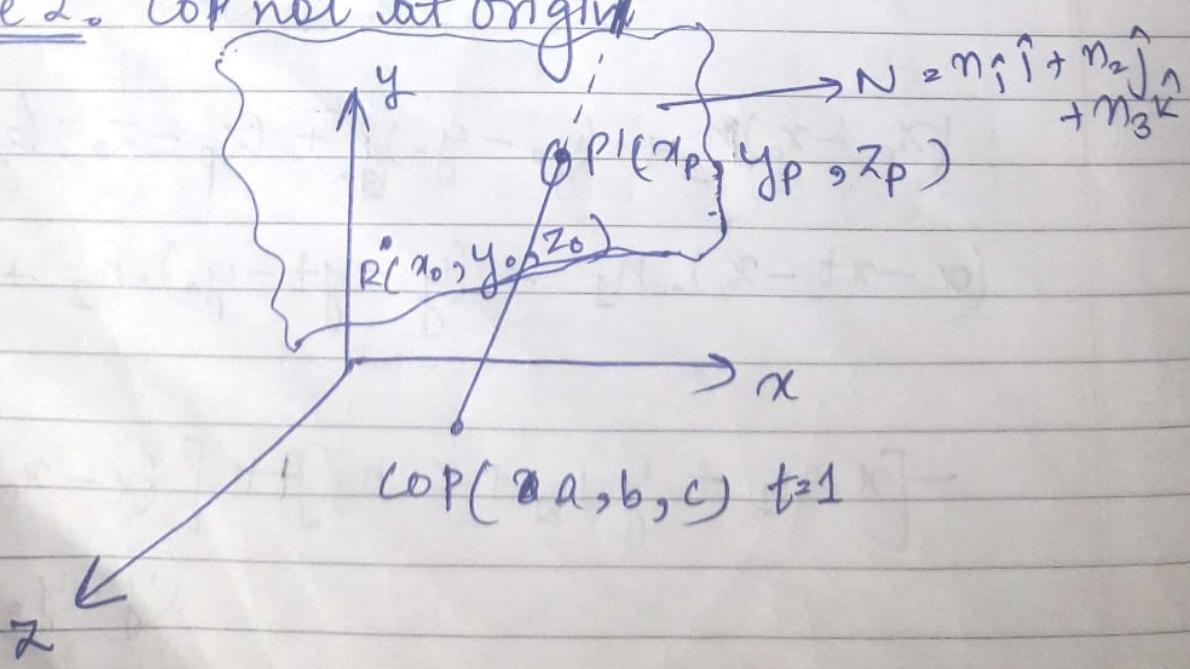
$$\text{Let } h = x \cdot n_1 + y \cdot n_2 + z \cdot n_3, d = x_0n_1 + y_0n_2 + z_0n_3$$

$$x_h = x_p \cdot h$$

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ n_1 & n_2 & n_3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P(x, y, z) \quad t=0$$

Type 2: COP not at origin



my companion

Parametric Eqn P - cop

$$\begin{aligned}x' &= x + (a-x)t \\y' &= y + (b-y)t \\z' &= z + (c-z)t\end{aligned}$$

For a point P

$$\begin{aligned}x_p &= x + (a-x)t \\y_p &= y + (b-y)t \\z_p &= z + (c-z)t\end{aligned}$$

$$\overset{\leftrightarrow}{P'R} \cdot \overset{\leftrightarrow}{N} = 0 \quad (\because \overset{\leftrightarrow}{P'R} \perp \overset{\leftrightarrow}{N})$$

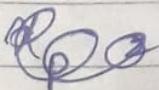
$$(x_p - x_0) \cdot n_1 + (y_p - y_0) \cdot n_2 + (z_p - z_0) \cdot n_3 = 0$$

$$t = \frac{(x - x_0) \cdot n_1 + (y - y_0) \cdot n_2 + (z - z_0) \cdot n_3}{(a - x) \cdot n_1 + (b - y) \cdot n_2 + (c - z) \cdot n_3}$$

$$= \frac{(x \cdot n_1 + y \cdot n_2 + z \cdot n_3) - (x_0 \cdot n_1 + y_0 \cdot n_2 + z_0 \cdot n_3)}{(a \cdot n_1 + b \cdot n_2 + c \cdot n_3)}$$

$$\left[\frac{(x \cdot n_1 + y \cdot n_2 + z \cdot n_3) - (x_0 \cdot n_1 + y_0 \cdot n_2 + z_0 \cdot n_3)}{(a \cdot n_1 + b \cdot n_2 + c \cdot n_3)} \right] \downarrow d_0$$

$$x_p = x + (a-x) \cdot \left[\frac{(x \cdot n_1 + y \cdot n_2 + z \cdot n_3) - (x_0 \cdot n_1 + y_0 \cdot n_2 + z_0 \cdot n_3)}{(a \cdot n_1 + b \cdot n_2 + c \cdot n_3)} \right]$$

 Find x_p, y_p, z_p & complete the matrix.

16th April, 2018

classmate

Date _____

Page _____

HIDDEN SURFACES

Backface Detection Method:

Tetrahedron

$$A(1,1,1)$$

$$B(3,1,1)$$

$$C(2,1,3)$$

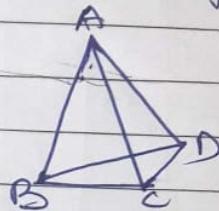
$$D(2,2,2)$$

Torch light is at $P(2,3,4)$

Find out which surfaces are illuminated & which are not illuminated (Backface)

Tetrahedron has 4 faces

ABC, BCD, CAD, ABD



Step ① Recognition of faces

Step ② find normal outward (Outward)

Edge	Outward	Mean position (centroid)
ABC	$\vec{AB} \times \vec{AC} = -\hat{j}$	$2\hat{i} + \hat{j} + 5/3\hat{k}$
BCD	$2\hat{i} + \hat{j} + \hat{k}$	$7/3\hat{i} + 4/3\hat{j} + 2\hat{k}$
ACD	$2\hat{i} - \hat{j} - \hat{k}$	$5/3\hat{i} + 4/3\hat{j} + 2\hat{k}$
ABD	$2\hat{i} - 2\hat{k}$	$2\hat{i} + 4/3\hat{j} + 4/3\hat{k}$

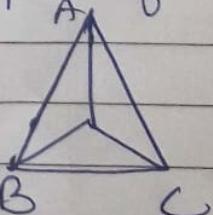
Step ③ (Centroid) mean position

Step ④ ($\vec{P}-\vec{M}$) (torch light pos to mean pos of every vector)

Edge	$(\vec{P}-\vec{M})$
ABC	$-2\hat{i} - 7/3\hat{k}$
BCD	$1/3\hat{i} - 5/3\hat{j} - 2\hat{k}$
ACD	$-1/3\hat{i} - 5/3\hat{j} - 2\hat{k}$
ABD	$-5/3\hat{j} - 8/3\hat{k}$

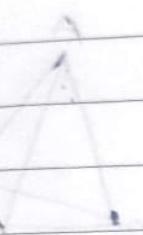
when

\vec{N} Outward & $\vec{P}-\vec{M}$ in same dirⁿ, that face won't be visible.



Step 5:

Check if direction of $\vec{P-M}$ & \vec{N} outward is same by using if $\vec{N} \cdot (\vec{P-M}) < 0$, visible
 > 0 , not visible



Frontal visibility need to be checked

$$\begin{aligned} & \vec{N} \cdot \vec{P-M} = \vec{N} \cdot \vec{P-M} \\ & \vec{N} \cdot \vec{P-M} + \vec{P-N} \\ & \vec{N} \cdot \vec{P-M} + \vec{N-P} \\ & \vec{N} \cdot \vec{P-M} + \vec{N-P} \end{aligned}$$

$$\begin{aligned} & \vec{N} \cdot \vec{P-M} = \vec{N} \cdot \vec{P-M} \\ & \vec{N} \cdot \vec{P-M} + \vec{P-N} \\ & \vec{N} \cdot \vec{P-M} + \vec{N-P} \\ & \vec{N} \cdot \vec{P-M} + \vec{N-P} \end{aligned}$$

Noticing down (biecto) (S)
not strong withdrawal (M-3) (D)

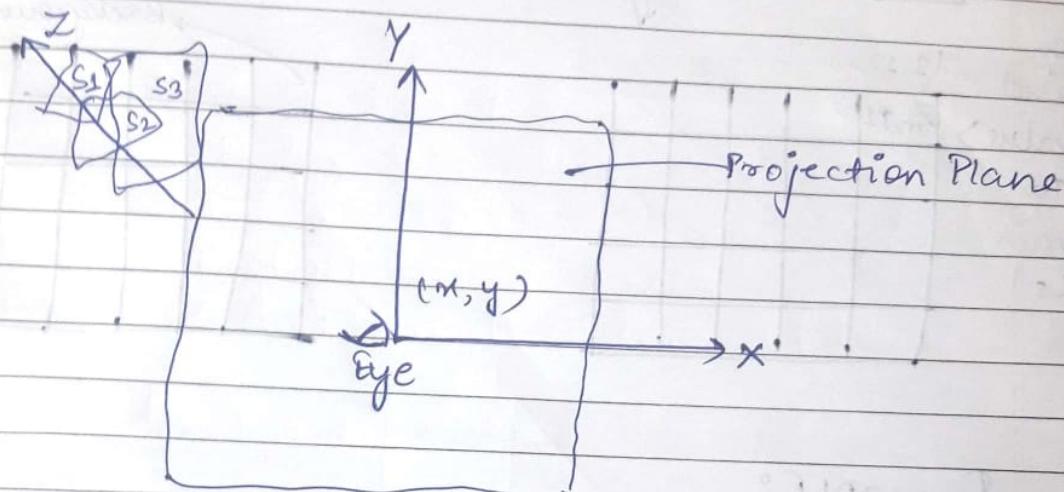
(M-3)

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HIDDEN SURFACES

- ① Backface Detection (3D object)
- ② Z-Buffer ^{image} (2D)



→ set of surfaces

$$S = \{ S_1, S_2, \dots, S_n \}$$

These are coordinates

$$S_1 (x_1, y_1, z_1)$$

$$S_2 (x_2, y_2, z_2)$$

$$S_3 (x_3, y_3, z_3)$$

Only min. values will appear

$$ax + by + cz + d = 0$$

$$z = -\frac{1}{c} [ax + by + d]$$

$$Z(x, y) = -\frac{1}{c} [ax + by + d]$$

Step ①:

$$Z(x+1, y) = Z(x, y) - \frac{a}{c}$$

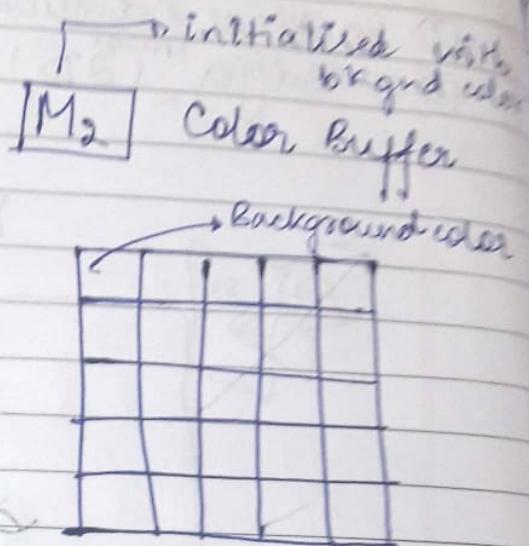
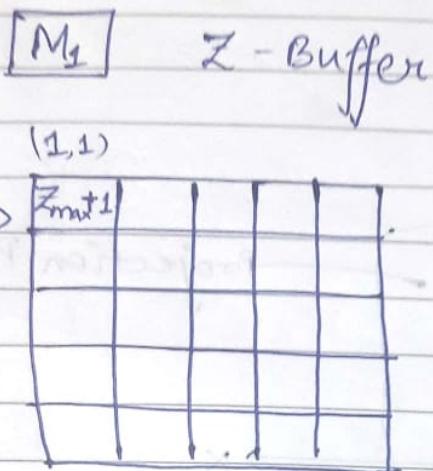
$$\{ x_{\min}, y_{\min}, z_{\min}; x_{\max}, y_{\max}, z_{\max} \}$$

(using S)

Step ②:

2D Buffer
(Z-values)

Every location initialised with value >
 Z_{\max} (to find min. of Z_s)



EXAMPLE:

Tetrahedron

A (1, 1, -1)
B (2, 1, -1)
C (2, 1, -3)
D (2, 2, -2)

Surface	color
ACD	RED(1)
CBD	BLUE(2)
BAD	CYAN(3)
ACB	GREEN(4)

Background BLACK(0)

Step ①:

$$x_{\min} = 1, y_{\min} = 1, z_{\min} = -3$$

$$x_{\max} = 3, y_{\max} = 2, z_{\max} = -1$$

Step ②: Find eqⁿ(es) of the planes.

Plane ACD:

$$\vec{AC} = \hat{i} + -2\hat{k}, \vec{AD} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{AC} \times \vec{AD} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2 \\ 1 & 1 & -1 \end{bmatrix} = 2\hat{j} + \hat{k}$$

$$\Rightarrow 2x\hat{j} + z = 0$$

$$\boxed{Z_{ACD} = -2x + y}$$

Plane CBD:

$$\vec{CB} = \hat{i} + 2\hat{k}, \vec{CD} = \hat{j} + \hat{k}$$

$$\vec{CB} \times \vec{CD} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} = -2x - y + z$$

$$\boxed{a = -2, b = -1, c = 1}$$

$$\begin{aligned} ax + by + cz &= d \\ -2(2) \quad 1(1) + 1(-3) &= d \\ \Rightarrow d &= -8 \\ \Rightarrow -2x - y + z &= -8 \end{aligned} \Rightarrow \boxed{Z_{CBD} = 2x + y - 8}$$

Plane BAD:

$$\vec{BA} = -2\hat{i}, \vec{BD} = -\hat{i} + \hat{j} - \hat{k}$$

$$\vec{BA} \times \vec{BD} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} = -2\hat{j} - 2\hat{k} \quad (d = 0)$$

$$\Rightarrow \boxed{-2y - 2z = 0}$$

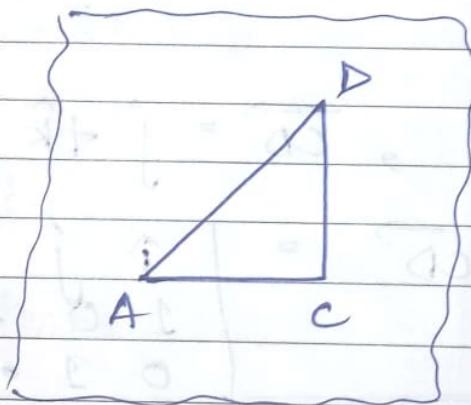
$$\Rightarrow \boxed{Z_{BAD} = -y}$$

Plane A_cB:

Step 2: (Image)

A_cD (Image):

$$A(1,1), C(2,1), D(2,2)$$



Inside Test: To determine if a pt. is inside or outside
 ≥ 0 , if inside. < 0 , if outside

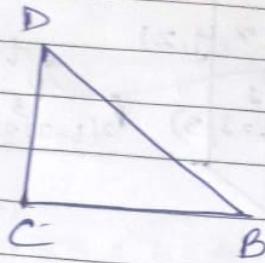
* line eq AC $y=1$ $f(x,y) : y - 1 \geq 0$

CD $x=2$ $f(x,y) : 2 - x \geq 0$

AD $y=x$ $f(x,y) : x - y \geq 0$

CBD (Image)

C(2, 1) B(3, 1), D(2, 2)



line eq CB $y = 1$ $f(x, y)_B : y - 1 \geq 0$

line eq BD $x + y = 4$ $f(x, y)_C : 4 - x - y \geq 0$

line eq DC: $x = 2$ $f(x, y)_D : x - 2 \geq 0$

x	y	z	Inside Test	
1	1	$Z_{ACD} = -1$ $Z_{BCD} = -5$	B T F	ACD (RED)
1	2	$Z_{ACD} = 0$ $Z_{BCD} = -4$	F F	Background
2	1			
2	2			
3	1			
3	2			

19/04/2018

- Font designing
- Automobile
- Ship building

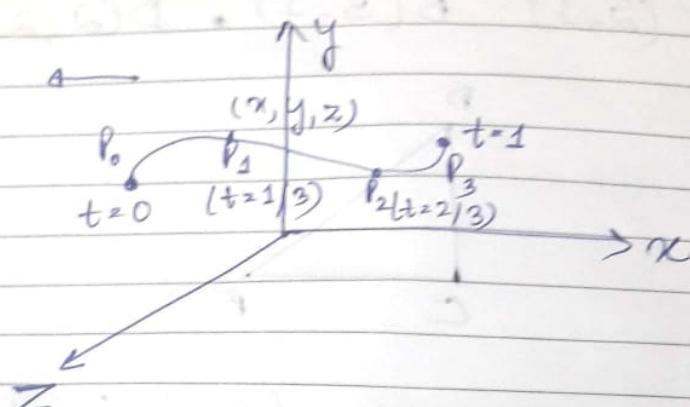
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3D CURVES

For
Cubic



$n=1$ line.
 $n=2$ Quadratic
 $n=3$ cubic curve
 $n=4$ quadratic space curve

Parametric form (curve segment)

$$P(t) = \{x(t), y(t), z(t)\} \quad 0 \leq t \leq 1$$

a_0, a_1, \dots
↓
constants

$$x(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n$$

$$y(t) = \dots$$

$$z(t) = \dots$$

For cubic curves ($n=3$), there are 4 control points. They are responsible for giving the shape to the curve, that is they control the shape of the curve.

Cubic curve segment

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$P(t) = \vec{a}t^3 + \vec{b}t^2 + \vec{c}t + \vec{d}$$

Page
Parametric
Form

$$P_0 = \vec{d}$$

$$P_1 = \frac{\vec{a}}{27} + \frac{1}{9}\vec{b} + \frac{1}{3}\vec{c} + \vec{d}$$

$$P_2 = \frac{b\vec{a}}{27} + \frac{4}{9}\vec{b} + \frac{2}{3}\vec{c} + \vec{d}$$

$$P_3 = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

$$\begin{pmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \\ \vec{d} \end{pmatrix} = \begin{pmatrix} -1/2 & 27/2 & -27/2 & 9/2 \\ 9 & -45/2 & 18 & -9/2 \\ -11/2 & 9 & -9/2 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

[By inverse of coefficients]

$$\begin{aligned}
 P(t) = & \left(-\frac{9}{2}P_0 + \frac{27}{2}P_1 - \frac{27}{2}P_2 + \frac{9}{2}P_3 \right) \vec{a}t^3 \\
 & + \left(9P_0 - \frac{45}{2}P_1 + 18P_2 - \frac{9}{2}P_3 \right) \vec{b}t^2 \\
 & + \left(-\frac{11}{2}P_0 + 9P_1 - \frac{9}{2}P_2 + P_3 \right) \vec{c}t \\
 & + P_0
 \end{aligned}$$

for $t=0$ to 1 skip of 0.0001

$$P(t) = \left[-\frac{9t^3}{2} + 9t^2 - \frac{11t}{2} + 1 \right] P_0 +$$

$$\left[\frac{27t^3}{2} - \frac{45t^2}{2} + 9t \right] P_1 +$$

$$\left[-\frac{27t^3}{2} + 18t^2 - \frac{9t}{2} \right] P_2 +$$

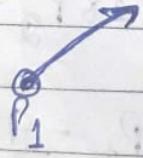
$$\left[\frac{9t^3}{2} - \frac{9t^2}{2} + t \right] P_3$$

→ Used in 'font' designing.

Problem 1: (Solⁿ given by Hermite).

A stone is thrown from a point P_0 with dirⁿ & magnitude. The stone is received at Point P_3 . It has 6a location as well as dirⁿ & magⁿ.

Dirⁿ &
Magn



Direction & magnitude



find the
parametric
form
of eqⁿ of
curve.

$$P_0(t=0) = \vec{d}$$

$$P_3(t=1) = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

$$P'(t) = 3\vec{a}t^2 + 2\vec{b}t + \vec{c}$$

$$P'_0(t=0) = \vec{c}$$

$$P'_3(t=1) = 3\vec{a} + 2\vec{b} + \vec{c}$$

$$P_3 = \vec{a} + \vec{b} + \vec{c} + P_0$$

$$P_3 = \vec{a} + \vec{b} + P_0' + P_0$$

$$\vec{a} = P_3 - \vec{b} - P_0' - P_0$$

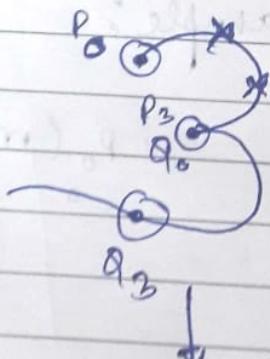
 P_3 P_1 P_2 P_0

! HERMITE

$$\begin{pmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \\ \vec{d} \end{pmatrix} = \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_0' \\ P_1' \end{pmatrix}$$

for $t = 0$ to 1 step 0.0001

$$P(t) = \underline{\hspace{1cm}}$$



This font can
be drawn
with 3
curves

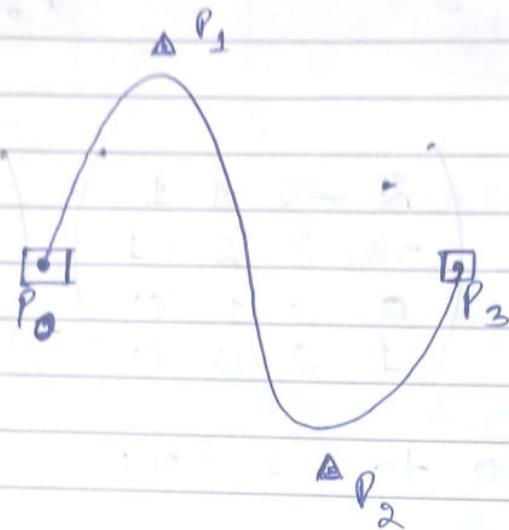
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BEZIER CURVE

(inventor → Pierre Bezier)

Cubic Curve:



△ Controlling point

□ End point

Example:

$P_0 (m_0)$

$P_1 (m_1)$

[P]

$P_2 (m_2)$

$P_3 (m_3)$

P =
(centre of mass)

$$P = \frac{m_0 P_0 + m_1 P_1 + m_2 P_2 + m_3 P_3}{m_0 + m_1 + m_2 + m_3}$$

- If point masses are fixed, the centre of mass is fixed.
- If point masses are variable, the centre of mass changes in value.

Bezier → Put masses as binomial functions

$$m_0 = (1-t)^3, m_1 = 3t(1-t^2), m_2 = 3t^2(1-t)$$

$$m_3 = t^3 \quad (0 \leq t \leq 1)$$

$$P(t) = \sum_{i=0}^3 P_i \cdot B_{i,3}(t)$$

Blending Function.

$$= P_0 \cdot B_{0,3}(t) + P_1 B_{1,3}(t) + P_2 \cdot B_{2,3}(t) + P_3 \cdot B_{3,3}(t)$$

$$P(t) = P_0 (1-t)^3 + P_1 \cdot 3t(1-t^2) + P_2 \cdot 3t^2(1-t) + P_3 \cdot t^3$$

$$= P_0 [-t^3 + 3t^2 - 3t + 1] P_0 +$$

$$[3t^3 - 6t^2 + 3t] P_1 +$$

$$[-3t^3 + 3t^2] P_2 +$$

$$[t^3] P_3$$

$$P_B(t) = T \cdot M_B \cdot G_B$$

for $t = 0$ to 1 skip = 0.0001

$$P_B(t) = [t^3 t^2 t 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ -3 & 6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} \rightarrow G_B$$

Rel^m b/w Beizer & Hermite:

(Transformation from T)

Hermite
to
Beizer

$$M_H \quad G_H$$

$$P_H(t) = [t^3 t^2 t 1] \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$G_H = M_{H-B} G_B$$

Transformation
Matrix ~~for~~
Hermite to
Beizer

$$G_H = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} =$$

Beizer:

$$P'(t) = [-3t^2 + 6t - 3] P_0 + [9t^2 - 12t + 3] P_1$$

$$+ [-9t^2 + 6t] P_2$$

$$+ 3t^2 P_3$$

$$P'(t=0) = -3P_0 + 3P_1$$

$$P'(t=1) = -3P_2 + 3P_3$$

$$G_H = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$G_H = M_{H-B} \cdot G_B$$

$$P_H(t) = T \cdot M_H \cdot G_H$$

$$= T \cdot M_H \cdot (M_{H-B} \cdot G_B)$$

$$= T \cdot (M_H \cdot M_{H-B}) \cdot G_B$$

To prove this

$$= T \cdot (M_B) \cdot G_B = P_B(t)$$

$$\begin{bmatrix} M_H \\ M_{H-B} \\ (M_B) \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & 6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(verified)

Question 1 Find eqn of Bezier curve which passes through pts $(0,0)$ & $(-2,1)$ and is controlled through pts $(7,5)$ & $(2,0)$.

$$P_B(t) = T \cdot M_B \cdot G_B$$

I eqn:

$$\begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 0 \\ -2 & 1 \end{bmatrix}$$

II Egn:

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 7 & 5 \\ -2 & 1 \end{bmatrix}$$

segment

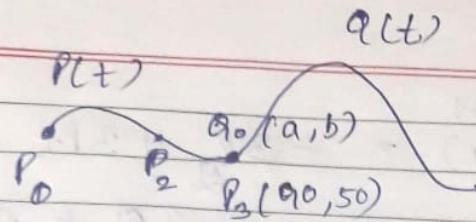
Ques 2 Cubic Beizer curve is described by pts:-

$$P_0(20, 20), P_1(40, 80), P_2(80, 80), P_3(90, 50)$$

Another curve segment is described as

$$Q_0(a, b), Q_1(c, 20), Q_2(150, 20), Q_3(180, 20)$$

Determine the value of a, b, c if the 2 curves are joined smoothly.



(Tangent at end pt. of joining \rightarrow equal)
 \Rightarrow Tangent at P_2, Q_0 is same.

\therefore Equate $P'(t=1)$ with $Q'(t=0)$

$$P'(t=1) = -3P_2 + 3P_3$$

$$Q'(t=0) = -3Q_0 + 3Q_1$$

$$\boxed{a = Q_0, b = 50}$$

$$-3 \times 80 + 3 \times 90 = -3 \times Q_0 + 3 \times C$$

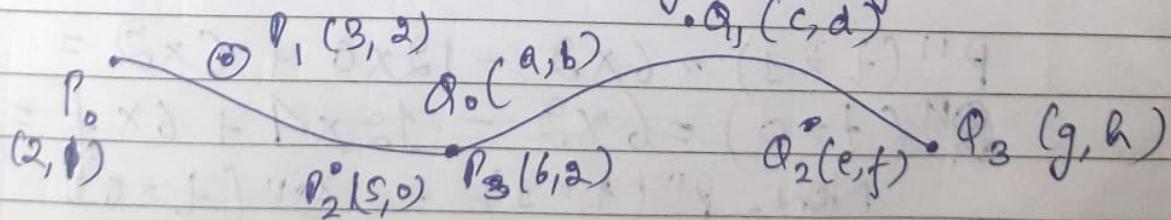
$$-120 = -120 + 3C \quad 30 = -270 + 3C$$

$$30 = -120 + 3C \Rightarrow 3C = 300$$

$$\boxed{C = 100}$$

Given 2 curve segments $P(t)$ & $Q(t)$.

where The 2 curve segments join smoothly.



Find a, b, c, d, e, f, g, h .

$$\boxed{a = 6, b = 2}$$

$$P'_0(t=0) = -3P_2 + 3P_1 = -3 \times 5 + 3 \times 6$$

$$Q'(t=0) = -3Q_0 + 3Q_1 = -3 \times 6 + 3 \times 5$$

$$\Rightarrow -3 \times 5 + 3 \times 6 = -3 \times 6 + 3 \times C$$

$$\Rightarrow 3 = -18 + 3C$$

$$\Rightarrow 21 = 3C$$

$$\Rightarrow C = 7$$

$$-3 \times 0 + 3 \times 2 = -3 \times 2 + 3 \times d$$

$$\Rightarrow d = 4$$

$$P''(t=0) = \begin{bmatrix} -6t+6 \\ -18t+6 \end{bmatrix} P_0 + \begin{bmatrix} 18t-12 \\ 6t \end{bmatrix} P_1$$

~~(Q''(t=0))~~

$$P''(t=1) = 6P_1 - 12P_0 + 6P_2$$

$$P''(t=1) = 6 \times 3 - 12 \times 5 + 6 \times 6 = -6$$

$$Q''(t=0) = 6Q_0 + -12Q_1 + 6Q_2$$

$$Q''(t=0) = 6 \times 6 - 12 \times 7 + 6 \times e = -48 + 6e$$

$$\text{Now, } P''(t=1) = Q''(t=0)$$

$$\Rightarrow -6 = -48 + 6e$$

$$\Rightarrow 6e = 42 \Rightarrow e = 7$$

$$P''(t=1) = 6 \times 2 - 12 \times 0 + 6 \times 2 = 24.$$

$$Q''(t=0) = 6 \times 2 - 12 \times 4 + 6 \times f = -36 + 6f$$

$$\text{Now, } P''(t=1) = Q''(t=0)$$

$$\Rightarrow 24 = -36 + 6f$$

$$\Rightarrow 60 = 6f$$

$$\Rightarrow f = 10$$

To find (g, h), use 3rd derivative $[P'''(t) & Q'''(t)]$.

B-Spline

Natural Spline:

Hermite
Bezier

Disadvantages:

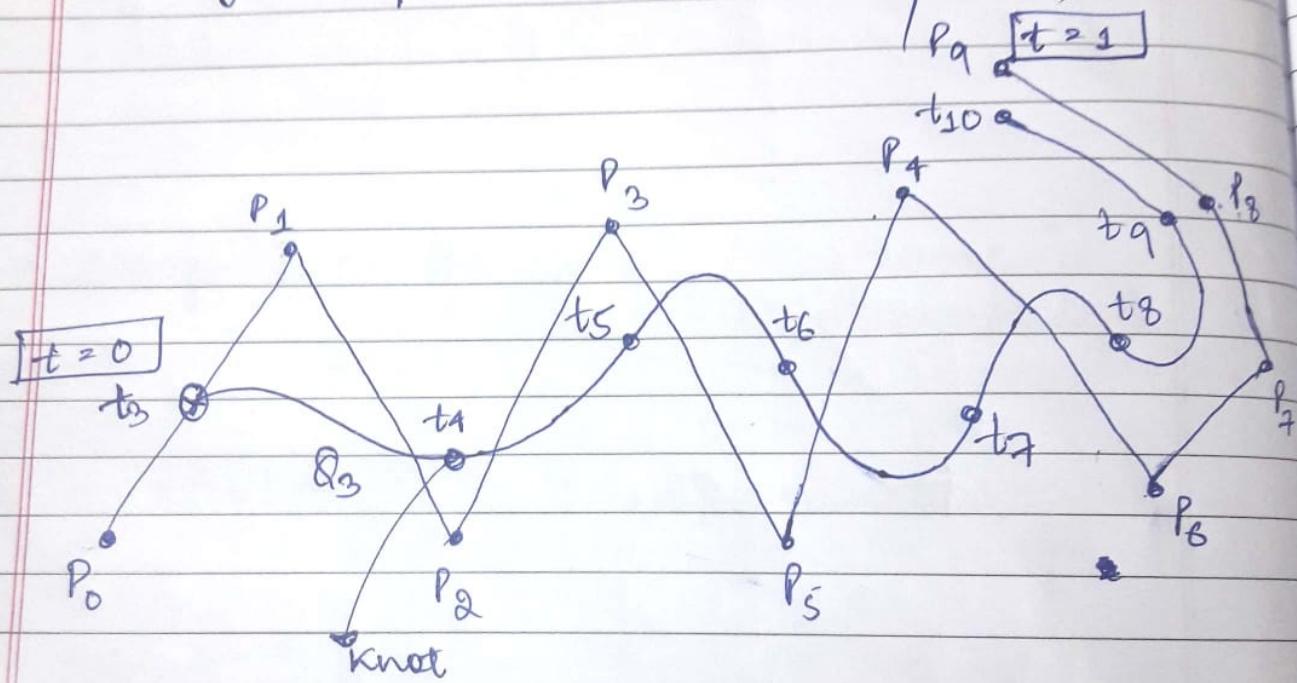
- ① Polynomial coefficients are dependent on all points (control & end points).
- Any movement of these points changes the shape of the entire curve.
- local control / end points (if $n+1$ control points, then changing any one changes all parts of curve)
- ② Computation time needed is large, Since it involves inverting of matrix.

Basis: Local Control / end points (Thus, the name B-Spline)

- for $n+1$, inverse needed of $n+1 \times n+1$ matrix every time we change even 1 point.
if local control pts., only some (local) control pts. need to be changed.

Global variables → affect whole fⁿ
 Local variables → "Only one f.ⁿ".

Similarly, we want to change only a part of curve! To advocate this disadvantage, we take a subset of all & say they affect only one part. (Local control
 end points)



B-Spline:

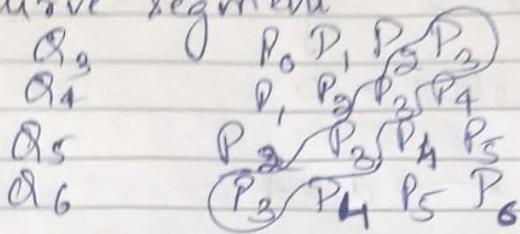
$$P(t) = \sum_{i=0}^n P_i * \text{Blending fn.}$$

$$0 \leq t \leq 1$$

$$\{t_0, t_1, t_2, \dots, t_n\}$$

If we see as cubic curve, 4 pts. needed.
 P_0, P_1, P_2, P_3 .

Curve segment



$$P(t) = \sum_{i=0}^n P_i * \text{Blending fn}$$

$t_{\min} \leq t \leq t_{\max}$

$$t_3 - t_4$$

$$t_{\max} - t_{\min}$$

In closed curve, every control point controls 4 curves.

e.g.: t_3 comes in Q_3, Q_4, Q_5, Q_6

if there are $(n+1)$ control pts., then there will be $(n-2)$ curve segments.

Blending fn

$t_3, t_4, t_5, \dots \rightarrow$ Knots

Must satisfy

Inherent property
of Recursion

if knot values
uniform

Uniform B-spline

$$\Rightarrow P(t) = \sum_{i=0}^n P_i * B_{i,d}(t)$$

if non-uniform

Non-uniform
B-spline

Cubic
 $d-1=3$

$d-1$ is a degree
 $t_{\min} \leq t \leq t_{\max}$
 $2 \leq d \leq n+1$

Cox - De - Boor Recursion funcⁿ

$$B_{i,d}(t) = \begin{cases} \frac{t - t_i}{t_{i+d-1} - t_i} B_{i,d-1}(t) + \frac{t_{i+d} - t}{t_{i+d} - t_{i+1}} B_{i+1,d-1}(t) \end{cases}$$

where $B_{i,1}(t) = 1$ if $t_i \leq t < t_{i+1}$

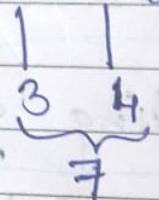
else $B_{i,1}(t) = 0$

Cubic B-spline $\leftrightarrow d = 4$

$$P(t) = P_0 * B_{0,4}(t) + P_1 * B_{1,4}(t) + \\ P_2 * B_{2,4}(t) + P_3 * B_{3,4}(t)$$

Sampling:

$t_{\max} - t_{\min}$ is divided into $n+d$ subintervals



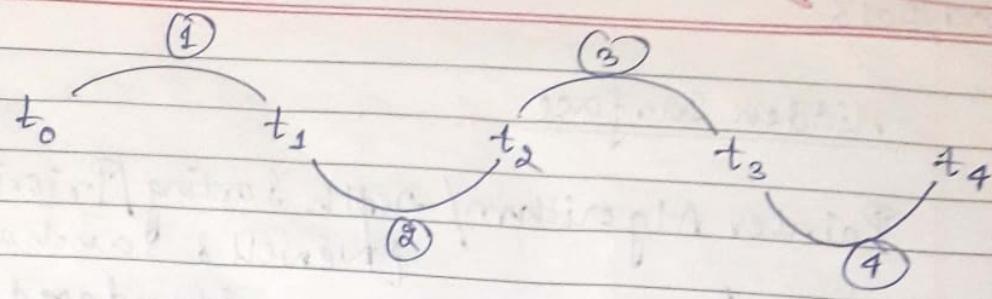
$$t : \{t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$$

↓ ↓
 0 1
 ↓
 7

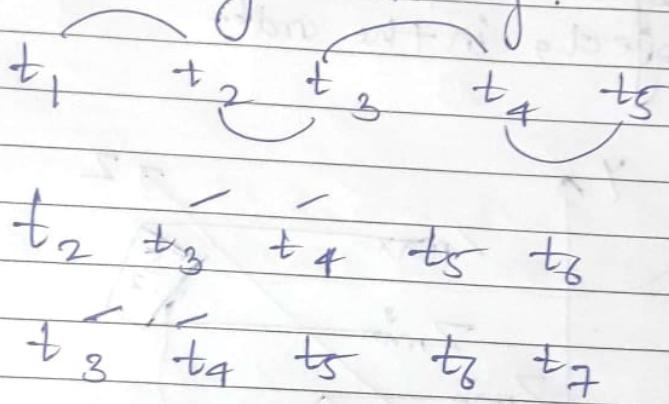
$$B_{0,4}(t) = \left\{ \frac{t-0}{3-0} \right\} B_{0,3}(t) + \left\{ \frac{4-t}{3} \right\} B_{1,3}(t)$$

$$B_{1,4}(t)$$

$$B_{2,4}(t)$$



In all 4, variation from $t_3 - t_4$ present in all segments. ∴ we draw that curve.
Proceeding similarly.



$$P(t) = [t^3 \ t^2 \ t^1] \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

question 4 points ① $P_0(1, b)$ ② $P_1(2, 5)$ ③ $P_2(4, 4)$
④ $P_3(7, c)$ are available for drawing a B-spline curve. Compute the value of a, b, c .
B-spline curve starts at point $(2, 4)$ & terminates with the slope $(-1/2)$.

t starts at 3 } as 3, 4 was common in
ends at 4 }

24/04/2018

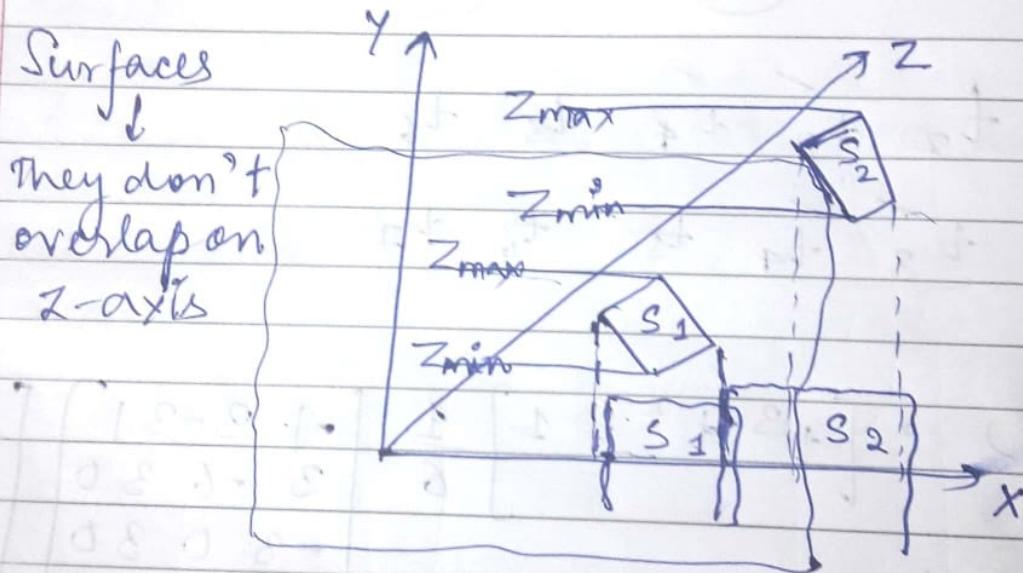
Date _____
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Hidden Surface

Painter Algorithm / Depth Sorting / Priority
(Newell & Sancha)

Standard practice of
a painter

- Distant objects are painted first
- nearer objects, in the order



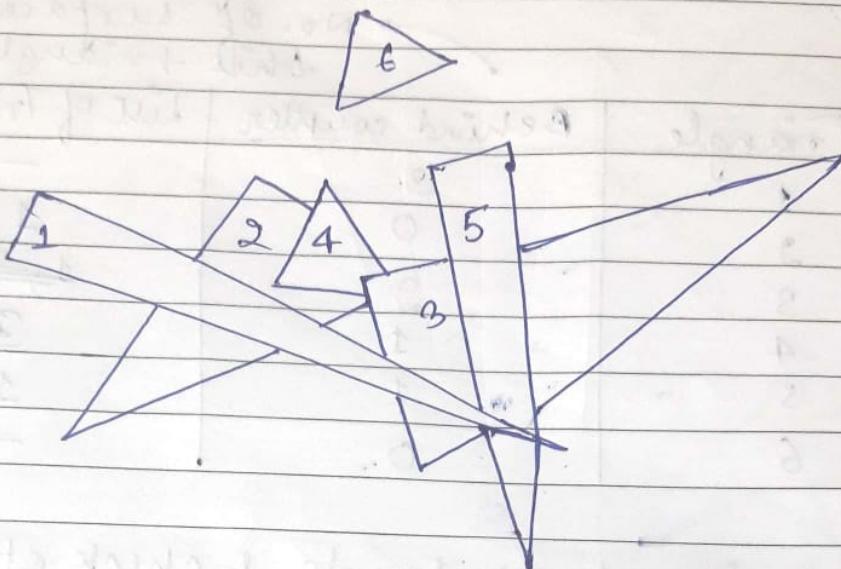
$$S : \{S_1, \dots, S_n\}$$

Sort (Depth)
↓

Cases

	I.	II
z-extent overlap	x	x x
y-extent overlap	x	x ✓
x-extent overlap	x	✓ x

Ex ①:



Set of Surfaces which don't have z-extent overlap

$$S: \{S_1, S_2, S_3, S_4, S_5, S_6\}$$

(x, y, z)

coordinates of all surfaces

Given: x, y, z of all surfaces

To find Blinnid counter

Step ①: Draw Bounding rectangles & see whose bounding rectangle are overlapping.

Solution: Construction of Table

No. of surfaces behind
this triangle

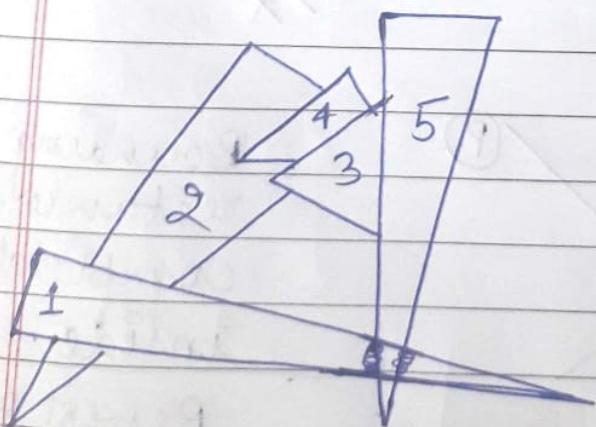
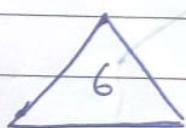
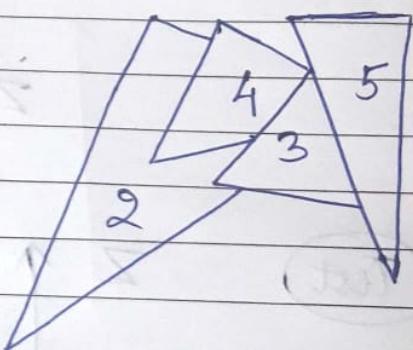
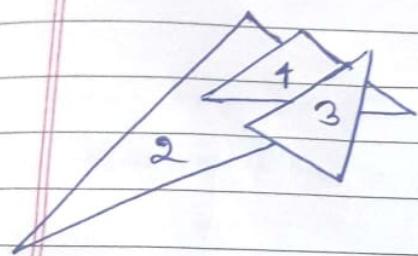
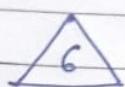
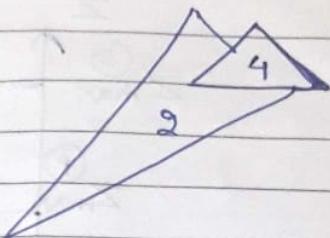
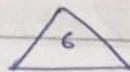
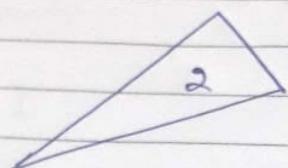
Triangle	Behind counter	List of Triangle in front
1	3	-
2	0	1, 3, 4
3	2	1, 2, 4 1, 5
4	1	3.
5	1	1
6	0	-

Take a bounding rectangle & check other bounding rectangles for no. of overlappings.

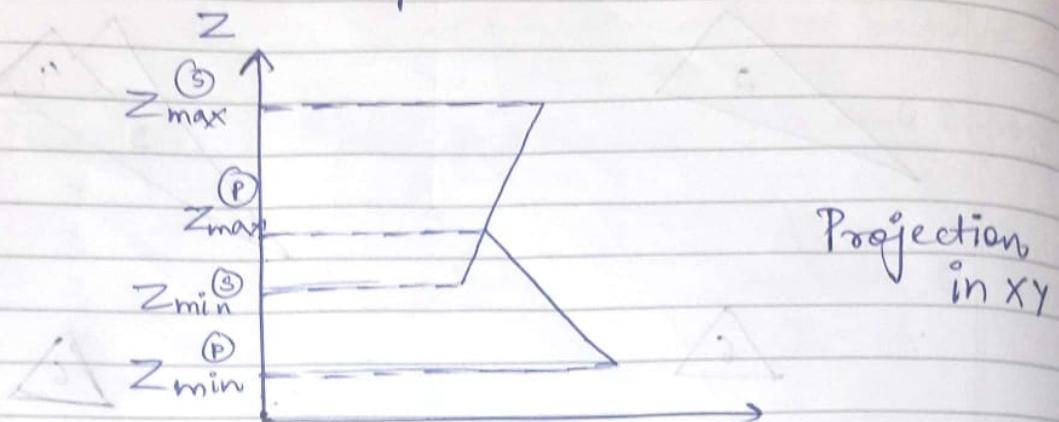
Contents of Table (Processed)

Triangle	Front list	Behind counter	Pass ①		Pass ②		
			↓	③	④	⑤	
1	↑	3	2	2	1	0	—
2	1, 3, 4	0 ✓					
3	1, 5	2	1	0 ✓			
4	3	1	0 ✓				
5	1	1	1	1	1	0 ✓	
6	-	0 ✓					

Take all with behind counter = 0 & draw. Then decrement the behind counter of all in their front list.

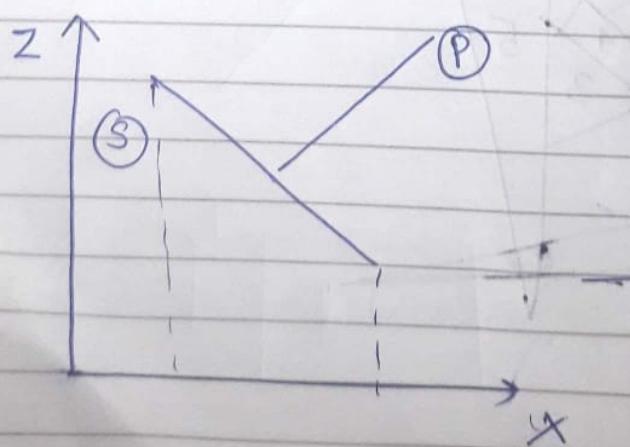
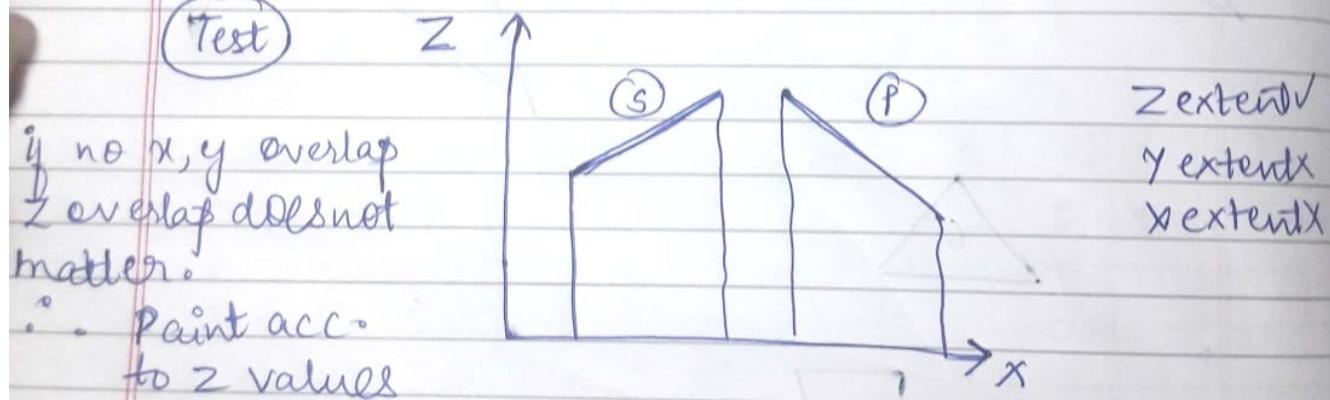


Z-extent overlap:



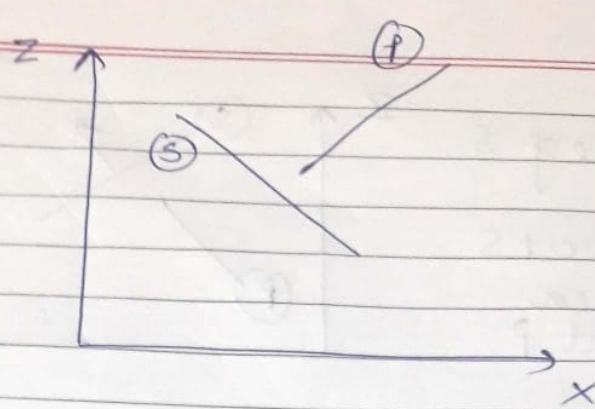
$$Z_{\min} < Z_{\max}$$

(Test)



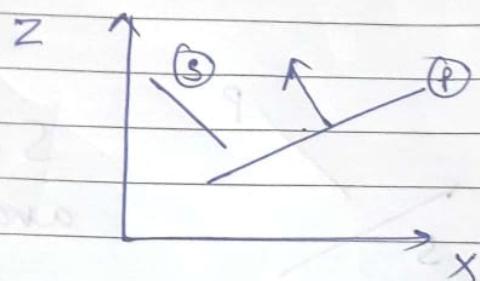
Bounding rectangle are completely inside..

Revert order
 Scan S
 ignore $P \rightarrow$
 NO
 need to draw P .



Partially obscured
↓
There are no. of cases

Cases :



Polygon S is "outside" of Polygon P

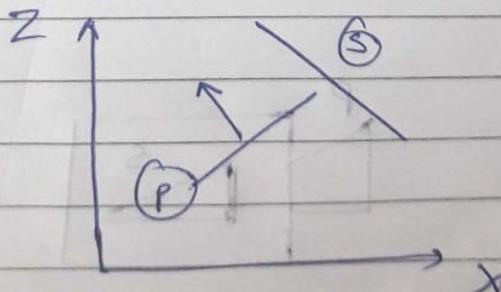
If Polygon S is outside of Polygon P relative to view plane

$$f(x, y) : Ax + By + (z + D)$$

P

(1) if putting point P in plane eqn gives > 0 , then S is outside the plane P.

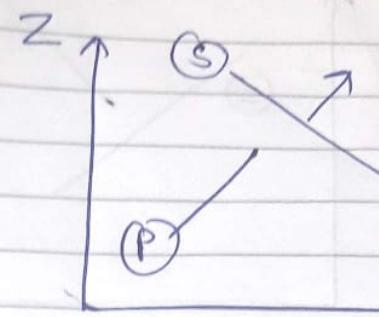
(2) Take normal of P. If both vertices of S on same side on normal then S is outside of S.



If case S "outside" of P fails then test to see if P is inside of S (again w.r.t. view plane).

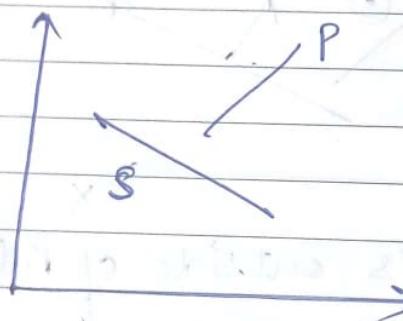
P is inside of S

so, scan converts
first & then
paint P .

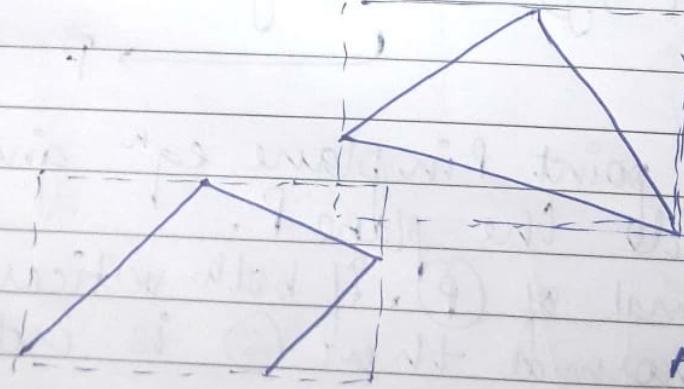


Bounding
rect-
overlap
but not
figure

Case:

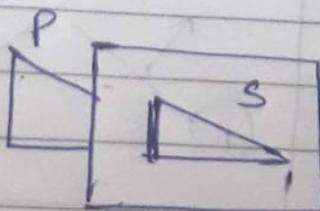


S is not inside of P
and P is not inside
of S



if there is
overlapping
with actual
surface S and P

apply polygon
clipping of P
& reapply
whole procedure
again.



tion.

Triangle ① $T_{11} (0, 7, 5)$
 $T_{12} (6, 7, 5)$
 $T_{13} (0, 1, 5)$

- Triangle ② $T_{21} (0, 6, 7)$
 $T_{22} (0, 1, 2)$
 $T_{23} (5, 1, 7)$