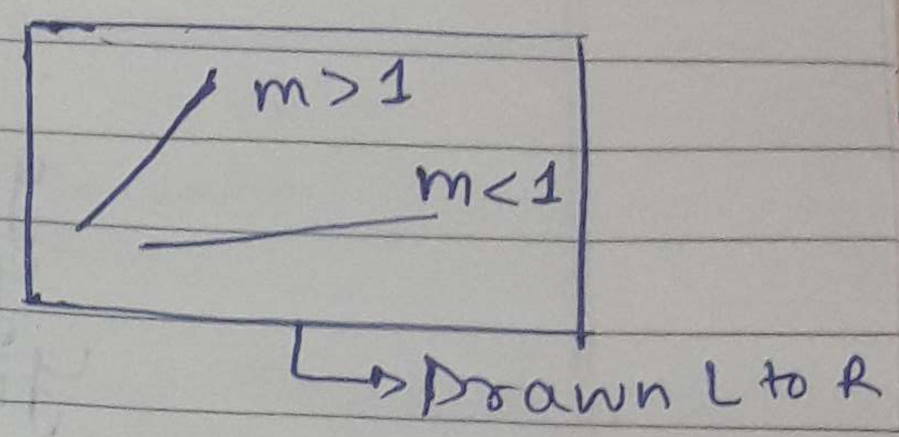
02/01/2018

Computer Graphics

2D on screen



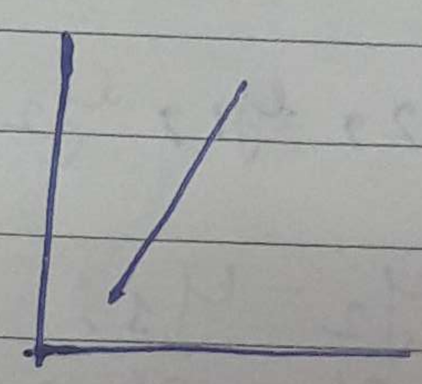
#include <graphics.h>

Primitives: Line, Circle

LINE:

$$y = mx + c$$

for $m > 1$



y changes more as compared to x.

$$x = y/m - c/m$$

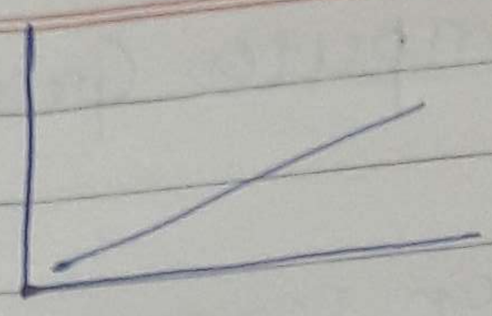
$$x_i = \frac{y_i}{m} - \frac{c}{m}$$

$$x_{i+1} = \frac{y_{i+1}}{m} - \frac{c}{m} \qquad (y_{i+1} = y_i + 1)$$

$$x_{i+1} = \frac{y_i}{m} - \frac{c}{m} + \frac{1}{m}$$

$$\boxed{x_i + 1 = x_i + \frac{1}{m}} \qquad [y_{i+1} = y_i + 1]$$

for $m < 1$



Changes in x are more than those in y

$y = mx + c$

$y_i = mx_i + c$

$y_{i+1} = mx_{i+1} + c$ $\quad (x_{i+1} = x_i + 1)$

$y_i + 1 = mx_i + c + m$

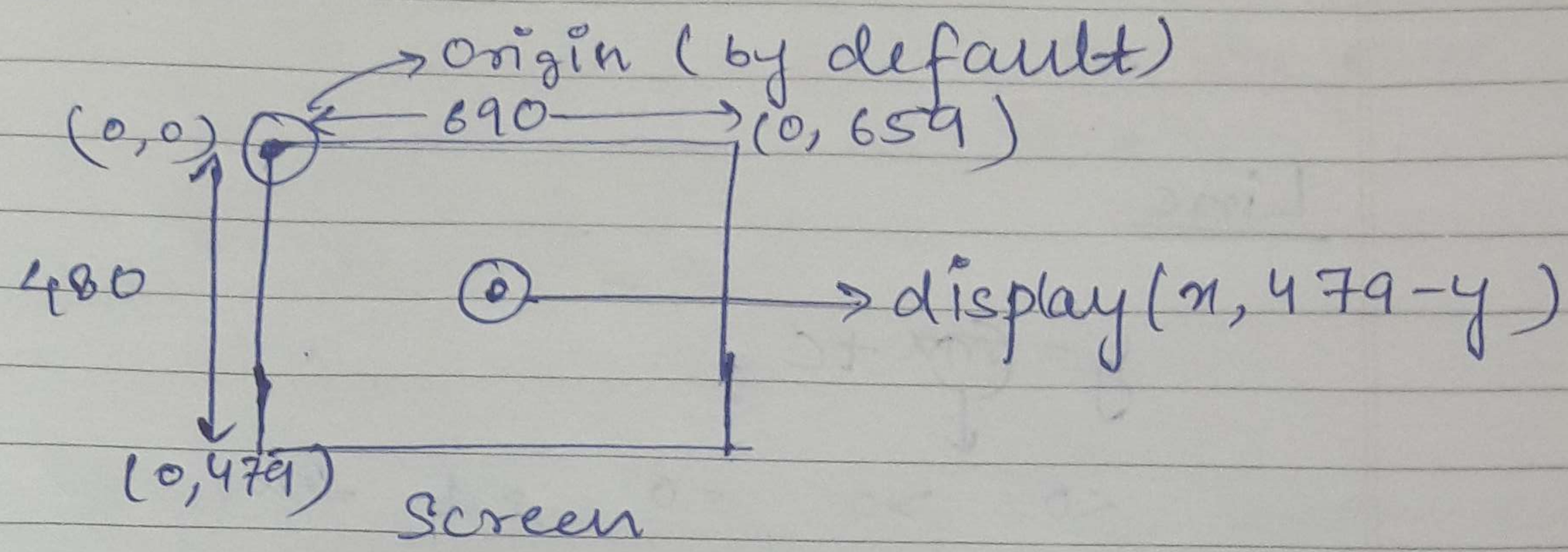
$\boxed{y_i + 1 = y_i + m}$ $\quad [x_{i+1} = x_i + 1]$

<u>Algorithm to draw a line:</u>

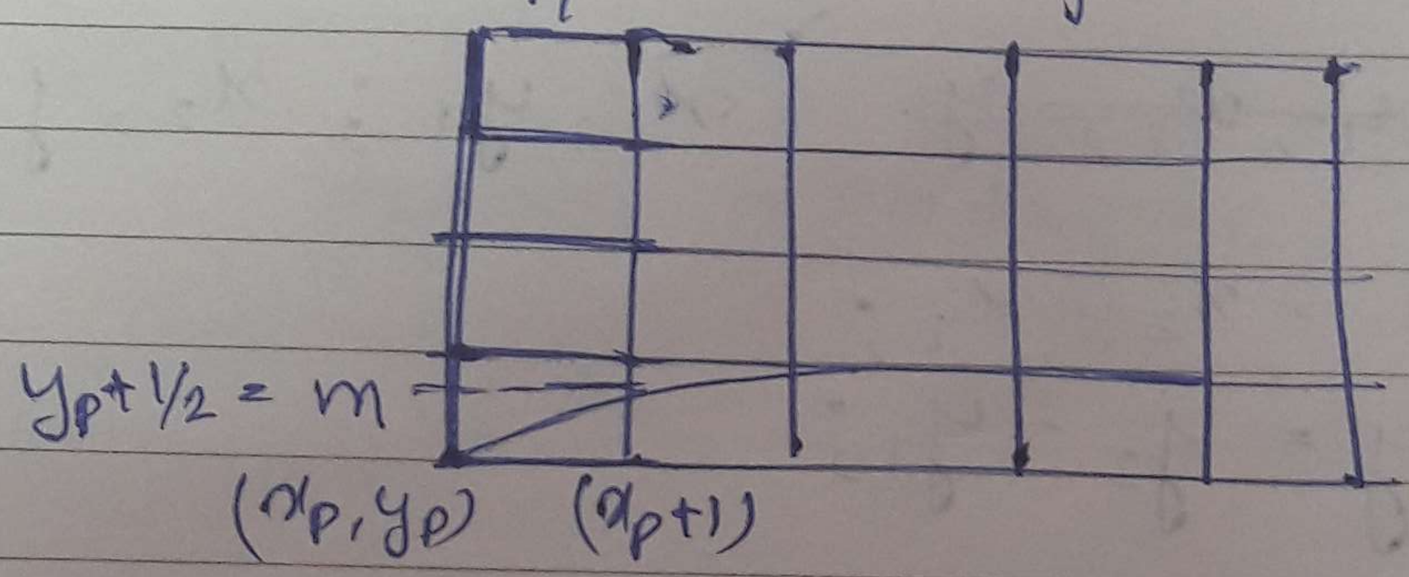
```
mline (x1, x2, y1, y2) {

    dy = y2 - y1;
    dx = x2 - x1;
    m = dy/dx
    if (abs(dy) < abs(dx)) {
        x = x1, y = y1;
        display (x, y);
        while (x < x2) {
            x++;
            y+= m;
            display(x, y);
        }
    }
    if (abs(dy) ≥ abs(dx)) {
        x = x1, y = y1;
        display(x, y);
        while (y <= y2) {
```

```
    y++;
    x+ = 1/m;
    display(x, y)
  }
}
```



Another approach of line (Midpoint Approach)



$ax + by + c = f(x, y)$

iff $f(x, y) = 0$, then point lies on the line

$$f(x_p, y_p) = ax_p + by_p + c$$

$$f(m) = a(x_p + 1) + b(y_p + 1/2) + c$$

deciding value

Case 1: if $f(m) = 0$, then lies on the line

Case 2: $f(m) > 0$

Case 3: $f(m) < 0$

04/01/2018

Primitive Drawing
→ Mathematical equation

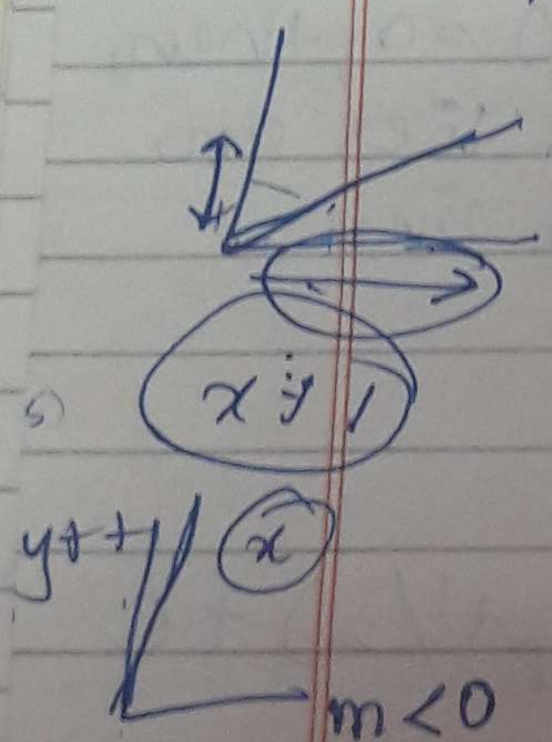
Line

$y = mx + c$

m: <0 >0 $=0$ $=1$ $=\infty$

Smallest unit: Pixel

Inputs: ~~x_1, x_2, y~~ $x_1, y_1 ; x_2, y_2$



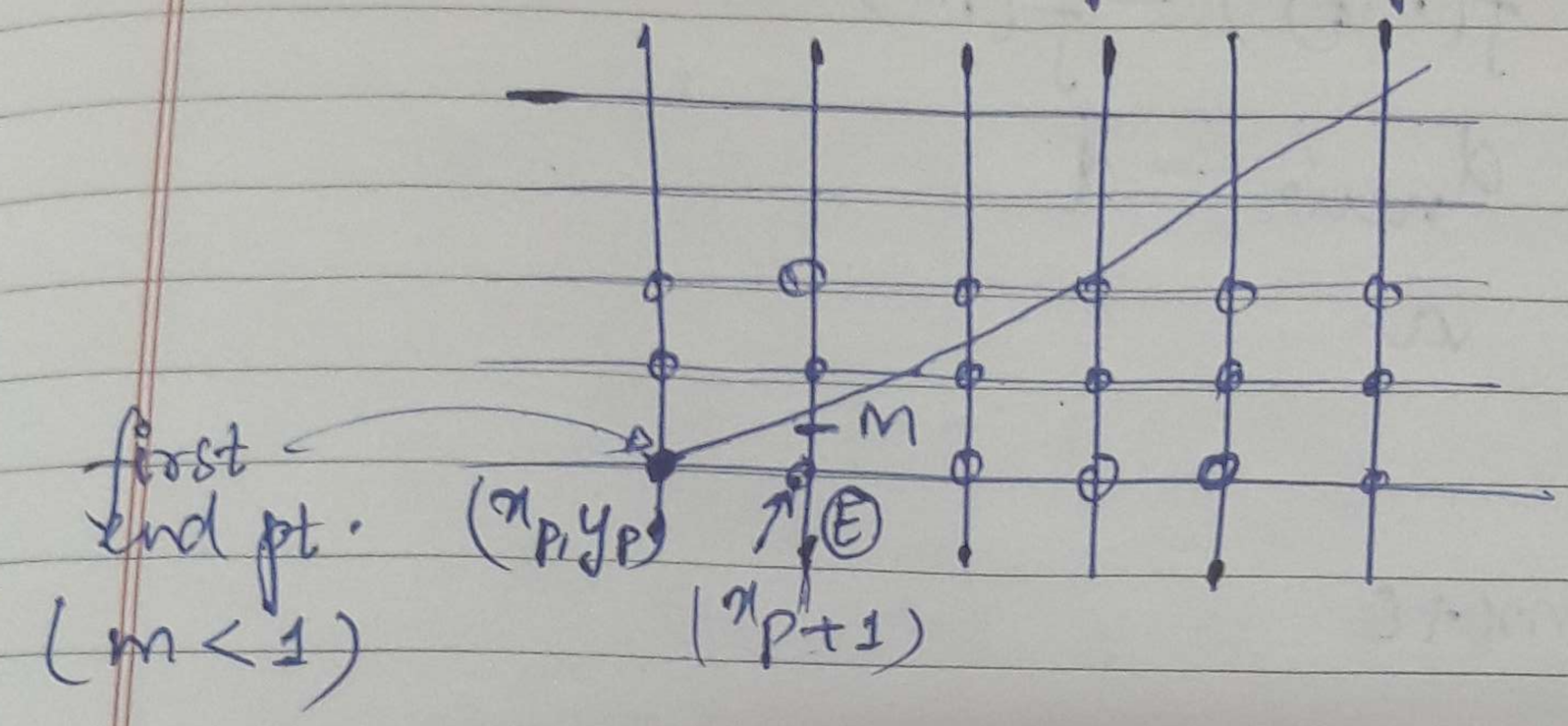

```
dx = x2 - x1;
dy = y2 - y1;
x = x1 , y = y1;
putpixel ( x, y, WHITE);
while ( x <= x2)
{
    y+ = m;
    x++;
    put pixel ( x, y, WHITE);
}
```

→ floating pt & floating pt. computation takes more time.

MidPoint Approach:

To avoid floating pt. computation

Screen is in the form of a grid



Every intersection is a pixel location

$\because$ $m<1$, $dy < dx$ $\Rightarrow$ x is incremented & accordingly y is computed (major movement in x).

$$f(x,y) = ax + by + c$$

<0 $=0$ >0

$$f(M) = f(x_p+1, y_p + 1/2)$$

d (decision) $= f(M) = a(x_p+1) + b(y_p+1/2) + c$

keeps on updating depending on value of $f(M)$

if $d<0$
selection is E

$$f(M_E) = f(x_p+2, y_p+1/2)$$

$$d_{new} = a(x_p+2) + b(y_p+1/2) + c$$

$\Delta E = f(M_{E}) - f(M)$

$\Delta E = d_{new} - d$

$= a$

$y = mx + c$

$\frac{dy}{dx}$ Replace m with dy/dx.

$dx \cdot y = dy \cdot x + dx \cdot c$

$f(x, y) = (dy)\, x - dx \cdot y + dx \cdot c = 0$

$\Downarrow$

(a)

$\Rightarrow \Delta E = a = dy$

else if $d > 0$

selection is (NE)

$f(M_{NE}) = f(x_p + 2, y_p + 3/2)$

$d_{new} = a(x_p + 2) + b(y_p + 3/2) + c$

$\Delta NE = d_{new} - d$

$\Delta NE = f(M_{NE}) - f(M)$

$\Delta NE = a + b$

$\Rightarrow \underline{a + b = dy - dx}$

$d = f(x_p + 1, y_p + 1/2)$

$= ax_p + a + by_p + b/2 + c$

$= \underbrace{ax_p + by_p + c}_{\Downarrow} + a + b/2$

This pt. is on line ; so it will be 0.

$\boxed{d_{in} = a + b/2}$.

To remove 1/2 from b/2, multiply it with a scalar qty, that is, 2.

$f(x, y) = 2(ax + by + c)$

$d_{in} = 2a + b = \underline{2dy - dx}$

If $d < 0$, $\Delta E = 2dy = 2a$.

else $\Delta NE = 2(dy - dx)$
$= 2(a + b)$

~~$d = f(x_p + 1,$~~

Changes in previous code:

```
d = ~~2*(dy-dx);~~  2*dy - dx;
while (x <= x2)
{
    if d<0
        d+ = 2*dy;
    else
        {d+ = 2*(dy-dx);
         y++;
        }

    x++;
    putpixel (x, y, WHITE);
}
```

Example:

$x_1 = 10$
$y_1 = 10$
$x_2 = 20$
$y_2 = 18$

$m < 1$

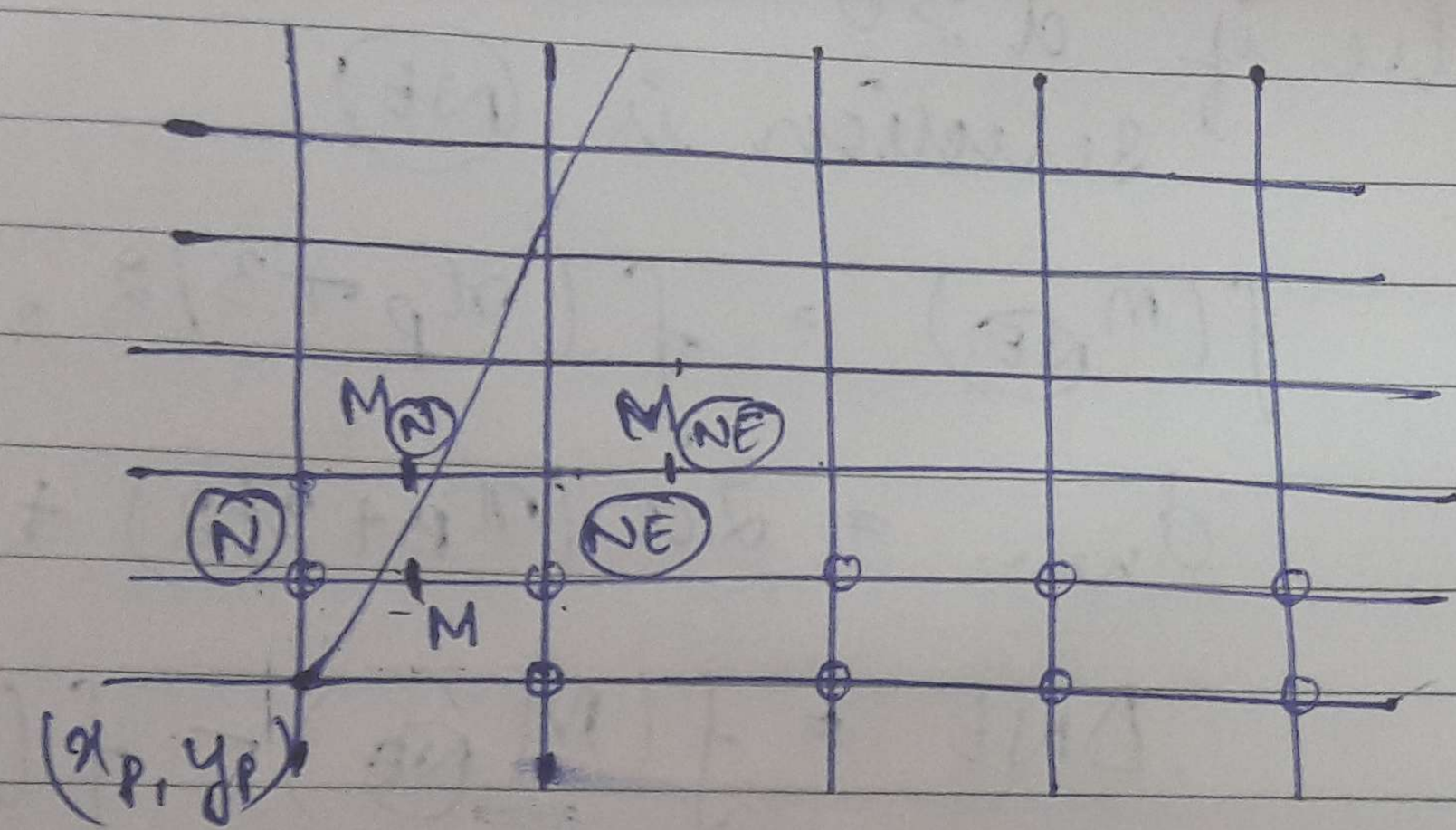
~~dy = 8~~
~~dx = 10~~
$x = 10$ (x_1)
$y = 10$ (y_1)

Check Table

x_1	y_1	d
10	10	6
11	11	2
12	12	-2
13	13	14

Complete the rest of

For $m > 1$:



At (x_p, y_p):

$$f(x_p, y_p) = 2(ax_p + by_p + c)$$

$$d = f(M) = f(x_p + 1/2 + y_p + 1)$$

$$d = 2a(x_p + 1/2) + 2b(y_p + 1) + 2c$$

if $d < 0$

selection is (N)

$f(M_{(N)}) = f(x_p + 1/2, y_p + 2)$

$d_{new} = 2a(x_p + 1/2) + 2b(y_p + 2) + 2c$

$\Delta N = f(M_{(N)}) - f(M)$

$\Delta N = d_{new} - d$

$\Delta N = 2b = -2dx$

else if $d > 0$

selection is (NE)

$f(M_{(NE)}) = f(x_p + 3/2, y_p + 2)$

$d_{new} = 2a(x_p + 3/2) + 2b(y_p + 2) + 2c$

$\Delta NE = f(M_{(NE)}) - f(M)$

$\Delta NE = d_{new} - d$

$\Delta NE = 2a + 2b = 2(dy - dx)$

$d = f(x_p + 1/2, y_p + 1)$

$d = ax_p + a/2 + by_p + b + c$

$d = \underbrace{ax_p + by_p + c}_{=0} + a/2 + b$

$$d = a/2 + b$$

$$f(x,y) = 2(ax + by + c)$$

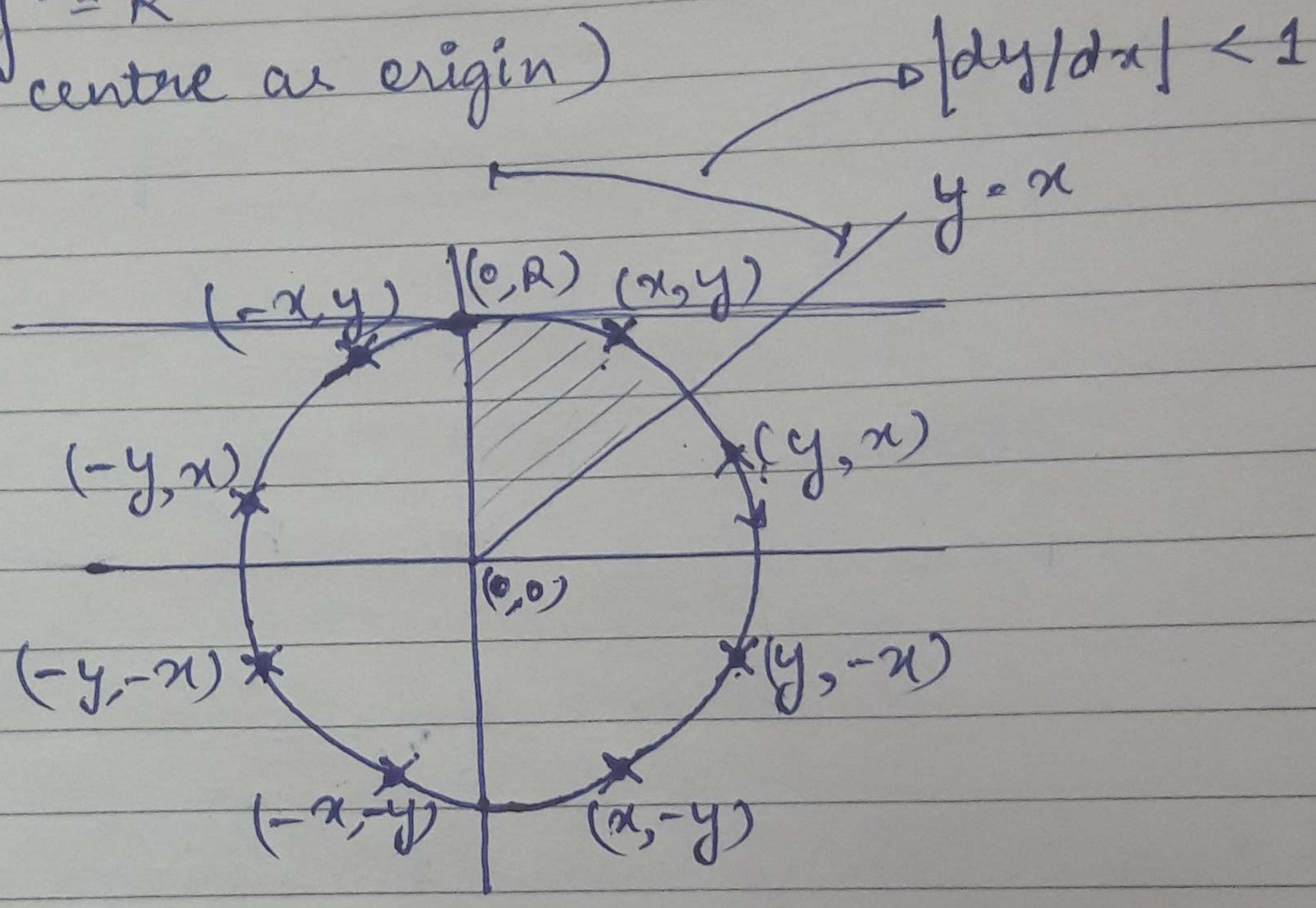
$$d_{in} = a + 2b = dy - 2dx$$

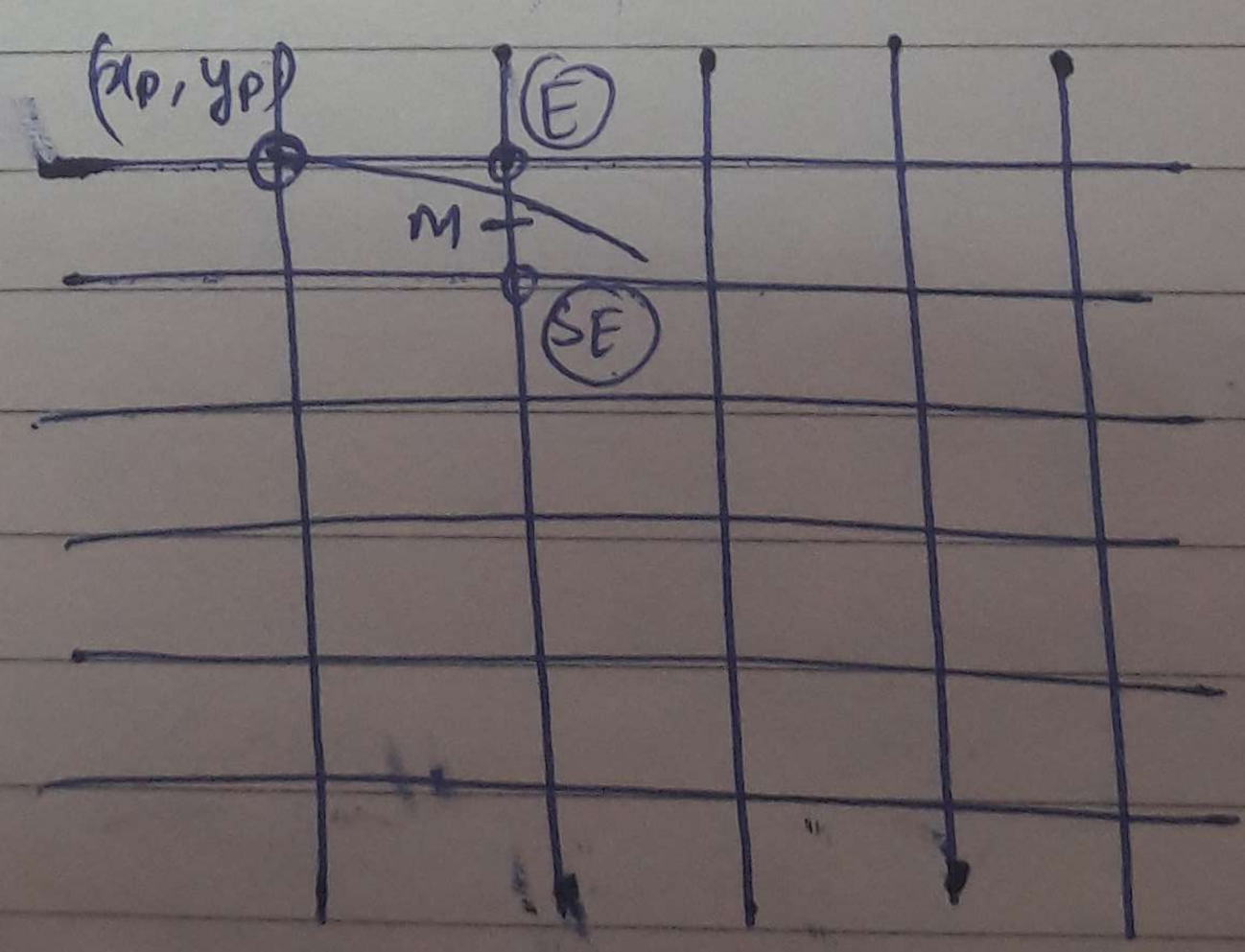
08/01/2018

Scan Conversion

Primitive Circle:

$x^2+y^2=R^2$
(with centre as origin)





At (x_p, y_p)

$$f(x_p, y_p) = x_p^2 + y_p^2 - R^2$$

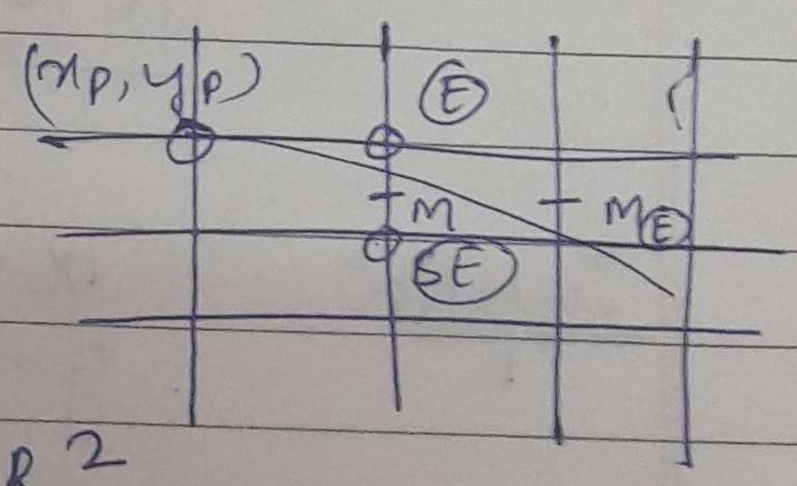
$$d = f(M) = f(x_p+1, y_p - 1/2)$$
$$= (x_p+1)^2 + (y_p - 1/2)^2 - R^2$$

<u>1st differential:</u>

if $d<0$

selection is (E) (assumed)

$$d_{new} = f(M_{(E)})$$
$$= f(x_p+2, y_p - 1/2)$$
$$= (x_p+2)^2 + (y_p - 1/2)^2 - R^2$$

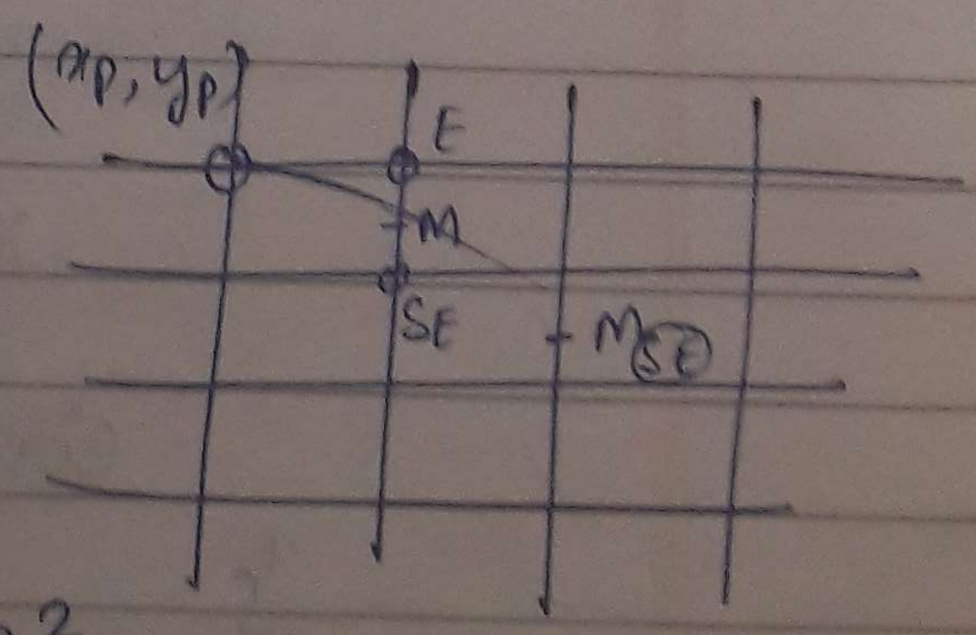


$$\Delta E = d_{new} - d = \boxed{3 + 2x_p}$$ (2 degree circle reduced to degree 1)

else

Selection is (SE)

$$d_{new} = f(M_{(SE)})$$
$$= f(x_p+2, y_p - 3/2)$$
$$= (x_p+2)^2 + (y_p - 3/2)^2 - R^2$$



$$\Delta SE = d_{new} - d = \boxed{2x_p - 2y_p + 5} = \boxed{2(x_p - y_p) + 5}$$

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initialization:

$$d = (x_p^2 + \cancel{y_p^2} - R^2) + 2x_p + 1 - y_p + 1/4 \quad (\to 0)$$

$$d = 2x_p - y_p + 5/4$$

$$\boxed{d = 5/4 - R}$$ (starting from (0,R))

$$\boxed{d \approx 1 - R}$$

Inputs:

```
x = 0;
y = R;
d = 1-R;
```

8 times putpixel

```
CircleSymmetry(x,y);

while (x <= y)
{
  if d<0
     d+= 3+2*x;
  else
  { d+= 2(x-y) +5;
    y--;
  }
  x++;      (becoz of major movement in x)
  circleSymmetry(x,y);
}
```

$R = 8$

x	y	d
0	8	-7
1	8	-4
2	8	-1
3	7	-6
4	7	3
5	6	2
6	5	5

* Rather ~~for~~ going for 1st diff, we go for 2nd diff. becoz time taken for multiplication is more than ~~arithmetic~~ addition so arithmetic computation time can be reduced.

Pre requisites for 2nd differential:

At (x_p, y_p), 1st differential are:

$\Delta E = 2x_p + 3$

$\Delta SE = 2(x_p - y_p) + 5$

If $d < 0$

Selection is (E)

At (E): $(x_p + 1, y_p)$

$$\Delta E_{new} = 2x_p + 5$$

$$\Delta SE_{new} = 2(x_p - y_p) + 7$$

$$\boxed{\Delta^2 E^{(E)}_{(\Delta E_{new} - \Delta E)} = 2}$$ → for the selection of E

$$\boxed{\Delta^2 SE^{(E)}_{(\Delta SE_{new} - \Delta SE)} = 2}$$

else

selection is (SE)

At (SE) : $(x_p + 1, y_p - 1)$

$$\Delta E_{new} = 2x_p + 5$$

$$\Delta SE_{new} = 2(x_p - y_p) + 9$$

$$\boxed{\Delta^2 E^{(SE)} = 2}$$

$$\boxed{\Delta^2 SE^{(SE)} = 4}$$

Now, we have to initialise ΔE & ΔSE.

$d = 1 - R$
$\Delta E = 3$
$\Delta SE = 5 - 2R$ } Starting from $(0, R)$

Changes in previous code:

```
x = 0;
y = R;
deltaE = 3;
deltaSE = 5-2R;

circle symmetry (x, y);
while (x <= y)        → for 1st octant only
{
  if d<0
  {
      d+ = deltaE;
      deltaE+ = 2;
      deltaSE+ = 2;
  }

  else
  {
      d+ = deltaSE;
      deltaE+ = 2;
      deltaSE+ = 4;   y--;
  }
  x++;
  circle symmetry (x,y);
}
```

$R = 8$

x	y	d	dettaE	dettaSE
0	6	-7	3	-11
1	8	-4	5	-9
2	8	1	7	-7
3	7	-6	9	-3
4	7	3	11	-1
5	6	2	13	3
6	5	5	15	7

$grad(E) = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$. If $\partial f / dx = \partial f / dy$ then $|dy/dx| = 1$

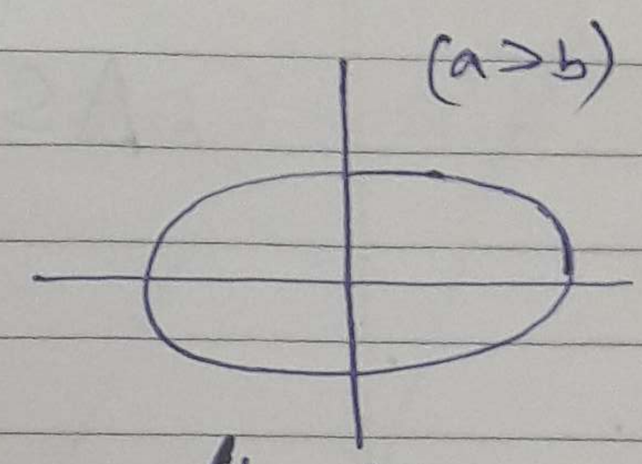
11/01/2018

Scan Conversion

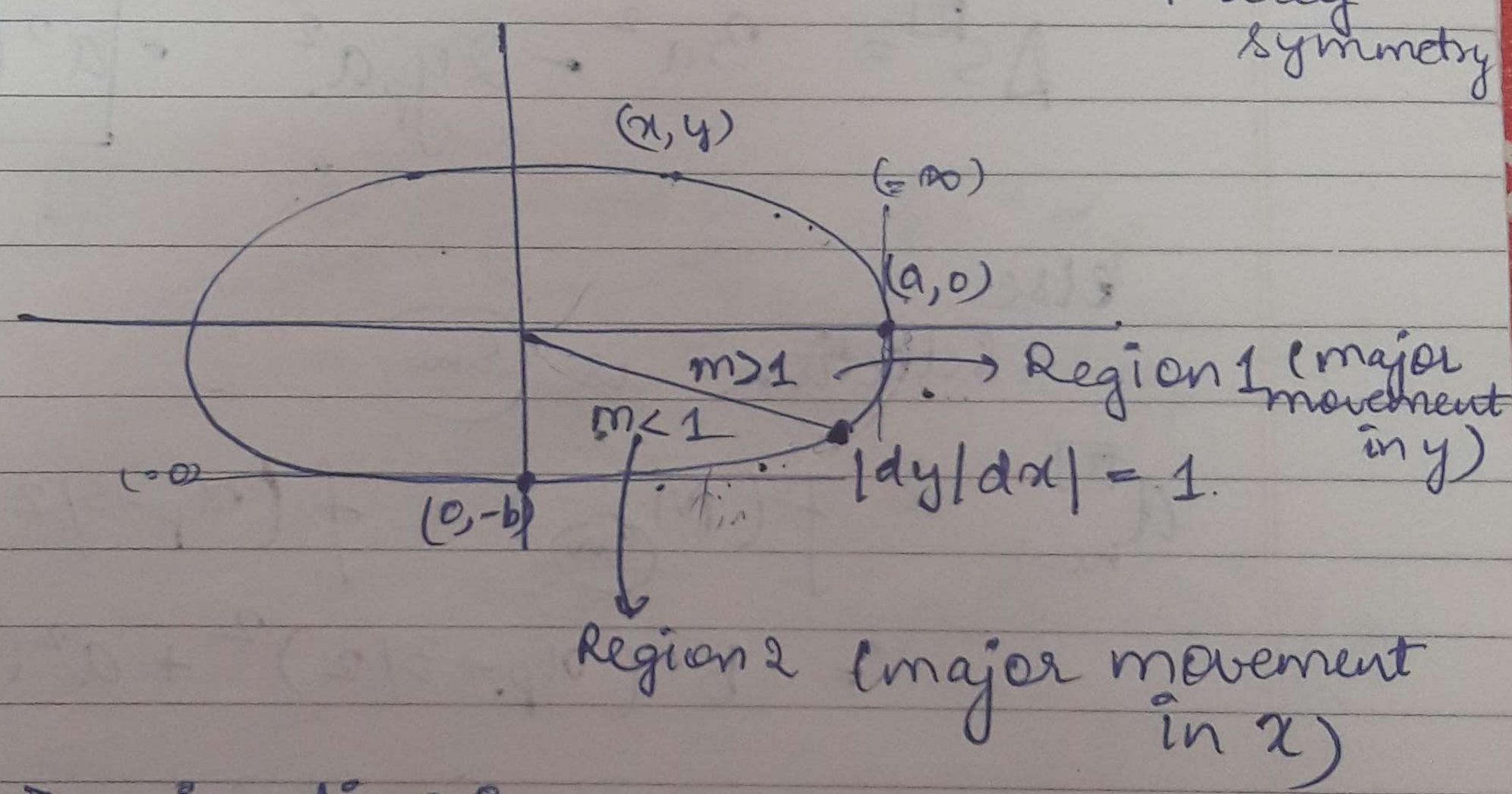
Ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$f(x,y) = b^2x^2 + a^2y^2 - a^2b^2$$

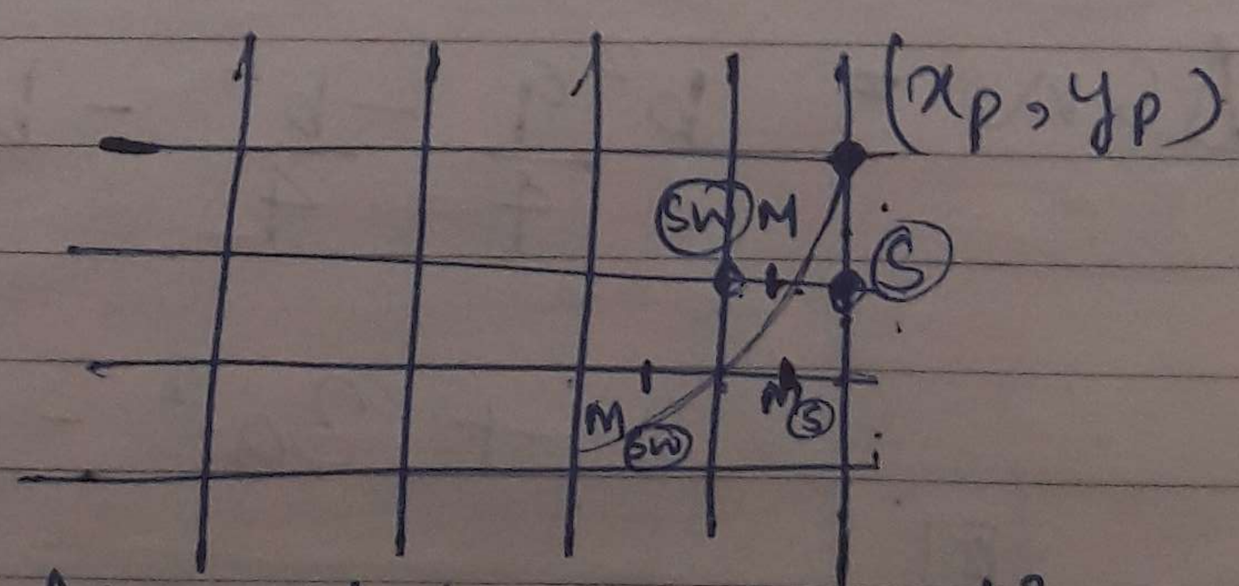


4-way symmetry



Ist Derivative:

Region 1:



$d^{I} = f(M) = f(x_p - 1/2, y_p - 1) = b^2(x_p - 1/2)^2 + a^2(y_p - 1)^2 - a^2b^2$

- If $d < 0$
  Selection is (S)

$d^{I}_{new} = f(M) = f(x_p - 1/2, y_p - 2)$

$$d_{new}^{\boxed{I}} = b^2(x_p - 1/2)^2 + b^2(y_p - 2)^2 - a^2b^2$$

$$\Delta S = d_{new}^{\boxed{I}} - d^{\boxed{I}}$$

$$\Delta S = \cancel{a^2 y_p^2} + 4a^2 - 4y_p a^2 - \cancel{a^2 y_p^2}$$
$$- a^2 + 2y_p a^2$$

$$\Delta S^{\boxed{I}} = 3a^2 - 2y_p a^2 = \boxed{a^2(3 - 2y_p)}$$

else
selection is SW

$$d_{new}^{\boxed{I}} = f(M_{SW}) = f(x_p - 3/2, y_p - 2)$$
$$= b^2(x_p - 3/2)^2 + a^2(y_p - 2)^2 - a^2b^2$$

$$\Delta SW = d_{new}^{\boxed{I}} - d^{\boxed{I}}$$

$$\Delta SW = b^2\frac{9}{4} - \frac{b^2}{4} - b^2 3x_p + b^2 x_p$$
$$+ 3a^2 - 2y_p a^2$$

$$\Delta SW_2^{\boxed{I}} = 2b^2 - 2b^2 x_p + 3a^2 - 2y_p a^2$$
$$= \boxed{a^2(3 - 2y_p) + b^2(2 - 2x_p)}$$

Starting from $(a,0)$ (Clockwise)

$$d_{in}^{\boxed{I}} = b^2(x_p - 1/2)^2 + a^2(y_p - 1)^2 - a^2b^2$$

$$\cancel{d_{in}^{\boxed{I}} = b^2(x - 1/2)^2 + x^2 - a^2b^2}$$

$$\cancel{d_{in}^{\boxed{I}} = a^2b^2 + \frac{b^2}{4} - ab^2 + a^2 - a^2b^2}$$

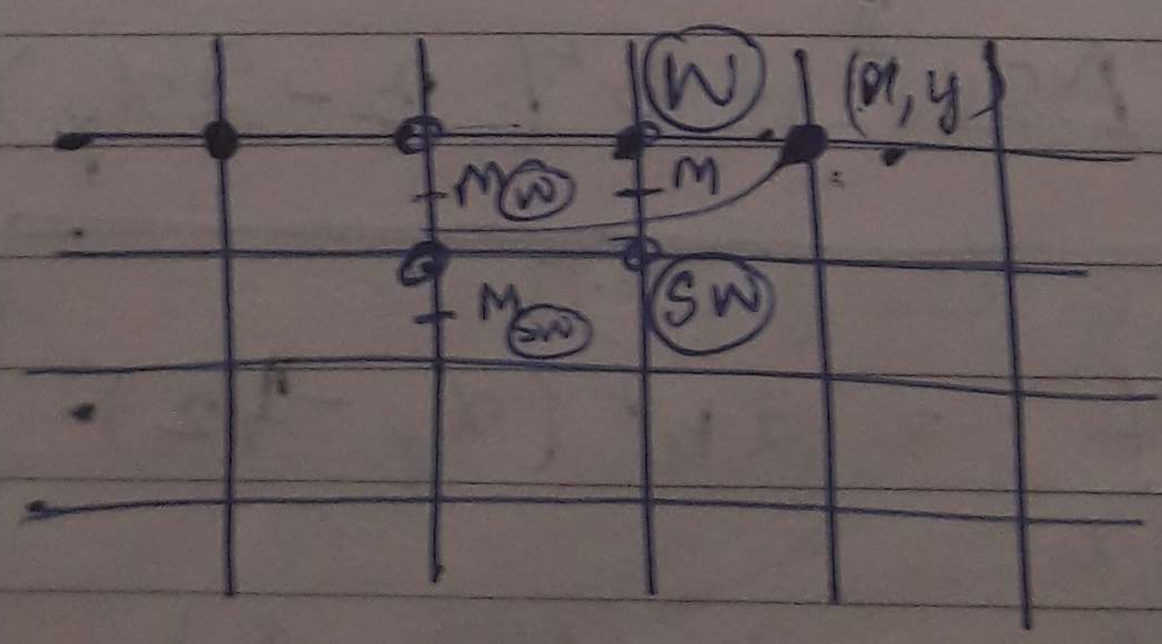
$$\cancel{d_{in}^{\boxed{I}} =}$$

$$d_{in}^{\boxed{I}} = b^2x_p^2 + \frac{b^2}{4} - b^2x_p + a^2y_p^2 + a^2 - 2a^2y_p - a^2b^2$$

$$d_{in}^{\boxed{I}} = \frac{b^2}{4} - b^2x_p + a^2 - 2a^2y_p$$

$$\boxed{d_{in}^{\boxed{I}} = \frac{b^2}{4} - b^2a + a^2}$$

Region II:



$$d^{\boxed{II}} = f(M) = f(x_p - 1, y_p - 1/2)$$

$$d^{\boxed{II}} = b^2(x_p - 1)^2 + a^2(y_p - 1/2)^2 - a^2b^2$$

Selection is (W)

$$d_{new}^{\boxed{II}} = f(M_{(W)}) = f(x_p - 2, y_p - 1/2)$$

$$d_{new}^{\boxed{II}} = b^2(x_p - 2)^2 + a^2(y_p - 1/2)^2 - a^2b^2$$

$$\Delta W^{II} = d_{new}^{\boxed{II}} - d^{\boxed{II}} = 3b^2 - 2b^2x_p$$

$$\boxed{\Delta W^{II} = b^2(3 - 2x_p)}$$

Selection is (SW)

$$d_{new}^{\boxed{II}} = f(M_{(SW)}) = f(x_p - 2, y_p - 3/2)$$

$$d_{new}^{\boxed{II}} = b^2(x_p - 2)^2 + a^2(y_p - 3/2)^2 - a^2b^2$$

$$\Delta SW^{II} = d_{new}^{\boxed{II}} - d^{\boxed{II}}$$

$$\boxed{\Delta SW^{II} = b^2(3 - 2x_p) + 2a^2(1 - y_p)}$$

$$\frac{\partial f}{\partial x} = 2b^2(x_p - 1/2)$$

$$\frac{\partial f}{\partial y} = a^2(y_p - 1)$$

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II Derivative :

Prerequisites → 1st derivative

Region I :

At (x_p, y_p) : 1st differential are :

$$\Delta S = a^2(3-2y_p).$$
$$\Delta SW = a^2(3-2y_p) + b^2(2-2x_p)$$

Selection is (S) :

At (S) : $(x_p, y_p - 1)$

$$\Delta S_{new} = a^2(3 - 2(y_p - 1))$$
$$\Delta S_{new} = a^2(3 - 2y_p + 2)$$
$$\Delta S_{new} = a^2(5 - 2y_p)$$
$$\Delta SW_{new} = a^2(3 - 2(y_p - 1)) + b^2(2 - 2x_p)$$
$$\Delta SW_{new} = a^2(5 - 2y_p) + b^2(2 - 2x_p)$$

$$\boxed{\begin{aligned} \Delta^2 S_{(I)}^{(S)} &= \Delta S_{new} - \Delta S = 2a^2 \\ \Delta^2 SW_{(I)}^{(S)} &= \Delta SW_{new} - \Delta SW = 2a^2 \end{aligned}}$$

Selection is (SW):

At (SW): $(x_p - 1, y_p - 1)$

$$\Delta S_{new} = a^2(3 - 2(y_p - 1))$$

$$\Delta S_{new} = a^2(5 - 2y_p)$$

$$\Delta SW_{new} = a^2(3 - 2(y_p - 1)) + b^2(2 - 2(x_p - 1))$$

$$\Delta SW_{new} = a^2(5 - 2y_p) + b^2(4 - 2x_p)$$

$$\Delta^2 S_{(I)}^{(SW)} = \Delta S_{new} - \Delta S = 2a^2$$

$$\Delta^2 SW_{(I)}^{(SW)} = \Delta SW_{new} - \Delta SW = 2a^2 + 2b^2$$

Region II:

At (x_p, y_p): 1st differential are:

$$\Delta W = b^2(3-2x_p)$$
$$\Delta SW = b^2(3-2x_p) + 2a^2(1-y_p)$$

Selection is W:

At W: $(x_p - 1, y_p)$

$$\Delta W_{new} = b^2(3-2(x_p-1))$$

$$\Delta W_{new} = b^2(5-2x_p)$$

$$\Delta SW_{new} = b^2(3-2(x_p-1)) + 2a^2(1-y_p)$$

$$\Delta SW_{new} = b^2(5-2x_p) + 2a^2(1-y_p)$$

$$\Delta^2 W^{W}_{II} = \Delta W_{new} - \Delta W$$

$$\boxed{\begin{aligned} \Delta^2 W^{W}_{II} &= 2b^2 \\ \Delta^2 SW^{W}_{II} &= \Delta SW_{new} - \Delta SW = 2b^2 \end{aligned}}$$

Selection is SW:

At SW: $(x_p - 1, y_p - 1)$

$$\Delta W_{new} = b^2(3 - 2(x_p - 1)) = b^2(5 - 2x_p)$$

$$\Delta SW_{new} = b^2(3 - 2(x_p - 1)) + 2a^2(2 - y_p + 1)$$

$$\Delta SW_{new} = b^2(5 - 2x_p) + 2a^2(2 - y_p)$$

$$\boxed{\begin{aligned} \Delta^2 W_{II}^{(SW)} &= \Delta W_{new} - \Delta W = 2b^2 \\ \Delta^2 SW_{II}^{(SW)} &= \Delta SW_{new} - \Delta SW = 2b^2 + 2a^2 \end{aligned}}$$

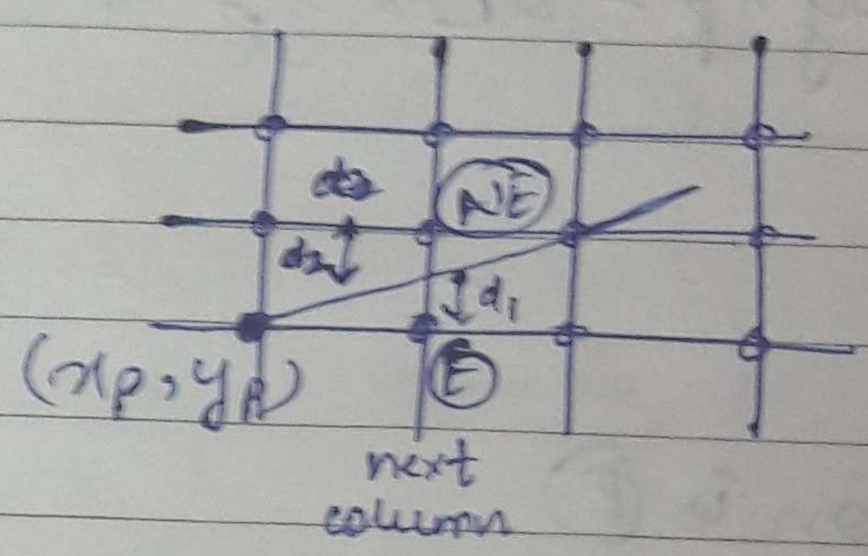
- Difference in measurable qty. for a primitive should either be +ve or -ve

15/01/2018

Bresenham's Approach

- → Find a measurable qty
- → Difference in measurable qty for decision making.

LINE:



$m < 1$

At (x_p, y_p): $y_p = m x_p + c$

→ For every primitive, there is a unique mathematical equation & unique measurable quantity.

→ Identify a measurable qty. for every primitive

→ Here, measurable qty is a distance.

At $(x_p + 1)$: $y = m(x_p + 1) + c$

$$d_1 = y - y_p \quad \text{—①}$$
$$d_2 = (y_p + 1) - y \quad \text{—②}$$

$$d = d_1 - d_2 = 2y - y_p - y_p - 1$$

$$\boxed{d = 2[m(x_p+1) + c] - 2y_p - 1}$$

→ floating pt. computation (because of m)

$$d \cdot \Delta x = 2[\Delta y(x_p+1) + c \cdot \Delta x] - 2y_p \Delta x - \Delta x$$

$$d = 2\Delta y x_p + 2\Delta y - 2y_p \Delta x - \Delta x + 2c\Delta x$$

$$d = 2\Delta y x_p - 2\Delta x y_p + \underbrace{[2\Delta y - \Delta x - 2c.\Delta x]}_{\text{constant}}$$

$$\boxed{d = 2\Delta y x_p - 2\Delta x y_p + C} \quad \text{—(3)}$$

if $d < 0$

selection is (E)

$$d_{new} = 2\Delta y (x_p + 1) - 2\Delta x y_p + C$$

$$\boxed{\Delta E = d_{new} - d = 2\Delta y}$$

else

selection is (NE)

$$d_{new} = 2\Delta y (x_p + 1) - 2\Delta x (y_p + 1) + C$$

$$\boxed{\Delta NE = d_{new} - d = 2(\Delta y - \Delta x)}$$

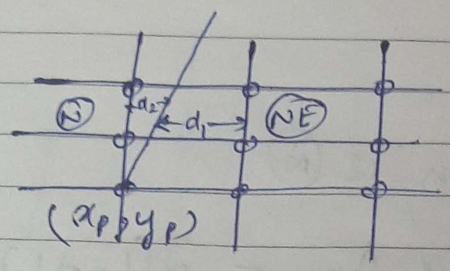
<u>For d_{in}</u> :

$$d = 2[m(x_p + 1) + c] - 2y_p - 1$$

$$d = 2\Delta x[m x_p - \cancelto{0}{y_p} + c] + (2\Delta y - \Delta x)$$

$$\boxed{d_{in} = 2\Delta y - \Delta x}$$

For $m > 1$:



$$d_1 = \cancel{x} - (x_p + 1) - x$$

$$d_2 = \cancel{x_p - x} \quad x - x_p$$

$$y = mx + c$$

$$x = \frac{1}{m}\left[(y_p + 1) - c\right] \qquad \frac{y}{m} - c = x$$

$$d = d_1 - d_2 = x_p + 1 - x - x + x_p$$

$$d = 2x_p - 2x + 1$$

$$d = 2x_p - 2\left[\frac{1}{m}\left[(y_p + 1) - c\right]\right] + 1$$

$$d \cdot \Delta y = 2x_p \Delta y - 2\left[(y_p + 1)\Delta x - c\Delta x + \Delta y\right]$$

$$d = 2x_p \Delta y - 2y_p \Delta x + \left[-2\Delta x + 2c\Delta x + \Delta y\right]$$

$$\boxed{d = 2x_p \Delta y - 2y_p \Delta x + C}$$

if $d < 0$

selection is NE

$$d_{new} = 2(x_p + 1)\Delta y - 2(y_p + 1)\Delta x + C$$

$$\boxed{\Delta NE = d_{new} - d = 2(\Delta y - \Delta x)}$$

else

selection is (N)

$$d_{new} = 2x_p \Delta y - 2(y_p + 1)\Delta x + C$$

$$\boxed{\Delta N = d_{new} - d = -2\Delta x}$$

<u>For d_{in}:</u>

$$d\Delta y = 2x_p \Delta y - 2y_p \Delta x - 2\Delta x + 2c\Delta y + \Delta y$$

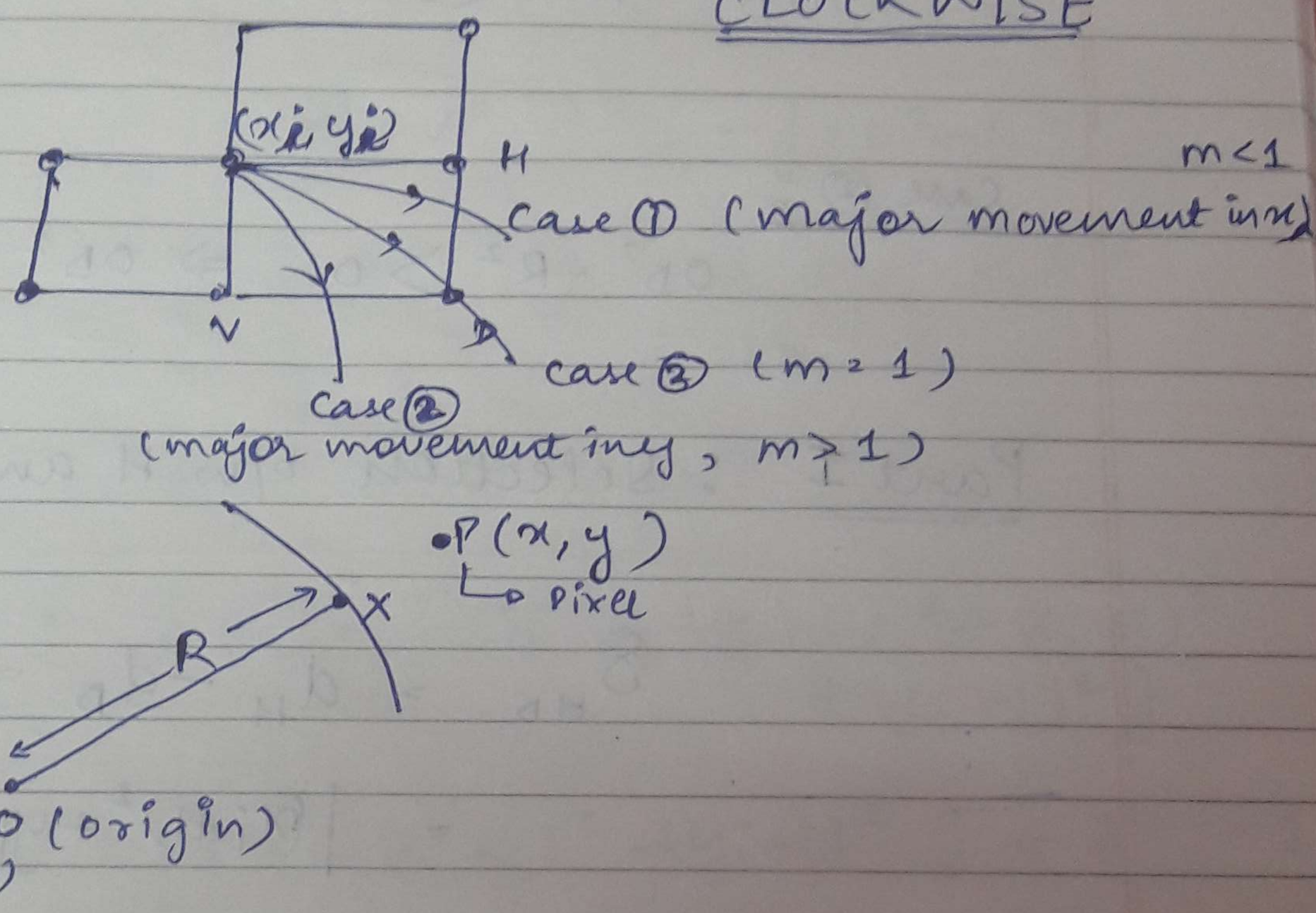
$$d = 2[x_p - y_p]$$

Calculate d_{in}.

16/01/2018

CIRCLE: $(x^2+y^2=R^2)$

CLOCKWISE



$$d_P = |OP^2 - OX^2|$$

$$d_P = |(x^2+y^2) - R^2|$$

measurable qty for a circle.

$$d_H = |(x_i+1)^2 + y_i^2 - R^2|$$

$$d_D = |(x_i+1)^2 + (y_i-1)^2 - R^2|$$

$$d_V = |x_i^2 + (y_i-1)^2 - R^2|$$

Diagonal (D) is reference which keeps on changing with every pixel selection.
ΔD_{i+1} is the differential (difference) in the change in D

Case (1):

$$OD^2 - R^2 < 0 \Rightarrow OD^2 < R^2$$

Case (2):

$$OD^2 - R^2 > 0 \Rightarrow OD^2 > R^2$$

Part I : Selection b/w H and D

$$\delta_{HD} = d_H - d_D = |OH^2 - R^2| - |OD^2 - R^2|$$

$$= |(x_i+1)^2 + y_i^2 - R^2| -$$

$$|(x_i+1)^2 + (y_i-1)^2 - R^2|$$

$\because$ $OD^2 < R^2$

$OH^2 > R^2$

$$\delta_{HD} = (x_i+1)^2 + y_i^2 - R^2 - [R^2 - (x_i+1)^2 - (y_i-1)^2]$$

$$\delta_{HD} = 2\underbrace{[(x_i+1)^2 + (y_i-1)^2 - R^2]}_{\Delta D_i} + 2y_i - 1$$

$$\delta_{HD} = \Delta D_i + 2y_i - 1$$

Part II: Selection b/w V & D

$$\delta_{VD} = d_V - d_D$$

$$= |OV^2 - R^2| - |OD^2 - R^2|$$

$$\because OD^2 > R^2 \;\&\; OV^2 < R^2$$

$$\delta_{VD} = 2R^2 - OV^2 - OD^2$$

$$= 2R^2 - [x_i^2 + (y_i - 1)^2 + (x_i + 1)^2 + (y_i - 1)^2]$$

$$= 2[R^2 - (x_i + 1)^2 - (y_i - 1)^2] + 2x_i + 1$$

(1) Pixel H is chosen

$$x_{i+1} \rightarrow x_i + 1$$
$$y_{i+1} \rightarrow y_i$$

$$\Delta D_i = (x_i + 1)^2 + (y_i - 1)^2 - R^2 \quad \text{— (1)}$$
$$\Delta D_{i+1} = (x_i + 2)^2 + (y_i - 1)^2 - R^2 \quad \text{— (2)}$$

(2) − (1)

$$\boxed{\Delta D_{i+1} = \Delta D_i + 2x_i + 3}$$

(2) Pixel V is chosen

$$x_{i+1} \rightarrow x_i$$
$$y_{i+1} \rightarrow y_i - 1$$

$$\Delta D_{i+1} = (x_i + 1)^2 + (y_i - 2)^2 - R^2 \quad \text{— (3)}$$

(3) − (1)

$$\boxed{\Delta D_{i+1} = \Delta D_i - 2y_i + 3}$$

(3) Pixel D is chosen

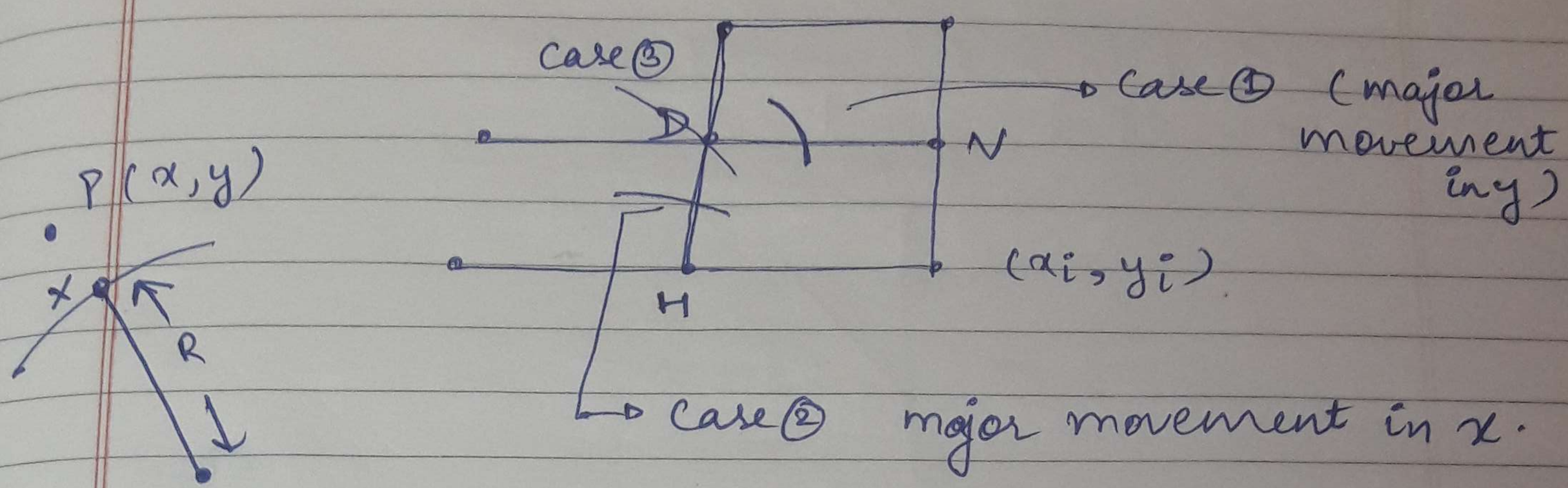
$$x_{i+1} \rightarrow x_i + 1$$
$$y_{i+1} \rightarrow y_i - 1$$

$$\Delta D_{i+1} = (x_i + 2)^2 + (y_i - 2)^2 - R^2$$

$$\boxed{\Delta D_{i+1} = \Delta D_i + 2(x_i - y_i) + 6}$$

$$\Delta D = 2(1 - R)$$ (Starting from 0, R)

ANTICLOCKWISE : (Starting from R, 0)



$$d_P = |OP^2 - OX^2|$$

$$d_P = |(x^2 + y^2) - R^2|$$

$$d_V = |x_i^2 + (y_i + 1)^2 - R^2|$$

$$d_D = |(x_i - 1)^2 + (y_i + 1)^2 - R^2|$$

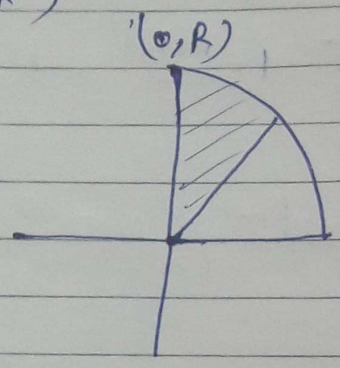
$$d_H = |(x_i - 1)^2 + y_i^2 - R^2|$$

18/01/2018

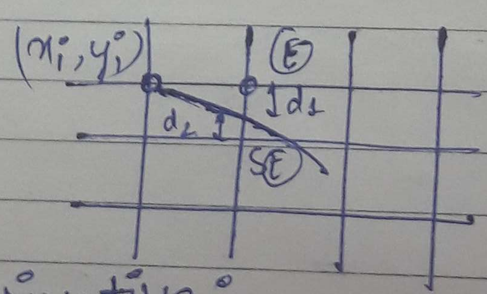
Bresenhem's Approach

Circle (with 1st quadrant):

$(x^2 + y^2 = R^2)$



1st octant : major movement in x.



1st derivative:

For column position $(x_i + 1)$:

$$d_1 = y_p^2 - y^2 \quad (\text{w.r.t } E)$$

$$d_2 = y^2 - (y_p - 1)^2 \quad (\text{w.r.t. } SE)$$

$$y^2 = R^2 - (x_p + 1)^2$$

$D = d_1 - d_2$ ⟶ Taking diff. in measurable qty.

$$D = y_p^2 - y^2 - y^2 + (y_p - 1)^2$$

$$D = y_p^2 - 2y^2 + y_p^2 + 1 - 2y_p$$

$$D = 2y_p^2 - 2y^2 - 2y_p + 1$$

$$\boxed{D = 2y_p^2 - 2(R^2 - (x_p+1)^2) - 2y_p + 1}$$

If $D < 0$

Selection is (E)

$$x_{p+1} \rightarrow x_p + 1$$

$$y_{p+1} \rightarrow y_p$$

$$D_{new} = 2y_p^2 - 2(R^2 - (x_p+2)^2) - 2y_p + 1$$

$$\cancel{D_{new} = 2y_p^2 - 2R^2}$$

$$\Delta E = D_{new} - D = 2(R^2 - (x_p+2)^2) - 2(R^2 - (x_p+1)^2)$$

$$\Delta E = 2\left[\cancel{R^2} - (x_p+2)^2 - \cancel{R^2} + (x_p+1)^2\right]$$

$$\Delta E = 2\left[-3 - 2x_p\right] = -2\left[3 + 2x_p\right]$$

$$\boxed{\Delta E = 2\left[2x_p + 3\right]}$$

else if $D \geq 0$
selection is (SE)

$$x_{p+1} \rightarrow x_p + 1$$
$$y_{p+1} \rightarrow y_p - 1$$

$$D_{new} = 2(y_p - 1)^2 - 2(R^2 - (x_p + 2)^2) - 2(y_p - 1) + 1$$

$$\Delta SE = D_{new} - D$$

$$\boxed{\Delta SE = 2(2x_p + 3) - 4(y_p - 1)}$$

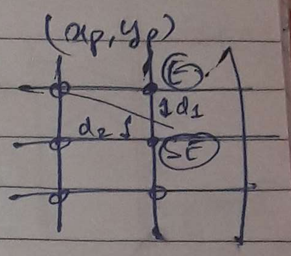
<u>For D_{in}</u> :

$$D_{(0,R)} = 2R^2 - 2(R^2 - 1) - 2R + 1$$

$$\boxed{D_{in} = 3 - 2R}$$

2nd Derivative:

$$\Delta E = 2(2x_p+3)$$
$$\Delta SE = 2(2x_p+3) - 4(y_p-1)$$



(P,R)

$\rightarrow \Delta E^i = 6$

$\rightarrow \Delta SE^i = 10 - 4R$

if $D<0$
selection is E

$$x_{p+1} \rightarrow x_p+1$$
$$y_{p+1} \rightarrow y_p$$

$$\Delta E_{new} = 2(2x_p+5)$$
$$\Delta SE_{new} = 2(2x_p+5) - 4(y_p-1)$$

$$\Delta^2 E_E = \Delta E_{new} - \Delta E$$

$$\boxed{\Delta^2 E_E = 4}$$

$$\boxed{\Delta^2 SE_E = \Delta SE_{new} - \Delta SE = 4}$$

if $D > 0$

selection is SE

$x_{p+1} \rightarrow x_p + 1$

$y_{p-1} \rightarrow y_p - 1$

$\Delta E_{new} = 2(2x_p + 5)$

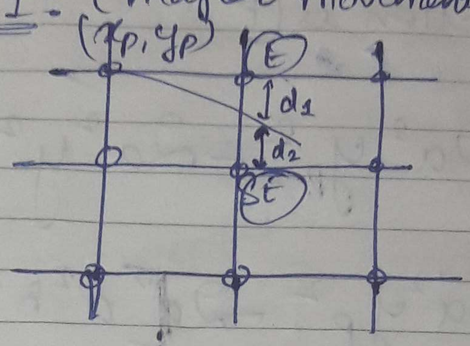
$\Delta SE_{new} = 2(2x_p + 5) - 4(y_p - 2)$

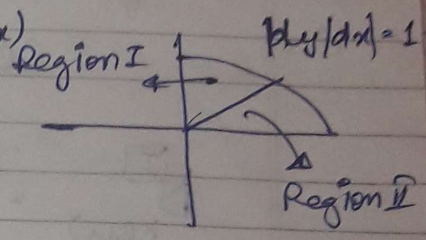
$$\boxed{\Delta^2 E_{SE} = \Delta E_{new} - \Delta E = 4}$$

$$\boxed{\Delta^2 SE_{SE} = \Delta SE_{new} - \Delta SE = 8}$$

Ellipse: $(b^2x^2 + a^2y^2 = a^2b^2)$

Region I: (major movement in x)





$$d_1 = a^2y_p^2 - a^2y^2$$
$$d_2 = a^2y^2 - a^2(y_p - 1)^2$$

$$a^2y^2 = a^2b^2 - b^2(x_p + 1)^2$$

$$D^{\boxed{I}} = d_1 - d_2$$

$$D^{\boxed{I}} = -2a^2y^2 + 2a^2y_p^2 - a^2 + 2y_pa^2$$

$$D^{\boxed{I}} = 2a^2y_p^2 + 2y_pa^2 - a^2$$
$$- 2(a^2b^2 - b^2(x_p+1)^2)$$

$$\cancel{D^{\boxed{I}} = a^2(2y_p^2 + 2y_p}$$

$$D^{\boxed{I}} = 2a^2y_p^2 - 2a^2y^2 + 2y_pa^2 - a^2$$

if $D < 0$

selection is E

$$D^{I}_{new} = 2a^2 y_p^2 - 2a^2 y^2 + 2y_p a^2 - a^2$$

$$D^{I}_{new} = 2a^2 y_p^2 - 2\left[a^2 b^2 - b^2 (x_p + 2)^2\right] + 2y_p a^2 - a^2$$

$$\Delta E^{I} = D^{I}_{new} - D^{I}$$

$$\Delta E^{I} = -2\left[a^2 b^2 - b^2 (x_p + 2)^2\right] + 2\left[a^2 b^2 - b^2 (x_p + 1)^2\right]$$

$$\Delta E^{I} = 2b^2 (x_p + 2)^2 - 2b^2 (x_p + 1)^2$$

$$\Delta E^{I} = 2b^2 (\cancel{x_p^2} + 4 + 4x_p - \cancel{x_p^2} - 2x_p - 1)$$

$$\boxed{\Delta E^{I} = 2b^2 (3 + 2x_p)}$$

else if $D >= 0$

selection is SE

$$D^{\boxed{I}}_{new} = 2a^2y_p^2 - 2a^2y^2 + 2y_pa^2 - a^2$$

$$D^{\boxed{I}}_{new} = 2a^2(y_p^2-1) - 2\left[a^2b^2 - b^2(x_p+2)^2\right] + 2(y_p-1)a^2 - a^2$$

$$\Delta SE^{\boxed{I}} = D^{\boxed{I}}_{new} - D^{\boxed{I}}$$

$$\Delta SE^{\boxed{I}} = 4a^2y_p^2 - 2\left[a^2b^2 - b^2(x_p+2)^2\right] + 2\left[a^2b^2 - b^2(x_p+1)^2\right]$$

$$\Delta SE^{\boxed{I}} = 2b^2\left[(x_p+2)^2 - (x_p+1)^2\right] + 4a^2y_p^2$$

$$\boxed{\Delta SE^{\boxed{I}} = 2b^2\left[3 + 2x_p\right] + \boxed{4a^2y_p^2}}$$

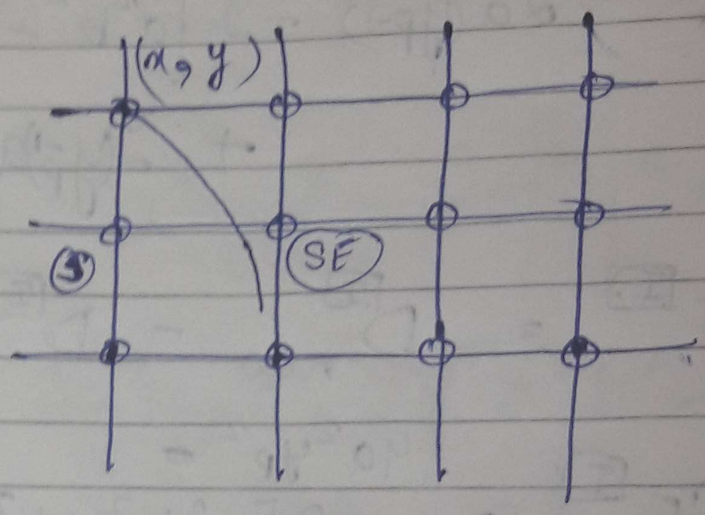
<u>For $D_{in}^{\boxed{I}}$:</u>

<u>Starting from $(0,b)$:</u>

$$D_{in}^{\boxed{I}} = 2a^2b^2 + 2ba^2 - a^2 - 2(a^2b^2 - b^2)$$

$$\Rightarrow \boxed{D_{in}^{\boxed{I}} = 2ba^2 - a^2 + 2b^2}$$

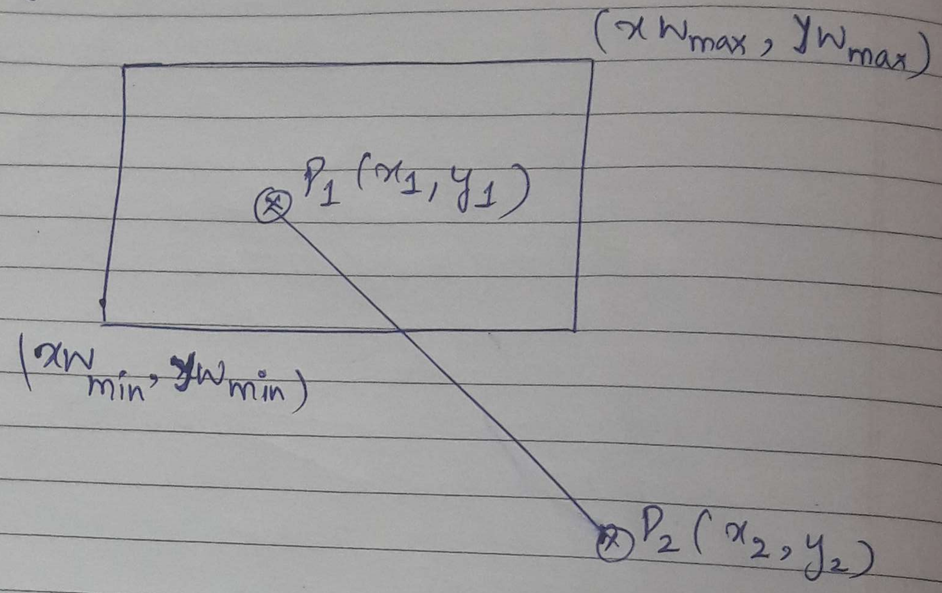
Region II: (major movement in y)



22/01/2018

CLIPPING

Rectangular window



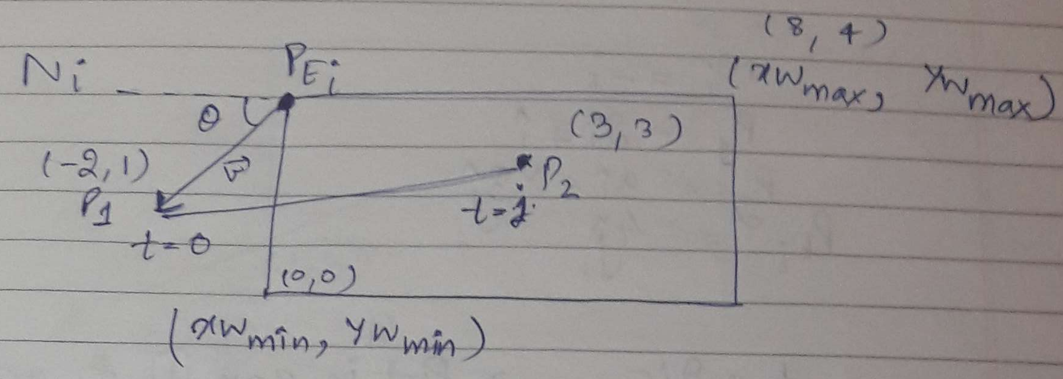
$min \le x \le max$
$min \le y \le max$

```
int inside ()
{
    Accordingly
        return (0) or (1)
}
```

// depending on whether it lies inside or outside.

0 0 (Both pts. out)
0 1
1 0
1 1 Both pts. in (no clipping reqd.) line retained.

Cyrus – Beck Method:



Parametric eqn: $P(t) = P_1 + (P_2 - P_1)t$ —— ①

Drop a vector from an edge & let it swing like a pendulum.

$$\vec{v} = [P(t) - P(E_i)]$$

$$\vec{v} \cdot \vec{N_i} \begin{cases} < 0 \\ = 0 \\ > 0 \end{cases}$$

$$\vec{v} \cdot \vec{N_i} = 0$$

$$[P(t) - P(E_i)] \cdot \vec{N_i} = 0$$

$$[P_1 + (P_2 - P_1)t - P_{E_i}] \cdot \vec{N_i} = 0$$

$$N_i \cdot (P_1 - P_{E_i}) + \vec{N_i} \cdot (P_2 - P_1)t = 0$$

$$\boxed{t = \frac{N_i \cdot (P_1 - P_{E_i})}{-N_i \cdot (P_2 - P_1)}}$$

Left Edge :

$N_i = -i$

$P_1 = 2\hat{i} + \hat{j}$

$P_2 = 3\hat{i} + 3\hat{j}$

$P_{E_i} = 4\hat{j}$

$t = 2/5 \longrightarrow$ Put it in eqn ① to get the intersection pt

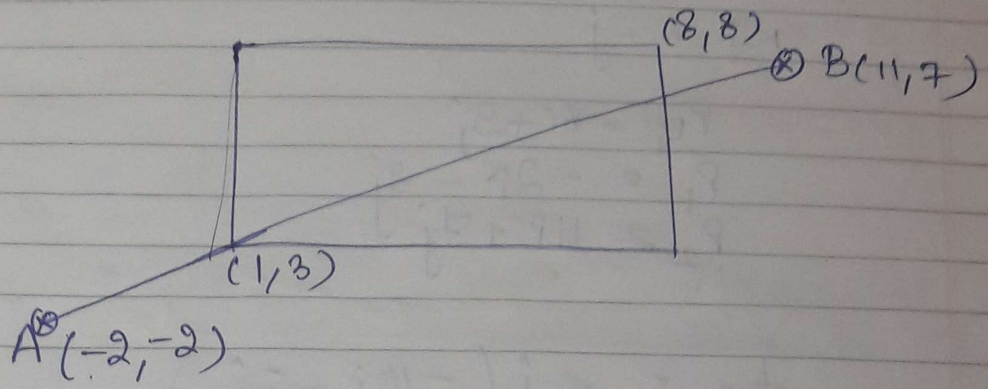
$-N_i \cdot (P_2 - P_1)$
- $\neq 0$
- < 0 — is potentially exiting
- > 0 — is potentially entering

If $\boxed{0 \leq t \leq 1}$

→ then only we have to find the intersection pt

Otherwise, the edge & the line ~~are~~ do not intersect. So, there is no need to find the intersection pt.

Example 1:



Left Edge:

$N_i = -i$

$P_{E_i} = (1,8) = i + 8j$

$P_1 = -2i - 2j$

$P_2 = 11\hat{i} + 7\hat{j}$

$$t_L = \frac{-i \cdot (-2\hat{i} - 2\hat{j} - i - 8j)}{+i \cdot (11\hat{i} + 7\hat{j} + 2\hat{i} + 2j)}$$

$$t_L = \frac{-i(-3\hat{i} - 10\hat{j})}{i(13\hat{i} + 9\hat{j})}$$

$$t_L = \frac{3 + 0}{13 + 0} = \boxed{\frac{3}{13}}$$

Bottom Edge:

$N_i = -j$

$P_{E_i} = 8i + 3\hat{j}$

$P_1 = -2\hat{i} - 2j$

$P_2 = 11\hat{i} + 7\hat{j}$

$$t_B = \frac{-\hat{j}\cdot(-10\hat{i} - 5\hat{j})}{-\hat{j}\cdot(13\hat{i} + 9\hat{j})}$$

$$\boxed{t_B = 5/9}$$

Right Edge:

$N_i = +\hat{i}$

$P_{E_i} = 8\hat{i} + 8\hat{j}$, $P_1 = -2\hat{i} - 2\hat{j}$, $P_2 = 11\hat{i} + 7\hat{j}$

$$t_R = \frac{\hat{i}(-10\hat{i} - 10\hat{j})}{-\hat{i}(13\hat{i} + 9\hat{j})}$$

$$t_R = \frac{-10}{-13} = \boxed{\frac{10}{13}}$$

Up Edge:

$N_i = +\hat{j}$

$P_{E_i} = 8\hat{i} + 8\hat{j}$, $P_1 = -2\hat{i} - 2\hat{j}$, $P_2 = 11\hat{i} + 7\hat{j}$

$$t_T = \frac{\hat{j}(-10\hat{i} - 10\hat{j})}{-\hat{j}(13\hat{i} + 9\hat{j})} = \frac{-10}{-9} = \boxed{10/9}$$

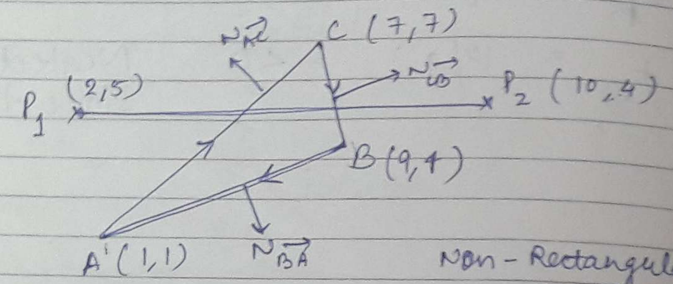
		Denominator values	
t_L	$= 3/13$	> 0	Max of those two
t_B	$= 5/9$	> 0	
t_R	$= 10/13$	< 0	
t_T	$= 10/9$	< 0	Neglected since $t > 1$.

28/03/2018

Line Clipping

Cyrus - Beck :

Example 3 :



Non - Rectangular window → can be convex or concave

This window is convex as all the angles $< 180°$

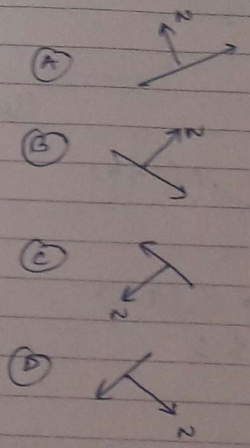
We can traverse the window either in ↷ or ↶ dir.

When ↷ → RHS is always towards the window, LH → outside

Edges	Normal (N_i)	P_{E_i}	$P_1 - P_{E_i}$	$N_i \cdot (P_1 - P_{E_i})$	$(P_2 - P_1) \cdot N_i$	t
$\vec{AC}$ $(6\hat{i}+6\hat{j})$	$-6\hat{i}+6\hat{j}$	$\hat{i}+\hat{j}$	$\hat{i}+4\hat{j}$	18	-54	1/3 ($P_{E_{ext}}$) ($P_{E_{ent}}$)
$\vec{CB}$ $(2\hat{i}-3\hat{j})$	$3\hat{i}+2\hat{j}$	$7\hat{i}+7\hat{j}$	$-5\hat{i}-2\hat{j}$	-19	22	19/22 ($P_{E_{ent}}$) (P_{Exit})
$\vec{BA}$ $(-8\hat{i}-3\hat{j})$	$3\hat{i}-8\hat{j}$	$9\hat{i}+4\hat{j}$	$-7\hat{i}+\hat{j}$	-29	32	29/32 ($P_{E_{ent}}$) (P_{Exit})

$P_2 - P_1 = 8\hat{i} - \hat{j}$

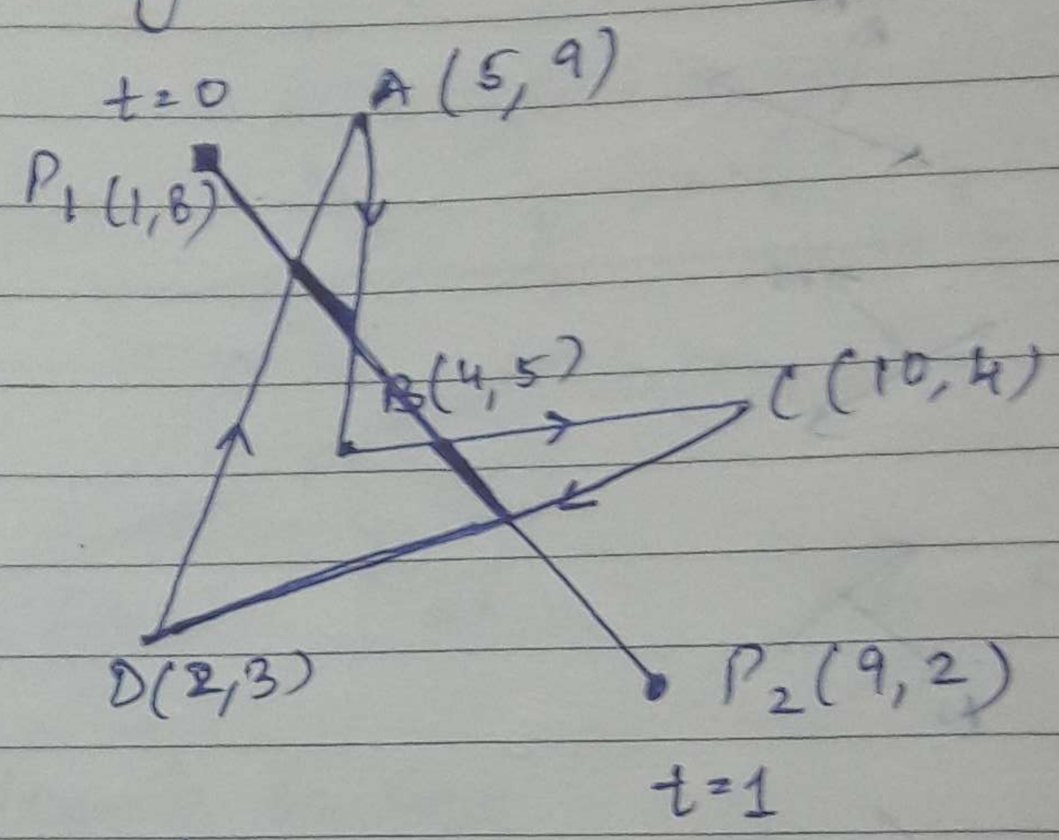
	Edge	Normal
(A)	+ +	− +
(B)	+ −	+ +
(C)	− +	− −
(D)	− −	+ −



25/01/2018

Line Clipping

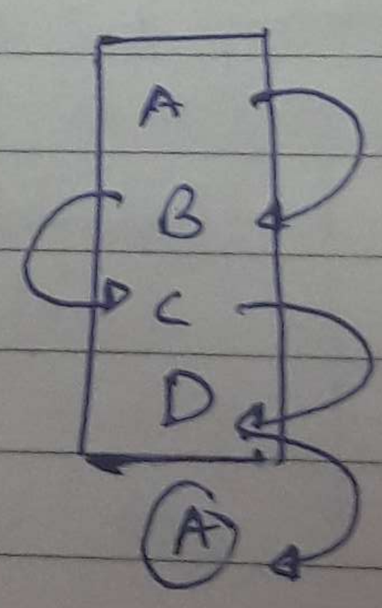
Non-Rectangular Window:



Clockwise

Concave window

arr[]



$P_1 = \hat{i} + 8\hat{j}$

$P_2 = 9\hat{i} + 2\hat{j}$

$P_2 - P_1 = 8\hat{i} - 6\hat{j}$

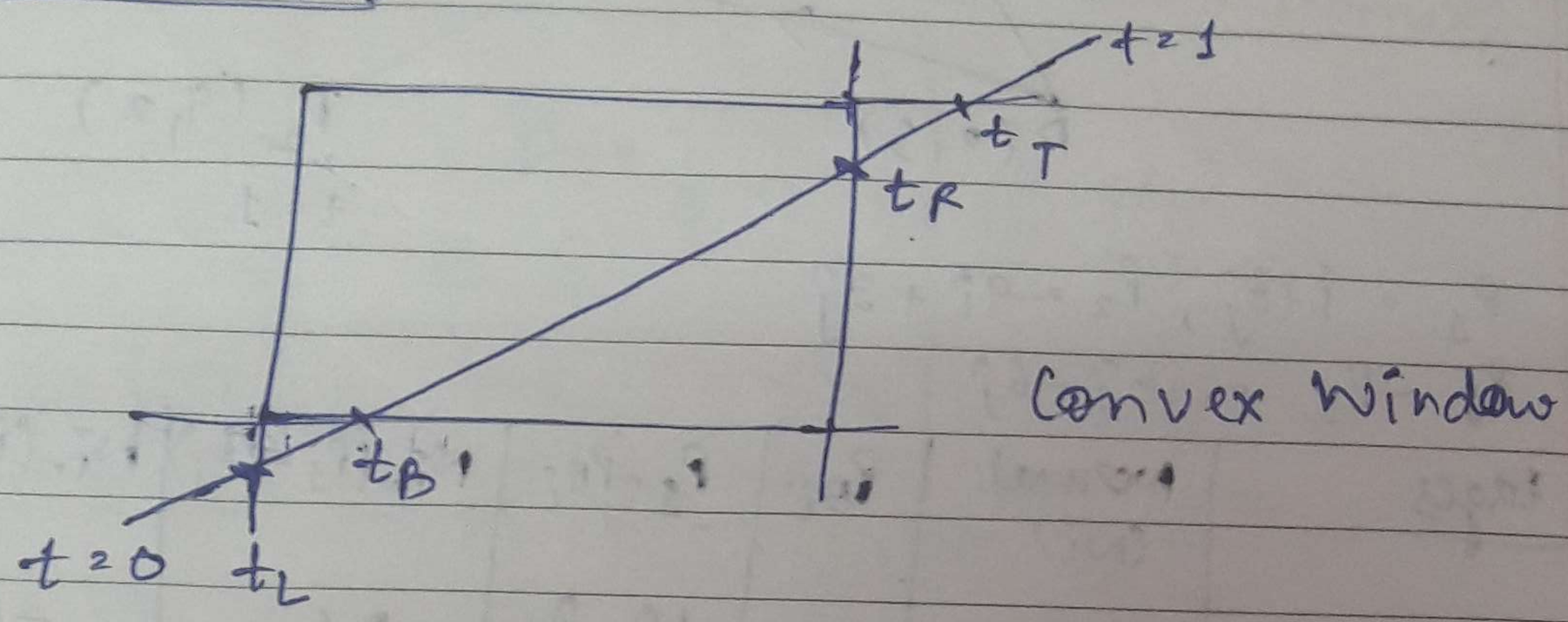
Edges	Normal (N_i)	P_{E_i}	$P_1 - P_{E_i}$	$N_i \cdot (P_1 - P_{E_i})$	$N_i \cdot (P_2 - P_1)$	t
$\vec{AB} = -\hat{i} - 4\hat{j}$	$4\hat{i} - \hat{j}$	$5\hat{i} + 9\hat{j}$	$-4\hat{i} - \hat{j}$	-15	38 (P_{Exit})	$15/38$
$\vec{BC} = 6\hat{i} - \hat{j}$	$\hat{i} + 6\hat{j}$	$4\hat{i} + 5\hat{j}$	$-3\hat{i} + 3\hat{j}$	15	-28 (P_{End})	$15/28$
$\vec{CD} = -8\hat{i} - \hat{j}$	$\hat{i} - 8\hat{j}$	$10\hat{i} + 4\hat{j}$	$-9\hat{i} + 4\hat{j}$	-41	56 (P_{Exit})	$41/56$
$\vec{DA} = 3\hat{i} + 6\hat{j}$	$-6\hat{i} + 3\hat{j}$	$2\hat{i} + 3\hat{j}$	$-\hat{i} + 5\hat{j}$	21	-66 (P_{End})	$21/66$

$\longrightarrow$ increasing order

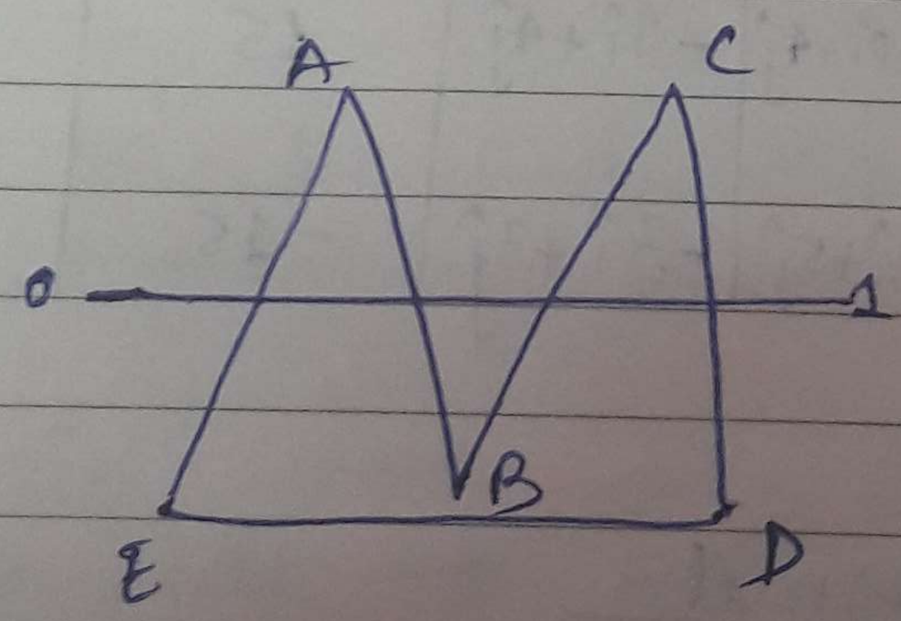
$$S : (\overset{t}{\underset{P_1}{0}} \quad 21/66 \quad 15/38 \quad 15/28 \quad 41/56 \quad \underset{P_2}{1})$$

P_{Ent} P_{Exit} P_{Ent} P_{Exit}

For Convex:



$$S : (0, \overset{t_L}{P_{Ent}}, \overset{t_B}{P_{Ent}}, \overset{t_R}{P_{Exit}}, \overset{t_T}{P_{Exit}}, 1)$$

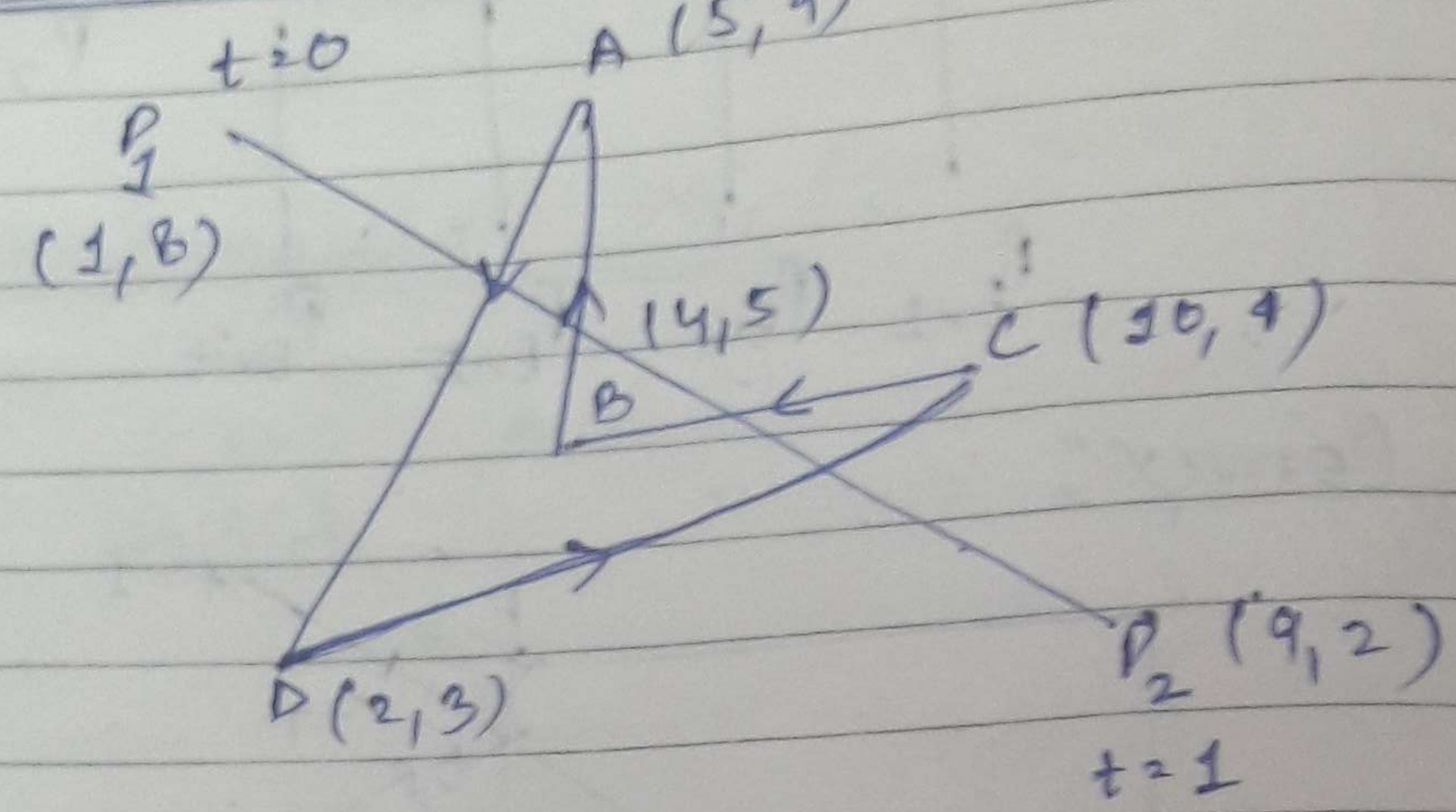


$\Longrightarrow$ Ent, Exit, Ent, Exit

If such pairs (Ent, Exit) are more than 1 b/w $t=0$ & $t=1$. $\Rightarrow$ then clipping in concave part.

$\Rightarrow t<0$ or $t>1$

Anticlockwise:



$P_1 = \hat{i} + 8\hat{j}$, $P_2 = 9\hat{i} + 2\hat{j}$

$P_2 - P_1 = 8\hat{i} - 6\hat{j}$

Edges	Normal (N_i)	P_{E_i}	$P_1 - P_{E_i}$	$N_i \cdot (P_1 - P_{E_i})$	$N_i \cdot (P_2 - P_1)$	t
$\overrightarrow{AD} = -3\hat{i} - 6\hat{j}$	$-6\hat{i} + 3\hat{j}$	$5\hat{i} + 9\hat{j}$	$-4\hat{i} - \hat{j}$	21	-66 (P_{Ent})	21/66
$\overrightarrow{DC} = 8\hat{i} + \hat{j}$	$\hat{i} - 8\hat{j}$	$2\hat{i} + 3\hat{j}$	$-\hat{i} + 5\hat{j}$	-41	56 (P_{Exit})	41/56
$\overrightarrow{CB} = -6\hat{i} + \hat{j}$	$\hat{i} + 6\hat{j}$	$10\hat{i} + 4\hat{j}$	$-9\hat{i} + 4\hat{j}$	15	-28 (P_{Ent})	15/28
$\overrightarrow{BA} = \hat{i} + 4\hat{j}$	$4\hat{i} - \hat{j}$	$4\hat{i} + 5\hat{j}$	$-3\hat{i} + 3\hat{j}$	-15	38 (P_{Exit})	15/38

Edge	Normal
+ +	+ −
+ −	− −
− +	+ +
− −	− +

$S: (0 \quad \underset{P_{Ent}}{21/66} \quad \underset{P_{Exit}}{15/38} \quad \underset{P_{Ent}}{15/28} \quad \underset{P_{Exit}}{41/56} \quad 1)$

/2018

Line Clipping

Cohen - Sutherland Approach :

(9) 1 0 0 1	(8) 1 0 0 0	(12) 1 1 0 0 (xw_{max}, yw_{max})
(1) 0 0 0 1	(0) $P(x,y)$ 0 0 0 0	(4) 0 1 0 0
(3) (xw_{min}, yw_{min}) 0 0 1 1	(2) 0 0 1 0	(6) 0 1 1 0

$$xw_{min} \le x \le xw_{max}$$
$$yw_{min} \le x \le yw_{max}$$

A rectangular window as 9 regions.
Point can be any where in these 9 regions.
Each region has a unique region code (address).

Left side → LSB 1.
Bottom side → 2nd bit 1
Right side → 3rd bit 1.
Upper side → MSB 1

Input: $P_1(x_1, y_1)$, $P_2(x_2, y_2)$

Method:

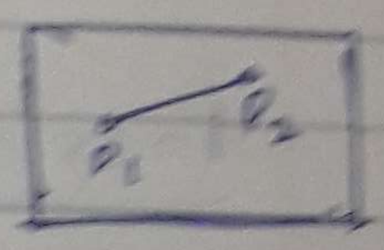
```
#define LEFT 0X1
#define RIGHT 0X1
#define BOTTOM 0X2
#define TOP 0X8

do{
    Code 1 = gencode(Point P1)        flag = 0; done = 0;
    Code 2 = gencode(Point P2)
```

Address assigned to end points

```
Code = 0X0;
gencode(Point P){

    if P.x <= xwmin
        Code |= LEFT;

    if P.x >= xwmax
        code | = RIGHT;

    if P.y <= ywmin
        code | = BOTTOM;

    if P.y >= ywmax
        code | = TOP;

}
```

Case I:

Code (P_1) = 0000
Code (P_2) = 0000



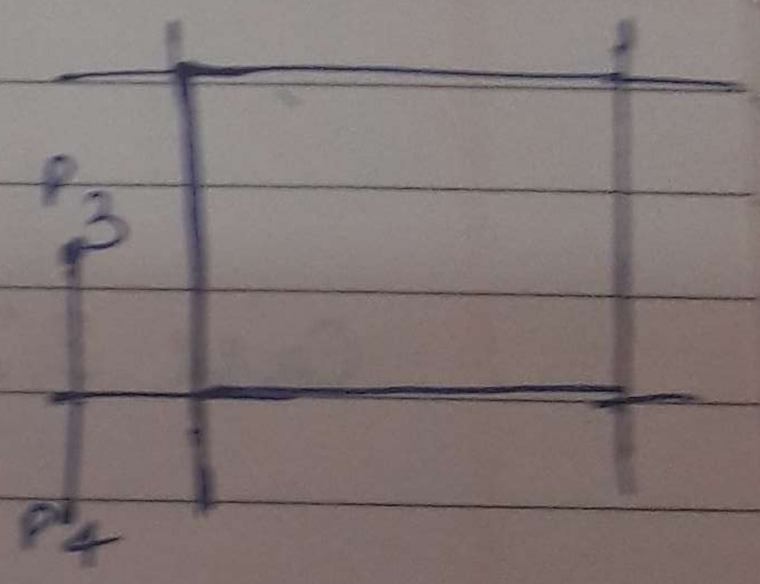
AND
OR

$\Downarrow$

$\boxed{\text{True}}$ ⇒ Line is inside, no clipping

Case II:

Code (P_3) = 0001
Code (P_4) = 0011

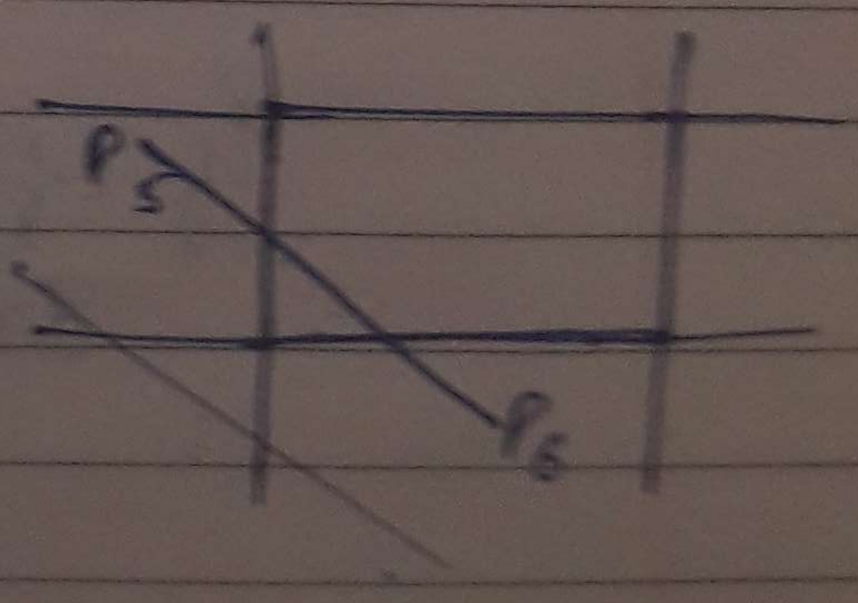


AND
$\Downarrow$

$\boxed{\text{NON-ZERO}}$ ⇒ Line is outside

Case III:

Code (P_5) = 0001
Code (P_6) = 0010



$\Downarrow$

AND → 0
OR → NON-ZERO

For case III:

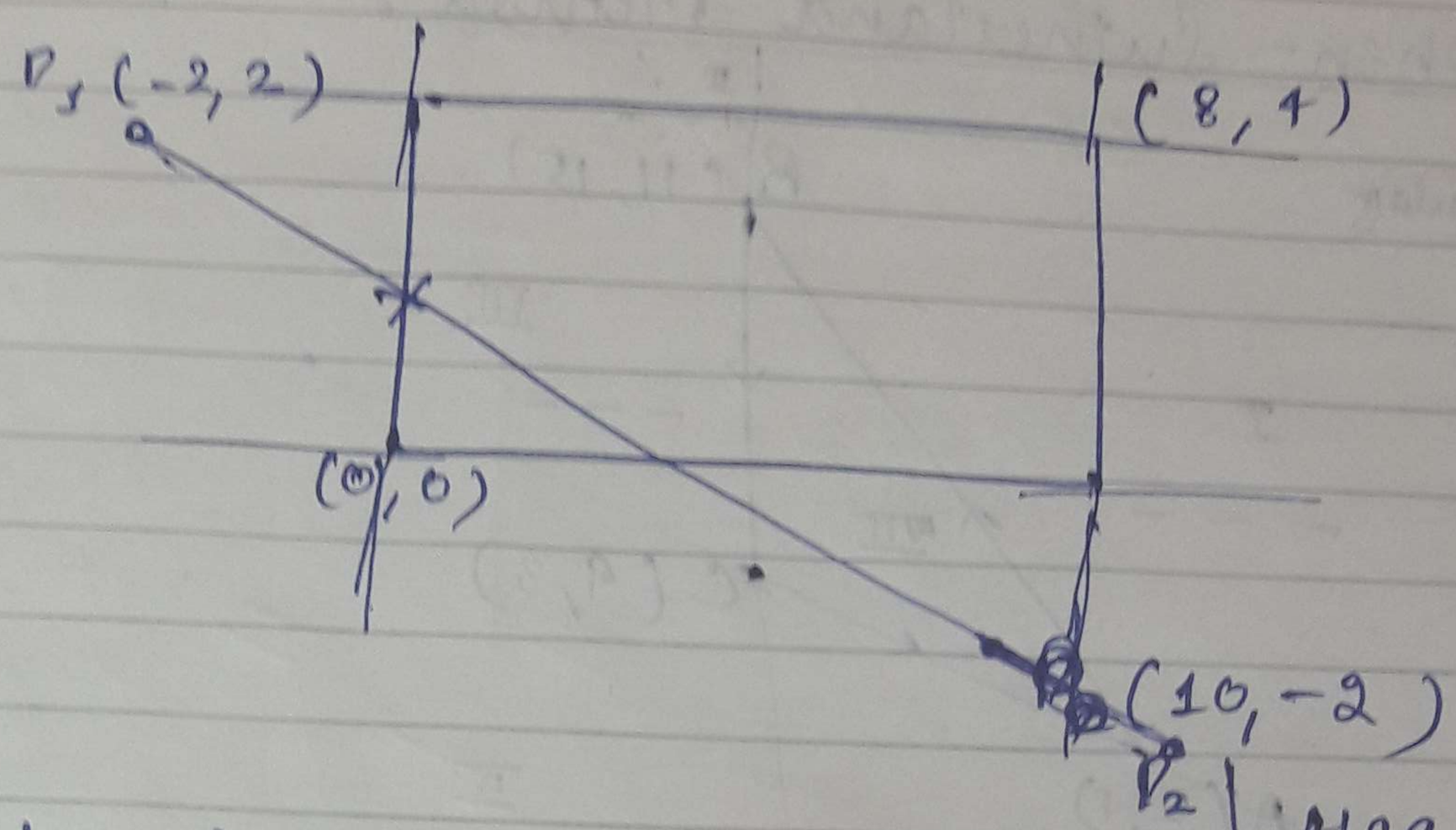
First clip B, then

* Keep checking iteratively whether after clipping line falls under case I or case II

Previous code:

```
case 1: if (code1 & code2 == 0) &&
                        (code1 | code2 == 0)
        {
            /* line is visible */
            flag = 1; done = 1;
            exit();
        }

case 2: if (code1 & code2 != 0)
        {
            /* line is not visible */
            exit(); done = 1;

Case 3: if (code1 & code2 == 0) && (code1 | code2 != 0)

            find intersection();

} while (done != 1)

    if (flag)  ---------> // line is inside
        drawline(P1, P2)
```

EXAMPE 1 :



flag	done
0	0

code (P_1) = 0001
code (P_2) = 0110

case (3
LEFT = 0x1;

if(LEFT & code P_1 == 1)
$x_I = xw_{min}$;
$y_I = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} * (x_1 - xw_{min})$

Clipped at left edge

$P_1 \leftarrow (x_I, y_I)$

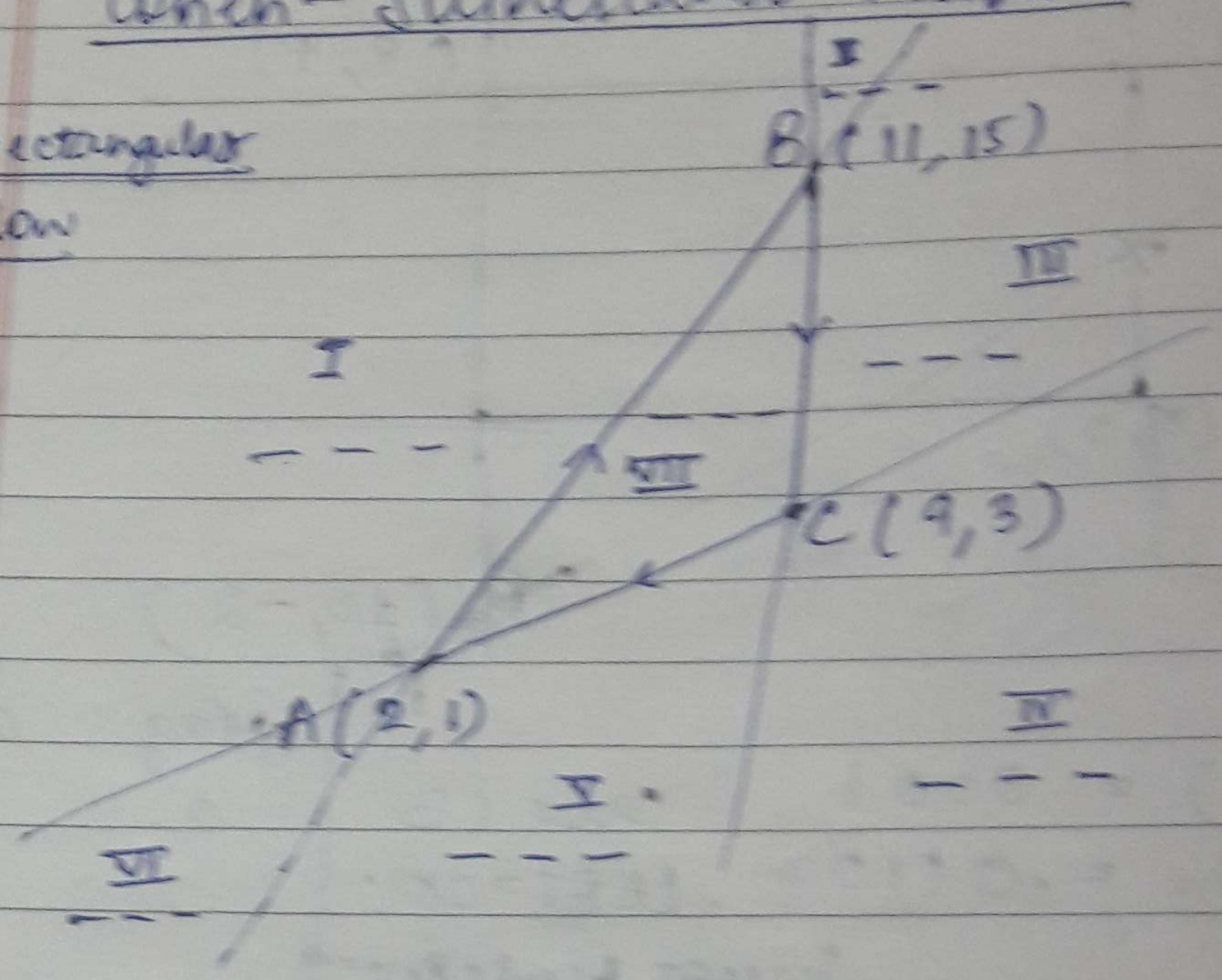
code (P_1) = 0000
code (P_2) = 0110

30/01/2018

Line Clipping

Cohen-Sutherland method:

Non-rectangular window



Inside Test

Equation	
AB $f(x,y) = 14x - 9y - 19$	$f(x,y)$: C(9,3) >0
BC $f(x,y) = -6x + y + 51$	$f(x,y)$: A(2,1) >0
CA $f(x,y) = -2x + 7y - 3$	$f(x,y)$: B(11,15) >0

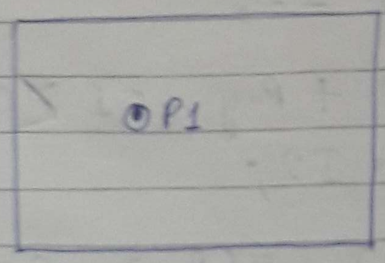
$>0 \rightarrow$ Point is inside the window.

```
#define I1 1
#define I2 2
#define I3 4

int code = 0;
int gencode (Point P)
{
    if ( 14(P.x) - 9(P.y) - 19 <= 0)
        code |= I1;

    if ( -6(P.x) + P.y + 51 <= 0)
        code |= I2;

    if ( -2(P.x) + 7(P.y) - 3 <= 0)
        code |= I3;
}
```

01/02/2018

Line Clipping

Nicholl - Lee - Nicholl :



Computation time faster than other algorithms but code length is large

Step 1:
Locate one of the end point of a line.

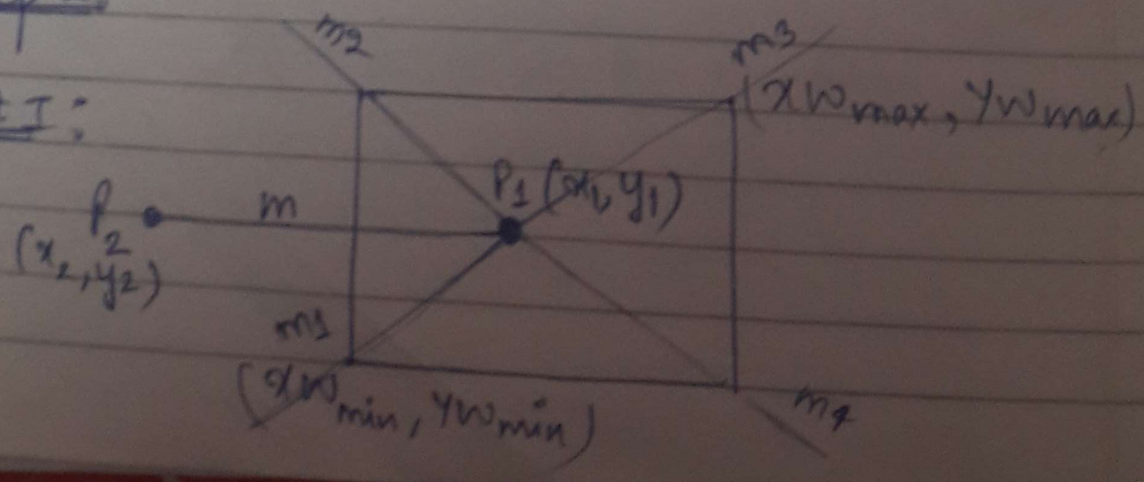
Part I :
End point is inside the rectangular window.

Part II :
End point is on the edge side.
(Left, Right, Top, Bottom)

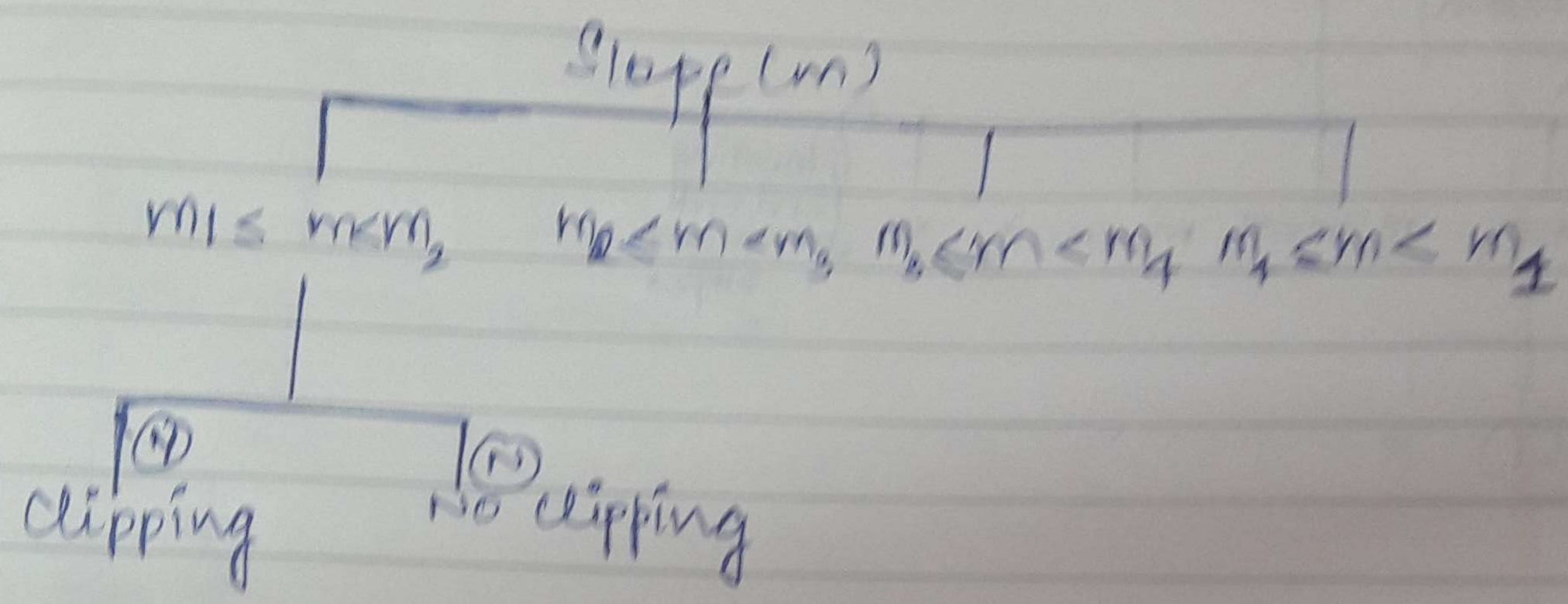
Part III :
End point is on the corner.

Step 2:

Part I :



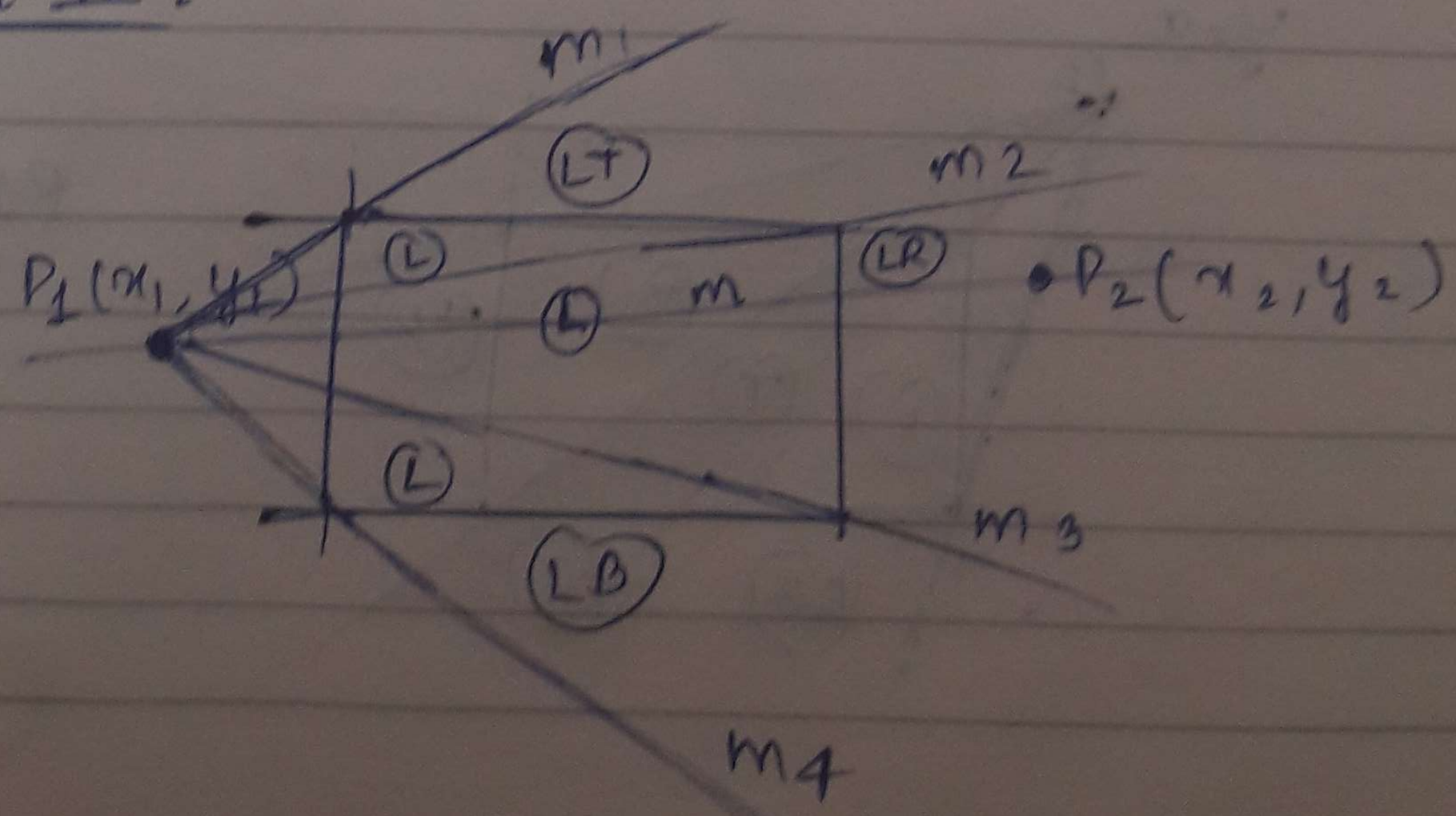
If line $(P_1 - P_2)$ is rotated, then its slope m will between any of the slopes m_1, m_2, m_3, m_4.

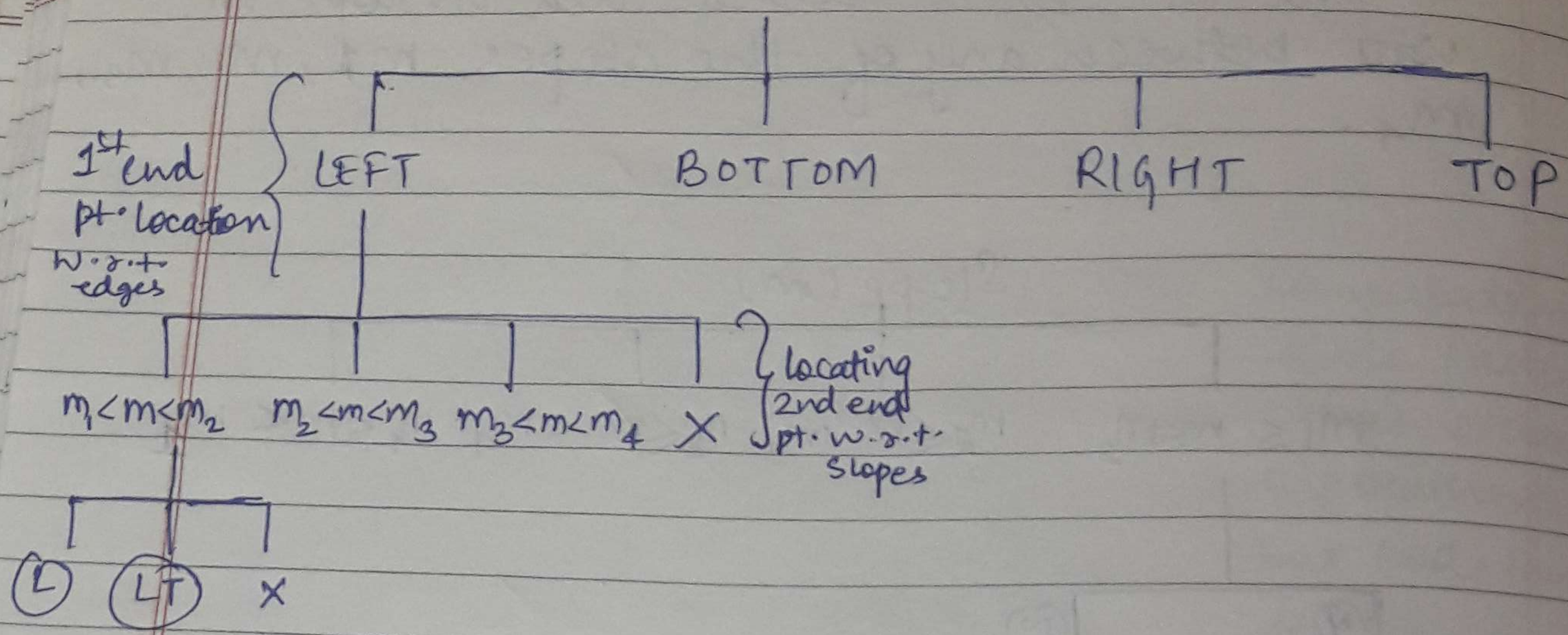



```
if ( (m > m2) && (m < m1) && (P1.x1 > P2.x2) )
{
    if (P2.x2 < xw_min)
        clipping

    ~~No cli~~
}
```

// Similarly for other cases.

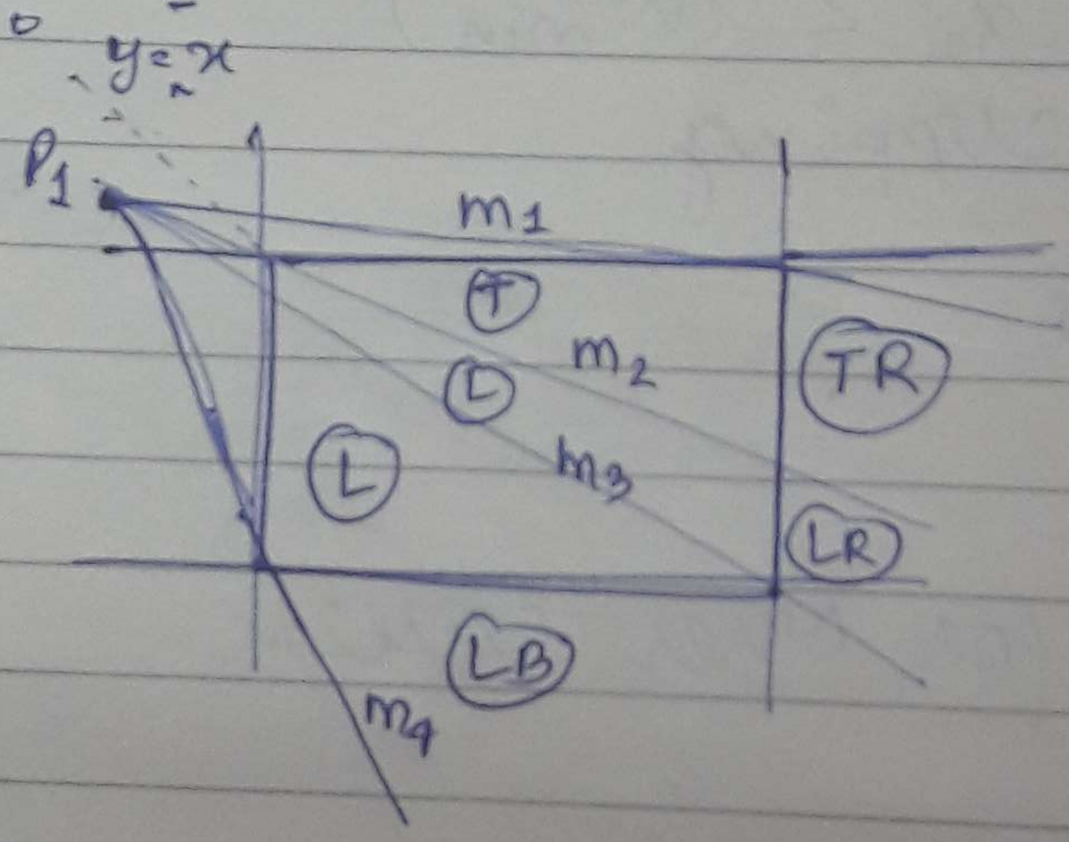
Part II:



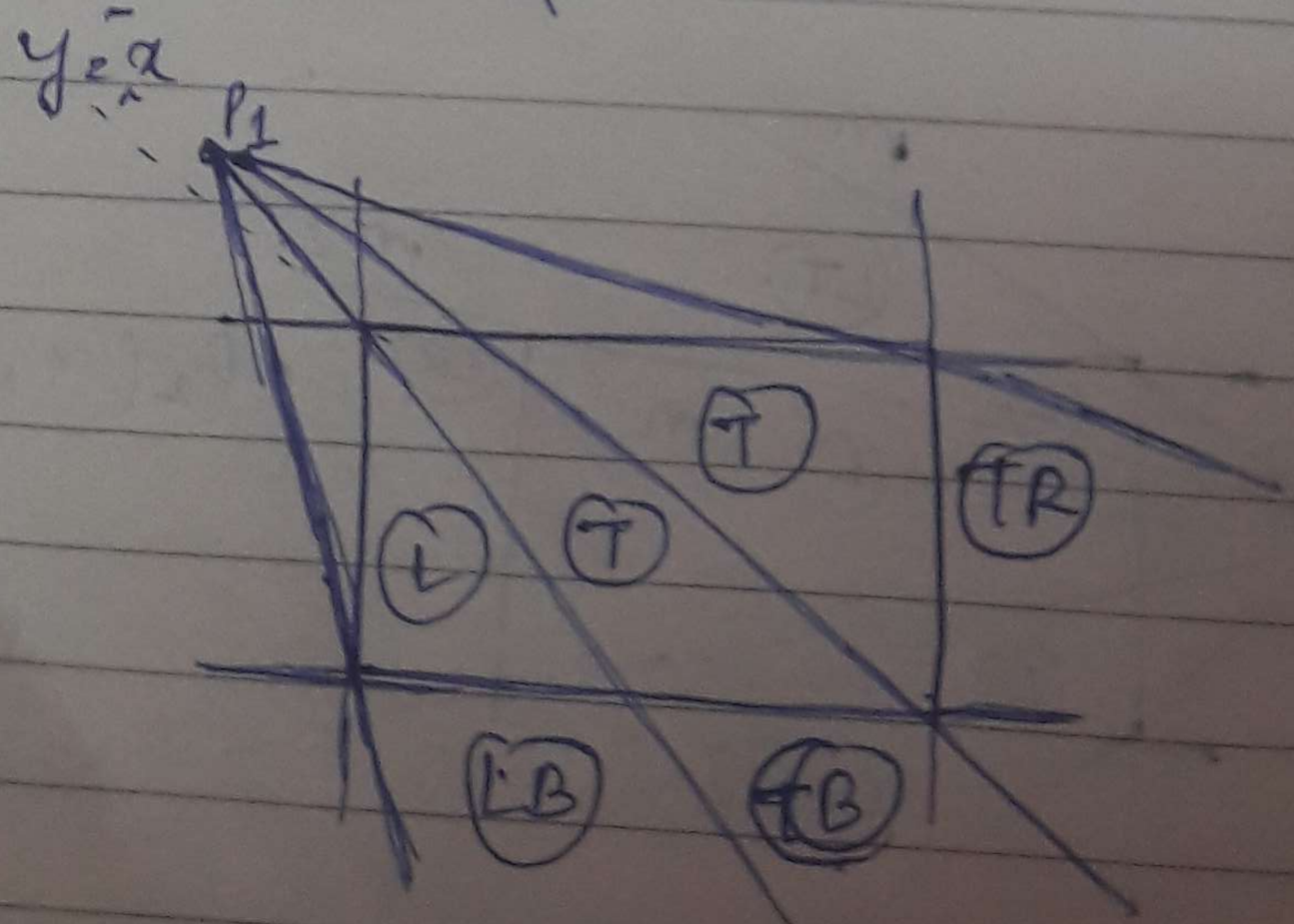


// Similarly for other cases.

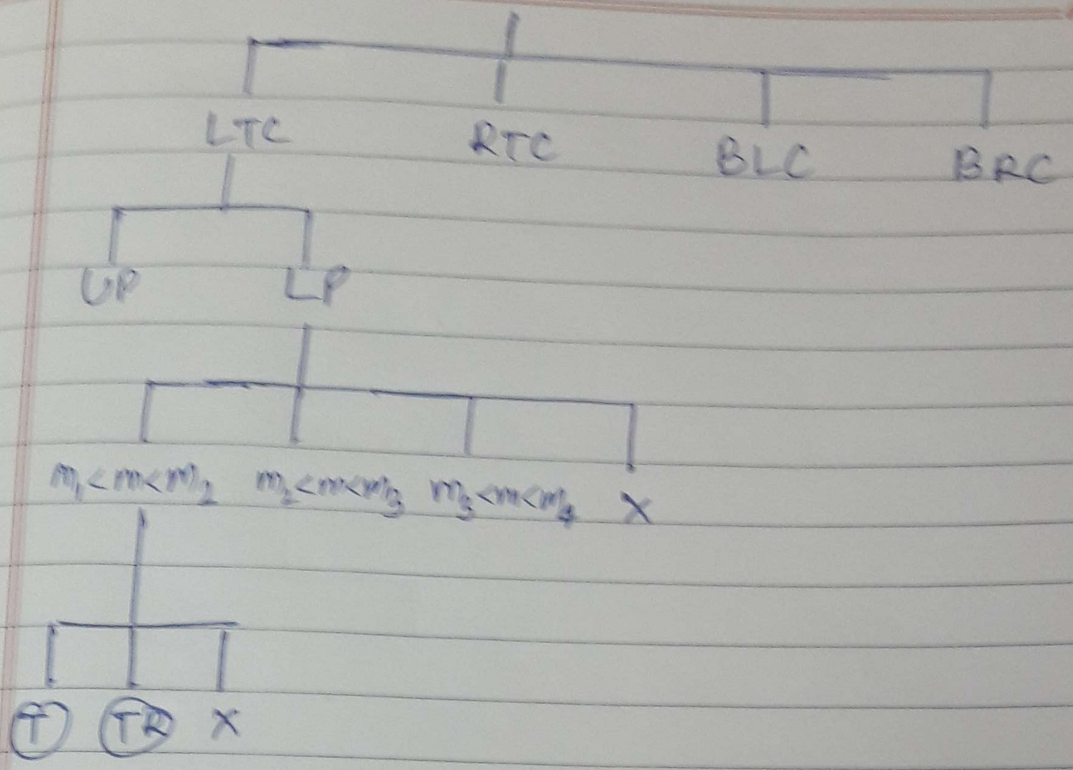
Part III:



P1 in lower part of y=x



P1 above y=x.



// Similarly for other cases.