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1. Computer Graphics - Hearn & Bucker
2. Computer Graphics - Foley Van Dam
3. Schaum Series.

Graphics header file : Pre-defined func.

Output Primitive

1) Point

Putpixel (x, y, COLOR);

Picture element.

2) Line.

input: x_1, y_1, x_2, y_2

method: $x = x_1$;

while $x <= x_2$

{

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) * (x - x_1) + y_1$$

x = x + 0.001;

}

Pre-defined func:

x = x1 ; y = y1 ;

putpixel (x, y, WHITE);

while x <= x2

{

x++;

y = y + dy/dx;

putpixel (x, round(y), WHITE);

}

$y_i = m x_i + c$

for next column:

$$y_{i+1} = m[x_{i+1}] + c$$
$$= m x_i + m + c$$
$$= \underline{m x_i + c} + m$$
$$= y_i + m$$

$$\boxed{y_{i+1} = y_i + m}$$

eg.

$x_1 = 10$ $x_2 = 20$

$y_1 = 10$ $y_2 = 18$.

x	y		
10	10		
11	10.8	11	
12	11.6	12	12
13	12.4	13	12
14	13.2	13	13
15	14.0	15	14
16	14.8		15
:			
20		20.	

$m < 1$ $\frac{8}{10}$

10 +

↑ increment x
compute y.

$y = mx + c$.

↑ Major movement in x

Cases :- $m = 0$, $m = \infty$, $m < 1$, $m > 1$, $m = 1$

↓ major movement in Y
compute X

input :- x_1, y_1, x_2, y_2

$dx = x_2 - x_1$

$dy = y_2 - y_1$

floating point computation.

Coherence eqn :- $y_{i+1} = y_i + m$

x_1, y_1, x_2, y_2

$dx = x_2 - x_1$

$dy = y_2 - y_1$

$x = x_1$; $y = y_1$

if abs(dy) < abs(dx) ← $\boxed{\text{Case 1}}$
{ putpixel (x, y, WHITE)
while (x <= x2)
{ x++;

1, 2 1, 3

$y = mx + c$
$1 = m \frac{dx}{dy}$
$\frac{dy}{dx} = m$
$y = mx + c$
$y - mx - c = 0$
$y - \frac{ay}{b} + c = 0$
$+ \frac{ay}{b}$
$a = m$

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y + = (float) (dy/dx) ← Staircasing comes.
putpixel (x, Round (y), WHITE)
}
}

$\boxed{m>1}$

int round (float p)
{
return (p+0.5)
}

$y = mx + c$

$\frac{y-c}{m} = x$

$x_i = \frac{y_i - c}{m}$

$y = mx + c$

$x = \frac{y-c}{m}$

$x_{i+1} = \frac{y_{i+1} - c}{m}$

$= \frac{y_i + 1 - c}{m}$

$= \frac{y_i}{m} x_1 + \frac{1}{m}$

$x_{i+1} = \frac{y_{i+1} - c}{m}$

$= \frac{y_i - c}{m} + \frac{1}{m}$

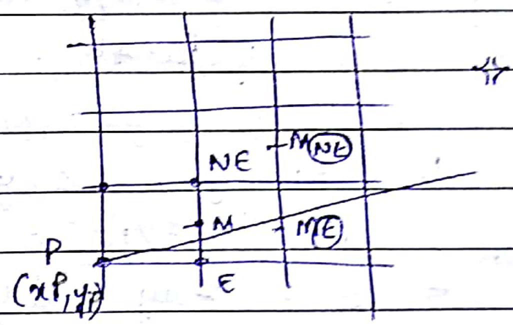
$x_{i+1} = x_i + \frac{1}{m}$

Method 2 Line Drawing

$f(x,y) = 2(ax + by + c)$

<0 >0 =0

{ Deciding factor }



M → midpoint in the next col.

$M = f(x_p+1, y_p + 1/2)$

$d = M = 2(a(x_p+1) + b(y_p+1/2) + c)$

if $d < 0$ /* Case E */

$ax + by + c$

$ax_1 + by_1 + c = 0$

$d_{new} = M(E) = f(x_p + 2, y_p + 1/2)$

$\boxed{\Delta E = d_{new} - d = a}$.

$a = 2dy$

$y = mx + B$

$dx \cdot y = dy \cdot x + B \cdot dx$

$dy \cdot x - dx \cdot y + B \cdot dx = 0$

$a = dy$ $b = -dx$

2a
avp+ 2a
am + . v
a

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$y = m$

if $d > 0$ /* case (NE) */

$d_{new} = M_{NE} = f(x_p + 2, y_p + \frac{3}{2})$

$= a(x_p+2) + b(y_p + \frac{3}{2}) + c$.

$ax_p + 2a + by_p + \frac{3}{2} + c - ax_p + a + by_p$

$d_{new} - d = \cancel{tan} = \cancel{rdx}$ $- \frac{by}{2} - c$

$\cancel{a + b}$

$\boxed{\Delta NE = d_{new} - d = a + b = 2(dy - dx)}$.

Initialise

$d = M = a(x_p + 1) + b(y_p + \frac{1}{2}) + c$.

$= \underbrace{ax_p + by_p + c}_{=0} + a + b/2$

$d_i = a + b/2$ ← so update to $2(ax + by + c)$

$= 2a + b$.

∴ x_1, y_1, x_2, y_2

$dx = x_2 - x_1$; $dy = y_2 - y_1$

Initialise decisn ← $d = 2dy - dx$.

$x = x_1$; $y = y_1$

putpixel (x, y, WHITE)

→ if (abs(dy) < abs(dx))

while (x <= x2)

{

if d <= 0

d += 2dy.

else

{ d += 2* (dy - dx);

y++;

}

x++;

putpixel (x, y, WHITE)

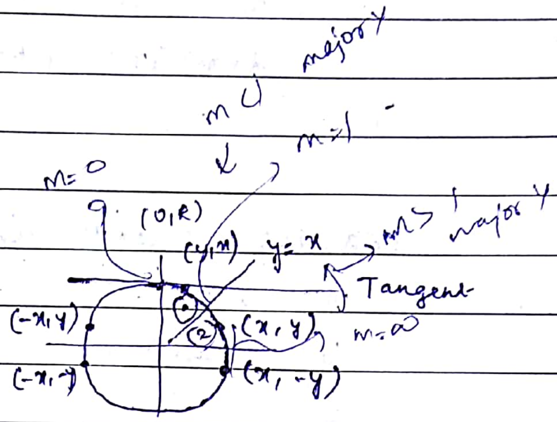
}

Ex:-

$x_1 = 10 \quad y_1 = 10 \quad x_2 = 20 \quad y_2 = 18$.

$dx = 10 \quad dy = 8$.

x	y	d
10	10	6
11	11	2
12	12	-2
13	12	14



Circle

$x^2 + y^2 = R^2$

Eight-way symmetry.

↓

```
void draw-circle ()
{
  → putpixel (x, y, WHITE);
  →    "     (x, -y,  "  );
  →          (-x, -y, "  );
  →          (-x, y,  "  ;
  :          (y, x)
}            (y, -x
             (-y, -x,
             (-y, x,
```

① Major move in X.

```
x = 0;
y = R;
draw-circle (x, y);
while (x, y);
while (x <= y)
{
   x++;
   y = sqrt(R * R - x * x);
   drawcircle
   putpixel (x, round(y)) WHITE)
}
```

$R = 8$

x	y
0	8
1	8
2	8
3	7
4	7
5	6

6 5

64 −36 28   64 −25 39   64 −16 48   64 −9 55

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Major movement in Y (2)

```
x = R;
y = 0;
drawcircle (x, y)
while ( y <= x)
{
    y++;
    x = sqrt (R*R - y*y);
    drawCircle (Round(x), y);
}
```

Drawing a circle in polar co-ordinates :-

$x = r\cos\theta$

$y = r\sin\theta$.

$x = r$

$\theta = 0$

```
x = r
θ = 0
while (θ <= 360)
{
   // To do
}
```

$1 + 2xp + \frac{1}{4} - yp$

$4 + 4xp + \frac{9}{4} - 4p$

$2xp + 3$

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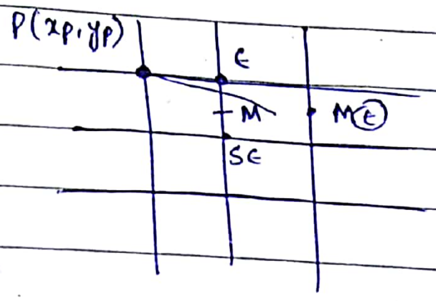
Mid point method



tangent slope

$\left|\frac{dy}{dx}\right| < 1$

major movement in x.

$f(xp, yp) = xp^2 + yp^2 - R^2$

$< 0 \quad = 0 \quad > 0.$

Initialise d.

$= xp^2 + 2xp + 1 + yp^2 - yp + \frac{1}{4} - R^2$

$= 1 - R + \frac{1}{4}$

$= \frac{5}{4} - R$

$d_{init} = 1 - R$

decision $\leftarrow d = \overbrace{f(xp+1, yp - \frac{1}{2})}^{M}$

$= (xp+1)^2 + (yp - \frac{1}{2})^2 - R^2$

if $d < 0$ /* select E */

$d_{new} = f(xp+2, yp - \frac{1}{2})$

$= (xp+2)^2 + (yp - \frac{1}{2})^2 - R^2$

First differential, if selection is E

$\Delta E = d_{new} - d$

$= 2xp + 3$

$1 + 2xp$

$4 + 4xp$

$+ \frac{9}{4} - 3yp$

$\frac{1}{4} - yp$

$2 - 2yp$

$3 + 2xp$

else $d > 0$ /* select SE */

$d_{new} = f(xp+2, yp - 3/2)$

$= (xp+2)^2 + (yp - \frac{3}{2})^2 - R^2$

First differential, if selection is SE

ΔSE

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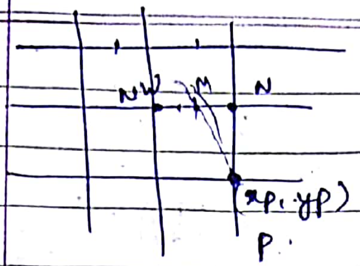
→ x = 0;
y = R
draw circle (x, y);
d = 1 - R;
while (x <= y)
{
if d <= 0
d += 2*x + 3;
else
{
d += 2*(x - y) + 5;
y - -;
}
x + +;
draw circle (x, y);
}

R = 8

x	y	d
0	8	-7
1	8	-4
2	8	1
3	7	-6 ✗
4	7	3
5	6	0
6	6	13
7	5	18

Major movement in X



$\cancel{f = xp^2}$ $f(x,y) = x^2 + y^2 - R^2$

$d = f(xp - \frac{1}{2}, yp + 1)$

if $d \le 0$ (* Select N *)

$d_{new} = f(xp - \frac{1}{2}, yp + 2)$

$$\begin{array}{r} 4 + 4yp \\ -1 - 2yp \\ \hline 3 + 2yp \end{array}$$

First diff $d_{new} - d = 3 + 2yp$

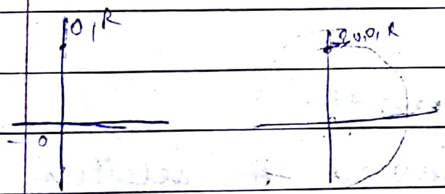
if $d > 0$ (* Select NW *)

$d_{new} = f(xp - \frac{3}{2}, yp + 2)$

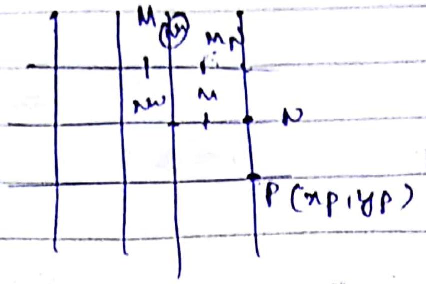
First diff $d_{new} - d =$

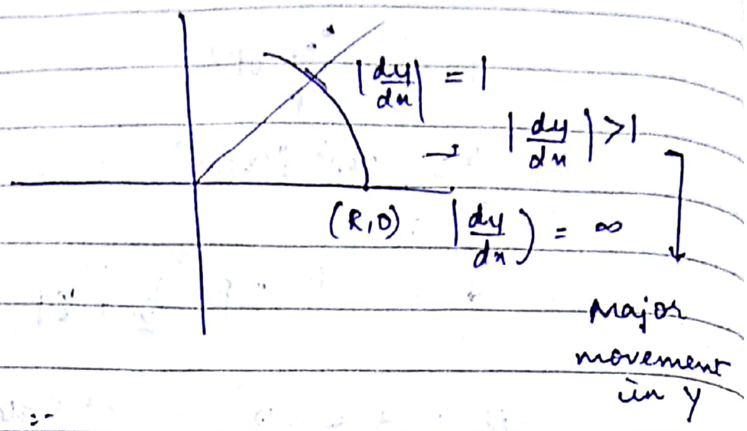
$$\begin{array}{rrrr} -3xp & + \frac{9}{4} & + 4yp & + 4 \\ -xp & + \frac{1}{4} & + 2yp & + 1 \\ + & - & - & + \\ \hline -2xp & + 2 & + 2yp & + 3 \end{array}$$

$= \boxed{-2xp + 2yp + 5}$



~~Polar~~ Circle drawing :-





Decision at mid point :-

$$d = f(M) = f(x_p - \tfrac{1}{2},\ y_p + 1)$$
$$= (x_p - \tfrac{1}{2})^2 + (y_p + 1)^2 - R^2 \quad -(1)$$

Case 1

$d < 0$ /* Select N */

Now decision is evaluated for selection M(N)

$$d_{new} = f(x_p - \tfrac{1}{2},\ y_p + 2)$$
$$= (x_p - \tfrac{1}{2})^2 + (y_p + 2)^2 - R^2 \quad -(2)$$

$4 + 4y_p \quad - 2 \quad - 2 \quad 2$

First differential of the decision at (N).

$$\Delta_N = d_{new} - d = 2y_p + 3$$

Case 2 :

$d > 0$ /* Select NW */

New decision is evaluated for selection M(NW)

$$d_{new} = f(x_p - \tfrac{3}{2},\ y_p + 2)$$

First differential at (NW)

$$\Delta_{NW} = d_{new} - d = 2(y_p - x_p) + 5$$

initialisation of $d = (x_p - \tfrac{1}{2})^2 + (y_p + 1)^2 - R^2$.

$$= x_p^2 + \tfrac{1}{4} - x_p + y_p^2 + 1 + 2y_p - R^2$$

$$\tfrac{1}{4} - R + 1 = \tfrac{5}{4} - R$$
$$= 1 - R$$

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```
x = R;
y = 0; d = 1 - R;
drawCircle(x, y);
while (y <= x)
{
    y++;
    if (d <= 0)
        d += 2y + 3
    else
    {
        d += 2(y - x) + 5
        x--;
    }
    y++;
    drawCircle(x, y);
```

R = 8

x	y	d
8	0	-7
8	1	-4
8	2	1
7	3	-6
7	4	3
6	5	2
5	6	5

2nd diff

$\boxed{\text{N selected}}$ if $d < 0$

~~First~~ diff $\not< 0$

$\boxed{N = x_p, y_p + 1}$

$\Delta_N = 2y + 3$

$= 2y_p + 5$ — (1)

$x_p^2 + (y_p^2) + 1 + 2y_p$

$\Delta_{NW} = 2(y_p - x_p) + 5$

$= 2(y_p + 1 - x_p) + 5$

$= 2(y_p - x_p) + 7$ — (2)

2nd diff at selection (N)

$\Delta^2 N^{(@N)} = 2$

$\Delta^2 NW^{(@N)} = 2$

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if $d > 0$ /* Select NW */

Location of NW $(xp-1, yp+1)$

$$\Delta N^{@NW} = 2y + 3$$
$$= 2yp + 5$$

$$\Delta NW^{@NW} = 2(y-x) + 5$$
$$= 2(yp+1 - xp+1) + 5$$
$$= 2(yp - xp) + 9$$

2nd differential at selection of NW :-

$$\Delta^2 N^{@NW} = 2$$

$$\Delta^2 NW^{@NW} = 4$$

Initialisation for 2nd differential

$d = 1 - R$

$\Delta N = 3$

$\Delta NW = 5 - 2R$

Algo:-

$x = R$

$y = 0$

$d = 1 - R$

$\Delta N = 3$

$\Delta NW = 5 - 2R$

drawCircle (x, y);

while $(y <= x)$

{ if $d <= 0$ /* Select N */

{ d += deltaN;

deltaN += 2;

deltaNW += 2;

}

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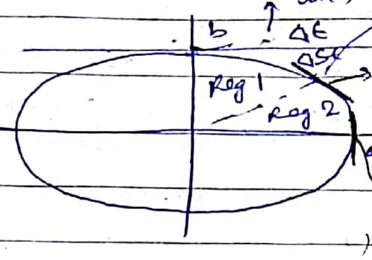
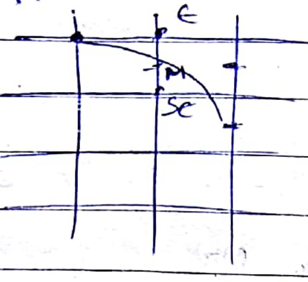
```
if d>0    /* select NW */
{
    d += deltaNW;
    deltaN += 2;
    deltaNW += 4;
    x--;
}
y++;
drawCircle (x, y);
}
```

x	y	d	delta N	delta NW
8	0	-7	3	-11
8	1	-4	5	-9
8	2	1	7	-7
7	3	-6	9	-3
7	4	3	11	-1
6	5	2	13	3
5	6	5	15	7

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 4 way symmetry



$2b^2x + 2a^2y\frac{dy}{dx} = 0$



$b^2x^2 + a^2y^2 - a^2b^2 = 0$

$\cancel{b^2x^2} + b^2 + 2b^2x_p + \cancel{a^2y^2} + \frac{a^2}{4} - a^2y$

$b^2x^2 + 4b^2 + 4b^2x_p \quad -a^2b^2$

Decision at midpoint M $(x_p+1,\ y_p - \frac{1}{2})$

$$d = f(M) = \frac{(x_p+1)^2}{a^2} + \frac{(y_p - 1/2)^2}{b^2} - 1 \quad \text{— (1)}$$

$$= \frac{1}{a^2} + \frac{2x_p}{a^2} + \frac{1}{4b^2} - \frac{y_p}{b^2}$$

Case 1

$d < 0$ /* Select E */

$$d_{new} = f(M_E) = \frac{(x_p+2)^2}{a^2} + \frac{(y_p - 1/2)^2}{b^2} - 1 \quad \text{— (2)}$$

$$= \frac{4}{a^2} + \frac{4x_p}{a^2}$$

$$\Delta_E = \frac{3}{a^2} + \frac{2x_p}{a^2} = \boxed{b^2(2x_p + 3)} \checkmark$$

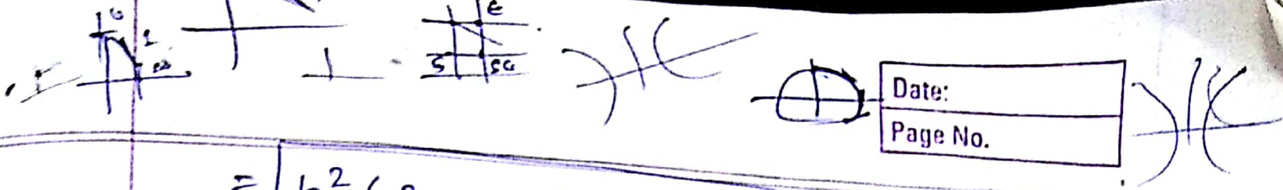
Case 2

$d > 0$ /* Select SE */

$$d_{new} = f(M_{SE}) = \frac{(x_p+2)^2}{a^2} + \frac{(y_p - 3/2)^2}{b^2} - 1$$

$$= \frac{3 + 2x_p}{a^2} \quad \frac{4}{4b^2} - \frac{3y_p}{b^2}$$

$$= \frac{3 + 2x_p}{a^2} + \frac{2(1 - y_p)}{b^2} \quad \Bigg| \quad \frac{8}{4b^2} - \frac{8y_p}{4b^2}$$



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$$= \boxed{b^2(2x_p+3) + 2a^2(1-y_p)}$$

Initialisation

$\frac{x_p^2}{a^2} + 1$ $\qquad M = x_p+1,\ y_p - \frac{1}{2}$

$x_p = 0$

$y_p = b$

$x_p+1,\ y_p-\frac{1}{2}$

$(1,\ b-\frac{1}{2})$

$$\frac{(2)^2}{a^2} + \frac{(b-\frac{1}{2}-\frac{1}{2})^2}{b^2} - 1$$

$$\frac{4}{a^2} + \frac{b^2+1-2b}{b^2} - 1$$

$$= \boxed{b^2 + \frac{a^2}{4} - a^2 b}$$

$d_{old} = b^2(x_p^2 + \frac{1}{4} + x_p) + a^2(y_p^2 + 1 - 2y_p)$

Gradient Vector

$$\text{Gradient}(f) = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$$

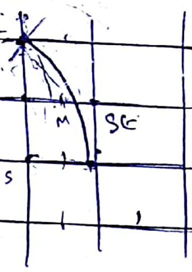
$x_p+1,\ y_p \quad (2x_p+5)$

Region 2

$f(x_p, y_p) = b^2 x_p^2 + a^2 y_p^2 - a^2 b^2$

$d = b^2 x_p^2 + \frac{b^2}{4} + b^2 x_p + a^2 y_p^2 + a^2 - 2a^2 b$

$d^{II} = f(M) = f(x_p + \frac{1}{2},\ y_p - 1)$



$d^{II} \le 0$ /* Select (S) */

$f(x_p + \frac{1}{2},\ y_p - 2)$

$d_{new} = b^2(x_p^2 + \frac{1}{4} + x_p) + a^2(y_p^2 + 4 - 4y_p)$

$\Delta S = a^2(3 - 2y_p)$

$\Delta SE = a^2(3 - 2y_p) + 2b^2(1 + x_p)$

$d_{new} = b^2(x_p^2 + \frac{9}{4} + 3x_p) + a^2(y_p + 4 - 4y_p)$

$2b^2 x\, i + 2a^2 y\, j$

$d_2 = b^2 x_p^2 + \frac{b^2}{4} + b^2 x_p + a^2 y_p + a^2 - 2a^2 y_p - a^2 b^2$

$2b^2 x + 2a^2 y \frac{dy}{dx} = 0$

$d_{2S} = b^2 x_p + \frac{b^2}{4} + b^2 x_p + a^2 y_p + 4a^2 - 4a^2 y_p$

$\frac{dy}{dx} = -1$

$b^2 x - a^2 y = 0 \qquad b^2 x = a^2 y \qquad \frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$

$3a^2 - 2a^2$

$a^2y^2 + b^2x^2 - a^2b^2$

$2a^2y\,\dot{}\; + 2b^2x$

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$x = 0,\ y = b$

a, b ;

d = b*b + 0.25*a*a - a*a*b;

drawellipse (x, y)

while((a²y) > (b²x)) ← Reg ①

{ if d < 0

d += b²*(2xp+3);

else {

d += b²*(2xp+3) + 2*a² (1 - yp)

x++; } y--;

drawellipse (x, y);

}

d = b*b + 0.25*a*a - a*a*y; ←

drawellipse (x,y)

{

←

}

Second differential for Ellipse :-

if d<0 → (E) selected. $(x_p+1,\ y_p)$

$\Delta E = b^2(2x_p + 3)$

$= b^2(2x_p + 5)$ — ①

$\Delta SE = b^2(2x_p + 3) + 2a^2(1 - y_p)$

$= b^2(2x_p + 5) + 2a^2(1 - y_p)$ — ②

$\Delta^2 E = 2b^2$

$\Delta^2 SE = 2b^2$.

$\Delta^2 E = 2b^2$

$\Delta^2 SE = 2b^2 + 2a^2$

if d≥0 (SE) selected. $(x_p+1,\ y_p-1)$

$\Delta E = b^2(2x_p + 2 + 3) = b^2(2x_p+5) =$

$\Delta SE = b^2(2x_p+3) + 2a^2(1 - y_p)$

$= b^2(2x_p+5) + 2a^2(2 - y_p)$

$\Delta^2 SE = 2b^2 + 2a^2$

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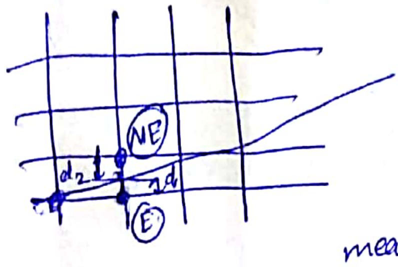
Bresenham's Approach :

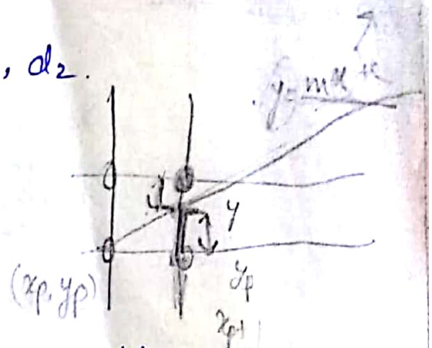


measurable quantity

$(m<1)$

major movement in x.

measurable quantity → d_1, d_2.



$y = mx + c$

$d_1 = y - y_p$

$= m(x_p + 1) + c - y_p$ —— ①

$= mx_p + m + c - y_p$

$d_2 = y_p + 1 - y$. —— ②

$= y_p + 1 - mx_p - m - c$.

measurable quantity in line can be distance.

Difference in measurable quantity gives me decision parameter

$D_p = d_1 - d_2$

$= 2y - 2y_p - 1$

$= 2(m(x_p + 1) + c) - 2y_p - 1$

<0   0   >0

↓

$d_2 > d_1$

E is selected

$D_p \Delta x = 2[\Delta y(x_p + 1) + c\Delta x] - 2y_p \Delta x - \Delta x$.

↓

decision multiplied by the number do not effect decision

↓

constants.

$D_p = 2\Delta y x_p + 2\Delta y + 2c\Delta x - 2y_p \Delta x - \Delta x$.

$\boxed{D_p = 2\Delta y x_p - 2\Delta x y_p + C}$ —— 1.

If $D_p < 0$ // Select E

$y_{p+1} = y_p$

$x_{p+1} = x_p + 1$

$D_{p+1} = 2\Delta y(x_p + 1) - 2\Delta x y_p + C$ —— ②

$\boxed{\Delta E = 2\Delta y}$

Else $D_p > 0$ // select NE.

$$y_{p+1} = y_p + 1$$
$$x_{p+1} = x_p + 1$$

$$D_{p+1} = 2\Delta y\,(x_p + 1) - 2\Delta x\,(y_p + 1) + \text{constant} \quad \text{--- (3)}$$

$$\boxed{\Delta NE = 2\Delta y - 2\Delta x}$$

Initialisation:

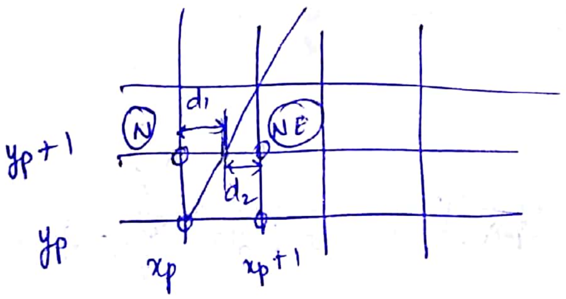
$$\cancel{D_p = 2\Delta y\, x_p - 2\Delta x\, y_p + 2\Delta y + 2c\Delta x - \Delta x}$$

$$D_p = 2[m(x_p+1) + c] - 2y_p - 1$$
$$= 2\underbrace{[m x_p + c - y_p]}_{0} + 2m - 1$$
$$= 2m - 1$$

$$\boxed{D_p \Delta x = 2\Delta y - \Delta x}$$

$(m > 1)$: Major movement in y

$$y = mx + c$$
$$x = \frac{y - c}{m}$$



$$d_1 = x - x_p$$
$$d_2 = x_p + 1 - x.$$

$$D_p = d_1 - d_2$$
$$= x - x_p - x_p - 1 + x$$
$$= 2x - 2x_p - 1$$
$$= 2\left(\frac{y_p + 1 - c}{m}\right) - 2x_p - 1$$

$$D_p \Delta y = 2\Delta x(y_p + 1 - c) - 2\Delta y\, x_p - 1$$

$$\boxed{\underbrace{\Delta y D_p}_{D_p} = 2y_p\Delta x - 2\Delta y\, x_p + \underbrace{2\Delta x - 2(\Delta x)c - 1}_{C}}$$

If $D_p < 0$ // $d_1 < d_2$, select N.

$$y_{p+1} = y_p + 1.$$
$$x_{p+1} = x_p$$

$D_{(new)}$

$$D_{p+1} = 2(y_p + 1)\Delta x - 2x_p\Delta y + C$$

If $D_p > 0$ // $d_1 > d_2$ select NE

$y_{p+1} = y_p + 1$

$x_{p+1} = x_p + 1$

$D_{p+1} = 2(y_p+1)\Delta x - 2(x_p+1)\Delta y + c$

$\boxed{\Delta NE = 2\Delta x - 2\Delta y}$

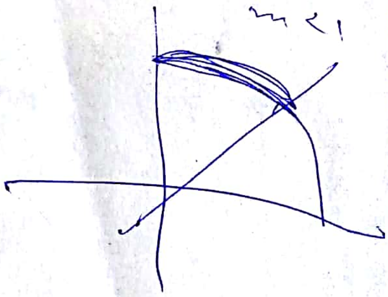
Initialisation :

$D_p = \frac{2}{m}(y_p + 1 - c) - 2x_p - 1$

$= \frac{2}{m}(\underbrace{y_p - c - m x_p}_{0}) + \frac{2}{m} - 1$

$= \frac{2}{m} - 1$

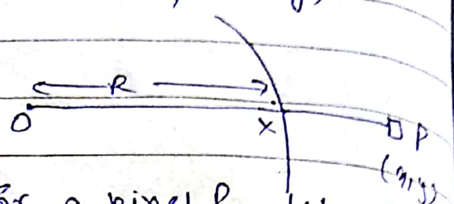
$\boxed{D_p \Delta y = 2\Delta x - \Delta y}$



Bresenham's Approach

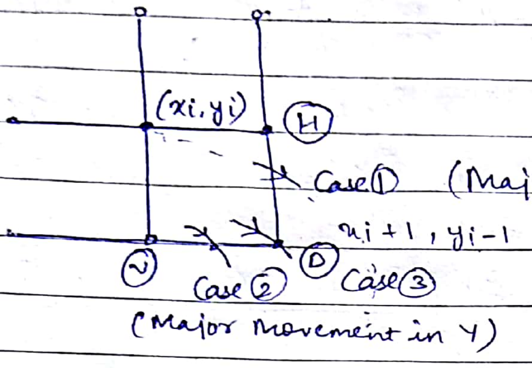
-(Measurable quantity)

Circle
(Clockwise)



For a pixel P, let the measurable quantity be d

$dp = |OP^2 - O\ldots$
$= |x^2 + y^2 - R^2|$



Known pixel (x_i, y_i)

the candidate pixel

→ H (x_i+1, y_i)
→ D (x_i+1, y_i-1)
→ V (x_i, y_i-1)

For a pixel H

$dH = |OH^2 - OX^2|$

$= |(x_i+1)^2 + y_i^2 - R^2|$

For case 1

$OD^2 < R^2$

$OD^2 - R^2 < 0$

For case 2

$OD^2 > R^2$

For case 3

$OD^2 = R^2$

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Case ①

To choose between (H) & (D)
- Difference in measurable quantity

$$\delta_{HD} = d_H - d_D$$
$$= |OH^2 - R^2| - |OD^2 - R^2| = (OH^2 - R^2) - (R^2 - OD^2)$$
$$= |[x_i+1]^2 + y_i^2 - R^2| + |[x_i+1]^2 + [y_i - 1]^2 - R^2|$$
$$= 2[(x_i+1)^2 + (y_i-1)^2 - R^2] + (2y_i - 1)$$

$$\boxed{\delta_{HD} = 2\Delta D_i + (2y_i - 1)}$$

Case ②

To choose between (V) & (D)
- Difference in measurable quantity

$$\delta_{VD} = d_V - d_D$$
$$= |OV^2 - R^2| - |OD^2 - R^2| \qquad \because OD^2 > R^2$$
$$= (R^2 - OV^2) - (OD^2 - R^2) \qquad OV^2 < R^2$$
$$= \cancel{R^2 - [x_i}$$
$$= [R^2 - (x_i)^2 - (y_i - 1)^2] - [(x_i+1)^2 + (y_i-1)^2 - R^2]$$
$$= 2[-R^2 + (x_i+1)^2 + (y_i-1)^2] + 1 + 2x_i$$

$$\boxed{\delta_{VD} = 2x_i + 1 - 2\Delta D_K}$$

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Part ①

Suppose Horizontal pixel (H)

$x_{i+1} = x_i + 1 \quad y_{i+1} = y_i$

$$\Delta D_i = (x_i+1)^2 + (y_i-1)^2 - R^2$$

$$\Delta D_{i+1} = [x_{i+1}+1)^2 + (y_{i+1}-1)^2 - R^2$$
$$= (x_i+2)^2 + (y_i-1)^2 - R^2$$

$$\boxed{\Delta D_{i+1} = \Delta D_i + 2x_i + 3}$$

Part ②

Suppose vertical pixel (V)

$x_{i+1} = x_i, \quad y_{i+1} = y_i - 1$

$$\Delta D_i = (x_i+1)^2 + (y_i-1)^2 - R^2$$

$-2y_i + 1$

$$\Delta D_{i+1} = (x_i+1)^2 + (y_i-2)^2 - R^2$$

$-4y_i + 4$

$$= \Delta D_i + 3 - 2y_i$$

$$\boxed{\Delta D_{i+1} = \Delta D_i - 2y_i + 3}$$

Part - ③

Suppose diagonal pixel (D) is chosen

$x_{i+1} = x_i + 1 \quad y_{i+1} = y_i - 1$

$$\Delta D_i = (x_i+1)^2 + (y_i-1)^2 - R^2$$
$$\Delta D_{i+1} = (x_{i+1}+1)^2 + (y_{i+1}-1)^2 - R^2$$
$$= (x_i+2)^2 + (y_i-2)^2 - R^2$$

$2 + 2x_i$

$$\boxed{\Delta D_{i+1} = \Delta D_i + 2(x_i - y_i) + 6}$$

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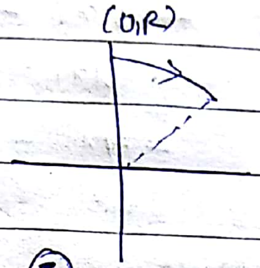
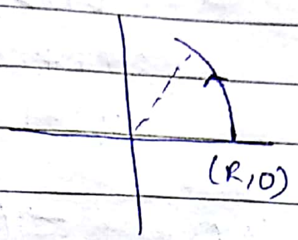
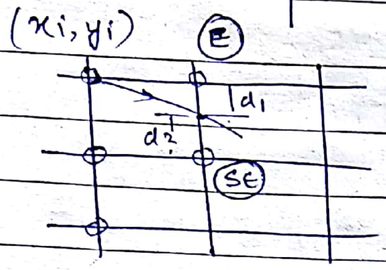
Initialisation







$d_1 = y_i^2 - y^2$

$d_2 = y^2 - (y_i - 1)^2$

y^2 at column $(x_i + 1)$

$\boxed{y^2 = R^2 - (x_i + 1)^2}$

$D_i = d_2 - d_1$

$D_i = 2y^2 - (y_i - 1)^2 - y^2$

$\boxed{D_i = 2[R^2 - (x_i + 1)^2] - (y_i - 1)^2 - y_i^2}$

$-1 \mp 2x_i$ $\quad +1 + 2y_i$

$2 + 4x_i$

if $D_i < 0$, select SE

{

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i - 1$

D_{i+1}

$\Delta SE = \cancel{2(x_i - y_i) + 6}$

$= 4(y_i - x_i) - 10.$

}

$D_{i+1} =$

$2[R^2 - (x_{i+1} + 1)^2]$

$- (y_{i+1} - 1)^2 - y_{i+1}^2$

$4 + 4x_i$

$= 2(R^2 - (x_i + 2)^2)$ $\quad 8 + 8x_i$

$- (y_i - 2)^2 - (y_i - 1)^2$

$y_i^2 + 4 + 4y_i$ $\quad -2x_i$

$= 6 + 4x_i$

$2y_i$

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else
{
$D1 > 0$

$x_{i+1} \rightarrow x_i + 1$

$y_{i+1} \rightarrow y_i$

$\Delta E = -(4x_i + 6)$

$2[R^2 - (x_i+2)^2] - (y_i - 1)^2 - (y$

$-8x_i - 8$

$-4x_i \quad 2$

$-6x_i - 6$

Parabola

1. $y^2 = 2px$
2. $y^2 = -2px$
3. $x^2 = 2py$
4. $x^2 = -2py$

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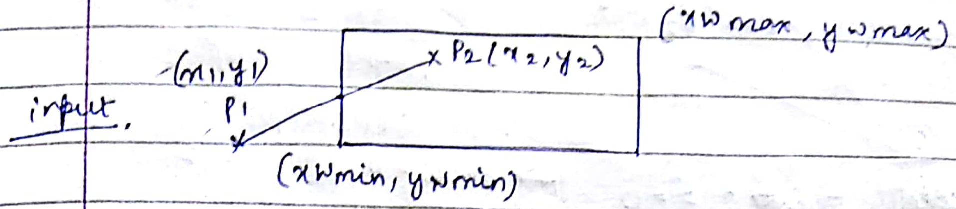
Clipping

Rectangular window



$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Left Edge

$$\frac{y-y_1}{x-x_1} =$$

$x_I = xw_{min}$

$$y_I = y_1 + \left(\frac{y_2-y_1}{x_2-x_1}\right) \times (xw_{min} - x_1) \qquad y = \left(\frac{y_2-y_1}{x_2-x_1}\right)x + c.$$

$$y = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1) + y_1$$

Where the end point lies?

0x9	0x8	0x12
1 0 0 1	1 0 0 0	1 1 0 0
0x1	0x0	0x4
0 0 0 1	0 0 0 0	0 1 0 0
0 0 1 1	0 0 1 0	0 1 1 0

0000
0001
1001
1000
1001

```
# define LEFT 0x1
# define BOTTOM 0x2
# define RIGHT 0x4
# define TOP 0x8
```

9 Areas (Region)

```
regcode = 0x0;
gencode (point p)
{
    if p.x <= xwmin
        regcode |= LEFT;
    if p.x >= xwmax;
        regcode |= RIGHT;
    if p.y <= ywmin
        regcode |= BOTTOM;
    if p.y >= ywmax
        regcode |= TOP;
}
```

flag = 0 | draw = 0
done = 0
do{

Task ①

Code₁ ← regcode(P₁) ✓
Code₂ ← regcode. (P₂) ✓

Task ②

line is visible → { code1 & code 2 == 0
code 1 | code 2 == 0 } → draw = 1, done = 1

line is not visible ← { code 1 & code 2 != 0 → done = 1

{ code 1 & code 2 == 0
&& code1 | code2 != 0
→ Task of clipping

} while (done != 1)

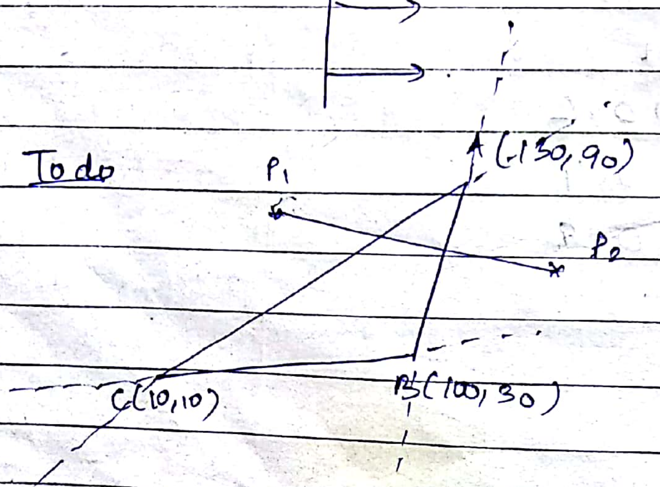
Task of clipping

→ Identify the region code.

P₁ ← 0×8 → 0no → regcode(P₁) & LEFT

x_I = xmin
y_I =

Todo

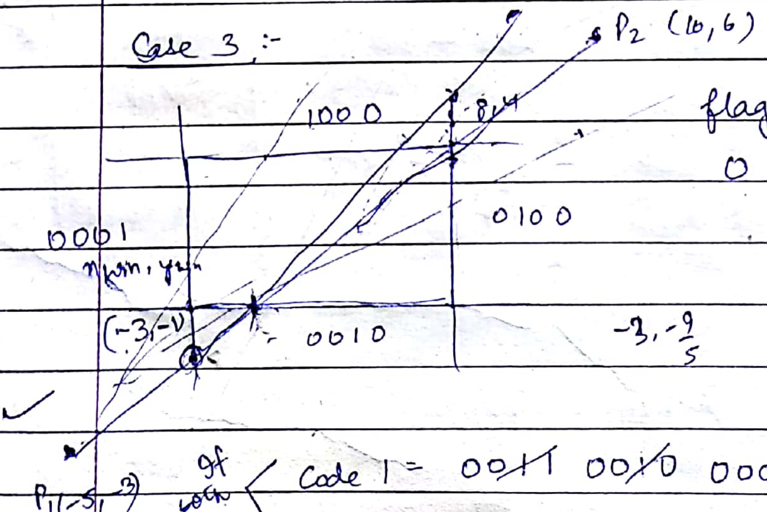


Cohen-Sutherland

- Iterative approach

```
flag = 0
done = 0
do
{
    code1 = regcode (P1); code 2 = regcode (P2);
    if (code1 & code 2 = = 0) && (code1 | code 2 = = 0)
    {
        flag = 1 ; done = 1 ;
                   ↳ exit
    }
    if (code 1 & code 2 ! = 0)
        done = 1 ;
            ↳ exit
    if (code1 & code 2 = = 0) && (code1 | code 2 ! = 0)
        /* clip process */
} while (done ! = 0)
```

Case 3 :-



flag	done	P_1	reg (P_1)	P_2	reg (P_1)
0	0	-5,-3	0011	10,6	1100
		$-3, -\frac{9}{5}$	0010	10,6	1100
		$-\frac{5}{3}$ (-1)	0000	10,6	1100
		$-\frac{5}{3}$ -1	0000	8,24	1000
		$-\frac{5}{3}$ -1	0000	$\frac{20}{3}, 4$	0000

If both non zeros, and else non zero

Code 1 = 0011 0010 0000 –

Code 2 = 1100

Code1 → Left, → Bottom, → Right, → Top

LEF

x 0001

~~0010~~

0011

0010

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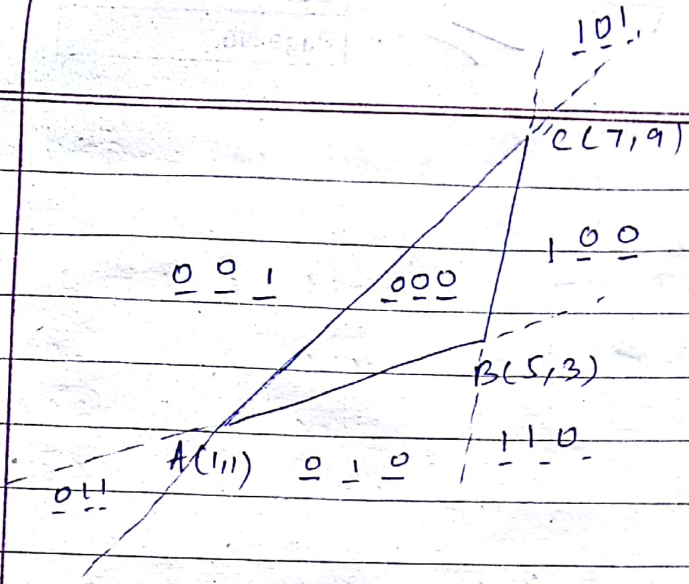


AB

$$\frac{y-1}{x-1} = \frac{\cancel{2}\,1}{\cancel{4}\,2}$$

$$y-1 = \frac{1}{2}(x-1)$$

$$y = \frac{x}{2} + \frac{1}{2}$$

$$(2y = x+1)$$

f(C) = +10

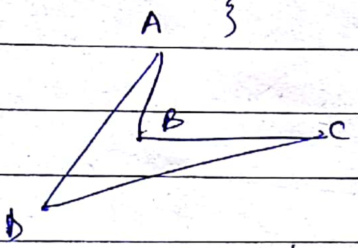
```
reg Code = 0x0;
genCode (Point P)
{

}
```

BC $\frac{y-3}{x-5} = \frac{\cancel{6}\,3}{\cancel{2}}$

$$y-3 = 3x-15$$

$$\boxed{y-3x+12=0}$$

f(A) = +10



AC

$$\boxed{4x-3y-1=0}$$

f(B) = +10.

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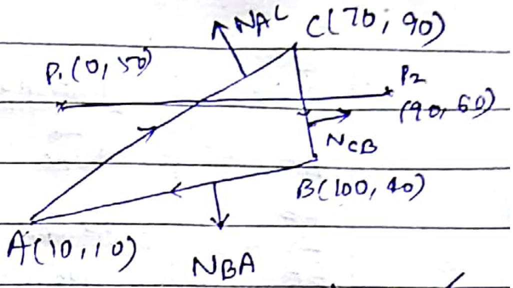
Line - Clipping (Cyrus - Beck)

Non - rectangular window

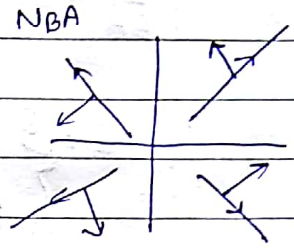


Edges

$\vec{AC}$ $60\hat{i} + 80\hat{j}$ $N_{\vec{AC}} = -80\hat{i} + 60\hat{j}$

$\vec{CB}$ $30\hat{i} - 50\hat{j}$ $\vec{N_{CB}} = 50\hat{i} + 30\hat{j}$

$\vec{BA}$ $-90\hat{i} - 30\hat{j}$ $\vec{N_{BA}} = 30\hat{i} - 90\hat{j}$



Edges x	y	Normal	
+	+	−	+
+	−	+	+
−	+	−	−
−	−	+	−

$10\hat{i} + 10\hat{j}$

$-60\hat{i} - 80\hat{j}$

$$t = \left(\frac{30\hat{i} - 90\hat{j}}{-30\hat{i} + 90\hat{j}}\right) \cdot \left(\frac{0,50 - -90,-30}{90\hat{i} + 10\hat{j}}\right)$$

$$= \frac{30\hat{i} - 90\hat{j} \cdot (-90\hat{i} + 80\hat{j})}{-30\hat{i} + 90\hat{j} \cdot 90\hat{i} + 10\hat{j}}$$

$i = \vec{AC}, \vec{CB}, \vec{BA}$

$$t = \frac{N_i \cdot (P_1 - E_i)}{-N_i \cdot (P_2 - P_1)}$$

$N_i \rightarrow$ Normal

$P_1 \rightarrow 0, 50$

$E_i \rightarrow A, B, C$.

$\vec{AC}$. $$t = \frac{(-80\hat{i} + 60\hat{j}) \cdot (0,50 - 60,80)}{(80\hat{i} - 60\hat{j})\ (90,60 - 0,50)}$$

$$= \frac{(-80\hat{i} + 60\hat{j}) \cdot (-60\hat{i} - 30\hat{j})}{(80\hat{i} - 60\hat{j}) \cdot (90\hat{i} + 10\hat{j})}$$

$$= \frac{+4800 - 1800}{7200 - 600} = \frac{3000}{6600} = \frac{+5}{11}$$

$\vec{CB} = \frac{(50\hat{i} + 30\hat{j})(0,50 - 30,-50)}{(-50\hat{i} - 30\hat{j})\ (90,60 - 0,50)}$

$$= \frac{50\hat{i} + 30\hat{j} \cdot (-30\hat{i} + 100\hat{j})}{-50\hat{i} - 30\hat{j}\ (90\hat{i} + 10\hat{j})} = \frac{-1500 + 3000}{-4500 - 300} = \frac{1500}{-4800} = -5/16$$

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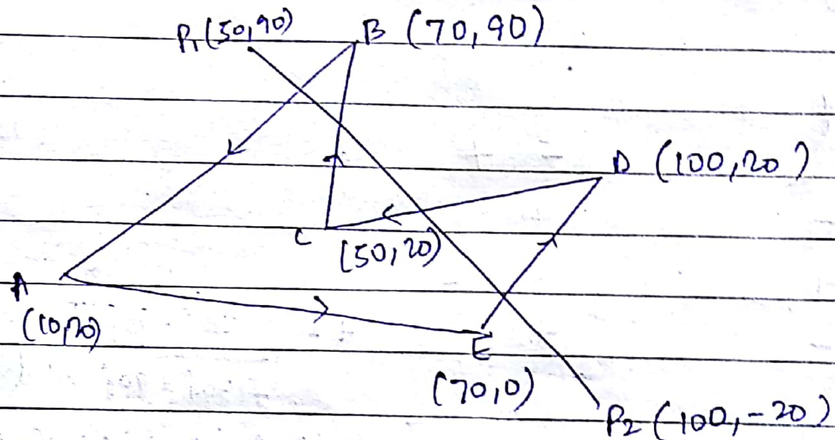
$t_{\vec{CB}} =$

Edges	Normal	$-N_i(P_2-P_1)$	t
$\vec{AC}$	$\vec{N_{AC}}$	> 0	16/33
$\vec{CB}$	$N_{\vec{CB}}$	< 0	47/48
$\vec{BA}$	$N_{\vec{BA}}$		+13/6

$S = \{ t_{\vec{AC}}, t_{\vec{CB}}, \cancel{t_{BA}} \}$

$\vec{P_{En}}$ $\vec{P_{Ex}}$

Ex:



\+ −   − −

− +   + +

Edges:		Normal		t	D
$\vec{AE}$	$60\hat{i} - 20\hat{j}$	$N\vec{AE}$ =	$-20\hat{i} - 60\hat{j}$	61/28	
$\vec{ED}$	$= 30\hat{i} + 20\hat{j}$	$N\vec{ED}$ =	$20\hat{i} - 30\hat{j}$	31/43	<0
$\vec{DC}$	$= 0 - 50\hat{j}$	$N\vec{DC}$ =	$-50\hat{i}$	7/11	>0
$\vec{CB}$	$= 20\hat{i} + 70\hat{j}$	$N\vec{CB}$ =	$70\hat{i} - 20\hat{j}$	14/57	<0
$\vec{BA}$	$= -60\hat{i} - 70\hat{j}$	$N\vec{BA}$ =	$\cancel{+}70\hat{i} + 60\hat{j}$	14/101	>0

$t_{ED} = \dfrac{(N_{ED}) \cdot (50,90 - 70,0)}{(-N_{ED}) \cdot (100,-20 - 50,90)}$

$= \dfrac{(20\hat{i} - 30\hat{j})(-20\hat{i} + 90\hat{j})}{(-20\hat{i} + 30\hat{j})(50\hat{i} - 110\hat{j})}$

$= \dfrac{-400 - 2700}{-1000 - 3300} = \dfrac{-3100}{-4300}$

$= \dfrac{31}{43}$

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$S: \{0, \cancel{t_{AE}}, t_{ED}, t_{DC}, t_{CB}, t_{BA}, 1\}$

<0, ≥0, >0, <0, >0

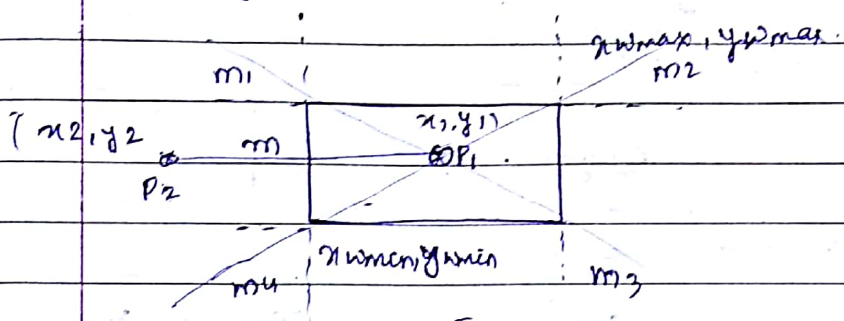
<0

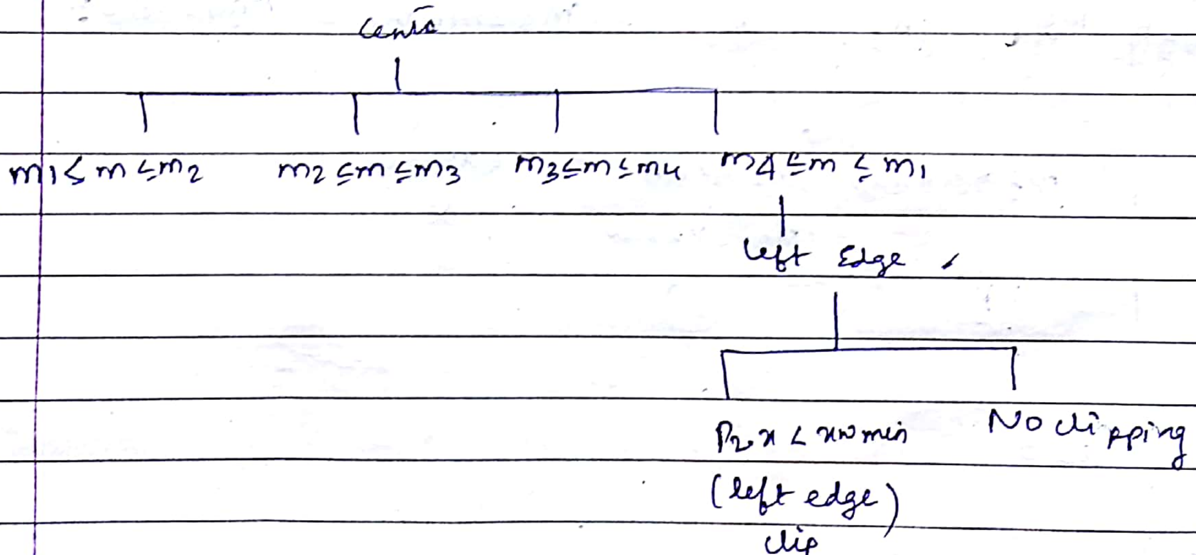
PEx PEn PEx PEn

Line Clipping

Nicholl-Lee-Nicholl

Rectangular window :-





if (($m_4 \cancel{\le} m \le m_1$) && ($\cancel{x_2} < x_1$))

{

Left

}

if (($m < m_4$) && ($m > m_1$) && ($x_2 < x_1$))

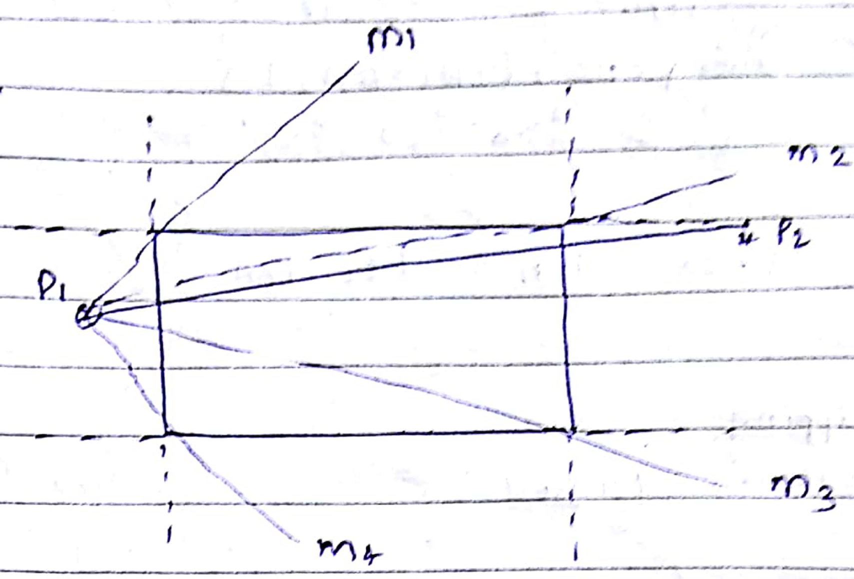
{

if ($x_2 < xwmin$)

{ —

Case 2:

Edges.



Edges

- Left
  - $m_1 < m < m_2$ ×
  - $m_2 < m < m_3$
    - $x_2 > x_{wmax}$ — LR
    - $x_2 > x_{wmin}$, $x_2 < x_{wmax}$ — L
    - $x < x_{wmin}$ ×
  - $m_3 < m < m_4$
  - $m_4 < m < m_1$
- Bottom
- Right
- Top.

Case 3:-

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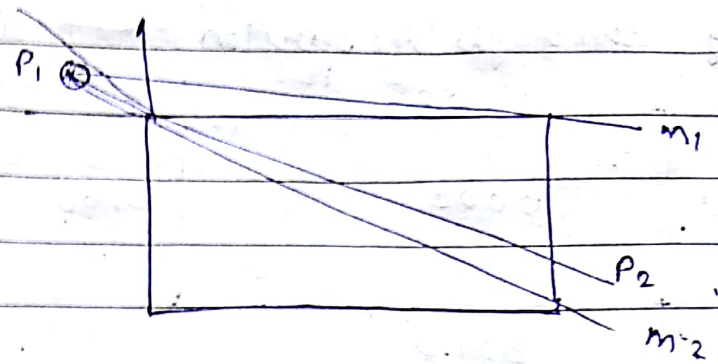
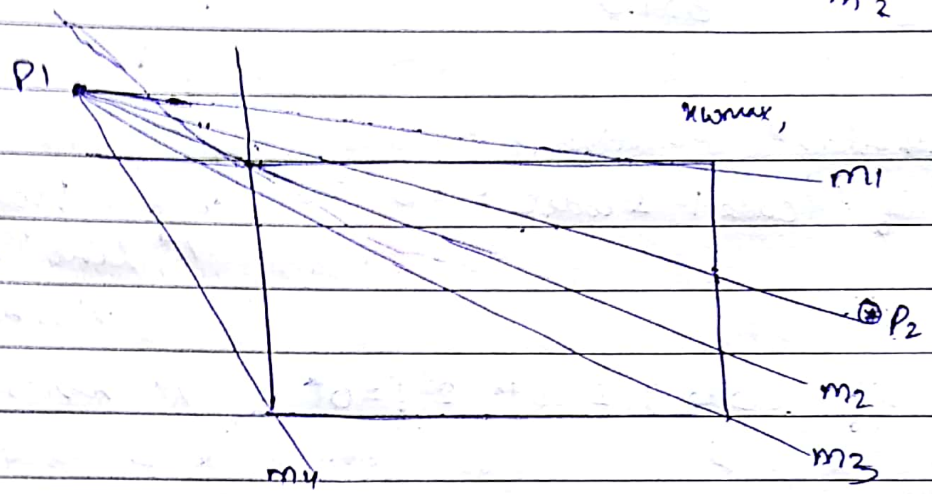
Case 3:





Corner

- Left Top
  - lower part
    - $m_1 < m < m_2$
    - $m_2 < m < m_3$
    - $m_3 < m < m_4$
    - x
  - Upper part
- Left Bottom
- Right Bottom
- Right top

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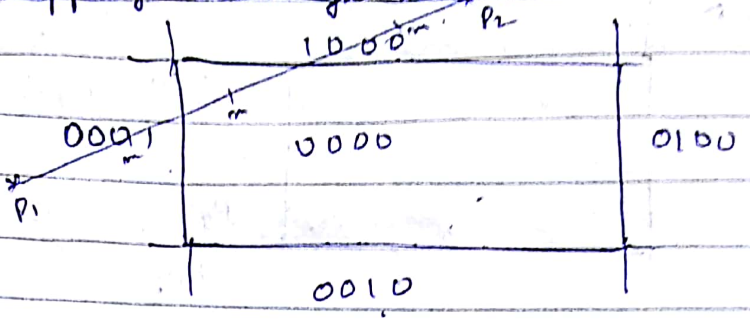
Clipping through recursion :- → dividing the line,

→ identify the end point region codes.



Cohen Sutherland iterative :-

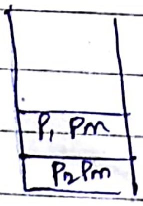
Case ① if (code 1 & code 2 = = 0) && (code 1 | code 2 = = 0)

/* line is totally visible */

Case ② if Code 1 & code 2 != 0 /* invisible */

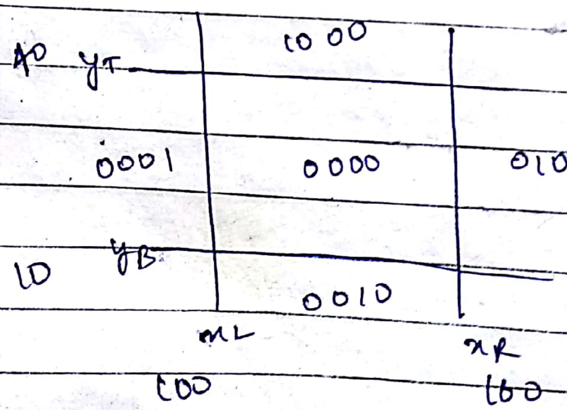
Case ③ if (code 1 & code 2 = = 0) && (code 1 | code 2 != 0)

/* compute intersect */



$P_m = \frac{P_1 + P_2}{m}$

Ex The visible portion of the line is P1 (120, 5), P2 (180, 30) wrt to a clipping window A (100, 10), B (160, 10) C (160, 40), D (100, 40)



1. Input $P_1(120,5)$ $P_2(180,30)$

$x_L = 100$, $x_R = 160$, $y_B = 10$, $y_T = 40$

P_1	Code (P_1)	P_2	Code (P_2)	P_m	Code (P_m)	
120,5	0010	180,30	0100	150,18	0000	consider P_1, P_m keep P_2, P_m
120,5	0010	150,18	0000	135,12	0000	
120,5	0010	135,12	0000	128,9	0010	
128,9	0010	135,12	0000	132,11	0000	
128,9	0010	132,11	0000	130,10	0000	
128,9	0000	130,10	0000	129,10	0000	

2.

P_1	Code (P_1)	P_2	Code (P_2)	P_m	Code (P_m)
120,5	0010	180,30	0100	150,18	0000
150,18	0000	180,30	0100	165,24	0100
150,18	0000	165,24	0100	158,21	0000
158,21	0000	165,24	0100	162,23	0100
158,21	0000	162,23	0100	160,22	

323 45

320 44

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Algorithm

Input $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, x_L, x_R, y_B, y_T.

1. Code 1 = Regcode (P_1), code 2 = Regcode (P_2)

2. if code 1, code both = 0 then the line is totally visible.

3. if code 1, code 2 <> 0, then the line is totally invisible.

4. $P_m = (P_1 + P_2)/2$, code M = Regcode (P_m).

5. if code M <> 0 then if code 1 & code M <> 0 then $P_1 = P_m$; goto step 1.

   else if code 2 & code M <> 0 then $P_2 = P_m$; goto step 1.

6. if code M = = 0 then
   if code 1, code M both = 0 then consider P_M, P_2
   else if code 2, code M both = 0 then consider P_1, P_M.

7. Consider $P_1 P_M$.

   do {
   $P_{M_1} = (P_1 + P_M)/2$, code M_1 = Reg Code (P_{M_1})
   if code M_1 <> 0 then $P_1 = P_{M_1}$, else $P_M = P_{M_1}$
   }
   while ($P_m.x$ <> x_L) && ($P_m.x$ <> x_R) &&
   ($P_m.y$ <> y_B) && ($P_m.y$ <> y_T)

   $P_1 = P_M$.

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8. Consider $P_M P_2$

P.

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Polygon Clipping

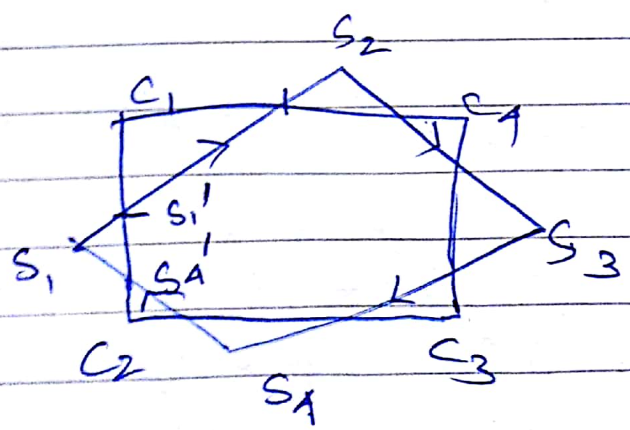
Sutherland - Hodgman — Convex, concave.

Subject Polygon (Polygon to be clipped)
input list : $\{S_1, S_2, S_3 \ldots S_n\}$

Clip polygon (Window against which Subject Polygon is clipped)
list of edges : $\{C_1 - C_2, C_2 - C_3, \ldots C_n - C_1\}$

① One side of the clipped polygon is extended infinitely in both directions

② subject Polygon ('Input list' is traversed). Vertices of input list are inserted into an o/p list if they lie on the visible side of the extended clipped polygon line & new vertices are added to the o/p list where the subject polygon path crosses the extended clipped polygon line



Rules

① Wholly inside visible region
- Save End point.

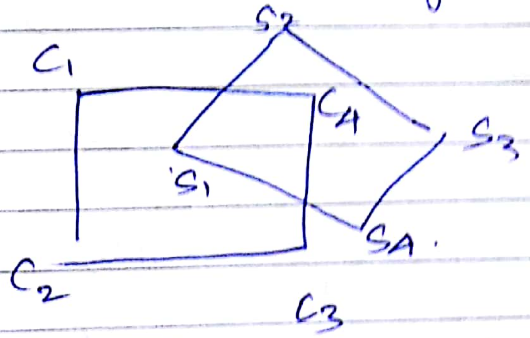
Edge Left

Edge	Left traversing	o/p
Left	$S_2 \to S_3$	S_3,
	$S_4 \to S_1$	S_4'

o/p list : { }.

② Exit visible region
- Save the intersection pt.

③ Wholly outside visible region
- Save nothing S_2



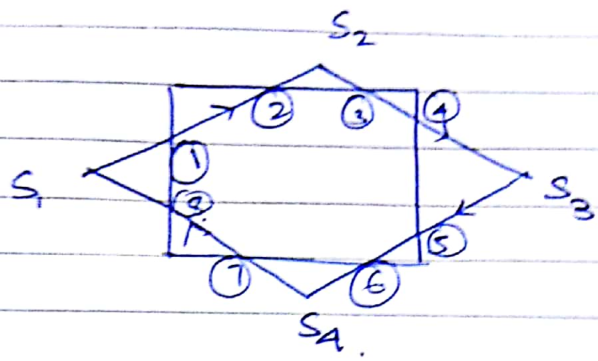
Right Edge	o/p
$S_3 \to S_4$	nothing

④ Enter Visible Region o/p list.

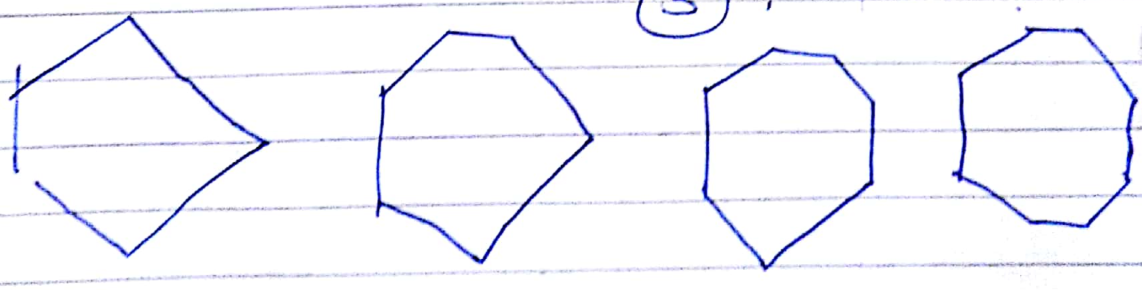
left edge	o/p list.	save intersection & end pt.
$S_1 \to S_2$	S_1' S_2	

Example

for i = left



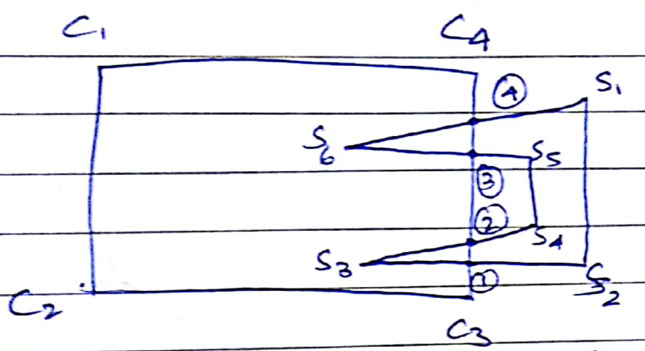
i/p list	w.r.t left edge	Top edge	Right edge	Bottom edge
S_1	1	2	3	4
S_2	S_2	3	4	5
S_3	S_3	S_3	5	6
S_4	S_4	S_4	S_4	7
	8	8	8	8
	1 (Repeat the first one)	1	1	1
		2	2	2
			3	3



S_1	S_2
0	0
0	1
1	0
1	1

```
int inside (Point P, Edge E)
{
   // ~~Detec~~ use Cross-Product.
}
```

Example ②



~~Left edge~~ Right edge

i/p	
S_1	
0→0	①
S_2	S_3
0→i	②
S_3	
i→0	③
S_4	S_6
0→0	
S_5	
0→i	④
S_6	
i→0	

Step ①

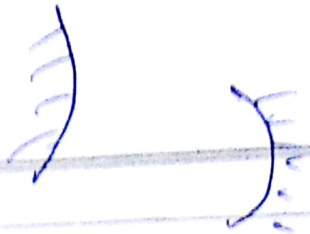
Filter the intersection pts

pt[ ]

sort also acc. to y-coordinate

①	→ 0→i
②	→ i→0
③	→ 0→i
④	→ i→0

if i→0
then print
line(pt[i] pt[i-1])



line (1, s_2)
line (s_3 2)
line (2, 1) as 2 was $i \to o$

line (3, s_6)
line (s_6, 4)
line (4 3) $\longrightarrow$ as 4 was $i \to o$

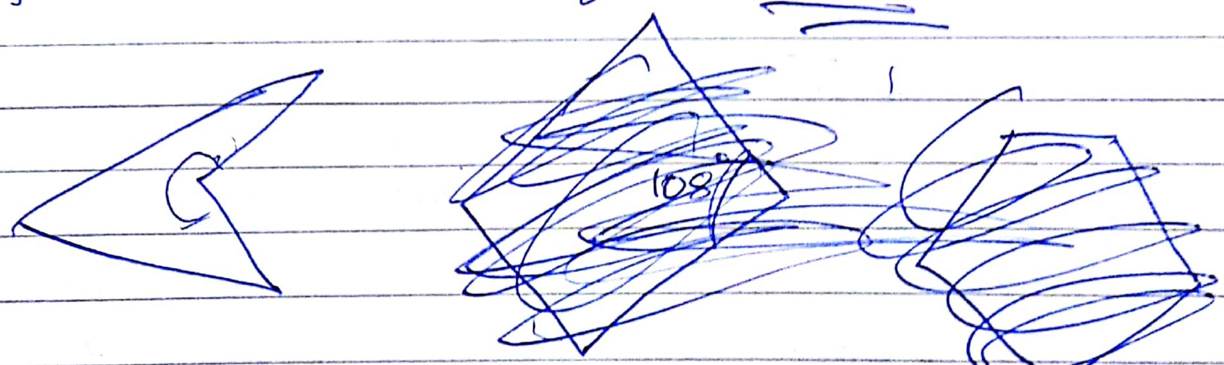
Midsem

Polygon filling
2-D Transformation.

Concave

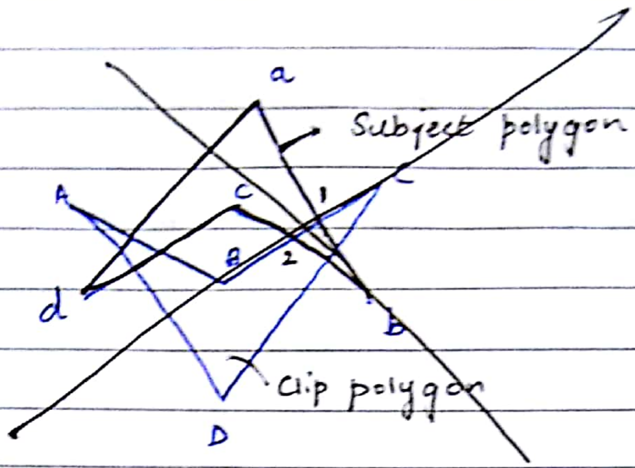
if one interior angle $> 180°$

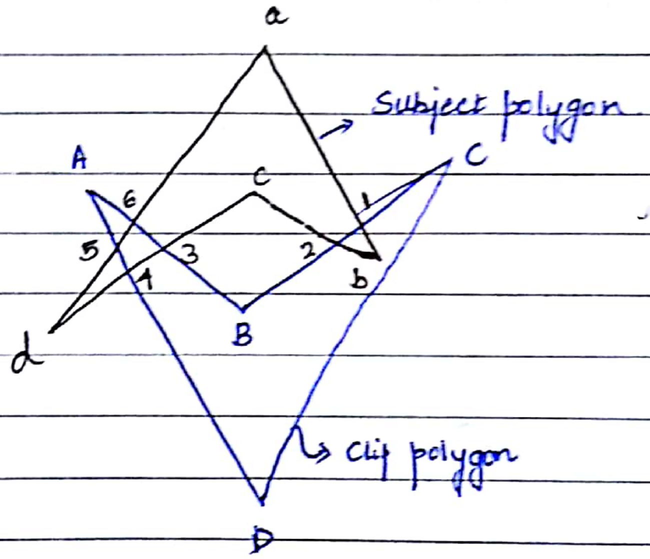
eg:



Polygon Clipping

Weiler - Atherton





① Assume the vertices of subject polygon are listed in clockwise order (Interior is on the right)

② Start an entering intersection

③ Follow the edges of the polygon being clipped until an existing intersection is encountered.

④ Turn right at the exiting intersection & follow clipped window edge until intersection is formed

⑤ Turn right and follow the subject polygon.

⑥ Continue until vertex already visited is reached

⑦ If entire polygon has not been processed, repeat.

Step 1 :

SP list : a b c d

CP list : A B C D

/* intersection evolve */

Step 2 : /* update the list in step 1 with intersection pt */

Apply cyrus beck for intersection pts

Outer loop	Inner loop		Outer loop	Inner loop	
a - b	A - B		d - a	AB	6
	B - C	1		BC	
	C - D			CD	
	D - A			DA	5
b - c	A - B				
	B - C	2			
	C - D				
	D - A				
c - d	A - B	3			
	B - C				
	C - D				
	D - A	4			

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1.

So,

SP list: a 1 b 2 c 3 4 d 5 6
1 0 1 0 1 0

CP list: A 6 3 B 2 1 C D 4 5
1 0 1 0 1 0

PE_n $0 \to i$ (Potential entry) → 1

PE_x $i \to 0$ (Potential exit) → 0

line (1, b)

line (b, 2)

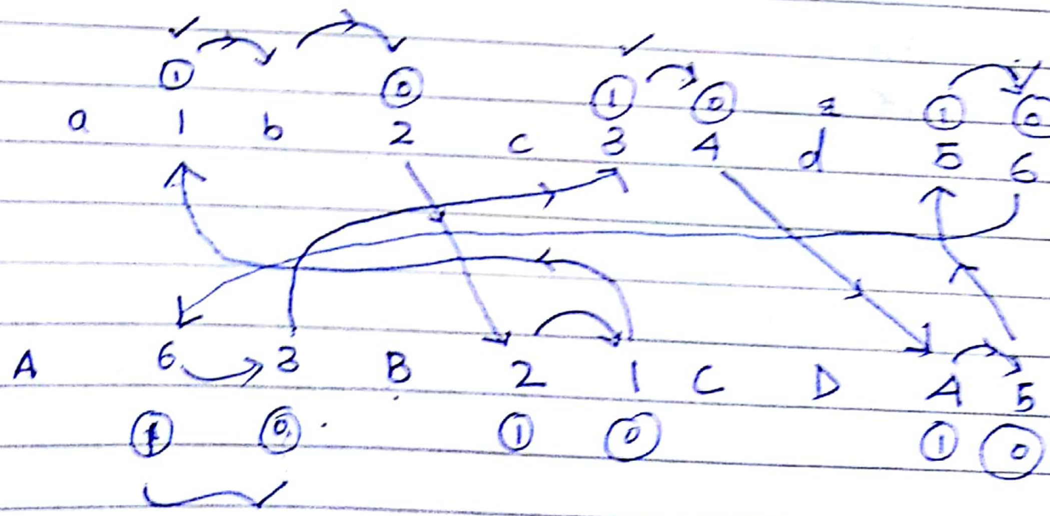
Now as 2 is 0 i.e. $i \to 0$ in SP list, so jump to 2 in CP list and connect to next

Line (2, 1)

line (3, 4)

line (5, 6)

line (6, 3)

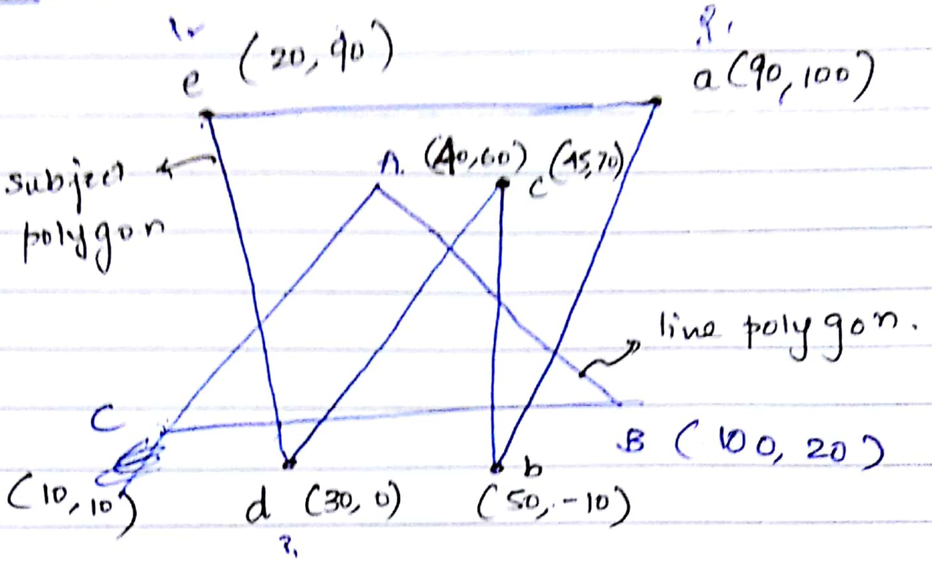


Here 6 came before 3 because of t (time) in cyrus beck

~~It~~ ~~there~~ is

1 1

Example 1 :



	Logic PE_n/PE_m $(0\to i)$ $(i \to 0)$ 1 0	t.
AB		
BC		
CA		

$\overline{AB} = 60\hat{i} - 40\hat{j}$

$N_{\overrightarrow{AB}} = 40\hat{i} + 60\hat{j}$

$\overrightarrow{BC} = -90\hat{i} - 10\hat{j}$

$N_{\overrightarrow{BC}} = 10\hat{i} - 90\hat{j}$

$\overline{CA} = 30\hat{i} + 50\hat{j}$

$= -50\hat{i} + 30\hat{j}$

$(P_2 - P_1)\,(ab) = -40\hat{i} - 110\hat{j})$

$(P_2 - P_1)\,(bc) = -5\hat{i} + 80\hat{j}$

$(P_2 - P_1)\,(cd) = -15\hat{i} - 70\hat{j}$

$(P_2 - P_1)\,(de) = -10\hat{i} + 90\hat{j}$

$(P_2 - P_1)\,(ea) = 70\hat{i} + 10\hat{j}$

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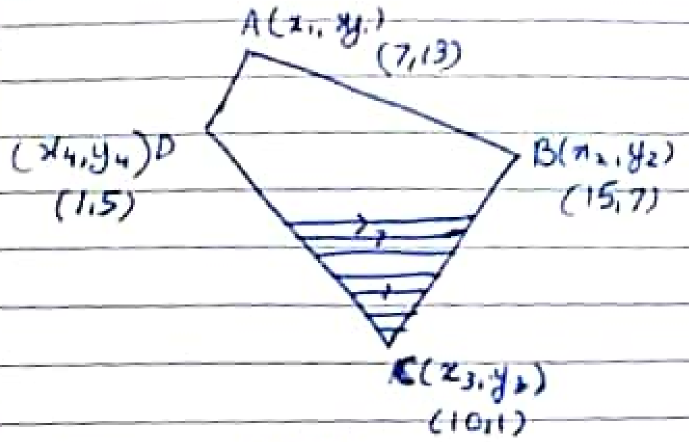
$$t_{dc} = \frac{\cancel{30}\,(-50\hat{i} + 30\hat{j}) \cdot \left(\overset{30\hat{i} - 10\hat{i} - 10\hat{j}}{\cancel{10\hat{i} + 10\hat{j} - 30\hat{i}}}\right)}{(50\hat{i} - 30\hat{j}) \cdot (-10\hat{i} + 90\hat{j})}$$

$$= \frac{-1000 - 300}{-500 - 2700} = \frac{1300}{2700} = \frac{13}{27}$$

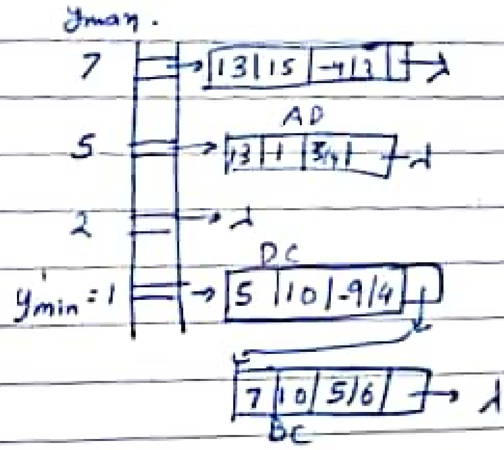
Polygon filling :- (Active Edge Table)
(From given constructed GET (Global Edge Table))

(a) Scan line Approach — (Horizontal)



Defining edges
i) y_{max}
(ii) $x_{y_{min}}$
(iii) $1/m$

$y_{i+1} \rightarrow y_i + 1$
$x_{i+1} \rightarrow x_i + \frac{1}{m}$



y_{max}	$x_{y_{min}}$	$1/m$

□ → λ

for $i = y_{min}$ to y_{max}.

$i = 1$ □ → [5 | 10 | -9/4] → [7 | 10 | 5/6] → λ

line (10, i, 10, i)

when first time new nodes are added to the list.

$i = 2$ □ → [5 | 10 | -9/4] → [7 | 10 | 5/6] → λ

$x_{i+1} \rightarrow x_i + 1/m$

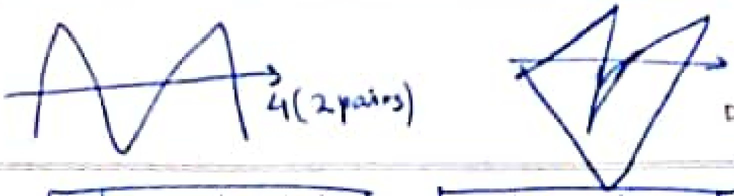
□ → [5 | 7.75 | -9/4] → [7 | 10.83 | 5/6] → λ.

-2.25

line (7.75, i, 10.8, i).

$i = 3$ □ → [5 | 5.50 | -9/4] → [7 | 11.66 | 5/6] → λ.

-20/3 -6.66



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i=4 ☐ → | 5 | 3.25 | -9/4 | → | 7 | 12.4 | 5/6 | → λ.

BUCKET SORT

i=5

☐ → | 13 | 1 | 3/4 | → | 5 | 3.25 | -9/4 | → | 7 | 12.46 | 5/6 | → λ

Rule(2). ☐ → | 13 | 1 | 3/4 | → | 5 | 1 | -9/4 | → | 7 | 13.29 | 5/6 | → λ.

(Rule 3).

i=6 ☐ → | 13 | 1.75 | 3/4 | → | 7 | 14.12 | 5/6 | → λ.

drawline( )

i=7 ☐ → | 13 | 1.5 | -4/3 | → | 13 | 2.25 | 3/4 | → λ.

drawline( )

i=8 ☐ → | 13 | ~~14.25~~ 13.67 | -4/3 | → | 13 | 3 | 3/4 | → λ

⋮

i=13 | 13 | 7 | -4/3 | → | 13 | 6.75 | 3/4 | → λ

for last node (i.e topmost pt.

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(y was back in prev.

$x_1 \le x \le x_2$

$y_1 \le y \le y_2$

Rules :-

(1) MERGE

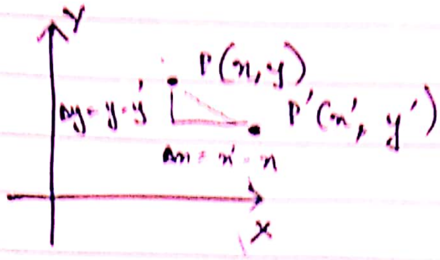
(2) if node are previously coming (update it)

(3) delete the node < i = y max (of some node)

(4) Draw line (in pair).

2-D Transformation

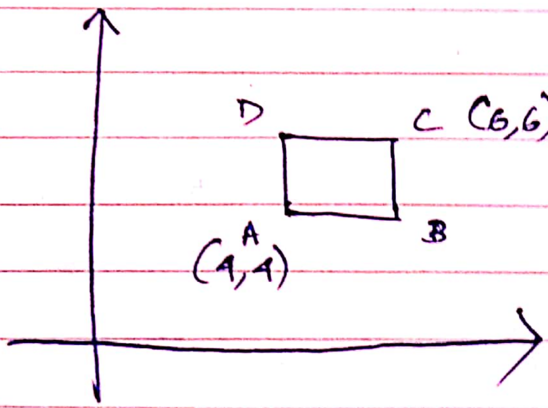
- Translation



$$\boxed{P' = P \ \text{op} \ ?}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \underbrace{\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}}_{\text{Translation entity}}$$

- Scaling
  - Scale up.
  - Scale down



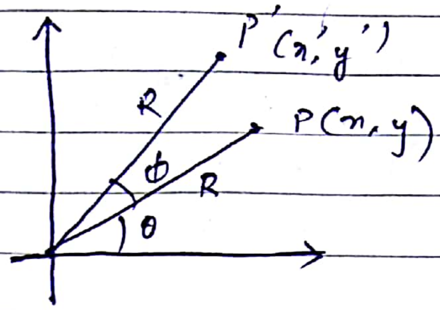
Scaling entity

S_x

S_y

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation



$x = R\cos\theta \qquad y = R\sin\theta$

$x' = R\cos(\theta+\phi) \qquad y' = R\sin(\theta+\phi)$

$x' = x\cos\phi - y\sin\phi$

$y' = y\cos\phi + x\sin\phi$

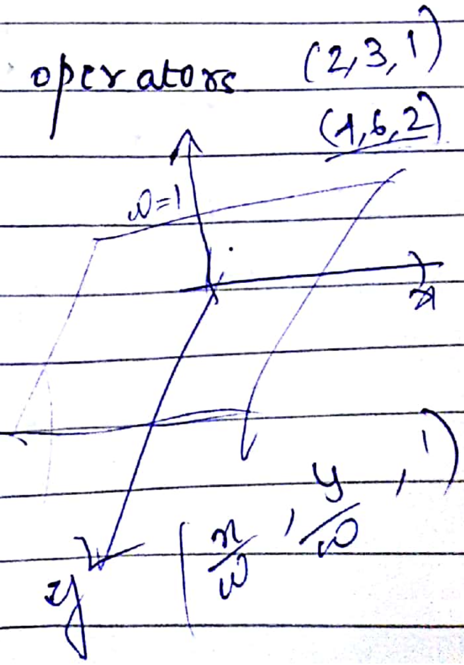
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

The operators are not same as in ~~first~~ translation " $\pm$ " are used while in other 2 " * " is used.

For Homogneity ~~of~~ of operators

For translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



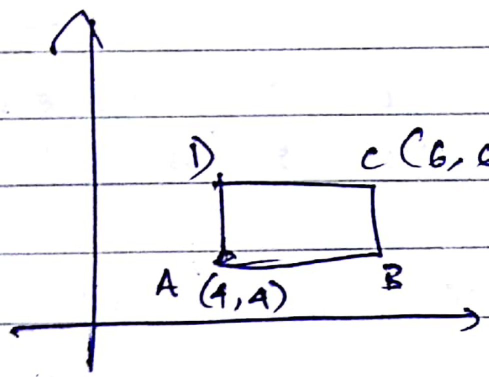
For Scaling

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

For rotation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Q. Given a square ABCD of 2 units



Transform into rectangle whose $l = 2b$ and position of A remains same.

* Scaling & Rotation are w.r.t to origin.

$$C$$

$$\begin{pmatrix} x_A' & y_A' \\ x_B' & y_B' \\ x_C' & y_C' \\ x_D' & y_D' \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \begin{pmatrix} x_A & y_A \\ x_B & y_B \\ x_C & y_C \\ x_D & y_D \end{pmatrix}$$

First translate square to Origin i.e. A to origin

$$\Rightarrow \qquad T_1 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

Then scale:

$$S = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then translate

$$T_2 = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

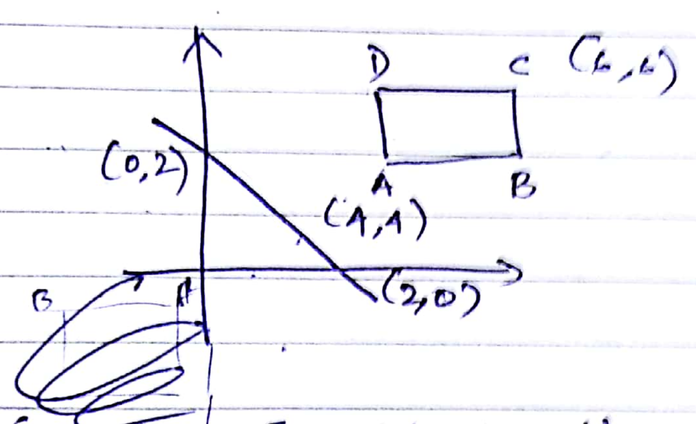
$$C = T_2 * S * T_1$$

$$= \begin{pmatrix} 1 & 0 & +4 \\ 0 & 1 & +4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & +4 \\ 0 & 1 & +4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C * r = \begin{pmatrix} 2 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{cccc} A & B & C & D \\ \end{array}$$
$$\begin{pmatrix} 4 & 6 & 6 & 4 \\ 4 & 4 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} \\ \\ \end{bmatrix}$$

Q: Given a square ABCD and line l. Find reflection about L, find C.



Translate the line to origin.

$$T_1 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$C = T_2 * R_4 * R_3 * R_2 * \cancel{T}_1$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 2 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * R_2 * T_1$$

$$= \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 2 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} * T_1$$

$$= \begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 & 4 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$