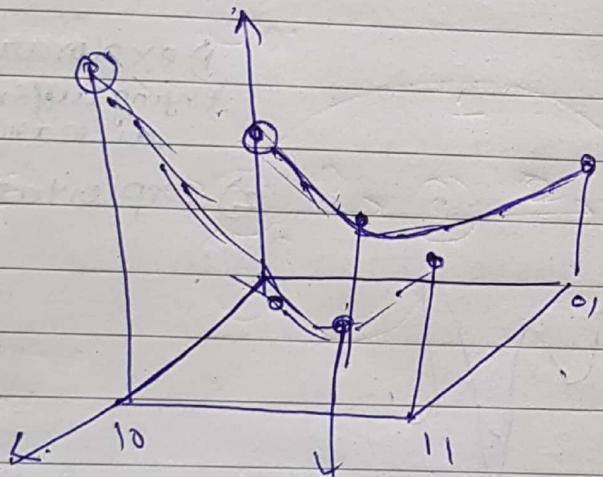


Von Newman architecture :

Intractable problems: Halting problem (can't be solved)

MIMD → Multiple Instructions multiple data streams  
 ↳ Homogeneous.

Array Processor = single instructions multiple data streams.



for minimisation problem  
 best solution.

by exhaustive, all possible soln : global maxima/  
 global minima

but in guess work - local minima / maxima.  
 (till we find optimised soln)

Meta heuristic - part of soft computing  
 using approximations

- 1) Allow fuzziness
- 2) Allow approximation
- 3) solve meta-heuristics

GENETIC ALGORITHMS - for approximation  
encoding:

chromosome → 

1	0	0	1	1	1	0	1	0
---	---	---	---	---	---	---	---	---

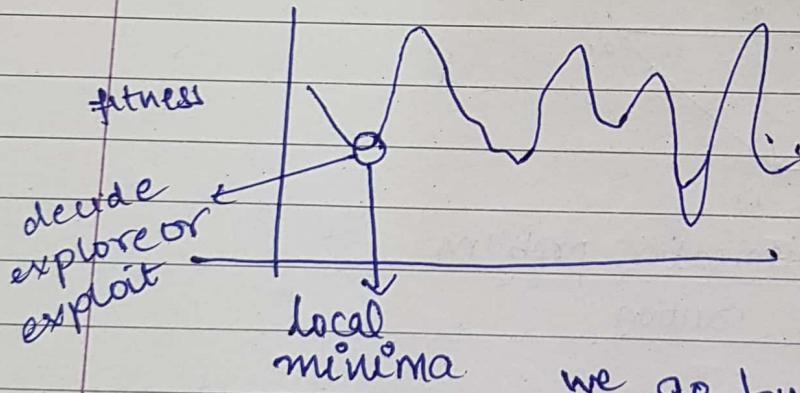
 (represents solution)  
↳ can be int, array, binary

You need some kind of code to encode your solution.

$$f = \frac{y + \cos 32y}{x + y}$$

1) exploration:  
trying different  $x, y$  in search space

2) exploitation:



we go by some heuristic &  
not making random guesses.  
we don't solve it analytically or numerically.

Exploration is a process of keeping on trying  
to make new guesses. The guesses are not  
random, they are problem specific & according to  
nature.

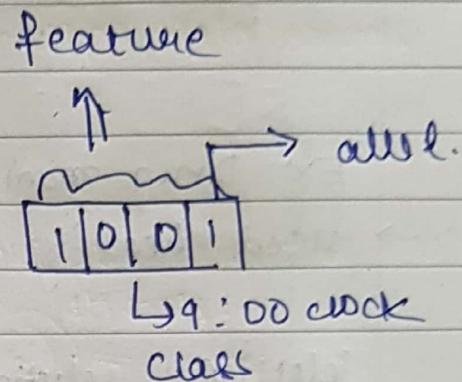
since tree problem  $\rightarrow$  Generic algorithm.

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Any metaheuristic can be applied to real time problems,  
generally used for design problem.

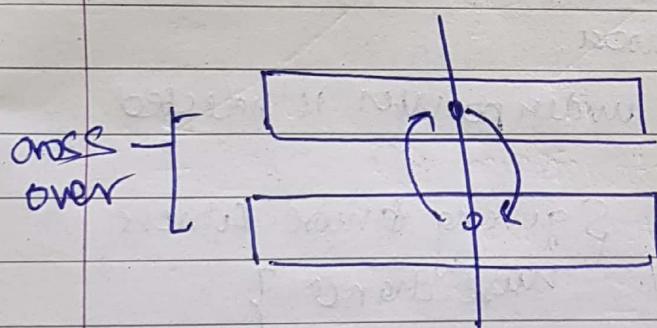
Once done, it is done.

- scheduling problem
- search engine optimization
- resource scheduling
- routing on a chip
- designing computer.



Genes  $\rightarrow$  A group. Gene - a group of alleles which represent a feature is called gene.

A group of genes  $\rightarrow$  genotype  
all features genes represent - phenotype.



Find a random pt and the exchange previous chromosome gene & new gene. chromosome

Select two chromosome, swap them.

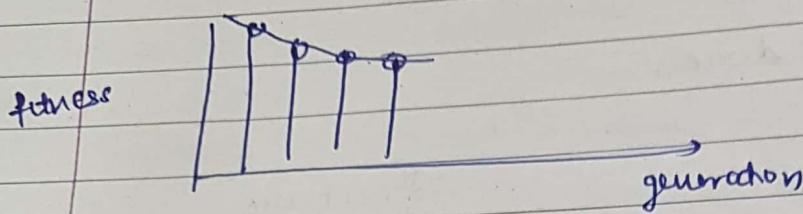
In breeding stops exploration, we need variety for exploration.

crossover - genetic operator, exploitation

selection - selecting two individuals for crossover.

HIGH PERFORMANCE COMPUTING

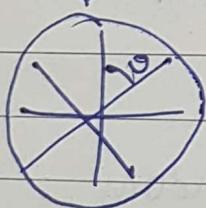
mutation - exploration.



Ex: Drug manufacture.  
→ used heavily in genetics.

Selection: selection of individuals who participate  
it in production of next generation  
(crossover)

$$F = \sum_{i=1}^N f_i$$



rotate N times  
and then choose.

Individual under pointer is selected.

$$E[i|t] = \frac{f(i,t)}{F(t)}$$

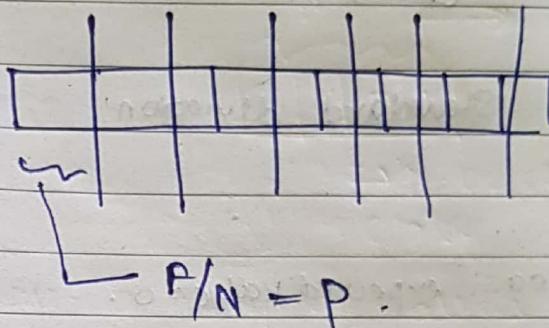
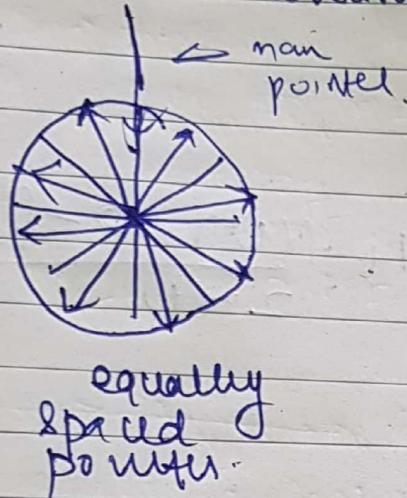
} giving more fitness  
more chance }

- ① IMPLEMENTATION:
- ② Write CDF (cumulative) form
- ③ Generate random no. b/w 0 & 1 + F
- ④ keep adding till  $\sum_{i=1}^t f(i) > r$ . (random no.):

After a period of time, fitness remains same over a period of time & evolution converges.

Stochastic Uniform Selection:

removes bias towards fitter generation.



& there is only single rotation

### IMPLEMENTATION

selector:

① select a random no ,  $\theta = \text{Rand}()$

②  $\{ \text{ptr} := \text{selector} + [ *P \mid i = 0 \dots N-1 ] \}$

③ For every individual

for ( $i=0$ ;  $i < N$ ;  $i++$ ) {

    for (sum +=  $f_{\text{fit}}(i, t)$ ; sum  $\geq$  ptr; sum -=  $f_{\text{fit}}(i, t)$ )

        select i;

for a individual :

minimum chance

maximum chance:

$f_{\text{fit}}(i, t)$

$f_{\text{fit}}(i, t)$

for selection

selection pressure at moderate level  
not too high not too low.

- Sigma scaling
- maintains constant selection pressure.
- function becomes non linear.

$$E[i,t] = p =$$

Standard deviation:

$$\frac{1 + f(c,t) - \text{avg}(f)}{2\sigma(t)}$$

least expected value  $\approx 0$ .

Those values beyond  $\sigma(t)$ , they get selected & standard deviation gets low,  
as a result standard deviation slips like  
\* Space or less fit also gets change

### Boltzmann Selection

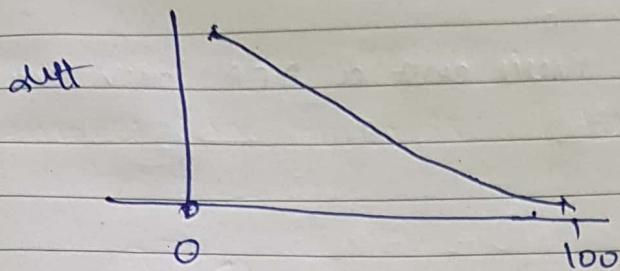
- make the expected value exponential
- difference b/w minima & maxima  
increases with help of temporary scheduling

$$E[i,t] = \frac{e^{f(i,t)}}{\langle e^{f(i,t)} \rangle}$$

$\underbrace{\langle e^{f(i,t)} \rangle}_{\text{avg}}$        $\xrightarrow{\text{temperature}}$

Annealing

The difference b/w maxima & minima increase as temperature is reduced.



→ controlled by temperature  
→ still low prob to select the less fit but still transition smooth & Temp. dependent

Space is distance based.

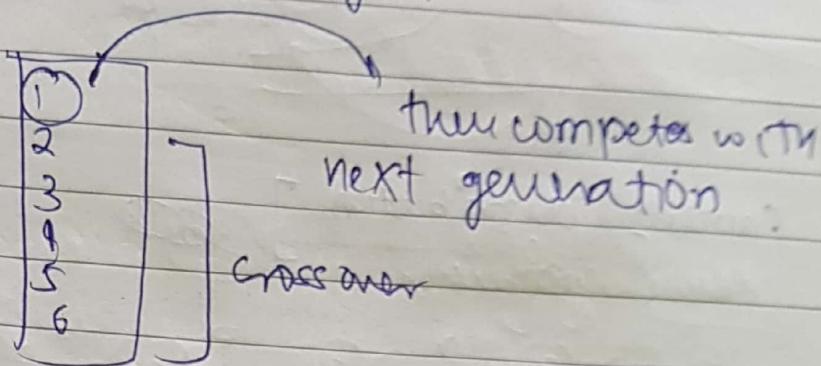
→ selection pressure can be varied with evolution

## SOFT COMPUTING

→ Cloning. → Schema theory.

### ⑤ Elitism.

You try to retain the best or second best chromosome of this generation as well as next gen.



### ⑥ ~~Tournament~~ Rank selection.

$$\text{Expected } (i, t) = \text{min} + \frac{(\text{max}-\text{min}) \text{Rank}(i, t) - 1}{N-1}$$

- Very effective method.
- Removed direct proportionality with fitness.
- Selection pressure low.

### ⑦ Tournament selection

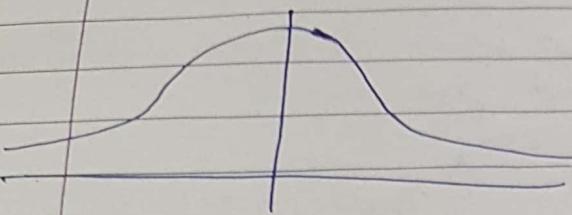
- select two individuals at random, then play knockout game.
- let there be parameter,  $k = 0.7$ .
- $x = \text{rand}(0,1)$
- If  $x > k$ , then less fit or else we choose more fit.

### ⑧ Steady State Selection

- Most fit are used for crossover with least fit

27  
C

## → CUCKOO SEARCH Lmeta heuristic



Cauchy distribution ] ~~new~~

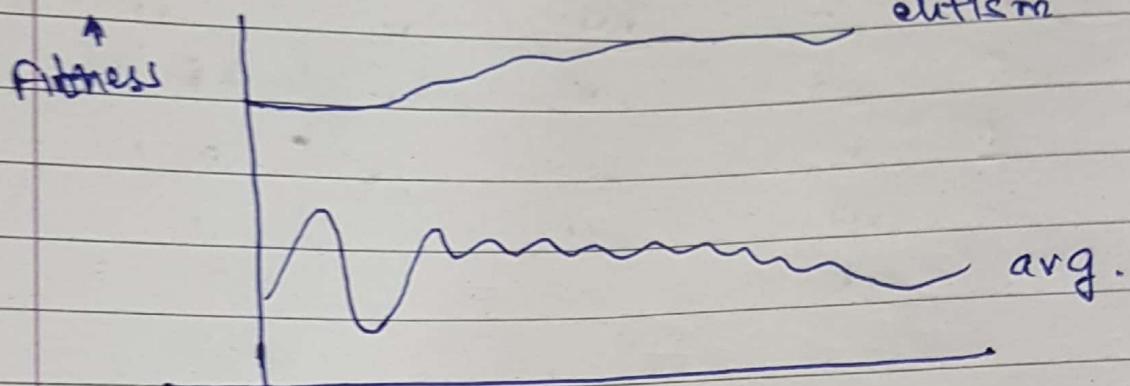
## MODELLING

- Q There are 50 students in a class. They have been awarded marks out of 100 in 5 different courses. Use R to visualize the data in different ways

Sc

- 1) calculate fitness of population and then sort the population according to the fitness.
- 2) selection based on elitism.

↳ leads to non decreasing function,

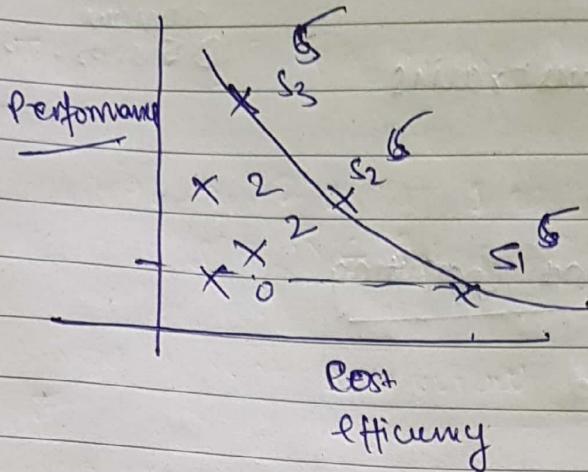


- Cauchy distribution
- Leap distribution

$$OF = \frac{w_1}{\sum w_i} f_1 + \frac{w_2}{\sum w_2} f_2$$

~~combi~~ effective obj function.

Non-inferior solutions, for these solutions you can't find a solution which is superior in ~~all~~ a aspect



Instead of assigning wts, go for ranks ?

Rank of the solution is the ~~no.~~ of solutions that ~~don't~~ supercede the soln

Rank of the solution ← use this method.

↳ is the no. that ~~doesn't~~ supercede in all direction

Pareto optimal set → needed for rank based selection.

Rank becomes your actual fitness.

~~COOKED~~

### 3). Ant - Colony optimization.

Two parameters

- 1) heuristic parameter  $\eta$
- 2) Pheromone  $\gamma$

$\eta$  → sort of local greedy search.

$\gamma$  → based on experience, quality of coin, fitness function

Combinatorial OP  $\langle S, C, f : s \rightarrow R_0 \rangle$

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$S$  = set of features

$S = \{x_i\}$

$C$  - set of constraints

It belongs

to some

domain

$D = \{U_1, U_2, \dots, U_n\}$

## High Performance Computing

PRAM

→ EREW

→ ERW

→ CRER

→ CREW

### Problem in concurrent write:

- 1) Priority
- 2) Arbitrary.
- 3) Common

→ processor who wish to write same things.

The read has no discrepancy while the concurrent write further divided into three cases.

→ Common: All processors are allowed to complete write if all the values written are equal.

Any algorithm for this model has to make sure that this condition is satisfied. If not the algorithm is illegal and machine state is undefined.

Formalization  
Set theory UML  
pseudo code.

$s_c^* \in S_c$

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## SOFT COMPUTING

$S_c$  → feasible soln.

↳ Ø assignment of  $f(x_i)$  } such that constraints are met.

$S_c^*$  → reached the global optimum.

$$f(f(x_i)) : S \rightarrow R$$

set of features - input

set of assigned features - output

for minimum  
optimal  
solution

$$f(S_c^*) \leq f(S_c)$$

$$S_c^* \in S_c$$

Initialize:

Set parameters

Initialize pheromone levels.

Loop:

Begin's

Construct Ant solutions

Synchronize (optional)

Update pheromone

either greedily  
or probabilisticly

End.

till stop criterion

Probability of choosing next node:

$$P(c_{ij} | s_p) = \frac{T_{ij}^\alpha n_{ij}^\beta}{\sum T_{ij}^\alpha n_{ij}^\beta}$$

$T_{ij}$  - pheromone level

$n_{ij}$  - distance b/w nodes.

$$\beta \otimes \alpha \frac{1}{\alpha}$$

$\gamma_{ij}$  - experience factor

$\eta_{ij}$  - local factor

- ⇒ In GA there was interaction through crossover
- ⇒ In ACO, there may or may not be synchronisation
- ↳ in case ant can't stay the wt of food item?

Update

all the ants that  
per next through the  
path with  
fitness.

$$T_{ij} \leftarrow (1-\rho) T_{ij} + \rho \sum_{S \in \text{Upd}} f(S)$$

evaporation  
rate.  
number b/w 0 & 1.

↑ uppermost

sharing  
experience.

$\rho$  - low : More weightage to past experience

& low weightage to new experience.

$S_{\text{upd}}$  = the best so far.

$$= \underset{S \in S_{\text{iteration}}}{\operatorname{argmax}} (f(S))$$

fitness of ants : inverse of length of path they  
traversed.

## Rough Set theory

→ A rough classifier.

U	a, a <sub>1</sub> , a <sub>2</sub>
b <sub>1</sub>	
b <sub>2</sub>	
b <sub>3</sub>	

a

c

U	Age	Height	Play Volleyball
x <sub>1</sub>	0-10	1-3	No
x <sub>2</sub>	0-10	3-4	Yes
x <sub>3</sub>	10-20	4-6	Yes
x <sub>4</sub>	10-20	4-6	No
x <sub>5</sub>	20-30	5-6.5	Yes
x <sub>6</sub>	20-30	5-6.5	Yes
x <sub>7</sub>	30-40	6-7	Yes
x <sub>8</sub>	20-30	5-6.5	Yes

$$B \subseteq A$$

$$\text{Ind}(B)$$

$$\begin{aligned} [x]_B \\ [x]_{\text{age}} \end{aligned}$$

$$\{ \{x_1, x_2\}, \{x_3, x_4\} \dots \}$$

$$\left\{ (x, x') \in U^2 \mid \forall a \in B, V_a(x) = V_a(x') \right\}$$

$$x_{Cj} = \{x \mid c(x) = v_j\}$$

$$x_Y = \{x_2, x_3, x_5, x_6, x_7\}$$

$$x_N = \{x_1, x_4\}$$

Now we define Lowerbound, Upperbound.

Lowerboundary,  $B(x)$ , whose equivalent classes are proper subset of classes

Want to collect all the features that define equivalent classes that surely belong  $Y$  or  $N$ .

$$Y = \{x_2, x_5, x_6, x_7\}$$

$$\underset{\substack{\text{equivalence} \\ \text{class}}}{B(x)} = \{x \mid [x]_B \subseteq X_Y\} \Rightarrow \text{lower equivalence}$$

$$c(\text{Age, Height})(x)_Y = \{x_2, x_5, x_6, x_7\}$$

$$c(\text{Age, Height})(x)_N = \{x_1\}$$

$$\overline{B_N(x)} = A \setminus B_Y(x) = \{x_2, x_3, x_4, x_5, x_6, x_7\}$$

↑  
upper      ↓  
equivalence    part outside  
                    part outside

$$\overline{B_N(x)} = \{x_1, x_3, x_4\}$$

$$\overline{B(x)} = \{x \mid [x]_B \cap X_Y \neq \emptyset\}$$

$$\overline{B(x)} = \overline{B(x)} - B(x) \Rightarrow \text{Boundary region}$$

$$\overline{Bx} = \{x_3, x_4\}$$

$$\overline{B^o(x)}_y = U - \overline{B(x)}_y$$

Now only feature is Age.

$$B(x)_y = \{x_5, x_6, x_7\}$$

$$B(x)_N = \emptyset$$

$$\overline{B(x)}_y = \emptyset \cup$$

{not externally defined as everything in the upper bound}

$$\overline{B(x)}_N = \{x_1, x_2, x_3, x_4\}$$

$$\overline{Bx}_y = \cancel{\{x_5, x_6, x_7\}} \emptyset$$

$$\overline{Bx}_N = \{x_5, x_6, x_7\}$$

$$\overline{Bx}_y = \{x_1, x_2, x_3, x_4\}$$

$$(*) \overline{Bx}_N = \{x_5, x_6, x_7, x_3, x_4\}$$

are  
Rules ~~were~~ becoming rougher i.e.  
lower boundary is shrinking & upper boundary  
is expanding!

Now feature is age, height, play.

$$B(x)_U = \{x_2 x_3 x_5 x_6 x_7\}$$

$$\overline{B(x)}_U = \{x_2 x_3 x_5 x_6 x_7\}$$

$$B^o(x)_U = \{x_1 x_4\}$$

$$\overline{B(x)}_U = \emptyset$$

• Rules are crisp but they decision <sup>variables</sup> not of interest as it leads to overfit.

5 classes of Rough set :

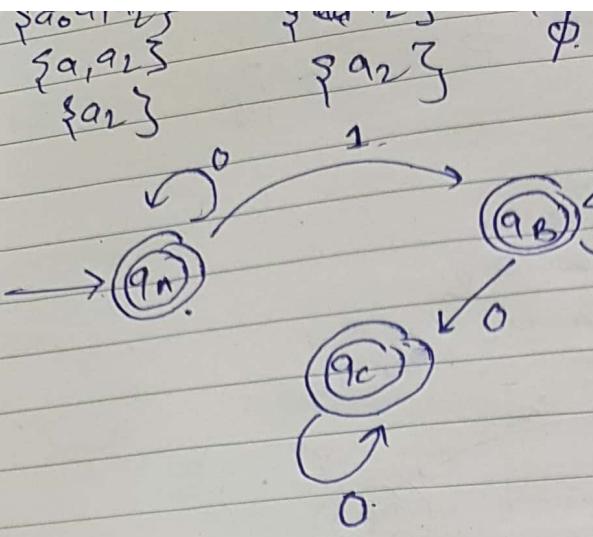
1) Roughly definable : lower boundary is not empty  
Upper boundary is not  $U$ .

2)  $B(x)_{c_j} = \emptyset$  and  $\overline{B(x)}_{c_j} \neq U$   
Not one in it is for sure but some are present  
Internally undefinable.

3).  $B(x)_{c_j} \neq \emptyset$  and  $\overline{B(x)}_{c_j} = U$   
Externally undefinable.  
some are definitely yes, but we can't say which  
are not present

4)  $B(x) = \emptyset$  and  $\overline{B(x)} = U$   
Totally undefinable - i.e. all are presentation boundary

5)  $B(x) = \overline{B(x)}$ , crisp, deterministic



<u>SC</u>	Dep	Exp	French	Ref	Decision
$x_1$	MBA	Mid	Y	Excellent	Accept
$x_2$	MBA	Low	Y	Neutral	Reject
$x_3$	MCE	Low	Y	Great	R
$x_4$	MSc	High	Y	Neutral	A
$x_5$	MSc	Mid	Y	Neutral	R
$x_6$	MSc	High	Y	Excellent	A
$x_7$	MBA	High	N	Good	A
$x_8$	MCE	Low	N	Excellent	R

$$x_{\text{accept}} = x_1 x_4 x_6 x_7$$

$$x_{\text{reject}} = x_2 x_3 x_5 x_8$$

$$x_{\text{DEFR}}: x_1 x_4 x_6 x_7$$

$$x_{\text{DEFR (reject)}}: x_2 x_3 x_5 x_7$$

$$\text{IND(DEFR)} = \{x_1\} \{x_2\} \dots \{x_8\}$$

$$\text{IND}(\text{SR}) = \{x_1\} \{x_2\} \{x_3\}$$

∴ reducing DEFR to ER, which reduces time.

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$
$x_1$	EVR			$d_{1,2}$				
$x_2$								
$x_3$								
$x_4$								
$x_5$								
$x_6$								
$x_7$								
$x_8$								

EOS

A : set of variables

→ define the sets

step 1:  $C_{ij} = \{a \in A \mid a(x_i) \neq a(x_j)\} \quad \forall (i,j) \in 1 \dots n$

↳ set of attributes which are different for  $i \neq j$

$$C_{12} = \{e, r\}$$

$$C_{13} = \{d, e, r\}$$

$$C_{14} = \{d, e, r\}$$

$$C_{15} = \{e, f, r\}$$

After this, we have all the features that distinguish any two pairs.

step 2: discernability function:  $\lambda \{V C_{ij} \mid 1 \leq i, j \leq n\}$

↳ Ans: {e, r}

We need minimum no. of attributes to distinguish b/w entities.

$$a(0+b) = a$$

$$a+ab = a$$

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	$x_1$	$x_2$	$x_3$	$\underline{\underline{x_4}}$	$x_5$	$x_6$	$x_7$	$x_8$
$x_1$	-	-	-	-	-	-	-	-
$x_2$ evr	-	-	-	-	-	-	-	-
$x_3$ evrvd	dvr	-	-	-	-	-	-	-
$x_4$ dvevr	dve	dve	vr	-	-	-	-	-
$x_5$ dvr	dve	dve	vr	e	-	-	-	-
$x_6$ dve	dvevr	dvevr	vr	evr	-	-	-	-
$x_7$ evfvr	evfvr	dvevr	dvfvr	dve	-	-	-	-
$x_8$ dverf	dvfvr	fvr	dvevfur	dvfie	dve	vr	vr	-

$$J_D = (evr) (dvevr) (dvr) (dve) (e)(fr) \\ (evfvr) (dvevf) (dvfvr) (fr) (dve)$$

using absorption, we get.

$$= fr$$

for absorption, you can use K-Map

de \ fr	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$
$x_1$	-	-	-	-	-	-	-	-	-
$x_4$	$\emptyset$	-	-	-	-	-	-	-	-
$x_6$	$\emptyset$	$\emptyset$	-	-	-	-	-	-	-
$x_7$	$\emptyset$	$\emptyset$	$\emptyset$	-	-	-	-	-	-
$x_2$	evr	dve	dverr	evfvr	-	-	-	-	-
$x_3$	evrvd	dvr	dverr	evrvd	$\emptyset$	-	-	-	-
$x_5$	dvr	e	evr	dve	$\emptyset$	$\emptyset$	-	-	-
$x_8$	dverf	dverfvr	dverf	dverr	$\emptyset$	$\emptyset$	$\emptyset$	-	-

$$\begin{aligned}
 I_D = & (evr)(dvervr)(dvr)(dverf) \\
 & (dve)(e)(dverfvr) \cancel{(evrv)} \\
 & (evfvr) \emptyset
 \end{aligned}$$

f.  $e(d+r)$

de		00	01	11	10
00					
01		1	1	1	
11		1	1	1	1
10			1		

1 class

ed + er

ed - Accept equivalence class.

$\{x_1\} \{x_2 x_6\} \{x_3 x_8\} \{x_4 x_5\}$  (3) rules

ed - Reject equivalence class

$\{x_2\} \{x_3 x_8\} \{x_5\}$  (3) rules

no. of rules  $\Rightarrow$  no. of equivalence classes.

D Make ID with respect to decision  $x_k$  2  
for each row.

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er - Accept.

$\{x_3 \{x_4\} \{x_6\} \{x_7\}$

er - Reject

$\{x_2\} \{x_3\} \{x_5\} \{x_8\}$

Total rules : 14

$$\sum_{i \in K} \left| \text{Eq}(\text{IND}(x_k)) \right| = \text{Total no of rules}$$

classes.

Total no of decisions - ~~of  $x_k$~~

info system/  
decision system  $R = (U, A \cup \{d\})$

Universal

attributes

decision

Say! every set is a function ;;

It has a domain & range.

all possible values  
 $d_1, d_2, \dots, d_m$

all values it will accept.

R defines the cardinality of <sup>image</sup> <sub>set</sub> of  $S_d$

$x'_1 \ x'_2 \ \dots \ x'^{r(d)}$

$\{d\} = f^{-1}(v'_1) \cup v'_2 \cup \dots \cup v'^k \cup \dots \cup v'^m$  ?

$x_i^k = \{x \in U \mid d(x) = v_d^k\}$

$x \in U$

$1 \leq k \leq R_d(d)$

no. of objects of the universe that are mapped to this decision

$[x]_B \leftarrow$  equivalence class of object  $x$  w.r.t to  $B$

attribute  $\rightarrow$  decision

$IND(x^k)$

We should also have decision class.  
no. of decision class = RANK.

$X(x) \rightarrow$  decision class of an object  $x$ .

$\downarrow \{x' \mid d(x') = d(x)\}$

$x' \in U$

POSITIVE REGION:

$\hookrightarrow$  union of all the lower region approx.

$$POS_B = Bx_d^1 \cup Bx_d^2 \dots \cup Bx_d^{R_d}$$

~~POS~~ which surely belong to an equivalent class

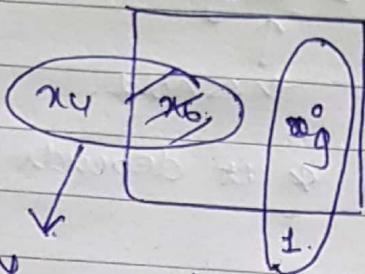
These positive regions are conservative rules.

If  $d(x)$  is crisp then positive region = universal.

~~Expectation~~ Expectation:- chance of occurrence of the event -

## SOFT COMPUTING

y



How to make it more

realistic?

Rough Membership - degree of overlap

Can be in Y  
or in N.

Is it okay  
to say that  
it can be in Y  
or N (equal prob)

$$\frac{|\{x\}_B \cap X|}{|\{x\}_B|} = \mu_x^B(x)$$

cardinality

We say that  $x_4$  &  $x_5$  has  
a probability of 50% belonging  
to Y.

	Age	Upbringing	Like FB
1	0-10	Rural	Y
2	10-20	Urban	Y
3	10-20	Semi-Urban	Y
4	10-20	Rural	N.
5	after 20	Urban	N.

IND.  $\{x\}_{Age} = \{1\} \quad \{2, 3, 4\} \quad \{5\}$

$$\mu_y^{Age}(x_1) = 1$$

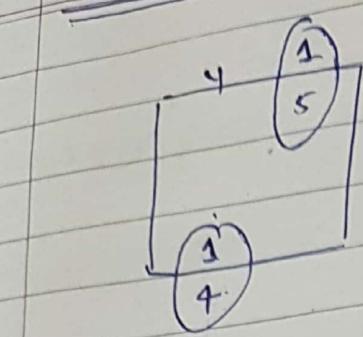
$$\mu_y^{Age}(x_4) = 2/3$$

$$\mu_y^{Age}(x_2) = 2/3$$

$$\mu_y^{Age}(x_5) = 0$$

$$\mu_y^{Age}(x_3) = 2/3$$

## Redefine upper & lower Approximation:



Should we relax our strictness?  
i.e. should we redefine our threshold for lower approximation.  
Yes, needed to avoid overfitting.

lower approx:  $B(x) = \{x \mid \mu_x^B(x) \geq \pi\}$

↑  
threshold

threshold is a hyperparameter & it depends upon application.

upper approx:  $\bar{B}(x) = \{x \mid \mu_x^B(x) \geq 1 - \pi\}$   
or  $\pi_2$ .

Earlier, we used threshold to be 1.

lower threshold is 60 &  $\pi = 1$ .

Positive region: Combination of all the  $\rightarrow$  lower approx of all decision classes.

We need a metric to determine the dependency of class on an attribute  $D$  ~~on~~ ~~of~~ ~~on~~ decision.

Rough dependency: what is dependency of  $D$  on  $C$ ?

$$D(C|D) = \frac{|\text{POS}_C(D)|}{|\text{U}|} \quad (\text{Kappa positive region metric})$$

attribute  $\hookleftarrow$   $\downarrow$  decision pair  
 $c$  say.

Set of decisions not  
no set of decision class

⇒ Frequently enough redux. also needs to  
roughness

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⇒ Actual redux  $\Leftrightarrow$  that doesn't change IND

B. threshold 1:

POS:  $f_{x_1, x_2}$  ~~pos~~  $f_{x_3}$

$$\text{rough dependency} = 2/5 = 40\%$$

threshold 70%.

Lower

$$\text{lower}(x) = f_{x_1} \quad \text{rough dependency} = 40\%$$

lower  $B(x)$

threshold 60%.

$$\text{rough dependency} = 100\% = 5/5$$

$C \Rightarrow D$

APPROXIMATE  
REDUX

So we try to look for approximate redux.

Significance of attribute  $a$ :

we remove ~~some other~~ that attribute  $a$  how are dependency  
on that feature gets affected.

GA prob./meta-heuristic

reduced  
features

$$\text{Significance } \sigma_{C,D}(a) = \frac{\gamma(C,D) - \gamma(C-\{a\},D)}{\gamma(C,D)}$$

subset  
of features.

$$\sigma_{C,D}(B) = 1 - \frac{\gamma(C-B,D)}{\gamma(C,D)}$$

subset of features.

$$\sigma_{C,D}(B) = \frac{\gamma(C,D) - \gamma(B,D)}{\gamma(C,D)}$$

error

Using small  $\Delta x \rightarrow$  increases accuracy.

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We need to have error threshold to see if it to determine error tolerance.

$E_{\text{tol}}(B)$



good approximate  
error if  
within permissible  
limit.

Should we make cluster of ages like 0-10 or simple 0, 1, 2 etc?

- Important not to loose that info that spoils my classification -
- It can be necessary to reduce the rules. Best method to reduce rules is to form clusters.
- We cluster in a way to ensure that we can make rules for that cluster  
(use K-mean clustering)
- clustering should be done in accordance with clustering

How to do decision related discretization of continuous functions?  
(NP-hard)

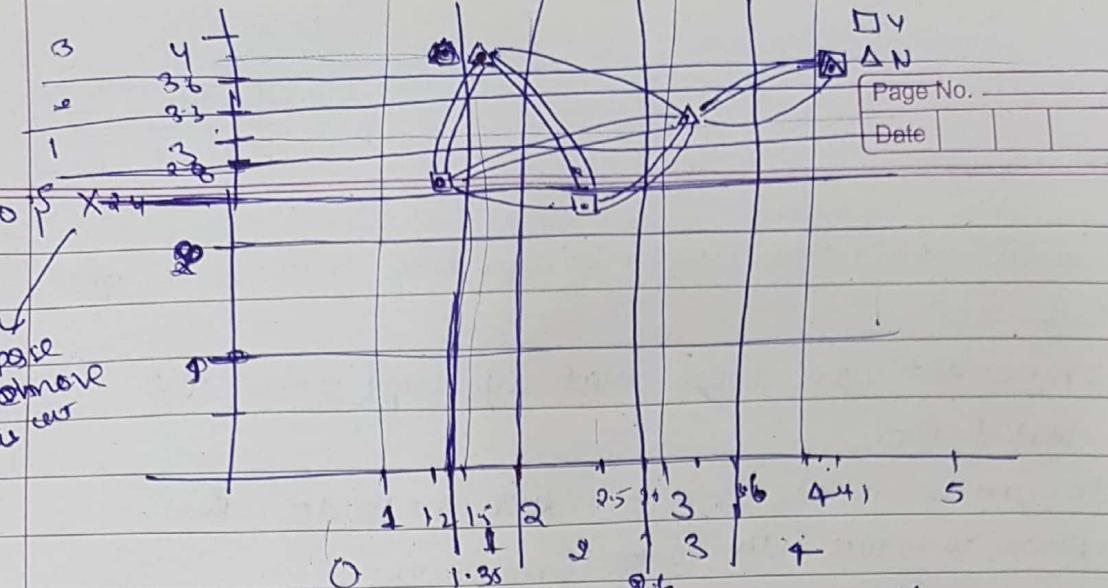
U	length	breadth	buy or not?
1	1.2	2.5	Y
2	2.5	2.3	Y
3	3.2	3.1	N
4	4.1	3.5	Y
5	4.1	3.5	N
6	1.5	3.7	N

length, breadth - real valued or continuous.

We have feature space, where each pt can be considered unique.

~~But~~ We need to partition our feature space.

- But we can't loose information
- we need minimum no. of partitions (optimization problem)
  - ↳ NP-hard.



(1) We adding cut at the moment irrespective of decisions

Aug My first cut is b/w obj 1 & 2.

2 & 6

2 & 3

3 & 4.

$$\begin{array}{r} 1.2 \\ 2.5 \\ \hline 3.7 = 1.35 \end{array}$$

breadth

$$1 \& 2 = 2.4.$$

$$1 \& 3 = 2.1$$

$$3 \& 4 = 3.3$$

$$4 \& 6 = 3.3$$

$$\begin{array}{r} 3.2 \\ 4.1 \\ \hline 7.3 \end{array}$$

$$\begin{array}{r} 2.5 \\ 3.1 \\ \hline 5.6 \end{array}$$

Make these cuts.

Do we need all the cuts?

(2) Makes edges, single line if no distinguish by cut & double line if there is distinguish by alt.

(3) All cuts through which double edge might be returned or single edge might be discarded

Now we need a heuristic to ~~not~~ have minimum cuts.

Remove cut (breadth) b/w 1 & 2 2.4

encode

(4) Then mark different region & then replace continuous features values with encoded region values.

U	length	breadth	Y/N.
1	0	0	Y
2	2	0	Y
3	3	1	N
4	4	2	N
5	4	2	Y
6	1	3	N

- ⑥ We should express the cuts as a boolean expression.  
we can also try empty heuristic

ex:  $(C_1 \sqcup C_2 \sqcup (C_3 \sqcup C_4))$

→  $C_1$  or  $C_2$  or  $C_3$  or  $C_4$  can distinguish.  $\{1, 2, 3\}$

- ⑦ We have maximum discernability heuristic

$P_1^l$  → remain  
 $P_1^b$  → cut

Make relative redux: (use pic from phone)

	$P_1^l$	$P_2^l$	$P_3^l$	$P_4^l$	$P_1^b$	$P_2^b$	$P_3^b$	$P_4^b$
$U_1 U_3$	1	1	1		1			
$U_1 U_5$			1			1	1	1

- ① We determine the cut/column which can distinguish maximum no. of points ( $P_3^b$ )  
strike away the rows & then select the column.  
Repeat the process till you obtain the minimum no. of cuts to partition the space.

$P_3^b \wedge P_3^e$   
↳ four regions.

Now we make table of gen.

	U	l	b	Y/N.
1	1	0	0	Y
2	0	0	0	Y
3	1	1	0	N
4	1	0	1	N
5	1	1	1	Y
6	0	1	1	N

(4 equivalent classes)

If value of attribute is symbol, not continuous values?

B	Y
R	N

B	Y
B	N

Both these  
symbols are  
required

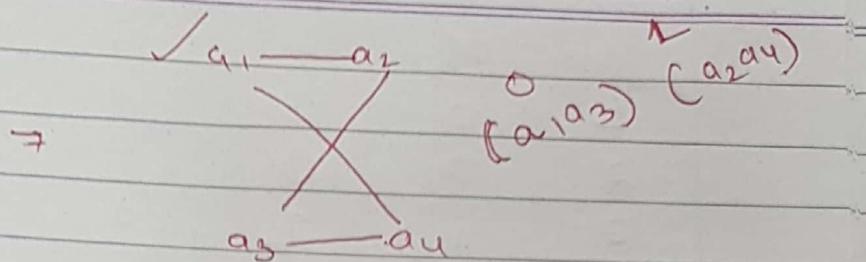
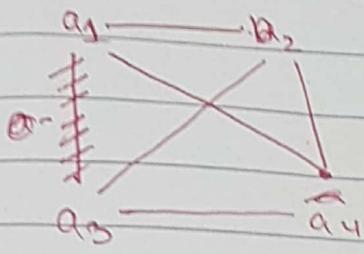
B is not required.

	B	Y
can be clipped together.	S	Y
	G	X
	B	N

	U	a	b	d	U <sub>1</sub>	U <sub>2</sub>	U <sub>3</sub>	U <sub>4</sub>
1	a <sub>1</sub>	b <sub>1</sub>	0	0	000			
2	a <sub>1</sub>	b <sub>2</sub>	0	0				
3	a <sub>2</sub>	b <sub>3</sub>	0	0				
4	a <sub>3</sub>	b <sub>1</sub>	0	0				
5	a <sub>1</sub>	b <sub>4</sub>	1	b <sub>4</sub> <sup>1</sup>	b <sub>4</sub> <sup>2</sup>	a <sub>1</sub> <sup>2</sup> b <sub>4</sub> <sup>3</sup>	a <sub>1</sub> <sup>3</sup> b <sub>4</sub>	
6	a <sub>2</sub>	b <sub>2</sub>	1	a <sub>2</sub> <sup>1</sup> b <sub>2</sub> <sup>1</sup>	a <sub>2</sub> <sup>1</sup> B	b <sub>2</sub> <sup>3</sup>	a <sub>2</sub> <sup>3</sup> b <sub>2</sub>	
7	a <sub>2</sub>	b <sub>1</sub>	1	a <sub>1</sub> <sup>2</sup>	a <sub>2</sub> <sup>1</sup> b <sub>1</sub> <sup>2</sup>	b <sub>1</sub> <sup>3</sup>	a <sub>2</sub> <sup>3</sup>	
8	a <sub>4</sub>	b <sub>2</sub>	1	a <sub>4</sub> <sup>1</sup> b <sub>2</sub> <sup>1</sup>	a <sub>4</sub> <sup>1</sup>	a <sub>4</sub> <sup>2</sup> b <sub>2</sub> <sup>3</sup>	a <sub>4</sub> <sup>3</sup>	
9	a <sub>3</sub>	b <sub>4</sub>	1	0 <sub>3</sub> <sup>1</sup> b <sub>4</sub> <sup>1</sup>	0 <sub>3</sub> <sup>2</sup> b <sub>4</sub> <sup>2</sup>	a <sub>3</sub> <sup>2</sup> b <sub>4</sub> <sup>3</sup>	a <sub>3</sub> <sup>3</sup> b <sub>4</sub>	
10	a <sub>2</sub>	b <sub>5</sub>	1	a <sub>2</sub> <sup>1</sup> b <sub>5</sub> <sup>1</sup>	a <sub>2</sub> <sup>1</sup> b <sub>5</sub> <sup>2</sup>	a <sub>2</sub> <sup>2</sup> b <sub>5</sub> <sup>3</sup>	a <sub>2</sub> <sup>3</sup> b <sub>5</sub>	

Symbol  
not discriminating

We can empty set ~~disjoint~~ Heuristic to  
so find the different cuts



∴ This is one  
such graph but  
not minimised

wrong illustrated

$V_{1,5}$   
 $V_{1,6}$   
 $V_{1,7}$   
 $V_{1,8}$   
 $V_{1,9}$   
 $V_{1,10}$

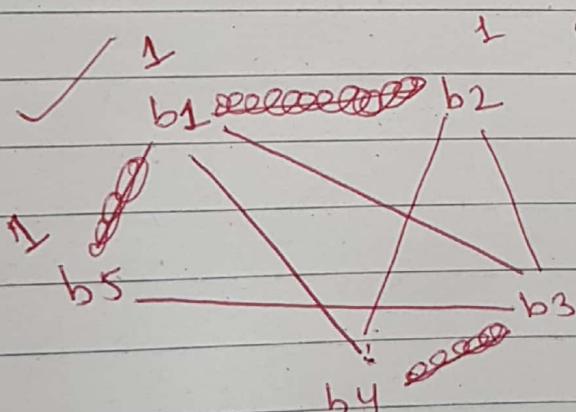
$V_{2,5}$

$a_2^1 b_2^1$   
 $a_1^1$   
 $a_4^1 b_2^1$   
 $a_2^1 b_4^1$   
 $a_2^1 b_5^1$

$b_4^2$   
 $b_4^2$   
 $a_2^1$   
 $a_1^1 b_1^2$   
 $a_4^2 b_4^2$   
 $a_2^1 b_5^2$

$a_2^1 b_4^3$   
 $a_2^3 b_1^1$   
 $b_2^3$   
 $a_2^3 b_2^1$   
 $a_2^3$   
 $a_4^2 b_2^3$   
 $a_3^2 b_4^3$   
 $a_2^2 b_5^3$   
 $b_4^1$   
 $a_2^3 b_5^1$

$V_{1,5}$   
 $V_{1,6}$   
 $V_{1,7}$   
 $V_{1,8}$   
 $V_{1,9}$   
 $V_{1,10}$   
 $V_{2,5}$   
 $V_{2,6}$   
 $V_{2,7}$



$\{b_1, b_2, b_5\}$   
 $\{b_3, b_4\} \perp$

Now assigning the code is reduced to  
graph colouring problem.

U	a <sub>0</sub>	b	P
1	0	0	0
2	0	0	0
3	1	1	0
4	0	0	0
5	0	1	1
6	1	0	1
7	1	0	1
8	0	0	1
9	0	1	1
10.	1	0	1