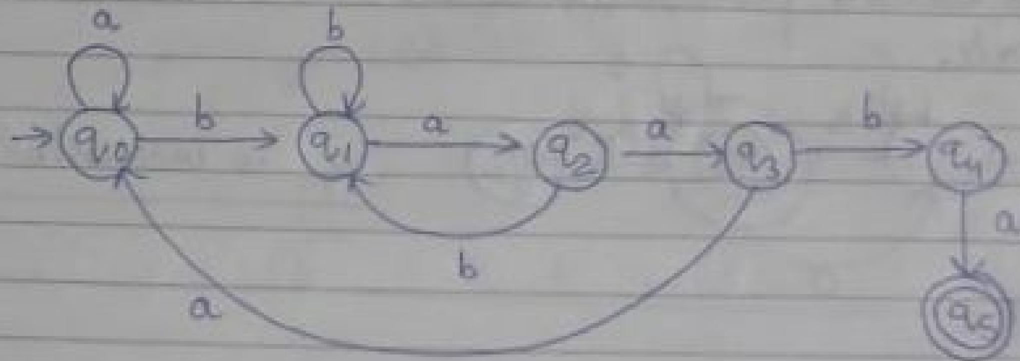
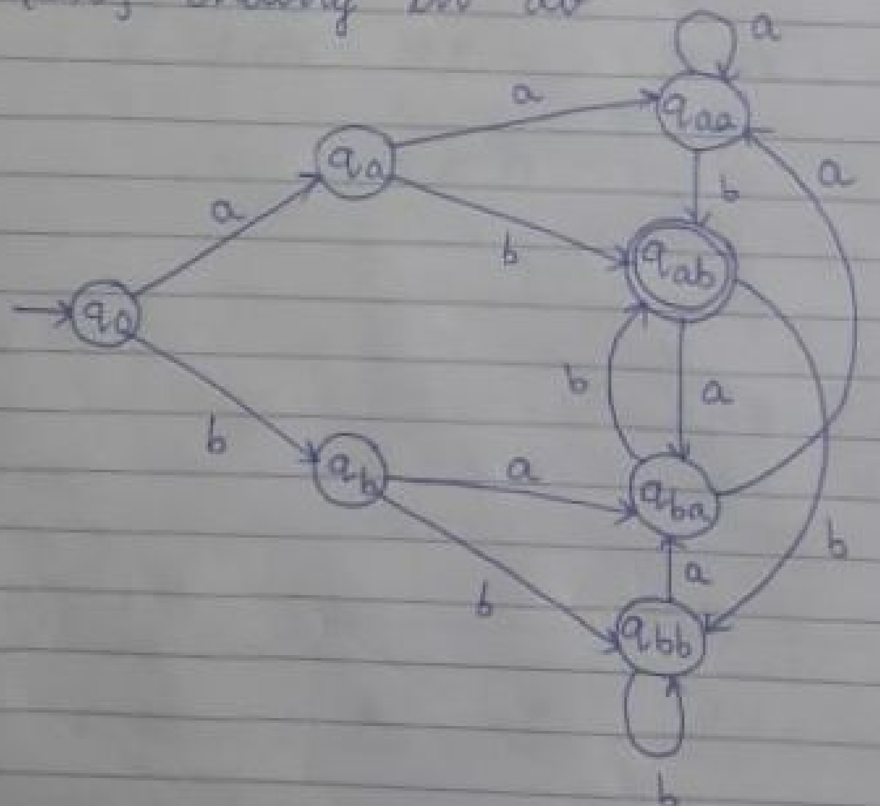
TUTORIAL-2

1. Construct a DFA which accepts strings which have substring "baab"

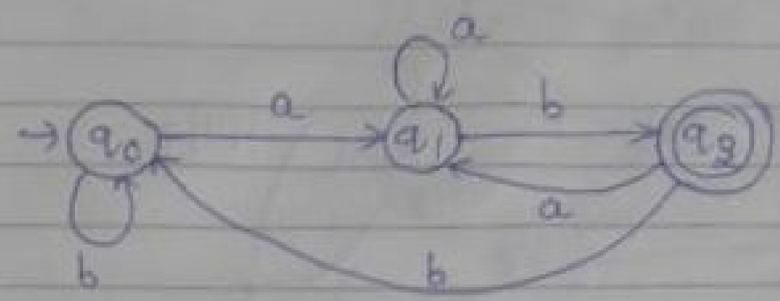


2. construct DFA accepting all strings over {a,b} ending in ab

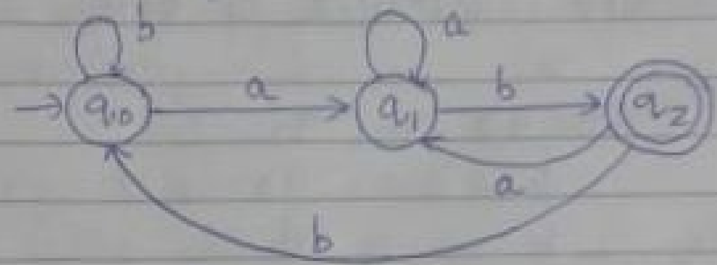


This is the basic one. There is a better solution to this

NOTE: ϵ : Unique string of length 0 and is called empty or null string (It is represented as input in automata)
ϕ : Represented as blank space



3 find T(M) for the DFA :



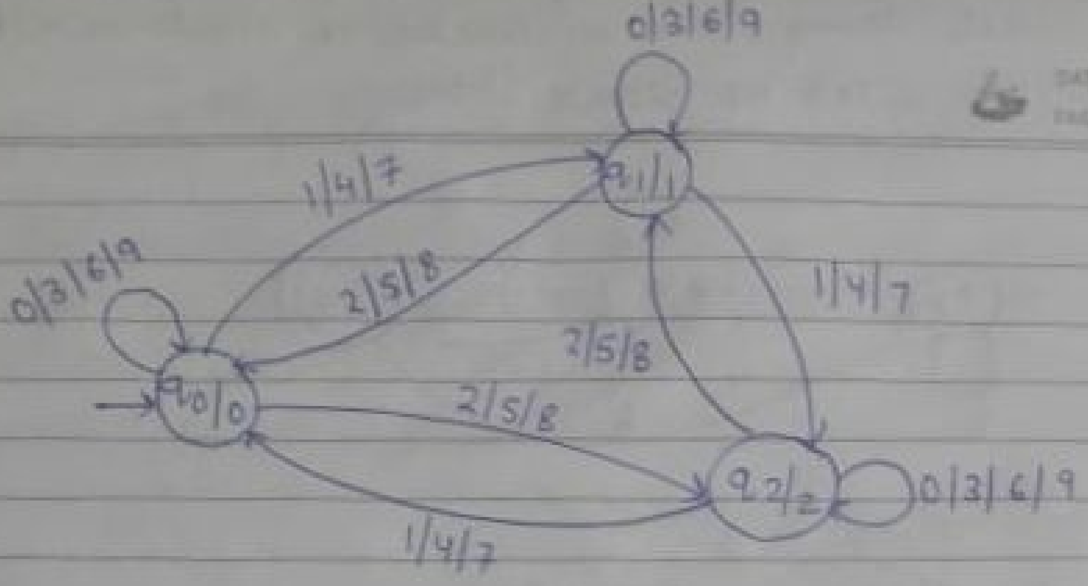
$\rightarrow$ $T(M) = \{ w \in \{a,b\}^* \}$ | w ends in substring "ab"

4 Residue mod 3 of decimal no. using mealy & moore machine

i	k	$10i+k$
$i/3$	$\Rightarrow p = 0,1,2$	

$$(10i+k)\,\%\,3 = p + k\,\%\,3$$

$p=0$	$k\,\%\,3$	$k\,\%\,3 = 0$	0, 3, 6, 9
$p=1$	$1+k\,\%\,3$	$k\,\%\,3 = 1$	1, 4, 7
$p=2$	$2+k\,\%\,3$	$k\,\%\,3 = 2$	2, 5, 8



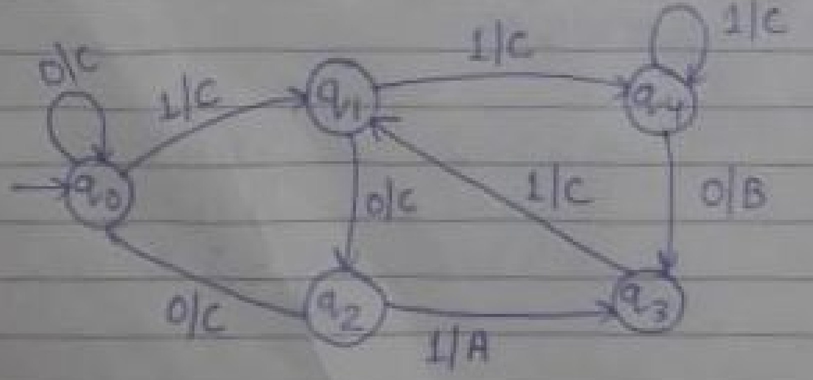
5. for the given NFA, construct eq. DFA

	0	1
$\rightarrow q_1$	q_1, q_2	q_2
q_2	ϕ	q_3
q_3	ϕ	ϕ

$\rightarrow$

	0	1
q_1	$q_1 q_2$	q_2
$q_1 q_2$	$q_1 q_2$	$q_2 q_3$
q_2	ϕ	q_3
$q_2 q_3$	ϕ	q_3
q_3	ϕ	ϕ

6. Construct mealy & moore machine in which if input ends with $101 \rightarrow A$, $110 \rightarrow B$ else C



MINIMIZATION OF FINITE AUTOMATA

(a) Hopecraft's method

(b) Myhill - Nerode theorem

- The pair of states (p,q) are distinguishable states if transition go to final states and non-final states respectively.

- The pair of states are non-distinguishable if both move to either final states group or non final states group

- Here, to minimize finite automata, all non-dist are to be merged and used as single state.

HOPECRAFTS METHOD ⇒

Equivalence classes: Groups of equivalent states.

→ For a DFA M, min. no. of states in an equivalent deterministic FA is same as no. of equivalence classes of M's states.

→ If we can find equivalence classes, we can use these as the states of the smallest equivalent machine

Construction of minimum automata:

Step 1: Construction of $\pi_0 \Rightarrow$

0 equivalence: $\pi_0 = \{Q_1^0, Q_2^0\}$

where Q_1^0 is set of all final states

$Q_2^0 = Q - Q_1^0$

Step 2: Construction of π_{K+1} and π_K

Let $q_i^{\pi k}$ be any subset in π_K. If q_1 and q_2 are in $q_i^{\pi k}$, they are $(K+1)$ equivalent provided $\delta(q_1, a)$ and $\delta(q_2, a)$ are (K) equivalent.

Find out whether $\delta(q_1, a)$ and $\delta(q_2, a)$ are in the same equivalence class in π_K for every $a \in \Sigma$. If so q_1 and q_2 are $(K+1)$ equivalent.

In this way, q_i^k is further divided into $(K+1)$ equivalent classes

Repeat this for every q_i^k in π_K to get all elements of π_{K+1}

Step 3:

Construct π_n where $n = 1, 2 \ldots$ until $\pi_n = \pi_{n+1}$

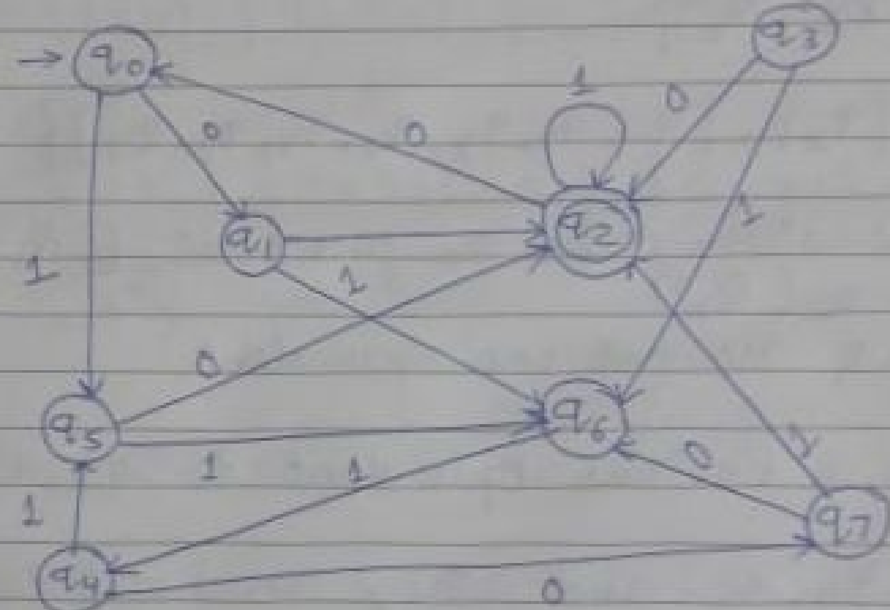
Step 4: Construction of minimum automata

For the required min state automata,

the states are equivalence classes obtained in step 3.

The state table can be obtained by replacing state queue with the corresponding equivalence class queue.

Ques: Construct a minimum state automata from the finite state automata given below:



state / Σ	0	1
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

Step 1:

$Q_1^0 = F = \{q_2\}$

$Q_2^0 = Q - Q_1^0$

$\pi_0 = \{Q_1^0, Q_2^0\}$

$= \{\{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\}$

Since on application of 1, q_0 goes to q_5 (Q_2^0) and q_1 goes to q_2 (Q_1^0)

$\therefore$ q_0 and q_1 are not one equivalent

Similarly q_0 is not one equivalent to q_3, q_5, q_7

We see q_0, q_4, q_6 and they come out to be one equivalent

$\{q_0, q_4, q_6\}$ is a subset in π_1

So $Q_2' = \{q_0, q_4, q_6\}$

Repeat the construction by considering q_1 and any of the states q_3, q_5, q_7

We see q_1 is one equivalent to q_7 but not q_3 & q_5

$\Rightarrow Q_3' = \{q_1, q_7\}$

$Q_4' = \{q_3, q_5\}$

$\pi_1 = \{\{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

q_0 and q_4, on application of 0 & 1 go to a state in Q_3' and Q_1'' respectively ⇒ they are two equivalent
But not with q_6

$\Rightarrow \pi_2 = \{\{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

Similarly,

$\pi_3 = \{\{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$

Since, $\pi_2 = \pi_3$ we stop

$M' = (Q', \{0,1\}, \delta', q_0', F')$

where $Q' = \{[q_2], [q_0, q_4], [q_6], [q_1, q_7], [q_3, q_5]\}$

$q_0' = [q_0, q_4]$, $F' = [q_2]$

δ'

State/Σ	0	1