# CSE400 — Probability Theory Lecture Scribe (Slides 1–28)

Instructor: Dhaval Patel, PhD

January 15, 2026

## 1 Motivation and Engineering Applications

### Why Learn CSE400?

Applications include:

- Daily life conversations
- Speech Recognition
- Radar Systems
- Communication Networks

### Speech Recognition System

Process:

- Speech input (example: “Hello”)
- Matching with stored vocabulary/templates
- Output (example: “Yes”, “No”)

Multiple speakers have different templates. Signals such as $L(t)$ and $r(t)$ are matched to stored references.

### Radar System

Steps:

- Transmit signal
- Receive reflected signal
- Decide target presence or false alarm

### Communication Network

Information propagates through connected nodes from a source.

## 2 Introduction to Probability Theory

**Definition (Experiment $E$):**

A procedure that produces some result.
Example: Tossing a coin five times ($E_5$).
**Definition (Outcome $\omega$):**
A possible result of an experiment.
Example:

$$\omega = HHTHT$$

**Definition (Event):**
A set of outcomes.
Example:

$$C = \{\text{all outcomes with even number of heads}\}$$

**Definition (Sample Space $S$):**
The set of all possible distinct outcomes.
Properties:

- Mutually exclusive
- Collectively exhaustive

Types of sample space:

- Discrete
- Countably infinite
- Continuous

Examples:

- Single coin flip
- Roll of die
- Two dice
- Flip until tails
- Random number in $[0, 1)$

## 3 Probability and Axioms

**Definition (Probability):**

A numerical measure of likelihood of an event.

**Axiom 1**

$$0 \leq Pr(A) \leq 1$$

**Axiom 2**

$$Pr(S) = 1$$

**Axiom 3 (Additivity)**

If $A \cap B = \emptyset$:

$$Pr(A \cup B) = Pr(A) + Pr(B)$$

For infinite sets:

$$Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} Pr(A_i)$$

**Corollary (Finite Case)**

For mutually exclusive $A_1, \ldots, A_M$:

$$Pr\left(\bigcup_{i=1}^{M} A_i\right) = \sum_{i=1}^{M} Pr(A_i)$$

## 4 Assigning Probability

**Classical Approach**

$$Pr(A) = \frac{n(A)}{N}$$

**Relative Frequency Approach**

$$Pr(A) = \lim_{N \to \infty} \frac{n(A)}{N}$$

## 5 Joint Probability

**Definition**

$$Pr(A \cap B)$$

Probability that both $A$ and $B$ occur.

### Example 1 — Card Deck

Let:

$$A = \text{heart}, \quad B = \text{king}$$

Only King of Hearts satisfies both:

$$Pr(A \cap B) = \frac{1}{52}$$

### Example 2 — Costume Party

Given:

$$Pr(A) = 0.6, \quad Pr(B) = 0.3, \quad Pr(A \cap B) = 0.2$$

Joint probability:

$$0.20$$

# 6 Conditional Probability

For $Pr(B) > 0$:

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

# 7 Product Rule

Two events:

$$Pr(A \cap B) = Pr(B)Pr(A|B)$$

Three events:

$$Pr(A \cap B \cap C) = Pr(A)Pr(B|A)Pr(C|A \cap B)$$

General form:

$$Pr(A_1 \cap \cdots \cap A_n) = Pr(A_1)Pr(A_2|A_1) \cdots Pr(A_n|A_1 \cap \cdots \cap A_{n-1})$$

# 8 Worked Examples

### Example 3 — Cards Without Replacement

Two cards drawn.

Let $A$: first Ace, $B$: second Ace.

Step 1:

$$Pr(A) = \frac{4}{52}$$

Step 2:

$$Pr(B|A) = \frac{3}{51}$$

Step 3:

$$Pr(A \cap B) = \frac{4}{52} \cdot \frac{3}{51}$$

### Example 4 — Poker (Exactly One Pair)

$$Pr = \frac{\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3}{\binom{52}{5}}$$

### Example 5 — Missing Key

10 keys, one correct.

Probability correct key is on $k$-th try:

$$Pr = \frac{1}{10}$$

# 9 Independence

**Definition**

$$Pr(A \cap B) = Pr(A)Pr(B)$$

**Conditional Independence**

$$Pr(A \cap B|C) = Pr(A|C)Pr(B|C)$$

# 10 Bayes' Rule

$$Pr(A|B) = \frac{Pr(B|A)Pr(A)}{Pr(B)}$$

# 11 Law of Total Probability

If $A_1, \ldots, A_n$ partition the sample space:

$$Pr(B) = \sum_{i=1}^{n} Pr(B|A_i)Pr(A_i)$$

**Bayes with Total Probability**

$$Pr(A_i|B) = \frac{Pr(B|A_i)Pr(A_i)}{\sum_{j=1}^{n} Pr(B|A_j)Pr(A_j)}$$

# 12 Final Summary

Topics covered:

- Sample space
- Events
- Probability axioms
- Joint probability
- Conditional probability

- Product rule
- Independence
- Bayes rule
- Total probability
- Worked examples