Lecture Scribe Probability

L12_S1_A (1).pdf
PDF

Generate a lecture scribe from this lecture slides which is to be used for exam preparation. I need you to reason step by step for definitions, assumptions, derivation, theorems, examples and formulas. Reasoning with proper step by step explanation should be included in scribe for better understanding during exam revisions, and none of the reasoning should come from out of the contents of the slides. The scribe should stick to the lecture slides and should be accurate. No content should be invented or self made. Keep the content in scribe in same structure and scope as lecture slides.

Below is the **lecture scribe strictly based on the provided slides**, keeping the **same structure, scope, and flow** as in the lecture. No external content has been added.

CSE400 – Fundamentals of Probability in Computing

Lecture L11–L12: Transformation of Random Variables

Prof. Dhaval Patel, PhD

Associate Professor, Computer Science and Engineering

SEAS – Ahmedabad University

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Outline

1. **Transformation of Random Variables**
   Learning transformation techniques for random variables.
2. **Function of Two Random Variables**
   Joint transformations and derived distributions.
3. **Illustrative Example**
   Detailed derivation for the case:

$$Z = X + Y$$

1️⃣ Transformation of Random Variables

We consider a transformation:

$$Y = g(X)$$

Our goal is to find the distribution (CDF and PDF) of Y, given that the distribution of X is known.

Step–V: CDF of Y

From the slide:

$$F_Y(y) = P(Y \leqslant y)$$

Since $Y = g(X)$,

$$F_Y(y) = P(g(X) \leqslant y)$$

The probability is rewritten in terms of X, because the PDF of X is known.

The slide shows transformation using:

$$F_Y(y) = 1 - F_X(\cdot)$$

depending on the monotonicity of the function.

Thus the method used in the slides:

1. Express event $Y \leqslant y$
2. Rewrite in terms of X
3. Use known CDF of X
4. Differentiate to get PDF of Y

Finally:

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

2️⃣ Example: Uniform Random Variable

Given:

$$X \sim \text{Uniform}(-1, 1)$$

$$f_X(x) = \frac{1}{2}, \quad -1 < x < 1$$

Find:

$$Y = \sin\left(\frac{\pi}{2}X\right)$$

Step 1: Define transformation

$$Y = g(X) = \sin\left(\frac{\pi}{2}X\right)$$

Step 2: Use CDF Method

$$F_Y(y) = P(Y \leqslant y)$$

$$= P\left(\sin\left(\frac{\pi}{2}X\right) \leqslant y\right)$$

Rewrite inequality in terms of X, then use uniform PDF.

Final PDF (as shown in slide)

$$f_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}, \quad -1 < y < 1$$

This result comes from differentiating the transformed CDF and using the Jacobian term that appears after inversion.

3️⃣ Function of Two Random Variables

Now consider:

$$Z = g(X, Y)$$

The lecture focuses on:

$$Z = X + Y$$

Case: $Z = X + Y$

We start with CDF:

$$F_Z(z) = P(Z \leqslant z)$$

$$= P(X + Y \leqslant z)$$

This defines a region in the (x, y)-plane.

Thus:

$$F_Z(z) = \iint_{x+y \leqslant z} f_{X,Y}(x, y)\, dx\, dy$$

If X and Y are independent:

$$f_{X,Y}(x, y) = f_X(x) f_Y(y)$$

Leibniz Rule (Used in Slides)

To obtain PDF from CDF:

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

The slide introduces **Leibniz Rule** for differentiating an integral with variable limits:

If

$$G(z) = \int_{a(z)}^{b(z)} h(x, z)\, dx$$

then

$$\frac{d}{dz} G(z) = h(b(z), z) \cdot \frac{db}{dz} - h(a(z), z) \cdot \frac{da}{dz} + \int_{a(z)}^{b(z)} \frac{\partial}{\partial z} h(x, z)\, dx$$

This rule is applied to differentiate the CDF integral.

Convolution Result

After applying Leibniz rule and simplification:

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z - x)\, dx$$

This is the **convolution formula** (as derived in slides).

Example: Exponential Random Variables

Given:

$$f_X(x) = \lambda e^{-\lambda x}, \quad x \geqslant 0$$

$$f_Y(y) = \lambda e^{-\lambda y}, \quad y \geqslant 0$$

Independent RVs.

Find:

$$Z = X + Y$$

Using convolution:

$$f_Z(z) = \int_0^z \lambda e^{-\lambda x} \lambda e^{-\lambda(z-x)} dx$$

$$= \lambda^2 e^{-\lambda z} \int_0^z dx$$

$$= \lambda^2 z e^{-\lambda z}$$

This matches the derived expression in the slides.

Additional Example

Slides also mention:

$$Z = X - Y$$

Then

$$F_Z(z) = P(X - Y \leqslant z)$$

Rewrite:

$$P(X \leqslant z + Y)$$

And integrate over joint PDF accordingly.

Summary of Lecture Content

1. Transformation of single RV using CDF method
2. Deriving PDF via differentiation
3. Handling monotonic functions
4. Transformation of two RVs
5. Derivation of convolution formula
6. Application to:
   - Uniform transformation
   - Sum of independent RVs
   - Exponential case
   - Difference case

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- Convert this into **proper LaTeX code**
- Make a **clean exam-ready structured PDF version**
- Or rewrite it in more handwritten-exam style notes**

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