Problem 3:

Derivation:

Given : $\Omega_p' = 1000\,Hz = 2000\pi$ rad/sec.

$\Omega_s' = 350\,Hz = 700\pi$ rad/sec.

$A_p = -3\,dB$

$A_s = -10\,dB$

$f = 5000\,Hz.$

$\therefore T = \frac{1}{f} = \frac{1}{5000} = 2\times10^{-4}$ sec.

Step 1: Finding Ω_p and Ω_s using pre-wrapping.

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\Omega_p' T}{2}\right)$$

$$= \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi\times2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.2\pi)$$

$$= 7265 \text{ rad/sec.}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\Omega_s' T}{2}\right)$$

$$= \frac{2}{2\times10^{-4}}\tan\left(\frac{700\pi\times2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.07\pi)$$

$$= 2235 \text{ rad/sec.}$$

step2: Finding 'A' & 'ε'

$$A_p = 20\log\frac{1}{\sqrt{1+\varepsilon^2}} \qquad A_s = 20\log\frac{1}{A}$$

$$-3 = 20\log\frac{1}{\sqrt{1+\varepsilon^2}} \qquad -10 = 20\log\frac{1}{A}$$

$$\therefore \varepsilon = 0.9977 \qquad A = 3.165$$

Step 3: Finding the order of filter

$$N = \frac{\log\left(\frac{\sqrt{A^2-1}}{\varepsilon}\right)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N = \frac{\log\left(\frac{\sqrt{(3.165)^2-1}}{0.9977}\right)}{\log\left(\frac{2235}{7265}\right)}$$

$$N = \frac{\log(3.0098)}{\log(0.3076)}$$

$N = 0.9345$

$N \simeq 1$

$\therefore$ Order of filter $= 1$

Step 4: Finding the Transfer function.

For 1^{st} order filter with $\Omega_c = 1$,

$$H(s) = \frac{1}{s+1}$$

For high-pass filter with $\Omega_c = 7265$ rad/sec

$$H(s) = \left.\frac{1}{s+1}\right|_{s \to \frac{7265}{s}}$$

$$\therefore H(s) = \frac{1}{\frac{7265}{s}+1}$$

$$H(s) = \frac{s}{s+7265}$$

Step 5: Using Bilinear Transformation:

$$H(z) = \left. H(s) \right|_{s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left.\frac{s}{s+7265}\right|_{s \to 10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$$

$$= \frac{10^4(1-z^{-1})}{10^4(1-z^{-1})+7265(1+z^{-1})}$$

$$= \frac{(1-z^{-1})}{(1-z^{-1}+0.7265+0.7265z^{-1})}$$

$$= \frac{1-z^{-1}}{1.7265-0.2735z^{-1}}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}} \quad //.$$