Задание:

1. $(\exists x)(\exists y)\phi \equiv (\exists y)(\exists x)\phi$

2. $(\forall x)\phi \equiv (\forall y)(\phi)^x_y$

Решение:

1. (a)
$$\cfrac{\cfrac{\cfrac{\cfrac{\phi \;\vdash\; \phi}{\phi \;\vdash\; (\exists x)\phi}\;\text{$\exists$ введ. справа)}}{\phi \;\vdash\; (\exists y)(\exists x)\phi}\;\text{($\exists$ введ. справа)}}{(\exists y)\phi \;\vdash\; (\exists y)(\exists x)\phi}\;\text{($\exists$ введ. слева)}}{(\exists x)(\exists y)\phi \;\vdash\; (\exists y)(\exists x)\phi}\;\text{($\exists$ введ. слева)}$$

(b) $$\cfrac{\text{Аналогично.}}{(\exists y)(\exists x)\phi \;\vdash\; (\exists x)(\exists y)\phi}$$

2. (a)
$$\cfrac{\cfrac{(\phi)^{xy}_{yx} \;\vdash\; \phi}{(\forall y)(\phi)^x_y \;\vdash\; \phi}\;\text{($\forall$ введ. слева)}}{(\forall y)(\phi)^x_y \;\vdash\; (\forall x)\phi}\;\text{($\forall$ введ. справа)}$$

(b)
$$\cfrac{\cfrac{(\phi)^x_y \;\vdash\; (\phi)^x_y}{(\forall y)\phi \;\vdash\; (\phi)^x_y}\;\text{($\forall$ введ. слева)}}{(\forall x)\phi \;\vdash\; (\forall y)(\phi)^x_y}\;\text{($\forall$ введ. справа)}$$