

Задание:

$(\forall z)((x + y) + z) \approx x + (y + z) \rightarrow (x + y) + s(z) \approx x + (y + s(z))$, т.е. индукционный шаг для $(x + y) + z \approx x + (y + z)$

Решение:

$$\frac{\frac{\frac{\vdash (x + y) + s(z) \approx s(x + y + z)}{\vdash s(x + y + z) \approx (x + y) + s(z)} \quad \frac{\vdash y + s(z) \approx s(y + z)}{\vdash s(y + z) \approx y + s(z)} \quad \frac{\vdash x + s(y + z) \approx s(x + y + z)}{\vdash s(x + y + z) \approx x + s(y + z)}}{\vdash (x + y) + s(z) \approx x + (y + s(z))} \text{ (замена равных)}$$
$$\frac{\frac{(x + y) + z \approx x + (y + z) \quad \vdash (x + y) + s(z) \approx x + (y + s(z))}{\vdash (x + y) + s(z) \approx x + (y + s(z))} \text{ (уточн.)}}{\vdash (x + y) + z \approx x + (y + z) \rightarrow (x + y) + s(z) \approx x + (y + s(z))} \text{ (}\rightarrow \text{ введ.)}$$
$$\vdash (\forall z) \left((x + y) + z \approx x + (y + z) \rightarrow (x + y) + s(z) \approx x + (y + s(z)) \right) \text{ (}\forall \text{ введ.)}$$