Задание:

$\Omega = (P^{(1)}, R^{(2)}, Q^{(3)}; f^{(2)}, g^{(1)})$

Вывести секвенции:

1. $(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \ \vdash \ (\forall x)P(x)$
2. $(\forall x)(\forall y)(\forall z)(R(x,y) \wedge R(y,z) \to P(y)), (\forall x)(\exists y)R(x,y) \ \vdash \ (\exists x)P(x)$

Решение:

1.
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(P(g(u)))^u_y, g(y) \approx x \ \vdash \ (P(u))^u_x}{P(g(y)), g(y) \approx x \ \vdash \ P(x)}}{P(g(y)), x \approx g(y) \ \vdash \ P(x)}\text{(замена эквив.)}}{P(g(y)), (\exists y)x \approx g(y) \ \vdash \ P(x)}\text{(}\exists\text{ введ. слева)}}{P(g(y)), (\forall x)(\exists y)x \approx g(y) \ \vdash \ P(x)}\text{(}\forall\text{ введ. слева)}}{(\forall y)P(g(y)), (\forall x)(\exists y)x \approx g(y) \ \vdash \ P(x)}\text{(}\forall\text{ введ. слева)}}{(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \ \vdash \ P(x)}\text{(замена эквив.)}}{(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \ \vdash \ (\forall x)P(x)}\text{(}\exists\text{ введ. справа)}$$

2.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(R(x,y))^y_x \vdash R(x,x)}{\left(\left(R(x,y)\wedge R(y,z)\to P(y)\right)^y_x\right)^z_x, (R(x,y))^y_x \vdash R(x,x)}\text{(утонч.)} \quad \dfrac{\dfrac{\left(R(x,y)\to P(y)\right)^y_x \vdash R(x,x)\to P(x)}{\left(\left(R(x,y)\wedge R(y,z)\to P(y)\right)^y_x\right)^z_x \vdash R(x,x)\to P(x)}\text{(замена эквив.)}}{\left(\left(R(x,y)\wedge R(y,z)\to P(y)\right)^y_x\right)^z_x, (R(x,y))^y_x \vdash R(x,x)\to P(x)}\text{(утонч.)}}{\left(\left(R(x,y)\wedge R(y,z)\to P(y)\right)^y_x\right)^z_x, (R(x,y))^y_x \vdash P(x)}\text{(удал. }\to\text{)}}{\left(\left(R(x,y)\wedge R(y,z)\to P(y)\right)^y_x\right)^z_x, (\exists y)R(x,y) \vdash P(x)}\text{(}\exists\text{ введ. слева)}}{\left(\left(R(x,y)\wedge R(y,z)\to P(y)\right)^y_x\right)^z_x, (\forall x)(\exists y)R(x,y) \vdash P(x)}\text{(}\forall\text{ введ. слева)}}{\left((\forall z)(R(x,y)\wedge R(y,z)\to P(y))\right)^y_x, (\forall x)(\exists y)R(x,y) \vdash P(x)}\text{(}\forall\text{ введ. слева)}}{(\forall y)(\forall z)(R(x,y)\wedge R(y,z)\to P(y)), (\forall x)(\exists y)R(x,y) \vdash P(x)}\text{(}\forall\text{ введ. слева)}}{(\forall x)(\forall y)(\forall z)(R(x,y)\wedge R(y,z)\to P(y)), (\forall x)(\exists y)R(x,y) \vdash P(x)}\text{(}\forall\text{ введ. слева)}}{(\forall x)(\forall y)(\forall z)(R(x,y)\wedge R(y,z)\to P(y)), (\forall x)(\exists y)R(x,y) \vdash (\exists x)P(x)}\text{(}\exists\text{ введ. справа)}$$