

Задание:

1.  $(\exists x)(\exists y)\phi \equiv (\exists y)(\exists x)\phi$
2.  $(\forall x)\phi \equiv (\forall y)(\phi)_y^x$
3.  $(\exists x)\phi \rightarrow (\forall x)\psi \vdash (\forall x)(\phi \rightarrow \psi)$

Решение:

$$1. \quad (a) \quad \frac{\frac{\frac{\phi \vdash \phi}{\phi \vdash (\exists x)\phi} \text{ } (\exists \text{ введ. справа)}}{\phi \vdash (\exists y)(\exists x)\phi} \text{ } (\exists \text{ введ. справа)}}{\frac{(\exists y)\phi \vdash (\exists y)(\exists x)\phi}{(\exists x)(\exists y)\phi \vdash (\exists y)(\exists x)\phi}} \text{ } (\exists \text{ введ. слева)}$$

$$(b) \quad \frac{\text{Аналогично.}}{(\exists y)(\exists x)\phi \vdash (\exists x)(\exists y)\phi}$$

$$2. \quad (a) \quad \frac{\frac{(\phi)_{yx}^{xy} \vdash \phi}{(\forall y)(\phi)_y^x \vdash \phi} \text{ } (\forall \text{ введ. слева)}}{(\forall y)(\phi)_y^x \vdash (\forall x)\phi} \text{ } (\forall \text{ введ. справа})$$

$$(b) \quad \frac{\frac{(\phi)_y^x \vdash (\phi)_y^x}{(\forall y)\phi \vdash (\phi)_y^x} \text{ } (\forall \text{ введ. слева)}}{(\forall x)\phi \vdash (\forall y)(\phi)_y^x} \text{ } (\forall \text{ введ. справа})$$

3.

$$\frac{\text{Неверно}}{\vdash (\forall x)\phi \vee (\forall x)\neg\phi} \text{ (уточн.)} \quad \frac{}{(\forall x)\phi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ ()} \quad \frac{\frac{\frac{\phi \vdash \phi}{\neg\phi, \phi, \neg\psi \vdash \phi} \text{ (утнч)} \quad \frac{\neg\phi \vdash \neg\phi}{\neg\phi, \phi, \neg\psi \vdash \neg\phi} \text{ (утнч)}}{\neg\phi, \phi \vdash \psi} \text{ } (\forall \text{ введ. справа)}}{\frac{(\forall x)\neg\phi \vdash \phi \rightarrow \psi}{(\forall x)\neg\phi \vdash (\forall x)(\phi \rightarrow \psi)}} \text{ (утонч.)}$$

$$\frac{(\exists x)\phi \rightarrow (\forall x)\psi \vdash (\forall x)\phi \vee (\forall x)\neg\phi}{(\exists x)\phi \rightarrow (\forall x)\psi, (\forall x)\phi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ (уточн.)} \quad \frac{(\forall x)\phi \vdash (\forall x)(\phi \rightarrow \psi)}{(\exists x)\phi \rightarrow (\forall x)\psi, (\forall x)\neg\phi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ (уточн.)} \quad \frac{}{(\exists x)\phi \rightarrow (\forall x)\psi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ (допустимое правило)}$$