

Задание:

$$(\forall z)((x+y)+z) \approx x+(y+z) \rightarrow (x+y)+s(z) \approx x+(y+s(z)), \text{ т.е. индукционный шаг для } (x+y)+z \approx x+(y+z)$$

Решение:

$$\begin{array}{c} \frac{\vdash (x+y)+s(z) \approx s(x+y+z) \quad \vdash y+s(z) \approx s(y+z) \quad \vdash x+s(y+z) \approx s(x+y+z)}{\vdash s(x+y+z) \approx (x+y)+s(z) \quad \vdash s(y+z) \approx y+s(z) \quad \vdash s(x+y+z) \approx x+s(y+z)} \text{ (замена равных)} \\ \frac{}{\vdash (x+y)+s(z) \approx x+(y+s(z))} \text{ (уточн.)} \\ \frac{(x+y)+z \approx x+(y+z) \quad \vdash (x+y)+s(z) \approx x+(y+s(z))}{\vdash (x+y)+z \approx x+(y+z) \rightarrow (x+y)+s(z) \approx x+(y+s(z))} \text{ (\rightarrow введ.)} \\ \vdash (\forall z)((x+y)+z \approx x+(y+z) \rightarrow (x+y)+s(z) \approx x+(y+s(z))) \text{ (\forall введ.)} \end{array}$$