

Задание:

$$\Omega = (P^{(1)}, R^{(2)}, Q^{(3)}; f^{(2)}, g^{(1)})$$

Вывести секвенций:

1.  $(\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (\exists y)(\exists z)R(z, y)$
2.  $(\forall x)(\forall y)(R(x, y) \rightarrow R(y, x), (\exists y)(\forall x)R(x, y) \vdash (\forall x)R(x, x)$
3.  $(\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (\forall x)(\exists y)(\exists z)R(z, y) \wedge R(y, x)$

Решение:

1. Убрать пару квантаров и получится аналогично правой ветви 3-го задания.
2. Подозрения на невыводимость.

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 ((R(x, u))_y^x)_t^u, t \approx y \vdash ((R(z, u))_y^z)_y^u
 }{(R(x, t))_y^x, t \approx y \vdash (R(z, y))_y^z} \text{(замена эквив.)}
 }{(R(x, t))_y^x, y \approx t \vdash (R(z, y))_y^z} \text{(замена эквив.)}
 }{(R(x, t))_y^x, y \approx g(y) \vdash (R(z, y))_y^z} \text{(перест.)}
 }{y \approx g(y), (R(x, g(x)))_y^x \vdash (R(z, y))_y^z} \text{(}\forall \text{ введ. слева)}
 }{y \approx g(y), (\forall x)R(x, g(x)) \vdash (R(z, y))_y^z} \text{(}\exists \text{ введ. слева)}
 }{(\exists y)y \approx g(y), (\forall x)R(x, g(x)) \vdash (R(z, y))_y^z} \text{(}\forall \text{ введ. слева)}
 }{(\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (R(z, y))_y^z}
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 ((R(x, t))_y^x)_t^u, t \approx u \vdash (R(y, u))_x^u
 }{(R(x, t))_y^x, t \approx x \vdash R(y, x)} \text{(замена эквив.)}
 }{(R(x, g(x)))_y^x, g(y) \approx x \vdash R(y, x)} \text{(замена эквив.)}
 }{(R(x, g(x)))_y^x, x \approx g(y) \vdash R(y, x)} \text{(перест.)}
 }{x \approx g(y), (R(x, g(x)))_y^x \vdash R(y, x)} \text{(}\forall \text{ введ. слева)}
 }{x \approx g(y), (\forall x)R(x, g(x)) \vdash R(y, x)} \text{(}\exists \text{ введ. слева)}
 }{(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash R(y, x)} \text{(}\forall \text{ введ. слева)}
 }{(\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash R(y, x)} \text{(}\wedge \text{ введ. справа)}
 }{
 (\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (R(z, y) \wedge R(y, x))_y^z \text{(}\exists \text{ введ. справа)}
 }{
 (\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (\exists z)R(z, y) \wedge R(y, x) \text{(}\exists \text{ введ. справа)}
 }{
 (\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (\exists y)(\exists z)R(z, y) \wedge R(y, x) \text{(}\forall \text{ введ. справа)}
 }{
 (\forall x)(\exists y)x \approx g(y), (\forall x)R(x, g(x)) \vdash (\forall x)(\exists y)(\exists z)R(z, y) \wedge R(y, x)
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 \end{array}$$