Задание:

1. $(\exists x)(\exists y)\phi \equiv (\exists y)(\exists x)\phi$
2. $(\forall x)\phi \equiv (\forall y)(\phi)^x_y$
3. $(\exists x)\phi \to (\forall x)\psi \ \vdash \ (\forall x)(\phi \to \psi)$

Решение:

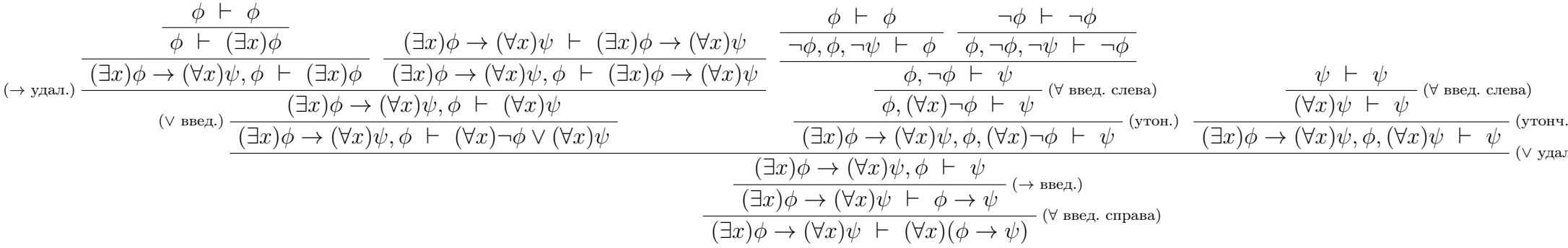
1. (a)
$$\dfrac{\dfrac{\dfrac{\dfrac{\phi \ \vdash \ \phi}{\phi \ \vdash \ (\exists x)\phi}\ \exists\text{ введ. справа)}}{\phi \ \vdash \ (\exists y)(\exists x)\phi}\ (\exists\text{ введ. справа)}}{(\exists y)\phi \ \vdash \ (\exists y)(\exists x)\phi}\ (\exists\text{ введ. слева)}}{(\exists x)(\exists y)\phi \ \vdash \ (\exists y)(\exists x)\phi}\ (\exists\text{ введ. слева)}$$

(b)
$$\frac{\text{Аналогично.}}{(\exists y)(\exists x)\phi \ \vdash \ (\exists x)(\exists y)\phi}$$

2. (a)
$$\dfrac{\dfrac{(\phi)^{xy}_{yx} \ \vdash \ \phi}{(\forall y)(\phi)^x_y \ \vdash \ \phi}\ (\forall\text{ введ. слева)}}{(\forall y)(\phi)^x_y \ \vdash \ (\forall x)\phi}\ (\forall\text{ введ. справа)}$$

(b)
$$\dfrac{\dfrac{(\phi)^x_y \ \vdash \ (\phi)^x_y}{(\forall y)\phi \ \vdash \ (\phi)^x_y}\ (\forall\text{ введ. слева)}}{(\forall x)\phi \ \vdash \ (\forall y)(\phi)^x_y}\ (\forall\text{ введ. справа)}$$

3.


$\phi \vdash \phi$
$\phi \vdash (\exists x)\phi$
$(\exists x)\phi \to (\forall x)\psi, \phi \vdash (\exists x)\phi$
(→ удал.)
$(\exists x)\phi \to (\forall x)\psi \vdash (\exists x)\phi \to (\forall x)\psi$
$(\exists x)\phi \to (\forall x)\psi, \phi \vdash (\exists x)\phi \to (\forall x)\psi$
(∨ введ.)
$(\exists x)\phi \to (\forall x)\psi, \phi \vdash (\forall x)\psi$
$(\exists x)\phi \to (\forall x)\psi, \phi \vdash (\forall x)\neg\phi \vee (\forall x)\psi$
$\phi \vdash \phi$
$\neg\phi \vdash \neg\phi$
$\neg\phi, \phi, \neg\psi \vdash \phi$
$\phi, \neg\phi, \neg\psi \vdash \neg\phi$
$\phi, \neg\phi \vdash \psi$
(∀ введ. слева)
$\phi, (\forall x)\neg\phi \vdash \psi$
(утон.)
$(\exists x)\phi \to (\forall x)\psi, \phi, (\forall x)\neg\phi \vdash \psi$
$\psi \vdash \psi$
$(\forall x)\psi \vdash \psi$
(∀ введ. слева)
$(\exists x)\phi \to (\forall x)\psi, \phi, (\forall x)\psi \vdash \psi$
(утонч.
(∨ удал
$(\exists x)\phi \to (\forall x)\psi, \phi \vdash \psi$
(→ введ.)
$(\exists x)\phi \to (\forall x)\psi \vdash \phi \to \psi$
(∀ введ. справа)
$(\exists x)\phi \to (\forall x)\psi \vdash (\forall x)(\phi \to \psi)$