

Задание:

1. $(\exists x)(\exists y)\phi \equiv (\exists y)(\exists x)\phi$
2. $(\forall x)\phi \equiv (\forall y)(\phi)_y^x$
3. $(\exists x)\phi \rightarrow (\forall x)\psi \vdash (\forall x)(\phi \rightarrow \psi)$

Решение:

$$\begin{array}{l}
 \text{1. (a)} \quad \frac{\frac{\frac{\phi \vdash \phi}{\phi \vdash (\exists x)\phi} \text{ } (\exists \text{ введ. справа})}{\phi \vdash (\exists y)(\exists x)\phi} \text{ } (\exists \text{ введ. справа})}{(\exists y)\phi \vdash (\exists y)(\exists x)\phi} \text{ } (\exists \text{ введ. слева}) \\
 \frac{(\exists y)\phi \vdash (\exists y)(\exists x)\phi}{(\exists x)(\exists y)\phi \vdash (\exists y)(\exists x)\phi} \text{ } (\exists \text{ введ. слева}) \\
 \text{(b)} \quad \frac{\text{Аналогично.}}{(\exists y)(\exists x)\phi \vdash (\exists x)(\exists y)\phi}
 \end{array}$$

$$\begin{array}{l}
 \text{2. (a)} \quad \frac{\frac{(\phi)_{yx}^{xy} \vdash \phi}{(\forall y)(\phi)_y^x \vdash \phi} \text{ } (\forall \text{ введ. слева})}{(\forall y)(\phi)_y^x \vdash (\forall x)\phi} \text{ } (\forall \text{ введ. справа}) \\
 \text{(b)} \quad \frac{\frac{(\phi)_y^x \vdash (\phi)_y^x}{(\forall y)\phi \vdash (\phi)_y^x} \text{ } (\forall \text{ введ. слева})}{(\forall x)\phi \vdash (\forall y)(\phi)_y^x} \text{ } (\forall \text{ введ. справа})
 \end{array}$$

3.

$$\begin{array}{c}
 \frac{\frac{\frac{\phi \vdash \phi}{\neg\phi, \phi, \neg\psi \vdash \phi} \text{ } (\text{утнч}) \quad \frac{\frac{\neg\phi \vdash \neg\phi}{\neg\phi, \phi, \neg\psi \vdash \neg\phi} \text{ } (\text{утнч})}{\neg\phi, \phi \vdash \psi} \text{ } (\rightarrow \text{ введ.})}{\frac{\neg\phi, \phi \vdash \psi}{(\forall x)\neg\phi \vdash \phi \rightarrow \psi} \text{ } (\forall \text{ введ. справа})}{\frac{(\forall x)\neg\phi \vdash (\forall x)(\phi \rightarrow \psi)}{(\exists x)\neg\phi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ } (\text{уточн.})} \text{ } (\text{уточн.}) \\
 \frac{\frac{\frac{\vdash (\forall x)\phi \vee (\forall x)\neg\phi}{(\exists x)\phi \rightarrow (\forall x)\psi \vdash (\forall x)\phi \vee (\forall x)\neg\phi} \text{ } (\text{уточн.}) \quad \frac{\frac{\text{Неверно}}{(\forall x)\phi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ } ()}{(\exists x)\phi \rightarrow (\forall x)\psi, (\forall x)\phi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ } (\text{уточн.})}{\frac{(\exists x)\phi \rightarrow (\forall x)\psi, (\forall x)\neg\phi \vdash (\forall x)(\phi \rightarrow \psi)}{(\exists x)\phi \rightarrow (\forall x)\psi \vdash (\forall x)(\phi \rightarrow \psi)} \text{ } (\text{допустимое правило})}
 \end{array}$$