

Задание:

1. $(\exists x)(\phi \wedge \psi) \vdash (\exists x)\phi \wedge \psi$
2. $\vdash t \approx t$

Решение:

1. В следствии правила свободной конкретизации: достаточно доказать для переменных.
 $\vdash x \approx x$
Аксиома.

$$\begin{array}{c} 2. \\ \text{(удал. } \wedge) \frac{\phi \wedge \psi \vdash \phi \wedge \psi}{\phi \wedge \psi \vdash \phi} \quad \frac{\phi \wedge \psi \vdash \phi \wedge \psi}{\phi \wedge \psi \vdash \psi} \text{(удал. } \wedge) \\ \text{(введ. } \exists \text{ справа)} \frac{\phi \wedge \psi \vdash \phi}{\phi \wedge \psi \vdash (\exists x)\phi} \quad \frac{\phi \wedge \psi \vdash \psi}{\phi \wedge \psi \vdash (\exists x)\psi} \text{(введ. } \exists \text{ справа)} \\ \frac{\phi \wedge \psi \vdash (\exists x)\phi \quad \phi \wedge \psi \vdash (\exists x)\psi}{\phi \wedge \psi \vdash (\exists x)\phi \wedge (\exists x)\psi} \text{(введ. } \wedge) \\ \frac{\phi \wedge \psi \vdash (\exists x)\phi \wedge (\exists x)\psi}{(\exists x)(\phi \wedge \psi) \vdash (\exists x)\phi \wedge (\exists x)\psi} \text{(} \exists \text{ введ. слева)} \end{array}$$