Generalized Pandemic Equation for Monitoring COVID-19

Michael Shur
Rensselaer Polytechnic Insitute
Troy, NY, USA
shurm@rpi.edu

***Abstract*— The generalized Pandemic Equation describes multiple waves of the COVID-19 pandemic, models the space dependence of the infection rate, and compares different pandemic evolution scenarios by extrapolating the pandemic evolution curves for the periods of time on the order of the Pandemic Equation instantaneous characteristic time constant. The parameter extraction for multiple locations and time periods could be used for uncertainty quantification of the Pandemic Equation predictions.**

Keywords—Pandemic Equation, COVID-19 pandemic, pandemic evolution, uncertainty quantification

I. Introduction

The number of COVID-19 deaths in the United States has exceeded one million and was near the top of per capita deaths compared to other countries (see Fig. 1).

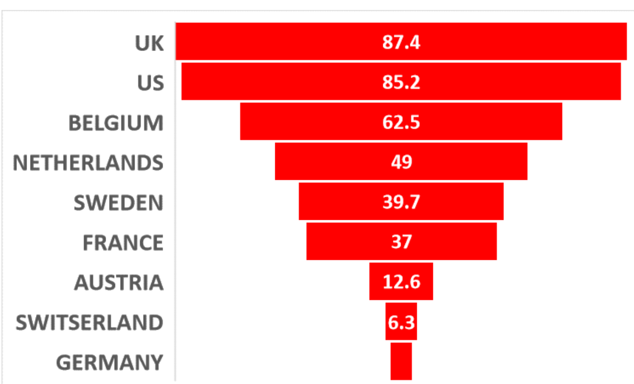

Fig. 1. Excess deaths from COVID-19 per 100,000 in 2020 (data from [1])

Even though the COVID-related death rate in the USA (exceeding 3,000 deaths a day during the 2020 peak) subsided to ~300 deaths a day from June to October 2022, the emerging spike in Europe is a precursor of the widely expected new pandemic spike in December-January of 2022. This new peak might be fueled by the emergence of new, more dangerous strains, better evading the antibodies that might be now present in the majority of the US population.

The COVID-19 infection rate, COVID-19-related hospitalizations, and COVID-19-related deaths all come in waves that rise, crest, dip, and depend on location (see Fig. 2).

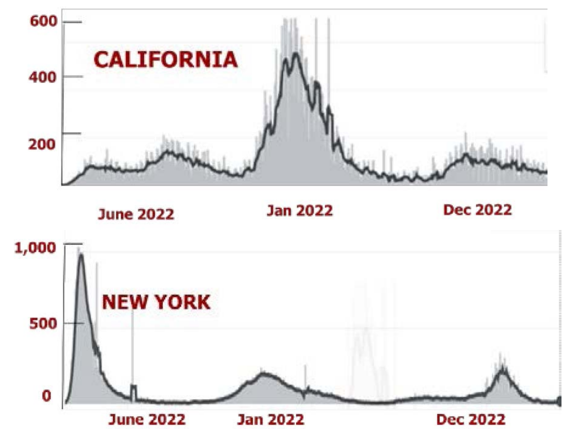

Fig. 2. Waves of the New York and California daily death rates shifted in time (data from [2]).

When the pandemic infections dip the masks come off, and businesses open up. But COVID-19 is not yet over, and there is a need for a transparent and easily understood pandemic monitoring tool.

The pandemic must be monitored and predicted both globally and locally at the state, county, and even at the school district, university campus, or company level. The generalized Pandemic Equation [3, 4] provides such a tool using an approach based on approximations similar to the Born-Oppenheimer approximation used in the quantum solid-state theory [5,6] and using the generalized Fermi-Dirac distribution functions to describe the COVID-19 mitigation or anti mitigation events. In comparison to many sophisticated and greatly varied pandemic prediction models [7–10], the Pandemic Equation is simple, easy to understand, and to use, and, therefore, well suited for both global and local applications.

II. Rate Equation and Pandemic Equation

The Pandemic Equation is based on the rate equation:

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$$\frac{dN}{dt} = \frac{(1 - N/N_t)N}{\tau_o} \quad (1)$$

Here N is the total number of infections to date, t is time, N_t is the total number of people in the relevant infection pool, and τ_o is the characteristic time constant of the initial pandemic growth. For the description of a single pandemic wave, the Pandemic Equation replaces τ_o in the solution of Eq. (1) by a time-dependent time constant of the pandemic $\tau(t)$. The linear dependence was sufficient for modeling the COVID-19 pandemic waves:

$$\tau = \tau_o + \alpha t \quad (2)$$

Here the characteristic time constant τ_o and parameter α are independent of time t.

Then the solution of the rate equation becomes

$$\Delta N = \frac{dN_{to}}{dt} = \frac{A_o e^{t/\tau}}{\left(1 + e^{t/\tau} f_o\right)^2 \tau} \quad (3)$$

Here $f_o = A_o / N_t$ is the ratio of the total initial infection number A_o to the relevant infection pool N_t. For α =0, the pandemic evolution curve becomes symmetrical, with the maximum of the symmetric curve reached at

$$t_{mo} = \tau_o \ln\left(1/f_o\right) \quad (4)$$

The parameter α is the curve flattening parameter as shown in Fig. 3 displaying the number of daily infections as a function of time and parameter α plotted using Eq. (3)

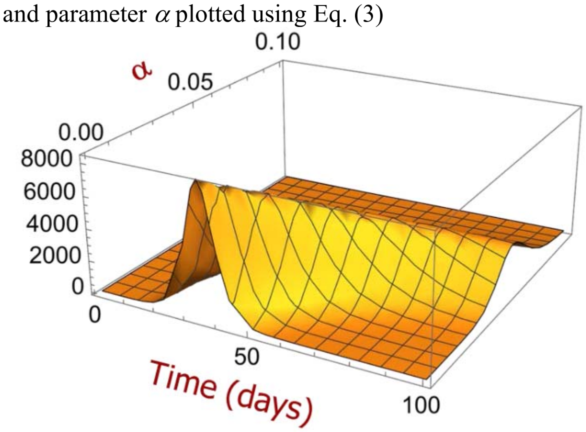

Figure 3. Effect of curve flattening parameter α.

The maxima of the daily infections are reached at approximately

$$t_m \approx \frac{\tau_o \ln(1/f_o)}{1 - \alpha \ln(1/f_o)} \quad (5)$$

(This expression is obtained assuming that $\alpha << \tau_o / t_m$.) As mentioned above, this approach is similar to the adiabatic (Born-Oppenheimer) approximation in the quantum theory of solids allowing two-time scales for slow (atomic) and rapid (electronic) motion. [5,6]

The generalized Pandemic equation accounts for different mitigation and anti-mitigation measures, such as mask mandates, social distancing, or closing or opening the economy. It also describes any number of the pandemic waves:

$$\Delta N_{wi} = \Delta N_{wio} \prod_{k=1}^{n_k} \left(1 - \beta_{wki} F_{FDG}\right) \quad (6)$$

Here

$$\Delta N_{wio} = \frac{A_{wi} \exp\left[\left(t - t_{wi}\right)/\tau_{wi}\right]}{\tau_{wi}\left(1 + f_{wi} \exp\left[\left(t - t_{wi}\right)/\tau_{wi}\right]\right)^2} \quad (7)$$

Each wave (numbered w) is described by the Pandemic Equation having the same functional dependence for events numbered i and corresponding to the daily number of hospital admissions (i=1), to the COVID-related daily number of occupied ICU beds (i=2), and to the COVID-related daily number of deaths (i=3). Eq. (6) accounts for n_k mitigation $\left(\beta_{wki} > 0\right)$ or anti-mitigation $\left(\beta_{wki} < 0\right)$ events. Here A_{wi} is the constant describing the initial infection number for wave w and event i, $f_{wi} = A_{oi} / N_{ti}$ is the ratio of the total initial infection number A_{oi} to the relevant infection pool N_{ti}, t_{wi} is the onset time of wave w, and event i, τ_{wi} is the time-dependent growth time constant of the event i and wave w (the current version of the Pandemic Equation uses a linear dependence:

$$\tau_{wi} = \tau_{wio} + \alpha_{wi} t \quad (8)$$

Here τ_{wio} the initial growth time constant of the event i and wave w. Parameters α_{wi} control the asymmetry of the pandemic evolution curve shape and are the flattening parameters responsible for "flattening" the pandemic evolution curves.

Mitigation parameters β_{wki} account for the mitigation or anti-mitigation measures, such as requirements of wearing masks, opening, or closing the economy, vaccinations, and booster shots. Each wave might have several such mitigation events described by the generalized Fermi-Dirac distribution function (see Fig. 4).

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$$F_{FDG} = \frac{1}{1+\exp\left[\left(t_{\beta wki} - t\right)/\tau_{\beta wki}\right]} \quad (9)$$

Here $t_{\beta wki}$ is the mitigation onset time, $\tau_{\beta wki}(t) = \tau_{\beta wkio} + \alpha_{\beta wki} t$ is the time-dependent slowly varying time constant of a mitigation event. Parameters $\alpha_{\beta wki}$ account for the curve flattening of the mitigation or anti-mitigation response. If $\alpha_{\beta wki} = 0$, F_{FDG} reduces to the Fermi-Dirac distribution function.

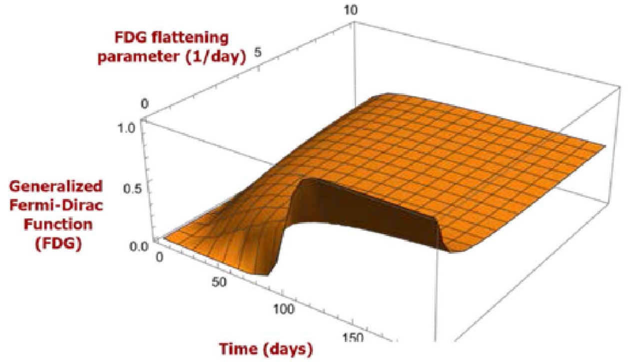

Fig. 4. Generalized Fermi-Dirac distribution function

As seen in Figure 5, the generalized Pandemic Equation describes all five waves of the COVID-19 pandemic. It could also model the space dependence of the infection rate and compare different scenarios of the pandemic evolution extrapolating the pandemic evolution curves for the periods on the order of the Pandemic Equation time constant. The parameter extraction for multiple locations and time periods could be used for uncertainty quantification for such pandemic evolution predictions.

The generalized Pandemic Equation could also describe the space dependence of the pandemic evolution (see Fig.6):

$$\Delta N_l = \sum_{w=1}^{n_w} \Delta N_{wl} \exp\left(-\frac{x - x_{\theta l}}{2\sigma_{xwl}^2} - \frac{y - y_{\theta l}}{2\sigma_{ywl}^2} \right) \quad (10)$$

Here ΔN_i is the COVID-related daily number of infections for wave $x_{\theta l}, y_{\theta l}$ are the coordinates of the maximum daily rate using the x-y coordinate system related by the angle θ with respect to the north-south direction σ_{xwl} and σ_{ywl} are the standard deviations.

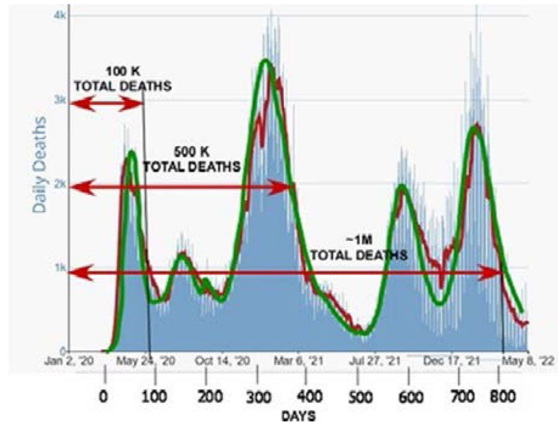

Fig. 5. Daily death rate in the USA fitted using the Pandemic Equation. [11]

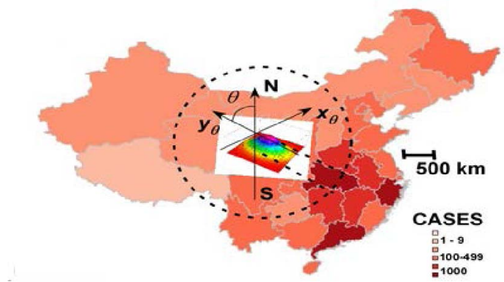

Fig. 6. Space-dependent solution of the Pandemic equation superimposed on the map of COVID-19 cases in China [11] (the map is from [12]).

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