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Adaptive Fuzzy Control for Synchronization of Coronary Artery System With Input Nonlinearity

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ABSTRACT In this paper, we propose a parametric adaptive control strategy for synchronization of Takagi–Sugeno (T-S) fuzzy coronary artery system. We use the T-S fuzzy model to represent the coronary artery system, because the coronary artery system has complicated nonlinear characteristic in reality. Based on the new model, a fuzzy parametric adaptive output feedback controller is designed to achieve the H_{∞} synchronization of coronary artery system with input nonlinearity and parameter perturbations. Some simulation results are given to illustrate the effectiveness of our control strategy.

INDEX TERMS Coronary artery system, adaptive control, fuzzy model, input nonlinear, H_{∞} synchronization.

I. INTRODUCTION

Chaos synchronization has been paid considerable attention among the scientists from biological engineering field such as epidemic diseases, nervous system and coronary artery system(CAS) [1], [2]. From the perspective of biology, CAS maintains our life by delivering oxygen and nutrition to myocardium. Once blood vessel of the coronary artery obstructed by thrombus, patients will suffering from a dangerous disease named myocardial infarction(MI). Therefore a lot of efforts have been done by the researchers among various areas. It's worth noting that Xu and Liu given the dynamics model of CAS in [3] which described CAS as a chaotic system:

$$\begin{aligned}\dot{x}_1(t) &= -bx_1(t) - cx_2(t), \\ \dot{x}_2(t) &= -(b+1)\omega x_1(t) - (c+1)\omega x_2(t) + \omega x_1^3(t) \\ &\quad + Ecos\sigma t\end{aligned}\quad (1)$$

where $x_1(t)$, $x_2(t)$ are the inner diameter and pressure changes of the coronary artery vessel, respectively. $Ecos\sigma t$ is used to describe the periodic perturbation.

Many existing works are based on the aforementioned model. In these researches, the treatment of MI are regarded as designing an appropriate control strategy to make the convulsionary vessel synchronize with a health one. In [4] and [5], backstepping approach and nonlinear state feedback

method are used to the CAS synchronization. Reference [6] utilizes sliding mode control method to achieve synchronization of CAS under the bounded uncertainties, takes full account of the presence of disturbances in the actual coronary artery system. Furthermore, the CAS synchronization in finite-time is achieved using high-order sliding mode adaptive control method in [7], makes the convulsionary vessel synchronize with a health one in finite-time,to ensure the control effect in the actual coronary artery system timeliness. Considering the time delay caused by medication time and drug absorption, a chaotic synchronization feedback controller with input time-varying delay is design to guarantee the control performance of CAS in [8]. The above articles are effective in considering the actual problems of CAS.

However, the CAS has complicated nonlinear characteristics. The nonlinear term in (1) will loss some information of the system. In the past two decades, T-S fuzzy model exhibited significant functions in approximating and describing complex nonlinear systems [9]–[14]. In this paper, we give a fuzzy CAS model which can retain much more information of nonlinear characteristics. Therefore, the study on CAS base on T-S fuzzy model compared to the previous research results is closer to the actual CAS.

Nonlinear effect widely exist in the natural phenomenon. The absorption and diffusion of drugs is also a nonlinear effect. Therefore, the medical efficacy is regarded as a

nonlinear inputs in our paper. Compared to [6], our study is closer to the actual CAS. Furthermore, the parameters uncertainties are considered in drive-response systems so that the research has stronger robustness. Recently, adaptive fuzzy feedback control approach [15]–[18] is proven to be effective in nonlinear system control. Previous studies on CAS relied on deterministic mathematical models. However, the existing model is the approximation of CAS, there is a certain error. For this reason, we design a fuzzy adaptive controller, so that in the case of nonlinear input signal, the coefficient matrix exists for the modeling uncertainty, the response system and the drive system to achieve synchronization. In recent years, the researchers have proposed some new control strategies based on adaptive control and fuzzy control for different nonlinear systems, such as adaptive fuzzy control [19]–[21], observer-based fuzzy adaptive output-feedback control [22], adaptive tracking Control [23]. Sliding mode control [24] and adaptive control are the general control theory of chaotic synchronization, there are some methods to combine sliding mode control, such as adaptive sliding mode control [25], optimal guaranteed cost sliding mode control [26], adaptive fuzzy hierarchical sliding mode control [27]. However, as we known, few researchers design control law based on fuzzy system which can better approach the real CAS with input nonlinearity and parameter perturbations.

Motivated by above discussions, we investigate the adaptive synchronization of CAS base on the T-S fuzzy model. An effective adaptive control strategy is proposed to the H_∞ synchronization of fuzzy CAS with the input nonlinear and parameter perturbations. The effectiveness of this strategy can be illustrated by the simulation in the following section.

A. CORONARY ARTERY FUZZY MODEL

In this paper, we use uncertain T-S fuzzy model to describe CAS as follows:

Plant rule k: IF $\phi_1(t)$ is M_{k1} , $\phi_2(t)$ is $M_{k2}, \dots, \phi_r(t)$ is M_{kr} , THEN

$$\begin{aligned}\dot{x}_m(t) &= (A_k + \Delta A_k)x_m(t) + (B_k + \Delta B_k)p(x_m(t), t) \\ &\quad + q(t) \quad (k = 1, \dots, v) \\ y_m(t) &= Cx_m(t)\end{aligned}\quad (2)$$

where $\phi_j(t) (j = 1 \dots r)$ is the premise variable. $M_{ij} (i = 1 \dots k, j = 1 \dots r)$ is the fuzzy set. r represents the number of the fuzzy rule, $x_m(t), y_m(t) \in \mathbb{R}^n$ are the state vector and output vector, respectively. $p(x_m(t), t)$ is the nonlinear term. $q(t)$ denotes a perturbation with certain period. $A_k, B_k, C \in \mathbb{R}^{n \times n}$ are constant real matrices. $\Delta A_k, \Delta B_k \in \mathbb{R}^{n \times n}$ represent the uncertainties of system which can be described as:

$$[\Delta A_k, \Delta B_k] = HF(t)[E_{ak}, E_{bk}] \quad (3)$$

where $H, E_{ak}, E_{bk} \in \mathbb{R}^n$ are known constant matrices and $F(t)$ is an unknown matrix function satisfying: $F^T(t)F(t) \leq I$.

Using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the dynamic fuzzy model in (2) can be represented by:

$$\begin{aligned}\dot{x}_m(t) &= \sum_{k=1}^v h_k(\phi(t))\{(A_k + \Delta A_k)x_m(t) + (B_k + \Delta B_k) \\ &\quad * p(x_m(t), t) + q(t)\} \\ y_m(t) &= Cx_m(t)\end{aligned}\quad (4)$$

where $h_k(\phi(t)) = \frac{\prod_{j=1}^r M_{kj}(\phi_j(t))}{\sum_{k=1}^v \prod_{j=1}^r M_{kj}(\phi_j(t))} (k = 1, \dots, v)$ is the normalized grade of membership and it satisfies: $\sum_{k=1}^v h_k(\phi(t)) = 1, h_k(\phi(t)) \geq 0$.

The fuzzy response system is given as follows:

Plant rule k: IF $\phi_1(t)$ is M_{k1} , $\phi_2(t)$ is $M_{k2}, \dots, \phi_r(t)$ is M_{kr} , THEN

$$\begin{aligned}\dot{x}_s(t) &= (A_k + \Delta \tilde{A}_k(t))x_s(t) + (B_k + \Delta \tilde{B}_k(t))p(x_s(t), t) \\ &\quad + q(t) + d(t) + E\Omega(u(t)) \quad (k = 1, \dots, v) \\ y_s(t) &= Cx_s(t)\end{aligned}\quad (5)$$

where $x_s(t), y_s(t) \in \mathbb{R}^n$ is the state vector and the output vector, respectively. $p(x_s(t), t)$ is the nonlinear term. $d(t)$ represents external disturbance. $E \in \mathbb{R}^{n \times n}$ is constant real matrix. $\Delta \tilde{A}_k(t), \Delta \tilde{B}_k(t) \in \mathbb{R}^{n \times n}$ denote the adaptive estimated value of $\Delta A_k, \Delta B_k$. Similar to (4), we infer the fuzzy response system (5) as:

$$\begin{aligned}\dot{x}_s(t) &= \sum_{k=1}^v h_k(\phi(t))\{(A_k + \Delta \tilde{A}_k(t))x_s(t) + (B_k + \Delta \tilde{B}_k(t)) \\ &\quad p(x_s(t), t) + q(t) + d(t)\} + E\Omega(u(t)) \\ y_s(t) &= Cx_s(t)\end{aligned}\quad (6)$$

Defining $e(t) = x_s(t) - x_m(t)$, the error system can be written as:

$$\begin{aligned}\dot{e}(t) &= \sum_{k=1}^v h_k(\phi(t))\{\Delta U_k x_s(t) + \Delta N_k p(x_s(t), t)\} \\ &\quad + \sum_{k=1}^v h_k(\phi(t))\{(A_k + \Delta A_k)e(t) + (B_k + \Delta B_k) \\ &\quad * p_e(t) + d(t)\} + E\Omega(u(t))\end{aligned}\quad (7)$$

where

$$\Delta U_k = \Delta \tilde{A}_k(t) - \Delta A_k = (a_{kij})_{n \times n} \quad (8)$$

$$\Delta N_k = \Delta \tilde{B}_k(t) - \Delta B_k = (b_{kij})_{n \times n} \quad (9)$$

$$p_e(t) = p(x_s(t), t) - p(x_m(t), t) \quad (10)$$

The system (4) and (6) will be asymptotically synchronized if the synchronization error $e(t)$ satisfies $\lim_{t \rightarrow 0} e(t) = 0$. In this paper, we design an adaptive output feedback controller as follows:

$$u(t) = -\frac{\gamma(t)}{2}Ce(t) \quad (11)$$

where $u(t) = [u_1(t) \dots u_n(t)]^T \in \mathbb{R}^n$ is the control input vector, $\Omega(u(t)) = \sum_{k=1}^v h_k(\phi(t))[\omega_1(u_1(t)) \dots \omega_n(u_n(t))]^T$

represents the nonlinear control input vector which satisfies the following inequality:

$$u_i(t)\omega_i(u_i(t)) \geq v_i(u_i(t))^2 \quad (1 \leq i \leq m) \quad (12)$$

$$v^* = \min v_i \quad (13)$$

ω_i is function, $\gamma(t)$ is an adaptive parameter and adjusted by the following adaptive law:

$$\dot{\gamma}(t) = v^* \delta \|Ce(t)\|^2, \gamma(0) > 0. \quad (14)$$

where δ and v are positive parameters. By applying of the above adaptive controller, synchronization error $e(t)$ will converge to zero asymptotically. To obtain the synchronization conditions, the following lemma and assumptions will be used during the proof.

Lemma 1 [28]: For a symmetric matrix Z and appropriately dimensional matrices D , G and $F(t)$ satisfying $F^T(t)F(t) < I$. Inequality $Z + He\{DF(t)G\} < 0$ is true, if and only if the following inequality $Z + \varepsilon DD^T + \varepsilon^{-1}GG^T < 0$ holds for any $\varepsilon > 0$.

Assumption 1: The nonlinear function $p(x(t); t)$ satisfies the Lipschitz condition:

$$|p(x_m(t), t) - p(x_s(t), t)| \leq |L(x_m(t) - x_s(t))| \quad (15)$$

where L is the Lipschitz constant matrix.

Assumption 2: Matrix $P > 0$ and satisfies the following equation:

$$E^T P = C \quad (16)$$

Remark 1: Assumption 1 is to deal with the nonlinear characteristics of chaotic systems. It is generally known that Assumption 2 is a matching condition in output feedback control of nonlinear systems, which is referenced in many papers [29], [30].

II. H_∞ SYNCHRONIZATION OF CAS

In this part, a parametric adaptive control strategy for synchronization of fuzzy CAS is proposed by utilizing above lemma and assumptions.

Theorem 1: Considering the fuzzy coronary artery drive and response systems (4) and (6), by applying the output feedback adaptive controller (11) with adaptive law (14), if existing symmetric positive definite matrix P and scalars α, ε_1 and $\varepsilon_2 > 0$ for given $\sigma > 0$, satisfying the following LMI:

$$\begin{bmatrix} \Gamma_1 & PB_k & -P & PH & PH & L^T \\ * & \Gamma_2 & 0 & 0 & 0 & 0 \\ * & * & -\sigma^2 I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1^{-1} I & 0 & 0 \\ * & * & * & * & -\varepsilon_2^{-1} I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (17)$$

where

$$\begin{aligned} \Gamma_1 &= PA_k + A_k^T P + I - \alpha PEE^T P + \varepsilon_1^{-1} E_{ak}^T E_{ak} \\ \Gamma_2 &= \varepsilon_2^{-1} E_{ak}^T E_{ak} - I \end{aligned} \quad (18)$$

the attention rate σ for H_∞ synchronization in the disturbance situation can be achieved.

Proof: In this segment, the Lyapunov-Krasovskii functional can be constructed as follows:

$$V(t) = V_1(t) + V_2(t) \quad (19)$$

where

$$\begin{aligned} V_1(t) &= e^T(t)Pe(t) + \int_0^t \sum_{k=1}^v h_k(\phi(s))\delta^{-1}\dot{\gamma}(s) \\ &\quad *(\gamma(s) - \gamma^*)ds \end{aligned} \quad (20)$$

$$V_2(t) = \sum_{k=1}^v \sum_{i=1}^n \sum_{j=1}^n [\frac{1}{2\theta_k} a_{ij}^2(t) + \frac{1}{2\varphi_k} c_{ij}^2(t)] \quad (21)$$

γ^* is a positive constants which will be defined later.

Taking the time derivative of $V(t)$:

$$\begin{aligned} \dot{V}_1(t) &= 2e^T(t)Pe(t) + \sum_{k=1}^v h_k(\phi(t))\delta^{-1}\dot{\gamma}(t)(\gamma(t) - \gamma^*) \\ &= \sum_{k=1}^v h_k(\phi(t)) \left\{ e^T(t) \left[PA_k + A_k^T P - \alpha PEE^T P \right] \right. \\ &\quad \left. * e(t) + e^T(t)PHF(k)E_{ak}e(t) + e^T(t)E_{ak}^T F^T(k) \right. \\ &\quad \left. * H^T Pe(t) + e^T(t)B_k p_e(t) + p_e^T(t)B_k^T e(t) + e^T(t) \right. \\ &\quad \left. * PHF(k)E_{bk} p_e(t) + p_e^T(t)E_{bk}^T F^T(k)H^T Pe(t) \right. \\ &\quad \left. + \alpha \|E^T Pe(t)\|^2 + 2e^T(t)PE\Omega(u(t)) + \delta^{-1}\dot{\gamma}(\gamma(t) - \gamma^*) \right. \\ &\quad \left. - e^T(t)Pd(t) - d^T(t)Pe(t) \right\} + 2e^T(t)P \\ &\quad * \sum_{k=1}^v h_k(\phi(t)) \{ \Delta U_k x_s(t) + \Delta N_k p(x_s(t), t) \} \end{aligned} \quad (22)$$

Utilizing (14), we can prove $\gamma(t)$ always remains positive. Assuming $Ce(t) = \Upsilon(t)_{m \times 1}$ and $\Upsilon_n(t)$ represents the $n - th$ element of $\Upsilon(t)$. Considering the following statements:

(1) If $\Upsilon_n(t) > 0$, it is easy to get $u_n(t) < 0$. Therefore, $\Upsilon_n(t)\omega_n(u_n(t)) \leq v_n \Upsilon_n(t)u_n(t)$ can be obtained by multiplying $\Upsilon_n(t)$ and dividing $u_n(t)$ by both sides of (11).

(2) If $\Upsilon_n(t) < 0$, it is easy to get $u_n(t) > 0$. Therefore, $\Upsilon_n(t)\omega_n(u_n(t)) \leq v_n \Upsilon_n(t)u_n(t)$ can be obtained by multiplying $\Upsilon_n(t)$, and dividing $u_n(t)$ by both side of (11).

We can find the following inequality will always holds:

$$\Upsilon_n(t)\omega_n(u_n(t)) \leq v_n \Upsilon_n(t)u_n(t). \quad (23)$$

Using Assumption 2 , (11) and (23) we obtain:

$$\begin{aligned} 2e^T(t)PE\Omega(v(t)) &= 2 \sum_{n=1}^m \Upsilon_n(t)\omega_n(u_n(t)) \\ &\leq 2 \sum_{n=1}^m v_n \Upsilon_n(t)u_n(t) \\ &\leq -v^* \gamma(t) \| Ce(t) \|^2 \end{aligned} \quad (24)$$

Let $\alpha = \nu^* \gamma^*$, incorporating Assumption 1, (15) and (22), we have:

$$\begin{aligned} V_1(t) = & \sum_{k=1}^v h_k(\phi(t)) \left\{ e^T(t) \left[PA_k + A_k^T P + L^T L - \alpha \right. \right. \\ & * PEE^T P + (\varepsilon_1 + \varepsilon_2) PHH^T P + \varepsilon_1^{-1} E_{ak}^T E_{ak} \Big] e(t) \\ & + e^T(t) PB_k p_e(t) + p_e^T(t) B_k^T Pe(t) - e^T(t) P d(t) \\ & \left. \left. - d^T(t) Pe(t) + p_e^T(t) (\varepsilon_2^{-1} E_{ak}^T E_{ak} - I) p_e(t) \right\} \right. \\ & + 2e^T(t) P \sum_{k=1}^v h_k(\phi(t)) \{ \Delta U_k x_s(t) + \Delta N_k p(x_s(t), t) \} \end{aligned} \quad (25)$$

Parametric adaptive laws are selected as follows:

$$\begin{aligned} \dot{a}_{1ij}(t) &= -h_1(\phi(t)) \theta_1 l_i x_{sj}(t) \\ &\vdots \\ \dot{a}_{vij}(t) &= -h_v(\phi(t)) \theta_v l_i x_{sj}(t) \\ \dot{b}_{1ij}(t) &= -h_1(\phi(t)) \varphi_1 l_i p_j(x_{sj}(t), t) \\ &\vdots \\ \dot{b}_{vij}(t) &= -h_v(\phi(t)) \varphi_v l_i p_j(x_{sj}(t), t) \end{aligned} \quad (26)$$

where l_i represents the i -th element of $2e^T(t)P$. θ_v and φ_v are known constants. The time derivative of $V_2(t)$ can be written as follows:

$$\begin{aligned} \dot{V}_2(t) &= \sum_{k=1}^v \sum_{i=1}^n \sum_{j=1}^n \left[\frac{1}{\theta_k} a_{kij}(t) \dot{a}_{kij}(t) + \frac{1}{\varphi_k} b_{kij}(t) \dot{b}_{kij}(t) \right] \\ &= - \sum_{k=1}^v \sum_{i=1}^n \sum_{j=1}^n h_k(\phi(t)) [a_{kij}(t) l_i x_{sj}(t) + b_{kij}(t) \\ &\quad * l_i p_j(x_{sj}(t), t)] \\ &= -2e^T(t) P \sum_{k=1}^v h_k(\phi(t)) (\Delta U_k x_s(t) + \Delta N_k p(x_s(t), t)) \end{aligned} \quad (27)$$

Combining equations (25) and (27), we have:

$$\begin{aligned} \dot{V}(t) = & \sum_{k=1}^v h_k(\phi(t)) \left\{ e^T(t) \left[PA_k + A_k^T P + L^T L - \alpha \right. \right. \\ & * PEE^T P + (\varepsilon_1 + \varepsilon_2) PHH^T P + \varepsilon_1^{-1} E_{ak}^T E_{ak} \Big] e(t) \\ & + e^T(t) PB_k p_e(t) + p_e^T(t) B_k^T Pe(t) - e^T(t) P d(t) \\ & \left. \left. - d^T(t) Pe(t) + p_e^T(t) (\varepsilon_2^{-1} E_{ak}^T E_{ak} - I) p_e(t) \right\} \right. \end{aligned} \quad (28)$$

To investigate H_∞ performance, we define J as follows:

$$J = \int_o^\infty [e^T(t)e(t) - \sigma^2 d^T(t)d(t)] dt \quad (29)$$

Using zero initial condition, we have:

$$\begin{aligned} J \leq & \int_o^\infty [\dot{V}(t) + e^T(t)e(t) - \sigma^2 d^T(t)d(t)] dt \\ = & \zeta^T(t)\Omega\zeta(t) \end{aligned} \quad (30)$$

where $\zeta^T(t) = [e^T(t) \quad p_e^T(t) \quad d^T(t)]$,

$$\Omega = \begin{bmatrix} \Gamma_1 & PB_k & -P \\ * & \varepsilon_2^{-1} E_{ak}^T E_{ak} - I & 0 \\ * & * & -\sigma^2 I \end{bmatrix}, \quad (31)$$

$$\begin{aligned} \Gamma_1 = & PA_k + A_k^T P + I + L^T L - \alpha PEE^T P \\ & + (\varepsilon_1 + \varepsilon_2) PHH^T P + \varepsilon_1^{-1} E_{ak}^T E_{ak}. \end{aligned} \quad (32)$$

when $\Omega < 0$, we can see that $J < 0$ and $\|e(t)\|_2 < \|d(t)\|_2$. Using Schur complement lemma, (31) can be transformed into (17), which completed the proof of Theorem 1.

Remark 2: Compared to the existing researches on chaotic synchronization of coronary artery system, our strategy fully considers input nonlinearity and parameter perturbations.

III. SIMULATION

The following numerical example is given to illustrate the effectiveness of our control strategy. Consider the T-S fuzzy coronary artery systems (4) and (6) with the following parameters:

$$\begin{aligned} A_1 = A_2 &= \begin{bmatrix} -0.15 & 1.7 \\ 0.575 & -0.35 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix}, \\ C &= \begin{bmatrix} 1.5 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad E = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \\ E_{a1} &= \begin{bmatrix} -0.025 & 0.05 \\ 0.025 & -0.05 \end{bmatrix}, \quad E_{a2} = \begin{bmatrix} 0.025 & -0.05 \\ -0.025 & 0.05 \end{bmatrix}, \\ E_{d1} &= \begin{bmatrix} 0 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad E_{d2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \\ F(t) &= \begin{bmatrix} 0.5\cos(t) & 0 \\ 0 & 0.5\sin(t) \end{bmatrix}, \quad d(t) = \begin{bmatrix} 0.05\cos(3t) \\ 0.1\cos(6t) \end{bmatrix}, \\ \Omega(u(t)) &= \begin{bmatrix} 1 + 0.1\sin(u_1(t))u_1(t) \\ 1 + 0.2\cos(u_2(t))u_2(t) \end{bmatrix}, \quad \sigma = 0.3. \end{aligned}$$

For the sake of analysis, we assume $\Delta N_k = 0$, $\theta_1 = 35$, $\theta_2 = 3$, The membership functions of the drive and response systems are selected as: $h_1(\phi(t)) = \frac{x_{m1}^2}{h^2}$, $h_2(\phi(t)) = 1 - \frac{x_{m1}^2}{h^2}$ and $h_1(\phi(t)) = \frac{x_{s1}^2}{h^2}$, $h_2(\phi(t)) = 1 - \frac{x_{s1}^2}{h^2}$ ($h^2 = 5$), respectively. According to Theorem 1 and Assumption 2, we can get $P = \text{diag}\{0.5 \quad \frac{0.8}{3}\}$, $\alpha = 15.5336$, $\varepsilon_1 = 17.9871$, $\varepsilon_2 = 17.0300$. Initial value of system (4) and (6) are selected as: $(x_{m1}(0), x_{m2}(0)) = (1.5, -0.2)$ and $(x_{s1}(0), x_{s2}(0)) = (0.5, 0.8)$, respectively. The following figures can explain the effectiveness of our control strategy. Phase portrait of the CAS (4) under aforementioned parameters are shown in Figure 1(a), we can observe that the trajectory of CAS exhibits a significant chaotic behavior. The errors between system (4) and (6) with different initial value and without any control are shown in Figure 1(b). It is obvious that the drive-response systems are nonsynchronous. Time response of estimation errors are displayed in Figure 1(c). We can see that the parameters of system (4) can't be estimated accurately. From Figure 2(c), we find that error systems can converge to zero by applying the proposed adaptive feedback controller which can be exhibited in Figure 2(a) and 2(b). Time response of

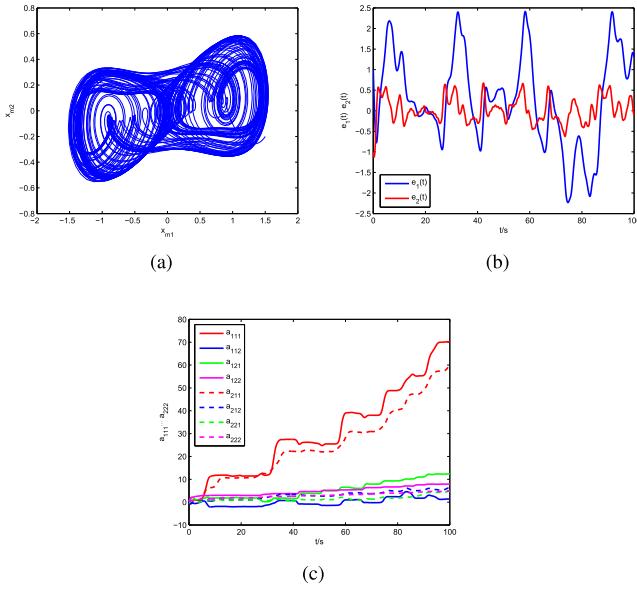


FIGURE 1. Behavior of the CAS (4) and (6) without control. (a) phase portrait of drive system. (b) synchronization errors. (c) time response of parameters estimation errors.

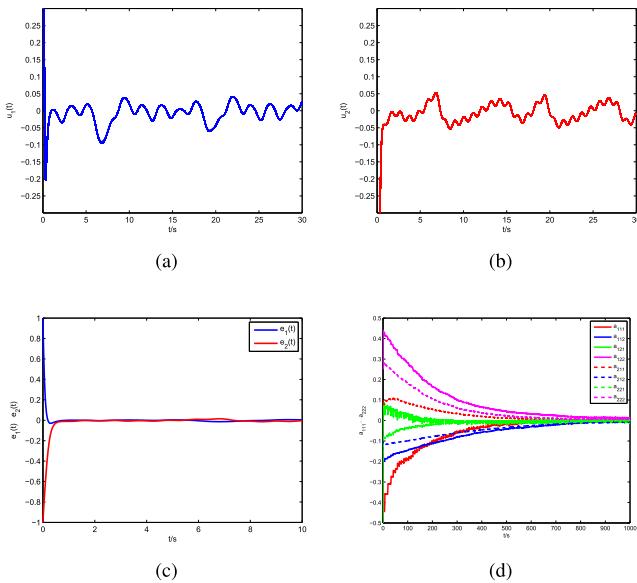


FIGURE 2. Behavior of the CAS (4) and (6) with control. (a) control input signal \$u_1(t)\$. (b) control input signal \$u_2(t)\$. (c) synchronization errors. (d) time response of parameters estimation errors.

parametric estimation errors are given in Figure 2(d). We can clearly see that system (4) can be estimated accurately. Above simulations demonstrated the effectiveness of our synchronization strategy under the input nonlinearity, parameters uncertainties and external disturbances.

IV. CONCLUSIONS

In this paper, we utilize T-S fuzzy model to describe the CAS and propose an adaptive synchronization strategy based on this model. To be more conformable to reality, the drug effect

of CAS is regarded as input nonlinear and the parametric adaptive control method is used to reduce the influence of parameter perturbations. The effectiveness of our control strategy can be demonstrated by the simulations. We investigated the coronary artery system based on T-S fuzzy model. However, for the uncertainties in the membership functions, the control strategies of the T-S fuzzy systems can not handle it well. Recently, type-2 T-S fuzzy model has been widely studied to deal with uncertain parameters existing in the membership functions [26]. In the future, we will focus on the chaos synchronization of the coronary artery system based on the type-2 T-S fuzzy model.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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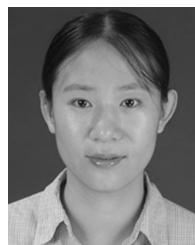
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