Acta Psychologica 255 (2025) 104962

Contents lists available at ScienceDirect

Acta Psychologica

journal homepage: www.elsevier.com/locate/actpsy

Developing pre-service teachers' noticing skills in mathematics PBL contexts: Effects of a video-based teacher education course

Sha Li, Chunxia Qi *, Ruisi Li, Ying Jin, Liuchang Cheng, Guanxiong Liu

Faculty of Education, Beijing Normal University, Beijing, China

ARTICLE INFO

Keywords:
Teacher noticing
Pre-service mathematics teacher
Video
Teacher education
Project-based learning

ABSTRACT

This study highlights the importance of developing teacher noticing skills in the context of mathematics project-based learning (PBL), an area critical for improving instructional quality. The research aims to explore how and the extent to which pre-service teachers (PSTs) develop noticing skills through a video-based teacher education course. Drawing on frameworks of teacher noticing (attending, reasoning, and decision-making) and PBL's six A's (e.g., authenticity and academic rigor), the study employed longitudinal analysis pre-post designs to evaluate changes in PSTs' noticing skills. The findings reveal significant improvements in reasoning skills, with PSTs increasingly adopting interpretive stances and integrating specific mathematical knowledge. Decision-making direction diversified, with a notable rise in proposing next instructional steps. However, PSTs prioritized teacher-centered strategies and struggled to accurately apply PBL principles. Developmental paths varied, with 40 % of PSTs expanding their noticing vision to integrate student thinking and PBL elements, while others retained narrow perspectives. These results underscore the value of structured video-based course in enhancing noticing skills but highlight the need for further research to optimize such programs for better instructional practices. This study contributes to the field by addressing the intersection of teacher noticing and PBL, offering insights into effective teacher preparation strategies.

1. Introduction

The central goals of mathematics education are to enhance the instructional quality and improve the effectiveness of teacher education programs. Being proficient in mathematics teaching requires the development of essential teaching competencies, such as teacher noticing, which is defined as a set of situation-specific skills mediating between individual dispositions and observable behaviors (Blömeke et al., 2015). Beyond planning lessons and designing classroom activities, teachers must be able to focus on and interpret classroom events—whether critical or routine—and make informed decisions in response to students' understanding or misconceptions. Furthermore, teacher noticing directly shapes classroom interactions by enabling student-centered, adaptive instruction. For instance, expert teachers use noticing to understand students' mathematical thinking and adjust explanations or tasks (Yang et al., 2021), which is in line with higher student achievement (König et al., 2021). Noticing individual learning trajectories helps teachers tailor mathematics activities (Ivars et al., 2020). Yet, not all teachers attend to what they should see within a complex teaching environment. Even when pre-service teachers (PSTs) have opportunities to observe and participate in their mentors' classes, they often fail to notice the critical insights that teacher educators aim to impart through these experiences (Star et al., 2011). This underscores the need to explicitly cultivate teacher noticing skills, particularly for PSTs, as part of mathematics education and professional development.

Unlike conventional teaching methods, project-based learning (PBL) highlights authentic, student-centered learning experiences, which is beneficial for fostering students' problem-solving skills but demanding for both students and teachers (Kokotsaki et al., 2016). In the context of PBL, students are encouraged to address real-world problems through teamwork, present their outcomes to audiences, and conduct self-assessment and peer assessment, inducing multiple challenges that teachers struggle to meet. Thus, preparing teachers for teaching mathematics in PBL contexts is a critical issue. To support students' learning in a complex classroom situation, teachers have to spontaneously make informed decisions according to their perception and analysis of student performance. The concept of teacher noticing (Kaiser et al., 2015) provides a key perspective for promoting their in-the-moment decisions.

* Corresponding author at: No. 19, Xinjiekouwai St, Haidian District, Beijing, China.
E-mail addresses: lis14@tsinghua.org.cn (S. Li), qichxia@126.com (C. Qi), ruisi_li@outlook.com (R. Li), kim24@mail.bnu.edu.cn (Y. Jin), chengliuchang@mail.bnu.edu.cn (L. Cheng), liugx0228@mail.bnu.edu.cn (G. Liu).

https://doi.org/10.1016/j.actpsy.2025.104962
Received 3 January 2025; Received in revised form 22 March 2025; Accepted 27 March 2025
Available online 30 March 2025

Therefore, it is viable to develop mathematics teachers' noticing in a PBL context to help them with teaching practices.

Research on teacher noticing has frequently employed videos as a key instructional tool (Santagata et al., 2021). Videos offer pre- and in-service teachers more extensive access to learning how to teach and making reflection on teaching practices than traditional classroom observation within a basically authentic educational setting (Sherin, 2004). The processes to teaching and learning could be slowed down in videos, which reveal more details about classroom interactions not easily available as teaching. Studies on video-based approaches in teacher education have demonstrated that classroom videos can promote context-specific, issue-focused learning and enhance teachers' ability to notice and analyze critical educational situations (Kleinknecht & Gröschner, 2016). However, videos alone are insufficient to fully develop noticing skills. Teacher educators play a critical role in guiding PSTs to notice and interpret significant classroom events.

In pre-service teacher education, videos selected by teacher educators based on the goals of methodological courses are often used to direct PSTs' attention to targeted aspects of teaching, such as instructional strategies and student thinking, thereby cultivating their noticing skills (Magiera & Zambak, 2021). Building on this approach, the present study develops a video-based teacher education course aiming to cultivate PSTs' noticing skills in a mathematics PBL context. This course integrates the frameworks of teacher noticing and PBL to provide PSTs with structured opportunities to develop their capacity to notice and respond to the complexities of PBL classrooms. Furthermore, the influence of this course on teacher noticing in a mathematics PBL context is explored to provide an empirical insight into effective teacher education.

2. Literature review

2.1. Conceptualizations of teacher noticing

Teacher noticing, recognized as a key part of teachers' professional competence in mathematics education, remains inconsistently conceptualized (Santagata et al., 2021). While early studies treated it as a holistic facet, recent research has increasingly differentiated it into distinct cognitive processes (König et al., 2022). For instance, van Es and Sherin's (2002) seminal three-phase model—(a) identifying noteworthy events in a classroom situation, (b) using one's expertise and experiences to make sense of the event, and (c) connecting the event with broader principles of teaching and learning—laid foundational groundwork. However, disagreements persist over the scope of “making sense” leading to fragmented sub-processes (Sherin et al., 2011). Making sense is either limited to interpreting or further differentiated. Thus, two common conceptualizations of teacher noticing have been identified (Weyers et al., 2024).

The first noticing construct includes two interrelated components of attending/perceiving and interpreting/reasoning. For instance, van Es and Sherin (2006) focused on (a) identifying what is important in a teaching situation and (b) drawing on one's knowledge of teaching and learning to reason about the situation. Sherin and van Es (2009) depicted teacher noticing as two components: (a) selective attention concerning the objects which a teacher decides to attend to, and (b) knowledge-based reasoning concerning the ways where a teacher reasons about what they noticed based on his/her knowledge and understanding.

The second noticing construct is conceptualized as the triad of attending/perceiving, interpreting/reasoning, and responding/decision-making. For instance, Jacobs et al. (2010) developed a three-facet framework of teacher noticing for a focus on children's thinking, including (a) attending to children's strategies, (b) interpreting children's understanding, and (c) deciding how to respond on a basis of children's understanding. Kaiser et al. (2015) constructed a conceptual model of teacher noticing with three situated facets: (a) perceiving particular events in an instructional setting, (b) interpreting the perceived activities in the classroom, and (c) decision-making, either as anticipating a response to students' activities or as proposing alternative instructional strategies.

Recent extensions, such as van Es and Sherin's (2021) addition of “shaping” (deliberately shaping interactions to glean insights), emphasize instructional enactment (Weyers et al., 2024). However, these advanced facets extremely depend on teaching experience and adaptive thinking, posing challenges for pre-service teachers (van Es & Sherin, 2021).

2.2. Video-based teacher education programs centered on teacher noticing

Videos have been widely used in teacher education to enhance noticing skills by providing authentic classroom scenarios (Santagata et al., 2021). They enable teachers to revisit and reflect on teaching moments missed during live instruction, helping PSTs link theory to practice. In mathematics education, videos can help PSTs understand students' cognitive processes, including mathematical reasoning and solution strategies, and improve their ability to design lessons that elicit and respond to student thinking (Star & Strickland, 2008). However, merely watching videos is inadequate. PSTs often focus on surface-level details due to limited experience, struggling to analyze deeper pedagogical interactions (Erickson, 2011).

Structured video training programs are critical for teacher development. These programs should incorporate theoretical frameworks with guided activities to sharpen noticing skills (Larison et al., 2024). Key design factors include video type, focus, and instructional support, which depend on training goals. Most educators use short clips (minutes or shorter) rather than full lessons to reduce cognitive load and target key teaching practices (Blomberg et al., 2011; Santagata et al., 2021). For PSTs, analyzing classroom videos of other teachers develops their noticing skills, while in-service teachers often review their own or peers' videos to refine instruction. Videos focused on mathematical problems also aid in deepening the understanding of students' problem-solving processes (Ivars et al., 2020).

Recent studies highlight emphasizes using cognitive prompts and structured guidance to help teachers focus on critical elements like student thinking and instructional strategies (Güler et al., 2023). Teacher educators often employ structured frameworks or viewing guides to help teachers notice and make sense of key video moments (Santagata et al., 2021), especially in mathematics education, where tailored frameworks address topics such as early numeracy or classroom dynamics (Mitchell & Marin, 2015; Schack et al., 2013). Structured prompts—such as guided analysis or video annotation tasks—prove effective in focusing responses to specific questions (Larison et al., 2024; Magiera & Zambak, 2021). While open-ended prompts (e.g., “What do you notice?”) initiate broader reflection during group discussions (van Es & Sherin, 2006, 2008), structured prompts are particularly beneficial for PSTs, enhancing their ability to analyze teaching practices systematically (Fisher et al., 2019).

Santagata et al. (2021) presented a systematic review of video-based programs centered on mathematics teacher noticing and found that scholars made efforts to examine *how* and *the extent to which* a program or intervention influenced the development of teacher noticing. In studies with PSTs, qualitative methods were used to address the former question most frequently (e.g., Ivars et al., 2020), whereas both qualitative and quantitative methods were used to address the latter one (e.g., van Es et al., 2017). A range of empirical studies were implemented based on different research designs, including cross-sectional designs (Roller, 2016), pre-post designs (Stockero & Rupnow, 2017), and longitudinal designs (van Es & Sherin, 2008). Only a few studies used true experiments with randomized groups. Studies concerning the impact of video-based interventions on decision-making skills are fewer than those on other components of teacher noticing (Weyers et al., 2024). Researchers tended to examine teachers' noticing through written reports (e.g., essay writing and narrative writing), video recordings (e.g., group discussions), and interviews (König et al., 2022).

2.3. Features and challenges of project-based learning

Project-based learning (PBL), grounded in constructivist and situated learning theories, emphasizes student-oriented problem solving (Kokotsaki et al., 2016). Its structured process involves three cyclical phases—design, implementation, and reflection—allowing iterative refinement of teaching practices (Condliffe et al., 2017). This phased approach supports holistic assessment, addressing both collaborative processes and tangible outcomes (e.g., final products), thereby mirroring authentic professional workflows (Thomas, 2000).

The design phase is critical in PBL, shaping meaningful learning experiences. In this phase, teachers define authentic problems, craft driving questions, and align activities with learning goals (Larmer & Mergendoller, 2015). However, challenges arise in balancing academic rigor with student relevance (Boss & Larmer, 2018), integrating projects into existing curricula (Kanter, 2010), and setting clear, measurable goals (Barron et al., 1998). Unclear objectives risk vague questions and unfocused student work.

During implementation, students engage in problem-solving, decision-making, and knowledge-building through sustained inquiry. This phase fosters hands-on learning through collaboration and active exploration, while teachers guide students to navigate complex challenges through scaffolds (Thomas, 2000). However, balancing student autonomy with classroom management remains difficult, particularly with varying engagement levels and time management struggles (Reeve et al., 1999). Teachers must also develop adaptive strategies to support diverse learner needs.

The reflection phase evaluates both learning processes and outcomes, prompting students to critically analyze their work and showcase final results. It reinforces student learning and offers feedback to improve future PBL designs. However, assessing if projects meet intended goals is challenging—teachers must weigh diverse factors like content mastery, critical thinking, collaboration, and creativity. Quantifying these outcomes across students or projects is difficult (Condliffe et al., 2017), and traditional assessments often fall short. Alternatives like rubrics, portfolios, and presentations are needed (Kokotsaki et al., 2016). Timely, actionable feedback remains vital for student growth.

Markham et al.'s (2003) six A's framework—Authenticity, Academic Rigor, Applied Learning, Active Exploration, Adult Connections, and Assessment Practices—offers a robust lens for analyzing three processes in PBL. *Authenticity* roots projects in real-world challenges (e.g., professional or community issues), enhancing students' learning motivation and engagement (Strobel et al., 2013). *Academic Rigor* ensures cognitive complexity, fostering critical thinking and subject-specific skills like mathematical analysis (Eliyasni et al., 2019). *Applied Learning* highlights practical problem-solving through hands-on experiences (Zhang & Ma, 2023). *Active Exploration* encourages inquiry-based learning via student-led research and iterative refinement of ideas (Larmer & Mergendoller, 2015). *Adult Connections* links projects to mentors or experts for real-world guidance and feedback (Thomas, 2000). *Assessment Practices* combines self-assessment, peer feedback, and teacher assessments to evaluate both learning processes and final outcomes (Bhat & Bhat, 2019).

2.4. Summary

PBL in mathematics helps students develop essential skills such as problem-solving, reasoning, and collaboration. However, implementing PBL in mathematics faces challenges, such as reconciling open-ended inquiry with academic depth and teachers' limited familiarity with PBL methods (Kokotsaki et al., 2016). Video-based teacher training programs show promise by exposing PSTs to authentic PBL examples, potentially strengthening their ability to translate PBL theory into practice through structured observation. However, in mathematics education, studies on teacher noticing have been limited in conventional classrooms and modeling tasks, which differ from the context of PBL highlighting the construction of real models.

A review of the related literature also reveals several key gaps that this study addresses. First, most research on teacher noticing has been conducted in North America and Europe (König et al., 2022), with limited exploration in other cultural contexts, such as China. Many studies in China mainly aim to explore the effect of different teaching experiences or cultural contexts on teacher noticing (Yang et al., 2019, 2021) rather than that of teacher education programs. Second, while video-based teacher education programs have proven effective in enhancing noticing skills, few studies have examined the trajectory of learning to notice, particularly through intervention-based designs (van Es & Sherin, 2008). Last, compared to attending and reasoning skills, the development of decision-making skill is relatively underexplored (König et al., 2022).

To address these gaps, an intervention study is conducted to investigate the effects of a video-based pre-service teacher education course on the development of teacher noticing skills in a mathematics PBL setting. This study aims to answer the following research questions (RQs) using a mixed approach:

RQ1: During the course, how do pre-service teachers develop their noticing skills in a mathematics PBL context?

RQ2: During the course, to what extent do pre-service teachers develop their noticing skills in a mathematics PBL context?

RQ3: Does the course improve pre-service teachers' noticing skills in a mathematics PBL context significantly?

3. Theoretical framework

The conceptualization of teacher noticing can affect the design of teacher learning activities, but the specific analytical framework of teacher noticing in a mathematics PBL context is lacking. The integrated construct of selective attention and knowledge-based reasoning (Sherin & van Es, 2009) seems to be prevalent in video-based studies on teacher noticing, and the decision-making skill has attracted more attention. Therefore, in this study, teacher noticing in a mathematics PBL context is conceptualized as (a) attending to noteworthy objects in a mathematics PBL context, (b) reasoning about what they noticed based on PBL principles and professional knowledge related to mathematics teaching and learning, and (c) decision-making, either as proposing alternative strategies to what they noticed (Santagata et al., 2007), or as advancing the next teaching in response to students' activities (Jacobs et al., 2010; Kaiser et al., 2015). The analytical framework of this study is presented in Table 1 including coding dimensions and categories adapted from previous studies.

The first component of noticing (attending skill) was coded as selective attention, including two dimensions: actor and topic. Actor refers to whom PSTs attended to. The original categories of actor (teacher, student, and other) were changed into five codes (Stockero et al., 2017) excluding "other", because the rough division into teacher and student is insufficient to depict the complex classroom interactions in a PBL context, and others (e.g., school leaders, and teacher educators) are not foci in our study. Specifically, *teacher/student* emphasizes a teacher's behavior (e.g., questioning or guiding) that promotes student learning, while *student/teacher* highlights a student's solution or misconception that elicits a teacher's response. Topic refers to what PSTs attended to, where *management* concerns issues of classroom organization (e.g., classroom discipline), *climate* represents the social environment of the classroom (e.g., the engagement of students in class), *pedagogy* captures moments related to instructional strategies, and *mathematical thinking* focuses on the process or method of students' problem-solving efforts.

The second component of noticing (reasoning skill) was coded as knowledge-based reasoning, including three dimensions: stance, mathematics, and PBL. Stance refers to how PSTs made sense of what they noticed. If a teacher just restated the observed details of the video, it was

Table 1
Analytical framework for coding PSTs' noticing in a mathematics PBL context.

Component	Dimension	Category
Selective attention	Actor	Teacher (T)
		Teacher/student (T/S)
		Student/teacher (S/T)
		Student group (SG)
		Individual student (IS)
	Topic	Management (MN)
		Climate (CL)
		Pedagogy (PD)
		Mathematical thinking (MT)
Knowledge-based reasoning	Stance	Describe (D)
		Evaluate (E)
		Interpret (I)
	Mathematics	Non-math (NM)
		Math (M)
	PBL	Authenticity (AU)
		Academic rigor (AR)
		Applied learning (AL)
		Active exploration (AE)
		Adult connections (AC)
		Assessment practices (AP)
Instant decision-making	Direction	Teaching improvement (TI)
		Feedback improvement (FI)
		Next plan (NP)
	Strategy	Teacher-centered move (TM)
		Student-centered move (SM)

coded as "describe"; If a teacher made judgements about the quality of classroom interactions without robust evidence, it was coded as "evaluate"; If a teacher made inferences about the reasons or/and potential impacts of what took place in the video with robust evidence, it was coded as "interpret". The original knowledge dimension was divided into mathematics and PBL to account for the dual complexity of a mathematics PBL context, encompassing both mathematical knowledge and salient features of PBL. Mathematics was further categorized into *non-math* and *math* based on whether specific mathematical knowledge was discussed, while "PBL" refers to six key PBL elements (Markham et al., 2003; see Section 2.3 for details).

The third component of noticing (decision-making skill) was coded as instant decision-making including two dimensions: direction and strategy. The direction dimension represents how PSTs responded to students or extended student learning. Categories include *teaching improvement* (highlighting general pedagogical improvements), *feedback improvement* (emphasizing student-specific responses), and *next plan* (advising on subsequent instructional steps). The strategy dimension was categorized into *teacher-centered moves* (alternative or direct instructional strategies) and *student-centered moves* (posing questions for students to address), which were adapted from Stockero and Rupnow (2017).

This study's theoretical framework integrates complementary theories to address the multifaceted nature of teacher noticing—attending, reasoning, and decision-making—within PBL contexts. Building on Sherin and van Es's (2009) foundational model of selective attention and knowledge-based reasoning, the framework expands to incorporate dynamic decision-making through an emphasis on adaptive response to student actions (Kaiser et al., 2015). To address limitations in evaluating PSTs' broad decision-making capacity (not just given moments), Jacobs et al.'s (2010) focus on responses to student cognition and Santagata et al.'s (2007) strategies for generating instructional alternatives are integrated. Stockero and Rupnow's (2017) categorization of teacher- and student-centered strategies refines decision-making analysis for PBL's unique demands, while Stockero et al.'s (2017) classification of classroom actors (students, teachers, content) sharpens the attending component. Markham et al.'s (2003) six A's framework is embedded within the reasoning component to ensure PSTs connect their interpretations to PBL's core principles.

Methodologically, these theoretical calibrations align with the study's mixed-methods design. Sherin and van Es's (2009) components provide the qualitative coding structure for analyzing PSTs' written responses, while actor categories from Stockero et al. (2017) and strategy categories from Stockero and Rupnow (2017) enhance the granularity of analysis. Markham et al.'s (2003) framework acts as a conceptual bridge, ensuring PSTs' reasoning is anchored in PBL-specific criteria rather than generic pedagogical knowledge. Santagata et al. (2007) and Jacobs et al. (2010) provide criteria for assessing adaptive decision-making in video-based tasks. This interdisciplinary synthesis enables nuanced exploration of PSTs' skill development (RQ1–RQ2) and statistical validation of the intervention (RQ3), bridging teacher noticing research with PBL implementation in mathematics education.

4. Methodology

4.1. Study context and course design

This study was conducted as a part of a *Middle School Mathematics Pedagogy* course within a teacher education program at a university in China. The course was offered during the first quarter of a three-quarter teacher education program, designed to equip PSTs with knowledge and skills for learning from mathematics teaching practices. Over a three-month period, the course met once a week on Wednesdays for three-hour sessions. The in-class module was designed and taught by one professor with support from a research associate and a PhD student, who assisted in selecting video clips of mathematics PBL classroom practices.

The course design was informed by prior research on video-based teacher noticing, mathematics teaching and learning, and PBL. The course had two main objectives: (1) to support PSTs' understanding of teaching practices and student thinking in the middle school mathematics curriculum, and (2) to develop PSTs' ability to notice in a mathematics PBL context. The course was structured into three distinct sections:

Introduction Section (weeks 1–2): This section introduced participants to the policy related to mathematics PBL, the six A's framework of PBL, and theoretical foundations of mathematical thinking and mathematics competencies.

Main Section (weeks 3–11): This section was divided into three parts based on major mathematics fields: geometry, algebra, and statistics, required by the Mathematics Curriculum Standards for Compulsory Schools (Ministry of Education, 2022). *Geometry (weeks 3–7)*: This part is central because geometry involves shapes, spatial reasoning, and tangible tasks (e.g., symmetry-based art) aligned with PBL's emphasis on observation and interdisciplinary connections (e.g., physics and engineering) (Remijan, 2017). In weeks 3–4, PSTs identified and described critical moments in PBL video clips. In weeks 5–6, PSTs interpreted moments using mathematical knowledge and PBL principles. In week 7, PSTs designed instructional strategies to enhance students' mathematical thinking. *Algebra (weeks 8–9)*: This part underscored students' computational competence through an algebra-focused PBL task. *Statistics (weeks 10–11)*: This part focused on students' data analysis competence through two videos of statistics-based PBL activities. Throughout this section, PSTs independently analyzed PBL video clips, followed by group discussions where the teacher educator guided evidence-based observations, critical analysis of instructional practices, and alternative strategy proposals. After that, selected PSTs and experienced middle school teachers shared insights with the class. Finally, the teacher educator concluded with targeted feedback to enhance PSTs' noticing skills.

Reflection Section (week 12): In this final section, PSTs worked in groups to design a mathematics PBL lesson. They were tasked with presenting their outputs, which could include handicrafts, proposals, or investigation reports resembling student projects. Groups chose themes related to middle school mathematics and were required to include instructional strategies and intentions, and anticipated classroom interactions in their lesson plans. These elements aimed to enhance PSTs' understanding of the classroom as an interactive space connecting teaching, students, and content. To ensure adequate preparation, PSTs were allowed to work on their lesson plans outside of class. At the end of this course, they revisited and reanalyzed the initial video clip to access the effect of the video-based intervention on teacher noticing.

4.2. Participants

A total of 28 PSTs (20 female, 8 male) participated in the study. All participants were undergraduate students in the final two years of their teacher education program. While most PSTs had prior experience with pedagogy or mathematics courses where they learned about middle school mathematical content knowledge (e.g., mathematical definitions and properties) and pedagogical content knowledge (e.g., knowledge of instructional strategies and students' misconceptions), none had previously engaged in video analysis or PBL. They were divided into eight groups with each consisting of three to four PSTs, submitted their PBL lesson plans, and presented their work in class. Groups were coded as G1–G8, and individual members were coded according to their group number and intra-group role (e.g., G802 referred to the second member of Group 8).

Participation in the study was voluntary, and all PSTs provided informed consent after being briefed on the research goals. The study followed ethical research protocols and was approved by the university's ethics committee.

4.3. Data collection

The main data source for this study was video analyses from PSTs who participated in the course. The video clips, each between 6 and 10 min long, were selected from an elementary education quality improvement project in China. The selection criteria included activity design characterized by PBL elements, high levels of classroom interactions between teacher and students or within student groups, and high-quality video recordings (e.g., classroom discourse and group presentation). Given that the abstractness of algebra and the monotony of statistics made it difficult for students to make active interactions in a mathematics PBL context, video clips on these two topics were excluded. In this study, four geometry-related video clips were used as they depicted consecutive stages of PBL within the same theme.

Table 2 provides a summary of these geometry-related clips. Notably, video 1 was viewed in both the first and last weeks of the course to facilitate pre- and post-analyses. Videos 2–4 represented three distinct stages of PBL course within a consistent thematic framework, offering a comprehensive view of the implementation process.

To complete the video analysis task, PSTs watched the video clip and responded to three prompts in the task table: (1) Which significant moments do you notice? (2) How do you reason about these moments using mathematical knowledge and PBL principles? (3) (If possible) What would you do next or which alternative suggestions would you provide for teacher in the video? Participants made notes when viewing the video clip, finished the analysis task within 30 min individually, and submitted them in class. Each of video clips was played once because it was served as a proxy for real teaching situations where a rewind or pause button did not exist. After submitting all of video analyses, one PST was removed in the phase for pre- and post-assessments because his data was missing in the first video analysis task.

Table 2
Overview of video clips included in the analysis tasks.

Video	Time	Topic	Stage of PBL	Foci of Video Clip
1	week 1 week 12	Circle	Entry	Drawing circles with three different tools; Comparing the difference among tools.
2	week 3	Axial symmetry	Entry	Summarizing the steps of making a kite; Designing and explaining patterns of a kite.
3	week 5	Axial symmetry	Inquiry	Exploring geometric properties of the kite; Teamwork and in-depth communication.
4	week 7	Axial symmetry	Exhibition	Displaying and elucidating their kites; Evaluating kites between groups.

4.4. Data analysis

4.4.1. Coding process

Qualitative method, an iterative and grounded approach, was used to analyze the PSTs' responses to task prompts. The video analyses were separately analyzed based on the coding scheme (Table 1). The coding process was conducted in three main phases:

The first phase: Pre-processing. PSTs' responses were segmented into idea units, with each unit representing a distinct idea or event. Inter-rater reliability for segmenting idea units exceeded 90 % for each video analysis. Discrepancies were resolved through discussion. To ensure credibility, video analyses from four PSTs were used as training cases, and their units were coded according to the framework. The resulting coding dictionary, including examples for each category, was finalized by the research team (see Table A.1 in Appendix A).

The second phase: Coding. Two researchers independently coded the remaining video analyses. Coding was conducted blind, achieving initial inter-rater reliability of 92.6 % or higher. Differences between coders were resolved through discussion, with the lead researcher consulted if necessary. Although the inherent complexity of PSTs' video analyses was not fully captured by the coding framework, all categories reflected PSTs' primary focus areas aligned with the study's research goals.

The third phase: Post-processing. To investigate developmental paths of teacher noticing, the overall vision of each PST at each phase indicating the focus of video analyses was coded based on van Es and Sherin (2008). First, the number and proportion of each category were calculated for each PST. Then, a PST's vision across each dimension was analyzed with two low-discriminability dimensions (*mathematics* and *strategy*) excluded. If a single category accounted for 50 % and exceeded other categories by >10 % within a dimension, a narrow vision was assigned; otherwise, a broad vision was assigned. For example, one PST coded as 60 % "teacher", 30 % "teacher/student", and 10 % "individual student" in the actor dimension was classified as "Narrow"; another PST with 51 % "teacher" and 49 % "teacher/student" was classified as "Broad". Finally, the overall vision of one PST was categorized as "Narrow" if three or more dimensions were narrow, and "Broad" if more than half of the dimensions were broad. For example, a PST with narrow visions in dimensions of actor, topic, and stance but broad visions in dimensions of PBL and direction, was classified as "Narrow" overall.

4.4.2. Standard-based ratings

Drawing on Jacobs et al. (2010) and van Es et al. (2017), a three-level framework (see Table A.2 in Appendix A) was developed to explore the variations in noticing levels of PSTs. Three types of criteria were established to distinguish levels of each PST at each dimension.

The first criterion was the dominance of key categories, especially in the attending and reasoning skills except for PBL. For instance, actor dimension focused on individual student's thinking and teacher-student interactions, using three categories ("teacher/student", "student/teacher", and "individual student"). If no mentions of these categories, a PST's level in the actor dimension was rated at Level 0; if categories mentioned but less emphasized (e.g., "teacher" discussed more), it was rated at Level 1; and if categories emphasized equally or more than others, it was rated at Level 2. Other dimensions (topic, stance, and mathematics) prioritized specific categories: “mathematical thinking”, “interpret”, and “math”.

The second criterion was the accuracy of PBL elements identified by PSTs compared with the research team. For instance, one video clip focused on authenticity and academic rigor in the dimension of PBL. If a PST missed both elements, his/her reasoning level in the PBL dimension was rated at Level 0; accurate identification of one element raised it to Level 1, and recognition of both resulted in Level 2. For video 3, where six categories were emphasized, PSTs achieving over 50 % accuracy in identifying relevant PBL elements were assigned Level 2.

The third criterion involved the richness in the direction and strategy dimensions. If no response to the third prompt, a PST's decision-making skill was rated at Level 0; if making decisions from a single perspective (e.g., “teaching improvement” or “next plan”) or proposing one strategy, it was rated at Level 1; and if making decisions from multiple perspectives or proposing two strategies, it was rated at Level 2.

4.4.3. Statistical analysis

To assess the impact of the video-based teacher education course on noticing skills, the Wilcoxon signed-rank test was used to compare pre- and post-analysis data. A Shapiro-Wilk test revealed that most categories did not conform to a normal distribution, disqualifying parametric tests such as the *Z*-test or *t*-test. The Wilcoxon signed-rank test, which does not assume normality, was selected for its robustness and focus on median differences, making it a suitable choice for this dataset.

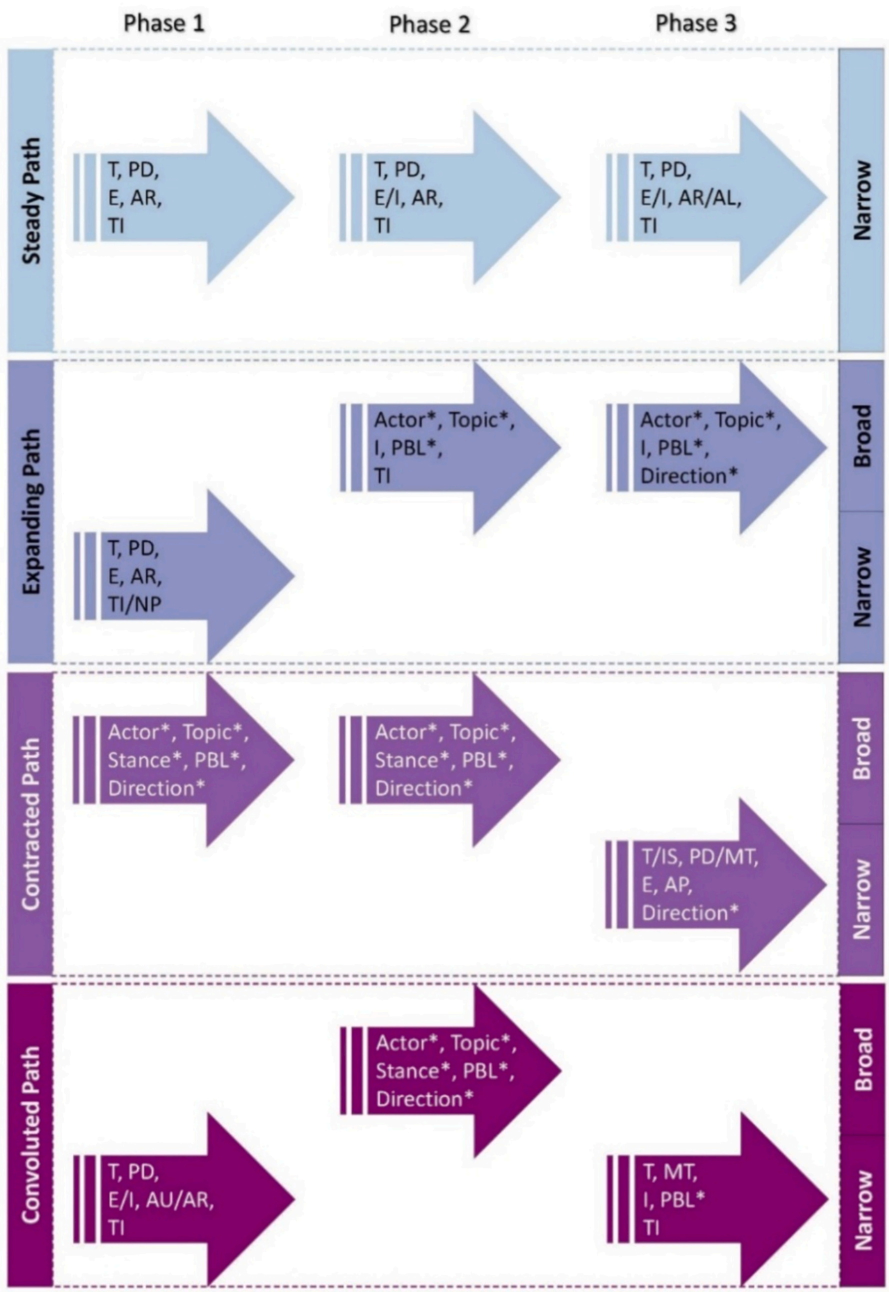

Fig. 1. Developmental paths along which pre-service teachers learned to notice. (* Three or more categories were primary in one dimension.)

5. Results

5.1. Development of teacher noticing

To answer the first research question, we examined how PSTs shifted their noticing vision during the course intervention based on their analyses of videos 2–4. Analyses of video 1 were excluded to minimize the impact of varying project themes and mathematics topics on teacher noticing. Based on a PSTs' overall vision at each phase, four developmental paths were identified to characterize how PSTs learned to notice: Steady Path, Expanding Path, Contracted Path, and Convoluted Path (Fig. 1). Notably, these paths represent dominant patterns of noticing development in a specific context and do not imply fixed or exclusive trajectories for PSTs across all video analyses. The primary categories for each dimension were explicitly listed. For example, "E/I" indicates that some teachers adopted an evaluative stance, while others used an interpretive stance, both of which were primary in the stance dimension. Visions on mathematics and strategy were consistently narrowed to "non-math" and "teacher-centered move" respectively, and were thus omitted in Fig. 1 for simplicity.

5.1.1. Steady path

Nine PSTs maintained a narrow vision (termed the Steady Path) through all phases, prioritizing the same categories within most dimensions (e.g., "teacher", "pedagogy", and "teaching improvement"), although shifts occurred in their evaluative stance and integration of PBL elements. For example, some PSTs initially evaluated teaching strategies and identified academic rigor (AR) but later adopted an interpretive stance in Phases 2–3. By Phase 3, some of them began incorporating other PBL elements such as applied learning (AL), reflecting gradual diversification within their narrow analytical framework (see examples 9–10 in Table A.1).

5.1.2. Expanding path

Approximately 40 % of PSTs demonstrated an Expanding Path, progressively widening their analytical focus from a limited perspective in Phase 1 to a broader lens by Phase 3. Unlike the Steady Path group, a part of these PSTs shifted toward anticipating next steps. During phases 2 and 3, they incorporated diverse classroom elements—such as actor dynamics, environment-related topics, and multiple PBL elements—-while adopting a more interpretive stance to analyze these moments. For instance, in phase 2, G503 wrote,

The teacher encouraged students to write the hypotheses on the property of the kite shape in the order of edge, conner, and diagonal. Students were enthusiastic about sharing their ideas. [T/S, CL, PD]

The teacher showed two kites to students before class, indicating objects in their real life and stimulating their interest in further learning. [I, AU]…*The teacher asked students to first think independently think and later collaborate in groups to complete task cards, fostering communication, problem-solving, and teamwork skills.* [I, AL]…*Throughout the activity, the teacher acted as a facilitator, allowing students to lead their own exploration.* [I, AE]

By phase 3, PSTs promoted classroom instruction from multiple perspectives. For instance, G503 noted,

The teacher should provide some suggestions for improvement after each group made their presentation, including the application of knowledge, and the quality of the kite. [FI]…*The teacher should allow students to think about the dimensions and scope of the assessment scales.* [TI]…*In the next class, the teacher could ask students to summarize their strengths and weaknesses during the project.* [NP]

5.1.3. Contracted path

Conversely, two PSTs demonstrated a contraction of their noticing skills, moving from a broad vision in phases 1 and 2 to a narrow vision in phase 3. This development is defined as the Contracted Path. During the first two phases, PSTs attended to multiple actors and topics, analyzed moments in different stances based on various PBL elements, and made decisions from diverse perspectives. However, by phase 3, their focus narrowed across most dimensions, centering on specific areas such as individual students' mathematical thinking and assessment practices, while maintaining a broader vision only in decision-making direction.

5.1.4. Convoluted path

The fourth developmental path, termed the Convoluted Path, is characterized by two transitions between narrow and broad visions across three video analyses. In phase 1, PSTs demonstrated narrow visions across all dimensions. For instance, they primarily focused on the teacher and pedagogy, evaluated or interpreted observations using specific PBL elements ("AU" or "AR"), and proposed teaching alternatives. In phase 2, PSTs shifted to a broader vision, attending to multiple categories within each dimension. Finally, in phase 3, PSTs narrowed their focus again, prioritizing mathematical thinking and interpreting what they noticed. Their focus on actors and decision-making directions reverted to phase 1 patterns, but their broader vision of PBL elements persisted from phase 2 to phase 3.

5.2. Variations in levels of teacher noticing

To investigate the extent to which PSTs developed their noticing skills in different phases of the course, we first rated their performance across different dimensions and summarized the number and proportions of participants at each level. In essence, the assessment aimed to determine whether PSTs attended to the elements we anticipated, analyzed the highlighted moments as expected, made decisions from diverse perspectives, and proposed a range of strategies to enhance instruction.

In terms of attending and decision-making skills (Table 3), none or very few PSTs performed outside of expectations entirely, but the majority were rated at Level 1. Most PSTs attended to teacher-student interactions, individual student, and pedagogy, and the number of highest-level PSTs increased continuously in the dimensions of actor and topic. Regarding the decision-making component, the majority of PSTs tended to adopt a single perspective and strategy during the phases 1 and 3. However, by phase 2, 61 % of PSTs were rated at Level 2 in the direction dimension, indicating that they had more foci and ideas on the implementation of PBL.

Table 4 highlights a progressive increase in the number of PSTs adopting a more interpretive stance, accompanied by a reduction in those rated at Level 0 during the three phases, reflecting an improvement in reasoning skills. Additionally, there was a notable increase followed by a decrease in the application of specific mathematical knowledge and unique PBL elements. This pattern suggests that PSTs recognized the importance of geometry knowledge, teamwork, and

Table 3
Distribution of rating for the attending and decision-making skills.

		Phase 1	Phase 2	Phase 3	Phase 1	Phase 2	Phase 3
		Actor			Topic		
Selective attention	L0	14 % (4)	7 % (2)	0 % (0)	11 % (3)	0 % (0)	0 % (0)
	L1	72 % (20)	75 % (21)	75 % (21)	82 % (23)	64 % (18)	57 % (16)
	L2	14 % (4)	18 % (5)	25 % (7)	7 % (2)	36 % (10)	43 % (12)
		Direction			Strategy		
Instant decision-making	L0	4 % (1)	7 % (2)	4 % (1)	4 % (1)	7 % (2)	4 % (1)
	L1	71 % (20)	32 % (9)	54 % (15)	75 % (21)	68 % (19)	86 % (24)
	L2	25 % (7)	61 % (17)	43 % (12)	21 % (6)	25 % (7)	11 % (3)

Note: Values in parentheses indicate the number of PSTs in each level (N = 28 PSTs, L0 = Level 0, L1 = Level 1, L2 = Level 2).

Table 4
Distribution of rating for the reasoning skill.

—	Stance			Mathematics			PBL		
	Phase 1	Phase 2	Phase 3	Phase 1	Phase 2	Phase 3	Phase 1	Phase 2	Phase 3
L0	46 % (13)	18 % (5)	11 % (3)	11 % (3)	4 % (1)	25 % (7)	7 % (2)	18 % (5)	7 % (2)
L1	18 % (5)	25 % (7)	25 % (7)	43 % (12)	43 % (12)	64 % (18)	57 % (16)	32 % (9)	54 % (15)
L2	36 % (10)	57 % (16)	64 % (18)	46 % (13)	54 % (15)	11 % (3)	36 % (10)	50 % (14)	39 % (11)

Note: Values in parentheses indicate the number of PSTs in each level ($N = 28$ PSTs, L0 = Level 0, L1 = Level 1, L2 = Level 2).

effective communication during the inquiry phase. However, their ability to incorporate specific PBL elements during the exhibition phase was limited, reducing the accuracy of their analyses. Meanwhile, specific mathematical knowledge was not a focus within the exhibition stage.

5.3. Effect of the video-based course on teacher noticing

To examine whether PSTs' noticing skills were significantly enhanced after the course, this study used Wilcoxon signed-rank tests to respectively compare the number of PSTs at Level 2 for each dimension between pre- and post-analyses. Table 5 shows a significant increase in the highest-level PSTs in three dimensions (stance, mathematics, and direction), indicating that more PSTs could analyze what they noticed with a more interpretive stance and specific mathematical knowledge, and make decisions from multiple perspectives. Additionally, improvements were observed in the dimensions of actor, PBL, and strategy, although these were not statistically significant. Notably, the number of PSTs who primarily focused on mathematical thinking decreased slightly, a trend that also did not reach significance.

To further investigate the source of variations in noticing skills, additional Wilcoxon signed-rank test was conducted for each category respectively. Results reveal significant differences in ten specific categories (see Table A.3 for all categories). Table 6 highlights that decreases in three categories (student group, describe, and non-math) and increases in seven others were statistically significant at the 0.05 level. When combined with the results in Table A.4, the findings suggest that the non-significant variation in the actor dimension may result from a greater focus on student/teacher interactions offset by a decrease in student group instances. Additionally, the increase in decision-making instances was primarily attributed to more frequent proposals of next plans. For the dimension of strategy, neither of the two categories showed a significant change.

Changes in the knowledge-based reasoning dimension were more complex. While there was a significant decrease in the descriptive stance, this reduction partially offset the increase in the interpretive stance, leading to an overall decrease in the total stance. Furthermore, the loss of non-math knowledge exceeded the gain in specific mathematical knowledge. Nevertheless, significant increases in three PBL-related categories (academic rigor, active exploration, and assessment practices) enhanced the overall application of PBL elements.

Table 5
Comparison of the highest-level PSTs in each dimension.

Dimension	Pre	Post	Z	p
Actor	3	7	−1.604	0.109
Topic	3	1	−0.707	0.480
Stance	5	12	−2.700	0.007*
Mathematics	3	10	−3.508	<0.001*
PBL	5	8	−1.567	0.117
Direction	14	21	−2.111	0.035*
Strategy	7	8	−0.302	0.763

* $p < 0.05$.

Table 6
Comparison of instances in specific categories.

Category	Pre	Post	Z	p
Student/teacher	9	23	−2.339	0.019
Student group	28	12	−2.181	0.029
Pedagogy	145	185	−2.411	0.016
Describe	63	20	−2.490	0.013
Interpret	25	63	−2.874	0.004
Non-math	123	75	−2.945	0.003
Math	25	56	−2.914	0.004
Academic rigor	17	37	−2.507	0.012
Active exploration	7	35	−3.553	<0.001
Assessment practices	2	11	−2.714	0.007
Next plan	14	26	−3.464	0.001

Note: Only categories with significant difference ($p < 0.05$) are shown.

These findings suggest that while noticeable improvements were achieved in several aspects of teacher noticing, certain areas, such as strategy and a focus on student mathematical thinking, warrant further investigation.

6. Discussion and conclusion

Pre-service teacher education aims to equip future educators with essential teaching skills. A critical competency, teacher noticing—particularly enhanced through video-based programs—improves educators' ability to analyze classroom interactions and interpret student responses (Fisher et al., 2019). Our study confirmed a shift among PSTs toward prioritizing teacher-student interactions over group dynamics, consistent with Qi et al. (2022). However, unlike Zuo et al. (2024) mathematical modeling research—which saw no pedagogical focus shift—our results highlight contextual differences in study designs or PBL implementations as potential explanations for these divergences.

This study explored how PSTs' noticing skills evolved during the intervention, revealing distinct developmental pathways across three PBL stages. While most PSTs started with a narrow perspective, only those on the Expanding Path broadened their vision by the final stage—contrasting with van Es and Sherin's (2008) findings where in-service teachers transitioned from broad to narrow perspectives, likely due to differences in experience and content familiarity (Bastian et al., 2022). Challenges in aligning PSTs' focus with PBL priorities may stem from video design and analysis methods, compounded by the dual demands of integrating mathematical rigor with PBL principles. These complexities likely contributed to divergent trajectories compared to prior studies focused on experienced educators.

Our study further investigated the extent to which PSTs developed their skills in three noticing components (attending, reasoning, and decision-making) during and after the course. Assessments revealed notable growth, particularly in their ability to interpret classroom interactions through robust evidence and specific mathematical knowledge. Structured prompts (e.g., “How do you reason about these moments using mathematical knowledge and PBL principles?”) and group discussions encouraged PSTs to move beyond evaluative judgements toward evidence-based interpretations. After discussion, selected

PSTs shared insights and questions with the class, while observing in-service teachers offered critiques. The teacher educator highlighted critical video moments and connected them to broader pedagogical principles, fostering deeper cognitive engagement and interpretive expression (Eilam & Poyas, 2009; Mena et al., 2017).

The course intervention significantly enhanced PSTs' decision-making skills, particularly in planning subsequent instructional steps. Structured prompts (e.g., "What would you do next or which alternative suggestions would you provide for teacher in the video?"), group discussions, and collaborative lesson design tasks encouraged multi-perspective thinking, such as adapting scaffolding strategies and reallocating resources. Emphasis on project continuity during video analysis also led PSTs to propose more actionable plans, consistent with evidence that explicit scaffolding strengthens connections between observation and practice (Barnhart & van Es, 2015).

Our study found varied development in PSTs' noticing skills, but several limitations merit attention. First, generalizability is constrained by the small sample size (28 PSTs), which limited detection of subtle patterns like the Contracted Path. Contextual factors—the course's design within China's structured mathematics pedagogy (focused on teacher authority and curricular goals)—may differ from student-centered PBL frameworks in Western settings. Additionally, a focus on geometry limits applicability to other mathematical domains (e.g., algebra). Second, the non-experimental design without randomized groups complicates isolating the intervention's effects, as skill improvements could stem from external factors like repeated video exposure or PSTs' growing familiarity with PBL principles. Future research should employ randomized trials comparing video-based and non-video PBL training, or test specific intervention components (e.g., prompts, discussions), to clarify causal impacts. Using distinct pre- and post-video clips for control groups could also mitigate content repetition bias. Third, the drivers of differing developmental paths (e.g., Steady Path where PSTs retained narrow visions) remain unclear. Potential explanations include prior socialization in teacher-centered systems, cognitive overload in PBL, insufficient scaffolds for noticing student thinking, or limited metacognitive reflection. Dominant voices in group discussions and PSTs' prior academic backgrounds (e.g., geometry proficiency) may further influence outcomes. To address these, programs could integrate dual-focused scaffolds that model balancing student cognition and PBL principles while employing metacognitive prompts to mitigate attention biases. Quantitative analysis of pre-course assessments (e.g., subject mastery) might also clarify these dynamics.

The limited development of PSTs' noticing skills across certain components stems from multiple factors. For *attending*, although some PSTs shifted focus toward teacher-student interactions and mathematical thinking, most highlighted teacher behaviors. This may reflect habitual framing from their teacher-centered educational background, compounded by video clips emphasizing teacher-led narration or group scenes, inadvertently sidelining individual student cognition. For *reasoning*, while PSTs improved in identifying specific PBL elements (e.g., academic rigor, active exploration), reaching high-level PBL integration remained rare. Evaluations prioritizing accuracy over depth likely reinforced treating PBL criteria as checklists rather than interconnected principles. Geometry-focused videos further narrowed application (e.g., reducing assessments to rubric design rather than student reflection). For *decision-making*, PSTs overwhelmingly favored teacher-directed strategies (e.g., scaffolding) over innovative approaches (e.g., exploratory discourse), likely hindered by PBL's complexity and their limited practical experience. Open-ended PBL tasks demanding simultaneous attention to inquiry, content, and dynamics challenged inexperienced PSTs, and video simulations offered limited impetus for proactive decision-making. Additionally, their socialization within China's teacher-centric system may have entrenched reliance on conventional pedagogical actions.

To enhance the impact of the video-based course on PSTs' noticing skills, targeted refinements are necessary across three areas. For attending skills, teacher educator should highlight individual student's cognition through taking students' written or handmade work as another analytic materials (Magiera & Zambak, 2021). Contrasting PBL video examples (e.g., effective vs. ineffective implementations) and annotation tools (e.g., timestamped comments) could clarify critical moments and externalize attentional shifts (Larison et al., 2024; Star & Strickland, 2008). To strengthen reasoning skills, expert-novice dialogue overlays could model interpretive thinking (van Es & Sherin, 2008) by verbalizing connections between classroom events and PBL principles (e.g., "This exemplifies academic rigor because…"). Domain-general video clips (e.g., algebra-based PBL) and scaffolded rubrics (e.g., differentiating applied learning from generic collaboration) would help PSTs avoid misattributing PBL elements and broaden their analytical scope (Schack et al., 2013). For decision-making skills, integrating micro-teaching simulations (e.g., responding to video-paused scenarios or live role-plays) and pairing PSTs with in-service teachers during PBL implementation could foster responsive, student-centered strategies. Templates linking common PBL challenges (e.g., uneven group participation) to evidence-based tactics (peer feedback prompts, adaptive scaffolding) would clarify actionable options. Analyzing student reflections on PBL experiences could further sensitize PSTs to learner-centered decision-making, counterbalancing teacher-directed tendencies.

To sum up, the present study contributes to the growing research on teacher noticing by demonstrating how and the extent to which a video-based teacher education program can enhance PSTs' skills in a mathematics PBL context. It highlights the importance of integrating subject-specific features and inquiry-based frameworks into noticing skill development. Furthermore, the study underscores the need to explicitly address decision-making skills in teacher education, an area often neglected in current programs. However, it is difficult to transform noticing skill to instructional practice in the real situation, because not only are the video analysis tasks unable to fully simulate the actual teaching situation, but also teachers with a high-level noticing skill may not address all the issues in classroom in the complex situation. Thus, more attention should be paid to researches on the associations between teacher noticing and instructional practice in the future.

CRediT authorship contribution statement

Sha Li: Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Chunxia Qi:** Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization. **Ruisi Li:** Writing – review & editing, Writing – original draft, Validation, Investigation, Conceptualization. **Ying Jin:** Writing – review & editing, Writing – original draft, Validation, Data curation. **Liuchang Cheng:** Writing – review & editing, Validation, Formal analysis, Data curation. **Guanxiong Liu:** Writing – review & editing, Validation, Formal analysis, Data curation.

Declaration of competing interest

The authors declare no potential conflict of interest with respect to the research, authorship, and/or publication of this article.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.actpsy.2025.104962.

Data availability

Data will be made available on request.

References

Barnhart, T., & van Es, E. (2015). Studying teacher noticing: Examining the relationship among pre-service science teachers' ability to attend, analyze and respond to student thinking. *Teaching and Teacher Education, 45*, 83–93. https://doi.org/10.1016/j.tate.2014.09.005

Barron, B. J. S., Schwartz, D. L., Vye, N. J., Moore, A., Petrosino, A., Zech, L., & Bransford, J. D. (1998). Doing with understanding: Lessons from research on problem- and project- based learning. *Journal of the Learning Sciences, 7*(3–4), 271–311.

Bastian, A., Kaiser, G., Meyer, D., Schwarz, B., & König, J. (2022). Teacher noticing and its growth toward expertise: An expert–novice comparison with pre-service and in-service secondary mathematics teachers. *Educational Studies in Mathematics, 110*(2), 205–232. https://doi.org/10.1007/s10649-021-10128-y

Bhat, B. A., & Bhat, G. J. (2019). Formative and summative evaluation techniques for improvement of learning process. *European Journal of Business & Social Sciences, 7*(5), 776–785.

Blomberg, G., Stürmer, K., & Seidel, T. (2011). How pre-service teachers observe teaching on video: Effects of viewers' teaching subjects and the subject of the video. *Teaching and Teacher Education, 27*(7), 1131–1140. https://doi.org/10.1016/j.tate.2011.04.008

Blömeke, S., Gustafsson, J. E., & Shavelson, R. (2015). Beyond dichotomies: Competence viewed as a continuum. *ZeitschriftF€ urPsychologie, 223*(1), 3–13.

Boss, S., & Larmer, J. (2018). *Project based teaching: How to create rigorous and engaging learning experiences*. Alexandria, VA: ASCD.

Condliffe, B., Quint, J., Visher, M. G., Bangser, M. R., Drohojowska, S., Saco, L., & Nelson, E. (2017). *Project-based learning: A literature review (working paper)*. New York, NY: MDRC.

Eilam, B., & Poyas, Y. (2009). Learning to teach: Enhancing pre-service teachers' awareness of the complexity of teaching–learning processes. *Teachers and Teaching: Theory and Practice, 15*(1), 87–107. https://doi.org/10.1080/13540600802661337

Eliyasni, R., Kenedi, A. K., & Sayer, I. M. (2019). Blended learning and project based learning: The method to improve students' higher order thinking skill (HOTS). *Jurnal Iqra': Kajian Ilmu Pendidikan, 4*(2), 231–248. https://doi.org/10.25217/ji.v4i2.549

Erickson, F. (2011). On noticing teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17–34). Routledge.

Fisher, M. H., Thomas, J., Jong, C., Schack, E. O., & Dueber, D. (2019). Comparing preservice teachers' professional noticing skills in elementary mathematics classrooms. *School Science and Mathematics, 119*(3), 142–149. https://doi.org/10.1111/ssm.12324

Güler, M., Taylan, R. D., Baki, M., Demirel, D., Çelik, D., Güzel, E. B., & Tutak, F. A. (2023). Supporting novice mathematics teachers: The impact of an ementoring and video-based professional development program on teachers' noticing skills. *International electronic journal of. Mathematics Education, 18*(2), Article em0737. https://doi.org/10.29333/iejme/13091

Ivars, P., Fernández, C., & Llinares, S. (2020). A learning trajectory as a scaffold for pre-service teachers' noticing of students' mathematical understanding. *International Journal of Science and Mathematics Education, 18*(3), 529–548. https://doi.org/10.1007/s10763-019-09973-4

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202. https://doi.org/10.5951/jresematheduc.41.2.0169

Kaiser, G., Busse, A., Hoth, J., König, J., & Blömeke, S. (2015). About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. *International Journal of Science and Mathematics Education, 13*(2), 369–387. https://doi.org/10.1007/s10763-015-9616-7

Kanter, D. E. (2010). Doing the project and learning the content: Designing project-based science curricula for meaningful understanding. *Science Education, 94*(3), 525–551. https://doi.org/10.1002/sce.20381

Kleinknecht, M., & Gröschner, A. (2016). Fostering preservice teachers' noticing with structured video feedback: Results of an online-and video-based intervention study. *Teaching and Teacher Education, 59*, 45–56.

Kokotsaki, D., Menzies, V., & Wiggins, A. (2016). Project-based learning: A review of the literature. *Improving Schools, 19*(3), 267–277. https://doi.org/10.1177/1365480216659733

König, J., Blömeke, S., Jentsch, A., Schlesinger, L., née Nehls, C. F., Musekamp, F., & Kaiser, G. (2021). The links between pedagogical competence, instructional quality, and mathematics achievement in the lower secondary classroom. *Educational Studies in Mathematics, 107*, 189–212. https://doi.org/10.1007/s10649-020-10021-0

König, J., Santagata, R., Scheiner, T., Adleff, A. K., Yang, X., & Kaiser, G. (2022). Teacher noticing: A systematic literature review of conceptualizations, research designs, and findings on learning to notice. *Educational Research Review, 36*, Article 100453. https://doi.org/10.1016/j.edurev.2022.100453

Larison, S., Richards, J., & Sherin, M. G. (2024). Tools for supporting teacher noticing about classroom video in online professional development. *Journal of Mathematics Teacher Education, 27*(2), 139–161. https://doi.org/10.1007/s10857-022-09554-3

Larmer, J., & Mergendoller, J. R. (2015). *Gold standard PBL: Essential project design elements*. Novato, CA: Buck Institute for Education.

Magiera, M. T., & Zambak, V. S. (2021). Prospective K-8 teachers' noticing of student justifications and generalizations in the context of analyzing written artifacts and video-records. *International Journal of STEM Education, 8*, 1–21. https://doi.org/10.1186/s40594-020-00263-y

Markham, T., Larmer, J., & Ravitz, J. (2003). *Project based learning handbook: A guide to standards-focused project based learning for middle and high school teachers* (2nd ed.). Novato, CA: Buck Institute for Education.

Mena, J., Hennissen, P., & Loughran, J. (2017). Developing pre-service teachers' professional knowledge of teaching: The influence of mentoring. *Teaching and Teacher Education, 66*, 47–59. https://doi.org/10.1016/j.tate.2017.03.024

Ministry of Education. (2022, April 8). *Mathematics curriculum standards for compulsory education* (2022 ed.). Beijing Normal University Press. Retrieved October 25, 2024, from http://www.moe.gov.cn/srcsite/A26/s8001/202204/t20220420_619921.html.

Mitchell, R. N., & Marin, K. A. (2015). Examining the use of a structured analysis framework to support prospective teacher noticing. *Journal of Mathematics Teacher Education, 18*, 551–575. https://doi.org/10.1007/s10857-014-9294-3

Qi, C., Liu, L., & Zuo, S. (2022). Fostering noticing in prospective teachers through a video-based course: Results of an intervention study from China. *Asian Journal for Mathematics Education, 1*(2), 204–220. https://doi.org/10.1177/27527263221107718

Reeve, J., Bolt, E., & Cai, Y. (1999). Autonomy-supportive teachers: How they teach and motivate students. *Journal of Educational Psychology, 91*(3), 537.

Remijan, K. W. (2017). Project-based learning and design-focused projects to motivate secondary mathematics students. *Interdisciplinary journal of problem-based Learning, 11*(1). https://doi.org/10.7771/1541-5015.1520

Roller, S. A. (2016). What they notice in video: A study of prospective secondary mathematics teachers learning to teach. *Journal of Mathematics Teacher Education, 19*, 477–498. https://doi.org/10.1007/s10857-015-9307-x

Santagata, R., König, J., Scheiner, T., Nguyen, H., Adleff, A. K., Yang, X., & Kaiser, G. (2021). Mathematics teacher learning to notice: A systematic review of studies of video-based programs. *ZDM Mathematics Education, 53*(1), 119–134. https://doi.org/10.1007/s11858-020-01216-z

Santagata, R., Zannoni, C., & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10*, 123–140. https://doi.org/10.1007/s10857-007-9029-9

Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. *Journal of Mathematics Teacher Education, 16*, 379–397. https://doi.org/10.1007/s10857-013-9240-9

Sherin, M. G. (2004). New perspectives on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education* (pp. 1–28). Oxford, UK: Elsevier.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 3–13). Routledge.

Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education, 60*(1), 20–37. https://doi.org/10.1177/0022487108328155

Star, J. R., Lynch, K. H., & Perova, N. (2011). Using video to improve mathematics teachers' abilities to attend to classroom features: A replication study. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 117–133). Routledge.

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*, 107–125. https://doi.org/10.1007/s10857-007-9063-7

Stockero, S. L., & Rupnow, R. L. (2017). Measuring noticing within complex mathematics classroom interactions. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: A hidden skill of teaching* (pp. 281–301). Cham: Springer.

Stockero, S. L., Rupnow, R. L., & Pascoe, A. E. (2017). Learning to notice important student mathematical thinking in complex classroom interactions. *Teaching and Teacher Education, 63*, 384–395. https://doi.org/10.1016/j.tate.2017.01.006

Strobel, J., Wang, J., Weber, N. R., & Dyehouse, M. (2013). The role of authenticity in design-based learning environments: The case of engineering education. *Computers & Education, 64*, 143–152. https://doi.org/10.1016/j.compedu.2012.11.026

Thomas, J. W. (2000). *A review of research on project-based learning*. San Rafael, CA: Autodesk Foundation.

van Es, E. A., Cashen, M., Barnhart, T., & Auger, A. (2017). Learning to notice mathematics instruction: Using video to develop preservice teachers' vision of ambitious pedagogy. *Cognition and Instruction, 35*(3), 165–187. https://doi.org/10.1080/07370008.2017.1317125

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571–596. https://www.learntechlib.org/primary/p/9171/.

van Es, E. A., & Sherin, M. G. (2006). How different video club designs support teachers in "learning to notice". *Journal of Computing in Teacher Education, 22*(4), 125–135. https://doi.org/10.1080/10402454.2006.10784548

van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers'"learning to notice" in the context of a video club. *Teaching and Teacher Education, 24*(2), 244–276. https://doi.org/10.1016/j.tate.2006.11.005

van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM Mathematics Education, 53*, 17–27. https://doi.org/10.1007/s11858-020-01211-4

Weyers, J., Koenig, J., Scheiner, T., Santagata, R., & Kaiser, G. (2024). Teacher noticing in mathematics education: A review of recent developments. *ZDM Mathematics Education, 56*, 249–264. https://doi.org/10.1007/s11858-023-01527-x

Yang, X., Kaiser, G., König, J., & Blömeke, S. (2019). Professional noticing of mathematics teachers: A comparative study between Germany and China. *International Journal of Science and Mathematics Education, 17*, 943–963. https://doi.org/10.1007/s10763-018-9907-x

Yang, X., König, J., & Kaiser, G. (2021). Growth of professional noticing of mathematics teachers: A comparative study of Chinese teachers noticing with different teaching experiences. *ZDM Mathematics Education, 53*(1), 29–42.

Zhang, L., & Ma, Y. (2023). A study of the impact of project-based learning on student learning effects: A meta-analysis study. *Frontiers in Psychology, 14*, 1202728. https://doi.org/10.3389/fpsyg.2023.1202728

Zuo, S., Liu, L., & Qi, C. (2024). Using video to develop pre-service teachers' noticing within a mathematical modelling context. *EURASIA Journal of Mathematics, Science and Technology Education, 20*(5), Article em2441. https://doi.org/10.29333/ejmste/14466