A Hybridized Stochastic SIR-Vasiček Model in Evaluating a Pandemic Emergency Financing Facility

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***Abstract*—In the context of the present global health crisis, we examine the design and valuation of a pandemic emergency financing facility (PEFF) akin to a catastrophe (CAT) bond. While a CAT bond typically enables fund generation to the insurers and re-insurers after a disaster happens, a PEFF or pandemic bond's payout is linked to random thresholds that keep evolving as the pandemic continues to unfold. The subtle difference in the timing and structure of the funding payout between the usual CAT bond and PEFF complicates the valuation of the latter. We address this complication, and our analysis identifies certain aspects in the PEFF's design that must be simplified and strengthened so that this financial instrument is able to serve the intent of its original creation. An extension of the compartmentalized deterministic epidemic model—which describes the random number of people in three classes: susceptible (S), infected (I), and removed (R) or SIR for short—to its stochastic analog is put forward. At time t, $S(t)$, $I(t)$, and $R(t)$ satisfy a system of interacting stochastic differential equations in our extended framework. The payout is triggered when the number of infected people exceeds a predetermined threshold. A CAT-bond pricing setup is developed with the Vasiček-based financial risk factor correlated with the SIR dynamics for the PEFF valuation. The probability of a pandemic occurrence during the bond's term to maturity is calculated via a Poisson process. Our sensitivity analyses reveal that the SIR's disease transmission and recovery rates, as well as the interest rate's mean-reverting level, have a substantial effect on the bond price. Our proposed synthesized model was tested and validated using a Canadian COVID-19 dataset during the early development of the pandemic. We illustrate that the PEFF's payout could occur as early as seven weeks after the official declaration of the pandemic, and the deficiencies of the most recent PEFF sold by an international financial institution could be readily rectified.**

***Index Terms*—Catastrophe bond, infectious-disease modeling, pandemic-trigger events, Poisson process, sensitivity analysis, Vasiček model.**

I. INTRODUCTION

CATASTROPHE (CAT) bonds are used by insurers as an alternative to traditional catastrophe reinsurance. They are structured as floating-rate bonds whose principal and coupons are partially or totally lost if specified trigger conditions are met. If triggered, the principal and coupons are paid to the sponsor. The triggers are linked to major natural catastrophes (e.g., earthquake, hurricane, tsunami and epidemic/pandemic).

We put forward a valuation framework for a pandemic bond. A synthesis of the Vasiček model [37] and a stochastic Kermack-McKendrick (KM) epidemic model [19] is developed with a general correlation structure. We provide ways to capture the probabilities of two types of occurrences: 1) pandemic over a certain period of time and 2) trigger activation for the pandemic bond. The synthesized model is implemented via a Monte Carlo simulation, and consequently, the bond price is determined. The proposed model is validated using real data on the evolution of Canadian COVID-19 infected cases and appropriate yields of the Canadian T-notes.

We recall some of the major pandemics in the world within the last century. The Asian pandemic flu of 1957 and 1958 was a global showing for influenza. The disease claimed more than one million lives. This pandemic was caused by a blend of avian flu viruses. In spring 2009, a new strain of H1N1 caused the swine flu pandemic, which originated in Mexico and then spread to the rest of the world. Some 1.4 billion people across the globe were infected within a year and between 1 51 700 and 5 75 400 people were killed. Between 2014 and 2016, Ebola ravaged West Africa with 28 600 reported cases, and 11 325 were dead. For the ongoing COVID-19 pandemic, we have so far more than 150 million confirmed cases worldwide with some three million deaths. It has caused business shutdown for many months globally.

Catastrophic risk has become an increasing focus for those involved in risk management largely due to the outbreak of the COVID-19 pandemic across the world. The essence of the catastrophic risks is of low frequency but high-loss severity. Traditionally, insurance companies hedge and transfer catastrophic risks by means of reinsurance contracts. However, such contracts are often less cost-effective due to the unpredictable nature of large catastrophic losses.

As both the financial demands and the difficulties of covering catastrophic losses by reinsurance companies significantly increase, securitization methods are considerably appropriate to protect and fit the needs of vulnerable individuals. With securitization mechanisms, insurance companies have a way to alleviate part of the catastrophic risks providing them with much-needed liquidity during a health crisis. Issuing CAT bonds is one of the common options to transfer the

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financial losses of catastrophic events from issuers to investors. A CAT bond aids in spreading the risks to the level of global financial markets. Investors of CAT bonds are offered an attractive investment rate in return for taking on a specific set of risks arising from a specified catastrophe. The investors will lose the principal they invested when the predefined catastrophic event occurs. The issuer (often insurance or reinsurance companies) will use that money to cover their losses.

There are three parties in a basic CAT-bond structure: a sponsor (insurer or re-insurer), a special purpose vehicle (SPV), and bond investors. The sponsor pays premiums to the SPV in order to obtain the coverage provided by the SPV. The SPV then issues CAT bonds to capital-market investors. Hence, the risk of disasters to sponsors can be mitigated by shifting risks to bond investors. The bond investors will receive a higher return in case the specified trigger events do not occur. Unfortunately, the bond investors will lose partially or fully the bond's face value and interest income when the predefined disaster risk materializes. Such CAT-bond instruments attract investors who are willing to take higher risks.

In particular, the SPV must manage the risk transfer between the sponsor and the CAT-bonds investors. Under the regular terms of such a deal, the sponsor needs to pay annual premiums to the SPV. The SPV invests the money that is raised from selling the bonds in a financial instrument, say the U.S. Treasury Money-Market Fund. Normally, the SPV is allowed to access broader financial markets and give investors the opportunity to earn an attractive return in exchange for assuming catastrophe-insurance risks.

The payoff from a CAT bond is linked to a predefined trigger event. There are three types of triggers commonly used in the CAT bond market: indemnity trigger, parametric trigger, and industry-claims index. The indemnity trigger is directly linked to the sponsor's actual damage cost as the triggering event after a specified catastrophe happens in a given region and line of business. The parametric trigger is designed based on the occurrence of a natural disaster, such as an earthquake exceeding a magnitude of 7.0 or the number of infected people during a pandemic exceeding a predefined threshold limit. This is easy to understand for investors and create the lowest moral-hazard risk. The industry-claims index is defined by the losses of the industry rather than the individual issuer's losses. This eliminates the issuer's moral hazard and brings greater transparency in loss estimation and shorter processing time although it makes it possible for an issuer to take extra basis risk (i.e., the difference between payout and actual loss).

When an industry trigger is applied for the catastrophe event in a CAT bond, an independent third-party catastrophe modeler will be in charge to determine the amount of the insurance industry losses covered under the deal. If the estimated industry losses are huge, it is possible to completely wipe out the investor's principal, which will be kept to compensate for the sponsor's disaster-risk coverage. On average, CAT bonds with indemnity triggers take two to three years to payout following a triggering loss, compared with three months for CAT bonds with industry-loss or parametric triggers. In this article, we will use the parametric trigger for ease of implementation and the above-mentioned practical considerations.

Despite the rising popularity of CAT bonds, the number of previous studies devoted to CAT-bond pricing, combining the modeling of its underlying trigger variables and a suitable valuation approach, is still rather limited. We take the view that the financial model must be incorporated in the modeling of a given catastrophe. In doing so, the results will be more beneficial from the practitioners' perspective.

Assessing catastrophe risks in relation to CAT-bond pricing will entail an incomplete-market framework (see Harrison and Kreps [17]). This is because it is not possible to replicate payoffs dependent on catastrophe risk by a portfolio of primitive securities. Such an assessment was in agreement with Cox *et al.* [6], Cox and Pedersen [7], and Vaugirard [38]. Young [40] further explained that there is no universal pricing theory, which can successfully resolve the issues of hedging-strategy specification or pricing robustness in an incomplete market. Attempts to address the said problem due to market incompleteness employed different approaches. For example, the Wang transform put forward in [39] was used and adopted by Galeotti *et al.* [13], Lin and Cox [25], and Pelsser [31]. Froot and Posner [12] created an equilibrium-pricing model that deals with uncertain parameters in multi-event risks. As well, a minimal-martingale measure for option pricing was introduced by Follmer and Schweizer [11], and a variance-optimal-martingale measure was pioneered by Schweizer [36].

Another common technique used in an incomplete-market environment is the principle of equivalent utility in carrying out indifferent pricing. Young [40] calculated the price of a contingent claim under a stochastic interest rate for an exponential utility function. A more complex payment structure based on the assumption of utility indifference was suggested by Egami and Young [10]. A CAT-bond model based on consumption was applied in Dieckmann [9], while an intertemporal equilibrium framework was used by Zhu [43] to ascertain the premium spread. Braun [3] analyzed bond premiums using the ordinary least-squares (OLSs) regression with robust standard errors. Among those contributions mentioned above, Cox and Pedersen [7] used a time-repeatable representative agent utility that was based on a model of the term structure of interest rates and a probability structure for catastrophe risks. It was assumed that the agent operates under a utility function in making choices about consumption streams. Such theoretical results were applied to Morgan Stanley, Winterthur, USAA, and Winterthur-style bonds. A similar setting for multiple-event CAT bonds was used for the first time by Reshetar [34]. Zimbidis [44] adopted the Cox–Pedersen framework [7] with the use of the equilibrium-pricing theory in the valuation of the Greek Government CAT Bond.

Stochastic processes were commonly used in pricing CAT bonds in several studies. One of the approaches under a continuous-time setup is to model the probability of credit default through the pricing of credit derivatives in finance. Baryshnikov *et al.* [2] presented, in continuous time, the no-arbitrage price of zero-coupon and non-zero coupon CAT bonds that incorporated a compound doubly stochastic Poisson process. One of the assumptions in [2] is that the risk-neutral measure and real-world measures coincide. Burnecki and Kukla [4] modified such an assumption and applied their results in the calculation of the arbitrage-free prices for zero-coupon and coupon-CAT bonds. The CAT-bond values were also computed by using loss data from the Property Claim Services data source when the flow of events is governed by an inhomogeneous Poisson process; see further Burnecki *et al.* [5]. Härdle and Cabrera [15] utilized the above method to CAT-bond pricing whose underlying is linked to earthquakes in Mexico. A simple closed-form CAT-bond solution with a LIBOR's term structure is presented by Jarrow [18].

Following Lin and Cox [26], a Markov-modulated Poisson process was employed to model catastrophe occurrences using a similar approach to that of Vaugirard [38]. In [24], a combined model of default, moral-hazard, and basis risks with stochastic interest rate was explored. Perez-Fructuoso [32] developed a CAT bond with indexed-link triggers. A mixed approximation method was proposed in [27] to simplify the distribution of aggregate losses and find the numerical solutions of CAT bonds with general pricing formulas. In addition, Nowak and Romaniuk [30] expanded Vaugirard's model and obtained CAT-bond prices using Monte Carlo simulations with different payoff functions and spot-interest rates.

Motivated by the above research undertakings, we make use of the Poisson process in this article to model the occurrences of pandemics. With the SIR model, pandemic events are simulated in regard to the number of susceptible and infected individuals. Most importantly, we make the Vasiček interest rate model correlated with the SIR model so that the interest rate can have a dependence on the behavior of a pandemic-event development. In addition, we define a parametric trigger embedded within the SIR model, which is the threshold for the number of infections, and monitor the activation status of the trigger throughout the entire Monte Carlo simulation experiment. As a result, we are able to use the empirical probability via simulation in the calculation of CAT-bond prices with different payoff functions and spot-interest rates.

On a high-level perspective, our motivation centers on bringing a greater understanding of a pandemic bond in the context of the COVID-19 event. This type of financial product, which is partly insurance, bond, swap, and cash grant, was sold by the World Bank (WB) to private-sector investors. It was WB's solution to attain global preparedness by building up public health care systems. Thus, it was meant to increase capacity in managing outbreaks through the creation of sustainable medical and clinical infrastructure, emergency response service, and extensive worker recruitment and training. Building this capacity and having robust health systems, however, require capital.

Although the emphasis of this article is the valuation of pandemic emergency financing facility (PEFF) and framework for the management of associated risks, a model for the evolution of COVID-19 infections and deaths is needed. To design and eventually market a PEFF, a starting modeling framework for the valuation and risk measurement is necessary. The SIR model is simple having only three compartments, and it is being employed here to showcase the fundamental ideas in transferring the risk from the poor countries into the capital market. It is worth noting that, more recently, there are various models aimed at modeling and forecasting the number of COVID-19 infections and deaths daily and cumulatively. Undoubtedly, the SIR's power to capture the evolving number of infections and deaths could be enhanced by recent model innovations and extensions. More sophisticated models could replace the SIR model in conjunction with other interest-rate models. In [21], their susceptible-asymptomatic-reported infected-unreported infected-quarantine-recovered (SAIUQR)-model simulations suggested that those compartments covering quarantine, reported, and unreported symptomatic individuals, as well as government intervention polices, such as media effect, lockdown, and social distancing are key in the mitigation of COVID-19 transmission.

There are other potential models that could be useful in describing the daily number of infections and deaths in lieu of the SIR model. The SAIU model put forward by Samui *et al.* [35] indicated that the disease transmission rate is important to control when attempting to contain the disease's basic reproduction number. A susceptible-exposed-infectious-recovered (SEIR) compartment model, refined by contact tracing and hospitalization, was developed in [22] where short-term prediction revealed increasing trend and long-term prediction signified oscillatory dynamics of COVID-19 cases (see [23]). Rai *et al.* [33] compartmentalized the population into susceptibles (S), exposed (E), symptomatic individuals (I_s), asymptomatic individuals (I_a), home quarantined asymptomatic individuals (Q), aware individuals (A), hospitalized (H), and recovered (R). Under the SEI$_s I_a$QAHR, it was found that non-pharmaceutical interventions should be implemented in an effort to decrease the basic reproduction number.

This article is organized as follows. In Section II, we introduce the Vasiček interest rate model with a correlation that links it explicitly to the SIR model. The main formulas for the pertinent event probabilities are also established. The synthesized mathematical model serves as the framework in our CAT-bond valuation. Our numerical solutions to price a three-year (maturity chosen in most test cases) zero-coupon CAT bond through the Monte-Carlo simulations are illustrated in Section III. This also includes sensitivity analyses that explore how CAT-bond prices vary with the changes in various model parameters. The SIR model is applied to the COVID-19 development data in Canada in Section IV. The purpose is to gauge the model's ability in replicating certain the salient features of the data and value a pandemic CAT bond. Section V summarizes our major findings along with some remarks on the natural directions of this research.

II. Mathematical Modeling and CAT Bond Pricing

A. Integrated SIR-Vasiček Model

The KM model is commonly known as the SIR model since it describes the dynamics of the numbers of suspected (S), infected (I), and removed (R) individuals at any time t for a population of size N. See [1] for the review and motivation of the original and deterministic model for the transmission dynamics of viral and bacterial infectious agents within population of hosts.

With $r(t)$ as notation for interest rate, our model combining SIR and Vasiček processes is

$$\begin{aligned}
\mathrm{d}r(t) &= \kappa(\theta - r(t))\mathrm{d}t + \sigma_r \mathrm{d}X(t) \\
\mathrm{d}S(t) &= -[\beta S(t)I(t)/N]\mathrm{d}t - \sqrt{\beta S(t)I(t)/N}\mathrm{d}Y(t) \\
\mathrm{d}I(t) &= [\beta S(t)I(t)/N - \gamma I(t)]\mathrm{d}t + \sqrt{\beta S(t)I(t)/N}\mathrm{d}Y(t) \\
&\quad - \sqrt{\gamma I(t)}\mathrm{d}Z(t) \\
\mathrm{d}R(t) &= \gamma I(t)\mathrm{d}t + \sqrt{\gamma I(t)}\mathrm{d}Z(t)
\end{aligned} \tag{1}$$

where $X(t)$ is a standard Brownian motion (BM) correlated with $Y(t)$ and $Z(t)$. Both $Y(t)$ and $Z(t)$ are independent BMs. In order to have a consistent correlation matrix, the BMs' dynamics are defined as follows:

$$\begin{aligned}
\mathrm{d}X(t) &= \mathrm{d}W_1(t) \\
\mathrm{d}Y(t) &= \rho_{12}\mathrm{d}W_1(t) + \sqrt{1-\rho_{12}^2}\mathrm{d}W_2(t) \\
\mathrm{d}Z(t) &= \rho_{13}\mathrm{d}W_1(t) + \rho_{23}\mathrm{d}W_2(t) + \sqrt{1-\rho_{13}^2-\rho_{23}^2}\mathrm{d}W_3(t),
\end{aligned}$$

where $W_1(t)$, $W_2(t)$, and $W_3(t)$ are independent standard BMs under the objective measure $\mathbb{P}$. The correlation coefficients ρ_{12} and ρ_{13} are chosen in such a way that $|\rho_{23}| \leq 1$, and

$$\rho_{23} = \frac{-\rho_{12}\rho_{13}}{\sqrt{1-\rho_{12}^2}}. \tag{2}$$

To express the SIR-Vasiček model more compactly and noting that $\mathbf{R(t)}$ is contained in $\mathbf{I(t)}$, the vector–matrix representation of the model is given as follows:

$$d\mathbf{G}(t) = (\mathbf{e} + \mathbf{A}\mathbf{G}(t))dt + \Sigma d\mathbf{W}(t) \tag{3}$$

where with $\eta(t) = \beta S(t)I(t)/N$ and $\mathbf{W}(t) = [W_1(t),\ W_2(t),\ W_3(t),\ 0]^\top$.

B. Trigger Process

The number of infected persons constitutes the trigger or threshold with a parametric form. The trigger event occurs when the number of infected persons exceeds the threshold. On or before the maturity date T, there are two scenarios: the threshold is surpassed or not surpassed. The payoff is structured in which if the trigger level is not exceeded, the investors get the face value M and coupons c. Otherwise, the investors receive αM and αc, where α is the recovery rate and $0 \leq \alpha < 1$.

C. Pandemic Probability in Valuation

Write $H{:}=$ pandemic event and $H'{:}=$ no pandemic. Also, write $C{:}=$ trigger is activated, and $C'{:}=$ trigger not activated. By the law of total probability,

$$\begin{aligned}
\mathbb{P}(C) &= \mathbb{P}(C|H)\cdot\mathbb{P}(H) + \mathbb{P}(C|H')\cdot\mathbb{P}(H') \\
\mathbb{P}(C') &= \mathbb{P}(C'|H)\cdot\mathbb{P}(H) + \mathbb{P}(C'|H')\cdot\mathbb{P}(H').
\end{aligned}$$

It is assumed that $H \sim \text{Poisson}(\lambda)$. It should be the case that $\mathbb{P}(C)+\mathbb{P}(C') = 1$ with both probabilities computed separately. If $n =$ number of pandemic occurrences in a century and T (in years) is the bond's maturity, then $\lambda = (nT/100)$.

Therefore, $\mathbb{P}(H') = e^{-\lambda}$, and the probability of at least one pandemic is $\mathbb{P}(H) = 1 - e^{-\lambda}$. The probability of trigger activation is empirically calculated through MC simulation based on SIR-Vasiček model. The total number of sample paths, wherein the trigger was activated, is examined. Thus, the probability of trigger activated during a pandemic is

$$\mathbb{P}(C|H) = \frac{\text{no. of } I(t) \text{ paths w/ trigger activated}}{\text{total no. of simulated paths}}. \tag{4}$$

D. Bond's Payout Structure

The pandemic bond price $B_{\text{pdmic}}(t,T)$ is

$$\begin{aligned}
&B_{\text{pdmic}}(t,T) \\
&\quad = E^{\mathbb{Q}}\left[\mathrm{M}e^{-\int_t^T r(s)ds}|\mathcal{F}_t\right]\cdot\mathbb{Q}\left(I(t) < I_j^B\right) \\
&\qquad + E^{\mathbb{Q}}\left[\alpha \mathrm{M}e^{-\int_t^{\tau_c} r(s)ds}|\mathcal{F}_t\right]\cdot\mathbb{Q}\left(I(t) \geq I_j^B\right)
\end{aligned} \tag{5}$$

where $\mathcal{F}_t$ is the joint filtration generated by $I(t)$ and $r(t)$, and $\mathbb{Q}$ is a pricing measure. Table I summarises the bond's payout structure. The market in this framework is incomplete; in other words, there is no unique risk-neutral pricing measure. We follow the actuarial approach suggested in [16], and considering that the product being valued is a form of an insurance contract, the pricing is carried under the objective measure $\mathbb{P}$ with an appropriate adjustment for the risk premium, which relates $\mathbb{P}$ to $\mathbb{Q}$.

For simplicity, our approach is consistent with a zero-risk premium assumption for the underlying catastrophe risks. This approach is justified by Cox *et al.* [6], Cox and Pedersen [7], Cummins and Geman [8], Lee and Yu [24], and Ma and Ma [27]. The zero-risk premium principle is argued in Merton [28] under an option-pricing context where the jump components of stock prices are likely caused only by company-specific events, and hence, these jump risks are not impacting the market. When this is the case, no risk premium is required.

Similarly, in CAT-bond valuation, it may be argued that the underlying catastrophic risks only have marginal influence on the overall economy. Hence, there is no systematic risk to the market. Thus, risk premiums could be set to zero; see further [24]. When localized catastrophes may cause the substantial and possibly systematic effect to the market, the treatment of non-zero risk premium is provided in Gürtler *et al.* [14]. Still, as the emphasis in this paper is to demonstrate how the integrated stochastic SIR-Vasiček model enables CAT-bond pricing, we suppose that the CAT-bond cash flows depend mainly on the catastrophic risk variables under the objective measure.

III. Numerical Demonstration

Five thousand simulated sample paths of the interest rate process and the number of susceptible and infected cases were generated. We shall also calculate the CAT-bond prices using the Monte Carlo simulation on the basis of the payoff structures and the CAT-bond pricing representation derived in

TABLE I
PAYOUT STRUCTURE

Trigger timing	CAT bond is triggered or not	Condition	Pay out
Not triggered	No	$I(t) < I_j^B$	M
before or at time T	Yes	$I(T) \geq I_j^B$	$\alpha \cdot M$

TABLE II
BASELINE-CASE PARAMETERS: VASIČEK MODEL

Parameter	Explanation	Value
r_0	initial interest rate	0.05
θ	long-term-mean level	0.1
κ	speed of reversion	0.2
σ_r	instantaneous volatility	0.25

Section II. In addition, sensitivity analyses are conducted to examine the attributes of the correlated Vasiček-SIR model designed to support the CAT-bond valuation. We assign model parameter values as baseline for comparison and further testings. We display in Table II the parameter values adopted in [42] for the interest rate-process, while Table III exhibits the chosen model parameters for the SIR model together with the correlation coefficients. Fig. 1 depicts the $I(t)$'s evolution showing several trends given certain parameter levels.

The parameters for the CAT-bond calculation are presented in Table IV. Using the parameter values from Tables II–IV, our 5000 sample-path simulations yield the numerical bond pricing results in Table V.

The bond price's behavior as parameters vary is examined. Fig. 2 shows the bond price movement as the correlation and volatility of interest rate change. The parameters β and γ are also varied, and the corresponding bond price's pattern is illustrated in Fig. 3. In Fig. 4, the pattern in the bond price is displayed in response to the changing correlation and bond's maturity. $\mathbf{G(t)}$ and Σ are shown at the bottom of the page.

IV. CASE STUDY: BENCHMARKING WITH THE OBSERVED CANADIAN COVID-19 PANDEMIC DATA

In this section, we customize our SIR model in an attempt to replicate the COVID-19 development in Canada. There are four major parameters in the SIR model. We set the values of these parameters in Table VI in accord with the actual Canadian COVID-19 data.

Regarding the infection and recovery rates, there are no official data that could match the definition in the SIR model. Thus, we simply set $\beta = 2$, meaning that the virus carrier could infect two persons daily. The recovery rate γ is assumed to be a little less than β for an outbreak to be observed.

From Fig. 5, the maximum number of active cases is around 35 000 happening at about $t = 105$ days. The data for the COVID-19 active cases in Canada, in actuality, are plotted in Fig. 6; the dataset was obtained from Worldometer (source: https://www.worldometers.info/coronavirus/country/canada/). The maximum number of active cases is 35 014, which happened on May 30, 2020, i.e., 103 days after the starting date (assumed February 15, 2020). Hence, we see that, by setting the parameters properly, our model could recreate closely the pandemic-cases situation unfolding in real life.

We now consider the CAT-bond price calculations with $T = 3$ based on the Canadian COVID-19 scenario. To determine the occurrence probability of COVID-19, a particular type of pandemic, we learned that there were six pandemics in the past century. They are the Spanish Flu (1918–1920), Asian Flu (1957–1958), AIDS (1981-present), H1N1 Swine Flu (2009–2010), West African Ebola (2014–2016), and Zika virus (2015–present). Taking into account the increasing frequency of pandemics in recent centuries, we add a buffer in our case study, by increasing the pandemic occurrences from 6 to 9. From the Poisson modeling discussed in the previous section, we get

$$\lambda = \frac{mT}{100} = \frac{(9)(3)}{100}. \tag{6}$$

Thus, the probability of no pandemic during the three-year life of the CAT-bond is

$$\mathbb{P}(H') = e^{-0.27} \quad \text{and } \mathbb{P}(H) = 1 - e^{-0.27}. \tag{7}$$

We also need to define the trigger threshold in terms of active cases or $I(t)$ in our model. The World Health Organization declared COVID-19 a pandemic on March 11, 2020 citing over 1 18 000 cases of coronavirus illness in over 110 countries

$$\mathbf{G(t)} = \begin{bmatrix} r(t) \\ S(t) \\ I(t) \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} \kappa\theta \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -\kappa & 0 & 0 \\ 0 & -\beta I(t)/N & 0 \\ 0 & \beta I(t)/N & -\gamma \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_r & 0 & 0 \\ -\rho_{12}\sqrt{\eta(t)} & -\sqrt{\left(1-\rho_{12}^2\right)\eta(t)} & 0 \\ \rho_{12}\sqrt{\eta(t)} - \rho_{13}\sqrt{\gamma I(t)} & \sqrt{\left(1-\rho_{12}^2\right)\eta(t)} - \rho_{23}^{*}\sqrt{\gamma I(t)} & -\sqrt{\left(1-\rho_{13}^2-\rho_{23}^{*2}\right)\gamma I(t)} \end{bmatrix}$$

TABLE III
BASELINE-CASE PARAMETERS: SIR

Parameter	Explanation	Value
N	total population	380000
$I(0)$	initial number of infections	38
β	infection rate	$\frac{1.1}{12}$
γ	recovery rate	$\frac{1}{12}$
ρ_{12}	correlation coefficient between number of susceptibles and interest rate	0.5
ρ_{13}	correlation coefficient between number of infections and interest rate	0.7
trigger	value of trigger threshold in the number of infections	1700

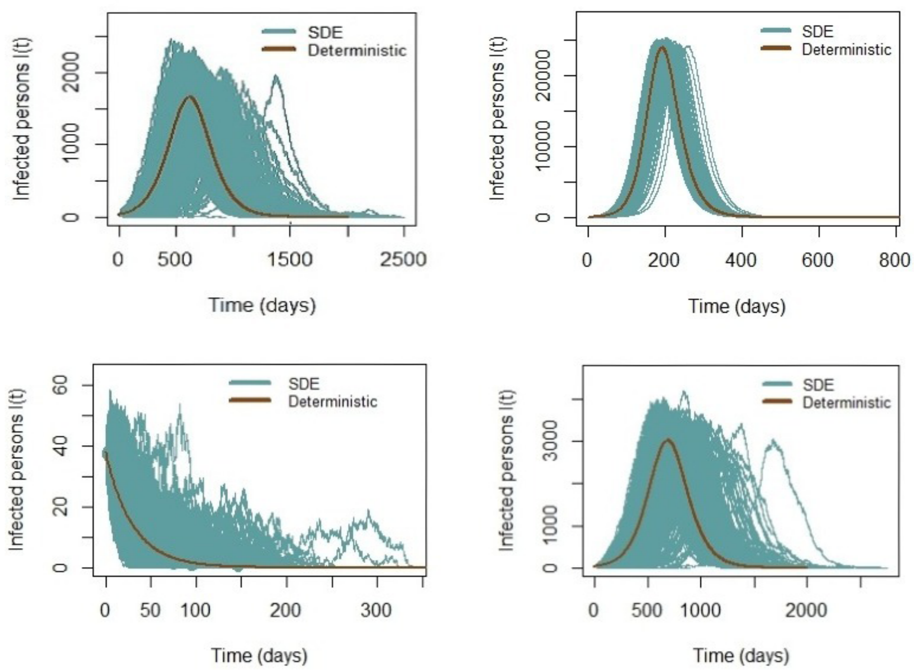

Fig. 1. Illustrating the evolution of $I(t)$. Upper left panel's parameters: $N = 3\,80\,000$, $I(0) = 38$, $\beta = 1.1/12$, $\gamma = 1/12$, $\rho_{12} = 0.5$, and $\rho_{13} = 0.7$. Upper right panel's parameters: $N = 3\,80\,000$, $I(0) = 38$, $\beta = 1.5/12$, $\gamma = 1/12$, $\rho_{12} = 0.5$, and $\rho_{13} = 0.7$. Lower left panel's parameters: $N = 3\,80\,000$, $I(0) = 38$, $\beta = 1.1/12$, $\gamma = 1/8$, $\rho_{12} = 0.5$, and $\rho_{13} = 0.7$. Lower right panel's parameters: $N = 7\,00\,000$, $I(0) = 38$, $\beta = 1.1/12$, $\gamma = 1/12$, $\rho_{12} = 0.5$, and $\rho_{13} = 0.7$.

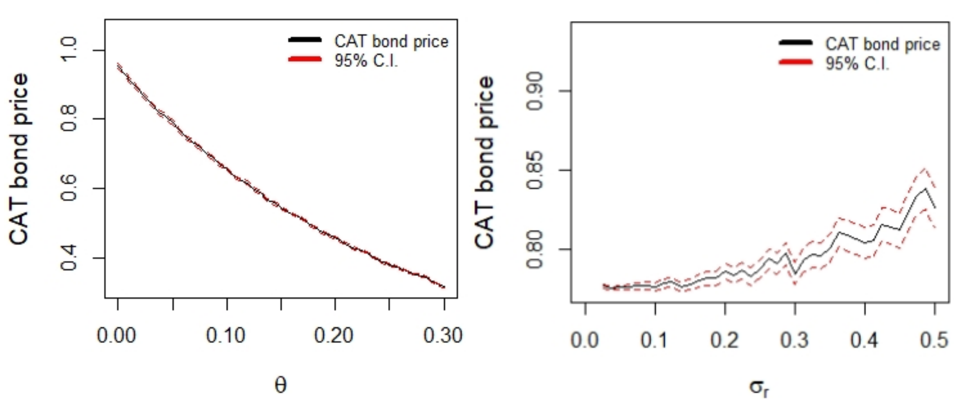

Fig. 2. Bond price's trend as parameters change (θ and σ_r).

and territories around the world. Thus, we set the average of $I(t)$ as the threshold in our model, which is 1072. For all other parameters, we rely on those set in our baseline-case experiment. To facilitate the presentation, the parameter values

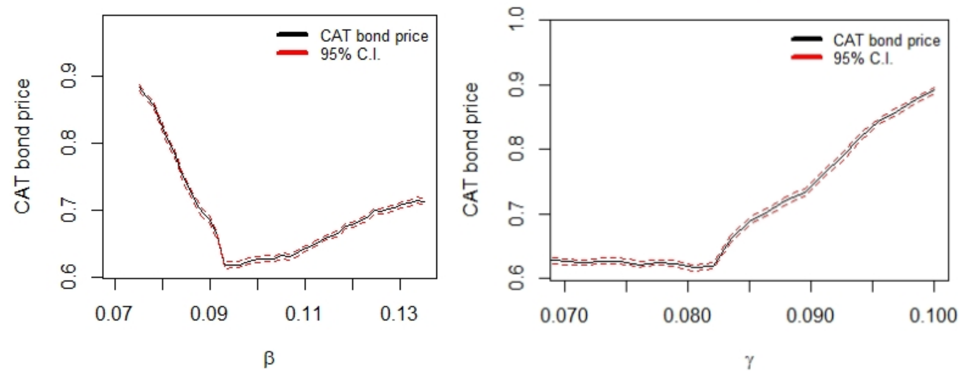

Fig. 3. Bond price's trend as parameters change (β and γ).

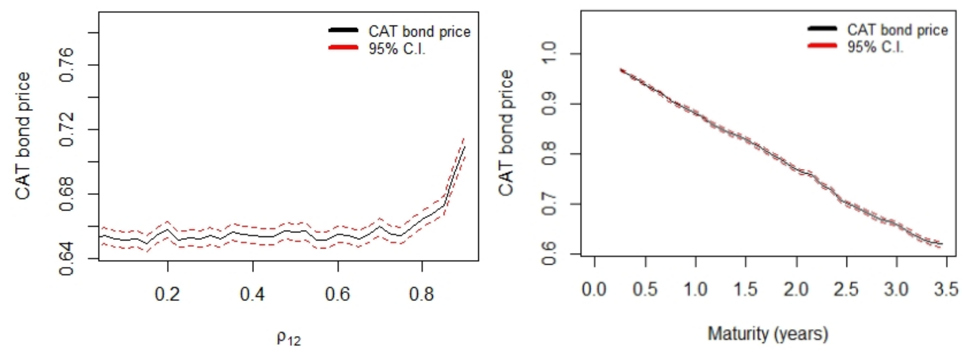

Fig. 4. Bond price's trend as parameters change (correlation and maturity).

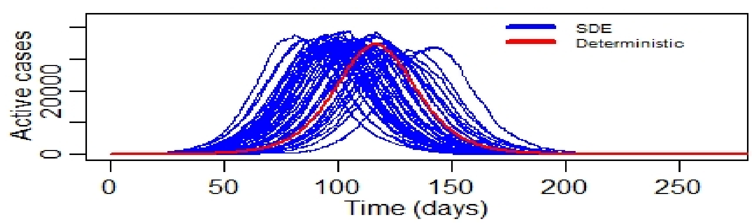

Fig. 5. Replicating the Canadian COVID-19 development data.

TABLE IV
BASELINE-CASE PARAMETERS: CAT BOND

Parameter	Explanation	Value
T	maturity (in years)	3
α	recovery rate	0.5
M	face value	1

for this case study are summarized in Table VII. We run our proposed Vasiček-SIR model 5000 times. The results are tabulated in Table VIII.

Our simple Vasiček-SIR model indicates that a bond's payout can be attained in about seven weeks on average after the pandemic. This is in stark contrast with the World Bank's PEFF that took about 3.5 months to pay out. The simple trigger that we set together with its transparency linked to the development of $I(t)$ made a big difference in the payout timing. This timing is critical as the pandemic could be contained if early and coordinated countermeasures are supported and financed.

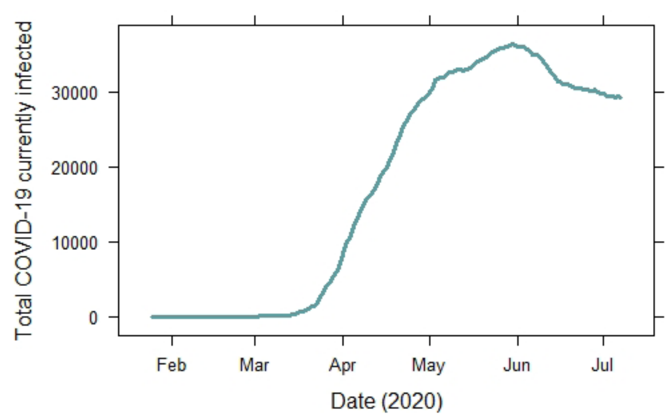

Fig. 6. Actual number of active cases of COVID-19 in Canada in 2020.

TABLE V

BOND PRICING RESULTS FOR $M = 1$, $\alpha = 0.5$, AND $T = 3$

Quantity's description	**Value**
τ_c - time at which trigger is activated	**50 (days)**
Total no. of triggered events	1296 out of 5000
$\mathbb{P}(C\|H)$:= prob of a trigger event conditional on pandemic	0.2592
$\mathbb{P}(H)$:= probability of a pandemic	0.23662
$\mathbb{P}(C)$	0.06133
$\mathbb{P}(C^{'})$	0.93867
Pandemic-bond price	**0.942976**
Standard error	0.000828

TABLE VI

MODEL PARAMETERS BASED ON CANADIAN DATA IN THE SIMULATION OF THE SIR MODEL

Parameter	**Explanation**	**Value**
N	Population in SIR model. We use the current population figure for Canada.	37,590,000
$I(0)$	Initial number of infections in SIR model. The first case report in Canada was on 15 Feb 2000 with 7 active cases.	7
β	Infection rate per day in SIR model	2
γ	Recovery rate in SIR model	1.915

TABLE VII

PARAMETERS OF CASE STUDY

Parameter	**Explanation**	**Value**
frequency (m)	average number of pandemics per century	9
trigger	trigger threshold - number of infected persons	1072
T	maturity (in years)	3
R_0	initial interest rate	0.05
θ	long-term mean interest-rate level	0.1
κ	speed of mean reversion	0.2
σ	instantaneous volatility	0.25
β	infection rate in SIR model	2
γ	recovery rate in SIR model	1.915
ρ_{12}	correlation coefficient between $S(t)$ and $r(t)$	0.5
ρ_{13}	correlation coefficient between $I(t)$ and $r(t)$	0.7

TABLE VIII

EMPIRICAL RESULTS OF CASE STUDY

Explanation	**Value**
τ_c - first time that trigger was activated	50 (days)
Total number of triggered events	1296 out of 5000
Probability of trigger events $\mathbb{P}(C\|H)$	0.2592
Probability of a pandemic $\mathbb{P}(H)$	0.23662
$\mathbb{P}(C)$	0.06133
$\mathbb{P}(C^{'})$	0.93867
CAT-bond price ($)	0.942976
Standard error	0.000828

Furthermore, the CAT-bond price is $0.942976, which is a little less than the CAT-bond price ($0.951997) without the correlation between $r(t)$ and SIR variables. The probability of a trigger event $\mathbb{P}(\mathbb{C})$ is 0.06133, which is small but not entirely negligible. This explains that the trigger event activation does not have a lot of impact on the CAT-bond-price calculation. However, when the threshold is crossed, there is a massive effect arising from the recovery rate α and, to a certain degree, some impact from $\mathbb{P}(\mathbb{C})$.

V. CONCLUSION

We highlight the practical significance of this study. First, our simple Vasiček-SIR model revealed that on average a bond's payout occurs about seven weeks after the pandemic. This offers a better alternative framework than that of the much-criticized World Bank's PEFF that took about 3.5 months to pay out. Second, our study illustrated that the risk factors that have the most impact on the bond price, in order of magnitude, are β, γ, and θ. These concomitant factors are associated with the reproductive number $R_0 = (\beta/\gamma)$, which indicates how contagious an infectious disease is. Third, this article provided the formulation and implementation demonstration of a much more realistic modeling setup with easily understood structure and payout mechanism. For the pandemic bond to be successful in its intent to provide the needed aid when the contingency arises, the proposed criteria for the payout trigger must be transparent and less burdensome. Our framework, as advocated in this study, does not require a third-party calculation of the exponential rate of infection in which such a calculation may be difficult to verify due to its proprietary nature. The parametric-trigger criterion built within our model separates capital and people and, thereby, delinks market activity that is susceptible to manipulation. Fourth, the difference between CAT bond and pandemic bond is implicitly underscored in our modeling framework. As a categorical reminder, the PEFF's structure revolves around deaths that are still in progress, while the CAT bond's payout is computed after a disaster happens. A financial instrument whose payoff depends on variables while a disaster is in progress presents the high complexity of its valuation and risk management. This is because of the scarcity of pandemic data at the outset considering especially the lack of public health functions such as testing, contact tracing, disease monitoring, and appropriate treatment.

Hence, we also recognize the main limitations of our study: 1) the reliability of the dataset collection since the period chosen was at the beginning of the pandemic with various reporting and data recording rules, from country to country, being continually revised at that time, and 2) the simplicity of the pricing assumption under an incomplete market with the determination of the appropriate risk premium requiring more research attention.

The development of a statistical estimation method to efficiently and dynamically calibrate the SIR-Vasiček model (i.e., recovery of parameters) via the improved maximum-likelihood, least-squares, or other techniques is worth pursuing. For a modeling setup that emphasizes reliance on basic reproduction numbers, see [20]; this is crucial since we showed that the pandemic bond price is sensitive to the parameters upon which the reproduction number depends. One way of validating a correlated SIR-Vasiček model using simulated and actual data is to use block bootstrapping to replicate correlation in the data. Of course, this would entail a multivariate daily time series dataset of interest rates in synchrony with the numbers for susceptible, infected, and removed persons in a given population. We do not pursue this here as our focus is on financial valuation, which takes the model setting and estimated parameters as starting points.

The SEIR framework and other related stochastic modeling extensions (e.g., [29] and those previously mentioned, such as the SAIU, SAIUQR, and SEI$_s$$I_a$QAHR) for the valuation and risk management of the CAT bond are also worth considering. Another possible improvement for this research work is the characterization (analytically or empirically) of the distribution of the first-passage time of the stochastic SIR's trigger process. This will aid the calculations of both the bond prices and corresponding risk metrics quicker and more accessible. In addition, this model could have an extended application to risk-measure computation for policy-claim reserving.

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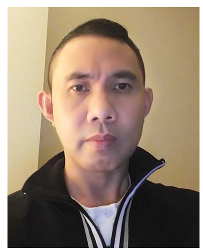

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