STEM Across Different Fields: Pandemic Modeling and Scaled Fermi Dirac Function

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***Abstract* – Rapidly evolving technology fields will require STEM students to develop problem-solving skills that could be applied to a wide variety of diverse fields and projects. This is possible to achieve because the same mathematical equations and functions find applications in diverse STEM areas. As shown in this paper, the key concepts of semiconductor physics such as the Fermi-Dirac distribution function and Vegard's Law (generalized to Scaled Fermi-Dirac function and Scaled Vegard's Law) are very useful in modeling and monitoring pandemics, such as COVID-19 or expected future mysterious pandemic Disease X discussed in DAVOS 24. In turn, these generalized concepts could find applications in modeling noise in radio frequency transistors and interpolating parameters of ternary compounds.**

Index Terms – STEM, COVID-19, Pandemic, Fermi-Dirac function

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INTRODUCTION

Rapidly evolving technology fields will require the STEM workforce to change jobs and even occupations more rapidly than ever before. Therefore, it has become more important than ever for STEM students to develop problem-solving skills that could be applied to a wide variety of projects. One approach to reaching this goal is to apply the concepts they learned in one field to an entirely different field. As an example, this paper describes how the modified concept of the Fermi Dirac function (taught in a class on the physics of semiconductor devices) finds application in monitoring and modeling a pandemic evolution.

Pandemic monitoring and modeling are of interest to most students because the COVID-19 pandemic is not yet over, and industrial nations discussed a potential new pandemic. They called it Disease X (see Figure 1) and the World Health Organization warned that Disease X could cause 20 times more deaths than COVID-19. [1] History teaches us that pandemics, such as COVID-19, Ebola, SARS, plague, HIV, and Spanish Flu, come and goes away. Their evolution is complex and the methods to describe this complexity could be borrowed from solid state physics theory.

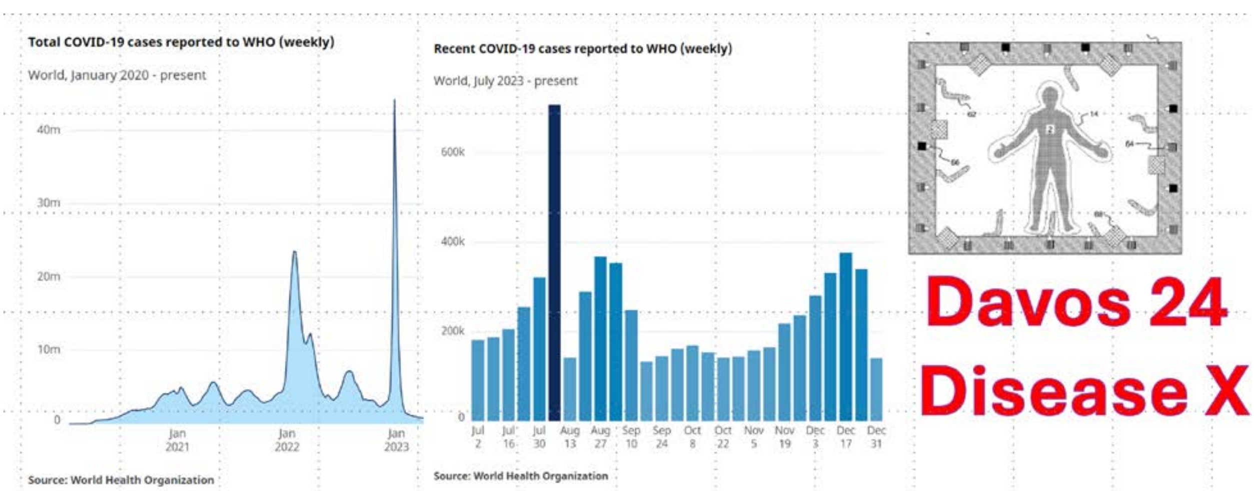

FIGURE 1. COVID-19 CASES REPORTED WEAKLY. ALSO SHOWN EBOLA PROTECTION SUIT AND DECONTAMINATION STATION [2] THAT MIGHT BE RELEVANT FOR FIGHTING DISEASE X DISCUSSED IN DAVOS 24.

As seen from Figure 1, the COVID-19 pandemic came in waves related to new variants. Each wave rises, rests, and decreases only to be replaced by the next wave. The Pandemic Equation [3-5] has been

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used to describe multiple pandemic waves. To the first order, each wave rise could be approximated by an exponential function. When the wave crests and starts decaying, the decay is more gradual. This wave behavior is called "curve flattening." [6] The Pandemic Equation accounts for a more gradual decay of each wave compared to its steeper rise by using slow and fast time scales. On a fast time scale, the numbers of pandemic events change exponentially. On a slow time scale, the exponent index changes. The characteristic time of the exponential growth or decay, τ, could be days, and the time scale on which τ changes could be weeks. This approach is the same as used in the solid state theory to describe fast electronic motion in comparison to a relatively slow nuclear motion (called Born-Oppenheimer approximation [7]). The Born-Oppenheimer approximation considers the electronic motion for fixed positions of nuclei. Electrons adjust to nuclei positions as a gadfly pursuing a horse quickly adjusts to a running horse position (see Figure 2.)

FIGURE 2. A HORSE PURSUED BY A GADFLY. [8]

The Pandemic Equation also accounts for the effects of vaccination and other pandemic mitigation measures. These measures gradually vary the numbers of pandemic events on different time scales. As seen from Fig. 1 (a), there are multiple pandemic waves. The transitions between the pandemic waves are also gradual. The Pandemic Equation describes gradual variation and gradual transitions using the Scaled Fermi-Dirac distribution functions and Scaled Vegard's Law described in the next Section.

SCALED FERMI-DIRAC FUNCTION AND SCALED VEGARD'S LAW

In the solid state theory, the Fermi Dirac function describes the probability of an electron occupying an energy state. Electrons occupy the lowest energy states first, and low energy states are all occupied by electrons. High energy states are all empty. The low occupied energy states and empty high energy states are separated by a transition region. The occupation probability within the transition region varies from 1 to zero. The energy level with an occupation probability of ½ is the Fermi level. The width of the energy level is determined by temperature (it is on the order of 6 thermal energies, $E_{th} = k_B T$, where k_B =1.38 10^{-23} Js is the Boltzmann constant and T is temperature. The Fermi-Dirac distribution function is

$$F_{FD} = \frac{1}{1 + \exp \dfrac{E_F - E}{k_B T}} \quad (1)$$

As seen in Figure 3, the Fermi-Dirac distribution function describes all the transitions varying from very gradual to abrupt and occurring at the energy position determined by the Fermi level. A similar function could approximate transitions in the pandemic events:

$$F_k = \frac{1}{1 + \exp\left(\dfrac{t_k - t}{\tau_k}\right)} \quad (2)$$

Index k refers to a specific event, time, t, and the time of transition t_k, and characteristic transition time constant τ_k are used instead of energy, E, the Fermi level, E_F, and thermal energy $k_B T$, respectively.

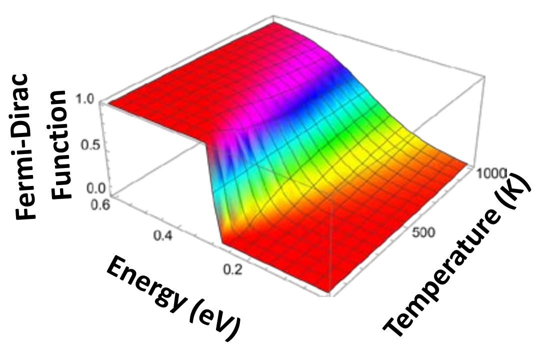

FIGURE 3. FERMI-DIRAC DISTRIBUTION FUNCTION.

The Pandemic Equation uses the Scaled Fermi-Dirac (SFD) distribution function accounting for a slow variation of the mitigation or transition event characteristic time constant with time:

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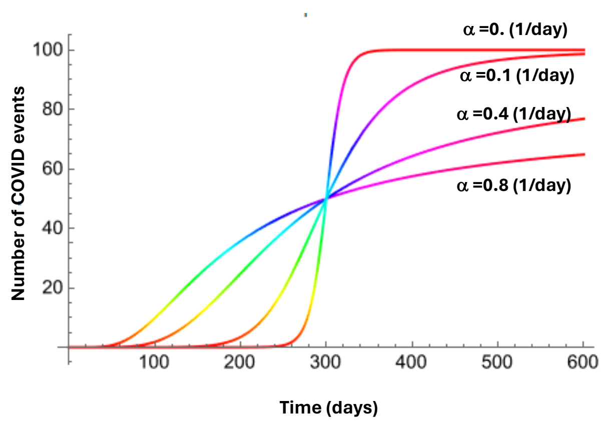
$$F_k = \frac{1}{1+\exp\left[\frac{t_k - t}{\tau_k(t)}\right]} \qquad (3)$$

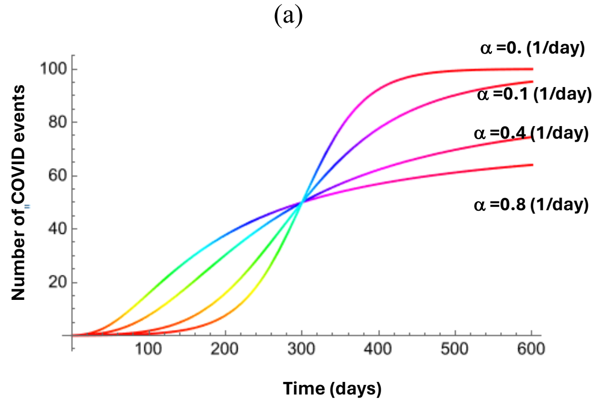
Here

$$\tau_k(t) = \tau_{ko}\left(1+\alpha_k t\right) \qquad (4)$$

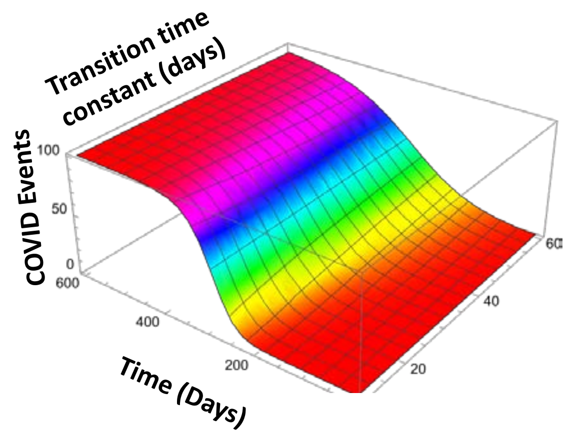
and α_k is the transition event flattening parameter. Figure 4 shows the effect of varying parameter α_k from 0.05 to 0.8. Students taking courses on semiconductor physics and semiconductor device physics could be asked to apply the concept of SFD function to describe transistor noise under bias since hot electrons concentrated in high electric field regions have a much higher effective temperature than the bulk of the electrons located in low field regions.



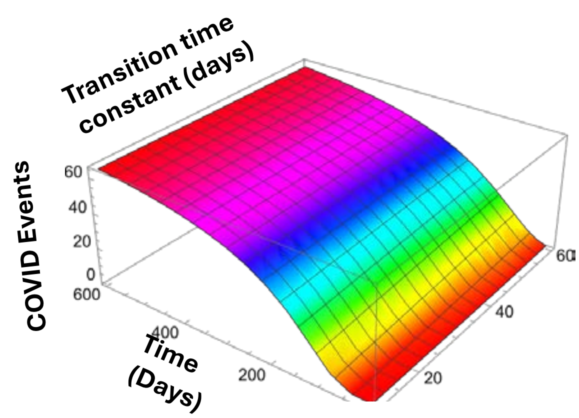
(a)



(b)



(b)



(c)

FIGURE 4. SCALED FERMI-DIRAC DISTRIBUTION FUNCTION USED TO INTERPOLATE COVID-19 EVENTS. (a) τ=10 days; (b) τ=40 days; (b) α=0.05; (c) α=0.8.

SFD functions also enable to accurately interpolate the transition between COVID-19 waves using the Scaled Vegard's Law (SVL). [9] Vegard's Law is used in solid state theory, material science, and chemistry for predicting the properties of mixtures and ternary materials in terms of the properties of their constituent components:

$$a = a_1 x + a_2\left(1-x\right) \qquad (5)$$

Here a is the unit constant of a ternary compound comprising binary components with unit constants a_1 and a_2 for molar fractions x and 1- x of compounds 1 and 2, respectively. SVL was used for COVID-19 modeling [9]:

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$$\Delta N_{l,l+1}(t) = 2\Delta N_l(1-x)F_l(t,t_1) + 2\Delta N_{l+1}xF_{l+1}(t,t_2) \qquad (6)$$

interpolates a large variety of transitions, from nearly linear to fairly abrupt transitions representing exponential decay and exponential rise with the gap in between (see Fig. 5). In Equation (6),

$$x = (t-t_1)/(t_2-t_1), \qquad (7)$$

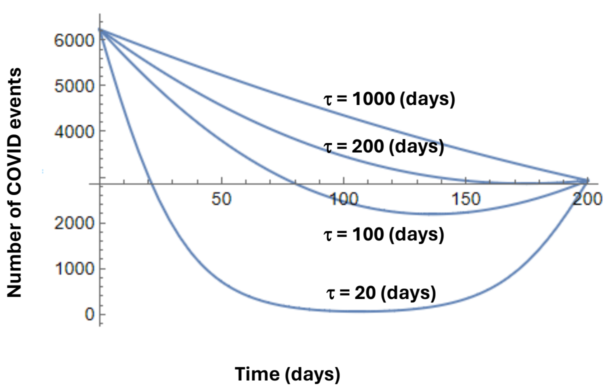
$$F_l(t,t_1) = \frac{1}{1+\exp\dfrac{t-t_1}{\tau_1}}, \qquad (8)$$

$$F_l(t,t_2) = \frac{1}{1+\exp\dfrac{t-t_2}{\tau_2}}, \qquad (9)$$

$\Delta N_{l,l+1}(t)$ is the interpolated number of COVID-19 events per day, ΔN_l and ΔN_2 are the numbers of COVID-19 events per day on times t_1 and t_2, respectively.

As seen in Figure 5, SVL could interpolate a large variety of transitions, from nearly linear (corresponding to the conventional Vegard's Law) to very nonlinear transitions representing exponential decay and exponential rise with the gap in between.

Figure 6 shows the fitting of the first three waves of the COVID-19 pandemic interpolated using the SVL between the first and second waves for days between 130 and 180 and between the second and third waves for days between 245 and 315.



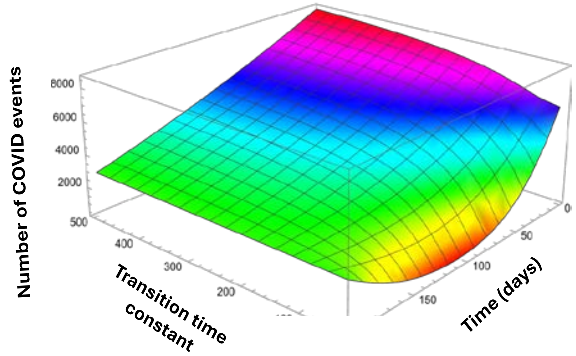


FIGURE 5. TRANSITION BETWEEN COVID-19 WAVES USING SVL.

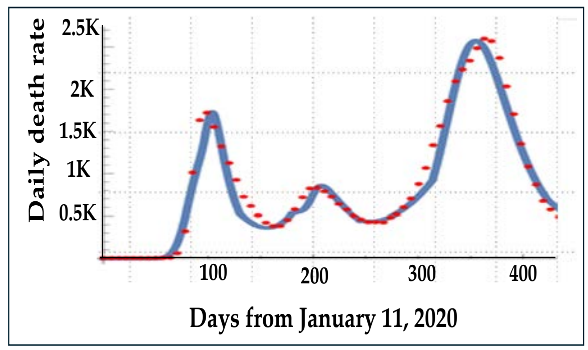


FIGURE 6. THE DAILY DEATH RATE IN THE USA FITTED USING THE PANDEMIC EQUATION SOLUTION AND SVL. RED DASHES ARE ACTUAL DATA, SOLID LINE THE SOLUTIONS OF THE PANDEMIC EQUATION WITH SVL INTERPOLATION.

CONCLUSIONS

STEM students need to develop problem-solving skills that can be applied to a wide variety of projects. The application of concepts learned in one field to an entirely different field helps achieve this goal. As an example, we showed how the modified concepts of the Fermi Dirac function and Vegard's laws, taught in a class on the physics of semiconductor devices, could be applied for modeling and monitoring COVID-19 pandemic evolution using the Pandemic equation. Pandemic monitoring and modeling are of interest to most students because the COVID-19 pandemic is not yet over, and industrial nations have already discussed at DAVOS 24 a potential new pandemic. The COVID-19 pandemic came in waves related to new variants. The Pandemic Equation has been used to describe multiple pandemic waves and SFD and SVL have been used to interpolate between the pandemic waves. This new approach to the Fermi Dirac distribution could, in turn, find applications in hot electron noise

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problems involved in modeling radio frequency transistors and interpolating parameters of ternary compounds.

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