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Three-way conflict analysis: Issue reduct based on incomplete fuzzy value information

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ABSTRACT

In the three-way conflict analysis (TWCA), certain core issues lead to the emergence, development, and resolution of conflicts. Issue reduct enables us to concentrate on key issues and more accurately identify the root causes of conflicts. Existing research primarily addresses issue reduct based on complete three-valued situation tables (TSTs), which have certain limitations. This paper discusses the issue reduct in TWCA based on incomplete fuzzy-valued situation tables (IFSTs). First, to deal with incomplete information, we introduce the Social Trust Network (STN) and the K-Nearest Neighbor (KNN) method, employing an iterative weighting method to fill in missing values. Second, by utilizing the matrix representation of relations among agents, we transform the relation matrix into constraint conditions and propose a recursive backtracking algorithm with pruning strategies to calculate conflict, neutrality, alliance, and global reducts. Finally, we use the development plan of the Gansu Provincial Government as a case study to illustrate the model's applicability and advantages through parameter and comparative analysis.

1. Introduction

This section will introduce the research background, progress, motivation, and innovation points of attribute (issue) reduct based on incomplete information from the perspective of three-way conflict analysis (TWCA).

1.1. Background

Conflict, a phenomenon prevalent in both human society and nature, serves as a force that not only promotes the development of individuals and society but also leads to contradictions and chaos. Therefore, recognizing, managing, and resolving conflicts is a significant challenge in the evolution of human society. In the realm of mathematics, research related to conflicts primarily focuses on areas such as game theory and discrete mathematics. To address the uncertainties present in complex conflict situations, researchers have introduced probability theory, fuzzy mathematics, and rough set theory to enhance conflict analysis models. In 1984, Pawlak [1] was the first to propose a conflict analysis model based on rough sets, which has since been widely applied in fields such as economics, politics, military strategy, and social management. Yao's three-way decision (TWD) theory [2] emphasizes analyzing and solving various problems through a three-way classification method. This theory serves as a mathematical tool for addressing

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uncertain information or problems and has garnered considerable attention from researchers across multiple fields, gradually being adopted for conflict analysis.

1.2. Research progress

In this subsection, we mainly introduce the research progress of TWCA, attribute (issue) reduct, and incomplete information processing.

1.2.1. TWCA

Conflict analysis is closely related to TWD. Lang et al. [3] proposed a TWD method based on decision-theoretic rough sets (DTRSs) for conflict analysis in dynamic information systems. Yao [4] developed a more general TWCA model by re-expressing and expanding Pawlak's model. Building on this foundation, many scholars have conducted in-depth explorations and research on TWCA, considering the more complex realities and needs.

Regarding the analysis of agents' ratings, Yi et al. [5] introduced a TWCA model based on hesitant fuzzy information systems, categorizing issues according to alliance and conflict measurements while analyzing the causes of conflicts. Yang et al. [6] developed a TWCA model grounded in a mixed situation table to better capture the complex and uncertain information encountered in the real world. Gao et al. [7] proposed a TWCA model utilizing interval set information to describe agents' preferences and hesitation psychology.

In the context of analyzing issue weights, Feng et al. [8] explored a TWCA model based on dual hesitant fuzzy information, objectively calculating issue weights using the Criteria Importance Through Intercriteria Correlation (CRITIC) method. Luo et al. [9] presented the N-bounded symmetric concave reciprocal weighting function to address the unclear weight distribution problem in existing TWCA models. Tang et al. [10] introduced a TWCA model that incorporates fuzzy multisets, considering both absolute and relative weights in determining issue weights. Zhu et al. [11] proposed a TWCA model that integrates the best-worst method with the correlation coefficient method to address the problem of weight distribution in incomplete and dispersed information systems.

For analyzing the relations among agents, Luo et al. [12] developed a TWCA model based on alliance and conflict functions, addressing semantic issues arising from representing two opposing aspects with a single evaluation function in existing frameworks. Wang et al. [13] proposed a multi-level analysis method for identifying coalition blocks in dynamic conflict analysis. Hu [14] developed a TWCA model based on preference relations among agents, simplifying the data collection process and enhancing the reliability and interpretability of conflict analysis. Zhang et al. [15] integrated TWCA into the role assignment method, enabling more effective classification and explanation of the complex relationships among agents.

In the context of threshold selection for defining relations between agents, Wang et al. [16] proposed a TWD model of composite risk appetite that integrates prospect theory and regret theory, applying it to conflict analysis. Mandal et al. [17] introduced a TWD model grounded in regret theory to address uncertain conflict problems in a q-rung orthogonal fuzzy information environment. Li et al. [18] proposed a dynamic TWCA model that calculates thresholds using an improved Bayesian algorithm. Jiang et al. [19] developed a TWCA model capable of managing the changing preferences of decision-makers due to future natural conditions and psychological behaviors in multi-issue conflicts.

Regarding the identification of root causes of conflict and approaches to conflict resolution, Sun et al. [20] applied TWD theory to a framework of probabilistic rough sets across two universes, proposing feasible consensus strategies and offering valuable tools for conflict resolution. Du et al. [21] introduced a method for analyzing and resolving conflict problems based on Pythagorean fuzzy information, incorporating the concepts of maximal strong alliances and maximal weak alliances to explain the root causes of conflicts. Liu et al. [22] examined feasible strategies for resolving conflicts from both consistent and inconsistent perspectives, proposing algorithms to identify these strategies and optimal solutions.

In addition, Chen et al. [23] explored a conflict analysis method based on the new TWD graph model within a hesitant fuzzy environment, providing decision-makers with a more reasonable preference ranking. Dou et al. [24] introduced a method to resolve conflicts between two sets of agents by maximizing the satisfaction of effective pairs, thereby reducing conflict. From the perspective of information fusion and based on component similarity, Zhi et al. [25] proposed a new method for achieving rapid conflict analysis.

1.2.2. Attribute (issue) reduct

Attribute reduct originated from rough set theory, aiming to extract key attributes from information systems to analyze and optimize models, thereby improving efficiency. Currently, research on attribute reduct primarily relies on the discernibility matrix [26]. Yao et al. [27] provided a logical foundation for constructing attribute reduct based on the simplification method of discernibility matrix and proposed optimizing the reduct process using a heuristic algorithm. Wang et al. [28] introduced an attribute reduct method based on covering rough sets, which simplified the construction process of the discernibility matrix and reduced computational complexity. Wei et al. [29] introduced the concept of compact decision tables and proposed an incremental attribute reduct algorithm based on the discernibility matrix for processing dynamic data. Sowkuntla et al. [30] developed an attribute reduct accelerator based on the fuzzy discernibility matrix to enhance efficiency when dealing with large datasets. Niu et al. [31] proposed a new single-sample attribute reduct method that calculates reducts by incorporating the discernibility matrix attribute vector and an algorithm based on vector matrix representation. Wen et al. [32] introduced a new unsupervised attribute reduct algorithm that accelerates calculations by transforming the fuzzy discernibility matrix into a two-dimensional form and utilizing a fuzzy deletion function. These contributions have significantly enriched the application of the discernibility matrix in attribute reduct.

In the field of TWCA, attribute reduct in rough sets corresponds to issue reduct that maintains the partition relations. Effective issue reduct can extract key issues and core information in conflict analysis, making it a critical component of the process. However, research in this area is relatively sparse. Lang [33] defined alliance, conflict, and neutral reducts based on three-valued situation tables (TSTs) and discernibility matrices. Zhang et al. [34] proposed a three-way issue reduct for conflict analysis, focusing on region preservation in both qualitative and quantitative models. Ran et al. [35] studied the issue reduct of weak alliances, weak neutrality, and weak conflict relations in incomplete three-valued situation tables (ITSTs), using discernibility functions. These contributions are essential for identifying problems and resolving conflicts.

1.2.3. Incomplete information processing

Conflict analysis is closely related to the determination of the information table. However, due to the complexity of real-world situations, data loss is likely to occur during the information acquisition stage. Therefore, studying the TWCA under incomplete information is significant. Suo et al. [36] conducted the first systematic study of incomplete situation tables in TWCA, addressing a gap in this field. In the realm of group decision-making research, several researchers have proposed more objective methods for handling incomplete information. Chen et al. [37] introduced the concept of additive consistency and utilized two conditions to estimate the missing preference information. Li et al. [38] proposed a collaborative filtering algorithm to estimate the missing preference information of opinion leaders within each subgroup. Wu et al. [39] introduced a new sequence K-nearest neighbor (KNN) interpolation method for estimating missing information. Shen et al. [40] employed an improved DeGroot model to obtain complete evaluation information by simulating the opinion formation process of decision-makers. Zhan et al. [41] introduced the KNN method, which comprehensively considers the trust levels of decision-makers and the similarities in preference relations, effectively filling in the missing preference information.

1.3. Research motivation and innovation points

In practice, there are numerous challenges in the TWCA, particularly when addressing complex conflict problems. The existing methods have several deficiencies that require urgent, in-depth research. In this paper, we primarily discuss the limitations in the following areas.

(1) **Limitations in handling incomplete information:** Traditional TWCA models are typically designed with the assumption that rating information is complete and certain. However, in practical applications, agents often provide incomplete information. This idealization restricts the model's effectiveness in real-world scenarios.

(2) **Limitations in the data structure of issue reduct:** Currently, issue reduct is mainly based on TSTs. However, as conflict analysis becomes more in-depth, the construction of situation tables becomes increasingly complex. Therefore, the problem of issue reduct under these complex situations needs further discussion.

(3) **Limitations in issue reduct method:** The current issue reduct method primarily involves calculating the conjunctive expression of the minimum disjunctive form of the discernibility function. However, these matrix-based reduct methods perform poorly with large-scale datasets and lack effective parallel algorithms, which prevents them from meeting the demand for rapid processing.

The above-mentioned restrictions indicate the urgent need for more effective methods to address the problem of issue reduct in TWCA under incomplete information. Therefore, we list the following main motivations:

(1) Dealing with incomplete information requires more effective tools to estimate the missing values.

(2) It is essential to propose a method for issue reduct that can be applied to complex situational tables.

(3) More effective algorithms are needed to obtain issue reduct results quickly and accurately.

In view of this, the main innovations of this study can be summarized as follows:

(1) **Introduce the Social Trust Network (STN) and KNN methods for estimating missing values:** We propose a method for estimating missing values using neighborhood information. This method establishes a trust relation based on the feature information of each agent. By analyzing the similarity and trust levels between agents and identifying the K nearest neighbors, we sequentially fill in the missing evaluation information, thereby enhancing the completeness and accuracy of conflict analysis.

(2) **Determine the constraint conditions based on the invariance of the agents' relations**: We establish the restrictive conditions for issue reduct by utilizing the distance matrix of agents and introduce the contribution matrix to simplify the discrimination conditions. The efficiency of issue reduct is enhanced through the parallel computation of invariance in relations among multiple agents.

(3) **Introduce the recursive backtracking algorithm for calculating issue reduct:** We apply the recursive backtracking algorithm with pruning strategies to calculate issue reduct. This algorithm is easy to implement quickly using computer programming software and is applicable to various complex situation tables. It ensures that all reduct results are obtained without omission.

1.4. Structure of the paper

The remaining part of this paper is structured as follows: Section 2 reviews the concepts of STN, KNN, and conflict analysis. Section 3 introduces a method for missing value estimation based on the KNN algorithm. Section 4 focuses on analyzing the issue reduct method and presents the corresponding algorithm. Section 5 provides a case analysis to verify the proposed model. Section 6 conducts parameter and comparative analyses to confirm the model's validity. Finally, Section 7 summarizes the paper and outlines directions for future research.

2. Preliminaries

This section reviews some essential concepts, such as STN, KNN, and conflict analysis.

2.1. STN and KNN

The STN is an effective tool for measuring the trust relation among agents. Since its proposal by Wasserman and Faust [42], it has been widely applied in multiple fields. Typically, the structure of the STN is presented in three forms: sociometry, algebraic, and graph theory. Although traditional sociometry is more intuitive and easier to understand in describing the internal relations of networks, it fails to capture the more subtle trust intensities among agents. To further analyze trust relations in complex social networks, Pérez et al. [43] and Dong et al. [44] proposed the concept of fuzzy sociometry.

Definition 2.1. [44] Let $T = (t_{u,v})_{m\times m}$ be the trust network relation matrix of the group of agents A, where $t_{u,v}$ is the trust degree of the agent x_u in the agent x_v, satisfying $0 \le t_{u,v} \le 1$ and $t_{u,u} = 1$.

The KNN algorithm is a classical machine learning method based on the principle that "birds of a feather flock together", meaning that similar samples are more likely to belong to the same category. Specifically, for a new sample, the algorithm identifies the K known samples closest to it in the feature space and predicts the properties of the new sample based on the categories (for classification tasks) or values (for regression tasks) of these neighbors. By utilizing local information and relations within the existing data, the algorithm enhances the accuracy and reliability of estimated missing values.

In regression tasks, for a target sample, we need to calculate the distances between it and other samples, sort these samples in ascending order of distance, and select the K samples with the smallest distances, which are referred to as the K nearest neighbors of the target sample. Among the sorted samples, the sample in the L-th $(1 \le L \le K)$ position is referred to as the L-th nearest neighbor of the target sample.

Definition 2.2. [45] Given a group of agents $A = \{x_1, x_2, \ldots, x_m\}$, let $d(\cdot,\cdot)$ be the distance between two agents under a given metric. Then, $\forall x_k \in A$, the K nearest neighbors of agent x_k under the given metric are defined as follows:

$$N_K(x_k) = \{x_l \in A | d(x_k, x_l) \le d(x_k, x_{n[K]}), k \neq l\},$$

where $d(x_k, x_{n[K]})$ represents the distance between the agent x_k and its K-th nearest neighbor $x_{n[K]}$.

For an agent with a missing value, we first find the K nearest neighbors. Next, we assign weights and perform a weighted summation based on the values of these nearest neighbors at the corresponding positions. This process enables us to estimate the value for the missing data.

2.2. Conflict analysis

Based on rough sets, Pawlak [1] introduced conflict analysis and utilized a TST to describe the ratings of agents w.r.t. issues.

Definition 2.3. [1] A 4-tuple $\mathfrak{T}^T = (A, I, V, f)$ represents a TST, where $A = \{x_1, x_2, \ldots, x_m\}$ is a non-empty finite set of agents, $I = \{i_1, i_2, \ldots, i_n\}$ is a non-empty finite set of issues, V is a set containing all possible ratings, and $f : A \times I \to V$ is a rating function. $\forall x \in A, i \in I$, $f_{x,i}$ denotes the rating of agent x w.r.t. issue i.

Then, Pawlak proposed the following auxiliary function to examine the relations between agents in conflict analysis.

Definition 2.4. [1] Let $\mathfrak{T}^T = (A, I, V, f)$ be a TST. $\forall i \in I$, $\forall x, y \in A$, the auxiliary function $\phi^i_{x\&y}$ between x and y w.r.t. i is defined as follows:

$$\phi^i_{x\&y} = \begin{cases} 1, & \text{if } f_{x,i} \cdot f_{y,i} = 1 \text{ or } x = y; \\ 0, & \text{if } f_{x,i} \cdot f_{y,i} = 0 \text{ and } x \neq y; \\ -1, & \text{if } f_{x,i} \cdot f_{y,i} = -1. \end{cases}$$

Based on the auxiliary function, Pawlak further defined the distance function between agents and proposed three types of relations.

Definition 2.5. [1] Let $\mathfrak{T}^T = (A, I, V, f)$ be a TST. For a non-empty set $I_c \subseteq I$, $\psi^{I_c} : A \times A \to [0, 1]$ represents a distance function. $\forall x, y \in A$, the three relations between x and y w.r.t. I_c are defined as follows:

(1) Alliance, if $\psi^{I_c}_{x\&y} < 0.5$;

(2) Neutrality, if $\psi^{I_c}_{x\&y} = 0.5$;

(3) Conflict, if $\psi^{I_c}_{x\&y} > 0.5$,

where

$$\psi^{I_c}_{x\&y} = \frac{\sum_{i\in I_c} \phi^{i*}_{x\&y}}{|I_c|},$$

$$\phi^{i*}_{x\&y} = \frac{1-\phi^{i}_{x\&y}}{2} = \begin{cases} 0, & \text{if } f_{x,i}\cdot f_{y,i} = 1 \text{ or } x = y; \\ 0.5, & \text{if } f_{x,i}\cdot f_{y,i} = 0 \text{ and } x \neq y; \\ 1, & \text{if } f_{x,i}\cdot f_{y,i} = -1. \end{cases}$$

To avoid the inconsistency in the treatment of the rating 0 in Pawlak's model, Yao [4] redefined the concepts of conflict, neutrality, and alliance relations and adopted the following distance function.

Definition 2.6. [4] Let $\mathfrak{T}^T = (A, I, V, f)$ be a TST. For a non-empty set $I_c \subseteq I$, $\forall x, y \in A$, the distance function $\chi^{I_c}_{x\&y}$ between x and y w.r.t. I_c is defined as follows:

$$\chi^{I_c}_{x\&y} = \frac{\sum_{i\in I_c} \chi^{i}_{x\&y}}{|I_c|},$$

where

$$\chi^{i}_{x\&y} = \frac{|f_{x,i} - f_{y,i}|}{2} = \begin{cases} 0, & \text{if } f_{x,i} = f_{y,i}; \\ 0.5, & \text{if } f_{x,i}\cdot f_{y,i} = 0 \text{ and } f_{x,i} \neq f_{y,i}; \\ 1, & \text{if } f_{x,i}\cdot f_{y,i} = -1. \end{cases}$$

In Pawlak's model, using 0.5 as a threshold to categorize relations between agents is often too strict for practical applications. In this context, Lang [3] and Yao [4] proposed using a more general threshold pair (τ_*, τ^*) to mitigate this limitation.

Definition 2.7. [4] Let $\mathfrak{T}^T = (A, I, V, f)$ be a TST. For a non-empty set $I_c \subseteq I$, $\forall x, y \in A$, $\chi^{I_c}_{x\&y}$ represents the distance between x and y w.r.t. I_c. Given a pair of thresholds (τ_*, τ^*) that satisfy $0 \le \tau_* < 0.5 < \tau^* \le 1$, the alliance relation $R^a_{I_c}$, neutrality relation $R^n_{I_c}$, and conflict relation $R^c_{I_c}$ are defined as follows:

(1) $R^a_{I_c} = \{(x, y) \in A \times A | \chi^{I_c}_{x\&y} < \tau_*\}$;

(2) $R^n_{I_c} = \{(x, y) \in A \times A | \tau_* \le \chi^{I_c}_{x\&y} \le \tau^*\}$;

(3) $R^c_{I_c} = \{(x, y) \in A \times A | \chi^{I_c}_{x\&y} > \tau^*\}$.

3. Missing value estimation based on STN and KNN

In this section, we will handle the missing values in incomplete fuzzy-valued situation tables (IFSTs) [36]. Specifically, in Subsection 3.1, we will construct the STN using feature information. In Subsection 3.2, we will present an estimation method for missing values based on the KNN algorithm.

3.1. The construction of STN

Ideally, when the data is complete, it allows for a comprehensive description of the phenomenon or problem under study and facilitates the establishment of a corresponding model, leading to more accurate predictions or decisions. However, in the real world, some data is often missing due to limitations in data collection, information loss or corruption, and concerns related to confidentiality or privacy. To address this problem, researchers have proposed various methods for estimating missing values, including statistical analysis, model prediction, multi-source data fusion, and uncertainty analysis, etc. Although each of these methods has advantages in specific fields, in TWCA, the rating data are multi-issue and single-source, and the individual differences among the agents that generate ratings result in unique characteristic information. Therefore, the KNN method based on the STN can better utilize these data features and effectively handle the incomplete fuzzy value information discussed in this paper.

In TWCA, the decision-making subjects or stakeholders involved in a conflict possess underlying characteristics—such as gender, age, and occupation—that significantly influence the judgments of individuals with similar traits, a phenomenon referred to as the "group characteristic effect". In this subsection, we will construct the STN based on the feature information of the agents.

For example, in one study, different groups of customers rated a particular neighborhood based on various factors that influence home buying, such as price, location, and school district. Not all customers conducted a comprehensive survey of the community, resulting in some missing evaluation information. When estimating these missing values, we cannot simply consider customer conformity psychology, as factors such as age, identity, and income can lead to different levels of acceptance. Instead, we should focus on the conformity behaviors within groups that share similar characteristics. Most surveys collect basic information about respondents, so using this data to build trust relations can facilitate more accurate estimations of missing values. For groups with similar characteristics, trust is likely to be higher.

To establish trust relations among agents based on multi-scale information, we introduce different auxiliary functions for encoding and quantifying data at various scales. This enables the analysis of all data at a consistent scale. By defining a single-scale trust calculation method and assigning corresponding weights based on the importance of different features, we can calculate the weighted trust degree among agents in multi-scale information.

Definition 3.1. A 4-tuple $\vartheta^F = (A, B, U, \upsilon)$ represents a feature information table (FIT), where $A = \{x_1, x_2, \ldots, x_m\}$ is a non-empty finite agent set, $B = \{b_1, b_2, \ldots, b_l\}$ is a non-empty finite feature set, U is a set containing all possible feature values, and $\upsilon : A \times B \to U$ is a feature function. $\forall x \in A, b \in B$, $\upsilon(x, b)$ denotes the feature value of agent x w.r.t. feature b.

Since different features have different scales, in order to construct the trust relation among agents at multiple scales, we need to normalize each feature and introduce a normalized mapping function, denoted by $g(\upsilon(x, b))$, satisfying $0 \le g(\upsilon(x, b)) \le 1$, where $\upsilon(x, b)$ represents the feature value of agent x w.r.t. feature b. The generality of the normalized mapping function depends on the encoding quantization method used for different features. In Section 5, we will demonstrate this through specific cases.

Definition 3.2. Let $\vartheta^F = (A, B, U, \upsilon)$ be a FIT and $g(\cdot)$ be a normalized mapping function. $\forall x, y \in A$, $\forall b \in B$, the trust degree $t^b_{x\&y}$ between x and y w.r.t. b is defined as

$$t^b_{x\&y} = 1 - |g(\upsilon(x, b)) - g(\upsilon(y, b))|. \tag{1}$$

Let ϕ_b represent the weight of b, satisfying $\sum_{b\in B} \phi_b = 1$. Then the weighted trust degree $t^B_{x\&y}$ is defined as

$$t^B_{x\&y} = \sum_{b\in B} \phi_b \cdot t^b_{x\&y}. \tag{2}$$

Thus, based on the trust degree among agents under multi-scale feature information, we can construct the corresponding STN for further analysis. The trust network relation matrix of the group of agents A is represented as $T = (t^B_{x\&y})_{m\times m}$.

3.2. Missing value estimation

In this paper, we primarily address the estimation of missing values in incomplete fuzzy-valued ratings. First, we provide the definition of IFSTs as follows.

Definition 3.3. A 4-tuple $\mathfrak{T}^{IF^*} = (A, I, R^*, f^*)$ represents an IFST, where $A = \{x_1, x_2, \ldots, x_m\}$ is a non-empty finite set of agents, $I = \{i_1, i_2, \ldots, i_n\}$ is a non-empty finite set of issues, $R^* = \bigcup\{R^*_i | i \in I\}$ represents the set of ratings w.r.t. issue i that contains unknown information, and $f^* : A \times I \to R^*$ is a rating function. $\forall x \in A, i \in I$, $f^*_{x,i}$ denotes the rating of agent x w.r.t. issue i, where $f^*_{x,i} \in [0, 1]$ or $f^*_{x,i} = \aleph$ ($\aleph$ denotes a missing value).

Based on the STN constructed in Definition 3.1, next, we will use the KNN algorithm to infer the missing rating information. Identifying the K nearest neighbors is a crucial step in this algorithm. Specifically, for a target agent, we need to calculate the trust degree between the target agent and other agents, sort these agents in descending order of trust degrees, and select the K agents with the highest trust degrees, referred to as the K nearest neighbors of the target agent. Among the sorted agents, the agent positioned at L $(1 \le L \le K)$ is called the L-th nearest neighbor of the target agent.

Definition 3.4. Let $\mathfrak{T}^{IF^*} = (A, I, R^*, f^*)$ be an IFST, $\vartheta^F = (A, B, U, \upsilon)$ be a FIT, and $t^B_{\cdot\&\cdot}$ be the weighted trust degree between two agents w.r.t. the feature set B. For an agent $x \in A$ with a missing rating $f^*_{x,i}$ under issue i, the K nearest neighbors of x are defined as follows:

$$N^{t^B}_K(x) = \{x_n \in A | t^B_{x\&x_n} \ge t^B_{x\&x_{n[K]}}, x_n \ne x, f^*_{x_n,i} \ne \aleph\}, \tag{3}$$

where $x_{n[K]}$ represents the K-th nearest neighbor of x.

Remark 3.1. If the trust level is the same, the nearest neighbors are selected in index order.

After identifying the K nearest neighbors, we estimate the target agent's rating by calculating a weighted average of the ratings from these neighbors. Typically, a higher level of trust suggests that a selected nearest neighbor holds greater reference value. Consequently, researchers use the trust degree as the weight in their calculations, giving more importance to the nearest neighbors with a higher trust degree. However, focusing solely on the trust degree may lead the model to overlook the overall structure and distribution among agents. As a result, some agents with low trust but similar characteristics to the target agent may be completely ignored, resulting in an irrational weight distribution. To address this problem, Zhan et al. [41] introduced feature similarity to enhance the model's accuracy and generalization ability. First, we give the definition of rating similarity.

Definition 3.5. Let $\mathfrak{I}^{IF^*} = (A, I, R^*, f^*)$ be an IFST and $\vartheta^F = (A, B, U, \upsilon)$ be a FIT. $\forall x \in A,\ x_n \in N_K^{t^B}(x)$, the similarity degree $s^{I^\kappa}_{x\&x_n}$ between x and x_n is defined as follows:

$$s^{I^\kappa}_{x\&x_n} = 1 - \frac{\sum_{j\in I^\kappa} |f^*_{x,j} - f^*_{x_n,j}|}{|I^\kappa|}, \tag{4}$$

where I^κ represents the set of issues for which both agent x and agent x_n have known ratings.

By calculating the geometric average of the trust degree and the similarity degree, we can obtain a new measure called the consensus degree. This method balances the influence of both metrics, preventing any single tool from disproportionately affecting the result.

Definition 3.6. $\forall x \in A,\ x_n \in N_K^{t^B}(x)$, let $t^B_{x\&x_n}$ represent the trust degree between x and x_n, $s^{I^\kappa}_{x\&x_n}$ denote the similarity degree between x and x_n. The consensus degree $c_{x\&x_n}$ between x and x_n is defined as follows:

$$c_{x\&x_n} = \sqrt{t^B_{x\&x_n} \times s^{I^\kappa}_{x\&x_n}}.$$

The normalized consensus degree is given by:

$$\tilde{c}_{x\&x_n} = \frac{c_{x\&x_n}}{\sum_{k=1}^{K} c_{x\&x_{n[k]}}}, \tag{5}$$

where $x_{n[k]} \in N_K^{t^B}(x)$ and $x_{n[k]}$ represents the k-th nearest neighbor of x.

Using the consensus degree as the weight, we can now calculate the missing values. To improve the accuracy of the estimation, we adopt an iterative solution method. We first deal with the missing values involved in the agents that have a relatively high total consensus degree, then update each value and repeat the solving process until all missing values are addressed. This approach gradually enhances the accuracy and stability of the overall estimation process.

Definition 3.7. Let $c_{x\&x_{n[k]}}$ represent the consensus degree between x and $x_{n[k]}$, $k = 1, \ldots, K$. The total consensus degree c^T_x for agent x can be computed as follows:

$$c^T_x = \sum_{k=1}^{K} c_{x\&x_{n[k]}}.$$

In each estimation process, the missing value with the highest total consensus degree is estimated first, and then the consensus degrees of the other missing values are updated, and the above process is repeated until all missing values are estimated.

Remark 3.2. For an agent whose rating is a missing value under an issue, if its K nearest neighbors cannot be found to meet the conditions, we do not deal with it temporarily because we adopt the iterative solution method. When some missing values in the iterative process are modified and the rating matrix is updated, the problem will be solved. In extreme cases, if the K nearest neighbors cannot be found for any agents involved in missing values, to minimize information loss, we use the average rating of agents under the issue where the missing values are located to estimate the missing values and then continue to estimate the remaining missing values according to the iterative solution method. In addition, when calculating similarity, if we cannot find an issue where both agents have known ratings, we use the trust degree instead.

Definition 3.8. Let $\mathfrak{I}^{IF^*} = (A, I, R^*, f^*)$ be an IFST and $\tilde{c}_{x\&x_{n[k]}}$ represent the normalized consensus degree between x and $x_{n[k]}$. The missing rating of agent x w.r.t. issue i can be calculated as follows:

$$f^*_{x,i} = \sum_{k=1}^{K} \tilde{c}_{x\&x_{n[k]}} f^*_{x_{n[k]},i}. \tag{6}$$

The algorithm for estimating the missing values is shown in Algorithm 1, which has a time complexity of $O(K^2mn)$.

Using the iterative solution method outlined in Algorithm 1, we can fill in the missing values in an IFST. This process transforms the original IFST into a complete fuzzy-valued situation table (FST), referred to as the complete FST induced by the KNN algorithm.

Algorithm 1: Estimate the missing values.

Input: $\vartheta^F = (A, B, U, \upsilon)$, $\mathfrak{T}^{IF^*} = (A, I, R^*, f^*)$, K.
Output: A complete fuzzy-valued situation table.
1 Compute the trust degree between agents and construct the STN;
2 Find the K nearest neighbors of the agents whose ratings for some issues are missing values;
3 Compute the similarity degree between the agents and the K nearest neighbors;
4 Compute the consensus degree between the agents and the K nearest neighbors;
5 Compute the total consensus degree of the agents;
6 Compute the agent's missing value with the highest total consensus degree based on Definition 3.8;
7 Update $\mathfrak{T}^{IF^*}$ and repeat the above process until all missing values are calculated.

4. Issue reduct in TWCA

In this section, we will discuss the issue reduct in TWCA. Specifically, in Subsection 4.1, we define and trisect the relations between agents. Subsection 4.2 provides the definition of issue reduct in an induced complete FST. Finally, in Subsection 4.3, we present a recursive backtracking algorithm to compute issue reduct efficiently.

4.1. The trisection of agents' relations

For the distance measure of ratings, Yao [4] extended the ratings from three-valued to many-valued and defined the distance function on $[-1, 1]$. We refine this function to enhance its application for measuring distances between fuzzy values.

Definition 4.1. Let $\mathfrak{T}^{IF^*} = (A, I, R^*, f^*)$ be an IFST. We refer to $\mathfrak{T}^{CF^*}$ as the induced complete FST. $\forall x, y \in A$, $\forall i \in I$, the distance function $D^i_{x\&y}$ between x and y w.r.t. i is defined as follows:

$$D^i_{x\&y} = |f^*_{x,i} - f^*_{y,i}|, \tag{7}$$

where $f^*_{x,i}, f^*_{y,i} \in [0, 1]$.

In multi-issue conflict analysis, different issues have different weights, leading to a weighted distance for multiple issues, as shown in Definition 4.2.

Definition 4.2. Let $\mathfrak{T}^{IF^*} = (A, I, R^*, f^*)$ be an IFST and $\mathfrak{T}^{CF^*}$ be the induced complete FST, a non-empty set $I_s \subseteq I$. $\forall x, y \in A$, $\forall i \in I_s$, $D^i_{x\&y}$ represents the distance between x and y w.r.t. i, ω_i represents the weight of i, satisfying $\sum_{i\in I} \omega_i = 1$. Then the weighted distance $D^{I_s}_{x\&y}$ is defined as follows:

$$D^{I_s}_{x\&y} = \sum_{i\in I_s} \omega'_i \cdot D^i_{x\&y}, \tag{8}$$

where $\omega'_i = \dfrac{\omega_i}{\sum_{j\in I_s} \omega_j}$.

Yao [4] proposed a framework for TWCA by reformulating and extending Pawlak's conflict analysis model. This framework categorizes the relations between agents into three levels. Below, we provide the definition of the trisection of agent pairs.

Definition 4.3. Let $\mathfrak{T}^{IF^*} = (A, I, R^*, f^*)$ be an IFST and $\mathfrak{T}^{CF^*}$ be the induced complete FST, a non-empty set $I_s \subseteq I$. Given the thresholds τ_* and τ^* that satisfy $0 \le \tau_* < \tau^* \le 1$. $\forall x, y \in A$, $D^{I_s}_{x\&y}$ represents the distance between x and y w.r.t. I_s. We define the conflict relation $R^c_{I_s}$, neutral relation $R^n_{I_s}$, and alliance relation $R^a_{I_s}$ as follows:

(1) $R^c_{I_s} = \{(x, y) | D^{I_s}_{x\&y} \ge \tau^*\}$;
(2) $R^n_{I_s} = \{(x, y) | \tau_* < D^{I_s}_{x\&y} < \tau^*\}$;
(3) $R^a_{I_s} = \{(x, y) | D^{I_s}_{x\&y} \le \tau_*\}$.

From Definition 4.3, we can derive the relation between any two agents — whether conflict, neutrality, or alliance. This understanding allows us to construct a complex social interaction network based on these relations.

4.2. The definition of issue reduct

With the rapid development of complex systems theory, TWCA, as an important tool to deal with multi-party relations, has made breakthroughs in the fields of social science, artificial intelligence, and strategic decision-making in recent years. However, most of the existing research focuses on the formal modeling of various relations, but there is still a key research gap in the underlying logic of theoretical construction — attribute reduct and core element extraction of issue sets. The solution to this problem will help us analyze the complex relations more deeply and provide a more explanatory theoretical framework for strategic decision-making. Lang [33] first proposed the method of issue reduct in TSTs, but with the development of society, the evaluation information is becoming increasingly complex. The existing methods are unable to handle more complex situation tables and are not suitable for the weighted methods commonly used in multi-issue conflict measurement. Furthermore, they require the pre-calculation of the discernibility matrix, followed by the solution of the reduction results, which is quite cumbersome. Therefore, we will improve the existing method and extend it to the induced complete FSTs for discussion.

Considering the superiority of matrix calculations, we first provide the matrix representation of the conflict, neutrality, and alliance relations.

Definition 4.4. For an agent set A and an issue set I, $\emptyset \neq I_s \subseteq I$, let $R^c_{I_s}$, $R^n_{I_s}$, and $R^a_{I_s}$ represent the conflict, neutrality, and alliance relations, respectively. $\forall x_p, x_q \in A$, the relation matrices $\mathbb{R}^C_{I_s}$, $\mathbb{R}^N_{I_s}$, $\mathbb{R}^A_{I_s}$, and $\mathbb{R}^G_{I_s}$ are defined as follows:

(1) Conflict matrix $\mathbb{R}^C_{I_s} = [r^c_{I_s}(p,q)]_{A\times A}$, where

$$r^c_{I_s}(p,q) = \begin{cases} 1, & \text{if } (x_p, x_q) \in R^c_{I_s}; \\ 0, & \text{otherwise.} \end{cases}$$

(2) Neutral matrix $\mathbb{R}^N_{I_s} = [r^n_{I_s}(p,q)]_{A\times A}$, where

$$r^n_{I_s}(p,q) = \begin{cases} 2, & \text{if } (x_p, x_q) \in R^n_{I_s}; \\ 0, & \text{otherwise.} \end{cases}$$

(3) Alliance matrix $\mathbb{R}^A_{I_s} = [r^a_{I_s}(p,q)]_{A\times A}$, where

$$r^a_{I_s}(p,q) = \begin{cases} 3, & \text{if } (x_p, x_q) \in R^a_{I_s}; \\ 0, & \text{otherwise.} \end{cases}$$

(4) Global matrix $\mathbb{R}^G_{I_s} = [r^g_{I_s}(p,q)]_{A\times A}$, where

$$r^g_{I_s}(p,q) = \begin{cases} 1, & \text{if } (x_p, x_q) \in R^c_{I_s}; \\ 2, & \text{if } (x_p, x_q) \in R^n_{I_s}; \\ 3, & \text{if } (x_p, x_q) \in R^a_{I_s}. \end{cases}$$

Remark 4.1. Generally, the elements in a relation matrix are represented using Boolean algebra. In this paper, we introduce a new global matrix that simultaneously satisfies the conditions of three general relation matrices. To enhance clarity and illustrate the relation between the global matrix and the others, we use the numbers 0, 1, 2, and 3 to represent different cases.

Based on these relation matrices, we can define the conflict, neutrality, alliance, and global reducts as follows.

Definition 4.5. For an issue set I, let $\mathbb{R}^C_I$, $\mathbb{R}^N_I$, $\mathbb{R}^A_I$, and $\mathbb{R}^G_I$ represent the conflict, neutrality, alliance, and global matrices, respectively.

(1) If $\mathbb{R}^C_{I_{cr}} = \mathbb{R}^C_I$ for $I_{cr} \subseteq I$, then I_{cr} is called a conflict weak reduct. If I_{cr} is a conflict weak reduct and $\mathbb{R}^C_{I_{cr}-\{i\}} \neq \mathbb{R}^C_I$ for any $i \in I_{cr}$, then I_{cr} is called a conflict reduct.

(2) If $\mathbb{R}^N_{I_{nr}} = \mathbb{R}^N_I$ for $I_{nr} \subseteq I$, then I_{nr} is called a neutrality weak reduct. If I_{nr} is a neutrality weak reduct and $\mathbb{R}^N_{I_{nr}-\{i\}} \neq \mathbb{R}^N_I$ for any $i \in I_{nr}$, then I_{nr} is called a neutrality reduct.

(3) If $\mathbb{R}^A_{I_{ar}} = \mathbb{R}^A_I$ for $I_{ar} \subseteq I$, then I_{ar} is called an alliance weak reduct. If I_{ar} is an alliance weak reduct and $\mathbb{R}^A_{I_{ar}-\{i\}} \neq \mathbb{R}^A_I$ for any $i \in I_{ar}$, then I_{ar} is called an alliance reduct.

(4) If $\mathbb{R}^G_{I_{gr}} = \mathbb{R}^G_I$ for $I_{gr} \subseteq I$, then I_{gr} is called a global weak reduct. If I_{gr} is a global weak reduct and $\mathbb{R}^G_{I_{gr}-\{i\}} \neq \mathbb{R}^G_I$ for any $i \in I_{gr}$, then I_{gr} is called a global reduct.

Proposition 4.1. Let $\mathbb{I}^w_{cr}$, $\mathbb{I}^w_{nr}$, $\mathbb{I}^w_{ar}$, and $\mathbb{I}^w_{gr}$ represent the conflict, neutrality, alliance, and global weak reduct sets, respectively. Let $\mathbb{I}_{cr}$, $\mathbb{I}_{nr}$, $\mathbb{I}_{ar}$, and $\mathbb{I}_{gr}$ represent the conflict, neutrality, alliance, and global reduct sets, respectively. The following statements are true.

(1) $\mathbb{I}_{cr} \subseteq \mathbb{I}^w_{cr}$, $\mathbb{I}_{nr} \subseteq \mathbb{I}^w_{nr}$, $\mathbb{I}_{ar} \subseteq \mathbb{I}^w_{ar}$, $\mathbb{I}_{gr} \subseteq \mathbb{I}^w_{gr}$;

(2) $\mathbb{I}^w_{gr} = \mathbb{I}^w_{cr} \cap \mathbb{I}^w_{nr} \cap \mathbb{I}^w_{ar}$.

Proof. (1) For any $I_{cr} \in \mathbb{I}_{cr}$, from Definition 4.5 (1), we have $I_{cr} \subseteq I$ and $\mathbb{R}^C_{I_{cr}} = \mathbb{R}^C_I$, i.e. $I_{cr} \in \mathbb{I}^w_{cr}$, thus, $\mathbb{I}_{cr} \subseteq \mathbb{I}^w_{cr}$ clearly holds. Similarly, $\mathbb{I}_{nr} \subseteq \mathbb{I}^w_{nr}$, $\mathbb{I}_{ar} \subseteq \mathbb{I}^w_{ar}$, and $\mathbb{I}_{gr} \subseteq \mathbb{I}^w_{gr}$ hold.

(2) From Definition 4.4, we have $\mathbb{R}_I^G = \mathbb{R}_I^C + \mathbb{R}_I^N + \mathbb{R}_I^A$. According to Definition 4.5, suppose I_{gr}^w is a global weak reduct, i.e. $\mathbb{R}_{I_{gr}}^G = \mathbb{R}_I^G$. Then, we have $\mathbb{R}_{I_{gr}}^C = \mathbb{R}_I^C$, $\mathbb{R}_{I_{gr}}^N = \mathbb{R}_I^N$, and $\mathbb{R}_{I_{gr}}^A = \mathbb{R}_I^A$ hold. That is, I_{gr}^w is also a conflict weak reduct, a neutrality weak reduct, and an alliance weak reduct. Thus, we have $\mathbb{I}_{gr}^w = \mathbb{I}_{cr}^w \cap \mathbb{I}_{nr}^w \cap \mathbb{I}_{ar}^w$. □

As a key tool that precisely focuses on core factors through reduct, the reduct core is becoming increasingly prominent in data processing and decision analysis. After addressing redundant issues using reduct methods, the reduct core can help us identify and concentrate on the core issues that play a decisive role in the conflict analysis process more accurately.

Definition 4.6. Let $\mathbb{I}_{cr}$, $\mathbb{I}_{nr}$, $\mathbb{I}_{ar}$, and $\mathbb{I}_{gr}$ represent the conflict, neutrality, alliance, and global reduct sets, respectively. Then, the reduct cores are defined as follows:

(1) Conflict reduct core: $C_{cr} = \bigcap\{I_{cr} \mid I_{cr} \in \mathbb{I}_{cr}\}$;

(2) Neutrality reduct core: $C_{nr} = \bigcap\{I_{nr} \mid I_{nr} \in \mathbb{I}_{nr}\}$;

(3) Alliance reduct core: $C_{ar} = \bigcap\{I_{ar} \mid I_{ar} \in \mathbb{I}_{ar}\}$;

(4) Global reduct core: $C_{gr} = \bigcap\{I_{gr} \mid I_{gr} \in \mathbb{I}_{gr}\}$.

4.3. Issue reduct based on recursive backtracking algorithm

The core of the issue weak reduct is to maintain the relation matrix unchanged before and after the reduct. In contrast, issue reduct aims to identify the minimum subset while ensuring the invariance of the relation matrix. Directly considering the invariance of the entire relation matrix can be challenging. Therefore, we recognize that the relation matrix comprises the relations of different agent pairs. When a specific reduct belongs to the reduct for each position in the matrix, we refer to this as an issue reduct that satisfies the invariance of the entire relation matrix. Consequently, the problem shifts from finding an issue reduct that maintains the invariance of the entire relation matrix to identifying an issue reduct that preserves the invariance of the relation division for each agent pair.

Furthermore, when focusing on the reduct of individual agent pairs, some subsets belong to issue reduct for the entire relation matrix but serve as issue weak reducts for individual agent pairs. As a result, they may be redundantly deleted during the calculation process. To avoid such omissions, we first calculate the issue weak reduct for a single agent pair, then perform the intersection operation to determine the issue weak reduct for the entire relation matrix, and finally compute the issue reduct for the entire relation matrix. This underscores the purpose behind our definition of issue weak reduct.

To better adapt an algorithm to calculate the issue reduct, we transform the contents of Definition 4.5 into constraint conditions.

For an issue set I, let $\mathbb{D}_I = [d_I(p,q)]_{A\times A}$ denote the distance matrix, where $d_I(p,q) = D^I_{x_p\&x_q} = \sum_{i\in I}\omega_i \cdot D^i_{x_p\&x_q}$, $\sum_{i\in I}\omega_i = 1$. For a reduct set $I_\diamond$, since the issue weights need to be re-normalized in the reduct process, we use $\mathbb{D}_{I_\diamond} = [d_{I_\diamond}(p,q)]_{A\times A}$ to denote the reduced distance matrix, where $d_{I_\diamond}(p,q) = D^{I_\diamond}_{x_p\&x_q} = \dfrac{1}{\sum_{j\in I_\diamond}\omega_j}\sum_{j\in I_\diamond}\omega_j \cdot D^j_{x_p\&x_q}$, $\diamond \in \{cr, nr, ar, gr\}$.

Given a pair of thresholds (τ_*, τ^*) that satisfy $0 \le \tau_* < \tau^* \le 1$. $\forall (x_p, x_q) \in A\times A$, the constraint conditions of conflict, neutrality, alliance, and global weak reducts are shown as follows:

(C1) If $d_I(p,q) \ge \tau^*$, $d_{I_{cr}}(p,q) \ge \tau^*$ must hold; if $d_I(p,q) < \tau^*$, $d_{I_{cr}}(p,q) < \tau^*$ must hold.

(N1) If $\tau_* < d_I(p,q) < \tau^*$, $\tau_* < d_{I_{nr}}(p,q) < \tau^*$ must hold; if $d_I(p,q) \le \tau_*$ or $d_I(p,q) \ge \tau^*$, at least one of $d_{I_{nr}}(p,q) \le \tau_*$ and $d_{I_{nr}}(p,q) \ge \tau^*$ must hold.

(A1) If $d_I(p,q) \le \tau_*$, $d_{I_{ar}}(p,q) \le \tau_*$ must hold; if $d_I(p,q) > \tau_*$, $d_{I_{ar}}(p,q) > \tau_*$ must hold.

(G1) If $d_I(p,q) \ge \tau^*$, $d_{I_{gr}}(p,q) \ge \tau^*$ must hold; if $\tau_* < d_I(p,q) < \tau^*$, $\tau_* < d_{I_{gr}}(p,q) < \tau^*$ must hold; if $d_I(p,q) \le \tau_*$, $d_{I_{gr}}(p,q) \le \tau_*$ must hold.

To improve clarity and efficiency, we construct the τ^*-contribution matrix $\mathbb{C}_i^{\tau^*} = [c_i^{\tau^*}(p,q)]_{A\times A}$, where $c_i^{\tau^*}(p,q)$ represents the τ^*-contribution of issue i and $c_i^{\tau^*}(p,q) = \omega_i \cdot d_i^{\tau^*}(p,q) = \omega_i \cdot (D^i_{x_p\&x_q} - \tau^*)$; the τ_*-contribution matrix $\mathbb{C}_i^{\tau_*} = [c_i^{\tau_*}(p,q)]_{A\times A}$, where $c_i^{\tau_*}(p,q)$ represents the τ_*-contribution of issue i and $c_i^{\tau_*}(p,q) = \omega_i \cdot d_i^{\tau_*}(p,q) = \omega_i \cdot (D^i_{x_p\&x_q} - \tau_*)$. Then, (C1) - (G1) can be further simplified as follows:

(C2) If $d_I(p,q) \ge \tau^*$, $\sum_{i\in I_{cr}} c_i^{\tau^*}(p,q) \ge 0$ must hold; if $d_I(p,q) < \tau^*$, $\sum_{i\in I_{cr}} c_i^{\tau^*}(p,q) < 0$ must hold.

(N2) If $\tau_* < d_I(p,q) < \tau^*$, both $\sum_{i\in I_{nr}} c_i^{\tau_*}(p,q) > 0$ and $\sum_{i\in I_{nr}} c_i^{\tau^*}(p,q) < 0$ must hold; if $d_I(p,q) \le \tau_*$ or $d_I(p,q) \ge \tau^*$, at least one of $\sum_{i\in I_{nr}} c_i^{\tau_*}(p,q) \le 0$ and $\sum_{i\in I_{nr}} c_i^{\tau^*}(p,q) \ge 0$ must hold.

(A2) If $d_I(p,q) \le \tau_*$, $\sum_{i\in I_{ar}} c_i^{\tau_*}(p,q) \le 0$ must hold; if $d_I(p,q) > \tau_*$, $\sum_{i\in I_{ar}} c_i^{\tau_*}(p,q) > 0$ must hold.

(G2) If $d_I(p,q) \ge \tau^*$, $\sum_{i\in I_{gr}} c_i^{\tau^*}(p,q) \ge 0$ must hold; if $\tau_* < d_I(p,q) < \tau^*$, both $\sum_{i\in I_{gr}} c_i^{\tau_*}(p,q) > 0$ and $\sum_{i\in I_{gr}} c_i^{\tau^*}(p,q) < 0$ must hold; if $d_I(p,q) \le \tau_*$, $\sum_{i\in I_{gr}} c_i^{\tau_*}(p,q) \le 0$ must hold.

The recursive backtracking algorithm systematically searches the solution space of a problem using recursion. The pruning strategy eliminates branches that are unlikely to yield effective solutions based on specific judgment conditions during the search process, enhancing the algorithm's efficiency. Therefore, in this paper, we will apply the recursive backtracking algorithm with a pruning strategy to solve the reduct problem under the constraint conditions (C2) - (G2).

The remaining maximum and minimum contributions, as key indicators for determining whether to perform pruning, are defined as follows.

Definition 4.7. The remaining maximum contribution represents the sum of the maximum possible positive contributions across all issues from the current processing location to the end. For a sorted issue set I_ε, suppose the u-th issue is currently being processed; then the $\star$-remaining maximum contribution from the u-th issue is defined as follows:

$$R^{[u]}_{\max\_\star} = \sum_{k=u}^{|I_\varepsilon|} \max(c^\star_{i^{(u)}}(p,q), 0), \tag{9}$$

where $c^\star_{i^{(u)}}(p,q)$ represents the $\star$-contribution of the u-th issue, with $\star \in \{\tau_*, \tau^*\}$, and $|I_\varepsilon|$ denotes the cardinality of I_ε.

Definition 4.8. The remaining minimum contribution represents the sum of the minimum possible negative contributions across all issues from the current processing location to the end. For a sorted issue set I_ι, suppose the v-th issue is currently being processed; then the $\star$-remaining minimum contribution from the v-th issue is defined as follows:

$$R^{[v]}_{\min\_\star} = \sum_{k=v}^{|I_\iota|} \min(c^\star_{i^{(v)}}(p,q), 0), \tag{10}$$

where $c^\star_{i^{(v)}}(p,q)$ represents the $\star$-contribution of the v-th issue, with $\star \in \{\tau_*, \tau^*\}$, and $|I_\iota|$ denotes the cardinality of I_ι.

The contribution sum, as a key indicator for determining whether the combined issue set satisfies the constraints, is defined as follows.

Definition 4.9. For an issue set I, let $I_\ni$ represent the issue set after sorting I. For a non-empty set $I_s \subseteq I_\ni$, the $\star$-contribution sum of I_s is defined as follows:

$$SC^\star_{I_s} = \sum_{i^{(k)} \in I_s} c^\star_{i^{(k)}}(p,q), \tag{11}$$

where $c^\star_{i^{(k)}}(p,q)$ represents the $\star$-contribution of issue $i^{(k)}$, with $\star \in \{\tau_*, \tau^*\}$.

Based on (C2) - (G2) and Definitions 4.7 - 4.9, for different reduct constraints, we present the four recursive backtracking algorithms with pruning strategies as follows:

I-recursive backtracking algorithm: For an issue set $I = \{i_1, i_2, \dots, i_n\}$, rank the τ^*-contributions under different issues from largest to smallest, the new issue sequence is denoted by $i^{(1)}, i^{(2)}, \dots, i^{(n)}$. Compute the τ^*-remaining maximum contributions $R^{[u]}_{\max\_\tau^*}$, where $u \in \{1, 2, \dots, n\}$. During the recursion process, if the τ^*-contribution sum $SC^{\tau^*}_{I_s}$ of an issue subset $I_s = \{i^{(k)}, \dots, i^{(l)}\}$ $(1 \le k < l \le n)$ satisfies $SC^{\tau^*}_{I_s} \ge 0$, the subset is recorded. If the τ^*-contribution sum of this subset satisfies $SC^{\tau^*}_{I_s} + R^{[l]}_{\max\_\tau^*} < 0$, prune the branch.

II-recursive backtracking algorithm: For an issue set $I = \{i_1, i_2, \dots, i_n\}$, rank the τ_*-contributions under different issues from largest to smallest, the new issue sequence is denoted by $i^{(1)}, i^{(2)}, \dots, i^{(n)}$. Compute the τ_*-remaining maximum contributions $R^{[u]}_{\max\_\tau_*}$, where $u \in \{1, 2, \dots, n\}$. During the recursion process, if the τ_*-contribution sum $SC^{\tau_*}_{I_s}$ of an issue subset $I_s = \{i^{(k)}, \dots, i^{(l)}\}$ $(1 \le k < l \le n)$ satisfies $SC^{\tau_*}_{I_s} > 0$, the subset is recorded. If the τ_*-contribution sum of this subset satisfies $SC^{\tau_*}_{I_s} + R^{[l]}_{\max\_\tau_*} \le 0$, prune the branch.

III-recursive backtracking algorithm: For an issue set $I = \{i_1, i_2, \dots, i_n\}$, rank the τ^*-contributions under different issues from smallest to largest, the new issue sequence is denoted by $i^{(1)}, i^{(2)}, \dots, i^{(n)}$. Compute the τ^*-remaining minimum contributions $R^{[v]}_{\min\_\tau^*}$, where $v \in \{1, 2, \dots, n\}$. During the recursion process, if the τ^*-contribution sum $SC^{\tau^*}_{I_s}$ of an issue subset $I_s = \{i^{(k)}, \dots, i^{(l)}\}$ $(1 \le k < l \le n)$ satisfies $SC^{\tau^*}_{I_s} < 0$, the subset is recorded. If the τ^*-contribution sum of this subset satisfies $SC^{\tau^*}_{I_s} + R^{[l]}_{\min\_\tau^*} \ge 0$, prune the branch.

IV-recursive backtracking algorithm: For an issue set $I = \{i_1, i_2, \dots, i_n\}$, rank the τ_*-contributions under different issues from smallest to largest, the new issue sequence is denoted by $i^{(1)}, i^{(2)}, \dots, i^{(n)}$. Compute the τ_*-remaining minimum contributions $R^{[v]}_{\min\_\tau_*}$, where $v \in \{1, 2, \dots, n\}$. During the recursion process, if the τ_*-contribution sum $SC^{\tau_*}_{I_s}$ of an issue subset $I_s = \{i^{(k)}, \dots, i^{(l)}\}$ $(1 \le k < l \le n)$ satisfies $SC^{\tau_*}_{I_s} \le 0$, the subset is recorded. If the τ_*-contribution sum of this subset satisfies $SC^{\tau_*}_{I_s} + R^{[l]}_{\min\_\tau_*} > 0$, prune the branch.

From the four recursive backtracking algorithms, we outline the following general steps to calculate the alliance, conflict, neutrality, and global reducts.

Step 1: (Calculate the contribution matrices) Based on the induced complete FST $\mathfrak{T}^{CF^*}$ and issue weights $\omega_1, \dots, \omega_n$, compute the distance matrices $\mathbb{D}_I$ and $\mathbb{D}_{i_1}, \dots, \mathbb{D}_{i_n}$. Next, based on the threshold pair (τ_*, τ^*), calculate the τ^*-contribution matrices $\mathbb{C}^{\tau^*}_{i_1}, \dots, \mathbb{C}^{\tau^*}_{i_n}$ and the τ_*-contribution matrices $\mathbb{C}^{\tau_*}_{i_1}, \dots, \mathbb{C}^{\tau_*}_{i_n}$. $\forall (x_p, x_q) \in A \times A$, the contributions under different issues are represented as $c^\star_{i_1}(p,q), \dots, c^\star_{i_n}(p,q)$, where $\star \in \{\tau_*, \tau^*\}$.

Step 2: (Calculate the local conflict weak reducts) $\forall (x_p, x_q) \in A \times A$, if $d_I(p,q) \ge \tau^*$, we execute I-recursive backtracking algorithm; if $d_I(p,q) < \tau^*$, we execute III-recursive backtracking algorithm. Thus, we can obtain the corresponding conflict weak reducts for different agent pairs.

Step 3: (Calculate the local neutrality weak reducts) $\forall (x_p, x_q) \in A \times A$, if $\tau_* < d_I(p,q) < \tau^*$, we execute II-recursive backtracking algorithm and III-recursive backtracking algorithm; the reduct set is the intersection of the results from the two functions. $\forall (x_p, x_q) \in$

$A \times A$, if $d_I(p,q) \leq \tau_*$ or $d_I(p,q) \geq \tau^*$, we execute IV-recursive backtracking algorithm and I-recursive backtracking algorithm; the reduct set is the union of the results from the two functions. Thus, we can obtain the corresponding neutrality weak reducts for different agent pairs.

Step 4: (Calculate the local alliance weak reducts) $\forall (x_p, x_q) \in A \times A$, if $d_I(p,q) \leq \tau_*$, we execute IV-recursive backtracking algorithm; if $d_I(p,q) > \tau_*$, we execute II-recursive backtracking algorithm. Thus, we can obtain the corresponding alliance weak reducts for different agent pairs.

Step 5: (Calculate the conflict, neutrality, alliance, and global weak reducts) For different reduct scenarios, we can obtain the issue weak reduct results for the entire relation matrix by taking the intersection of the weak reduct results obtained at each position in the relation matrix. Since the distance matrix is symmetric and all diagonal elements are 0, we only consider the upper triangular part of the matrix (excluding the diagonal elements). $\forall (x_p, x_q) \in A \times A$, $1 \leq p < q \leq m$, we denote the set of all issue weak reducts for the agent pair (x_p, x_q) as $\mathbb{SI}^{\diamond}_{p,q} = \{I^{\diamond}_1, I^{\diamond}_2, \cdots\}, \diamond \in \{cr, nr, ar\}$. Then, the sets of conflict, neutrality, and alliance weak reducts can be calculated as:

$$\mathbb{WI}^{\diamond} = \bigcap_{p=1}^{m-1} \bigcap_{q=p+1}^{m} \mathbb{SI}^{\diamond}_{p,q}, \diamond \in \{cr, nr, ar\}. \tag{12}$$

According to Proposition 4.1, the set of global weak reducts can be calculated as:

$$\mathbb{WI}^{gr} = \mathbb{WI}^{cr} \cap \mathbb{WI}^{nr} \cap \mathbb{WI}^{ar}. \tag{13}$$

Step 6: (Calculate the conflict, neutrality, alliance, and global reducts) For a set of issue weak reduct $\mathbb{WI}^{\diamond}, \diamond \in \{cr, nr, ar, gr\}$. $\forall I_o, I_e \in \mathbb{WI}^{\diamond}$, we can calculate the issue reducts as follows:

$$\mathbb{MI}^{\diamond} = \{I_o \in \mathbb{WI}^{\diamond} \mid \nexists\ I_e \in \mathbb{WI}^{\diamond} \backslash \{I_o\}, I_e \subseteq I_o\}, \diamond \in \{cr, nr, ar, gr\}. \tag{14}$$

To clarify the proposed model, Fig. 1 shows the framework of the issue reduct process. From the aforementioned methods and steps, we design the corresponding algorithms as follows:

Algorithm 2: Conflict reduct.

```
Input: 𝔗^{CF*}, A = {x_1, ..., x_m}, I = {i_1, ..., i_n}, W = {ω_1, ..., ω_n}, (τ_*, τ^*).
Output: The conflict reducts.
Compute the distance matrices D_I and D_{i_1}, ..., D_{i_n};
Compute the τ^*-contribution matrices C^{τ^*}_{i_1}, ..., C^{τ^*}_{i_n};
Define I-recursive backtracking algorithm and III-recursive backtracking algorithm;
for p = 1 to m − 1 do
    for q = p + 1 to m do
        if d_I(p, q) ≥ τ^* then
            Execute I-recursive backtracking algorithm;
        end
        else
            Execute III-recursive backtracking algorithm;
        end
    end
end
Calculate the intersection of conflict weak reducts at different positions;
Calculate the conflict reducts.
```

Algorithm 2 outlines the process for obtaining all conflict reducts; the time complexity is $O(m(m-1)2^{n-1})$. Algorithm 3 details the process for obtaining all neutrality reducts; the time complexity is $O(m(m-1)2^n)$. Algorithm 4 describes the process for obtaining all alliance reducts; the time complexity is $O(m(m-1)2^{n-1})$. In the actual running process, the pruning efficiency of different algorithms will be different, which will affect the complexity of the algorithm to a certain extent.

Remark 4.2. In Algorithms 2-4, we traverse all possible subsets to ensure that we identify all reduct results that meet the specified conditions. This approach significantly reduces the number of invalid search paths by calculating the remaining maximum or minimum contributions. Additionally, since the time complexity of the backtracking method increases exponentially with the number of issues, the algorithms mentioned above are not suitable for addressing problems that involve a larger number of issues.

5. An illustrative example

In this section, we use an example about the attitudes of fourteen cities in Gansu Province on eleven issues [20] to demonstrate the implementation steps of the scheme and verify the effectiveness of the adopted method. Unlike the original case, considering the rigor of urban government departments in decision-making, it is necessary to conduct quantitative evaluations of different policies by combining multiple factors and then making judgments. Thus, we use fuzzy values in $[0, 1]$ instead of three values to express the

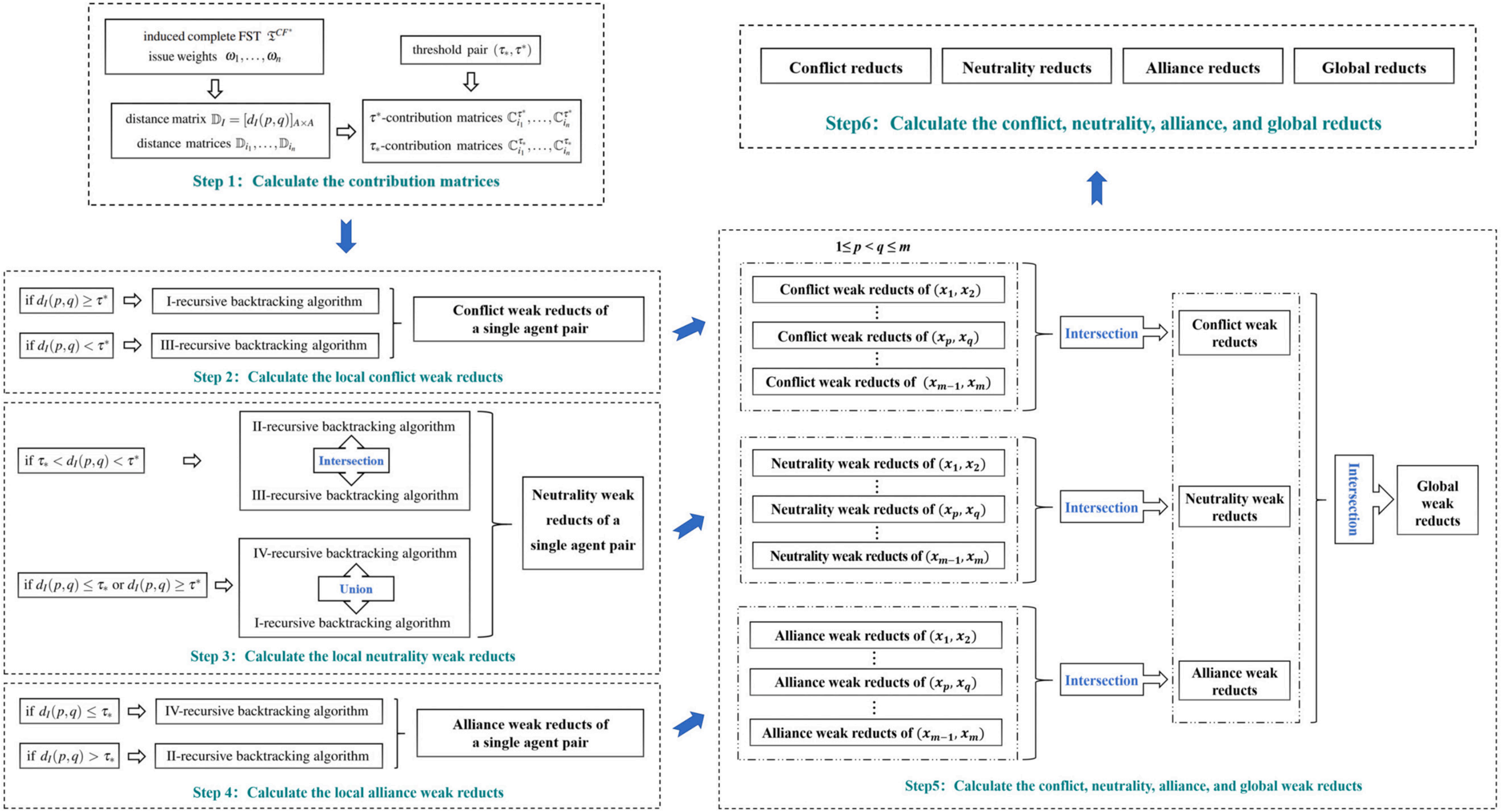


Fig. 1. Framework for issue reduct.

Algorithm 3: Neutrality reduct.

Input: $\mathfrak{T}^{CF^*}$, $A=\{x_1,\ldots,x_m\}$, $I=\{i_1,\ldots,i_n\}$, $W=\{\omega_1,\ldots,\omega_n\}$, (τ_*,τ^*).
Output: The neutrality reducts.

Compute the distance matrices $\mathbb{D}_I$ and $\mathbb{D}_{i_1},\ldots,\mathbb{D}_{i_n}$;
Compute the τ^*-contribution matrices $\mathbb{C}_{i_1}^{\tau^*},\ldots,\mathbb{C}_{i_n}^{\tau^*}$ and τ_*-contribution matrices $\mathbb{C}_{i_1}^{\tau_*},\ldots,\mathbb{C}_{i_n}^{\tau_*}$;
Define I-recursive backtracking algorithm, II-recursive backtracking algorithm, III-recursive backtracking algorithm, and IV-recursive backtracking algorithm;
for $p=1$ to $m-1$ **do**
    for $q=p+1$ to m **do**
        if $\tau_*<d_I(p,q)<\tau^*$ **then**
            Execute the intersection of II-recursive backtracking algorithm and III-recursive backtracking algorithm;
        end
        else
            Execute the union of IV-recursive backtracking algorithm and I-recursive backtracking algorithm;
        end
    end
end
Calculate the intersection of neutrality weak reducts at different positions;
Calculate the neutrality reducts.

Algorithm 4: Alliance reduct.

Input: $\mathfrak{T}^{CF^*}$, $A=\{x_1,\ldots,x_m\}$, $I=\{i_1,\ldots,i_n\}$, $W=\{\omega_1,\ldots,\omega_n\}$, (τ_*,τ^*).
Output: The alliance reducts.

Compute the distance matrices $\mathbb{D}_I$ and $\mathbb{D}_{i_1},\ldots,\mathbb{D}_{i_n}$;
Compute the τ_*-contribution matrices $\mathbb{C}_{i_1}^{\tau_*},\ldots,\mathbb{C}_{i_n}^{\tau_*}$;
Define II-recursive backtracking algorithm and IV-recursive backtracking algorithm;
for $p=1$ to $m-1$ **do**
    for $q=p+1$ to m **do**
        if $d_I(p,q)\leq\tau_*$ **then**
            Execute IV-recursive backtracking algorithm;
        end
        else
            Execute II-recursive backtracking algorithm;
        end
    end
end
Calculate the intersection of alliance weak reducts at different positions;
Calculate the alliance reducts.

evaluation result of different policies by different cities under the consideration of multiple interests. Additionally, considering the limitations of subjective and objective factors, we allow the existence of partially missing values.

5.1. Problem description

Gansu Province solicited opinions on 11 driving factors (Roads i_1, Factories i_2, Entertainment i_3, Educational Institutions i_4, Total Population of Residence i_5, Ecological Environment i_6, Number of Senior Intellectuals i_7, Traffic Capacity i_8, Mineral Resources i_9, Sustainable Development Capacity i_{10}, and Water Resource Carrying Capacity i_{11}) from 14 cities (Lanzhou x_1, Jinchang x_2, Baiyin x_3, Tianshui x_4, Jiayuguan x_5, Wuwei x_6, Zhangye x_7, Pingliang x_8, Jiuquan x_9, Qingyang x_{10}, Dingxi x_{11}, Longnan x_{12}, Linxia x_{13}, Gannan x_{14}) under its jurisdiction when formulating its development plan for the coming year.

Owing to the differences among cities in economic conditions, infrastructure, environmental conditions, and so on, they hold different attitudes towards various driving factors, as shown in Table 1. At the same time, to grasp the development status of these cities, we investigated five basic pieces of information: Economic level (Per Capita GDP) b_1, Scientific and technological level b_2, Infrastructure level b_3, Education level b_4, and Location advantage b_5. The survey results are shown in Table 2.

(1) For the basic development status information used to build a trust relation, it is difficult for us to decide which information will have a greater impact on the trust relation, so we assign equal weights for the basic information ϕ_b. Similarly, the weight of each driving factor ω_i is also given the same value.

(2) The parameter K determines the number of nearest neighbors considered in the KNN algorithm. In this paper, we determine that $K=5$ is more appropriate through the experimental analysis of the parameter K.

(3) The stability of the model proposed in this paper is not affected by the thresholds, and different thresholds should be selected according to the requirements for different application scenarios. In this paper, we set the division ratio of the three relations as 3:4:3; that is, $\tau^*=0.7$ and $\tau_*=0.3$, to reflect the case's stricter requirements for conflict and neutral relations while avoiding being too extreme and thus affecting the universality of the model.

Table 1
The attitude scores of 14 cities on 11 driving factors.

A	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}	i_{11}
x_1	0.90	0.10	0.85	0.00	0.95	0.10	1.00	ℵ	0.00	0.10	0.95
x_2	ℵ	1.00	0.10	0.85	0.30	0.90	0.10	0.70	0.90	0.80	0.10
x_3	0.10	0.80	0.00	0.70	0.10	0.90	ℵ	0.95	0.90	0.80	0.20
x_4	0.75	0.10	0.90	0.10	0.95	0.20	0.85	0.10	0.20	0.00	1.00
x_5	0.10	0.80	0.00	ℵ	0.10	1.00	0.20	0.75	0.80	1.00	0.30
x_6	0.20	0.95	0.20	0.90	0.20	0.80	0.10	ℵ	0.80	0.90	0.20
x_7	0.20	0.90	ℵ	1.00	0.20	0.70	0.40	0.80	0.30	0.90	0.40
x_8	0.20	0.60	0.10	0.80	ℵ	0.60	0.90	0.90	0.10	0.70	1.00
x_9	0.80	0.20	0.70	0.10	0.70	0.20	0.85	0.10	0.30	0.20	0.80
x_{10}	0.70	0.10	0.85	0.20	0.90	ℵ	0.80	0.10	0.90	0.10	0.20
x_{11}	0.10	0.60	0.20	0.40	0.10	0.30	0.00	0.05	0.10	ℵ	0.00
x_{12}	0.10	ℵ	0.15	0.70	0.00	0.10	0.10	0.80	0.10	0.20	0.80
x_{13}	0.80	0.30	0.70	0.25	0.80	0.20	0.80	0.00	0.10	0.20	0.80
x_{14}	0.00	0.10	0.10	0.80	0.10	0.20	0.15	0.90	0.10	0.10	ℵ

Table 2
The survey results of 14 cities on five basic information.

A	b_1	b_2	b_3	b_4	b_5
x_1	85000	Leading	Top	A	I
x_2	115000	Developed	Medium	E	V
x_3	46000	Developing	Medium	D	IV
x_4	38000	Weak	Good	B	III
x_5	148000	Developed	High	D	II
x_6	48000	Lagging	General	D	V
x_7	56000	Developing	Good	B	IV
x_8	43000	Lagging	Medium	D	IV
x_9	78000	Weak	High	C	II
x_{10}	45000	Lagging	Medium	C	III
x_{11}	32000	Lagging	General	E	V
x_{12}	30000	Lagging	General	E	V
x_{13}	28000	Lagging	General	E	V
x_{14}	25000	Lagging	General	E	V

5.2. Solving process

In this subsection, we will analyze the cases mentioned above using the established model. The specific process is outlined as follows.

Step 1: Information preprocessing

To estimate the missing values, we first need to preprocess the basic information across different scales and then construct the STN.

For the Economic level (Per Capita GDP) b_1, since the difference in Per Capita GDP is significant, direct normalization is influenced by extreme values, resulting in a right skew. To address this problem, we first perform a logarithmic transformation to compress the magnitude differences and reduce the right bias. After the transformation, we use the Min-Max method for normalization. The specific formula is:

$$g(v(x,b_1)) = \frac{\ln(v(x,b_1)) - \max_{\ln(v(x_a,b_1))}}{\max_{\ln(v(x_a,b_1))} - \min_{\ln(v(x_a,b_1))}}, x_a \in A.$$

For the Scientific and technological level b_2, we do normalized assignments in the order Leading ≻ Developed ≻ Developing ≻ Weak ≻ Lagging. The specific formula is:

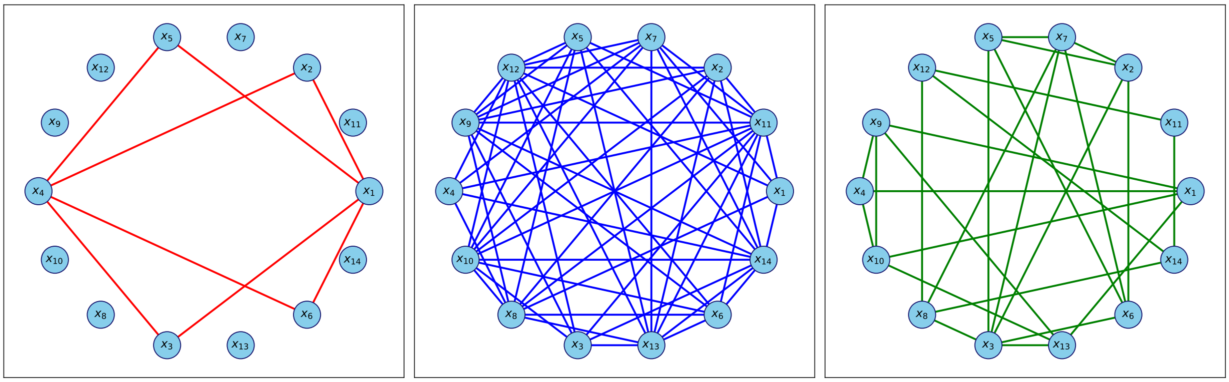
$$g(v(x,b_2)) = \begin{cases} 1, & \text{if } v(x,b_2) = \text{“Leading”}; \\ 0.75, & \text{if } v(x,b_2) = \text{“Developed”}; \\ 0.5, & \text{if } v(x,b_2) = \text{“Developing”}; \\ 0.25, & \text{if } v(x,b_2) = \text{“Weak”}; \\ 0, & \text{if } v(x,b_2) = \text{“Lagging”}. \end{cases}$$

Similarly, for the Infrastructure level b_3, the Education level b_4, and the Location advantage b_5, we do normalized assignments in the order Top ≻ High ≻ Good ≻ Medium ≻ General, A ≻ B ≻ C ≻ D ≻ E, and I ≻ II ≻ III ≻ IV ≻ V, respectively.

Step 2: Construct the STN

Table 3
The trust matrix between 14 cities on five basic information.

A	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
x_1	1.0000	0.3660	0.3309	0.5595	0.6376	0.1857	0.5031	0.2734	0.6903	0.3785	0.0901	0.0829	0.0751	0.0624
x_2	0.3660	1.0000	0.6970	0.5255	0.6716	0.6517	0.5691	0.6394	0.5563	0.5445	0.6561	0.6489	0.6411	0.6284
x_3	0.3309	0.6970	1.0000	0.7285	0.5686	0.8452	0.8279	0.9424	0.6406	0.8475	0.7592	0.7519	0.7442	0.7314
x_4	0.5595	0.5255	0.7285	1.0000	0.5971	0.5737	0.8564	0.6861	0.7691	0.7810	0.5307	0.5234	0.5156	0.5029
x_5	0.6376	0.6716	0.5686	0.5971	1.0000	0.4234	0.5407	0.5110	0.8280	0.5161	0.3278	0.3205	0.3127	0.3000
x_6	0.1857	0.6517	0.8452	0.5737	0.4234	1.0000	0.6827	0.8876	0.4954	0.7927	0.9044	0.8971	0.8894	0.8766
x_7	0.5031	0.5691	0.8279	0.8564	0.5407	0.6827	1.0000	0.7703	0.7127	0.7754	0.5871	0.5798	0.5720	0.5593
x_8	0.2734	0.6394	0.9424	0.6861	0.5110	0.8876	0.7703	1.0000	0.5830	0.8949	0.8168	0.8095	0.8017	0.7890
x_9	0.6903	0.5563	0.6406	0.7691	0.8280	0.4954	0.7127	0.5830	1.0000	0.6881	0.3998	0.3925	0.3848	0.3720
x_{10}	0.3785	0.5445	0.8475	0.7810	0.5161	0.7927	0.7754	0.8949	0.6881	1.0000	0.7117	0.7044	0.6966	0.6839
x_{11}	0.0901	0.6561	0.7592	0.5307	0.3278	0.9044	0.5871	0.8168	0.3998	0.7117	1.0000	0.9927	0.9850	0.9722
x_{12}	0.0829	0.6489	0.7519	0.5234	0.3205	0.8971	0.5798	0.8095	0.3925	0.7044	0.9927	1.0000	0.9922	0.9795
x_{13}	0.0751	0.6411	0.7442	0.5156	0.3127	0.8894	0.5720	0.8017	0.3848	0.6966	0.9850	0.9922	1.0000	0.9873
x_{14}	0.0624	0.6284	0.7314	0.5029	0.3000	0.8766	0.5593	0.7890	0.3720	0.6839	0.9722	0.9795	0.9873	1.0000



(a) Conflict relationship (b) Neutrality relationship (c) Alliance relationship

Fig. 2. The three types of relations between 14 cities.

From Definition 3.2, suppose the weights of different features in the basic information set B are equal. $\forall x, y \in A$, the weighted trust degree $t^B_{x\&y}$ can be calculated as:

$$t^B_{x\&y} = \frac{1}{5}\sum_{j=1}^{5} t^{b_j}_{x\&y}.$$

Then, we can obtain the trust matrix between 14 cities on five basic information, as shown in Table 3.

Step 3: Estimate the missing values

Through the information collected during the investigation, we determined the specific locations of 11 missing values. M_v represents the set of all missing values.

$$M_v = \{\ \mathrm{Na} \mid f^*_{x_2,i_1}, f^*_{x_{12},i_2}, f^*_{x_7,i_3}, f^*_{x_5,i_4}, f^*_{x_8,i_5}, f^*_{x_{10},i_6}, f^*_{x_3,i_7}, f^*_{x_1,i_8}, f^*_{x_6,i_8}, f^*_{x_{11},i_{10}}, f^*_{x_{14},i_{11}}\ \}.$$

From Equations (3) to (6), the missing values can be calculated as:

$$\{0.1216, 0.4929, 0.4431, 0.4465, 0.2297, 0.6146, 0.4310, 0.2884, 0.5508, 0.4085, 0.5578\}.$$

Therefore, we can obtain a complete attitude score information table, referred to as the induced complete attitude score information table by the KNN algorithm. Next, we can calculate the issue reduct based on this table.

Step 4: Issue reduct

From Definition 4.3, we can derive the relations among the 14 cities, as illustrated in Fig. 2. The color-coded lines indicate how the cities relate to one another across 11 driving factors. To determine the core issues that sustain these relations, we will discuss the issue reduct of the corresponding relations based on the invariance of relation partitioning.

Conflict reduct: According to Algorithm 2, we can calculate all conflict reducts as follows:

$$\{i_1,i_{10},i_{11}\},\{i_3,i_6,i_{11}\},\{i_5,i_6,i_{11}\},\{i_1,i_2,i_9,i_{10}\},\{i_2,i_3,i_5,i_6\},\{i_2,i_3,i_5,i_9\},\{i_2,i_3,i_9,i_{10}\},$$
$$\{i_2,i_3,i_{10},i_{11}\},\{i_2,i_5,i_6,i_9\},\{i_2,i_5,i_9,i_{10}\},\{i_3,i_4,i_5,i_6\},\{i_3,i_4,i_9,i_{10}\},\{i_1,i_2,i_3,i_9,i_{11}\},$$
$$\{i_3,i_4,i_7,i_{10},i_{11}\},\{i_3,i_7,i_9,i_{10},i_{11}\},\{i_4,i_5,i_6,i_9,i_{10}\},\{i_5,i_7,i_9,i_{10},i_{11}\},\{i_2,i_5,i_6,i_7,i_8,i_{10}\},$$
$$\{i_2,i_5,i_7,i_8,i_9,i_{10}\},\{i_3,i_4,i_5,i_7,i_8,i_{10}\},\{i_3,i_4,i_6,i_7,i_8,i_{10}\}.$$

The resulting conflict reducts offer multiple perspectives for the Gansu provincial government to analyze the core issues. If the government wants to focus on a few key factors among them for in-depth research and analysis, the following reduct sets should be prioritized: $\{i_1,i_{10},i_{11}\}$, $\{i_3,i_6,i_{11}\}$, and $\{i_5,i_6,i_{11}\}$, with i_{11} being the core contradiction. If the reduct sets with fewer factors are insufficient for resolution, we can gradually consider the reduct sets with more factors. Obviously, as the number of factors in the reduct set increases, the complexity of the considerations also escalates.

Neutrality reduct: According to Algorithm 3, we can calculate the neutrality reduct as follows:

$$\{i_2,i_3,i_5,i_7,i_8,i_9,i_{10}\}.$$

The neutrality relation is generally less discussed and is usually regarded as a vague boundary to be further dealt with. Here, we only show that this reduct set $\{i_2,i_3,i_5,i_7,i_8,i_9,i_{10}\}$ covers the core factor to maintain a neutral relation between cities.

Alliance reduct: According to Algorithm 4, we can calculate all alliance reducts as follows:

$$\{i_1,i_3,i_6,i_7,i_8,i_9,i_{10}\},\{i_2,i_3,i_5,i_7,i_8,i_9,i_{10}\},\{i_1,i_2,i_3,i_5,i_6,i_8,i_9,i_{10},i_{11}\},\{i_1,i_2,i_4,i_5,i_6,i_7,i_8,i_9,i_{10}\}.$$

These reduct sets indicate the core factors that maintain the alliance relations between agents. From Definition 4.6, the alliance reduct core can be calculated as $\{i_8,i_9,i_{10}\}$, which should be given more attention.

Global reduct: From Equations (13) and (14), we can calculate the global reduct as follows:

$$\{i_2,i_3,i_5,i_7,i_8,i_9,i_{10}\}.$$

That is, if we want to ensure that all three relations remain unchanged at the same time, the set of factors we need to focus on is $\{i_2,i_3,i_5,i_7,i_8,i_9,i_{10}\}$.

It can be observed that after the initial reduct, some results still contain numerous factors, making it difficult to focus on the core issue. Therefore, in our future work, a secondary reduct can be considered. On one hand, while the amount of information decreases after the initial reduct, the key features from the original dataset remain intact. This allows the situation table created from this information to continue reflecting the main characteristics of the data, providing a foundation for further reduct. On the other hand, each reduct serves to purify the data. Through successive reducts, secondary factors can be progressively eliminated, allowing us to focus more sharply on the most essential factors. This approach aligns with the cognitive process through which individuals analyze and comprehend complex information.

6. Parameter and comparison analysis

In this section, we will discuss the parameters and algorithms of our model, comparing it to other models to illustrate its advantages and capabilities.

6.1. Analysis of the parameter K

For the model proposed in this paper, the parameter that significantly affects its performance is primarily K, which is used in missing value estimation. Our goal is to ensure that the selected K value enhances the model's performance while maintaining optimal computational efficiency. Therefore, we mainly reverse-deduce the optimal K selection from the results of missing value estimation.

At the relative level, we calculated the estimated results of each missing value under different K values, as shown in Fig. 3. We found that when $K = 1, 2, 3$, the estimation of most missing values is poor; when $K = 4$, the estimation of some missing values remains unsatisfactory; but when $K \geq 5$, the estimation of all missing values demonstrates a relatively stable trend. At the absolute level, we calculated the Euclidean distance between the estimated value of each missing value and its average estimate under different K values to assess the deviation of the estimated missing values. From Fig. 4, we can see that the results are better when $K = 5$ or 7. After considering the computational cost, we decided to set K to 5 to achieve a balance between the estimated results and computational efficiency.

6.2. Analysis of the reduct algorithms

In Subsection 4.3, we discussed the time complexity of the three reduct algorithms. Without considering pruning conditions, the time complexities are $O(m(m-1)2^{n-1})$, $O(m(m-1)2^{n})$, and $O(m(m-1)2^{n-1})$, respectively. Through the example in Section 5, we now compare the actual time complexities of pruning and non-pruning methods to illustrate the effects of pruning. From Table 4, we observe that the efficiency of different reduct algorithms improves after pruning. Notably, the effect of neutral reduct is the most significant, with time complexity reduced by approximately 22% compared to the non-pruning scenario.

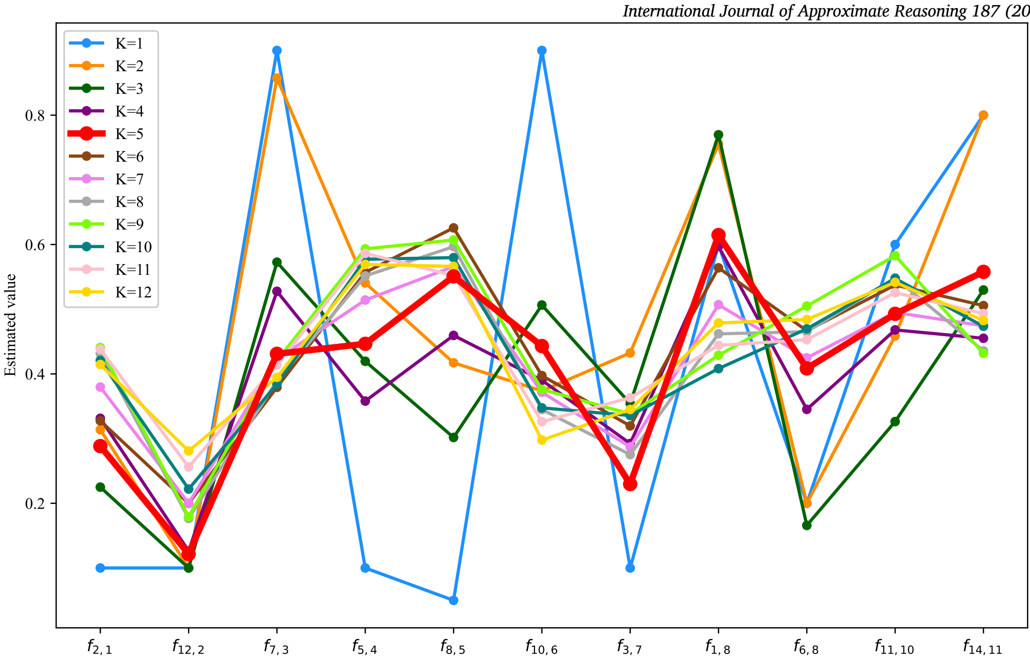


Fig. 3. Estimation of each missing value under different K values.

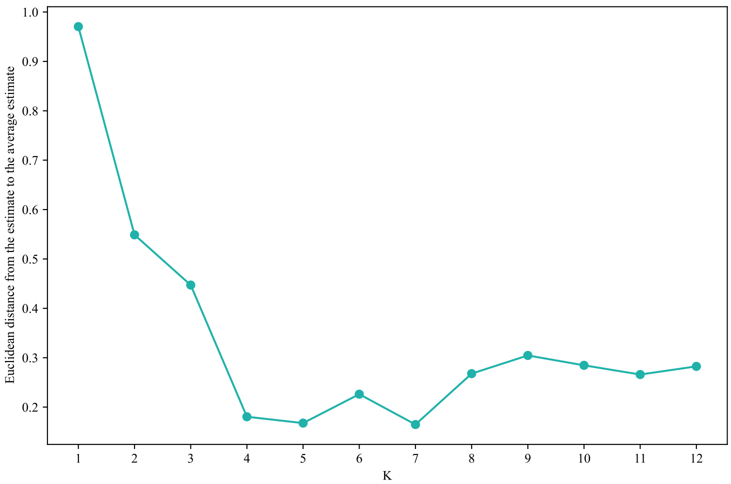


Fig. 4. The Euclidean distance between the estimated values and its average estimate under different K values.

Table 4
Comparison of pruning and non-pruning time complexities about the three reducts.

	Conflict reduct	Neutrality reduct	Alliance reduct
Non-pruning	186368	372736	186368
Pruning	**184202**	**292070**	**178915**
Efficiency	98.84%	78.36%	96.00%

6.3. Compare with other models

In existing studies, there are relatively few discussions on the issue reduct of TWCA. Lang [33] first proposed an alliance, conflict, and neutrality reduct method based on the discernibility matrix, offering a new perspective for identifying key issues. Zhang et al. [34] aimed to determine the minimum reduct set using a heuristic strategy, gradually eliminating redundant issues and verifying regional invariance. Ran et al. [35] examined the issue reduct of weak alliances, weak neutrality, and weak conflict relations in

Table 5
Comparison of our model and some existing models.

Models	Research environment	Estimation of missing values	Issue reduct algorithm	Scalability
Lang [33]	TSTs	/	Disjunction of discernibility matrices	limitation
Zhang et al. [34]	TSTs	/	Deletion strategy	limitation
Ran et al. [35]	ITSTs	Uncertainty analysis	Disjunction of discernibility matrices	limitation
Our model	**IFSTs**	**STN and KNN**	**Recursive backtracking algorithm**	**extensible**

ITSTs by utilizing discernibility functions. As shown in Table 5, the model proposed in this paper has several advantages compared to current research:

(1) The situation table discussed involves incomplete information and is more in line with the actual situation. Meanwhile, the KNN algorithm adopted in this paper does not require a training process and can calculate missing values based on feature similarity. It is easier to implement and apply on the basis of objectivity.

(2) The discussed situation tables are no longer limited to three-valued ones but fuzzy-valued ones, and can be directly applied to situation tables of other data types. That is to say, for any data type, as long as the distance matrix is calculated, we can use the model proposed in this paper to calculate the issue reduct. Therefore, we call it is extensible.

(3) Compared with the matrix method and deletion strategy, the proposed recursive backtracking algorithm is more efficient for handling issue reduct and can quickly produce results using a computer programming software.

7. Conclusion

This paper proposed a new model for identifying and resolving the core issues of conflicts in complex social systems, expanding the existing research on issue reduct in TWCA under incomplete information.

Our study contributes to the field in several ways. First, by utilizing the STN and KNN methods, we employ consensus degree to balance trust and rating similarity, construct trust relations, and fill in missing values through iterative weighting. Second, we propose a recursive backtracking algorithm based on the contribution matrix. This algorithm transforms the relation matrix into constraint conditions and incorporates a pruning strategy to reduce computational complexity. Additionally, parallel computing of different issue reducts is achieved.

However, our study has several limitations. For instance, the complexity of the recursive backtracking algorithm leads to exponential growth when addressing large-scale issues; some reduct results are less than ideal; the relation division relies on fixed thresholds and does not account for adaptive adjustments in dynamic environments; the issue weights are assigned statically by experts instead of conducting a more in-depth analysis, which will affect the applicability of the model to a certain extent. Therefore, future research should focus on the method of secondary reduct, the selection of automated thresholds, the optimization of computational efficiency, the allocation of issue weights, and the model's generalization ability in conflicts across different domains.

CRediT authorship contribution statement

Hai-Long Yang: Writing – review & editing, Writing – original draft, Methodology. **Sheng Gao:** Writing – original draft, Visualization, Software, Methodology. **Zhi-Lian Guo:** Writing – review & editing, Validation, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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