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A Mathematical Model Based on Salinity of Ocean for Climate Change

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***Abstract*—As we are aware that sea ice production, heat uptake, ocean circulation patterns and density are efficiently impacted by ocean salinity, therefore it plays a vital role in the earth climate system. Thus, in order to prevent agriculture, forest and public safety, the accurate and more reliable estimation of climate becomes particularly important. Hence, the ocean salinity must be included in the ocean climate change models. This paper proposes an integer order climate change model for the prediction of climate by incorporating the salinity feature. This strengthens the model dependability and yields more precise findings on the relationship between depth and time in the setting of climate change. Also, the complex and constantly shifting features of marine ecosystems are largely determined by the interaction between salinity and temperature. The numerical solution of the model is obtained by employing a modified Laplace Adomian decomposition method. The estimated results are validated by the comprehensive comparisons.**

***Index Terms*—Salinity, Adomian Decomposition Method, Climate change.**

I. Introduction

Climate change is a pressing global issue that carries significant consequences for both human society and the environment [1]. The scientific evaluations carried out by the Intergovernmental Panel on Climate Change (IPCC) consistently highlight the increasing consequences of elevated global temperatures and sea levels resulting from human activities, specifically the release of greenhouse gases like carbon dioxide [2]. The IPCC is the premier worldwide organisation for evaluating climate change and their research offer a thorough summary of the most recent findings in science about the origins, effects, and dangers of climate change [3].

The concentration of carbon dioxide (CO_2), a significant contributor to the greenhouse effect has experienced a notable increase since the pre-industrial period [4]. This rise of CO_2 is primarily attributed to human activities, including the combustion of fossil fuels and alterations in land use [5]. Thus, increase in carbon dioxide (CO_2) concentrations have resulted in a global warming phenomenon, which is manifested in various observable consequences including the thawing of permafrost, melting of glaciers, and the elevation of sea levels. The study conducted in Alaska has established a connection between higher temperatures and the heightened occurrence of seismic events [6]. It is also emphasizing the seismic implications of global warming. Furthermore, alongside these climate-related issues, the Earth also demonstrates fluctuations through microseismic activity. Therefore, this activity is highly influenced by atmospheric storms and can be understood through a phenomenological model as explained by Correig et al. [7]. Wilson [8] suggested that the ambient noise in the ocean is also impacted by the presence of wind-generated noise as explained in his theory of wave height spectra.

Nowadays, the research in this field has been driven by the increasingly strained relationship between humans and the ocean and comes under the domain of ocean and human health (OHH). In this field, the researchers combine the disciplines of marine science with human beings [9]. Thus, this holistic approach employed in this context plays a significant contribution to the interdependence between human health and the health of the ocean. Talley et al. [10] demonstrated that the salinity fluctuations in oceanic water are impacted by various factors such as precipitation, river inflow, and evaporation, and therefore possess important implications for the regulation of global climate and the water cycle. Hedgpeth et al. [11] discuss the global salinity of ocean water and exhibits the extreme variations in salinity levels with certain regions. Salinity is conventionally quantified in units of parts per thousand (ppt) with average magnitudes typically hovering around 35ppt. The relationship between salinity with temperature and density illustrating a key role for the properties of seawater, and in turn affect ocean currents and the freezing point of seawater [12]. Thus, the role of salinity in the Earth energy balance and climate regulation is of utmost importance [13]. Moreover, it is worth noting that the storms and hurricanes, which are intensified by the presence of warm surface waters, can exert a substantial influence on the dynamics of the ocean by facilitating the mixing of surface and deeper waters [14].

Currently, there are two basic reasons for considering the salinity of ocean water for the prediction of climate change. First, it has a direct impact on seawater density, which turn in affects the movement of ocean currents from the tropics to the poles together with temperature. Therefore, the way that heat is transported across the seas is ultimately controlled by these currents and affect the global climate. Second, the amount of freshwater that evaporates and precipitates into the seas as well as the Earth total water cycle is directly related to sea surface salinity [15]. In order to gain a comprehensive understanding of the dynamic behaviour of the ocean, it is imperative to utilize a model that incorporates the temporal evolution of ocean depth. However, for improving the precision of these models the inclusion of the salinity feature is being contemplated and acknowledging the influence of salinity on oceanic dynamics over various depths and time periods. Moreover, the utilization of the modified Laplace Adomian decomposition approach is considered a sophisticated technique for solving these models as it eliminates the requirement for linearization, perturbation, and extensive computing [16]. The Adomian decomposition method (ADM) can be used to solve any type of nonlinear differential equations in several areas of research such as physics, engineering, and other disciplines [17].

In this paper, an integer order climate change model based on salinity features is presented for the prediction of climate. The mathematical numerical solution of the model is determined with the help of the modified Laplace Adomian Decomposition method. In this method, the complicated equation is divided into a number of smaller equations according to the decomposition principle, which forms the foundation of ADM. Hence, after solving each of these equations separately, the solution of the original equation can be found as a linear combination of all these solutions. This approach is consistent with the continuous endeavours to improve model adaptability and enhance the ability to forecast. The estimated results are compared with the existing models.

The rest of the paper is organised as follows: The preliminary discussions are covered in Section II, and the mathematical climate change model along with relevant formulations are discussed in Section III. Section IV illustrates the comprehensive numerical solution of the model. Section V provides the experimental results and a brief discussion. Section VI concludes the article and offers future work recommendations.

II. Preliminaries

Salinity: The quantity of salt dissolved in seawater is measured by the ocean salinity, which varies greatly throughout the world [18]. The primary determinants influencing the variations in ocean salinity encompass evaporation, precipitation, ocean current, and marine depth. These are defined as:

Evaporation: It is the conversion of a liquid into a gas. Extreme regions of the ocean, such as the subtropics, see higher rates of evaporation, leading to increased water loss and subsequent salt accumulation. The subtropical regions exhibit a notable evaporation rate, leading to an increased depletion of freshwater resources and subsequently elevated levels of salinity. The upshot of this is a salinity level of 37 parts per thousand (ppt) [19].

Precipitation: Precipitation refers to the process by which water formed in the atmosphere descends to the Earth's surface. The water can exist in either a liquid or frozen state. In the polar regions and near river mouths, heavy rainfall leads to an increase in freshwater influx, which in turn dilutes the seawater and reduces its salinity. The Baltic Sea has an average salinity of around 7 parts per thousand (ppt), despite the considerable amount of freshwater from rivers and copious precipitation in the area. This phenomenon is mostly ascribed to the increase in the availability of freshwater and the subsequent process of dilution, which in turn causes a reduction in salinity levels due to precipitation [20].

Ocean Currents: The constant, directed movement of seawater caused by wind, gravity, and water density (the Coriolis Effect) is known as an ocean current and regional differences are a result of the movement of water masses with varying salinities around the planet by ocean currents. As an illustration, the Gulf Stream raises the salinity of the North Atlantic ocean by transporting warm, salted water from the Gulf of Mexico northward [21].

Depth: Seawater exhibits a higher density compared to freshwater. Therefore, the weight of water increases proportionally with the concentration of dissolved salt, such as sodium chloride, inside a given volume. Hence, the increase in depth leads to a corresponding increase in salinity and reach to a specific threshold value.

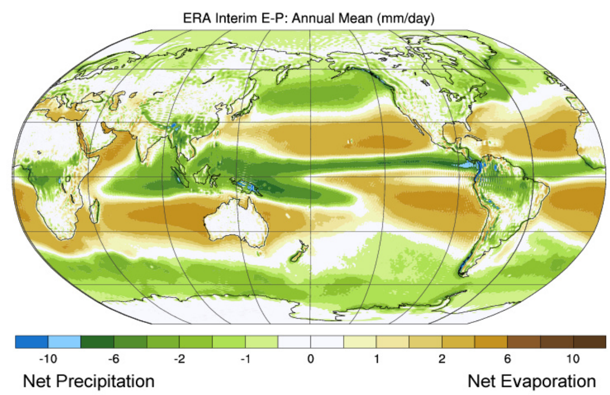

Fig. 1. Annual mean global differences in evaporation and precipitation [22].

III. Mathematical Modeling of Climate Change Model

In order to improve the model adaptability and provides more precise prediction of climate, a regularisation parameter can be included to incorporate the salinity feature of the ocean into the conceptual model of tropical Pacific oscillations as

$$\frac{du}{dt} = -T + \lambda + \gamma S \tag{1}$$

and

$$\frac{dT}{dt} = -aT + (u + bu^3) + k \cos \omega t + \delta S \tag{2}$$

where, positive upper ocean temperature anomalies create shallow thermocline depth anomalies. Here, the symbol u, T, λ and t represent ocean depth, upper ocean temperature, noise and time, respectively. Moreover, $k \cos \omega t$ denotes solar forcing and $u+bu^3$ shows thermocline displacement. Also, a & b are the coefficients associated with damping and nonlinearity, respectively. The variable S represents salinity, and γ and δ denote the regularization parameter for changes in depth and temperature changes(regularisation parameter regularise the effect of evaporation and precipitation), respectively.

Now, differentiating expression in (1) with respect to t and replacing it in (2), we get

$$\frac{d^2u}{dt^2} = aT - (u + bu^3) - k\,cos\,\omega t \tag{3}$$

Substituting the expression of (1) in (3) and using the $\lambda = \frac{1}{f^{\alpha^*}}$ as suggested in [7], we have

$$\frac{d^2u}{dt^2} = a\left(\frac{1}{f^{\alpha^*}} - \frac{du}{dt}\right) - (u + bu^3) - k\,cos\,\omega t + \gamma S - \delta S \tag{4}$$

Incorporating human carbon dioxide (CO_2) emissions radiative forcing into the model is assumed, as it is widely recognized as the primary contributor to greenhouse gas emissions in the atmosphere and has a significant impact on global warming. In the field of climate science, the notion of radiative forcing is employed to quantitatively assess the alteration in the energy equilibrium inside the Earth's atmosphere resulting from several sources, including variations in greenhouse gas concentrations, aerosols, and fluctuations in solar radiation. It is defined as

$$R = \frac{6.3\beta_1(1-\phi)}{C_h}\log\left(\frac{C}{C'}\right) \tag{5}$$

where, C_h is the Earth specific heat capacity and it is to be considered as $16.7\,\mathrm{Wm}^{-2}K^{-1}$. Here, C, C', ϕ and β_1 are the concentration of CO_2 in the atmosphere after the increase, concentration of CO_2 in the atmosphere before the increase, amount of heat produced by the greenhouse effect that is absorbed by the seas and transferred from the upper layers to the deep sea and regularization parameter, respectively. The value of β_1 is taken in between 1.1 and 3.4 and ϕ is used as 0.23 as suggested by Farmer et al. [23]. Hence, in order to take into account the radiative forcing brought on by an increase in CO_2, the expression in equation (4) can be rewritten as follows

$$\frac{d^2u}{dt^2} + \frac{du}{dt} + u + bu^3 = af^{-\alpha*} + R - k\,cos\,\omega t + \gamma S - \delta S \tag{6}$$

where, k, ω and f represent the amplitude, natural frequency of solar forcing and frequency of seismic activities, respectively. Thus, in order to find the solution of above equation (6), the following initial conditions(These conditions are taken as depth and change in depth equal to zero at some initial stage.) are considered as

$$u(0) = \dot{u}(0) = 0 \tag{7}$$

The detailed solution of equation (6) is discussed in next section.

IV. Numerical Solution

The solution of the differential equation given in (6) is determined with the help of Laplace transform [24]. Thus, the following expression is obtain

$$\begin{aligned} L(u) = &- \frac{ks}{(s^2+\omega^2)(s^2+as+1)} + \frac{af^{-\alpha*} + R + \gamma S - \delta S}{s(s^2+as+1)} \\ &+ \frac{-bL(u^3)}{s^2+as+1} \end{aligned} \tag{8}$$

Let us assume that $u(t) = \sum_{n=0}^{\infty} u_n(t)$ and $u^3 = \sum_{n=0}^{\infty} A_n$, we get

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(t) = L^{-1}\Bigg(&\frac{af^{-\alpha*} + R + \gamma S - \delta S}{s(s^2+as+1)} - \\ &\frac{ks}{(s^2+\omega^2)(s^2+as+1)}\Bigg) + L^{-1}\left(\frac{-bL(\sum_{n=0}^{\infty} A_n)}{s^2+as+1}\right) \end{aligned} \tag{9}$$

On simplifying the equation (9), we get the following recurrence relations

$$\begin{aligned} u_0(t) =& L^{-1}\left(\frac{af^{-\alpha*} + R + \gamma S - \delta S}{s(s^2+as+1)} - \frac{ks}{(s^2+\omega^2)(s^2+as+1)}\right) \\ =& (af^{-\alpha*} + R + \gamma S - \delta S)\Bigg(1 - e^{-at/2}\cos\left(\frac{\sqrt{4-a^2}t}{2}\right) \\ &- e^{-at/2}\left(\frac{a}{\sqrt{4-a^2}}\right) sin\left(\frac{\sqrt{4-a^2}t}{2}\right)\Bigg) + \sin\left(\frac{\sqrt{4-a^2}t}{2}\right) \\ &\left(\frac{ae^{-at/2}(\omega^2-1)}{\sqrt{4-a^2}}\right) - \frac{k}{(\omega^2-1)^2 + \omega^2 a^2}\Bigg((\omega^2-1)\cos\omega t \\ &- a\omega\sin\omega t - (\omega^2-1)e^{-at/2}\cos\left(\frac{\sqrt{4-a^2}t}{2}\right)\Bigg) \end{aligned}$$

and

$$\sum_{n=0}^{\infty} u_{n+1}(t) = L^{-1}\left(\frac{-bL(\sum_{n=0}^{\infty} A_n)}{s^2+as+1}\right) \tag{10}$$

Now, the following expression is used to construct the Adomian polynomials for the nonlinear term u^3 as given in [17]:

$$\begin{aligned} A_n =& \frac{1}{n!}\frac{d}{d\lambda^n} N\left[\sum_{k=0}^{n} y_k \lambda^k\right] \\ A_0 =& u_0^3, A_1 = 3u_0^2 u_1, \cdots \end{aligned}$$

Hence, the final solution of climate change model (6) is given as

$$\begin{aligned} u_1 &= L^{-1}\left[\frac{-bL(u_0^3)}{s^2+as+1}\right] \\ u_2 &= L^{-1}\left[\frac{-bL(3u_0^2 u_1)}{s^2+as+1}\right] \\ &\vdots \\ u &= u_0 + u_1 + u_2 + ... \end{aligned} \tag{11}$$

V. Experimental results and Discussion

In order to comprehend the effects of climate change in relation to Earth dominant fluctuations and make strong predictions about future climate patterns, we have estimated the results in terms of graphs based on the approximate solution obtained from (11). The solutions have been determined as per the assessments conducted by the Intergovernmental Panel on Climate Change (IPCC), which suggested the radiative forcing value of $R = 3.7\,\mathrm{W/m^2}$ [3]. As per the suitability, the frequency interval is taken from $0.04\,\mathrm{Hz} - 0.3\,\mathrm{Hz}$, which also corresponds to the frequency range of microseismic activities that best approximates the background equilibrium fluctuations as observed globally [25]. Additionally, the exponent $\alpha*$ has varied within the range of $0.1-2.0$, and the damped constant within the range of $0.0-2.0$. The estimated results of the integer order proposed model in terms of lack of damping are compared with model of Eze et al. [16] in Fig. 2. It is observed that the sinusoidal curve exhibits a sustained amplitude. Fig. 3 demonstrates a relationship between the depth and time for a damped constant value of $a = 0.2$. Also, the Fig. 4 represents the similar relationship between the depth and time for the damped value $a = 0.5$. These comparisons indicate that the solution does not eliminate its oscillatory characteristics. It means that there is a fluctuation in climate change. The implication is that irrespective of any measures taken to address climate change, its consequences will endure as a result of Earth natural variations. Fig. 5 illustrates the comparison where the frequency f is set to $0.05Hz$, while Fig. 6 depicts the situation when f is increased to $0.2Hz$. It can be observed that as the value of f increases from $0.05Hz$ to $0.2Hz$, the amplitude gradually diminishes over time but albeit not entirely. Thus, the aforementioned phenomenon can also be ascribed to the influence of Earth oscillations. From Figs.7-8, it was observed that an increase in $\alpha*$ from 0.5 to 1.25 results in increase of oscillation. This implies that the outcome has the potential to result in the more occurrence of flooding.

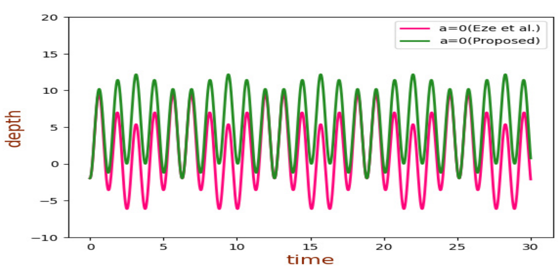

Fig. 2. Results comparison in terms of lack of damping: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.1$ Hz, $\omega = 5$ Hz, $\alpha^* = 0.6$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0$.

VI. Conclusion and Future Scope

In this work, an integer order model has been introduced to predict the potential effects of climate change by considering the predominant fluctuations in the Pacific Ocean. The fluctuations have been considered in terms of ocean salinity. The numerical solution of the model has been derived through the utilization of the modified Laplace Adomian decomposition

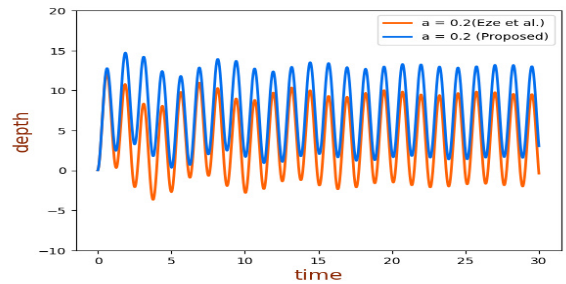

Fig. 3. Results comparison in terms of damping: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.1$ Hz, $\omega = 5$ Hz, $\alpha^* = 0.6$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0.2$.

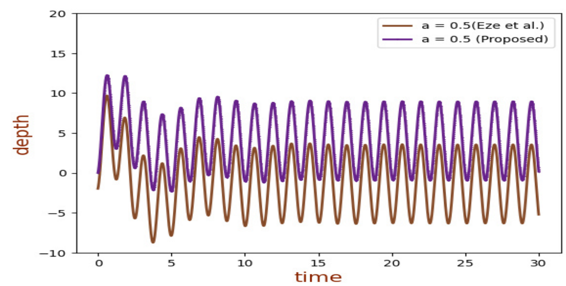

Fig. 4. Results comparison in terms of damping: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.1$ Hz, $\omega = 5$ Hz, $\alpha^* = 0.6$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0.5$.

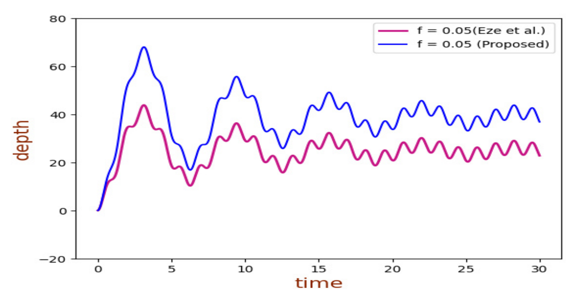

Fig. 5. Results comparison in terms of frequency: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.05$ Hz, $\omega = 5$ Hz, $\alpha^* = 0.6$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0.2$.

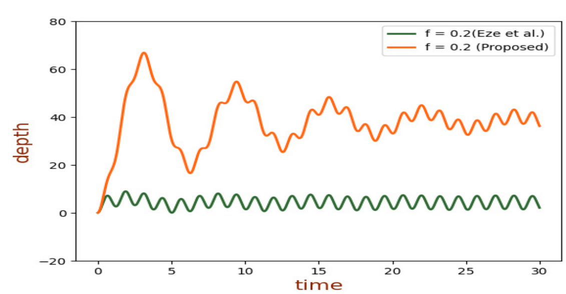

Fig. 6. Results comparison in terms of frequency: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.2$ Hz, $\omega = 5$ Hz, $\alpha^* = 0.6$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0.2$.

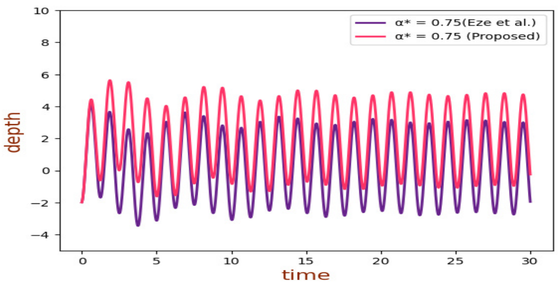

Fig. 7. Results comparison in terms of α^*: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.1$ Hz, $\omega = 5$ Hz, $\alpha^* = 0.75$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0.2$.

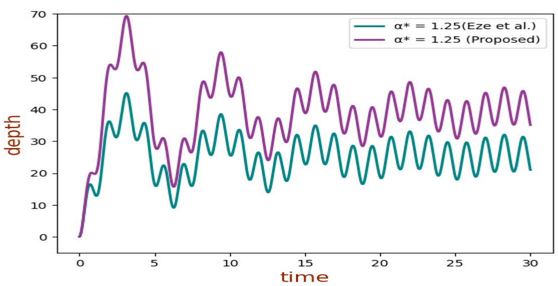

Fig. 8. Results comparison in terms of α^*: Proposed model vs Eze et al. [16] for $b = 1$, $f = 0.1$ Hz, $\omega = 5$ Hz, $\alpha^* = 1.25$, $S = 35$, $\gamma = 2$, $\delta = 1$, $k = 6$ m, and $a = 0.2$.

method. The inclusion of the salinity feature in our proposed model was found to enhance its robustness and accuracy, leading to improved results in comparison to existing models. However, these results suggested that irrespective of the mitigation strategies employed, the impact of climate change will persist and indicate that the complete eradication of this effect through mitigation practices is still unattainable. In the present study, the utilization of average salinity was necessitated by the computational constraints. In future, it will be more feasible to explore the variable salinity data and incorporate fractional order derivatives into our modified model. This strengthens the model dependability and yields more precise results for climate change. Additionally, the existing dataset of different regions can be employed to validate the proposed model. Hence, it can be inferred that salinity serves as a significant parameter in the examination of oceanic depth and provides more accurate outcomes when assessing the influence of climate change in conjunction with the prevailing fluctuations of the Earth.

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