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'Control' and its significance for the modern world

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SUMMARY

The Science of Control achieved its majority during World War II, when the need to control guns, radar antennae and other weapon systems prompted a careful study of the control and feed-back processes which are embodied in servo-control loops of all kinds. Since then developments in electronics have permitted the complex control problems encountered in the automation of industry to be handled.

The paper seeks to explain the simple basis of servo-devices and how the control of unstable systems is now possible by the use of digital computers. An inverted pendulum with two degrees of freedom has been treated as a simple example of such a system. The experimental and theoretical investigations performed to set up a computer strategy for the control of this system are described.

Examples of automated control in various parts of industry are given and the application of computer control techniques in industrial economic systems is touched upon.

This lecture was illustrated by a colour film showing working models of experiments and the performance of equipment described in the text.

OPSOMMING

Beheerwetenskap het mondig geword gedurende die tweede wêreldoorlog toe die behoefte om kanonne, radaranten nes en ander wapenstelsels te beheer aanleiding gegee het tot 'n grondige studie van beheer- en terugvoerprosesse wat gebruik word in alle tipes servobeheerlusse. Sedertien het ontwikkelings in elektronika dit moontlik gemaak om deingewikkelde probleme te hanteer wat voorkom in die outomatisasie van die nywerheid.

Die referaat poog om te verduidelik wat die eenvoudige basis van servo-toestelle is en hoe die beheer van onstabiele stelsels nou moontlik is deur syferrekenaars te gebruik.

'n Omgekeerde slinger met twee vryheidsgrade is behandel as 'n eenvoudige voorbeeld van 'n dergelike stelsel. Die eksperimentele en teoretiese ondersoeke wat uitgevoer is om rekenaarstrategie vir die beheer van hierdie stelsel op te stel, word beskryf.

Voorbeelde van geoutomatiseerde beheer in verskeie afdelings van die nywerheid word aangehaal en die toepassing van rekenaarbeheertegnieke in industriële ekonomiese stelsels word genoem.

Hierdie voordrag is geïllustreer met 'n kleurfilm oor werkende modelle in eksperimente en die gedrag van apparaat wat in die referaat beskryf is.

Africa is a fascinating continent. Not only is it of enormous size but it contains vast natural resources and, straddling the equator as it does, experiences those variations of climate that have favoured the evolution of all manner of creatures, including man himself. Certainly, primitive men existed in East Africa many millions of years ago, as shown by the patient researches of Dr Leakey at Olduvai Gorge. But the relevance of this discovery to our subject this evening is that one piece of evidence testifying to the essentially human origin of these fossil remains was the presence of artifacts in the shape of primitive tools. Man has been variously described as a worshipping animal, as an artistic animal and also as a creator of tools. It is upon man's capability as a toolmaker that I wish to concentrate this evening, for this is the reason for his success in the struggle for existence.

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We are all familiar with the marvellous story of evolution, of the process of selection by the environment of those variants of a species which are better suited for survival in it. These are the individuals from which the future generations stem, and so we have a process of gradual adaptation of a species to suit the circumstances of its habitat.

The same process happened during the evolution of man, but the reason for man's dominance over other forms of life was that he gradually learned how to control his environment and to adapt it to his needs. He learnt how to shape stone, wood and bone into tools that could be used as weapons for defence against wild animals and also as the means of obtaining food. With his tools he built simple shelters, made clothes and so, with the discovery of fire, could avoid the worst effects of cold. Fire also made available to him other foods and opened the way to the use of metals.

In all of this we see the gradual growth of a simple technology which, coupled with the emergence of agriculture, permitted primitive man to become increasingly independent of the climatic and other vagaries in his surroundings. In modern phraseology we would say that by means of his progress in technology man was achieving a higher standard of living.

This process of technological discovery and the subjugation of the environment took place over vast periods, and during this time man gave little thought to the niceties of control theory. Nevertheless, he practised control both consciously and unconsciously, as a moment's reflection will show. In all his bodily movements there was co-ordination of muscles and senses, which is the essence of control as we wish to consider it this evening. He invented the rudder to steer his primitive boats and learnt to feather an arrow the better to direct it to its mark, and so made an early conscious application of control as a way of ordering a system to achieve a desired end.

Mankind is not alone among the creatures of the earth which exercise control in their daily life. When we watch a bird alighting on the swaying branch of a tree on a windy day, it is apparent that control of the most subtle kind is being exercised, The bird has to sense its position, which it does by means of the eye, assisted by what we might term its gyroscope or balancing organ, which is the canal structure of the inner ear. Signals of position are then communicated to the brain and a computation is performed which results in an executive command being transmitted to the muscles of the wing. tail or leg. This results in a stable posture being assumed.

It is our objective to trace briefly the path by which man has built up a science of control and to indicate the important part which control theory can play in the modern world and in the technology which sustains it.

The science of mechanics and the First Industrial Revolution

The discovery of the wheel is shrouded in the mists of time, but structures such as Stonehenge and the Pyramids show that man had mastered certain building skills a few thousand years ago. In later historic times particularly in the Graeco-Roman period and the succeeding centuries, the rate of discovery was greatly accelerated. Men such as Archimedes, Leonardo da Vinci, Stevinus of Bruges, Copernicus, Galileo and Newton all made their contributions and by the 17th century the science of mechanics was well established. Many types of mechanisms were also in common use, such as levers, windlasses, capstans, cranes and pulleys, all of which were usually designed to increase the strength of man's arm by the mechanical advantage that the machine conferred—paid for by the velocity ratio. Sources of power were also employed, such as the water-wheel and windmill, and some element of automatic control was sometimes embodied, as in the post mill in which a wind vane kept the plane of the mill at right angles to the wind's direction.

Thus, by the early 18th century the stage was well prepared for the First Industrial Revolution. This was based essentially on the use of the steam engine to drive the many ingenious machines which the inventors of that era devised to meet the product needs of the growing populations of Europe.

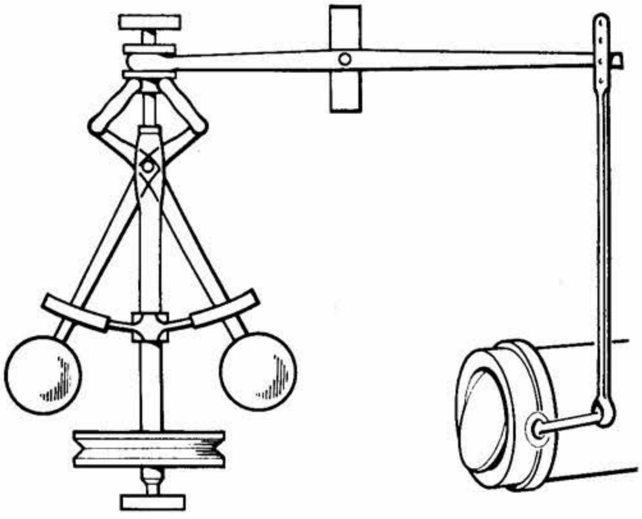
The Second Industrial Revolution and the science of control

Although the machines of the First Industrial Revolution were power-driven they were not automated in the modern sense. The machines were set up to deliver a product, but the state of the output could be monitored only manually and adjustments to the machine had to be made by hand. The output was not measured continuously, nor was such information fed back immediately to the input so as to affect the nature of the output while production was in progress. It is this process of measurement and feedback of data from the output of a system over an appropriate communication link to the input or to a control centre that is the essential feature of automation, for in this way the performance of the system may be continuously evaluated and adjusted to meet the demand.

The process of comparison between the present state of an automated system and the planned objective may be complex. Calculations have to be performed both rapidly and accurately to derive the necessary control instructions. They must also be repeated at short intervals but this is just the function which a modern digital computer is admirably suited to fulfil. Modern society needs the fruits of automation—it is no coincidence that the technologists of our day, following the example of their forebears, have created the tool to solve the problem.

Automation in this era of nuclear power, space technology and micro-electronics is the exercise of control in its broadest aspect on behalf of man and his works. Mechanics was the lifeblood of the First Industrial Revolution, but it is upon the science of control that the automation age depends.

Let us now look briefly at the methods used to control some simple devices and then see how the computer



Steam Governer

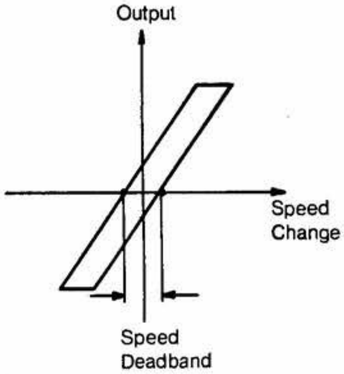


Fig 1 Basic Watt governor and its characteristics; showing effects of static friction in introducing a speed dead-band

permits us to apply similar concepts of control to unstable systems.

Speed control

The motorist controls the speed of his car by adjusting the engine throttle via the accelerator pedal. This is open-loop control with no feedback. But if the driver observes his speedometer and adjusts his throttle and brakes so as to maintain a constant speed of 40 mph, no matter the gradient of the road, then we have an example of closed loop or 'feedback control' with the man himself acting as the communication link and computer. The flyball governor was applied to the steam engine by James Watt and fulfilled automatically these same four functions of speed measurement, feedback, comparison and throttle adjustment, and so may be regarded as the prototype automatic regulator of the machine era.

The film illustrates such a governor controlling the speed of an old beam engine at Wearmouth and also a modern version of the same device as used to control a modern small steam turbine (typically 1 MW).

A serious limitation of Watt's simple system is that the force required to move the levers and actuate the steam valve must be provided by the flyball head from the centrifugal force when the speed increases, and from gravity when the speed falls. Using the flyball head in this way causes high bearing loadings and, hence, gives large friction forces which introduce a dead-band in the characteristic, as shown in Fig 1. This dead-band causes the speed control to be rather crude and discontinuous, and the accuracy of control may be improved by introducing a hydraulic force amplifier between the flyball head and the steam valve. The governor head has now only to move the pilot valve to the hydraulic or fluidic amplifier, whose output opens and closes the steam valve on the turbine, so adjusting the speed of the turbine by controlling the flow of steam through the machine. This combined control device for speed measurement, force amplification and steam valve operation we call a servomechanism.

Model of a simple servomechanism

You will have noticed that considerable complications have now been added to the original Watt regulator in order to obtain more precise control of the turbine. We shall find that the quality of control can be greatly influenced by the characteristics of the forward and feedback linkages in a servomechanism. These effects can be conveniently explored in the two models shown in Figs 2a and 3a.

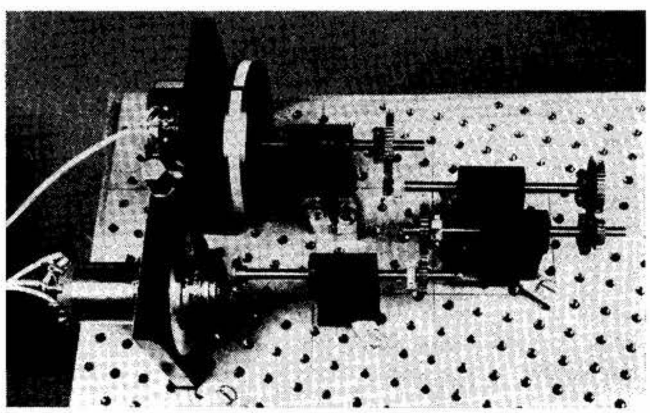


Fig 2 (a) Mechanical model of a servomechanism with stiff coupling

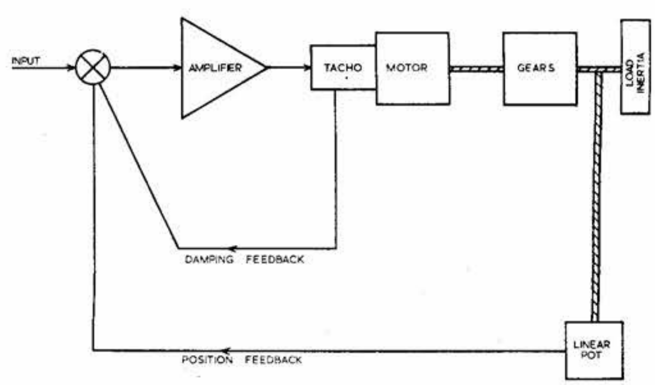


Fig 2 (b) Schematic diagram of model

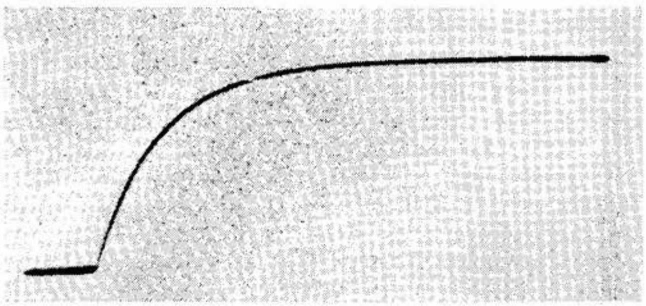


Fig 2 (c) Oscillogram showing response of the stiff model to a step input

In Fig 2a the controlled element is a brass disc of large inertia whose angular position is measured by the voltage delivered from the wiper arm of a potentiometer mounted on the same shaft. This voltage is fed to a transistor amplifier which drives a small servo-motor linked through a gear train to the disc shaft. A tachometer generator is contained within the motor and supplies a voltage proportional to the motor speed (Fig 2b). It is convenient when studying the performance of the servo to switch the amplifier off, move the potentiometer track away from the null position and re-energize the amplifier. The wiper will now deliver a voltage to the amplifier and the servo-motor will seek to drive the disc and potentiometer until the wiper voltage is reduced to zero.

In the language of the control engineer we have just applied a step function of position to the servo, in order to move the disc from one equilibrium position to another You will see from Fig 2c that in this non-elastic system the disc moves smoothly and gradually to the final position. In the second model, shown in Figs 3a and 3b, an elastic coupling in the form of a spring has been introduced into the linkage. The interaction of this coupling with the inertia of the brass wheel is indicated in Fig 3c, which is a recording of the motion of the wheel when, with the servo amplifier disconnected, it is rotated away from the equilibrium position, in opposition to the spring reaction, and then allowed to return to its initial position. The arrangement behaves like a normal pendulum and swings to and fro, until the cumulative effect of the frictional losses at the pivot and elsewhere brings the device gradually to its initial equilibrium position. The linkage now possesses a natural frequency of oscillation.

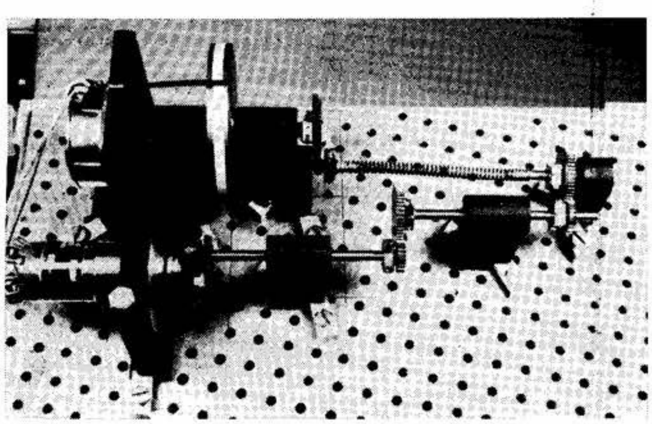


Fig 3(a) Mechanical model of a servomechanism with an elastic coupling (note the helical spring)

With the servo amplifier reconnected, the behaviour of the system, when a change of position is suddenly asked for, may be investigated. In Fig 4a is shown the response when the system is over-damped. You will note that the movement of the disc is gradual, reaching the desired position slowly. You may also note a small oscillation superimposed. When the degree of damping is reduced, the result is as shown in Fig 4b. Here we have achieved the so-called critical damping condition, in which the response of the system is as fast as can be achieved without actual oscillations taking place. In Fig 4c the degree of damping has been reduced still further, with the result that the small oscillations, instead of dying down, increase gradually and make the system uncontrollable.

The linkage of a servomechanism is clearly not so simple as appears at first sight but may exhibit rather complex behaviour. We wish the control process to be rapid, accurate and stable. But the prime function of the linkage is to transmit energy to the controlled

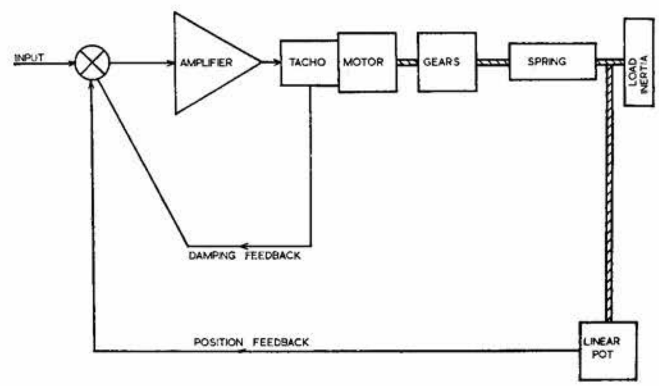


Fig 3 (b) Schematic diagram of model

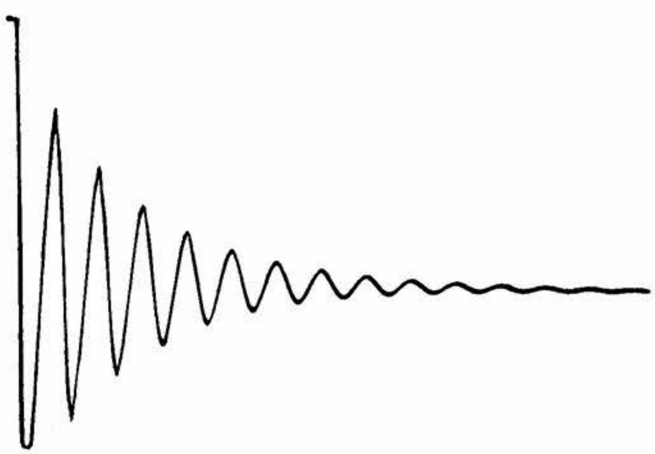


Fig 3 (c) Oscillogram showing damped, free oscillation of elastic model

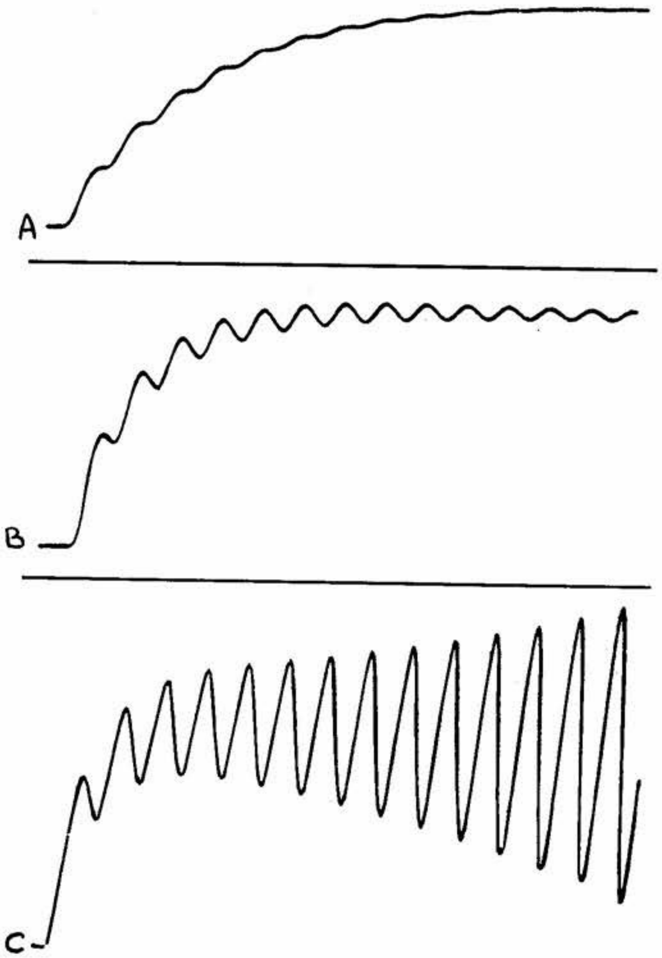


Fig 4 Oscillograms showing response of elastic model
(a) Overdamped
(b) Critically damped
(c) Underdamped, resulting in build-up oscillation

element, and so we recognize that instability may occur by reason of the oscillation of some of this energy between the kinetic and potential forms. Potential energy will be associated with capacitors in electrical circuits and with the elastic elements in the mechanical members. Similarly, kinetic energy will be associated with electrical inductors or with the velocities and rotations of masses, while the existence of friction, air drag or electrical resistance will cause energy losses which tend to damp the oscillations. These facts are implicit in the second-order differential equations which describe such systems.

In the energy transfer process a number of frequencies is involved and these signals will pass in both the forward and reverse directions. Thus there is a bandwidth associated with the link transmission and the gain and phase change of the signals round this loop will vary with the frequency. To avoid instability it is necessary that none of these signals shall be fed back to the input in a phase and amplitude relation that will cause an oscillation of the associated frequency to be built up. This is the Nyquist condition for stability and must be observed in any servomechanism design.

The analogue computer

The curves represent the performance of the two models shown. They were taken not on the models themselves but on an electrical network, which behaves in the same way as the models. This device, which we call an analogue computer, is illustrated in Fig 5. It is made up of a series of functional circuit blocks, interconnected as required, to simulate an idealized system, which may be made to behave like the equipment to be controlled. In this way the designer may study the performance of a device without actually building it, thus saving considerable time and money. It permits modifications to be introduced to the design and their effect measured, until final performance is satisfactory. It also permits a number of important tests which would not normally be possible on the real system. An analogue simulation may not appear to be very significant for the models shown, but when we have to deal with large structures such as a satellite tracking aerial (Fig 13) then analogue simulation is essential.

We may now see how these principles of servoloop design find application in two important types of control problem.

Speed control of dc electric motors

As a boy I often visited the Lancashire cotton spinning mill of which my father was the manager. I used to admire greatly the rope race, which was the assembly of rope drives that extended from the 30-ft flywheel of the twin-cylinder steam engine in the basement to the six floors of the factory, there to power the carding machines, slubbers and spinning mules that composed the mill. The steam engine was controlled by its Watt's governor to a constant speed, and the different speed requirements of the various machines had to be met by cunning arrangements of belt drives and gear trains—

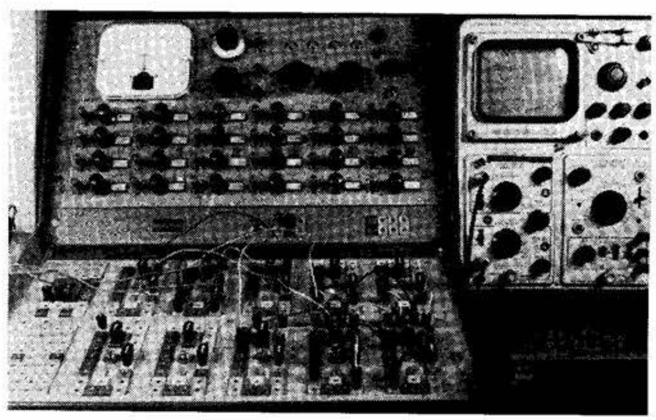


Fig 5 Analogue computer with simulation of servomechanism of Fig 3 (a) set up

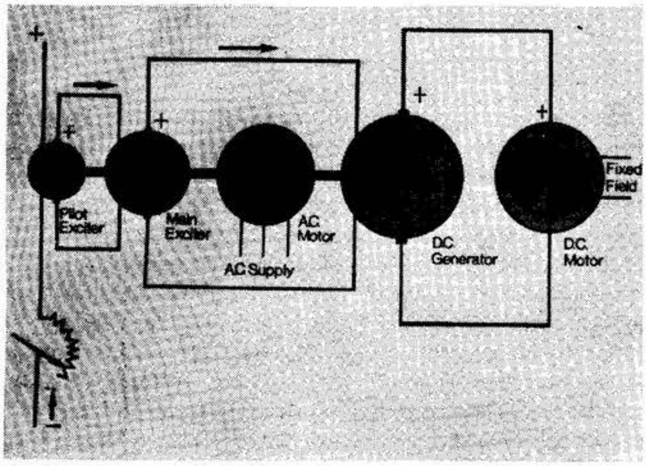


Fig 6 Ward-Leonard system of speed control for large dc electric motors (open loop)

all very wasteful of energy. The modern method of powering the mill is much to be preferred for, with the availability of electric power from the grid, it is now convenient to fit individual electric motors to all process machines, each with its own speed controller programmed to the spinning or other process to be performed. With dc motors the speed may conveniently be changed by the adjustment of armature voltage and/or field current, although the resistances which have to be used waste power. The Ward-Leonard system, shown in Fig 6, is more complex but it is capable of controlling very large motors with relatively high efficiency; it should be noted that no feedback is employed and the system is open-loop.

The modern speed control system for dc motors shown in Fig 7 is of the closed-loop type and consists of a tachometer-generator which provides a voltage proportional to the speed; a stabilized power unit

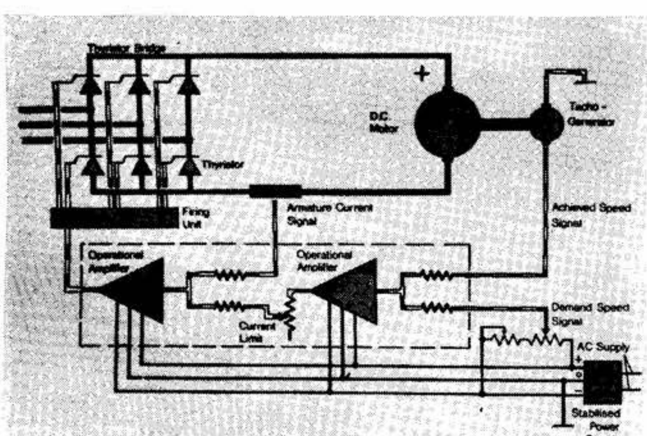


Fig 7 Modern speed control system for large dc motors using semiconductor amplifiers and rectifiers (closed loop)

to provide a voltage reference; a dual amplifier servo module in the dotted section, consisting of two cascaded amplifiers and associated functional networks; a firing unit which controls the operation of the thyristor diodes; and finally the thyristor bridge itself, which provides current to the dc motor armature.

The major feedback loop includes the tachogenerator, the output from which is compared with the stable voltage which represents the required speed. The difference or error is applied to the first operational amplifier which provides a time integral of this error, i.e. the output of this amplifier increases as long as an error of the right polarity exists. When an error of opposite polarity appears the output decreases. The output is constant when the error reduces to zero. The following amplifier is also an integrating amplifier, the input to which is the difference between the integral of the speed error and the motor armature current. Since this current is a measure of the motor torque, it follows that the input to the second integrator is a demand for motor torque, that demand being consistent with zero speed error.

The performance of this motor control system may be described by a family of curves similar to those established for the model servomechanism. Fig 8 compares the large fluctuations in speed which occur as

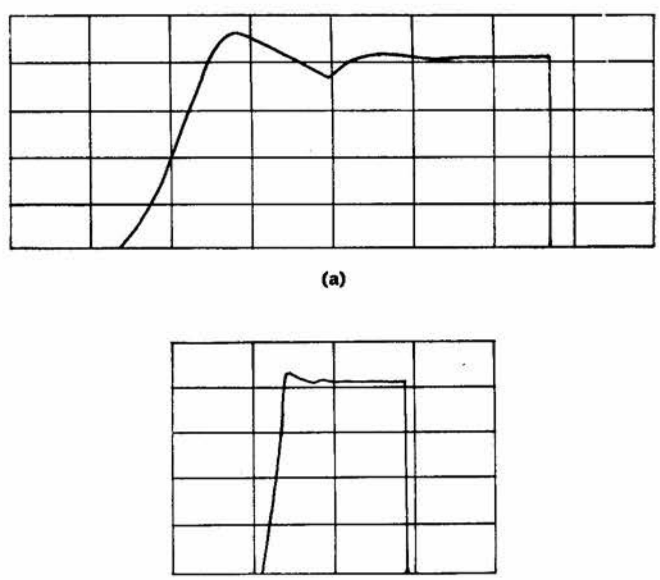


Fig 8 Speed fluctuation when bringing the motor to its operational speed (a) manually—no feedback (b) with closed-loop control

the motor is brought to speed manually with the small changes resulting from the application of feedback. When the damping is heavy the response characteristic is as in Fig 9, and the final speed is approached only very gradually. As damping is reduced the response time is reduced, with optimum conditions reached for critical

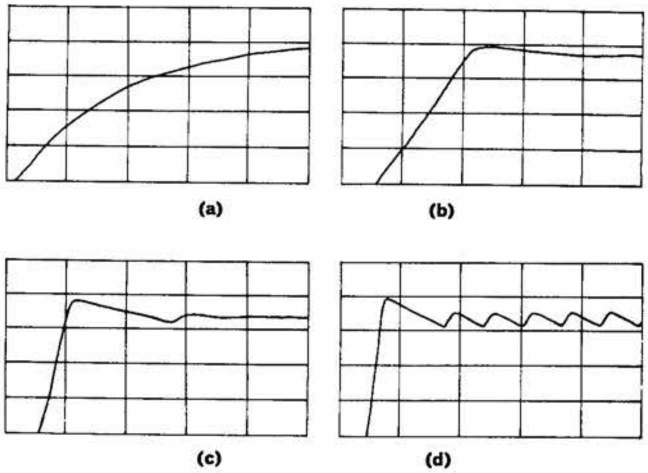


Fig 9 Response of dc motor with closed-loop control (a) heavy damping (b) critical damping (c) underdamped-showing fluctuation (d) oscillatory

damping, as shown in Fig 9b. Further reduction in damping causes overshoot and speed fluctuations, as indicated in Fig 9c, and finally sustained oscillations as shown in Fig 9d.

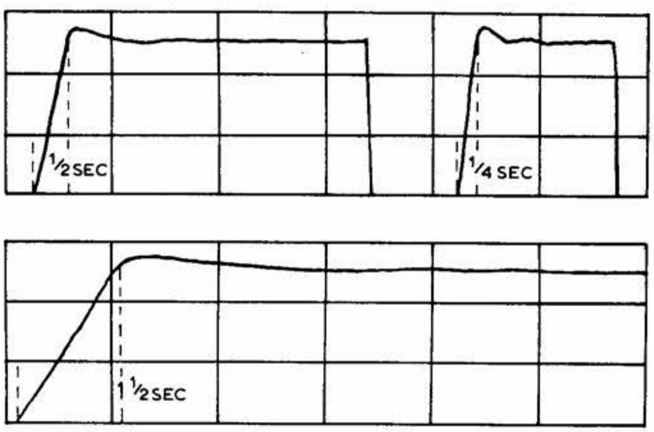


Fig 10 Reduction of response time by adjustment of feed back loop, critical damping conditions being maintained

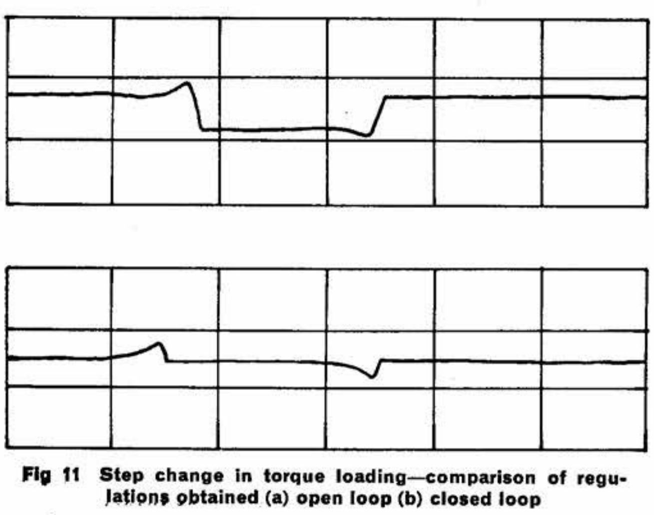


Fig 11 Step change in torque loading—comparison of regulations obtained (a) open loop (b) closed loop

It is possible, by the addition of subsidiary feedback loops and adjustment of loop gain, to maintain critical damping but to reduce the response time (Fig 10). The application of feedback not only improves the motor starting conditions; it also improves its response to disturbances occurring during running. For example, a change of torque loading results in a much smaller reduction of speed than when no feedback is applied, as indicated in Fig 11.

It is interesting to compare present-day semiconductor control devices with the valved units of only ten years ago (Fig 12). The power-to-space ratio of the equipment has increased, accompanied by important advances in robustness, reliability and maintainability. Control capability and speed of response have also been greatly improved. Feedback systems of this kind are now widely used to control motors driving printing presses, elevators in automatic car parks and machine tools, as illustrated in the film.

Position control—Pointing the ground aerial of a satellite communication system

The problem of controlling remotely the angular position of a shaft is encountered in many electro-mechanical systems. Thus we need to control the azimuth and elevation of aerials in radar, radio astronomy and satellite communications (Fig 13); but aerials are elastic structures and are subject to wind stresses. So, to avoid behaviour similar to that of the model examined above, the control system must be designed with great care.

A satellite tracker is required to keep the aerial pointing to a selected satellite with an accuracy of one or two minutes of arc, even when the satellite is moving. This accuracy is essential since, with the large gains required for communication purposes, the aerial beamwidth is low and a small angular error can cause a large signal loss. For example, a typical 85-ft satellite aerial operating at a frequency of 8 Gc/s has a half-power beamwidth of only six minutes of arc. Consequently, a misalignment of only three minutes of arc can cause a signal loss of 50 per cent. The aerial must be moved about two axes, typically in azimuth and elevation, usually by dc motors of 10-60 hp armature controlled by thyristor amplifiers, as described above.

A block diagram of the closed-loop formed by one of the axis tracking systems is shown in Fig 14. Transistor networks are used to amplify low-power signals and to perform signal shaping and compensating functions. The input to the control system is the position of the satellite in the sky relative to some arbituary axis passing through the earth station (Fig 15), usually the geometrical axis of the dish. The input is the error it is required to minimize, namely the angular difference between the satellite direction and the aerial axis. This error is measured at the aerial by means of a beam sensor device, operating on a tracking signal radiated by the satellite. It is then amplified by the tracking receiver and converted to a dc signal proportional to

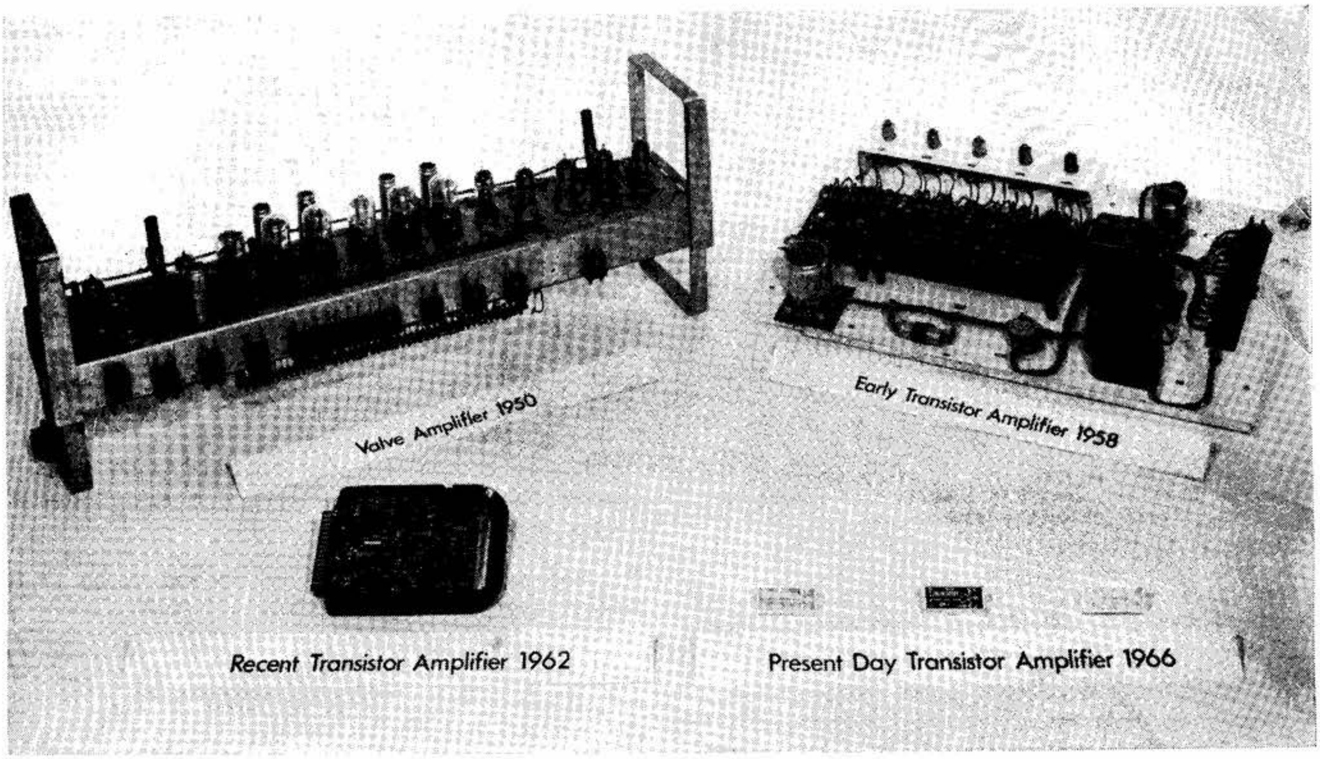


Fig 12 Progress in servo amplifier units 1950-1966

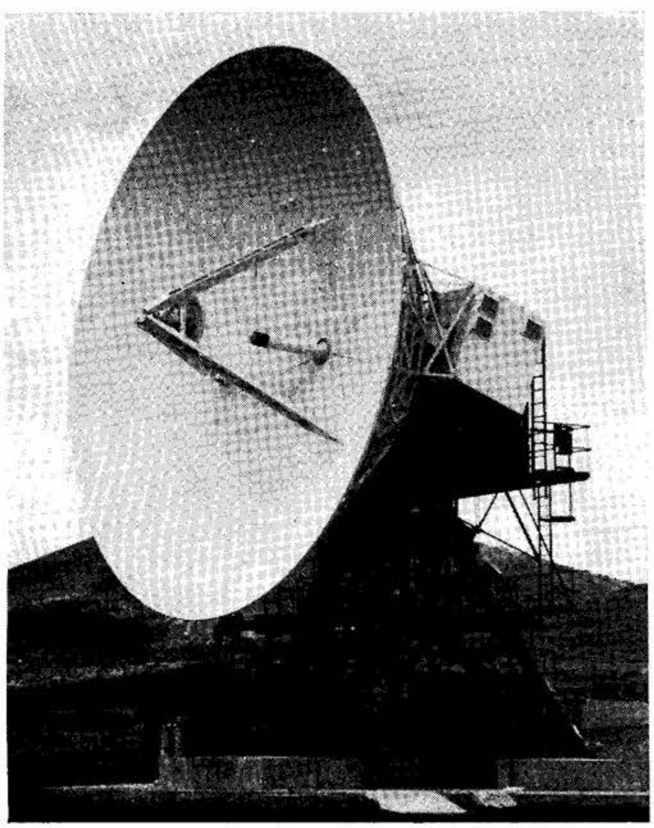


Fig 13 Computer-controlled satellite communication aerial

pointing error. It is further amplified by the servo and power amplifier to provide a correcting torque that moves the aerial in a direction tending to minimize the error. A tachogenerator provides velocity feedback to the servo input, thus ensuring the stability of the position control loop, under certain conditions, as will be indicated later.

The mechanical coupling from driving motor to aerial has characteristics very similar to those of the elastic model of a servomechanism as previously described, i.e. it has inertia, elasticity and damping. We therefore expect that oscillations may be induced in the aerial system, and catastrophic consequences could result unless the feedback loop is designed to ensure stability.

Stabilization

It is necessary that the aerial should be capable of slewing from one satellite to another in the minimum time, i.e. we need as fast a response as possible to sudden demands. We have seen that the speed of response can be improved by increasing the gain in the feedback loop, but this gain must not be increased indefinitely or oscillation will result.

The rate loop shown in Fig 14 provides a means of increasing the speed of response without incurring instability. Since the aerial possesses inertia it takes a certain time to reach a new position. An error will exist up to the moment the aerial passes through the required orientation, when the error will change sense, The faster the aerial is moved the larger the overshoot. The rate loop obviates this difficulty by reducing the torque according to the speed of rotation of the aerial. In this way the overshoot is reduced and the stability increased without sacrifice of the speed of response.

By the use of two feedback loops, in this case transmitting velocity and position, the resultant control of the aerial is made more precise. This example will serve to indicate that the control of very complex systems may require a number of interlocking loops, the design of which can become complicated indeed.

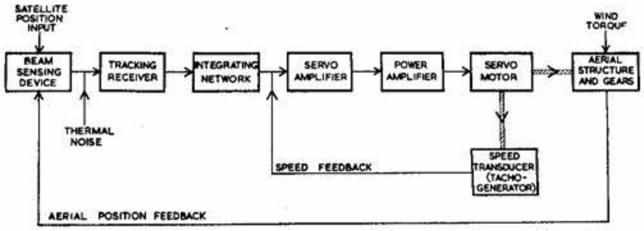


Fig 14 Diagram of two-loop control system for a satellite communication aerial (compensation for velocity lag error by inclusion of an integrating network)

Reduction of steady state and random errors

A two-loop control system designed as above is adequate if the satellite is almost stationary in space, and only fine adjustment of the aerial is required to resist disturbance due to receiver noise and wind. If, however, the satellite is not stationary but orbiting at a constant angular velocity, say, it will be found that the aerial is following with a constant angular lag behind the satellite, never quite catching up. It is this lag which gives rise to the angular error and so provides the drive to the slewing motors.

It is possible to reduce this lag to zero by adding an integrator in the forward path of the feedback loop, as indicated in Fig 14. A constant motor torque is obtained when the integrator output voltage is constant and this occurs when its input is zero—that is, when the error voltage is reduced to zero. The output voltage increases when the input (i.e. the error) is positive and decreases when it is negative. Thus, whenever an error exists between the satellite and aerial directions, the torque increases if the tracker is lagging and decreases if the tracker is ahead, thus reducing the error in either case.

The aerial collects radio noise which is added to that produced by the first circuit of the receiver. These two noise signals are amplified to yield a random voltage which acts upon the motors to cause jitter of the aerial in both azimuth and elevation. Wind gusts also produce random pointing errors. The design requirements for correcting errors due to wind and receiver noise respectively have to be matched to each other; the inevitable compromise is most conveniently found with the analogue computer—an indispensable aid to the study of all control problems.

The control of unstable systems

In the development of the science of control it was natural for engineers to cope first with the problem presented by the regulation of essentially stable systems. In such cases a form of open-loop control is usually possible even when the feedback link is disabled. But there are many devices and systems which are unstable and which are functional only by reason of a complex control subsystem. We would like to know something about the control of unstable systems, for many important automation applications in industry are of this type. Also, the more generalized theory of control to which such studies lead is of great importance in non-engineering applications.

Fortunately, you can be confident that control of an unstable device is frequently possible, for your ability to walk into the hall this evening is adequate proof of the fact. If you have cycled, then the demonstration is even more complete. The human body is an unstable system par excellence. Many inventors, authors and composers have been fascinated by the theme of the animated doll and have found that it is not too difficult to imitate the mechanics of the human body, but the control system has defied simulation. Nevertheless, we can see that the components and functions found to be necessary in man-made control systems are also present in the human case. Thus we have vision and other senses for measurement, the central nervous system for communication and the 'brain' for computation, with the muscles as powered elements. We have to remember, of course, that the human body has taken many millions of years to evolve, while the programming of the brain—inadequately regarded as a computer in this simple analogy—is a continuous process from birth onwards. You will note that the programme we have written to illustrate computer control of an unstable system is hardly so perfect!

The example of an unstable device I have chosen is the apparently simple one of an inverted pendulum (Fig 16). An ordinary pendulum has its centre of gravity below the pivot and hangs in stable equilibrium. If

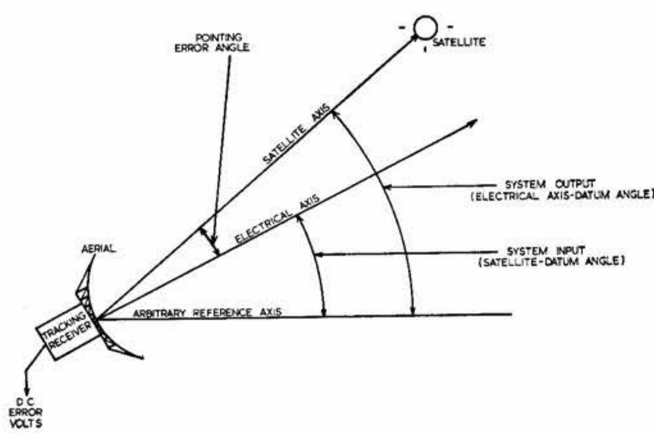


Fig 15 Extraction of satellite aerial pointing error

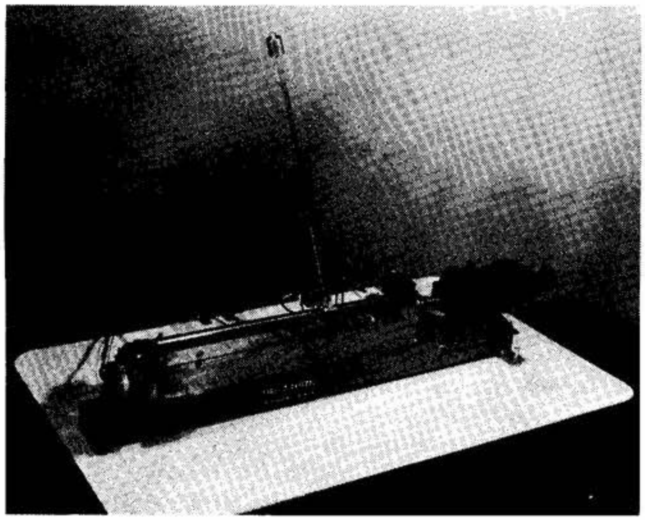


Fig 16 Inverted pendulum—to be stabilized by a high-speed digital computer—showing displacement potentiometer and velodyne drive to lead-screw shaft

displaced it returns to the vertical position after performing a damped oscillation. The inverted pendulum, although constrained to motion in one plane only, is clearly unstable and can be maintained in an upright position only by appropriate movement of the pivot at its foot. How is the correcting force at the pivot determined and applied? In otherwords, how is the control of such an unstable pendulum achieved?

The aluminium pendulum is supported by a horizontal axle fitted to a carriage which can be moved along a lathe-like bed by means of a very smooth lead screw (Fig 17). Attached to the axle is a potentiometer which measures the tilt of the pendulum. The lead screw shaft also operates a potentiometer to measure the displacement of the carriage and is rotated by a velodyne motor which provides a fast response.

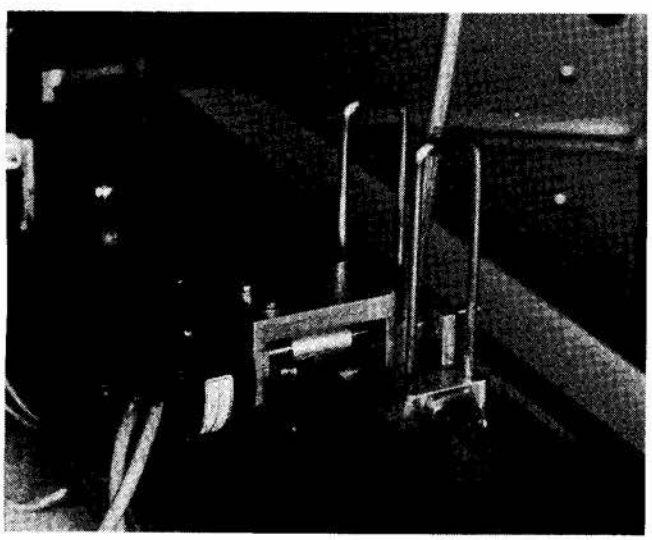


Fig 17 Foot of the inverted pendulum showing pivot and tilt potentiometer, movable carriage, slide-bed and lead screw

The angular velocity of the pendulum and the linear velocity of the carriage are obtained by differentiation of the measured displacements, using an electrical network located in the analogue computer (Fig 18). These measurements, after passage through an analogue-to-digital converter, are communicated to a Marconi Myriad digital computer at intervals of about 1/20 sec.

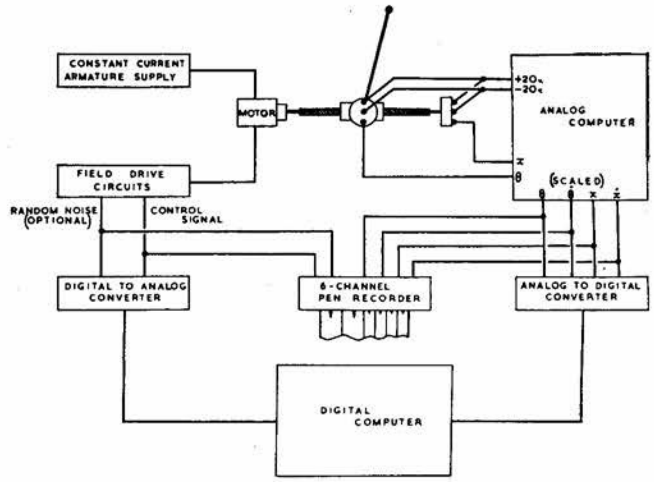


Fig 18 Block diagram of complete system used to control the inverted pendulum

Similarly, the output instructions from the computer are converted back from digits to analogue signals before operating the motor drive to the lead screw.

We wish to maintain the pendulum vertical with the carriage situated at a given position along the lead screw. The measurements we have taken tell us the amount by which the pendulum is tilted away from the vertical and the displacement of the carriage from the desired position. They also tell us how fast the pendulum and carriage are moving towards or from the required configuration. From these four state variables the computer can calculate the force required on the block to restore the pendulum to the upright position at the desired point. The dynamical equations of the system are:

$$\ddot{\theta} = A_{12}\,\theta + D_2 P$$
$$\ddot{X} = A_{14}\,\theta + D_4\,P$$

where P is the existing force on the carriage, X and θ are the position variables as shown in Fig 19. The coefficients are defined as

$$A_{12} = \frac{(M+m)\,\dfrac{g}{L}}{(M+m)\,\dfrac{I}{mL^2} + M}$$

$$A_{14} = \frac{-gm}{(M+m)\,\dfrac{I}{mL^2} + M}$$

$$D_2 = \frac{-\frac{g}{2}}{(M+m)\frac{I}{mL^2} + M}$$

$$D_4 = \frac{g(1 + \frac{I}{mL^2})}{(M+m)\frac{I}{mL^2} + M}$$

m = mass of pendulum
M = mass of carriage
I = moment of inertia of pendulum about its centre of gravity
L = distance from CG of pendulum to the pivot
g = acceleration due to gravity

The characteristic equation $\ddot{\theta} - A_{12}\theta = 0$ has positive real roots which indicate that the system is inherently unstable and that the inclination θ tends to increase indefinitely, unless checked by an applied force. A solution for the correcting force P may be obtained by considering the equilibrium of the system over a series of small consecutive intervals. The condition of the system at the beginning of each interval is defined by the measured values of positions and velocities.

We may postulate a final value for only one of these four quantities and derive a value of P which will give us this result. Thus we may elect to make the pendulum upright at the end of a chosen interval and calculate the restoring force P that will ensure this. The other quantities, i.e. the lateral position and lateral and angular velocities, will then be determined and their value can be computed. In general these will not be zero. We therefore have to continue the computation into the next interval, always finding that we can make right one quantity only. So there is no certainty that we shall ever be able to reach our final goal, nor do we know that we have approached it in the best possible way.

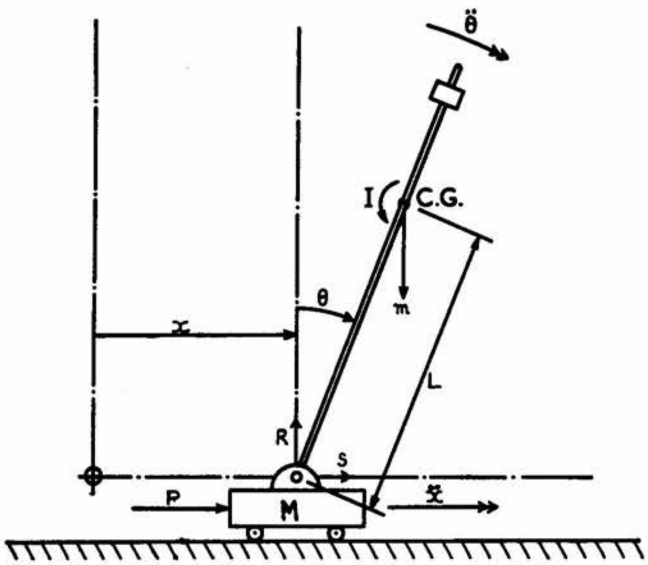


Fig 19 Schematic of the inverted pendulum showing the definitions of the state variables x, $\dot{x}$, θ, and $\dot{\theta}$ also the mechanical parameters of the system

For example, if the state of the pendulum happens to be as indicated in Fig 19 at the beginning of the first interval considered, and we decide to apply P towards the left in order to bring the pendulum to the centre, then it will surely fall. On the other hand, if the pendulum happens to be rotating towards the vertical in the position indicated, then we may conceivably have a chance of righting it.

It is clear that we must programme the computer to execute a definite control strategy, i.e. to fulfil certain selected criteria in a prescribed sequence. Since our aim is to stabilize the pendulum in the vertical position in the centre of the slide, we may select as our criterion that the sum of θ, the angular displacement from the vertical, and x, the lateral displacement, shall be a minimum. Since, however, we recognize that we cannot apply an infinite restoring force P, we may decide more realistically to include this in the minimization process. Thus we could decide to minimize what might appropriately be termed a performance index Jo, made up as follows:—

$Jo = a\,\theta + bx + c\,P$ (where a, b and c are weighting factors to be selected)

This criterion would not be satisfactory, however, since Jo could be minimized by having θ large and positive, with x also large but negative, i.e. the pendulum would be lying on its side, near the end of the slide.

We therefore choose as our performance index a quantity J defined more nearly in accordance with the usual Gauss definition of the most probable value of a variable, i.e. the value which makes the sum of the squares of the errors a minimum:—

$J = a\theta^2 + bx^2 + cP^2$

Our performance index J introduces constants a, b and c which are weighting factors or measures of the relative importance we attach to the various parameters. If, for example, a is large, then a small change in θ would give a large change in the performance index.

Dynamic programming

Unfortunately, this is not quite the end of the story. If we refer again to Fig 19 we find that under certain conditions (i.e. when the pendulum is nearly vertical, but slightly tilted to the right) we need to apply an inward force to minimize the performance index. This will increase the tilt until its weighted effect is greater than the weighted effect of the lateral displacement, when the force reverses to drag the pendulum outward, i.e. the control system is not stable.

We are able to invoke a mathematical technique proposed by Bellman to help us, however. He called it Dynamic Programming. This involves the analysis of the system, starting from the final time interval of control, at the end of which the pendulum is in equilibrium, and calculating P to minimize the performance index in that stage. We then pass to the previous stage

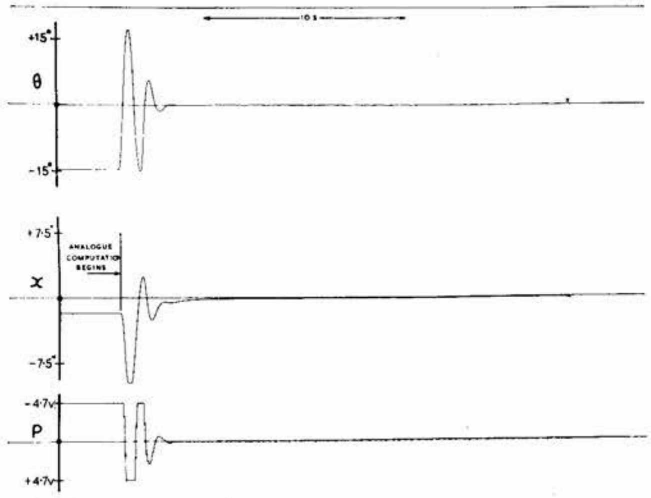


Fig 20 Variations of θ, x and P when the computer controls the analogue simulation of the pendulum

and minimize the performance index over the last two stages. Thus we may proceed back to the actual situation and derive a restoring force P, which will minimize the performance index over the complete operation and not just in the individual intervals. Given then the parameters of the system, the definitions of a performance index and an optimal control policy based on dynamic programming, it is possible to derive a control matrix which, when multiplied by the four state variables, gives the restoring force P.

Effectiveness of computer control

The result of applying the above strategy to the control in real time by Myriad of the analogue simulation of the inverted pendulum is indicated in Fig 20. In the upper trace is shown the variation in the inclination θ of the pendulum, in the middle trace the variation x in lateral position, and in the lower trace the magnitude of P, the restoring force. It is quite clear that the strategy selected is capable of controlling this electrical system effectively.

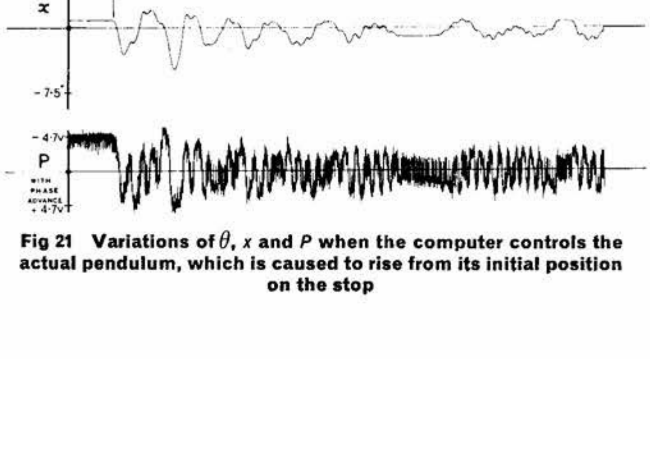


Fig 21 Variations of θ, x and P when the computer controls the actual pendulum, which is caused to rise from its initial position on the stop

The curves resulting from the control of the actual pendulum by the computer are shown in Fig 21. It will be noted immediately that, although the control is quite adequate, it is not as effective as when the pendulum was simulated by the analogue circuits.

The analogue simulator gives a good representation of the dynamic equations we have set up, but these equations are only a mathematical model which, we hope, describes to a close approximation the mechanical properties of the pendulum system. Thus the difference between the behaviour of the actual pendulum and that of its analogue simulation measures the inadequacy of the equations to describe the real system. Complete correspondence is too much to hope for in the unstable systems with which we have to deal, as there are usually too many unknown factors present, but approximate solutions are none the less valuable.

Of interest also is the response of this computerized control system to an external disturbance. Fig 22 shows

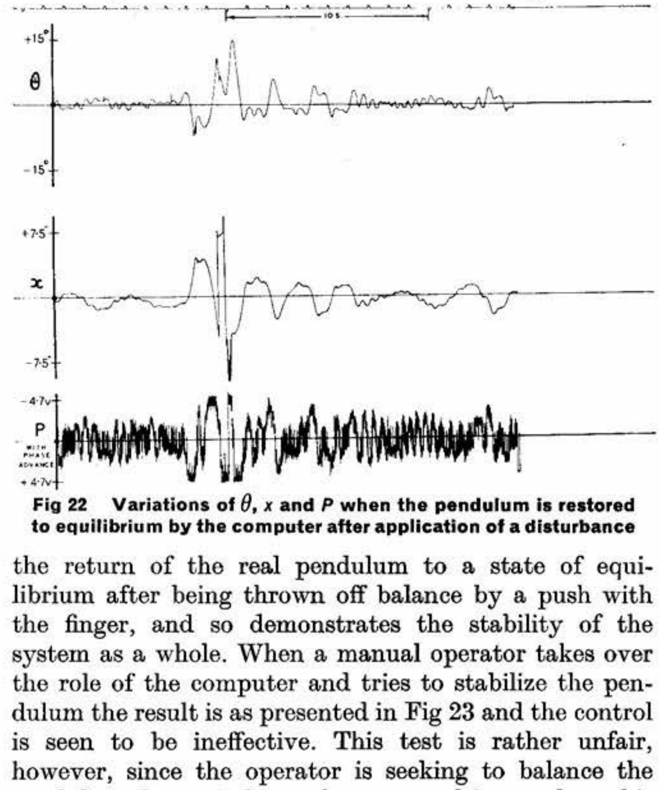


Fig 22 Variations of θ, x and P when the pendulum is restored to equilibrium by the computer after application of a disturbance

the return of the real pendulum to a state of equilibrium after being thrown off balance by a push with the finger, and so demonstrates the stability of the system as a whole. When a manual operator takes over the role of the computer and tries to stabilize the pendulum the result is as presented in Fig 23 and the control is seen to be ineffective. This test is rather unfair, however, since the operator is seeking to balance the pendulum by switching the motor drive and so his control suffers from the restrictions of motor delay and single valued thrust. It is a sobering thought that he would probably have no difficulty in balancing the pendulum on his finger, and might even do so blindfold, so testifying to the goodness of a human's sensing and computing ability. On the other hand, the computer control system I have described could cope with about ten pendula simultaneously, and would continue to do so without the deterioration of performance which a human operator always shows, owing to fatigue.

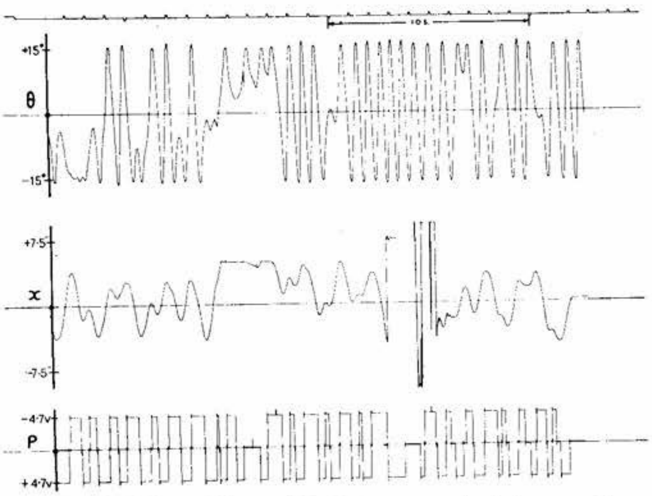


Fig 23 Variations of θ, x and P when an operator takes over the role of the digital computer and tries to stabilize the pendulum

A practical example of the stabilization of an unstable, pendulum-like system is the control of a Blue Streak rocket at the moment of launch. It will be remembered that the Blue Streak is the first stage of the satellite launcher system which is under development in the European launcher organization. Blue Streak has been developed by Hawker Siddeley Dynamics and is a twin-jet rocket which is 72 ft long and 10 ft in diameter. The film shows how precisely this huge vehicle is maintained vertical as it gradually builds up speed, control being effected by adjusting the directions of thrust of the twin rocket motors.

Control in automation and society

As with so much of modern technology, the science of control has developed very rapidly during the last decade. There are many reasons for this; not least the recognition that highly complex control systems will be needed if our dreams of comprehensive automation of production are to be turned into reality. A further influence has been the surge forward in electronic techniques during and since World War II. Oscillography and other methods of measurements have advanced rapidly, sensors of all kinds have been greatly developed and their sensitivities improved; but most important of all for the future of control in automation and society has been the breathtaking progress in digital computers.

We have seen that the control of pseudo-stable dynamical systems, such as satellite aerials, depends upon the basic processes of measurements, communication of feedback of data from the output to the input in order to extract an error signal with which to adjust the output to the demand. The objective has been to obtain accuracy and speed of control but to avoid instability through the build-up of oscillation via the feedback path. In fact, optimum response in many cases lies quite close to loss of control. It is the arrival of the fast, on-line digital computer which has made it possible to extend the science of control to devices that are naturally unstable, and it was to illustrate this point that we have studied in detail the behaviour of the inverted pendulum and how an adequate strategy of control could be built up. The essential feature of this process was to exploit the enormous speed of the computer to calculate the corrections to be applied to the system on a cyclic basis from the state variables continuously monitored and fed into the computer.

The three filmed experiments we have seen this evening have all underlined the importance of the analogue computer for simulating the behaviour of a complex control system. The further development of the theory of control will depend heavily on the experimental investigation of digital computer control in real time of complex systems, the mathematical models for which will be set up on analogue simulators. It was in this way that the inverted pendulum was first studied in the Nelson Research Laboratory, using the Marconi Myriad for real time control of the system simulated on a Solartron Analogue machine. Other problems under investigation in this laboratory include comprehensive electrical power generation and transmission systems, multiple stand rolling mills for steel billet production, combined boiler and turbine control systems, and steam-raising control with nuclear piles.

We have seen from the film how the principles of control science apply to human beings when performing the ordinary movements of daily life as well as when engaged in gymnastics. In fact, it is astonishing to find how widespread is the use of feedback-control in association with either stable or unstable systems. Thus the question arises of whether the principles of control which have emerged from careful analysis of the behaviour of mechanical systems can also be applied to the less ponderable aspects of human affairs. We would like to analyse as a control problem the total behaviour of an industrial company, including all the various interacting activities such as purchasing, stock holding, manufacturing, selling, shipping, etc. Even more ambitious would it be to subject the economy of a country as a whole to this kind of computer-assisted, analytical study. Cambridge University is making such an attempt.

In such studies it is necessary first to identify as many as possible of the contributory variables and to express their interrelationships in terms of mathematical models which can be simulated on the computer. Our experimental demonstration of the inverted pendulum showed that even a four-variable problem was not easy to solve. An adequate simulation of the economy would involve two hundred variables at least and its solution would obviously take much longer than our little problem—assuming that the required computers are ever built.

Some of you may well question whether it will ever be possible to set up such a multi-variable simulation that may even approximately reproduce the intricacies of a country's economic system. I agree that the problem is difficult, but the effort is surely worthwhile for the deeper understanding of the workings of the economy that such a progressive study will surely bring. After