# Autonomous Excavation Trajectory Generation for Trenching Tasks Based on Skills of Skillful Operator

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Abstract—In this paper, a new autonomous excavation trajectory generation system is proposed. Instead of controlling the excavator to follow the teaching path precisely, our system replans the skillful operator's arbitrary bumpy digging and slow trajectory into a topological equivalent trajectory to ensure a fast and smooth excavation process. In this paper, the waypoints which are equivalent to the teaching path topology are found by several manual excavation tracks. The trajectory is then parameterized to obtain the minimum time-jerk trajectory under kinodynamic constraints. The trajectory generation method proposed in this paper is integrated into a complete autonomous excavation platform and the feasibility of this system is verified in a field test.

Keywords—autonomous excavation, trajectory generation, manual excavation modeling, teach and plan

# I. INTRODUCTION

Excavator is one of the most widely used construction machinery, excavator operation relies heavily on skillful operators, and there are risks when excavating in harsh environments. To reduce the dependence on skillful operators and improve the stability of excavation operations, analyzing the skills of skillful operators, and automatically generating excavation trajectory is an important process for developing an autonomous excavation system [1].

The trajectory generation of autonomous excavation is usually to study the motion trajectory of the bucket teeth in three stages: penetrating the soil, bucket dragging, and bucket lifting [2]. Trajectory generation based on time-optimal has been the focus of research. Guan et al. [3] interpolates critical excavation waypoints with 5-order NURBS curves and solve the time-optimal excavation trajectory problem iteratively through the sequential quadratic programming method. Sun et al. [4] adopted a differential evolution algorithm to optimize the trajectory parameterized by 4-3-3-3-4 segment polynomials to obtain the optimal excavation time. Li et al. [5] used particle swarm optimization to optimize the trajectory time to shorten the time used for a single excavation. Zhang et al. [6] established the time-jerk optimal trajectory generation model with the given waypoints. Kim et al. [7] proposed a method that uses the line search method to find the optimal excavation time. Yang et al. [8, 9] interpolated the waypoints using a mixed-order segmented polynomial, and an optimal trajectory generation model was established considering the swept volume, bucket pose, and motion constraints. On the other hand, minimizing jerk can decrease the error of trajectory tracking and reduce the damage caused to the actuator. Studies have shown that the smoothness of joint motion is associated with a jerk (i.e., the derivative of acceleration) [10]. Gasparetto et al. [11] assign different weight coefficients to time and jerk to ensure that the optimal trajectory is generated while the feasibility constraint of joint kinematics is obtained, and the jerk-time optimal trajectory is obtained. Besides, many researchers have used other optimization methods for trajectory generation. Weng et al. [12] interpolated the waypoints using five non-uniform rational B-splines to generate a smooth excavation trajectory. Zhong et al. [13] parameterize excavation trajectories by a cubic polynomial for obstacle avoidance.

The environment of excavation operations is usually complex and varied, which requires autonomous excavation systems to have a higher level of decision-making and planning abilities when faced with changes in the working environment or operating conditions. For this ability to adapt to changes in the operation environment, the decision-making and coping skills of skilled operators on specific tasks are conducive to path planning [14]. On the other hand, in a specific working environment, it is generally desirable to be able to generate efficient, fast, and stable motion. Therefore, it is necessary to establish a model that can make full use of the topological structure of the path of skilled operators to generate a smooth and fast excavation path.



Fig. 1. Overview of the autonomous excavation system.

Based on the topological structure of skilled operators' excavation trajectory (i.e., skills of skillful operators), this paper proposes a new autonomous trajectory generation method for trench excavation. Then, this method was integrated into the autonomous excavation system, as shown in Fig. 1, and field tests were conducted. Finally, the comparative analysis of the actual trajectory and the planned trajectory proves the feasibility of this method.

# II. EXCAVATION PATH MODEL

# A. Excavation Kinematic Model

The complete trench excavation operation can be roughly divided into two parts: trench excavation (i.e., bucket penetrating, bucket dragging, and bucket lifting), and swing dumping. Fig. 2 shows the trench excavation process.

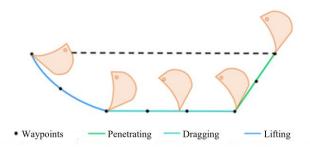


Fig. 2. Schematic diagram of trench excavation path.

The generation of the optimal excavation trajectory is generally carried out in the joint space, and the joint angles of the swing, boom, stick, and bucket are generally used  $(\theta_1, \theta_2, \theta_3, \theta_4)$  to represent the joint space. However, the excavation trajectory is usually planned in the pose space according to the excavation task, and according to the position of the bucket teeth  $O_4(x, y, z, \xi)$ , so it is necessary to establish a kinematic model of coordinate transformation from joint space to pose space. The relevant D-H coordinate system method expresses the transformation relationship between joint space and pose space. The D-H coordinate system of the excavator is shown in Fig. 3.

According to the D-H coordinate system, the excavator bucket teeth pose  $O_4(x, y, z, \xi)$  can be calculated by Eq. (1) as follows:

$$\begin{cases} x = a_4 \cos(\theta_2 + \theta_3 + \theta_4) \cos \theta_1 \\ + a_3 \cos(\theta_2 + \theta_3) \cos \theta_1 \\ + a_2 \cos \theta_2 \cos \theta_1 \\ + a_1 \cos \theta_1 \end{cases}$$

$$y = a_4 \cos(\theta_2 + \theta_3 + \theta_4) \sin \theta_1 \\ + a_3 \cos(\theta_2 + \theta_3) \sin \theta_1$$

$$+ a_2 \cos \theta_2 \sin \theta_1$$

$$+ a_1 \sin \theta_1$$

$$z = a_4 \sin(\theta_2 + \theta_3 + \theta_4)$$

$$+ a_3 \sin(\theta_2 + \theta_3)$$

$$+ a_2 \sin \theta_2$$

$$+ d_1$$

$$\xi = \theta_2 + \theta_3 + \theta_4$$

$$(1)$$

where  $a_1, a_2, a_3, a_4$  represent the link lengths of swing, boom, stick and bucket joint, respectively, and  $d_1$  represents the link offset between the  $X_0$  and  $X_1$  axes.

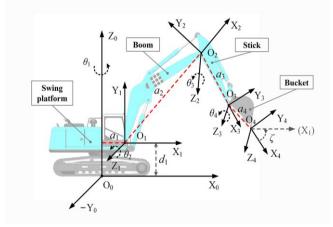


Fig. 3. D-H coordinate system of the excavator.

## B. Manual Excavation Modeling

Before obtaining the trajectory that can be tracked by the controller, the critical waypoints of the manual excavation path need to be extracted, which usually consists of the following steps:

- (1) Extract the topological structure of the trajectory in joint space. First, the data is processed by timing signal alignment so that the data length after the trajectory of the skilled operator is consistent. Then, the moving average filter can reduce the interference generated by noise signals (such as the jerk of hydraulic cylinders) on the trajectory of the excavator. Finally, the excavation path is obtained by computing averages of multiple groups' excavation trajectories of a skillful operator.
- (2) Kinematic transformation. Since the excavation task and topological structure of the skillful operator's teaching trajectory requires the planned path in the pose space, the joint

space trajectory obtained in step (1) is transformed into the pose space through Eq. (1).

(3) Search for the waypoints. Given that the geometry and temporal distribution of the excavation motion obtained by skilled operators are far from the optimal trajectory, thus, they are useless for trajectory optimization and cannot be used for trajectory tracking. To retain the topological information of skilled manual paths and improve the efficiency and quality of generating traces, this paper uses the Douglas-Peucker algorithm to automatically generate sparse excavation paths [15], which provides a lot of freedom for the following trajectory optimization process.

After getting the critical waypoints of the topological equivalent path of the skillful operator's teaching trajectory through the above method, the following trajectory optimization can be completed to generate the trajectory that can fully exploit the performance (i.e., fast and smooth trajectory) of the autonomous excavation system.

# III. EXCAVATION TRAJECTORY OPTIMIZATION

#### A. Formulation

Through the proposed methods in Section II, the topological equivalent excavation waypoints can be obtained. However, the optimal trajectory cannot be generated if path smoothing and time optimization cannot be performed. Therefore, this paper uses a smooth polynomial curve to connect these waypoints sequentially, so that the trajectory optimization problem can be established under the constraints of kinodynamic feasibility.

Trajectories consisting of segmented polynomials are parameterized by the time variable t in each joint space. The nsegment k-order polynomial trajectory of the jth joint can generally be expressed in the following form:

$$Q_{j}(t) = \begin{cases} c_{1}^{T} \beta(t), & t \in [t_{0}, t_{1}] \\ c_{2}^{T} \beta(t), & t \in [t_{1}, t_{2}] \\ \vdots & \vdots \\ c_{n}^{T} \beta(t), & t \in [t_{n-1}, t_{n}] \end{cases}$$
(2)

where  $c_i^T = (c_{i,0}, c_{i,1}, \dots, c_{i,k})$  is the coefficient vector of the *i*th segment excavation trajectory, and  $\beta(t) = (1, t, \dots, t^k)^T$  is the time vector.

Generally, excavation track requirements must meet the following requirements:

- (1) Waypoints constraint: It is necessary to ensure that the value is interpolated at the endpoint of each section to ensure that the trajectory passes through the critical waypoints.
- (2) Boundary constraint: Considering the start and end operation states of the excavator, it is necessary to set the velocity and acceleration of the trajectory at the beginning and end points to meet the requirements of the excavation operation states.

(3) Continuity constraint: To obtain a smooth trajectory, it is necessary to ensure that the n-1 derivative of the two connected segments is continuous at the breakpoint.

This research expects that the excavation operation time will always be as small as possible, but this will cause the end state of the excavation to be kinodynamic infeasible (i.e. exceed the velocity limit or acceleration limit), such as the acceleration of the endpoint is not zero, resulting in a jerk on the mechanism. Therefore, this article chooses the objective function as a trade-off between minimum jerk and time, and the trajectory generation problem described above is expressed as follows:

min 
$$J = w_1 \int_0^{t_n} (\ddot{Q}_j(t))^2 dt + w_2 t_n$$
  
s.t.  $Q_j(t_0) = q_{j,0}, \ Q_j^{(1)}(t_0) = v_{j,0}, \ Q_j^{(2)}(t_0) = a_{j,0}$   
 $Q_j(t_n) = q_{j,n}, \ Q_j^{(1)}(t_n) = v_{j,n}, \ Q_j^{(2)}(t_n) = a_{j,n}$   
 $|\dot{Q}_j(t)| \le V_{j\text{max}}, \ |\ddot{Q}_j(t)| \le A_{j\text{max}}, \ \forall t \in [t_0, t_n]$   
 $Q_{i,j}^{(s)}(t_i) = Q_{i,j+1}^{(s)}(t_i), \ s \in \mathbb{N}, \ i = 1, \dots, n-1$ 

where  $w_1$  is the weight coefficient of the jerk,  $w_2$  is the weight coefficient of the time,  $q_{j,0}$ ,  $v_{j,0}$  and  $a_{j,0}$  are the position, velocity, and acceleration at the beginning, respectively,  $q_{i,n}$ ,  $v_{i,n}$  and  $a_{i,n}$  are the position, velocity, and acceleration at the end, respectively,  $V_{j\,\mathrm{max}}$  and  $A_{j\,\mathrm{max}}$  are the kinodynamic limits of velocity and acceleration, respectively.

## B. Algorithms

The problem in Eq. (3) can be solved by a nonlinear optimization algorithm, and the solution of the optimization problem can be completed by using the solver (reference the [11, 16] for details), and the corresponding optimal time interval and trajectory sequence can be obtained to achieve autonomous excavation trajectory tracking. The procedures for solving the trajectory optimization problem are summarized below:

- (1) Start from the given waypoints, the kinodynamic limits of excavation motion.
- (2) Choose the appropriate solver and the corresponding optimization algorithm. In this research, trajectory optimization problems by iteratively and locally approximating the original problem with a sequence of quadratic optimization subproblems [17].
- (3) Put the objective and constraints into the Mosek solver [18]. Finally, the solution to the optimization problem and trajectory sequence is obtained.

# IV. EXPERIMENTAL RESULTS

In this section, the generated trajectory is compared with the trajectory generated by an experienced operator to verify the effectiveness of method proposed in this paper. Firstly, the weight  $w_1$  and the weight  $w_2$  were 0 and 1, respectively, which means that the excavator expected to excavate as fast as possible. The order k is set as 3, and 15 groups of repeated teaching were performed to obtain the excavation waypoints by

the method in Section II. Then, the optimal trajectory is generated by the method in Section III. Finally, field tests are carried out to test feasibility of the proposed method. The D-H parameters used to find waypoints are shown in Table I, and the kinodynamic limit is shown in Table II.

TABLE I.	D-	H PARAMETERS OF	EXCAVATOR

i	$d_i(mm)$	$a_i(mm)$	<b>a</b> <sub>i</sub> (°)	$ heta_i$ (°)
1	1125	860	90	[-180, 180]
2	0	2845	0	[-60, 65]
3	0	1945	0	[-15, -25]
4	0	835	0	[-135, 40]

TABLE II. KINODYNAMIC LIMITS OF THE EXCAVATOR

Constraints	Swing	Boom	Stick	Bucket
Velocity (°/s)	70	845	50	90
Acceleration (°/s²)	160	140	135	135

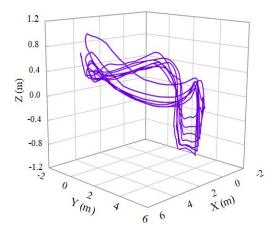


Fig. 4. Excavation trajectory of the bucket teeth in pose space.

The excavation trajectory of the bucket teeth generated by the optimization method is shown in Fig. 4. A complete trenching process consists of seven cycles, where the purple line reflects the five excavation phases of the teeth motion such as bucket penetrating, bucket dragging, bucket lifting, swing to the dumping point and swing to the penetration point.

In this test, the time taken for autonomous excavation was 80.8 s. By comparing the planned trajectory of the proposed method with the actual measurement trajectory, as shown in Fig. 5, it can be seen that compared with the planned trajectory, the swing angle of the actual measurement has been overshot many times (such as multiple rotations less than 0°), resulting in a weak swing tracking effect. The measured trajectory change trend of the stick is quite different from the ideal planned trajectory. However, the changing amplitude of the boom and bucket cylinder is consistent with changing trend of the planned trajectory.

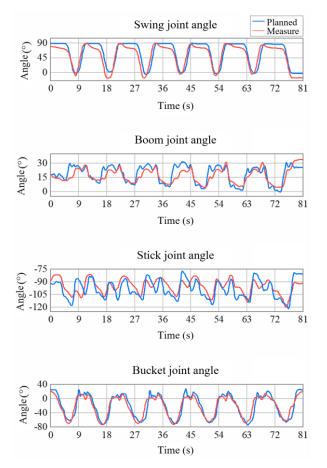


Fig. 5. Comparison of autonomous excavation planned trajectory and measure trajectory.

# V. CONCLUSION

Aiming at the trajectory generation problem of autonomous excavation, this paper establishes a system for trajectory generation by searching for the topological characteristics of the excavation trajectory of skillful operators, and the main conclusions of this paper are as follows: (1) Extract the topological equivalent path of skillful manual excavation, and generate a time-jerk optimal excavation trajectory when the physical performance limit of the excavator is satisfied. (2) Integrate the sensing module, trajectory generation module, and control module into the autonomous excavator platform, and the feasibility of autonomous trajectory generation method was verified in field tests.

The trajectory generation system established in this work provides a reference for the autonomous operation of construction machinery, although the system shows some feasibility, it is not enough to apply to the real environment. Also, in the future, the changes in kinematic and kinodynamic parameters of autonomous trajectory planning and skillful operators in the excavation process under different soil conditions are compared and analyzed, and the trajectory generation method is further optimized, so that the system in this paper can be applied to the actual excavation operation and the automation level of construction machinery can be improved.

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