



Research article

By-production modeling of technical and environmental inefficiency in Brazilian dairy farms

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ARTICLE INFO

JEL classification:

C11
D24
Q12
Q56

Keywords:

Stochastic frontier analysis
By-production
Copulas
Bayesian estimation
Technical & environmental inefficiency
Brazilian dairy farms

ABSTRACT

This study develops a by-production stochastic frontier model to assess farm technical and environmental inefficiency, applied to a panel of Brazilian dairy farms observed from 2014 to 2021. The model specifies a good output (mainly milk production) and a bad output (CO_2 -equivalent emissions) equation, capturing the dependence between the inefficiency terms and between the noise terms across the two equations using two distinct copulas. A Bayesian estimation framework is introduced for parameter estimation. Results indicate that milk production is primarily driven by feed inputs, while emissions are mostly influenced by herd size. Technical inefficiency averages 0.066, while environmental inefficiency averages 0.061, reflecting relatively high efficiency levels in both dimensions. Strong upper tail dependence between the inefficiency terms and between the noise terms across the two equations highlights the interconnected nature of these processes, suggesting shared inefficiency drivers and external shocks.

1. Introduction

The growing demand for food, driven by rapid population growth, combined with societal expectations to mitigate negative environmental externalities, highlights the urgent need for sustainable agricultural practices. Farms are increasingly challenged to adopt production methods that balance the dual objectives of meeting rising food demands while minimizing environmental impacts. Assessing farms' inability to fully maximize production (referred to as technical inefficiency) and simultaneously minimize pollution (referred to as environmental inefficiency) is critical for identifying their shortcomings in achieving these objectives and aligning with broader sustainability targets. These goals are explicitly outlined in the United Nations Sustainable Development Goals (SDGs), introduced in 2015 (United Nations, 2020). This discussion is particularly relevant for dairy farms, which must address the increasing demand for milk while minimizing their environmental footprint, including reducing greenhouse gas emissions such as methane. For instance, the Brazilian dairy sector, which serves as the empirical application of this paper, accounted for approximately

10% of the agri-food sector's greenhouse gas emissions in 2019, with methane contributing nearly three-quarters of these emissions (Vogel et al., 2023).

In this study, we employ a state-of-the-art by-production stochastic frontier model that distinguishes between two sub-technologies, enabling the simultaneous measurement of farm technical inefficiency and environmental inefficiency. This approach provides a comprehensive understanding of the trade-offs and challenges farms face in achieving sustainable production. Within this modeling framework, we (i) apply the model to Brazilian dairy farms - a sector characterized by significant methane emissions, (ii) separately account for the dependence between the inefficiency terms and between the noise terms across the two sub-technologies, enabling a detailed representation of the underlying stochastic processes, and (iii) implement a Bayesian estimation approach, which effectively incorporates uncertainty in parameter estimation.

In contrast to conventional stochastic frontier models (Aigner et al., 1977; Meeusen and Van den Broeck, 1977) that evaluate technical

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<https://doi.org/10.1016/j.jenvman.2025.126606>

Received 21 February 2025; Received in revised form 3 June 2025; Accepted 15 July 2025

Available online 28 July 2025

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and environmental inefficiency based on a single, unified production technology (see Lin et al., 2013; Skevas et al., 2018; Adenuga et al., 2019), Førsund (2009) and Murty et al. (2012) propose a more comprehensive approach that specified two sub-technologies. The first sub-technology focuses on the production of good outputs, such as milk or other agricultural products, while the second sub-technology models the production of bad outputs, such as greenhouse gas emissions, which arise as by-products of the production process. In this by-production technology framework, it is intuitive to assume that the two sub-technologies are inherently interconnected. External shocks, such as weather conditions, economic fluctuations, or policy changes, often affect both the production of good outputs and the production of bad outputs. Similarly, farm-level management practices, including input allocation, farm size, and waste management systems, can simultaneously influence the inefficiency of both sub-technologies. By accounting for these linkages, the by-production framework offers a comprehensive understanding of the joint production process, capturing both synergies and trade-offs between the production of good and bad outputs.

Certainly, modeling the inherent dependence between these sub-technologies poses some econometric challenges, which may explain why two-equation stochastic frontier models have seen limited application to date. The few studies that have implemented such models can broadly be categorized into two groups based on their specification approach. The first category includes studies such as Lai and Kumbhakar (2021) and Wang et al. (2024), which specified a good output sub-technology linking the good output to the utilized inputs, alongside a bad output sub-technology that links the bad output to both the good output and the utilized inputs. The second category follows the approach proposed by Førsund (2009) and Murty et al. (2012) more closely. Studies such as those by Repkine (2023) and Skevas (2025) define a good output sub-technology relating the good output to the utilized inputs, while specifying the bad output sub-technology in terms of the polluting inputs. These studies account for the dependence between the two equations by linking the corresponding inefficiency and composite error terms across the equations, respectively, providing a more integrated representation of the joint production process.

In this study, we adopt the latter approach, as it aligns more closely with the by-production framework of Murty et al. (2012), which explicitly separates the technological relationships governing good and bad outputs. Our objective is to measure farm technical and environmental inefficiency using this framework, allowing for a clearer distinction between technical and environmental inefficiency. We contribute to the literature in three ways. First, we apply the by-production model to Brazilian dairy farms, a context where this model has not been used before. Second, we advance previous research by separately accounting for dependence in both the inefficiency terms and in the noise terms, offering deeper insights into how stochastic processes interact across the two sub-technologies - unlike prior studies that focused solely on inefficiency dependence (e.g., Repkine, 2023) or composite error term dependence (e.g., Skevas, 2025). Third, we introduce a Bayesian framework for estimating the by-production stochastic frontier model.

The structure of the paper is as follows: Section 2 introduces the modeling framework and details the estimation process. Section 3 describes the application of the model, including the data, specified variables, and empirical specifications. Section 4 presents the results, while Section 5 offers concluding remarks.

2. Modeling framework & estimation

2.1. Construction of the by-production frontier

Following Murty et al. (2012), the by-production technology T is defined as the intersection of a good output sub-technology, denoted by T^g , and a bad output sub-technology, denoted by T^b :

$$T = T^g \cap T^b \quad (1)$$

This structure reflects the fact that the production of good and bad outputs involves different input-output relationships and performance objectives. The good output sub-technology T^g describes the production of a good output g_{it} by firm i at time t , using both non-polluting inputs \mathbf{x}_{it}^{np} and polluting inputs \mathbf{x}_{it}^p . In contrast, the bad output sub-technology T^b describes the production of a bad output b_{it} , which arises directly from the use of polluting inputs \mathbf{x}_{it}^p as suggested by Førsund (2021). This input separation reflects the asymmetric roles of different inputs: while both polluting and non-polluting inputs contribute to productive output, only polluting inputs are associated with environmental externalities. Accordingly, the good output sub-technology is modeled using a production frontier, as the objective is to maximize good output. The bad output sub-technology, in contrast, is modeled using a cost frontier, as the goal is to minimize bad output. To ensure that each sub-technology satisfies economically and environmentally sound principles, we assume that the following axiomatic properties hold. For the good output sub-technology T^g , these include: no free lunch and inactivity, input essentiality and attainability, non-emptiness and closeness, boundedness, positive monotonicity in inputs, free disposability of good output, reverse nestedness, and convexity in inputs. For the bad output sub-technology T^b , we assume: boundedness, polluting input essentiality, convexity in inputs and output, positive monotonicity in the bad output, and negative monotonicity in polluting inputs. In fact, the above indicate the weak disposability of the bad output, meaning that its reduction is not costless. These properties are consistent with the axiomatic framework of by-production technology, and one can consult (Skevas et al., 2023) for formal definitions and discussion.

Based on the above framework, the functional forms of the two frontiers are specified as follows:

$$\begin{aligned} g_{it} &= \alpha_i^g + \mathbf{x}_{it}^{np} \boldsymbol{\beta} + \mathbf{x}_{it}^p \boldsymbol{\gamma} - u_{it}^g + v_{it}^g \\ b_{it} &= \alpha_i^b + \mathbf{x}_{it}^p \boldsymbol{\delta} + u_{it}^b + v_{it}^b \end{aligned} \quad (2)$$

In the specified production frontier for the good output, α_i^g captures unobserved heterogeneity, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of parameters associated with non-polluting and polluting inputs, respectively, while u_{it}^g represents good output inefficiency, and v_{it}^g accounts for random noise. Similarly, in the bad output production frontier, α_i^b captures unobserved heterogeneity, $\boldsymbol{\delta}$ is the vector of parameters associated with polluting inputs, and u_{it}^b represents bad output inefficiency, with v_{it}^b capturing the corresponding noise. The unobserved heterogeneity terms for each farm (α_i^g and α_i^b) are modeled as Normal distributions. Specifically, α_i^g and α_i^b follow $\mathcal{N}(0, \frac{1}{\omega_1})$ and $\mathcal{N}(0, \frac{1}{\omega_2})$, respectively, where ω_1 and ω_2 are precision parameters. The inefficiency terms for the good output (u_{it}^g) and bad output (u_{it}^b) are modeled as Half-Normal distributions, that is, $u_{it}^g \sim \mathcal{HN}(0, \frac{1}{\phi_1})$ and $u_{it}^b \sim \mathcal{HN}(0, \frac{1}{\phi_2})$, reflecting their non-negative nature. Here, ϕ_1 and ϕ_2 represent the precision parameters. Finally, the noise terms for the good output (v_{it}^g) and bad output (v_{it}^b) are assumed to follow Normal distributions, with $v_{it}^g \sim \mathcal{N}(0, \frac{1}{\tau_1})$ and $v_{it}^b \sim \mathcal{N}(0, \frac{1}{\tau_2})$, where τ_1 and τ_2 are precision parameters.

2.2. Copula dependence of inefficiency and noise across equations

In this study, we introduce a flexible dependence structure for the inefficiency terms and for the noise terms across the two frontier equations. This reflects the possibility that firms may exhibit correlated inefficiencies in the production of good and bad outputs due to shared technological constraints, or persistent managerial practices. Similarly, the noise terms may be correlated due to common external influences such as weather, disease outbreaks, or other shocks. To capture these dependencies, we use a copula-based framework, which enables the modeling of the joint distribution of the latent variables by combining their known marginal distributions with a flexible specification of their dependence structure. Unlike linear correlation, copulas allow us to accommodate asymmetric or non-linear relationships, which are often observed in empirical studies. For a comprehensive discussion on the

application of copulas in stochastic frontier analysis, refer to [Amsler and Schmidt \(2021\)](#) and [Mamonov et al. \(2022\)](#).

Two different copulas are employed to model the dependence between the inefficiency terms and the noise terms. For the inefficiency terms u_{it}^g and u_{it}^b , the copula $C_{\text{inefficiency}}$ is used to capture their dependence. The joint distribution of these inefficiency terms, $F(u_{it}^g, u_{it}^b)$, is constructed using the copula $C_{\text{inefficiency}}(F_{u_{it}^g}(u_{it}^g), F_{u_{it}^b}(u_{it}^b); \theta_u)$, where $F_{u_{it}^g}(u_{it}^g)$ and $F_{u_{it}^b}(u_{it}^b)$ are the individual marginal cumulative distribution functions (cdfs) of the inefficiency terms, while θ_u governs the strength of their dependence. Similarly, the copula C_{noise} captures the dependence between the noise terms v_{it}^g and v_{it}^b . The joint distribution $F(v_{it}^g, v_{it}^b)$ is modeled using the copula $C_{\text{noise}}(F_{v_{it}^g}(v_{it}^g), F_{v_{it}^b}(v_{it}^b); \theta_v)$, where $F_{v_{it}^g}(v_{it}^g)$ and $F_{v_{it}^b}(v_{it}^b)$ are the individual marginal cdfs of the noise terms, whilst θ_v controls the strength of their dependence. This separation into two distinct copulas reflects the different nature and likely sources of dependence in inefficiency and noise. Inefficiency terms may exhibit correlation due to structural or managerial factors, whereas noise correlations are more likely to arise from short-term external shocks. Modeling these components independently provides greater flexibility and allows for a more accurate representation of the underlying processes affecting each latent variable.

2.3. Bayesian estimation

We estimate the model using Bayesian methods, as in [Wanke et al. \(2020\)](#). This approach is well-suited to our setting, as it provides a coherent framework for handling latent variables such as inefficiency, facilitates the incorporation of prior information, and handles the complex structure introduced by the copula-based dependence. The likelihood function is constructed from the joint distribution of all observed and unobserved variables, conditional on the model parameters and covariates. It includes the density contributions from the good and bad output equations, the inefficiency and noise components, the unobserved heterogeneity terms, and the copula density functions that model the dependence between inefficiency terms and between noise terms across the two frontier equations. Specifically, the model's likelihood is:

$$\begin{aligned}
 & p(\{g_{it}\}, \{b_{it}\}, \{\alpha_i^g\}, \{\alpha_i^b\}, \{u_{it}^g\}, \{u_{it}^b\} \mid \mathbf{x}_{it}^{np}, \mathbf{x}_{it}^p, \beta, \gamma, \delta, \omega_1, \omega_2, \phi_1, \phi_2, \tau_1, \\
 & \quad \tau_2, \theta_u, \theta_v) = \\
 & \prod_{i=1}^N \prod_{t=1}^T \left[\frac{1}{\sqrt{2\pi/\tau_1}} \exp\left(-\frac{\tau_1}{2} (g_{it} - \alpha_i^g - \mathbf{x}_{it}^{np}\beta - \mathbf{x}_{it}^p\gamma + u_{it}^g)^2\right) \right. \\
 & \quad \times \frac{1}{\sqrt{2\pi/\tau_2}} \exp\left(-\frac{\tau_2}{2} (b_{it} - \alpha_i^b - \mathbf{x}_{it}^p\delta - u_{it}^b)^2\right) \\
 & \quad \times \frac{1}{\sqrt{2\pi/\omega_1}} \exp\left(-\frac{\omega_1}{2} (\alpha_i^g)^2\right) \times \frac{1}{\sqrt{2\pi/\omega_2}} \exp\left(-\frac{\omega_2}{2} (\alpha_i^b)^2\right) \\
 & \quad \times \sqrt{\frac{2\phi_1}{\pi}} \exp\left(-\frac{\phi_1}{2} (u_{it}^g)^2\right) \times \sqrt{\frac{2\phi_2}{\pi}} \exp\left(-\frac{\phi_2}{2} (u_{it}^b)^2\right) \\
 & \quad \times \frac{1}{\sqrt{2\pi/\tau_1}} \exp\left(-\frac{\tau_1}{2} (v_{it}^g)^2\right) \times \frac{1}{\sqrt{2\pi/\tau_2}} \exp\left(-\frac{\tau_2}{2} (v_{it}^b)^2\right) \\
 & \quad \times c_{\text{inefficiency}}(F_{u_{it}^g}(u_{it}^g), F_{u_{it}^b}(u_{it}^b); \theta_u) \times c_{\text{noise}}(F_{v_{it}^g}(v_{it}^g), \\
 & \quad \left. F_{v_{it}^b}(v_{it}^b); \theta_v) \right] \quad (3)
 \end{aligned}$$

where the terms $c_{\text{inefficiency}}(F_{u_{it}^g}(u_{it}^g), F_{u_{it}^b}(u_{it}^b); \theta_u)$ and $c_{\text{noise}}(F_{v_{it}^g}(v_{it}^g), F_{v_{it}^b}(v_{it}^b); \theta_v)$ are the copula probability density functions (pdfs) for the inefficiency and noise components, respectively.

Regarding the prior distributions, we adopt standard choices that are either weakly informative or consistent with established practice in the stochastic frontier literature. The coefficient vectors β and γ in the good output equation, as well as δ in the bad output equation, are assigned Multivariate Normal priors. Specifically, β is given a prior

$\mathcal{N}(\mathbf{0}, (0.001 \cdot \text{diag}(K^{np}))^{-1})$, where K^{np} denotes the number of non-polluting inputs, and γ is given a prior $\mathcal{N}(\mathbf{0}, (0.001 \cdot \text{diag}(K^p))^{-1})$, where K^p represents the number of polluting inputs. Similarly, δ , the coefficient vector for the bad output equation, is assigned a prior $\mathcal{N}(\mathbf{0}, (0.001 \cdot \text{diag}(K^p))^{-1})$. The precision parameters τ_1 for the noise term of the good output equation, and τ_2 for the noise term of the bad output equation, follow Gamma priors with shape and rate parameters of 0.001, specifically, $\tau_1, \tau_2 \sim \text{Gamma}(0.001, 0.001)$. The precision parameters ϕ_1 and ϕ_2 , associated with the inefficiency terms u_{it}^g and u_{it}^b , respectively, are assigned Gamma priors with a shape parameter of 7.0 and a rate parameter of 0.5, such that $\phi_1, \phi_2 \sim \text{Gamma}(7.0, 0.5)$. This specification aligns with the approach proposed by [Van den Broeck et al. \(1994\)](#), yielding a reasonable prior median inefficiency of 0.2. Additionally, the precision parameters ω_1 and ω_2 , which govern the variability in the unobserved heterogeneity terms for the good output and the bad output, follow Gamma priors with shape and rate parameters of 0.001: $\omega_1, \omega_2 \sim \text{Gamma}(0.001, 0.001)$. Finally, the copula parameters θ_u and θ_v are assigned Uniform priors, with the corresponding hyperparameters explained in the next section where the copula specifications are discussed.

The posterior distribution is obtained by combining the likelihood and the prior distributions through Bayes' theorem. It represents the full probabilistic characterization of the model parameters and latent variables, conditional on the observed data. Specifically, the posterior distribution is proportional to the product of the likelihood function and the prior densities assigned to each model parameter:

$$\begin{aligned}
 & \pi(\beta, \gamma, \delta, \omega_1, \omega_2, \phi_1, \phi_2, \tau_1, \tau_2, \theta_u, \theta_v, \{\alpha_i^g\}, \{\alpha_i^b\}, \{u_{it}^g\}, \{u_{it}^b\} \mid \{g_{it}\}, \{b_{it}\}, \\
 & \quad \mathbf{x}_{it}^{np}, \mathbf{x}_{it}^p) \propto \\
 & \quad p(\{g_{it}\}, \{b_{it}\}, \{\alpha_i^g\}, \{\alpha_i^b\}, \{u_{it}^g\}, \{u_{it}^b\} \mid \mathbf{x}_{it}^{np}, \mathbf{x}_{it}^p, \beta, \gamma, \delta, \omega_1, \omega_2, \phi_1, \phi_2, \\
 & \quad \tau_1, \tau_2, \theta_u, \theta_v) \\
 & \quad \times p(\beta) \times p(\gamma) \times p(\delta) \times p(\omega_1) \times p(\omega_2) \times p(\phi_1) \times p(\phi_2) \times p(\tau_1) \times p(\tau_2) \\
 & \quad \times p(\theta_u) \times p(\theta_v) \quad (4)
 \end{aligned}$$

The terms $p(\cdot)$ in the last two lines represent the prior distributions for each parameter in the model as these are described above. Posterior inference is conducted using Markov Chain Monte Carlo simulation, which allows for sampling from the joint posterior distribution and estimation of all quantities of interest.

3. Application

3.1. Data & specified variables

The data for this study come from the Educampo extension program in Minas Gerais, Brazil, a privately funded technical assistance initiative supported by farmers and dairy processors in collaboration with the Brazilian Micro and Small Business Support Service. The dataset is an unbalanced panel of 701 Brazilian dairy farms observed from 2014 to 2021, comprising a total of 2108 observations. On average, each farm is observed for approximately 4 years, with some appearing for the entire period and others for shorter periods. The Educampo extension agents collect the data through regular farm visits, where they record detailed financial, production, and input usage information directly from farm records and farmer interviews as part of their technical assistance program. The good output, g_{it} , is measured as total farm income (in BRL), encompassing all revenue streams from the dairy enterprise, including milk, young stock, cull animals, and surplus feedstock. The bad output b_{it} is represented by the carbon footprint (kg CO₂-eq), calculated using Life Cycle Assessment to account for on-farm emissions, including CH₄ from enteric fermentation and manure management, N₂O from nitrogen compounds in manure, and CO₂-eq emissions from feed production and its transport to the farm. Details on the calculation of the bad output can be found in the Appendix.

Table 1
Summary statistics for specified variables.

Variable	Unit	Mean	Standard deviation	2.5%	97.5%
Good output	BRL	1,739,212	1,841,447	274,607	7,264,038
Bad output	KG	1,222,747	1,069,123	254,740.7	4,273,968
Capital	BRL	109,954.7	101,135.8	21,896.9	408,057.3
Labor	BRL	207,016.8	205,932.5	40,611.5	813,485.6
Land	HA	96.9	95.7	13.5	325.9
Animals	HEADS	237.1	194.4	54.7	735.9
Intermediate inputs	BRL	409,205.8	458,827	64,614.8	1,734,283
Feed	BRL	781,037.7	829,874.6	103,021	3,179,019

The vector of non-polluting inputs, x_{it}^{np} , specified only in the good output sub-technology, includes the following variables: capital, representing the opportunity cost of farm capital (interest costs on total capital, in BRL); labor expenses, which include both hired and family labor (in BRL); land, defined as the area dedicated to the dairy enterprise (in hectares); and intermediate inputs, covering total operating expenses excluding labor and feed costs (in BRL). The vector of polluting inputs, x_{it}^p , specified in both the good output sub-technology and the bad output sub-technology, includes: animals, representing the total number of animals in the dairy enterprise; and feed costs (in BRL). The monetary data used in the study were deflated using the General Price Index – Internal Availability (IGP-DI), a metric developed and calculated by Fundação Getúlio Vargas. The IGP-DI for February 2022 was employed as the deflation reference. Table 1 presents summary statistics for the specified variables.

3.2. Frontier specifications

The production frontier for the good output and the cost frontier for the bad output are specified using a translog functional form, allowing for interaction and nonlinearity among inputs. A time trend is also included in the two equations (say in the vector of polluting inputs that appears in both equations). Prior to estimation, the outputs and inputs are normalized by their respective geometric means, while the trend variable is normalized by its arithmetic mean. This normalization ensures that the first-order term coefficients can be interpreted as output elasticities evaluated at the geometric mean of the data. The model equations are as follows:

$$\begin{aligned}
 g_{it} &= \alpha_i^g + \sum_{k=1}^{K^{np}} \beta_k x_{kit}^{np} + \sum_{j=1}^{K^p} \gamma_j x_{jit}^p + \frac{1}{2} \sum_{k=1}^{K^{np}} \sum_{l=1}^{K^{np}} \beta_{kl} x_{kit}^{np} x_{lit}^{np} \\
 &\quad + \frac{1}{2} \sum_{j=1}^{K^p} \sum_{m=1}^{K^p} \gamma_{jm} x_{jit}^p x_{mit}^p + \sum_{k=1}^{K^{np}} \sum_{j=1}^{K^p} \gamma_{kj} x_{kit}^{np} x_{jit}^p - u_{it}^g + v_{it}^g \\
 b_{it} &= \alpha_i^b + \sum_{j=1}^{K^p} \delta_j x_{jit}^p + \frac{1}{2} \sum_{j=1}^{K^p} \sum_{m=1}^{K^p} \delta_{jm} x_{jit}^p x_{mit}^p + u_{it}^b + v_{it}^b,
 \end{aligned} \quad (5)$$

Regarding the copula specification, this study employs two widely used copulas, namely the Gumbel and the Clayton, due to their ability to account for asymmetric tail dependence. In the context of farming, theoretical motivation leads us to believe that tail dependence plays a crucial role. For the inefficiency components, one can expect upper tail dependence when both outputs (milk and emissions) exhibit high inefficiency levels, which is often the case in underperforming farms where managerial shortcomings simultaneously affect multiple production dimensions. Conversely, lower tail dependence may emerge when both inefficiencies are low, as seen in efficient farms with effective practices leading to high output and low emissions. These contrasting scenarios justify the use of asymmetric copulas with different tail sensitivities.

A similar logic applies to the noise components, which reflect random shocks or external factors beyond the farm's control. Theoretical considerations suggest that upper tail dependence in noise may arise

Table 2
Comparison of models with Gumbel and Clayton copulas for inefficiency and noise.

Model	u_{it} copula	v_{it} copula	Prior	Marginal log \mathcal{L}	Posterior	DIC
Model 1	Gumbel	Gumbel	0.250	4,252.526	1.000	−9,564.260
Model 2	Clayton	Clayton	0.250	3,121.999	0.000	−5,078.925
Model 3	Gumbel	Clayton	0.250	3,108.905	0.000	−5,180.832
Model 4	Clayton	Gumbel	0.250	4,108.169	0.000	−8,926.752

when extreme events, such as adverse weather or disease outbreaks, simultaneously increase variability in both outputs. In contrast, lower tail dependence may occur during periods of stable environmental and market conditions, when both outputs experience minimal variation. As with the inefficiency terms, the Gumbel copula is well-suited to capture upper tail dependence, while the Clayton copula is appropriate for modeling dependence in the lower tail. From a technical perspective, to avoid numerical underflow and overflow issues, as highlighted by Schmidt and Kneib (2023), we limit the dependence parameters θ_u and θ_v to narrower ranges. Specifically, θ_u and θ_v are restricted to 1 to 17 for the Gumbel copula and 1×10^{-16} to 28 for the Clayton copula, even though their theoretical parameter spaces are larger. We explore all possible combinations of these copulas for the inefficiency and noise components. The final model specification is selected based on Bayes factors and an information criterion, which identify the copula combination that provides the best fit to the data.

4. Results

The sampling scheme used to derive the empirical findings presented in this section consisted of a burn-in phase of 400,000 iterations, followed by an additional 600,000 iterations, with a thinning interval of 10 to reduce autocorrelation in the posterior samples. First, we estimate four models, each with distinct copula specifications for the inefficiency (u_{it}) and noise (v_{it}) components. These models are compared based on their marginal log likelihoods, approximated using the Laplace-Metropolis estimator (Lewis and Raftery, 1997). To quantify the relative evidence for one model over another, we compute Bayes factors, defined as the ratio of the marginal likelihoods of two models. Assuming equal prior probabilities of 0.250 for all models, posterior probabilities are derived using Bayes' theorem, with the constraint that they sum to one. The model with the highest posterior probability is considered the most strongly supported by the data. Additionally, the Deviance Information Criterion (DIC), introduced by Spiegelhalter et al. (2002), is also reported for each model, with lower values indicating a better fit.

Table 2 summarizes the results, including the prior probabilities, marginal log likelihoods, posterior probabilities, and DIC values for each model. Based on the posterior probabilities, Model 1, which uses a Gumbel copula for both inefficiency and noise, is overwhelmingly favored, indicating that the data strongly support its specification. Importantly, Model 1 also exhibits the lowest DIC value, providing further evidence in its favor. This finding highlights the presence of upper tail dependence between the inefficiency components across the two equations, as well as between the noise terms of the two equations. Therefore, the subsequent findings are based on this model specification.

Table A1 in the Appendix provides the complete set of the model's parameter estimates. Table 3 presents the parameter estimates for the first-order terms of the good output and bad output sub-technologies. Regarding the good output sub-technology, the credible intervals for the coefficients of capital, land, and trend include zero, while the credible intervals for the remaining coefficients are strictly positive. Among the latter, the elasticity with respect to feed exhibits the largest magnitude, highlighting its critical role in the production of the good output. This finding is consistent with the results reported by Ferreira et al. (2024) for Brazilian dairy farms and Skevas (2025) for

Table 3
Parameter estimates of first-order terms from the two sub-technologies.

Good output g_{it}			
Parameter	Mean	Standard deviation	95% credible interval
β_0	-3.976	0.077	[-4.127, -3.824]
β_{capital}	0.022	0.017	[-0.012, 0.055]
β_{labor}	0.048	0.014	[0.020, 0.077]
β_{land}	-0.017	0.010	[-0.036, 0.002]
$\beta_{\text{intermediate}}$	0.176	0.017	[0.143, 0.209]
γ_{herd}	0.088	0.022	[0.046, 0.130]
γ_{feed}	0.699	0.019	[0.663, 0.736]
γ_{trend}	0.002	0.003	[-0.003, 0.008]
ω_1	203.774	7.747	[145.180, 292.152]
ϕ_1	140.835	2.306	[118.164, 166.444]
τ_1	194.299	4.718	[166.573, 224.311]
Bad output b_{it}			
Parameter	Mean	Standard deviation	95% credible interval
δ_0	-3.780	0.052	[-3.883, -3.682]
δ_{herd}	0.581	0.016	[0.551, 0.611]
δ_{feed}	0.410	0.012	[0.385, 0.434]
δ_{trend}	-0.003	0.002	[-0.008, 0.001]
ω_2	309.102	6.073	[209.787, 465.882]
ϕ_2	164.445	3.577	[139.406, 192.535]
τ_2	141.498	7.815	[127.357, 157.252]

Dutch dairy farms. The sum of the output elasticities results in a scale elasticity of 1.011, indicating slightly increasing returns to scale in the production of the good output. Such behavior could result from efficiencies gained through input complementarities or economies of scale inherent in the production process.

For the bad output sub-technology, the coefficients for both polluting inputs are positive, with their corresponding credible intervals excluding zero. Among these inputs, herd size has the greatest impact on the production of the bad output. Our findings are broadly consistent with those of Skevas (2025), who identifies feed as the main contributor to bad output, closely followed by herd size. While our results reverse this order, highlighting herd size as the most influential factor, the overall conclusion that both inputs are key drivers of bad output is shared across the two studies. The sum of the elasticities for the polluting inputs yields a scale elasticity of 0.991, indicating slight decreasing returns to scale in the production of the bad output. This suggests that proportional increases in polluting inputs lead to less than proportional increases in pollution. This result may be due to several factors, such as improved resource utilization or more efficient management practices at larger input scales.

Fig. 1 presents a copula density plot, illustrating the dependence between the inefficiency terms of the good output and bad output sub-technologies. The plot reveals a strong upper tail dependence because the data points cluster closely along the upper right portion of the plot, indicating that farms with high inefficiency in one output are likely to exhibit similarly high inefficiency in the other. This relationship points to the presence of shared inefficiency drivers, such as suboptimal feed management or poor herd management practices, which simultaneously affect both outputs. This interdependence highlights the interconnected nature of inefficiency in dairy farms, suggesting that addressing inefficiencies in one aspect of production, such as milk production, could lead to efficiency gains in the other area, like emissions.

Fig. 2 shows the copula density plot for the dependence between the noise components of the good and bad outputs. The plot also reveals strong upper tail dependence, indicating that extreme random shocks or external factors that heavily affect one output are likely to simultaneously impact the other. Such dependence may arise from shared external drivers of noise, including environmental events like severe weather conditions (e.g., droughts or floods). These shared influences highlight the interconnected nature of external uncertainties in dairy

Table 4
Dependence metrics.

	θ	Kendall's τ	Spearman's ρ	Tail dependence coefficient
Inefficiency terms	9.029	0.889	0.778	0.920
Noise terms	16.986	0.941	0.882	0.958

farms, where large-scale disruptions tend to propagate across different aspects of production.

To quantify the relationships depicted in Figs. 1 and 2, Table 4 presents key dependence metrics for both the inefficiency and noise components. These metrics include the estimated copula dependence parameters, θ_u and θ_v , along with derived measures such as Kendall's τ , Spearman's ρ , and the tail dependence coefficient. While the copula dependence parameters suggest a strong positive association for both inefficiency and noise components, with a stronger association observed in the noise components, their direct interpretation is not particularly meaningful. To better interpret the strength of the associations, we compute Kendall's τ , defined as $1 - \frac{1}{\theta}$, and Spearman's ρ , calculated as $1 - \frac{2}{\theta}$. Both of these measures are rank-based and quantify the strength of the association between two variables. For the Gumbel copula, their parameter space lies within $[0, 1]$, where values close to 0 imply little to no association, and values close to 1 indicate a strong positive association. In our case, both metrics are very high for both inefficiency and noise components, highlighting strong positive associations, especially in the noise components. The tail dependence coefficient, defined as $2 - 2^{1/\theta}$, provides insights into the extent of upper tail dependence, measuring the likelihood that extreme values in one variable are associated with extreme values in the other. The coefficient is also bounded between 0 and 1, with values near 0 indicating no upper tail dependence and values near 1 reflecting strong upper tail dependence. In both inefficiency and noise components, the observed tail dependence coefficients are close to 1, indicating strong upper tail dependence in both cases.

The stronger dependence observed in the noise components compared to the inefficiency components suggests that random external factors influencing the good and bad outputs are more closely linked than the inefficiencies specific to the production process. This might imply that shared external drivers, such as environmental conditions (e.g., weather events like droughts or floods), play a more significant role in jointly affecting the noise terms across the two outputs. In contrast, while the inefficiency components also exhibit strong dependence, they reflect internal, farm-specific inefficiencies (e.g., feed and manure management, herd practices) that may vary slightly more independently between the good and bad outputs. The distinction between the two could highlight that noise, as an external factor, is inherently more synchronized across outputs due to its origin in shared external conditions, whereas inefficiencies, driven by farm-specific practices, may retain a degree of variability.

Fig. 3 illustrates the estimated inefficiency densities for the good output (u_{it}^g , shown in blue) and the bad output (u_{it}^b , shown in green). The estimated average inefficiency is 0.066 for the good output and 0.061 for the bad output. These low inefficiency levels highlight that farms, on average, perform rather well in both dimensions, maintaining a high level of efficiency in the production of both the good and bad outputs. The slight shift in the good output's density toward higher inefficiency values could highlight marginally greater challenges in optimizing the production of the good output compared to mitigating inefficiencies in the bad output.¹ The reported low inefficiencies in both

¹ A sensitivity analysis was performed using alternative inefficiency distributions, including the Exponential and the Truncated-Normal. The main conclusions of the study remain robust across these distributions, while the

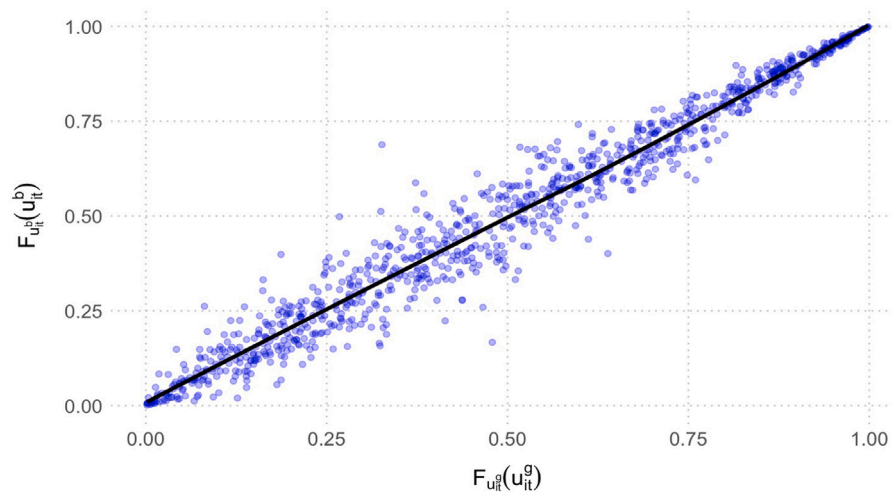


Fig. 1. Copula density plot for inefficiency dependence.

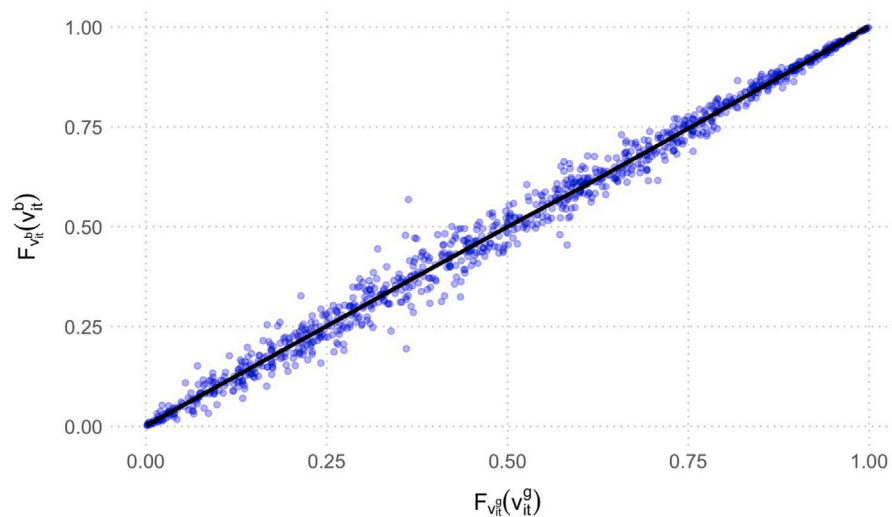


Fig. 2. Copula density plot for noise dependence.

dimensions are consistent with findings from previous studies. Alem (2021) and Ferreira et al. (2024) report low inefficiency in good output production for Norwegian and Brazilian dairy farms, respectively. Similarly, Vogel and Beber (2022) find low emissions per unit of output in certain intensive dairy systems in Brazil, while Skevas (2025) report low bad output inefficiency for Dutch dairy farms. It is worth noting that the high performance of the sampled farms, which is comparable even to advanced dairy sectors such as the Dutch, is likely attributable to their participation in the Educampo program. This initiative provides farms with substantial support through technical assistance, training, and performance monitoring. The capital-intensive nature of these farms, combined with continuous access to managerial guidance, likely contributes to the low inefficiency observed in both the good and bad output dimensions. The efficiency-enhancing role of extension services is well documented in the literature. For instance, Ahikiriza et al. (2025) provide evidence that participation in such programs significantly improves dairy farm productivity and efficiency.

Finally, we perform a model comparison to address the ongoing debate in the literature regarding whether non-polluting inputs should

be included in the bad output frontier. One perspective argues for inclusion, given the potential indirect effects of non-polluting inputs on pollution through improved production practices (Skevas et al., 2023). The alternative view, which we adopt, asserts that bad outputs result exclusively from polluting inputs, in accordance with the materials balance principle (Førsund, 2021). To this end, we also estimate the proposed model with the inclusion of non-polluting inputs in the bad output frontier and compare the results with our original model specification. Table A6 in the Appendix presents the full set of results from the robustness check, which includes non-polluting inputs in the bad output frontier. Notably, the 95% credible intervals for the parameter estimates of these inputs in the bad output frontier all contain zero, indicating that they do not impact the production of the bad output. This finding provides empirical support for our original specification. Moreover, the estimates of the copula dependence parameters remain virtually unchanged relative to those in the main model, as do the inefficiency scores reported in Table A7. Table 5 reports the prior probabilities (set to 0.5 for each model), the marginal log likelihoods, the corresponding posterior probabilities, and DIC values. The results clearly indicate that our original model specification, which excludes non-polluting inputs from the bad output frontier, fits the data best. This is evidenced by its posterior probability of 1, supporting the conclusion that non-polluting inputs should only be included in the good output frontier. Notably, this conclusion is further reinforced by

Half-Normal distribution is favored by the data, as indicated by the Bayes factors and the DIC. Detailed results and comparisons are presented in Tables A2–A5 in the Appendix.

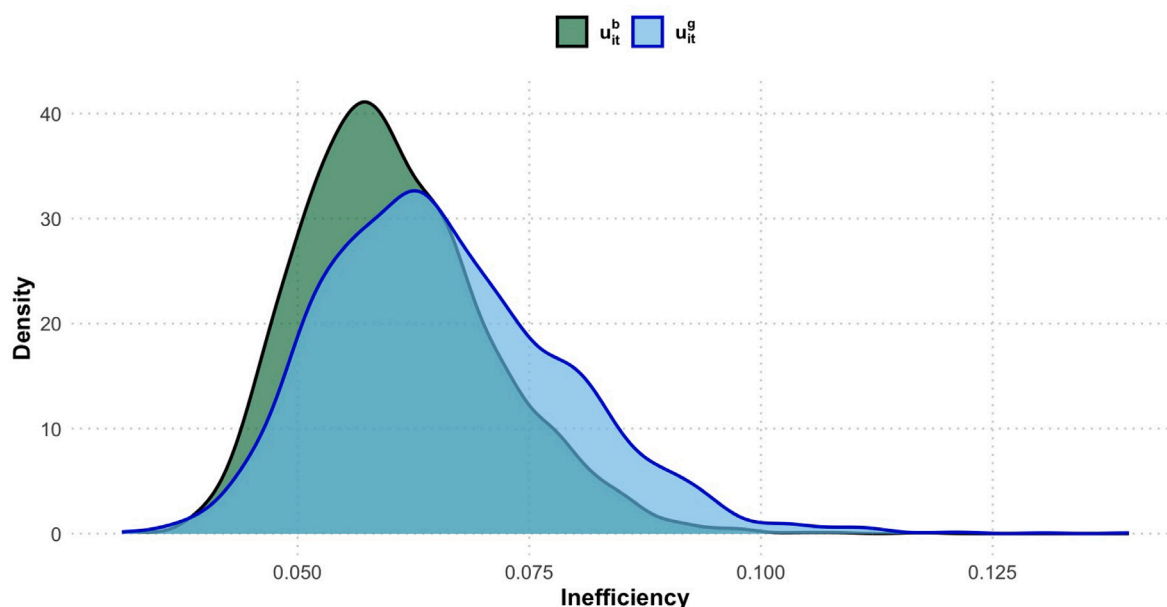


Fig. 3. Densities of good and bad output inefficiencies. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 5
Comparison of different model specifications for the bad output frontier.

Model specification	Prior	Marginal log \mathcal{L}	Posterior	DIC
Original model (Non-polluting inputs only in good output frontier)	0.5	4252.526	1	-9564.260
Robustness specification (Non-polluting inputs included also in bad output frontier)	0.5	4104.399	0	-9092.784

the DIC values, with the original specification also exhibiting the lowest DIC, thus confirming its superior fit from an information-theoretic perspective.

5. Concluding remarks

In this study, we develop a by-production stochastic frontier model comprising a good output and a bad output equation to assess farm technical and environmental inefficiency, while accounting for dependence across the two equations. The dependence structure employed is more flexible than in previous studies, as we separately link the inefficiency terms as well as the noise terms across the two equations using two distinct copulas. We apply this model to Brazilian dairy farms, which is a sector characterized by significant methane emissions (Vogel and Beber, 2022) - marking the first use of the by-production stochastic frontier model in this context. Additionally, we advance the existing literature by introducing a Bayesian framework for estimating the specified model. The dataset used in this study is a panel of Brazilian dairy farms observed from 2014 to 2021. The good output primarily represents milk production, which is associated with a combination of non-polluting and polluting inputs. The bad output is measured in terms of CO₂-equivalent emissions, which are linked exclusively to polluting inputs. To model the dependence between the inefficiency terms and between the noise terms of the good and bad output equations, we employ two separate copulas, selecting the best-fitting copula based on Bayes factors and DIC.

The empirical findings reveal that milk production is primarily driven by feed inputs, with production exhibiting increasing returns to scale. In contrast, emissions are predominantly influenced by herd size,

with production exhibiting decreasing returns to scale. The inefficiency terms across the good and bad output equations exhibit strong positive upper tail dependence, indicating that high inefficiency in milk production is likely to coincide with high inefficiency in emissions production, highlighting the interconnected nature of inefficiencies across the two outputs. Moreover, the noise terms across the two equations exhibit even stronger positive upper tail dependence compared to the inefficiency terms. This suggests that random shocks or external factors, such as weather conditions or market fluctuations, have a more pronounced simultaneous impact on the production of both outputs.

The average inefficiency in milk production is estimated at 0.066, while the average emission inefficiency is slightly lower at 0.061. These estimates indicate that Brazilian dairy farms in this sample operate with high efficiency in both maximizing milk production and minimizing emissions. This aligns with findings that dairy farms can achieve high technical efficiency levels in the presence of incentive programs and is consistent with Ferreira et al. (2024), who focus on capital-intensive dairy farms in Brazil. However, the density of milk production inefficiency reveals a greater number of outliers, suggesting that there is more variability and room for improvement in this dimension compared to emissions. Furthermore, a formal model comparison using Bayes factors and DIC indicates that the original model specification - where non-polluting inputs are included exclusively in the good output frontier - provides the best fit to the data. This finding supports the argument that non-polluting inputs primarily contribute to the production of good outputs and that their influence on emissions is adequately captured through their effect on polluting inputs.

The findings of this study have important policy implications for improving both technical and environmental efficiency in Brazilian dairy farming. First, the identification of feed as the primary driver of milk production suggests that policies aimed at optimizing feed use, such as subsidies for high-quality feed or training programs on feed management, could significantly enhance milk production. Simultaneously, recognizing herd size as the key determinant of emissions highlights the need for policies that encourage better herd management practices, such as herd size optimization and investment in low-emission livestock technologies. The strong upper tail dependence between inefficiencies in milk production and emissions highlights the interconnected nature of these processes, suggesting that addressing inefficiencies in one area (e.g., milk production) could lead to spillover efficiency gains in emissions reduction. Additionally, the even stronger dependence

observed in the noise terms suggests that external factors like weather variability or economic shocks play a substantial role. This finding calls for policies that enhance the resilience of farms to such external shocks, for example, through climate-smart agriculture initiatives or risk management programs.

The main limitation of this study is that these findings may not be fully generalizable to Brazilian dairy farms. The sample analyzed consists of farms participating in the Educampo program, which are likely more resource-efficient and environmentally conscious than the average dairy farm in Brazil. Educampo provides tailored technical assistance, management training, and strategic tools that enhance productivity and sustainability. For instance, in 2021 the average milk yield per cow in our sample was 8162 liters while the national average for Brazil was 2213 liters (IBGE, 2022). As a result, this sample may not fully reflect the broader Brazilian dairy sector, where such support is less widespread and efficiency levels tend to be lower (Mareth et al., 2019; Vogel et al., 2023). However, this also suggests that expanding similar programs nationwide could help improve the sustainability of the dairy sector, which is predominantly composed of small producers (Beber et al., 2021).

Future research could expand on this study by addressing several enhancements to deepen the understanding of farm inefficiency. First, the inclusion of inefficiency determinants, such as farm-specific characteristics or managerial practices, could provide insights into the factors driving inefficiency and offer targeted recommendations for improvement; nevertheless, such indicators are not available in the current dataset. Second, the estimation of cost efficiency would offer a complementary perspective by assessing how effectively farms minimize costs relative to their output levels. However, this approach typically requires detailed farm-level price data, which is not available. Third, specifying time-invariant inefficiency using the generalized true random effects model would allow for a more detailed decomposition of inefficiency into persistent and time-varying components, offering a clearer picture of structural inefficiency. Nevertheless, this extension would require the introduction of a third copula to model the dependence between the time-invariant inefficiency terms across the two equations, adding complexity to the model but enabling a more comprehensive analysis.

CRedit authorship contribution statement

Ioannis Skevas: Writing – review & editing, Methodology, Data curation, Writing – original draft, Validation, Investigation, Visualization, Formal analysis, Conceptualization. **Everton Vogel:** Methodology, Formal analysis, Conceptualization, Writing – review & editing, Data curation. **Andre Rozemberg Peixoto Simões:** Supervision, Funding acquisition, Conceptualization, Writing – review & editing, Resources, Investigation, Validation, Project administration, Data curation. **Marcelo Dias Paes Ferreira:** Writing – review & editing, Resources, Supervision, Data curation, Validation, Project administration, Funding acquisition, Investigation, Conceptualization. **Caetano Luiz Beber:** Validation, Resources, Investigation, Project administration, Funding acquisition, Writing – review & editing, Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The views expressed are purely those of the authors and may not in any circumstances be regarded as stating an official position of the European Commission. A version of this article was presented at the Department of Agricultural Economics, Texas A&M University, USA (January 2025). This research was partly funded by the following

grants: Greenhouse gas emissions, reduction potential, and abatement costs in dairy farming in Minas Gerais, funded by Fundação de Amparo à Pesquisa do Estado de Minas Gerais (The Research Support Foundation of the State of Minas Gerais), Brazil, grant id APQ-05444-24; Productivity Analysis in Agriculture: Environment, Technologies, and Public Policies, funded by National Council for Scientific and Technological Development—CNPq, Brazil, grant id 422542/2021-0; Joint Research Centre – EU Commission.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jenvman.2025.126606>.

Data availability

The authors do not have permission to share data.

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