Multi-fractal analysis on Neolithic Sites around the Sanxia Reservoir Area

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***Abstract*—Applying probability moment method this areticle calculated 143 *q*-value of the Paleolithic sites and the Neolithic sites respectively, and values were ranged in [-40, +45] and [-9.3, +9.3], fractal dimensions were within [-52.88, +33.16] and [-12.37, +5.86]. The regression results for probability moment revealed that the spatial distribution of the sites were in correspondence with the multifractal power-law other than simple fracal. The generalized dimension spectra were determined with ΔD(Paleo) = 0.5359 and ΔD(Neo) = 0.4940, implying that the Paleolithic sites in height were fastly variant but sluggish for Neolithic sites. The multifractal spectra showed that the spatial characteristics of the Paleolithic sites were partly clustered and dispersed on the whole. Contrarily the Neolithic sites are little variance in height.**

***Keywords*—multifractal spectrum; the Sanxia area; paleolithic and Neolithic sites;**

I. INTRODUCTION

Multi-fractals are generated by different physical processes such as multiplicative cascade processes, diffusion-limited aggregation (DLA), turbulence and Brownian motion[1, 2]. In a multi-fractal view, some related probability distributions can be considered as special cases of multi-fractal models providing new insight into the interrelationships between systems. Presently, multifractal dimensions were calculated by moment method[3]. Few studies devoted into the spatial distribution of archaeological sites with multi-fractal spectra. Because of the predominance of multifractal method for inadequate information matters, we believe that it be beneficial using the multifractal method to discuss the spatial distribution of sites in the Sanxia area. Notably is the hidden mechanism of trends for spreading and clustering of the sites in the process of evolution in parallel to the environmental factors shown by spectra of generalized dimensions. In addition, a clear-cut relationship among multifractal spectra, generalized dimension curve and spatial distribution of the sites should be more interesting and important for introducing the multifractal view into geo-archaeology.

II. STUDY AREA AND METHOD

A. Study Area

The study region is located in Chongqing City. The south of the region with few rivers is sharply precipitous Mt. Qiyao and Mt. Fangdou, whereas the north part is relatively moderate ranges of Mt. Daba. This pattern presents many branches eventually pouring into the Yangtze such as Jianglingjiang River, Xiaojiang River, Caotang River, Daning River and Xiangxi River etc. Mt. Wu sits in the east of the region. Being one of the transitional sections between the East and the West in China, from the sea-facing to the inland-depending, this area performs comparatively independent geographical unit, therefore the most of down hill and little of plain form peculiar gorge geomorphology, this influenced life styles and productions of ancient humankind deeply.The focus of the study is the found 95 Neolithic sites and 48 Paleolithic sites, and they are stochastically scattered along the Yangtze River and its tributaries. These sites were left along the Yangtze, and most of them lies on the second, third order terraces.

B. Method

The multifractal formalism is a particularly useful tool for the study of non-uniform distributions [4]. A given set of sites, represented by points distributed in space (in 2D or 3D), is the object we wish to investigate for multifractality. The multifractal formalism adopted makes use of the box-counting procedure. Box counting is used to calculate the normalized probabilities, P_i, of box occupancy by sites. Assuming the

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number $N(\varepsilon)$ of boxes needed to cover an iso-α_i set varies as $\varepsilon^{-f(\alpha)}$, one can define a fractal dimension. The quantities $f(\alpha)$ and $\alpha(q)$ can be determined by $\tau(q)$(linear-regression fitting slopes of $\mathrm{Ln}(\chi_q)$-$\mathrm{Ln}(\varepsilon)$) from following equation :

$$\tau(q) = q\alpha - f(\alpha) \qquad (1)$$

in combination with Equation 2,

$$D_q = \frac{\tau(q)}{q-1} \qquad (2)$$

Therefore, known spectrum for $f(\alpha) \sim \alpha$, generalized dimensions D_q can be solved, similarly, we can also obtain formula for singularity value α based on relevant formula, which is given by

$$\alpha(q) = \tau'(q) \qquad (3)$$

III. RESULTS

A. Determination of multifractality of the sites

According to distributional probability density of sites, q-values were determined within [-40, +45] and [-9.3, +9.3] for Paleolithic sites and Neolithic sites(Fig. 1). Through double-logarithm linear-regression fitting between $\chi_q(\varepsilon)$ and ε, we obtained different values mass exponents from partition function, and it appeared highly multifractal by fractal dimensions within [-52.88, +33.16] and [-12.37, +5.86].

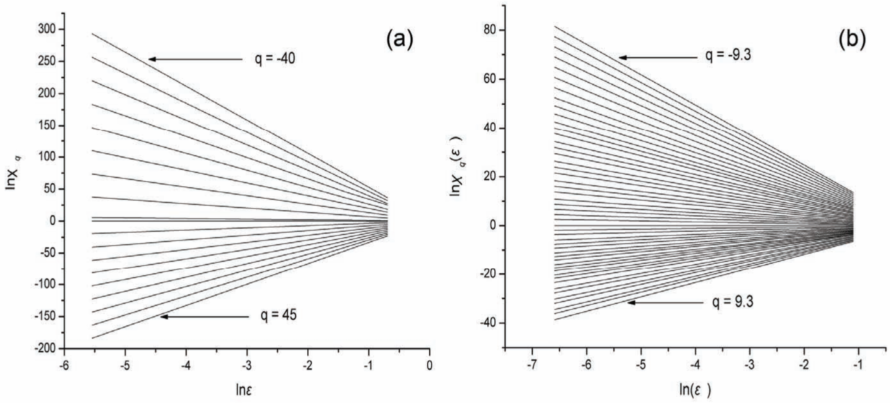

Figure 1 Results of q-values for the Paleo-sites(a) and Neolithic sites(b).

B. Generalized dimensions

Using Eq. (2) and values of $\tau(q)$, generalized dimensions of the sites can be determined (Fig.2). Width value of D_{max}—D_{min}, is proportion to disparity in height of the sites. In Fig. 1 ΔD(Paleo) = 0.5359, ΔD(Neo) = 0.4940. Inversely, the distribution density of the Paleolithic sites fell sharply from -15 to +15, meanwhile, its fractal dimensions ranging from -19.833 to 11.051. The parameters of D_{max} (Paleo) = 1.2897, D_{max} (Neo) = 1.2007; D_{min}(Neo) =0.7066 and D_{min} (Paleo)= 0.7537.

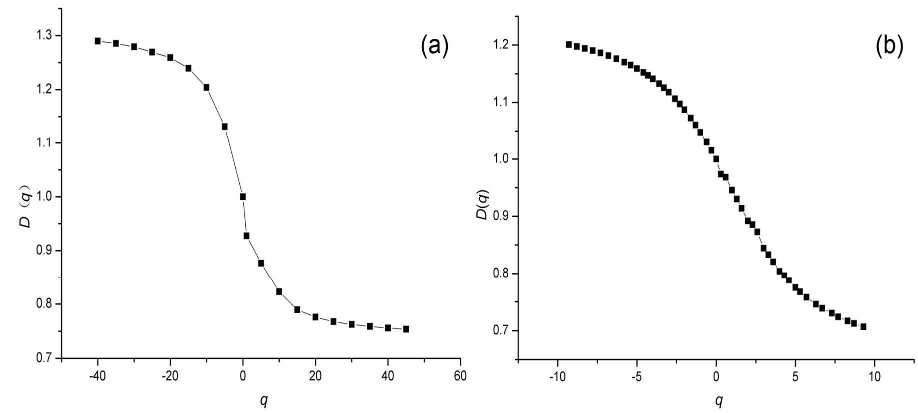

Figure 2. Curves of generalized dimension of the Paleolithic (a) and the Neolithic sites (b)

C. Multifractal spectrum for the sites in various epochs

Important are Curves for $f(\alpha) \sim \alpha$, i.e. multifractal spectrum that can be applied to describe singularities of the sites. Between variants $f(\alpha)$ and α, the former illustrates the changing trend of different subsets for q-th order moment probability, while $\Delta\alpha$ depicts non-uniform degree of spatial distribution for the sites. As compared with the Paleolithic sites , the value for the Neolithic sites $\Delta\alpha$(Neo) = 0.457 suggested that the spatial distribution of the Neolithic sites remain more uniform, however, both the sites in different stages own equal maximum of $f(\alpha) = 1.0$ means that the coverage degree of the site distribution keep little variant as long as over 6000 years.

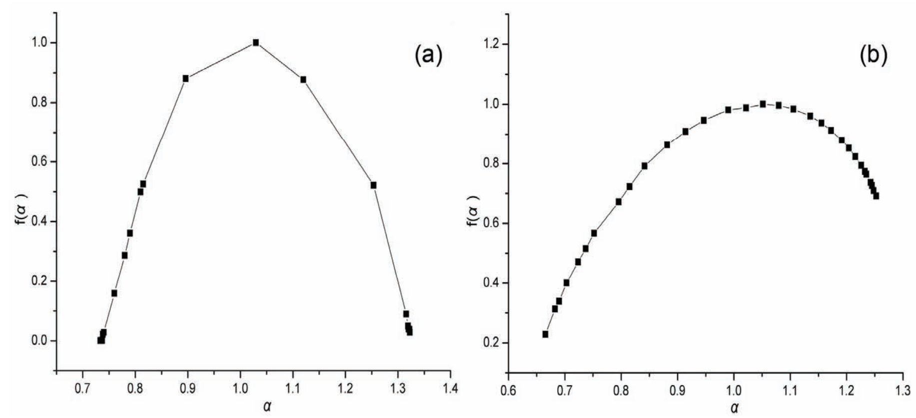

Figure 3. Multifractal spectra of the spatial distribution for the Paleo (a) and the Neolithic sites(b)

IV. DISCUSSION

The spatial distribution of the sites studied are well conformed to multifractality, reflecting regularity veiled by stochastic sequences. The shape of multifractal spectrum for Neolithic sites is asymmetrical convex curve suggesting a site-overlapped process, not apparent for paleolithic sites instead.

He and Zhao [5] determined the fractal dimension of the Sanxia area of the Yangtze River between 1.58~1.66. Comparatively, the range Dq for Paleolithic sites came at 0.7537~ 1.2897 and 0.7066~1.2007 for Neolithic sites indicating both the periods in spatial occupancy in process far from perfect state, because a fractal object grows by a law of filling space as sufficiently as possible[6]. However, does that the maximum Dq-value for Paleo-sites is larger than that for the Neo-sites just mean the spreading scope of the Paleo-sites much faster? It is obviously paradoxical to the evolution theory either for nature or for society. Simultaneously, there is no difficulty understanding the anomaly if it is involved in the abrupt climatic events (around 5000 yrs. B.P. and 4000 yrs. B.P.) in the Holocene[7, 8].

As regards the shape of spectrum for Neo-sites shown in Fig. 3 (b), the right is not so integrated as the left expressing small probability subsets are dominated in distribution format, however, Allen et al. [9] suggested it is less reliable on account of inaccurate estimations for small probabilities. On the other hand, it may as well indicate there are undiscovered sites elsewhere. In fact, that the mouth width of $f(\alpha)$ curve for Paleo-sites is larger than that of Neo-sites means that the former own a stronger singularity, which conveys that the Paleo-sites are distributed in locally concentrative way but scattered as a whole; whereas the latter is just adversely. On the other hand, it explained the evolution processes of ancient sites with advances of technology and settlements' proliferation.

Mathematically, the left part of spectral curve stands for characteristics of $q \geqslant 0$, actually reflecting features of larger probability subsets[10], While the right half denotes the properties of $q < 0$ including those of smaller subsets and the finest structures of system. Furthermore, we can extract the left part only: $\Delta\alpha_{\text{Paleo}} = 0.296$; $\Delta\alpha_{\text{Neo}} = 0.386$, showing that the degree of concentration of Neo-sites concerning to large probability subsets is prior to that of Paleo-sites. In sum, the multifractal spectra comprehensively depict basic forms of Paleolithic and Neolithic sites in the Sanxia area.

V. CONCLUSION

As a summary, the study concerned the following items: (1) The spatial distribution of the sites in Sanxia area abided by multifractal law with the fractal dimensions range of [-52.88, +33.16] and [-12.37, +5.86]. (2) The featrue of the Paleolithic sites was clustered ($\Delta\alpha$(Paleo)=0.599) as well as scattered ($D_{\max}$ (Paleo) = 1.2897)on the whole but, that of the Neolithic sites appeared on the contrary. (3) The various ratio of generalized dimensions (Dq) for sites corresponded to vicissitude of ancient sites. And (4) there is less variance in height of sites (both $f(\alpha)$-values at 1.0) in different periods suggesting few floods or disastrous events in the Sanxia area.

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