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Original article

Dynamic identification of historic masonry towers through an expeditious and no-contact approach: Application to the "Torre del Mangia" in Siena (Italy)

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A B S T R A C T

The paper presents a synergic and multidisciplinary approach where laser scanner survey, radar interferometric monitoring and finite element (FE) numerical modelling are used for expeditious and no-contact dynamic identification of monumental masonry towers. The methodology is applied to a real case of great historical interest: the "Torre del Mangia" (Mangia's tower) in Siena (Italy). The tower geometry was acquired through Terrestrial Laser Scanning (TLS) techniques. The tower oscillations were detected using an interferometric radar in "Piazza del Campo", the square facing the Mangia's Tower, along three alignments, and movement of the structure at several heights were recorded. A FE model, built on the basis of the geometry acquired through the TLS, was used to interpret and verify the physical meaning of the experimental results. Through the discussion of the case study, the paper shows that the proposed approach can be considered as an effective and expeditious method for assessing the dynamic behavior of monumental buildings (and to plan interventions) on territorial scale.

1. Research aims

Health monitoring of historic masonry towers, because of their intrinsic vulnerability, is a priority in Italy as in many other countries. A complete structural analysis of such architectonic structures need preliminarily the identification and characterization of the vibration modes, i.e. the dynamic behavior of the structure. The aim of this paper is to present an expeditious method, based on laser scanner and interferometric radar, for dynamic identification of masonry towers that could be used even on territorial scale.

2. Introduction

The number of ancient masonry towers in Europe and worldwide is huge. Each of these deserves a specific structural analysis, based on an accurate survey and a series of dynamic tests, to assess its safety level with respect to both static and dynamic loads (i.e. the assessment of their static and seismic vulnerability) [1]. The experimental survey, if developed with traditional techniques, could be expensive and time-consuming; moreover in some cases the need to preserve the integrity of historic constructions makes difficult the development of an extensive experimental investigation with the traditional techniques. Nevertheless local authorities and agencies devoted to Cultural Heritage preservation need complete and updated databases for planning interventions on territorial scale and also for preventing possible damages. For these reasons there is a great interest in expeditious and no-contact methods. Laser scanner is undoubtedly a versatile and powerful tool for rapid no-contact survey, but does not provide information about the dynamic characteristics of the structure under test. Interferometer radar is a more recent equipment [2–5] able to remotely detect the dynamic behavior of a large structure [6,7], but it does not provide an image able to be used as a geometric survey. The combination of both provides rapid survey and dynamic characterization operating quickly without physical contact with the structure under test. The information obtained through laser scanner and interferometer radar need to be interpreted according to a model of the historic construction able to reproduce and to assess its structural behavior. In this context the Finite Element (FE) techniques has been shown to be an effective tool for the interpretation of physical behavior of historic *fabricæ* [1,8,9]. FE models can be used as a numerical laboratory where the sensitivity of the results to unknown input material parameters, boundary conditions

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and actions can be efficiently analyzed, offering invaluable information in the understanding of in-situ survey and dynamic monitoring. In addition, as any damage changes the physical properties and then the modal response changes accordingly, a dynamic investigation repeated over time combined with a FE model of the structure (able to assess the sensitivity of the modal parameters according to changes in the structural system) can be used for a Structural Health Monitoring (SHM) [10,11].

Based on this background, the paper proposes a synergic and multidisciplinary approach composed of laser scanner survey, radar interferometric monitoring and FE numerical modeling for expeditious and no-contact dynamic identification of monumental towers. The methodology is applied to a real case of great historical interest: the “Torre del Mangia” (Mangia’s tower) in Siena (Italy). The paper is organized as follows: in section 2 the main historical and architectonical features of the tower are briefly sketched. In section 3 the laser scanning survey together with the main results is reported, while the experimental dynamic investigation by means of the interferometer radar is described in section 4. Next section 5, finally, describes the identified FE model employed to interpret the results of the experimental investigation and to assess the actual dynamic behavior of the tower.

3. The Mangia’s tower

The Mangia’s tower in Siena is famous all over the world together with the shell-shaped square (named “Piazza del Campo”) in front of it. The tower’s curious name comes from the nickname of the first bell-ringer.

It is one of the tallest Italian medieval towers, about 88 m height from the ground level on the northern corner, and it has a square section of about 7 m. The tower is substantially a masonry structure with a belfry in white travertine. Over the belfry a metallic structure supports the huge bell named “Campanone” (Big bell). Light rods bring the overall height of the tower up to about 102 m.

The technical and executive plan of the tower is attributed to Muccio and Francesco di Rinaldo, masters from Perugia, but there is no definite information about neither the construction stages nor their chronology. What is certain is that the laying of the foundation stones took place in 1325, simultaneously at the beginning of the last enlargement phase of the Palace, and it is assumed that the travertine crowning dates back to 1348 [12], at the end of construction works. The tower and the Palace became a model for the later civic buildings, marking the transition from the fortress to the palace; therefore they play a very important role in the history of architecture [13].

During its life, some restoration and maintenance interventions were carried out in order to fix the damages caused by lightning and fires. However, the tower seems to have stood up well over the centuries, even against a disastrous earthquake that hit Siena in 1798, which did not cause substantial damages to the structure.

4. The architectural survey

The survey of the tower was carried out through Terrestrial Laser Scanning (TLS) techniques. Three-dimensional scanning techniques combine accuracy and sampling density. This allows the creation of a three-dimensional database from which the needed information can be extracted [14–16].

The tower’s geometry and its position within the historical center required special attentions. The most critical aspect is related to the tower’s height and to the non-availability of enough elevated accessible places from which data could be acquired. The tower’s sides overlooking the market square and the City Hall are within the buildings up to about 20 m in height while the visible part is about 68 m high. The other two tower’s sides are outside the buildings and they are about 88 m high (towards Piazza del Campo and via Salicotto). These conditions led to use two different instruments: a phase-based scanner (Leica HDS6000) for the lower sides and a pulse-based scanner (RIEGL LMS-Z420i) for the taller ones (Fig. 1). The range of the first is not enough to reach the top of the higher sides, but the later has lower resolution. The phase-based scanner has a nominal accuracy on positioning of $\pm$ 10 mm (up to 50 m distance, 1sigma) and sampling density of about 1.5 cm (at 100 m distance). The pulse-based scanner has a nominal accuracy on positioning ranging from $\pm$ 5 mm to $\pm$ 10 mm ($\pm$ 20 ppm up to 100 m distance) while the sampling density is about 5 cm (at 100 m distance).

Data have been referred to a local reference system determined by a topographic network. The alignment was solved at first with an automatic procedure using topographic targets as reference points. After that, an ICP algorithm was performed to add constraints between scans couples. The root mean square error of the alignment is lower than 1 cm. The overlapping percentage between point clouds is dependent on the scan positions, as we can identify two groups of point clouds–the one in the square and the one on the reverse side of the tower. The overlapping percentage is about 70% between scans belonging to the same group. We made this choice in order to guarantee the completeness of data and to reduce lacks of information. The overlapping between the two data sets is whereas about 10%. This difference is due to the conformation of the site.

A first analysis of the fronts showed that the exterior surfaces are out of plumb and the tower is tilted towards North. These preliminary results had led to perform more detailed analyses.

From the 3D points model of the tower’s upper part — located on top of the building — ten horizontal sections were extracted from every 4 m vertical distance. Once imported into a CAD-software, maintaining their referencing, the geometric center of each one was positioned, in order to observe its relative displacement between different levels. The obtained graph is shown on Fig. 2. The detected out-of-plumb, although interesting from a geometrical point of view, is very small: 18 cm in the North/East-South/West direction, 8 cm in the North/West-South/East direction. This corresponds to about 0.31° that, taking into account the aim of the structural analyses herein developed, can be considered as not significant. Nevertheless the possibility to have instruments that expeditiously and precisely allow for the correct evaluation of the actual tilt of such typology of structures can be very important in those cases where it becomes significant from a structural point of view.

5. The interferometric radar measurements

In order to detect the dynamic characteristics of the tower under environmental loads (mainly a light wind as downtown is closed to vehicular traffic), a radar survey was planned in May 2012 to record ambient vibrations. The measurements were carried out by placing the radar in Piazza del Campo, facing the Mangia’s Tower, at the three positions indicated with the letters A, B, C in Fig. 3. The aim of the measurement is to observe the movement of the structure at several heights, along both the direction orthogonal to the faces of the structure and one of the two diagonals. Each measurement position provides information about a projection of the real movement and by combining all information an estimate of the mode shape and the direction of the movement at each resonance frequency can be obtained. As measurements from different points of view were carried out at different times, possible changes on the environmental conditions (noise sources, weather conditions,

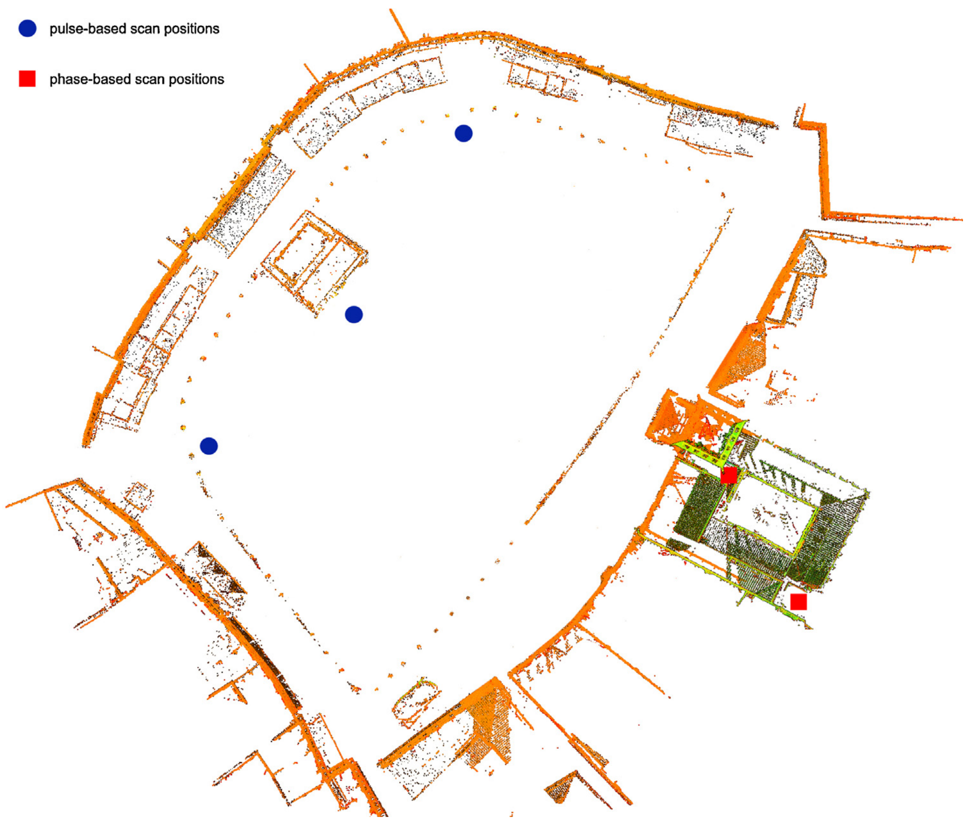

Fig. 1. Top view of the 3D point model showing the TLS scans scheme.

etc.) could occur. These affect the amplitude and phase of detected modes, but not their shape and frequency that are rather insensitive to the environmental conditions. The tower, with a rough surface and several corbels, arches and cornices at the top, easily reflects the radar signal all over its height. Indeed, rarely an architectonic structure does not provide several radar targets (just as a example, also in the very smooth surface of the steel pillar of a wind turbine, the solders of discontinuities are able give a detectable signal [18]). In Fig. 4 there is a visible example of the radar profile, that is the amplitude of the microwave signal backscattered by the tower as function of the height, obtained from the position A. The pixels of the radar image used for the following modal analysis are highlighted with vertical dotted lines.

For long-time measurements the wind turbulence stimulus can be compared to a white noise for low frequencies (from 0 to few Hz), the range where the resonance frequencies of large structures take place. Therefore the period for each measurement was assumed longer than 30 minutes.

During the measurements, the wind intensity was quite low and the tower moves in the order of magnitude of few tenths of millimetre, with amplitude obviously increasing with the height.

A spectral estimation of the displacements measured at the selected heights was performed by using the Welch modified periodogram, with 100 s window 60 s superposed. In particular the window used is a Kaiser with 4.68 as β coefficient. The starting resolution due to the length of the window (100 seconds) is 0.02 Hz (the width of the main lobe of the sinc function after the FFT) and it becomes 0.036 Hz using the Kaiser window. The power spectral density (PSD) is also interpolated by using a zero-padding before the FFT calculation.

The PSD of signals recorded from position A shows two resonant frequencies all over the height of the structure, corresponding to 0.35 Hz and 0.39 Hz, while only for the sections above 70 m an additional peak at about 1.91 Hz appears from the noise floor (Fig. 5).

Measurements from position B point out the same resonance frequencies of 0.35 Hz and 0.39 Hz, whereas in this case the highest frequency is 1.80 Hz. From position C only one resonance frequency is visible at 0.35 Hz (the 0.39 Hz peak does not appear in the PSD graph).

By means of the Frequency Domain Decomposition (FDD) method, a mode shape of each resonant frequency was estimated. The FDD provides a mode shape estimation by an output-only identification procedure. The only one assumption for the input exciting the structure is that it must be similar to a white noise. As explained above, this is true for ambient excitation like wind at low frequencies.

Focusing on the measurement carried out from position A, the mode shape for the resonance frequency of 0.35 Hz and 0.39 Hz are shown in Fig. 6a and Fig. 6b respectively. Both the resonant frequencies show the typical shape of the first order vertical bending mode. The same shape is obtained from the measurements from positions B and C. About the resonant frequency at 1.80 Hz and 1.91 Hz, the amplitude of the signal over the noise floor is too much low. The noise affects heavily the calculated mode shape, which does not offer significant information.

Resuming, the radar measurements bring to the following conclusions:

- the tower has got at least four resonance frequencies (0.35 Hz, 0.39 Hz, 1.80 Hz, 1.91 Hz);
- the tower at the two lower frequencies moves with a vertical bending shape mainly along the diagonals of its base and the two directions of the movement are approximately orthogonal. This can be argued by considering that the two lower frequencies are

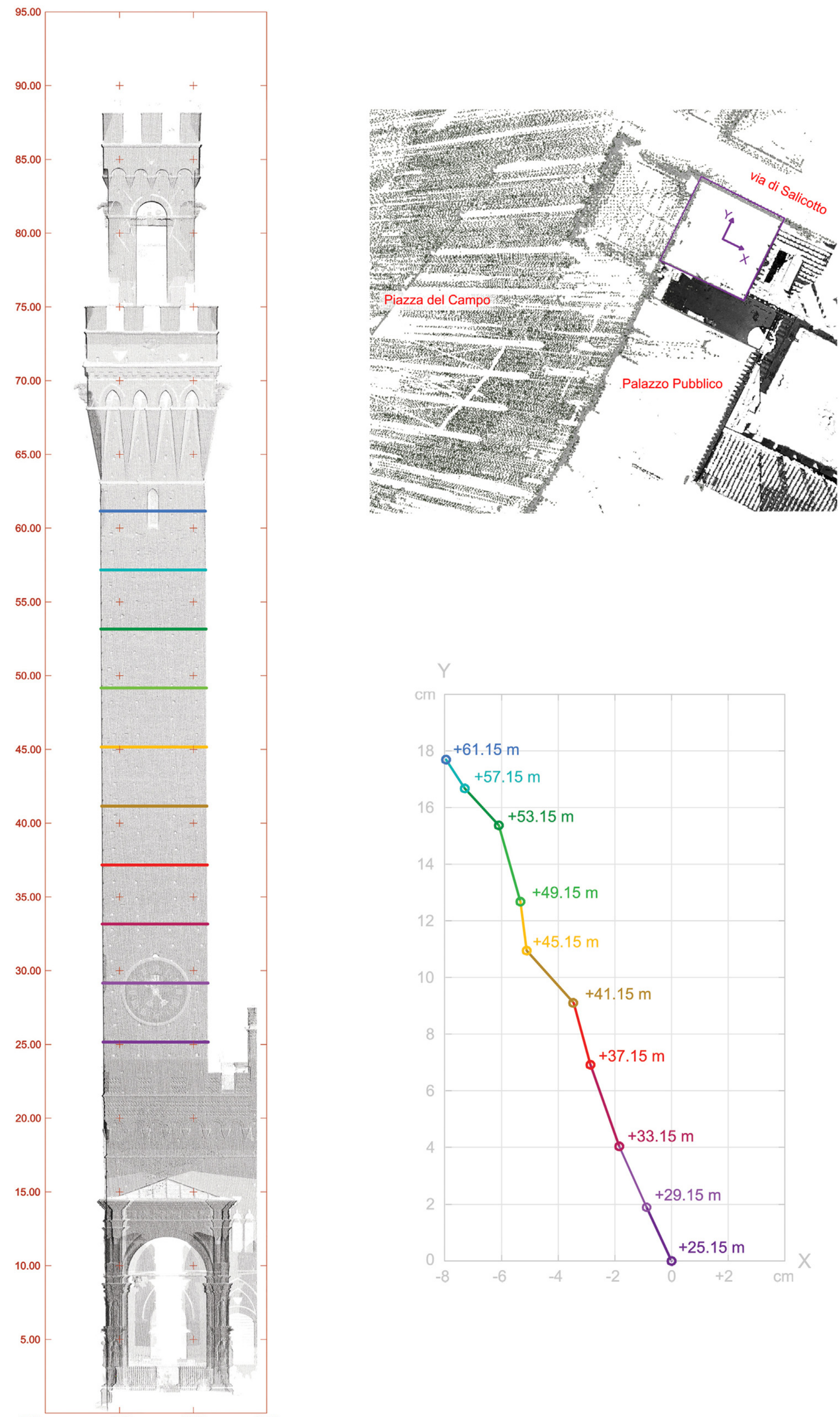

Fig. 2. On the left: scheme of the 10 horizontal sections, drawn on the front facing Piazza del Campo. On the top right, plan with the local reference system chosen to analyze the relative displacements of the geometric center. On the lower right the graph of displacements of the geometric center with height of the sections at different levels.

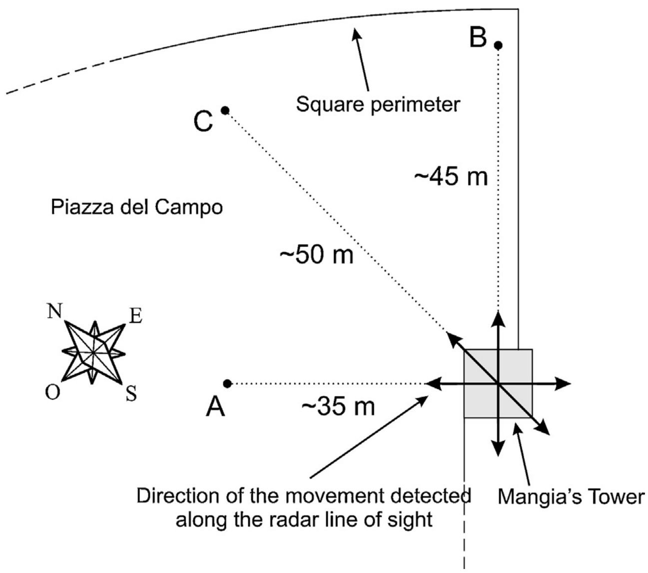

Fig. 3. Sketch of the experimental measurements in Piazza del Campo: the radar was installed at the positions A, B, and C, in order to detect the projections of the Tower movement indicated with black arrows.

visible from both the positions A and B, whereas only one of them is visible from the position C.

6. The numerical modeling

The experimental measurements allowed evaluating the main frequencies of the tower. To validate the physical meaning of these frequencies, a numerical model of the tower was built on the basis of the 3D laser scanner survey by means of commercial CAD software. TLS data redundancy and reduction methods to provide a simplified numerical model with an adequate level of detail is one of the issues on which the international scientific community is focusing, proposing some interesting results [17]. We are concentrating on this aspect in order to identify a semi-automatic procedure of model's simplification. Since the 3D point model obtained with TLS techniques was very dense, about 4 million points on the tower's external surfaces, we extracted the primitives to built the 3D model to be processed in the commercial FE code ANSYS.

Since TLS techniques allow for expedite surveying of only the external surface of the tower, size and dimensions of the internal part of the tower (together with the wall thickness, measured in correspondence of the openings) were directly measured during an inspection with traditional topographic technique. It must be pointed out that, due to both the relatively simple geometry of the structure (a tower) and the aim of the modeling herein developed, only few data were required to complete the survey.

To build the model three dimensional isoparametric solid elements were used, which reproduces as accurately as possible (i.e., in comparison with the available geometrical data: walls, for instance, that are of multi-layer type — two facing masonry walls with internal filling whose thickness is not known — were modeled through a macro-model approach) the geometry of the structure. A specific attention was paid to the upper part of the Tower (Fig. 7) where the corbels were modeled in detail thanks to the results of the laser scanner survey. Specific attention was also paid to model the boundary condition since the Tower at lower level is incorporated in the neighbor "Palazzo Comunale" (Town Hall).

The final 3D model consisted of 33,691 joints, 69,647 3D solid eight-node elements (main masonry walls), corresponding to 99,243 degrees of freedom. Apart from the effects of the neighboring buildings at the lower level (that were modeled as linear springs), the structure was modeled by assuming fixed restraints

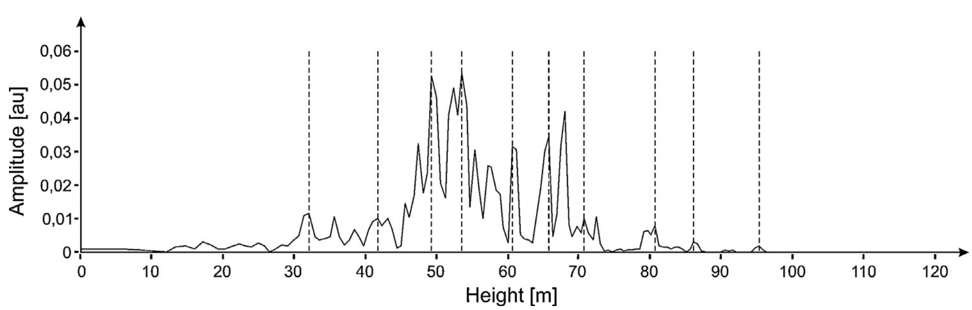

Fig. 4. Radar profile obtained from the position A, with the pixel selected for the modal analysis.

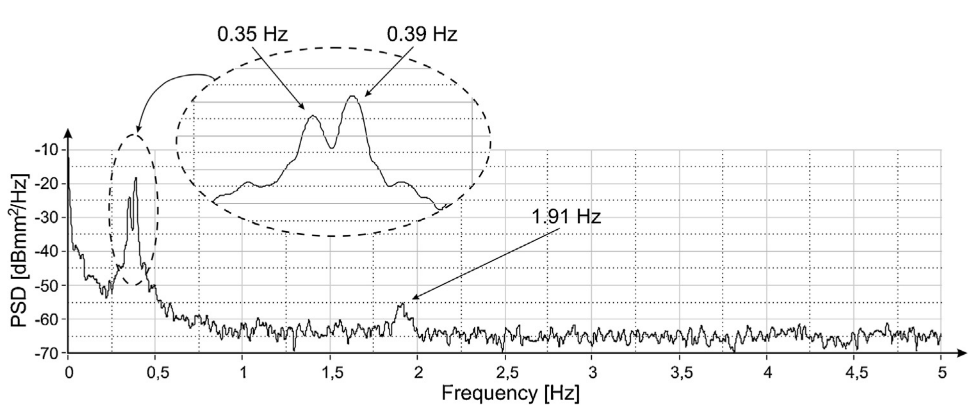

Fig. 5. PSD vs frequency of the displacement measured at 80 m height with the radar at the position A; three resonant frequencies are visible at 0.35 Hz, 0.39 Hz and 1.91 Hz.

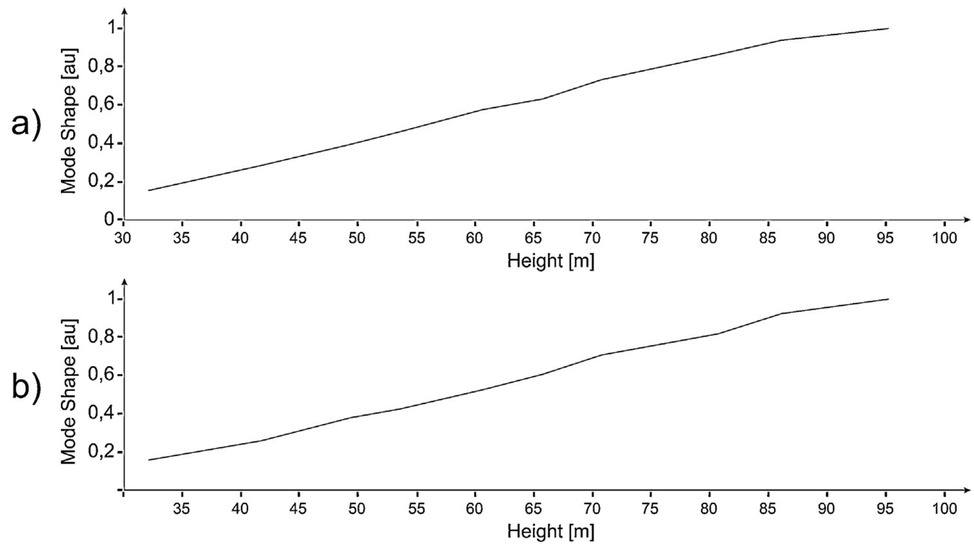

Fig. 6. Mode shapes of the tower related to the measurement of the radar at the position A. The mode shape a) corresponds to the resonance frequency of 0.35 Hz and mode shape and b) corresponds to the resonance frequency of 0.39 Hz.

at the base as the soil-structure interaction was considered not significant for the analysis herein developed.

Mechanical parameters of the numerical model (mainly the elastic modulus) were parametrically identified in order to compare the dynamic simulation with the results of the dynamic measurement experimental campaign according to a procedure also adopted by other authors [19,20] but based on the results of a traditional investigation (i.e. full topographic survey and comparison with the results of accelerometer acquisitions). In particular, to take into account the dynamic effects, the elastic modulus of the masonry materials was varied within an admissible range until the first experimental frequency was reproduced. It is noteworthy to outline that, due to large number of uncertainties affecting the material parameters (mainly their spatial distribution and the masonry multi-leaf internal texture), the assessment procedure herein followed was mainly devoted to match the first experimental modal frequency.

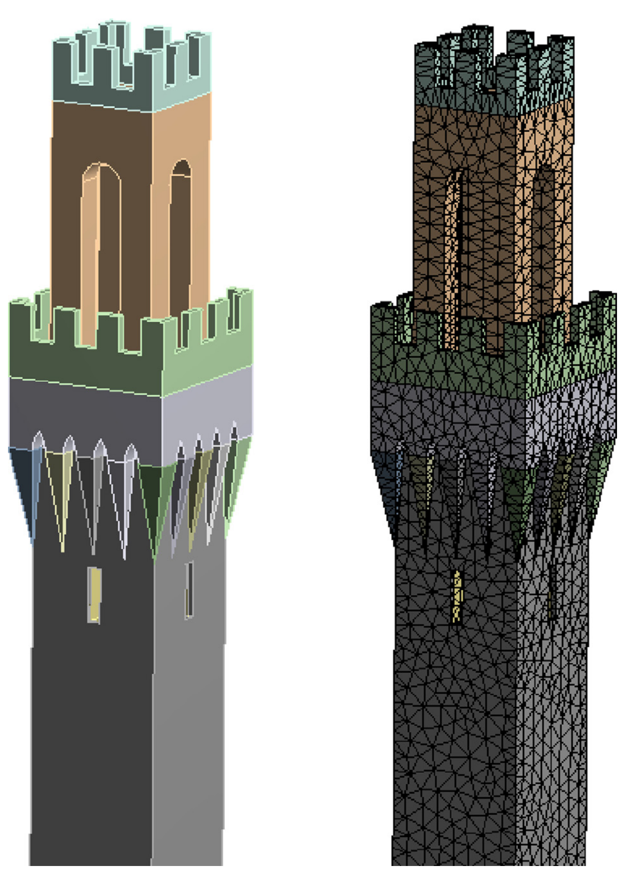

Fig. 7. Detail of the upper part of the numerical model of the Tower: volume representation (left); mesh detail (right).

Based on this approach the mean elastic modulus was estimated about 2400 N/mm^2, which is closer to values already utilized in other works [5] [21]. The Poisson's ratio was assumed equal to 0.2; however by varying this value to 0 or 0.49 no change appeared in the mode shape, and the corresponding frequency variation was less than 1%, well below the uncertainty level. Specific weight of the material (masonry brick) was assumed equal to about 1500 kg/m^3. Fig. 8 shows the first three numerical modal shaped of the Tower. The first two are bending mode along (approximately) the two main directions of the tower. The third is again a bending one, but is a superior mode. The first bending mode involves translation mainly in the Y-direction (direction North/East-South/West in Fig. 2) while the second one involves translation in the X-direction (direction North/West-South/East in Fig. 2) with a component of translation also in the Y direction as shown in Fig. 9, where are plotted the results as obtained (output) with the numerical model of the tower.

Despite the symmetry of the section of the tower (the section is approximately a square section whose dimension are about 7.0 × 7.1 m with the sustaining walls, with an even thickness of about 2.2 m) the slight asymmetries introduced by the connection with the neighbor Town Hall originates a small difference in the first two frequencies. Numerical frequencies are reported in Table 1 where they are compared with the experimental ones. It is possible to observe that the numerical model substantially confirms the experimental values with respect to the first, third and fourth frequency but it remains uncertain the assessment of the second frequency, experimentally positioned at 0.39 Hz. This difference may be due to different factors, but mainly:

- to the uncertainties in the modeling, in particular with respect to the actual extent and location of the constraints offered by the adjacent Town Hall;

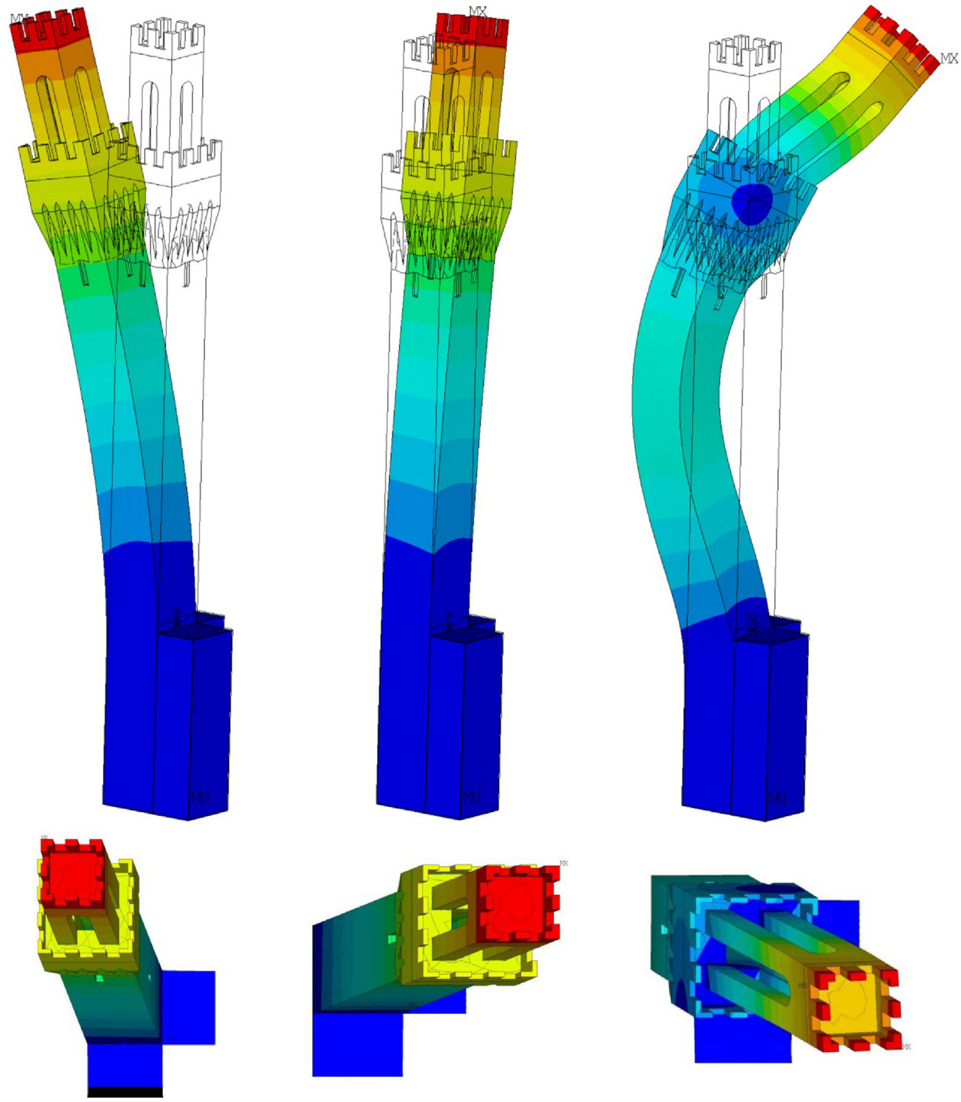

Fig. 8. Perspective and zenithal view of the first three main modal shapes of the Tower (different colours refer to different displacement amplitudes).

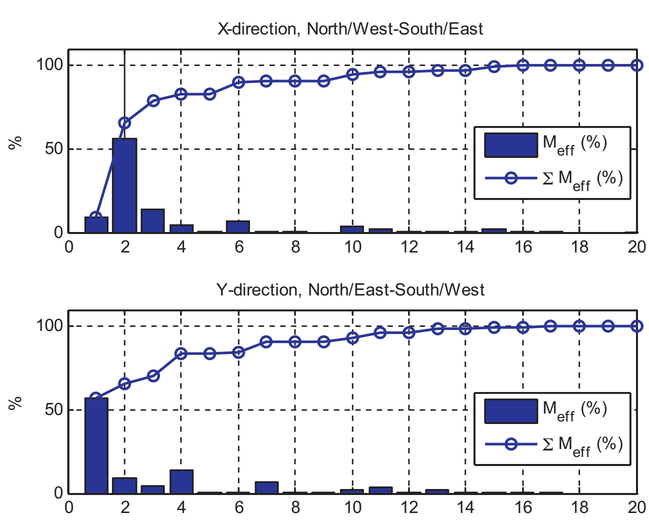

Fig. 9. Modal effective and cumulative masses for X (North/West-South/East) and Y (North/East-South/West) direction.

Table 1
Experimental and numerical frequencies.

Modal shape no.	Experimental Frequency (Hz)	Numerical Frequency (Hz)	δ (%)
f_1 (dir. NE–SW)	0.35	0.355	−1.43
f_2 (dir. NW–SE)	0.39	0.366	6.15
f_3 (dir. NE–SW)	1.80	1.883	−4.61
f_4 (dir. NW–SE)	1.91	1.912	−0.10
Modal shape no.	Experimental results (−)	Numerical results (−)	δ (%)
f_2/f_1	1.114	1.031	7.45
f_4/f_3	1.061	1.015	4.34
f_3/f_1	5.143	5.304	−3.13
f_4/f_2	4.897	5.224	−6.68

- to a different non-uniform spatial distribution of the mechanical parameters of the materials constituting the Tower.

7. Conclusions

The paper has presented the results of a multidisciplinary approach composed of laser scanner survey, radar interferometric monitoring and F.E. numerical modeling aimed at testing and proposing expeditious methodologies (i.e. without direct contact with the masonry building) to evaluate the main structural characteristics of historic masonry buildings. Such an approach is needed for planning interventions on territorial scale and, for instance, for the subsequent seismic assessment. The approach was applied to an emblematic case study of great historical interest: the Mangia's tower in Siena (Italy). After having acquired the geometry of the structure through the laser scanner technique, a F.E. model of the tower was built. Subsequently the numerical model was identified according to the results of the experimental investigation performed by means of an interferometric radar, that is able to remotely detect the dynamic behavior of the tower, subject to compliance with certain geometric and environmental constraints. The tuned F.E. model was used to reproduce the dynamic behavior of the tower and to evaluate the physical evidence of the experimental results. After tuning, the model is a good candidate to be used in subsequent investigations to assess the tower's structural behavior under severe loading (it must be however pointed out that such assessment requires further targeted experimental investigations aimed at both evaluating structural details and characterizing nonlinear mechanical behavior of masonry texture) or, in case of dynamic investigations repeated over time, to be used for structural health monitoring. The procedure discussed in this article (where TLS and interferometric radar are used as inputs for the subsequent numerical modelling) is intended as a tool for expeditious and no-contact dynamic identification of monumental towers to be employed on territorial scale.

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