A Fuzzy Hierarchical Attributed Graph Approach for Handwritten Egyptian Hieroglyphs description and matching.

Denis Arrivault
Laboratoire SIC - CNRS - FRE 2731
SP2MI - BP30179
F-86 962 Futuroscope Cedex, France
denis.arrivault@sic.sp2mi.univ-poitiers.fr

Noël Richard
Laboratoire SIC - CNRS - FRE 2731
SP2MI - BP30179
F-86 962 Futuroscope Cedex, France
noel.richard@sic.sp2mi.univ-poitiers.fr

Philippe Bouyer
RC-SOFT
Domaine de la Combe - BP39
F-16 710 Saint-Yriex, France
philippe.bouyer@rcsoft.fr

Abstract

For a complex writting as egyptian hieroglyphs, combining the works done in hierarchical modelizations and fuzzy grammar definitions seems natural. This paper introduce the hierarchical-fuzzy-attributed graph (FHAG), extended from fuzzy-attributed graph, which modelize attributes by fuzzy-tree grammar. We give a formal definition of FHAGs and introduce adapted measurements for matching them. FHAG is then integrated in a recognition system based on single models comparisons for handwritten egyptian hieroglyphs.
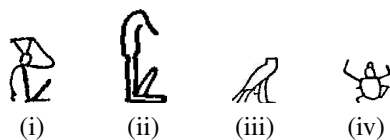
1. Introduction

Figure 1. Handwritten Hieroglyphs Examples.

The handwritten character recognition problem is studying since many decades [11]. In general, the character recognition systems consist of two steps : features extraction and classification of the feature vectors into a number of class. Then the classification can be done reffering to a learning base if such a base exists or reffering to single models if it does not. The models can furthermore come from an expert or from a clustering phase of prototypes (see [2] for exemple).

In ancient egyptian language, the number of hieroglyphs grew to approximately 6,000. As a matter of fact, having a robust learning base for recognition will take a long time. A handwritting hieroglyphic recognition system should work in a model-recognition phase at least till a robust learning base is built.

Starting from this point of view, we are presenting in this paper a new approach for modelling and recognizing handwritten characters based on single models comparisons. According to the complexity of egyptian hieroglyphs and the huge variations between models and characters to recognize (figure 1), a fuzzy structural description has been adopted.

Introduced by L.A.Zadeh ([15]), the fuzzy logic is a very powerfull tool for describing uncertainty, ambiguity and vagueness. Moreover structural approaches for pattern recognition have improved pattern descriptions by introducing topological and contextual informations. That is why combining structural technics and fuzzy logic became natural. We can cite Chan & Cheung ([3]) who first used Fuzzy-Attributed Graphs (FAG) for chinese character recognition. An other very interesting work on character description by combining fuzzy and structural informations is the one made by Malaviya & Peters ([8], [9]). They built a complete fuzzy language for the syntactic description of on-line handwritten symbols. The characters are decomposed into semantic features and fuzzy logic techniques are used to describe their syntactic relations. The power of such a lan-

guage lies in the possibility to fine tune uncertainty. Nevertheless such a description is limited to on-line writings (temporal aspects) or at least simple characters.

Our approach, presented in this article, is to consider an egyptian hieroglyph as a set of pattern primitives. Each primitive is described not only by classical fuzzy attributes but also, following the idea of Malaviya & Peters, by linguistic fuzzy trees. Then a FAG is built with primitives as vertices and relations between them as arcs. A similarity measurement between the linguistic fuzzy trees is used for performing an inexact graph matching between FAGs, and a global degree of matching gives a recognition posssibility. This paper is structured as follows. At the beginning, definitions of FAGs and extended Fuzzy Hierarchical Attributed Graphs (FHAG) are given. Then our character decompositions and primitive descriptions are explained in a second section. In the third section we present the similarity measurements and the matching process. And finally we give some results and conclusions.

2. Fuzzy Hierarchical Attributed Graph

2.1. Attributed Graphs and Fuzzy Attributed Graphs

Attributed graph (AG) was introduced by Tsai and Fu for pattern analysis [14]. The vertices of the graph represent pattern primitives describing the pattern while the arcs are the relations between these primitives. A FAG can be defined as a generalization of the AG since a crisp set can always be represented as a special case of a fuzzy set ([3]).

Each vertex may take attributes from the set $Z = \{z_i | i = 1, ..., I\}$. For each attribute, z_i will have possible values taken from the set $S_i = \{s_{ij} | j = 1, ..., J_i\}$. $\tilde{L}_v = \{(z_i, \tilde{A}_{S_i}) | i = 1, ..., I\}$ denotes the set of all possible attribute-value pairs of vertices where $\tilde{A}_{S_i}$ is a fuzzy set on the attribute-value set S_i. A valid pattern primitive is then a subset of $\tilde{L}_v$ in which each attribute appears only once, and $\tilde{\Pi}$ is the set of all those valid pattern primitives. Thus each vertex will be represented by an element of $\tilde{\Pi}$.

Similarly, for the arcs, the attribute set is denoted by $F = \{f_i | i = 1, ..., I'\}$ in which each f_i may take values in $T_i = \{t_{ij} | j = 1, ..., J_i'\}$. $\tilde{L}_a = \{(f_i, \tilde{B}_{T_i}) | i = 1, ..., I'\}$ denotes the set of possible relational attribute-value pairs, with $\tilde{B}_{T_i}$ as a fuzzy set on the relational-attribute value set T_i. A valid relation is a subset of $\tilde{L}_a$ in which each attribute appears only once. The set of all valid relation is denoted $\tilde{\Theta}$.

Definition 1 : *A fuzzy attributed graph $\tilde{G}$ over $\tilde{L} = \{\tilde{L}_v, \tilde{L}_a\}$ with an underlaying graph structure $H = (N, E)$, is defined to be an ordered pair $(\tilde{V}, \tilde{A})$ where $\tilde{V} = (N, \tilde{\sigma})$ is called a fuzzy vertex set and $\tilde{A} = (E, \tilde{\delta})$ is called a fuzzy arc set. The mappings $\tilde{\sigma} : N \to \tilde{\Pi}$ and*

$\tilde{\delta} : E \to \tilde{\Theta}$ *are called fuzzy vertex and fuzzy arc interpreters, respectively.*

The pattern primitives used by Chan & Cheung for describing chinese characters are strokes. For egyptian hieroglyphs, the primitives are more complex and the attributes have to be more elaborate.

2.2. Fuzzy Hierarchical Attributed Graphs

In our primitive descriptions we introduce a hierarchy and attributes become trees. In figure 2 (a) the primitive attribute "TYPE" of objects 1 and 2 can take values in $\tilde{A_{S_1}} = \{circle, ellipse\}$ or $\tilde{A'_{S_1}} = \{circle, left\_ellipse, right\_ellipse\}$. For an object comparison both value sets will give completly different results. Actually the first description will give objects 1 and 2 similar, but the second description will give them different. A more intuitive description can be given by a linguistic fuzzy tree as shown in figure 2 (b).
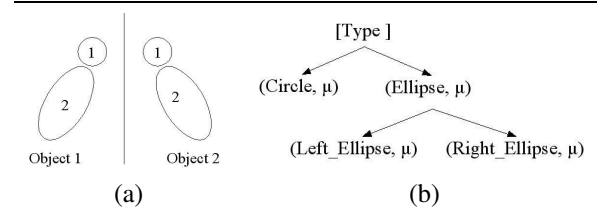

Figure 2. Linguistic Fuzzy Tree illustration

A linguistic fuzzy tree can be performed as a sample of a fuzzy tree grammar ([5],[13]).

Definition 2 *:*

1. *A tree t is a set of nodes satisfying :*
    (i) a unique node is called the root of tree t;
    (ii) other nodes are divided into disadjoining sets $t_1, ..., t_n$ where t_i is called a subtree of tree t.

2. *A fuzzy tree $\tilde{t}$ is $\tilde{t} = (t, \mu)$, where t is a tree and $\mu \in [0, 1]$.*

Definition 3 *:*

1. *A fuzzy tree grammar is a 4-tuple $G_t = (V_N, V_T, P, S)$, where*
    (i) V_N is a finite set of nonterminals;
    (ii) V_T is a finite set of terminals;
    (iii) P is a set of productions of the form : $t_i \xrightarrow{\mu} t_j$, where t_i, t_j are trees, $\mu \in [0, 1]$ is called the membership of the production.
    (iv) $S \in V_N$ is a starting symbol.

2. *The Language $L(G_t)$ generated by G_t is defined as $L(G_t) = \{(t, \mu) | S \xRightarrow{*} t\}$.*

Then for FHAG, each vertex may take hierarchical attributes from the set $\hat{Z} = \{\hat{z}_i | i = 1, ..., I\}$. For each hierachical attribute, $\hat{z}_i$ will have possible samples taken from the set $G_{v_i} = \{\hat{s_{ij}} | j = 1, ..., J_i\}$, where G_{v_i} is a fuzzy tree grammar and $\hat{s_{ij}}$ are linguistic fuzzy trees. $\hat{L}_v = \{(\hat{z}_i, \hat{s_{ij}}) | i = 1, ..., I; j = 1, ..., J_i\}$ denotes the set of possible attribute-linguistic_fuzzy_tree value pairs of vertices. A valid pattern primitive is just a subset of $\hat{L}_v$ in which each attribute appears only once, and $\hat{\Pi}$ is the set of all those valid pattern primitives. Thus each vertex will be represented by an element of $\hat{\Pi}$.

Similarly, we define $\hat{F} = \{\hat{f}_i | i = 1, ..., I'\}$, $G_{a_i} = \{\hat{t_{ij}} | j = 1, ..., J'_i\}$, $\hat{L}_a = \{(\hat{f}_i, \hat{t_{ij}}) | i = 1, ..., I'; j = 1, ..., J'_i\}$ and $\hat{\Theta}_i$ for the arcs. And finaly :

Definition 4 *: A fuzzy hierarchical attributed graph $\hat{G}$ over $\hat{L} = \{\hat{L}_v, \hat{L}_a\}$ with an underlaying graph structure $H = (N, E)$, is defined to be an ordered pair $(\hat{V}, \hat{A})$ where $\hat{V} = (N, \hat{\sigma})$ is called a hierarchical fuzzy vertex set and $\hat{A} = (E, \hat{\delta})$ is called a hierarchical fuzzy arc set. The mappings $\hat{\sigma} : N \rightarrow \hat{\Pi}$ and $\hat{\delta} : E \rightarrow \hat{\Theta}$ are called hierarchical fuzzy vertex and hierarchical fuzzy arc interpreters, respectively.*

3. Character Decomposition and FHAG Construction

The decomposition of a character into a FHAG is made in two steps. First the character is skeletonized and singular (intersections and end points) and inflexion points are extracted. Then the primitives are selected and the FHAG is built.

3.1. Skeletonisation and points extraction

A skeleton is a synthetic representation of a shape ([1]) set up with unit-thickness strokes. Among the numerous skeletonization methods ([7]), only the strictly 8-connected results are interesting for a fast and simple forward computing. We choosed the algorithm proposed by Zhang and Wang [16] because of its good properties and speed.

The singular points (end points and intersection points) are extracted with simple morphological "hit or miss" transforms ([12]). The inflexion points are calculated with a Bezier parametrization.

3.2. FHAG Construction

The FHAG construction principle is exposed with less details about attribute calculation which is not the main topic of this article. We focus on the hierarchical attributes definitions.

3.2.1. Vertices Hierarchical Attributes Calculation
Each vertex of the FHAG stands for a primitive and is associated with two hierarchical attributes. The first one, called V_TYPE, details the type of the primitive. The second one, called V_LENGTH, details its length.

The fuzzy tree grammar of V_TYPE is outlined in figure 3, the terminals set $V_T = \{$Stroke (ST), Simple Curve (SC), Complex Curve (CC), Positive Stroke (PS), Negative Stroke (NS), Vertical Stroke (VS), Horizontal Stroke (HS), C-like Curve (C), D-like Curve (D), A-like Curve (A), U-like Curve (U), Loop(LOO), Others (OTH), Circle (CIR), Ellipse (ELL), Positive Ellipse (PE), Negative Ellipse (NE), Vertical Ellipse (VE), Horizontal Ellipse $(HE)\}$.
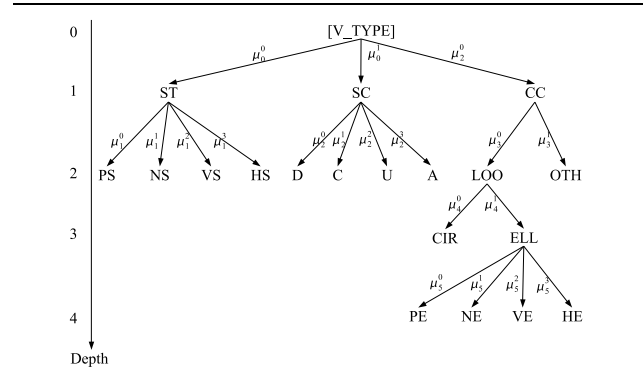

Figure 3. V_TYPE representation.

The μ_0^i are calculated with a fuzzification of arcness$=\sqrt{(1 - d/l)}$, where d is the distance between the end points of the primitive and l is its length. Then, the μ_1^i are obtained with a fuzzification of the slope function. For the μ_2^i, we use the slope of the stroke defined by the mean point of the curve and the middle of end points. The fuzzification is made on its slope. The μ_3^i come from a fuzzification of the correlation between the curve and its first order Fourier reconstruction. The μ_4^i are simply extracted from the fuzzification of the ratio between the big and the little axis of the shape. And finally, the fuzzification of the big axis slope gives the μ_5^i.

V_LENGTH is a 1-depth attribute which describes the $length/total\_length$ ratio where the total length is the sum of the lengthes of all the primitives of the graph. The description is made by using the linguistic terms of Malaviya and Peters ([8]) and the terminals set $V_T =\{$ Zero Z, Very Very Low VVL, Very Low L, Low L, Medium M, High H, Very High VH, Very Very High VVH, Excellent $E\}$. Its tree representation is given in figure 4-(a).

3.2.2. Arcs Hierarchical Attributes Calculation The arcs describe the relations between primitives. For a bet-

ter topological representation, we use oriented arcs. As a matter of fact, the relation between two primitives is always decsribed by a pair of arcs (a and a_b with oposite orientations). Then the graph underlaying from definition 4 is overwritten into $H\prime = (N, E\prime)$ where $E\prime = E \times E_b$, E is the set of arcs from V_i to V_j, $i \leq j$, and Eb is the set of arcs from V_j to V_i. And finally AG, FAG and FHAG are called oriented AG, FAG and FHAG.

Two hierarchical attributes are associated to an arc. E_POS defines the relative position of the two primitives. E_PROX defines its proximity.

E_POS is a 1-depth attribute with $V_T =\{$ Above AB, On the left LE, Under UN, On the right $RI\}$. The tree representation is illustrated in figure 4-(b). The μ^i are calculated with the fuzzy relative position calculation of Bloch ([6]).

E_PROX is a 1-depth attribute with $V_T =\{$0-Proximity $0P$, 1-Proximity $1P$, 2-Proximity $2P$, 3-Proximity $3P\}$. The tree representation is illustrated in figure 4-(c). The proximity is the fuzzy connectivity of primitives. The μ^i are obtained by a fuzzification of the distances between end points of both primitives.
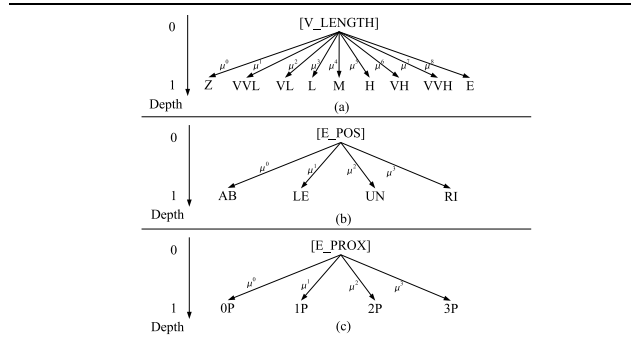

Figure 4. (a) V_LENGTH ; (b) E_POS ; (c) E_PROX

4. Fuzzy-Hierarchical-Attributed Graph Matching

The recognition problem in the FHAG approach consists of defining a degree of matching between the unknown graph $\hat{G}_x$ and some models $\hat{G}_i$. It is a classical inexact graph matching problem which requires similarity measurements between graph elements.

4.1. Degree Of Matching

Man and Poon ([10]) give a similarity measurement between two FAGs matched by a isomorphism and Chan and Cheung ([3]) defined a degree of matching γ between two monomorphic FAGs. By combining both approaches we define :

Definition 5 : α_1 is a similarity function between vertices v_1 and v_2 of two FAGs $\tilde{G}_1$ and $\tilde{G}_2$. If $\tilde{A_{1S_i}}$ is the fuzzy subset that gives the attribute value for z_i of v_1 and $\tilde{A_{2S_i}}$ for z_i of v_2, α_1 can be defined as in eq.1.

$$\alpha_1(v_1, v_2) = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J_i}(\mu_{\tilde{A_{1S_i}}}(s_{ij}) \wedge \mu_{\tilde{A_{2S_i}}}(s_{ij}))}{\sum_{i=1}^{I}\sum_{j=1}^{J_i}(\mu_{\tilde{A_{1S_i}}}(s_{ij}) \vee \mu_{\tilde{A_{2S_i}}}(s_{ij}))} \quad (1)$$

Definition 6 : α_2 is a similarity function between arcs a_1 and a_2 of two FAGs $\tilde{G}_1$ and $\tilde{G}_2$. If $\tilde{B_{1T_i}}$ is the fuzzy subset that gives the attribute value for f_i of a_1 and $\tilde{B_{2T_i}}$ for f_i of a_2, α_2 can be defined as in eq.2.

$$\alpha_2(a_1, a_2) = \frac{\sum_{i=1}^{I\prime}\sum_{j=1}^{J\prime_i}(\mu_{\tilde{B_{1T_i}}}(t_{ij}) \wedge \mu_{\tilde{B_{2T_i}}}(t_{ij}))}{\sum_{i=1}^{I\prime}\sum_{j=1}^{J\prime_i}(\mu_{\tilde{B_{1T_i}}}(t_{ij}) \vee \mu_{\tilde{B_{2T_i}}}(t_{ij}))} \quad (2)$$

For oriented FAGs, a similarity between pairs of arcs becomes :

Definition 7 : $\alpha_2\prime$ is a similarity function between couples of arcs $P_{a_1} = (a_1, a_{1b})$ and $P_{a_2} = (a_2, a_{2b})$ of two oriented FAGs $\tilde{G}_1$ and $\tilde{G}_2$. It can be defined as in eq.3

$$\alpha_2\prime(P_{a_1}, P_{a_2})) = \quad Max \{\alpha_2(a_1, a_2), \alpha_2(a_1, a_{2b}), \\ \alpha_2(a_{1b}, a_2), \alpha_2(a_{1b}, a_{2b})\} \quad (3)$$

As the degree of matching defined by Man and Poon is too restrictive for our complex structures, we define a γ adapted for oriented FAGs as :

Definition 8 : Let $\tilde{G}_1$ and $\tilde{G}_2$ be two FAG with underlying graphs $H_1\prime = (N_1, E_1\prime)$ monomorphic to $H_2\prime = (N_2, E_2\prime)$ by h. Then a degree of matching is eq.4 with $n_v = \min\{N(N_1), N(N_2)\}$ and $n_e = \min\{N(E_1), N(E_2)\}$.

$$\gamma(\tilde{G}_1, \tilde{G}_2) = \quad \frac{1}{n_v \times n_e} \sum_{i \in N_1} \alpha_1(i, h(i)) \sum_{(i,j) \in E_1\prime} \\ \alpha_2\prime(P_a(i, j), P_a(h(i), h(j))) \quad (4)$$

If a depth d is given, a set of fuzzy attributes can be extracted from a fuzzy hierarchical attribute. For example, for V_TYPE with $d = 2$, the extracted set of

fuzzy attributes is $\{(ST|PS, (\mu_1^0 \wedge \mu_0^1)), (ST|NS, (\mu_1^1 \wedge \mu_0^1))\}, (ST|VS, (\mu_1^0 \wedge \mu_0^1))...\}$. Then the degree of matching defined in definition 8 can be used for FHAGs.

4.2. Matching and recognition

Finding homomorphism between two FHAGs is a classical inexact matching problem. Many algorithms were proposed for solving it. Gold & Rangarajan [4] designed a fast algorithm which directly calculates the matching matrix by a graduated assignment. The assignment is made by minimizing an energy function. The low order computationnal complexity ($O(l, m)$, where l and m are the number of arcs in the two graphs), and the robustness in the presence of noise are a main advantage for this approach.

The recognition of a FHAG $\hat{G}_x$ over a set of models $\hat{G}_i$ for a given depth d is made in four steps :

1. the fuzzy attributes of depth d are built for every hierarchical attributes in all the graphs.

2. α_1 and $\alpha_2\prime$ are computed for every vertices and arcs of all the graphs.

3. by using α_1 and $\alpha_2\prime$, a monomorphism between $\hat{G}_x$ and every $\hat{G}_i$ is found with the Gold & Rangarajan algorithm.

4. $\hat{G}_x$ is recognized to represent the same character than $\hat{G}_k$ if $k = \max_i(\gamma(\hat{G}_x, \hat{G}_i))$

5. Results

Some tests have been computed with a model basis of 296 handwritten hieroglyphs. Figure 5 give some examples which illustrate the interest of our approach. The models were written by an egyptologist and the unknown characters were extracted from a handwritten document made by another egyptologist. The interest of our system is its capability to classify big families ("Man", "Calvary", "Bird" in figure 5).
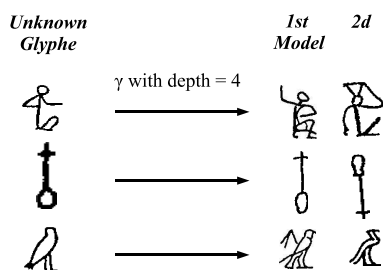

Figure 5. Results

6. Conclusions

In this paper we propose an extension to the fuzzy-attribute graph by introducing a hierarchy on the attributes. The fuzzy-tree grammars give the possibility to modelize complex structures at different depth. We also integrate the FHAG in a recognition system based on single models comparisons for handwritten egyptian hieroglyphs. The first results are very encouraging but we now have to include an iterative learning process for building a robust hieroglyph recognition system.

References

[1] H. Blum. A transformation for extracting new descriptions of shape. In *Models for the Perception of Speech and Visual Form*, pages 362–380. MIT Press, 1967.

[2] H. Bunke. Recent advances in structural pattern recognition with applications to visual form analysis. In *VF01*, page 11 ff., 2001.

[3] K. Chan and Y. Cheung. Fuzzy-attribute graph with application to chinese character recognition. *SMC*, 22(2):402–410, 1992.

[4] S. Gold and A. Rangarajan. A graduated assignment algorithm for graph matching. *PAMI*, 18(4):377–388, April 1996.

[5] R. Gonzalez and M. Thomason. *Syntactic Pattern Recognition: An Introduction*. Addison-Wesley, 1978.

[6] B. I. Fuzzy relative position between objects in image processing : a morphological approach. *PAMI*, 21(7):657–664, July 1999.

[7] L. Lam, S.-W. Lee, and C. Y. Suen. Thinning methodologies - a comprehensive survey. *PAMI*, 14(9):869–885, september 1992.

[8] A. Malaviya and L. Peters. Fuzzy feature description of handwriting patterns. *PR*, 30(10):1591–1604, October 1997.

[9] A. Malaviya and L. Peters. Fuzzy handwriting description language: Fohdel. *PR*, 33(1):119–131, January 2000.

[10] G. Man and J. Poon. A fuzzy-attributed graph approach to handwritten character recognition. In *Second IEEE int. Conf. on Fuzzy Systems*, pages 570–575, San Francisco, March 1993.

[11] R. Plamondon and S. Srihari. On-line and off-line handwriting recognition: A comprehensive survey. *PAMI*, 22(1):63–84, January 2000.

[12] J. Serra and P. Soille. *Mathematical Morphology and Its Applications to Image Processing*. Kluwer, August 1994.

[13] L. Shu. An inference method for fuzzy tree grammars. *Fuzzy Sets Syst.*, 112(1):173–176, 2000.

[14] W. Tsai and K. Fu. Subgraph error-correcting isomorphisms for syntatic pattern recognition. *SMC*, 13(1):48–62, January 1983.

[15] L. Zadeh. Fuzzy sets. *InfoControl*, 8:338–353, 1965.

[16] Y. Y. Zhang and P. S. P. Wang. A new parallel thinning methodology. *PRAI*, 8:999–1011, 1994.