International Journal of Approximate Reasoning 187 (2025) 109528

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

journal homepage: www.elsevier.com/locate/ijar

Explainable multi-criteria decision-making: A three-way decision perspective

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ARTICLE INFO

Keywords:
Multiple criteria analysis
Explainable artificial intelligence
Three-way decision theory
Symbols-meaning-value space
Trilevel thinking

ABSTRACT

This paper proposes an Explainable Multi-Criteria Decision-Making (XMCDM) framework that constructs trilevel explanations with respect to classic multi-criteria decision-making methods. The framework consists of explainable data preparation, explainable decision analysis, and explainable decision support, which integrates ideas from three-way decision and symbols-meaning-value spaces. First, we briefly introduce the key concepts at each level and list potential issues to be resolved, including gathering multi-criteria data, interpreting multi-criteria decision-making working principles, and offering effective outcome presentation. We examine existing literature that solves part of those questions and point out that rule-based explanations may be applicable and efficient to explain ranking/ordering results. Then, we discuss two methods that generate three-way rankings with respect to an individual criterion and integrate three-way rankings with multi-criteria ranking. We modify the Iterative Dichotomiser 3 algorithm to build rule-based explanations. Finally, after giving a small illustrative example, we design experiments on five real-life datasets, test explainability of three classic multi-criteria decision-making methods, and tune the thresholds. The experimental results demonstrate that our proposed framework is feasible and adaptable to various data characteristics.

1. Introduction

Multi-criteria decision making (MCDM) has been acknowledged as a valuable tool in assisting complex human decision making. The key process of MCDM involves amalgamating the diverse aspects or possibly conflicting viewpoints into a single scale of measurement. These multiple viewpoints, aspects, or perspectives are specifically treated as criteria. An MCDM technique may define a specific way to evaluate the decision alternatives under these criteria. It usually provides a ranking or an ordering of the alternatives and supports the user to select the best alternative(s) efficiently. Although MCDM techniques are powerful tools that provide decision makers with suggested decisions, the explainability and transparency remain a challenge. The recent studies on explainable artificial intelligence (XAI) argue that the complexity of AI models makes it hard for people to trust or use them effectively [7,12,15,26,27,31,45]. The necessity of explanations is emphasized because providing explanations can help users to understand, trust, and manage AI applications. The same arguments are equally applicable to MCDM systems. The lack of explainability limits trust and adoption of MCDM solutions and leads to insufficient decision support, especially in certain domains where understanding the rationale behind the recommended decisions is crucial. Motivated by the growing influence of XAI, it may be the right time to investigate the issue of explainable MCDM (XMCDM).

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https://doi.org/10.1016/j.ijar.2025.109528
Received 27 January 2025; Received in revised form 23 May 2025; Accepted 13 July 2025

Available online 18 July 2025

Research across various domains reflects and uses different understandings of the term "explainability." Erwig and Kumar [11] argued that the process of explaining multi-criteria decision-making is to understand why a certain alternative or a certain subset of alternatives would be suggested. However, the limited explanations of MCDM techniques partially meet users' requirements and lead to a gap in understanding. Human decision-makers may need more interpretable or explainable descriptions of the given alternatives in order to infer why an alternative would be recommended over another or why an alternative would be ranked higher than the remaining. Another notable obstacle is that MCDM does not offer adequate information to help us determine the evaluation of a new alternative, in other words, the knowledge obtained from MCDM methods cannot help us to make predictions about the future. While the user may know the ranking of existing alternatives, MCDM techniques do not fully facilitate the ranking of new alternatives under the same environment of multiple criteria. In order to explicitly determine the ranking of new alternatives or the rankings of a set of new alternatives, the user has to repeat the experimental process, which might consume more resources and potentially result in significant changes to the established rankings.

There have been several studies that attempt to combine explainable artificial intelligence and MCDM. Yusuf, Yang, and Panoutsos [46] worked on developing a data-driven MCDM framework that allows for both textual and graphical explanations in classification problems. They suggested that combining with AI-based methods can make an MCDM system interpretable. Yusuf [47] expanded the work to fuzzy logic and neutrosophic logic and investigated a novel category of interpretable MCDM models. Lozano et al. [25] illustrated a case study of employing fuzzy MCDM models to deal with actuarial problems. Their explainable framework integrated a fuzzy inference system with a modified MCDM method. Černevičienė and Kabašinskas [6] outlined the advantages of MCDM in the field of financial data mining and decision-making. They further pointed out that the development of hybrid MCDM with AI aligns with the principles of XAI development. These recent investigations showcase the potential of integrating XAI with MCDM.

The term of "explainability" has multiple interpretations by different authors and there is no an agreeable precise definition. This may be, in a large part, due to the fact that explainability is a multi-faceted, domain-dependent, and context-sensitive notion. Nevertheless, it is possible to examine some important aspects. For example, we may consider three necessary features of a good explanation, namely, user-oriented, domain-knowledge-based, and evidence-supported. First, the audience of an explanation is a user, and hence explanations must be user-oriented. This requires that an explanation must be given in human understandable terms and ways, prioritizing the simplicity, clarity and intuitive rationales and helping users to understand the reasoning or motivations. Second, a good explanation needs to exploit the domain knowledge and background information of various user groups. By grounding explanations on the established knowledge and principles of a particular field, one may enhance the credibility and understandability of an explanation. Third, a good explanation must be backed up by useful data and solid evidence. Evidence-supported explanations rely on real-world data and testing results, showing how an AI system derives its decisions or recommendations and why they are trustworthy. Data-driven and evidence-based explanations build the trust of users. Any conceptual model for XAI must consider all or at least some of the many facets of explainability and explanation. Yao [45] pointed out two broad classes of issues for building such a conceptual framework. One class focuses on the explanations themselves, concerning, for example, functionality of an AI system and its properties, the process of the system, and the meaning of the results from the system. The other class concerns the communication of an explanation. Based on ideas in three-way decision theory, he suggested using triadic structures for constructing and communicating explanations.

The fundamental ideas of the theory of three-way decision are thinking, problem-solving, and computing in threes [45]. They are about effective use of triadic structures [42–45]. A cognitive basis of three-way decision is our limited capacity of short-term memory, which forces us to think in a very small number of items. Our brain works well with three things. We do not need to spend much effort or time to understand, organize/prioritize, and remember them. A theory, model, or system with three main components is therefore simple to understand, easy to remember, and practical to use. As an example, we consider a model of trilevel thinking. It involves the use of top, middle, and bottom levels, representing various abstractions or understandings. The advances of trilevel thinking are clearly evident in intelligent data analysis, theory construction, and modeling. For example, we can understand an information system at the abstract theory, algorithm, and physical implementation levels. We may also understand a computer system at hardware, system software, and application levels. By integrating ideas from communications, management science, information systems, and psychology, Yao [43] introduced the notion of Symbols-Meaning-Value (SMV) spaces for trilevel thinking. He has applied the notion of SMV spaces to build concept models of data science [43], human-machine co-intelligence [44], and XAI [45].

A recent review of three-way decision [35] demonstrates the benefits of thinking in threes in addressing complex decision-making problems. In particular, many authors have made proposals to combine three-way decision and MCDM [4,13,17,18,22,24,36,38,39,48–51]. In this paper, we take a different direction of research by looking at the potentials of three-way decision for XMCDM. According to the principles of three-way decision, when constructing a conceptual model of XMCDM, we have a preference for a tripartite framework and triadic explanations. In addition, the existing studies give us the confidence to study the explainability of MCDM based on ideas of three-way decision and SMV spaces. In this paper, we propose a framework for XMCDM, aiming to solve the interpretability issues by providing a trilevel structure that aligns with the SMV spaces. The SMV space serves as a carrier of XMCDM by adding or providing explanations according to the three levels, respectively. The proposed framework integrates processes that allow human users to better comprehend and trust the results produced by MCDM. We review existing literature on SMV space and identify potential projections onto this space. The framework firstly projects the whole process of MCDM onto the SMV space and involves the trilevel explanations with the decomposition. It not only highlights the importance of explanation at each level of MCDM but also presents a transparent structure for embedding explanations correspondingly. We further introduce the concept of three-way rankings and focus on explaining the ranking result generated by a traditional MCDM method. It presents two methods for generating three-way rankings: a statistical method generating symmetric three-way rankings, and a prospect-theory based method creating asymmetric three-way rankings. By combining three-way rankings with an MCDM ranking, we can extract rules using a modified

Table 1
MCDM and SMV space (adapted from [44]).

SVM space	Weaver's communication [34]	DKW interpretation [41]	PCA trilogy [28]	MCDM
Symbols	Symbols	Data	Perception	MC data preparation
Meaning	Meaning	Knowledge	Cognition	MC decision analysis
Value	Effectiveness	Wisdom	Action	MC decision support

decision tree algorithm, demonstrating the effectiveness and efficiency of rule-based explanations. Our work provides a generalizable framework for creating XMCDM models, contributing to the advancement of rule-based explainability in decision-making contexts and multiple criteria environments.

The remainder of this paper is organized as follows. In Section 2, we propose a universal conceptual framework for XMCDM that is grounded in trilevel data analytics and symbols-meaning-value spaces. We identify the fundamental components at each level. Section 3 explores the detailed objectives at the meaning level by utilizing three-way decision methods and specific MCDM approaches. In Section 4, we develop an algorithm based on Iterative Dichotomiser 3 (ID3), and we illustrate its application by using a concrete example to generate rule-based explanations. Section 5 marks the experimental portion of our study, where we select datasets, adjust parameters, visualize the results, and analyze the performance. In the final section, we conclude our proposed XMCDM framework and suggest a number of future research opportunities.

2. A general framework of explainable multi-criteria decision-making

This section introduces the concepts of an SMV space and proposes an XMCDM framework. We consider an SMV space as the foundation and decompose MCDM at the symbols, meaning, and value levels. Within this context, we explore how trilevel explanations can be added to this framework.

2.1. The Symbols-Meaning-Value (SMV) spaces based on three-way decision

The ever-growing number of academic achievements in utilizing trilevel thinking has demonstrated its comparative advantages. From our points of view, the SMV space is a specific variant and a tangible execution of trilevel thinking. Within its structure, a problem is broken down into three levels: the bottom Symbols-level, the middle Meaning-level, and the top Value-level. An example to illustrate the wide applicability of the SMV space is the conception of human-machine co-intelligence by mapping certain existing studies onto the SMV spaces. The basic ideas of an SMV space are summarized in Table 1. According to Weaver's [34] decomposition of the communication problem, the three levels are built as the Symbols-level of technical issues, the Meaning-level of the semantic problem, and the Value-level of effectiveness of communication. The data-knowledge-wisdom (DKW) hierarchy is a trilevel reinterpretation of the DIKW hierarchy or the DIKW pyramid, in which data, information/knowledge, and wisdom correspond to hierarchical elements in decision-making. The Symbols-level focuses on data, the Meaning-level concentrates on the information/knowledge embedded in data, and the Value-level emphasizes the utilization of knowledge. Another way to map onto the SMV space is through the interpretation of the perception-cognition-action trilogy, in which perception corresponds to data collection, cognition is the meaning and understanding, and action connects to human decision-making. The last column is our new projection of MCDM onto an SMV space, corresponding to data preparation, decision analysis, and decision support.

In earlier discussions on MCDM problems, we thoroughly examined the existing studies and had a bird view of the entire decision-making process. For convenience and readability, we use MC as the short form of multi-criteria. Within the structure of the SMV space, we divide the whole MCDM process into three levels as the symbols-level of MC data preparation, the meaning level of MC decision analysis, and the value-level of MC decision support. It is important to note that these three levels grant us a general understanding of MCDM. They broadly sketch out the decision-making process under a multi-criteria environment, as per the SMV space. In detail, we may encounter a number of various tasks to be accomplished at the three levels. For example, at the symbols-level, the task of data preparation may have the following sub-tasks: (i) problem definition that aims to clearly define the final objective of decision-making; (ii) identification of decision alternatives; and (iii) selection of criteria. While, at the meaning-level, our emphasis is on analyzing the data collected at the previous level. The primary tasks might be (i) data pre-processing, (ii) selecting an appropriate MCDM method, and (iii) implementing the experiments. Following the decision analysis stage, the concentration transitions to supporting the decision-making in formulating an informed and well-scrutinized decision. There are several potential scenarios, such as (i) supplier selection: where an MCDM method may yield a ranking of all alternatives, and the role of decision support stage is to provide the decision maker with the most fitting supplier; (ii) top-k problem: we may need select the top k number of alternatives from the ranking; (iii) students performance assessment: the decision-maker may require the total ranking to understand each student's position.

2.2. An explainable MCDM framework

In reference to the projection onto SMV space, we divide the process of MCDM into a trilevel structure consisting of data preparation at the symbols-level, decision analysis at the meaning-level, and decision support at the value-level. Basically, we have a straightforward way to incorporate explanations at the three levels: respectively, explain symbols at the symbols-level, explain meaning at the meaning-level, and explain value at the value-level. As depicted in Fig. 1, the leftmost dotted rectangle describes the

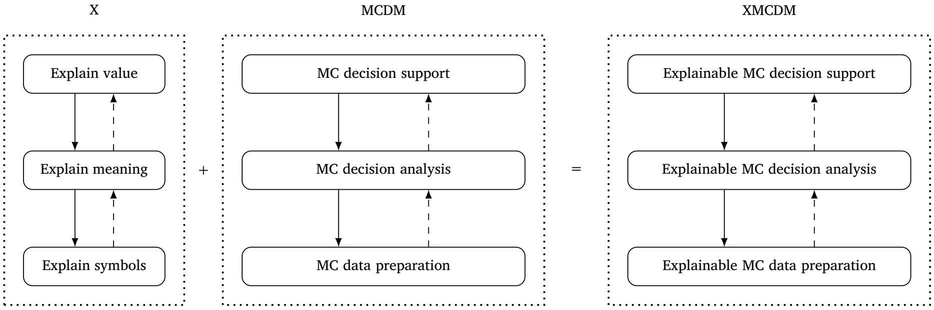


Fig. 1. The trilevel architecture of XMCDM.

Table 2
The tasks of XMCDM at three levels.

SMV space	Trilevel explanations	XMCDM
Symbols	Explain symbols	Explainable MC data preparation • Explain the definition of problem • Explain the ways to identify decision alternative • Explain the selection of criteria • …
Meaning	Explain meaning	Explainable MC decision analysis • Explain the process of data analysis • Explain the method working principles • …
Value	Explain value	Explainable MC decision support • Explain the presentations of results • Explain the usage/action of results • …

trilevel explanations and is labeled by "X". Following that, we use an addition operator to signify the addition of trilevel explanations to the trilevel MCDM, resulting in a new XMCDM framework. XMCDM also follows a trilevel structure, including explainable MC data preparation, explainable MC decision analysis, and explainable MC decision support.

In Table 2, we outline a series of tasks to be done corresponding to the trilevel explanations inherent in the XMCDM framework. Since the table provides a structured starting point, the related tasks are not limited to those in the table. Users may adapt, extend, or redefine them based on domain-specific requirements. We examine the listed tasks through the lens of a general decision-making scenario. At the symbols-level, the primary objective in conventional MCDM is data preparation. While in terms of XMCDM, we elevate the goal to explaining data preparation. The key tasks here may include explaining the definition of the problem and the motivation behind that particular problem definition; explaining the methods used to identify decision alternatives and criteria, which contributes to explainable data collection. These elements converge to create a more transparent and interpretable process of data collection. Moving to the meaning-level, we carry out decision analysis, more specifically, once the necessary data is gathered, we must choose an appropriate MCDM method to analyze this data. The tasks related to explaining this level may include explaining the data processing, such as elucidating the mechanisms of the selected MCDM method, and providing justification for why this particular method is appropriate for the given problem. Finally, at the value-level, our attention shifts to interpreting the MCDM results from the meaning level and providing clarity on the subsequent actions or decisions that should be made based on these results. Such explainable decision support is not only about presenting the outcomes but also about making them understandable, allowing users to understand why certain decisions are preferred, thus bringing users greater confidence and trust in the decision-making process.

Explainable MC data preparation: The issue of preparing data is usually described in a special information table that contains the decision alternatives and the criteria used to evaluate them. This information table provides the foundation for subsequent multi-criteria decision analysis. In Definition 1, the MCDM table is defined as a specific information table with decision alternatives as rows and criteria as columns. Each tuple in the table represents the performance of an alternative under a particular criterion.

Definition 1. An MCDM problem is described in an MCDM table (MCDMT) which is a triplet $\langle A, C, p\rangle$ where $A = \{a_1, \ldots, a_n\}$ is a finite and non-empty set of n alternatives, $C = \{c_1, \ldots, c_m\}$ is a finite and non-empty set of m criteria, and $p : A \times C \to V$ is a function that maps a decision alternative a_i and a criterion c_j into a value $p_j(a_i) \in V$.

Example 1. As seen in Table 3, the problem of city ranking is identified by 15 decision alternatives $\{a_1, a_2, \ldots, a_{15}\}$ and 6 criteria $\{c_1, c_2, \ldots, c_6\}$.

Table 3
An MCDMT of City-Ranking.

Alternatives	Criteria					
	c_1	c_2	c_3	c_4	c_5	c_6
a_1	70.46	19.07	51.01	86.16	31.87	76.45
a_2	81.89	49.70	82.86	34.31	76.77	24.22
a_3	82.43	54.30	75.00	85.59	60.28	21.35
a_4	65.18	11.25	44.44	83.45	8.61	85.70
a_5	78.12	32.91	67.49	78.07	43.89	33.22
a_6	71.91	22.91	59.55	30.55	40.51	44.53
a_7	80.74	51.24	84.85	18.40	83.76	16.67
a_8	84.52	80.72	36.66	82.08	77.13	30.21
a_9	86.00	63.28	88.43	43.08	90.08	15.34
a_{10}	76.16	33.69	61.67	68.21	57.01	18.18
a_{11}	90.45	50.13	41.12	30.54	65.27	48.31
a_{12}	78.84	44.03	72.69	29.86	52.08	46.59
a_{13}	81.44	52.91	79.63	68.93	80.87	42.45
a_{14}	79.63	49.51	75.28	6.78	78.52	32.08
a_{15}	83.31	68.77	54.17	38.64	65.53	68.58

Unlike in traditional MCDM frameworks, where the data collection process is often overlooked or assumed to be complete, the XMCDM framework emphasizes the importance of thoroughly explaining the data gathering process. This guarantees the delivery of a clear understanding of how necessary information is obtained. The following concrete case of explainable data preparation should give a clearer picture of the steps involved and the importance of a transparent and understandable data collection process. Dang et al. [10] developed a specific MCDM model for site selection of island photovoltaic charging station (IPVCS), which effectively resolves criteria conflicts. After reviewing a wide array of existing research, they concluded that both photovoltaic charging station and power station site selections must take into account cost factors, such as construction and operational expenses, social aspects like policies, and environmental factors, including pollution. They stated that photovoltaic power stations primarily focus on natural elements such as sunshine duration and temperature, while charging stations need to consider technical factors like power demand, charging distance, and grid-connection distance. Beyond these basic metrics, they further summarized evaluation indicators for photovoltaic power station site selection into categories like resource criteria, environmental criteria, technical risks, energy factors, physical assessments, and economic factors. Regarding charging station site selection, they classified the criteria into social benefits, societal criteria, and technical factors.

From the perspective of XMCDM, they had a unique and significant contribution to the explainable data preparation by building a novel evaluation criteria system for the site selection of IPVCS, which comes with two new criteria. They elaborated on the necessity of analyzing criteria and convincingly demonstrated that their criteria system is completely applicable, stable, and rational.

Explainable MC decision analysis: Ranking or ordering is the majority purpose of MCDM [14], as it provides decision-makers with a structured and ordered hierarchy of alternatives. The ranking process involves assigning a relative importance or weight to each criterion and subsequently computing an aggregate score for each alternative. The alternatives are then ranked based on these scores, while the highest-ranking alternative was deemed the most preferable solution. Several techniques have been developed to rank and order alternatives in MCDM, including Analytic Hierarchy Process (AHP) [33], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [16], and Elimination and Choice Expressing Reality (ELECTRE) [3]. These methods differ in their approaches to aggregating criteria and generating preference structures but share the common goal of establishing a comprehensive assessment of multiple alternatives. At the meaning-level, we generally treat the assessments, the ranking, or the ordering as the results of decision analysis.

As suggested in Table 2, we may expect explanations not only for data processing but also the working mechanism behind MCDM methods. Chen et al. [8] implemented a case study of eco-friendly material production using a fuzzy-rough approach. Their main contribution laid in the development of a novel fuzzy-rough MCDM model which integrates extension stepwise weight assessment ratio analysis and additive ratio assessment methods with fuzzy-rough numbers. In their specific case study, they applied their model to compute the weights of 10 green criteria. According to their research, linguistic evaluations are often favored over numerical ratings when it comes to assessing criteria importance in this case. They further explored the ten criteria, including economic and ecological criteria, to determine their relative importance. From the viewpoint of XMCDM, they provided clear explanations for the assignments of criteria weights. An instance in their work is that two criteria were labeled as dominant criteria because experts considered that procuring high-quality and environment-friendly materials from suppliers is crucial.

Explainable MC decision support: As we progress from MC decision analysis to MC decision support, the unexplained ranking/ordering produced by MCDM highlights the significance of explanations. Because of the lack of explanation, the suggested decisions may not provide sufficient decision support, and it may hinder decision-makers' trust and confidence in MCDM. One illustrative situation is when facing novel alternatives that have not been evaluated before, the existing results may not be able to provide a meaningful ranking or comparison. Consequently, the decision maker may need to restart the experiment to assess the novelty and new results. In Example 1, we assume that an MCDM method suggests that city a_2 is preferred to a_1. We notice that, as detailed in Table 3, a_2 comes with better evaluations under $\{c_1, c_2, c_3, c_5\}$ than a_1 but performs worse than a_1 under $\{c_4, c_6\}$. The explainable decision support is expected to explain what factors contribute to the decision $a_2 \succ a_1$. A potential rule-based explanation such as "given two alternatives a_i and a_j, if $p_2(a_i) > p_2(a_j)$ and $p_3(a_i) > p_3(a_j)$, then $a_i \succ a_j$," offers sufficient explainability. Additionally, the rule might be applicable to when we have a novel alternative and we require the preferences between it and existing alternatives.

The conception of SMV spaces is an abstract notion. It suggests a three-level explanation that focuses on the data, the meaning of data, and the value of data. This general idea is applicable to any data analysis and decision-making methods, of which MCDM is an example. This study is only a preliminary step towards a large and long-term goal of building explainable MCDM. The notion of SMV spaces serves as providing the useful high-level guidance and hints on building explanations. Substantial work is needed for building a more comprehensive framework of XMCDM based on the conception of SMV spaces. Data-level explanation needs information about a particular domain. Meaning-level explanation needs domain-specific knowledge. Value-level explanation needs application details. It may be difficult to construct a very concrete model without direct reference to a particular application. Therefore, our trilevel framework calls for the need for different explanations, as well as a high-level architecture for building explanations. The discussions in the rest of the paper are examples, such as three-way ranking, decision tree explanation of a ranking method, etc, which are meant for illustration of the main ideas. Each example only touches upon one particular aspect of explanation at a specific level. For example, decision trees are used for explaining the meaning of a ranking produced by an MCDM method. Through these examples, it is hoped that the notion of SMV spaces and the associated trilevel framework may provide useful hints when constructing particular models of explainable MCDM. With the SMV conception, we may also easily see the focuses of an explainable MCDM method. In summary, the proposed framework raises up the questions and does not provide a complete answer. It also provides a setting for future research.

This section constructs a conceptual framework for XMCDM and discusses the details at the three levels. Although the three levels are equally important, we prioritize the investigations at the meaning and value levels. Section 3 delved deeply into the meaning level by analyzing rankings and involving three-way decision to build rankings. Section 4 bridges the meaning level and the value level by extracting decision rules and providing human-understandable decision support. Finally, the experiments in Section 5 showcase the concrete applications of XMCDM and demonstrate its practical value.

3. MCDM rankings and three-way rankings

One of the key challenges in XMCDM is to effectively transfer the unexplained results into explainable results at the meaning level. Regarding classical multi-criteria decision analysis, Roy [32] categorized the main problematic into four sub-problems, namely, choice, sorting, ranking, and description. Among these, the type of ranking is very common in practical applications, for instance, the supply chain ranking. In this section, we select the ranking problem as the main issue and discuss the means to generate rankings and three-way rankings.

3.1. Explaining MCDM working principles and evaluating MCDM rankings

TOPSIS stands for Technique for Order of Preference by Similarity to Ideal Solution. We select TOPSIS as a concrete instance of solving the ranking problem, which is also an easy-to-understand method used to rank alternatives.

The main process of TOPSIS consists of six steps:

1. Normalize the decision matrix;
2. Construct the weighted normalized decision matrix;
3. Calculate the ideal and worst solutions;
4. Calculate the distance of each alternative from the ideal and worst solutions;
5. Calculate the relative closeness to the ideal solution;
6. Rank the alternatives based on the relative closeness.

The alternatives are ranked based on their relative closeness to the ideal solution, and the alternative with the highest relative closeness is considered as the best option. Afterwards, we use $\succ_{TOPSIS}$ to denote the ranking generated by TOPSIS:

$$\succ_{TOPSIS} = \{(a_i, a_k) \in A \times A \mid a_i \text{ has higher relative closeness than } a_k\}. \tag{1}$$

That is, $\forall a_i, a_k \in A$, if TOPSIS ranks a_i ahead of a_k, then we have $a_i \succ_{TOPSIS} a_k$.

3.2. Constructing three-way rankings based on pairwise comparison

Pairwise comparison is a useful tool that involves comparing two alternatives to determine a preferred one, usually coming with a higher amount or a higher quantity of a certain characteristic. There are many research studies on pairwise comparison [5,12,20,21,30]. Pairwise comparison matrix is often used in MCDM [14] because such a technique can be used to evaluate each pair of alternatives based on single one criterion at a time. Instead of a comprehensive evaluation, this method provides the decision maker with informative and individual preferences and attitudes towards multiple and possibly conflicting criteria.

For simplicity, we assume that every pair of alternatives in a given MCDMT is comparable. The pairwise comparison matrix is then defined by directly placing the alternative pairs as the rows and the multiple criteria as the columns.

Definition 2. A pairwise comparison matrix is a triplet $PCM = \{A \times A, C, r\}$, where A is the set of decision alternatives, C is the set of criteria, and r is a function: $r_j(a_i, a_k) = p_j(a_i) - p_j(a_k)$.

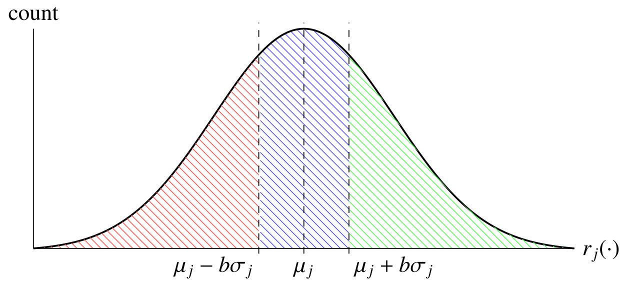


Fig. 2. A normal distribution of a column in pairwise comparison matrix.

Each cell in the matrix contains the difference value, which reflects the comparison result between a specific pair of alternatives under a specific criterion. In our example, in the pairwise comparison matrix used to evaluate different cities based on various criteria such as quality of life, pollution, crime rate, and purchase power, the rows may represent the pair of cities being compared, while the columns may represent the criteria being selected to evaluate them.

By taking into account the idea of thinking in threes, we construct three-way rankings that are built on such a pairwise comparison matrix. Yao and Shi [42] have defined the three-way rankings and the trisection over the set of alternatives. For any two alternatives $a_i, a_k \in A$, the three-way rankings refer to three possible situations:

1. $a_i \succ a_k$: a_i is better than / preferred to a_k,
2. $a_i \prec a_k$: a_i is worse than / inferior to a_k.
3. $a_i \approx a_k$: a_i is approximately the same as a_k.

In the context of three-way rankings, we recognize that there may exist a relatively small margin of difference in evaluation values between a certain pair of alternatives. In real-life applications, the small difference is possibly resulted by statistical errors and measurement variance, so that such a pair of decision alternatives may be considered practically and approximately the same as each other. This small margin of difference can also be considered within the acceptable range of tolerance and doesn't sufficiently have an impact on their rankings from the user's view. That is, to acknowledge that a pair of alternatives is truly different, we require that there exists a significant or observable difference between them.

Definition 2 provides the basic ingredients for us to construct three-way rankings. Once the pairwise comparison matrix is calculated, the next step is to determine the thresholds for the three possible situations between alternative pairs. These thresholds are related to the determination of how much difference is considered significant or observable, and how much difference is considered as an acceptable nuance. In other words, the thresholds in three-way rankings help to define the degree of difference that sufficiently has an impact on the overall ranking of the alternatives. Thus, the determination of thresholds is the most crucial step in three-way rankings and ensures that the rankings are suitable and accurate.

There have been outstanding studies that have been conducted to contribute to finding appropriate thresholds [23,37]. Basically, the thresholds are calculated based on the range of values in the pairwise comparison matrix. In this subsection, we take two methods as two typical cases: one is the statistic-based model, which produces a pair of symmetric thresholds, and the other is the prospect-theory-based model generating a pair of asymmetric thresholds.

3.2.1. Statistic-based three-way rankings

Yao and Gao [40] proposed the statistical interpretations for constructing three regions in terms of evaluation-based three-way decision models. They investigated the statistical methods to effectively calculate the thresholds based on some fundamental notions such as percentile, median, mean, and standard deviation. According to their proposed findings, the special combo of mean and standard deviation is widely used in applications with numerical values, for example, IQ score classifications and blood pressure classifications.

Within the pairwise comparison matrix described by Definition 2, we formulate the calculations of the mean μ and standard deviation σ with respect to a criterion c_j:

$$\mu_j = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{k=1}^{n} r_j(a_i, a_k),$$

$$\sigma_j = \left(\frac{1}{n^2}\sum_{i=1}^{n}\sum_{k=1}^{n}(r_j(a_i, a_k) - \mu_j)^2\right)^{\frac{1}{2}}. \tag{2}$$

We observe that, in the example of city rankings, the distribution of each column in such a pairwise comparison matrix is a normal distribution as shown in Fig. 2. From a normal distribution, we give a simple method for constructing a three-way ranking matrix based on the mean μ and the standard deviation σ. To apply the method, we first need to check whether the distribution is a normal distribution. For a non-normal distribution, the proposed method may not be entirely meaningful. Alternately, we may consider other statistical methods. For example, we may use the median and percentile values [40] in place of the mean and standard deviation.

The validation of the hypothesis on normal distribution and the statistical associated method need further evaluation using more real-world data. It may be interesting to examine other types of distribution under which the proposed method does work too.

The lower threshold is calculated by $\mu_j - b\sigma_j$ and the higher threshold is given by $\mu_j + b\sigma_j$ where $b \geqslant 0$. A three-way ranking matrix determined by a pair of symmetric thresholds is defined as follows.

Definition 3. A statistic-based three-way ranking matrix is a three-valued matrix $S3VM = \{A \times A, C, \upsilon\}$, where A is the set of decision alternatives, C is the set of criteria, and υ is a three-valued mapping: $\upsilon : A \times A \times C \longrightarrow \{\succ, \prec, \approx\}$. For $c_j \in C$, we have

$$\upsilon_j(a_i, a_k) = \begin{cases} \succ, \text{ if } r_j(a_i, a_k) > \mu_j + b\sigma_j, \\ \prec, \text{ if } r_j(a_i, a_k) < \mu_j - b\sigma_j, \\ \approx, \text{ if } \mu_j - b\sigma_j \leqslant r_j(a_i, a_k) \leqslant \mu_j + b\sigma_j, \end{cases} \tag{3}$$

where $b \geqslant 0$.

Definition 3 actually translates the pairwise comparison matrix into a three-valued comparison matrix by introducing a pair of thresholds $(\mu_j - b\sigma_j, \mu_j + b\sigma_j)$ for every single column c_j. For simplicity, in the rest of the paper, we assume that the underlying distribution for each criterion is a normal distribution, and we adopt the entire matrix for illustrative examples and experiments. In this way, the mean value μ_j is always 0 and the pair of thresholds is actually $(-b\sigma_j, +b\sigma_j)$. The symmetry of the thresholds enables that the following properties of statistic-based three-way rankings are true: for a given criterion c_j, $\forall (a_i, a_k) \in A \times A$,

1. if $\upsilon_j(a_i, a_k) = \succ$, then $\upsilon_j(a_k, a_i) = \prec$;
2. if $\upsilon_j(a_i, a_k) = \prec$, then $\upsilon_j(a_k, a_i) = \succ$;
3. if $\upsilon_j(a_i, a_k) = \approx$, then $\upsilon_j(a_k, a_i) = \approx$.

Algorithm 1 The algorithm to generate statistic-based three-way rankings.

Input: A multi-criteria decision-making table MCDMT $= \{A, C, p\}$, parameter b
Output: A statistic-based three-way ranking matrix

for each $a_i \in A$ **do**
  for each $a_k \in A$ **do**
    $r_j(a_i, a_k) = p_j(a_i) - p_j(a_k)$;
  end for
end for
for each $c_j \in C$ **do**
  Calculate the mean μ_j and standard deviation σ_j using Equation (2);
  for each $(a_i, a_k) \in A \times A$ **do**
    Calculate three-way rankings $\upsilon_j(a_i, a_k)$ using Equation (3) and parameter b;
  end for
end for
return the statistic-based three-way ranking matrix;

Algorithm 1 consists of three main components: 1. generate the pairwise comparison matrix, 2. calculate the mean and standard deviation for each criterion, and 3. compute the statistic-based three-way ranking matrix. Its time complexity is $O(m \cdot n^2)$, where $m = |C|$ and $n = |A|$.

Example 2. Table 4 is an implementation of Algorithm 1 on Table 3 by setting the threshold parameter as $b = 0.4$. We demonstrate the effectiveness of Algorithm 1 for generating the three-way rankings through the application on an MCDMT. The results obtained from applying Algorithm 1 are partially displayed in Table 4 due to the large size of 225 pairs of alternatives in total.

Each row in Table 4 represents a pair of alternatives, and columns show the criteria and corresponding three-way rankings from an individual view. Overall, the results of this example study indicate the usefulness and clearness that provide the decision maker with informative comparisons and specific preferences between each pair of alternatives.

3.2.2. Prospect-theory-based three-way rankings

Yao and Shi [42] proposed a three-way ranking-based MCDM approach by reformulating the cumulative prospect theory and the TODIM method. They integrated the idea of thinking in threes and built a family of three-way rankings in the form of trisections over the set of decision alternatives. The cumulative prospect theory is given by a two-part value function:

$$\theta(x) = \begin{cases} x^{\alpha}, & \text{if } x \geqslant 0, \\ (-\lambda)(-x)^{\beta}, & \text{if } x < 0. \end{cases} \tag{4}$$

Given a threshold $t \geqslant 0$, we describe the three-way rankings:

Table 4
The statistic-based three-way rankings with $b = 0.4$.

$A \times A$	c_1	c_2	c_3	c_4	c_5	c_6
(a_1, a_1)	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
(a_1, a_2)	$\prec$	$\prec$	$\prec$	$\succ$	$\prec$	$\succ$
(a_1, a_3)	$\prec$	$\prec$	$\prec$	$\approx$	$\prec$	$\succ$
(a_1, a_4)	$\succ$	$\approx$	$\approx$	$\approx$	$\succ$	$\approx$
(a_1, a_5)	$\prec$	$\prec$	$\prec$	$\approx$	$\approx$	$\succ$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(a_{15}, a_{11})	$\prec$	$\succ$	$\succ$	$\approx$	$\approx$	$\succ$
(a_{15}, a_{12})	$\succ$	$\succ$	$\prec$	$\approx$	$\succ$	$\succ$
(a_{15}, a_{13})	$\approx$	$\succ$	$\prec$	$\approx$	$\succ$	$\succ$
(a_{15}, a_{14})	$\succ$	$\succ$	$\prec$	$\succ$	$\prec$	$\succ$
(a_{15}, a_{15})	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$

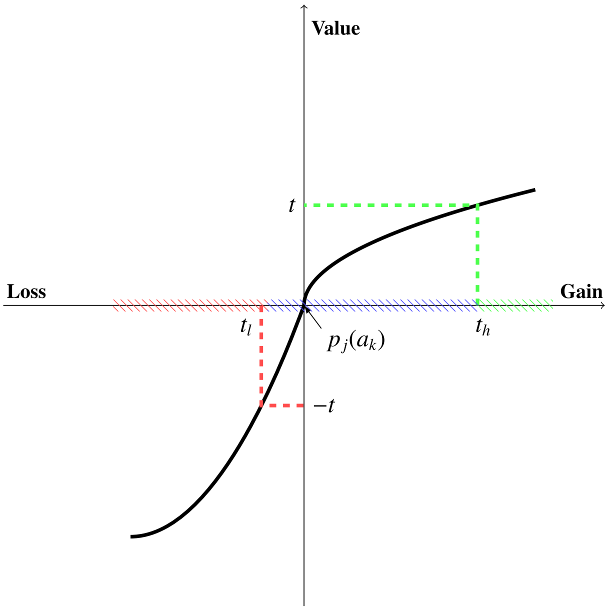


Fig. 3. The prospect theory value function (adapted from [42]).

$$
\begin{aligned}
&a_i \succ_j a_k, \text{ if } \theta(r_j(a_i, a_k)) > t,\\
&a_i \prec_j a_k, \text{ if } \theta(r_j(a_i, a_k)) < -t,\\
&a_i \approx_j a_k, \text{ if } -t \leqslant \theta(r_j(a_i, a_k)) \leqslant t.
\end{aligned} \tag{5}
$$

Among these three situations, we may substitute the threshold t with a pair of thresholds (t_l, t_h) where $|t_h| > |t_l| \geqslant 0$. According to the Equation (4), we have $t_l = -\left(\frac{t}{\lambda}\right)^{\frac{1}{\beta}}$ and $t_h = t^{\frac{1}{\alpha}}$. The asymmetry of the pair of thresholds disables the properties discussed in the previous statistic-based model. The positions of the two involved alternatives in a pair make a difference to their orderings. For example, the given pair (a_i, a_k) assumes that the user takes a_k as the reference point and the main task is to determine whether a_i is better than, worse than, or similar to a_k. We observe this possibility that, even if the two pairs (a_i, a_k) and (a_k, a_i) involve the exact same alternatives, different references from the user's viewpoint may produce different results. This is suggested by the prospect theory, which provides an explanation for the deep reason behind the observed phenomenon. The prospect theory states that people evaluate potential gains and losses relative to a reference point, rather than in absolute terms, which results in different perceptions of gains and losses, as shown in Fig. 3.

A three-way ranking matrix determined by a pair of asymmetry thresholds is defined as follows.

Definition 4. A prospect-theory-based three-way ranking matrix is a three-valued pairwise comparison matrix $P3VM = \{A \times A, C, v\}$, where A is the set of decision alternatives, C is the set of criteria, and v is a three-valued mapping: $v : A \times A \times C \longrightarrow \{\prec, \approx, \succ\}$,

Table 5
The prospect-theory-based three-way rankings with ($t_l \approx -2.4779$, $t_h \approx 6.2271$).

$A \times A$	c_1	c_2	c_3	c_4	c_5	c_6
(a_1, a_1)	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
(a_1, a_2)	$\prec$	$\prec$	$\prec$	$\succ$	$\prec$	$\succ$
(a_1, a_3)	$\prec$	$\prec$	$\prec$	$\approx$	$\prec$	$\succ$
(a_1, a_4)	$\approx$	$\succ$	$\succ$	$\approx$	$\succ$	$\prec$
(a_1, a_5)	$\prec$	$\prec$	$\prec$	$\succ$	$\prec$	$\succ$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(a_{15}, a_{11})	$\prec$	$\succ$	$\succ$	$\succ$	$\approx$	$\succ$
(a_{15}, a_{12})	$\approx$	$\succ$	$\prec$	$\succ$	$\succ$	$\succ$
(a_{15}, a_{13})	$\approx$	$\succ$	$\prec$	$\prec$	$\prec$	$\succ$
(a_{15}, a_{14})	$\approx$	$\succ$	$\prec$	$\succ$	$\prec$	$\succ$
(a_{15}, a_{15})	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$

$$v_j(a_i, a_k) = \begin{cases} \succ, \text{ if } r_j(a_i, a_k) > t_h, \\ \prec, \text{ if } r_j(a_i, a_k) < t_l, \\ \approx, \text{ if } t_l \leqslant r_j(a_i, a_k) \leqslant t_h, \end{cases} \tag{6}$$

where $t_l = -\left(\frac{t}{\lambda}\right)^{\frac{1}{\beta}}$, $t_h = t^{\frac{1}{\alpha}}$, and $|t_h| > |t_l| \geqslant 0$.

Algorithm 2 The algorithm to generate prospect-theory-based three-way rankings.

Input: A multi-criteria decision-making table $\text{MCDMT} = \{A, C, p\}$, parameters $t, \lambda, \alpha, \beta$
Output: A prospect-theory-based three-way ranking matrix
for each $a_i \in A$ **do**
  for each $a_k \in A$ **do**
    $r_j(a_i, a_k) = p_j(a_i) - p_j(a_k)$;
  end for
end for
Calculate t_l and t_h using parameters $t, \lambda, \alpha, \beta$;
for each $c_j \in C$ **do**
  for each $(a_i, a_k) \in A \times A$ **do**
    Calculate three-way rankings $v_j(a_i, a_k)$ using Equation (6);
  end for
end for
return the prospect-theory-based three-way ranking matrix;

From Fig. 3 and Definition 4, the selection of a reference point in the given pair of alternatives may affect the user's perception and evaluation and lead to the differences in their ranking depending on the positions of the alternatives in the pair. Algorithm 2 shows how to derive a prospect-theory-based three-way ranking matrix. Its time complexity is $O(m \cdot n^2)$, where $m = |C|$ and $n = |A|$.

Example 3. Table 5 is the results when applying Algorithm 2 with the set of parameters: $\lambda = 2.25$, $\alpha = 0.88$, $\beta = 0.88$, and $t = 5$. By Definition 4, we obtain $t_l \approx -2.4779$ and $t_h \approx 6.2271$.

Through the Example 3, we can easily find that the prospect theory gives us a pair of asymmetry thresholds. That is, the threshold t_h to determine one alternative is better than another requires a greater strength or value of the positive difference than the threshold t_l to determine one is worse than another.

4. Rule-based explanation of MCDM results

In this section, we aim to extract rule-based explanations at the value level in XMCDM. To achieve this, we develop an algorithm by modifying the ID3 decision tree to generate rules. The induced rules can be considered as a powerful tool to explain MCDM results.

4.1. Combining an MCDM ranking and three-way rankings

In Section 3, we presented the concept of a comprehensive ranking generated by a classic MCDM method. To integrate this ranking into the three-way rankings obtained from individual evaluations of the decision alternatives, we define a decision attribute that reflects the overall performance of each alternative based on the criteria used in the MCDM method.

This decision attribute is used as a reference in the three-way decision-making process, allowing us to investigate the relationship between individual rankings and the comprehensive one. That is, referring to the value of the decision attribute, decision-makers can gain insights into how each criterion impacts the overall performance of each alternative. In this case, decision-makers can gain a more detailed understanding of the strengths or weaknesses of each alternative, as well as the relative importance of each criterion. The decision attribute is defined as follows.

Definition 5. In addition to the three-way ranking matrix, a decision attribute is denoted by d and determined by a ranking $\succ_{MCDM}$ produced by an MCDM method. With respect of d, we have

$$v_d(a_i, a_k) = \begin{cases} \prec, \text{ if } a_k \succ_{MCDM} a_i, \\ \approx, \text{ if } \neg(a_i \succ_{MCDM} a_k) \wedge \neg(a_k \succ_{MCDM} a_i), \\ \succ, \text{ if } a_i \succ_{MCDM} a_k, \end{cases} \tag{7}$$

For the illustrative example in this section, we use the TOPSIS ranking $\succ_{TOPSIS}$ to generate the values for the decision attribute.

4.2. Construction of a ternary decision tree using ID3 algorithm

The ID3 algorithm, invented by Quinlan [29], is a widely used algorithm for constructing decision trees by recursively selecting the best attribute that splits the instances into subsets with the most distinct values, until all instances are classified into classes or no further attributes are available. In our case, we use the comprehensive ranking generated by a classic MCDM method as well as the individual three-way rankings as input for the ID3 algorithm. The resulting decision tree represents the decision logic and provides a clear and concise way to interpret the comprehensive ranking through the individual three-way rankings. By analyzing the structure and performance of the decision tree, we can obtain new insights into the criteria and preferences used to make decisions.

At first, we introduce two key concepts in the ID3 algorithm: information entropy and information gain. Information entropy is a measure of the uncertainty of a set of examples. It is calculated as the sum of the negative logarithms of the probabilities of each class label, in our case, $\prec$, $\approx$, or $\succ$.

$$Ent(G) = -\sum_{u \in \{\prec, \approx, \succ\}} Pr_u \log_2 Pr_u, \tag{8}$$

where

$$Pr_u = \frac{|\{(a_i, a_k) \in A \times A \mid v_d(a_i, a_k) = u\}|}{|A \times A|}.$$

Information gain is a measure of the reduction in uncertainty achieved by splitting the instances based on a criterion, and is calculated as the difference between the entropy before and after the split:

$$IG(G, c) = Ent(G) - \sum_{u \in \{\prec, \approx, \succ\}} \frac{|G_{cu}|}{|D|} Ent(G_{cu}), \tag{9}$$

where G is the input set and G_{cu} is the subset of G in which criterion c has value of u.

The ID3 algorithm consists of the following steps:

1. If all examples in the dataset belong to the same ranking $\succ_d$, $\approx_d$, or $\prec_d$, return a leaf node labeled with that ranking;
2. If the dataset is empty, return a leaf node labeled with the most common class in the parent dataset;
3. Calculate the information gain for each criterion c_j in the dataset using Equations (8) and (9);
4. Choose the criterion c_j with the highest information gain as the splitting criterion, denoted by c_{best};
5. Create a decision node based on the splitting criterion c_{best};
6. For each possible ranking of the splitting criterion c_{best}, create a branch from the decision node and recursively apply the algorithm to the subset of the dataset that has the exact ranking for the splitting criterion;
7. Upon the recursion terminates, return the decision tree.

Algorithm 3 modifies the classic ID3 decision tree approach to construct a ternary decision tree. It starts with the initial matrix M and recursively applies the main function MODIFIEDID3, which partitions the matrix by selecting the criteria c_{best} with maximum information gain. At each step, it creates a node c_{best} that splits the input matrix according to $u \in \{\succ, \prec, \approx\}$. Then, it calls function MODIFIEDID3 by taking each sub-matrix G_u as a new input. The recursive calls return sub-trees and the sub-trees are added to T as branches. The recursion halts when all alternatives share the same ranking or no criteria remain by returning a leaf node. The time complexity of Algorithm 3 is $O(m \cdot n^2 \cdot \log_3 n^2)$, where $n = |A|$ and $m = |C|$.

By applying Algorithm 3, we can construct a ternary decision tree, in which each node has a maximum of three branches representing three distinct relationships ($\prec$, $\approx$, $\succ$). When using such a ternary decision tree to provide users with rule-based explanations, it is important to evaluate the performance of the tree to ensure that it accurately reflects the preference problem and offers useful insights. One common metric for evaluating the performance of a decision tree is accuracy for each leaf node, which measures the percentage of alternative pairs in the three-way ranking matrix that are correctly ranked by the tree. Another metric is coverage for

Algorithm 3 The algorithm to create a ternary decision tree.

Input: A three-way ranking matrix with added decision attribute $M = (A \times A, C \cup \{d\}, v)$
Output: An unpruned ternary decision tree

function MODIFIEDID3(G)
  if all examples in G have the same ranking for d **then**
    return leaf node labeled with d.
  end if
  if C is empty **then**
    return leaf node labeled with majority d in G.
  end if
  for each $c_j \in C$ **do**
    Calculate the entropy using Equation (8) and information gain using Equation (9) for c_j;
  end for
  Let c_{best} be the criterion with highest information gain;
  Create a tree T with node c_{best};
  for each $u \in \{\succ, \prec, \approx\}$ **do**
    Let G_u be the subset of G with the condition of the ranking over c_{best} equals to u;
    Let G_u drop the criterion c_{best};
    Add a branch (u, MODIFIEDID3(G_u)) to T;
  end for
  return T;
end function
Call function MODIFIEDID3(M);

each leaf node, which measures the percentage of alternative pairs in the matrix that are covered by each leaf node in the tree. Each leaf node provides a unique rule to determine the preference between two decision alternatives, denoted by $r : LHS \to RHS$.

Definition 6. Given a decision tree, each leaf node generates a rule in the form of $r : LHS \to RHS$,

$$LHS : \bigwedge_{c_j} (v_j = \text{a certain value from } \{\prec, \approx, \succ\}),$$
$$RHS : v_d = \text{a certain value from } \{\prec, \approx, \succ\}.$$

The left-hand side (*LHS*) is composed of the conditions that must be met starting from the root node to the leaf node. As the decision tree is traversed, each branch represents a different condition. The right-hand side (*RHS*) is the final outcome of the leaf node, which represents the final decision. A higher accuracy and a higher coverage indicate that the generated rule is more effective at predicting the preference of two given decision alternatives from two aspects.

Definition 7. Given a decision tree, the accuracy and coverage of a rule $r : LHS \to RHS$ are calculated as follows:

$$\begin{aligned} accuracy(r) &= \frac{|m(LHS) \cap m(RHS)|}{|m(LHS)|}, \\ coverage(r) &= \frac{|m(LHS)|}{\text{total \# of examples}}, \end{aligned} \tag{10}$$

where $m(\cdot)$ is the set of examples that satisfy the specific conditions.

Another essential metric is the average accuracy for the entire tree, which measures the overall effectiveness of the tree at capturing average prediction accuracy and can provide insights into the overall quality of the rule-based explanations generated by the tree. By evaluating the performance of the decision tree using these metrics, decision-makers can gain a better understanding of the strengths and weaknesses of the resulting rule-based explanations and can make more informed decisions.

Definition 8. Given a decision tree, the family of rules is denoted by R. The average accuracy of the tree is calculated as follows:

$$average\ coverage = \sum_{r \in R} \text{accuracy}_r * \text{coverage}_r. \tag{11}$$

Example 4. As shown in Fig. 4, we generated a decision tree with 39 rules and an average accuracy of 98.22% for the matrix in Table 6 by implementing Algorithm 3. In the decision tree, the top node is the root, represented by a circle. The internal nodes are also represented by circles, and the decision nodes are illustrated as squares. Due to the symmetry of thresholds in statistic-based three-way rankings, the decision tree in Fig. 4 is mirror-symmetric.

To evaluate the individual quality of each rule, we calculated the accuracy and coverage for each leaf node in the tree. We found that the coverage varied widely among the rules, with some rules having a high coverage while others had a very low coverage. This indicates that the tree may be overfitting to the training data and may not generalize well to new instances. To improve the

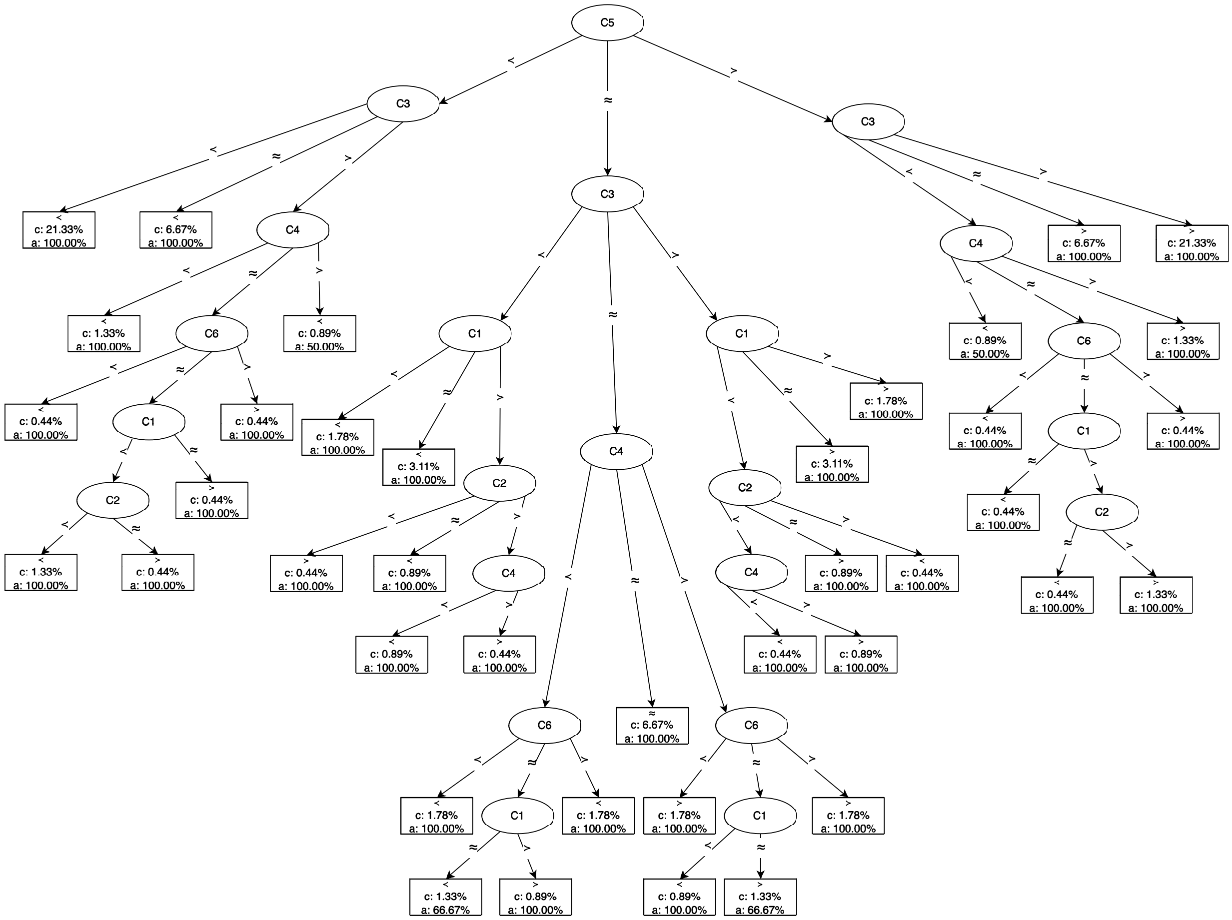


Fig. 4. The decision tree with accuracy and coverage.

Table 6
The statistic-based three-way rankings with $b = 0.4$ and added decision attribute.

$A \times A$	c_1	c_2	c_3	c_4	c_5	c_6	d
(a_1, a_1)	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
(a_1, a_2)	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$
(a_1, a_3)	$\prec$	$\prec$	$\prec$	$\approx$	$\prec$	$\prec$	$\prec$
(a_1, a_4)	$\succ$	$\approx$	$\approx$	$\approx$	$\succ$	$\approx$	$\succ$
(a_1, a_5)	$\prec$	$\prec$	$\prec$	$\approx$	$\approx$	$\prec$	$\prec$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(a_{15}, a_{11})	$\prec$	$\succ$	$\succ$	$\approx$	$\approx$	$\prec$	$\prec$
(a_{15}, a_{12})	$\succ$	$\succ$	$\prec$	$\approx$	$\succ$	$\prec$	$\prec$
(a_{15}, a_{13})	$\approx$	$\succ$	$\prec$	$\succ$	$\prec$	$\prec$	$\prec$
(a_{15}, a_{14})	$\succ$	$\succ$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$
(a_{15}, a_{15})	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$

interpretability of the tree and reduce the risk of overfitting, we may consider pruning the tree or reducing the complexity of rules, for example, restricting the depth. The decision tree can serve as a valuable tool for identifying the most relevant criteria and help users reach a better understanding of the preferences among alternatives. From the leftmost leaf node to the rightmost one, we show a selection of the first five rules:

(r1). **If** $(c_5, \prec) \wedge (c_3, \prec)$, **then** $(d, \prec)$.
(r2). **If** $(c_5, \prec) \wedge (c_3, \approx)$, **then** $(d, \prec)$.
(r3). **If** $(c_5, \prec) \wedge (c_3, \succ) \wedge (c_4, \prec)$, **then** $(d, \prec)$.
(r4). **If** $(c_5, \approx) \wedge (c_6, \prec)$, **then** $(d, \prec)$.
(r5). **If** $(c_5, \approx) \wedge (c_6, \approx) \wedge (c_1, \prec) \wedge (c_2, \prec)$, **then** $(d, \prec)$.

Table 7
The descriptions of real-life datasets.

Dataset ID	Number of alternatives	Number of criteria
Pollution	60	16
City-Ranking	216	6
Winequality-red	1599	11
Quake	2178	4
Winequality-white	4898	11

In the case of predicting the ranking of new alternatives, for example a_{new}, we may have if $a_{new} \prec_{c_5} a_1 \wedge a_{new} \prec_{c_3} a_1$, then we can decide $a_{new} \prec_d a_1$. By examining the extracted rules, we observe that the decision tree is mirror-symmetric because the statistic-based three-way ranking matrix is symmetric. We can infer that an asymmetric three-way ranking matrix may result in an asymmetric decision tree, for example, using the prospect-theory-based three-way rankings. It is reasonable because the preferences of one alternative over another one may not be reciprocated in an asymmetric three-way ranking matrix, which leads to a non-symmetric tree structure and different decision rules.

5. Experimental case studies and analysis

In this section, we conducted a series of experiments to evaluate the performance of our proposed XMCDM framework. At first, we give a simple description of selected datasets and explain the designation of experiments, including how we generate three-way rankings, what MCDM methods are chosen, and how we set up the thresholds tuning. Then, we collect the experimental results, visualize them in a proper way, and conclude the performance.

5.1. Experiment design and setup

The experimental datasets comprised five real-life datasets (see Table 7), which provided a comprehensive and diverse testing environment to assess the robustness and effectiveness of our approach.

The five real-life datasets were sourced from various domains: "Pollution" and "Quake" are gathered from Keel repos[1] [1,2], "City-Ranking" is downloaded from Kaggle and MoveHub,[2] and "Winequality-red" and "Winequality-white" are collected from UCI repos[3] [9]. These datasets were chosen for their relevance to practical applications of MCDM and to demonstrate the versatility of our method across different fields. Each real-life dataset was preprocessed, ensuring optimal compatibility with our proposed XMCDM framework. To ensure the reliability and validity of our experimental results, we employed a rigorous evaluation methodology, including the total number of generated rules, maximum complexity, and average accuracy.

Additionally, we selected three popular MCDM methods (TOPSIS, VIKOR, and COPRAS) to rank the alternatives at the meaning level, and tuned the threshold in the statistic-based three-way rankings. The parameter b serves as a multiplier of the standard deviation. In normal distributions, the three-sigma rule shows that values within 1σ, 2σ, and 3σ of the mean approximately cover 68%, 95%, and 99.7% of data, respectively. To assess its impact, we select the range of $b \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6\}$. Furthermore, we restricted the maximum conditions to 30%, 40%, 50%, 60%, 70%, 80%, 90%, and 100%. This constraint directly controls both the decision tree's depth and the rules' structural complexity. For instance, in datasets with numerous criteria, a decision tree can be overly deep and produce complicated decision rules. By tuning this parameter, we can analyze its effect on average accuracy and computational time.

All experiments were carried on a PC equipped with an Intel Core i9-12900 K CPU and 128 GB RAM. The algorithms were implemented by Python 3.10. The selected TOPSIS, VIKOR, and COPRAS methods were implemented by using Python library pymcdm [19].

5.2. Experiment results and analysis

As shown in Tables 8 and 9, we present the average accuracies under various settings: the sub-tables display experimental results with threshold b ranging from 0.1 to 0.6; in each sub-table, rows represent different datasets, while columns represent combinations of maximum conditions and selected MCDM methods. Figs. 5, 6, 7, 8, and 9 respectively visualize the trends of average accuracies in each real-life dataset by explaining three MCDM methods and adjusting the threshold and maximum conditions.

The experimental results demonstrated that our proposed XMCDM framework consistently achieved competitive performance on the three MCDM methods in five real-life datasets, highlighting its robustness and adaptability to various data characteristics. By examining the computational costs, we found that it mainly depends on the maximum conditions, actually, the depth of the decision trees. In order to show the effects of maximum conditions on accuracies and computational costs, we collect the results and visualize them in Fig. 10.

[1] KEEL: https://sci2s.ugr.es/keel/index.php.

[2] Kaggle: https://www.kaggle.com/datasets/blitzr/movehub-city-rankings and MoveHub: https://www.movehub.com/city-rankings.

[3] UCI Machine Learning Repository: https://archive.ics.uci.edu/.

Table 8
The average accuracy of generated rules from real-life datasets ($b \in \{0.1, 0.2, 0.3\}$).

(a) With threshold $b = 0.1$.

Dataset ID	conditions ⩽ 30%			conditions ⩽ 40%			conditions ⩽ 50%			conditions ⩽ 60%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	84.17%	77.72%	90.56%	90.50%	85.50%	93.72%	95.61%	91.83%	96.61%	96.94%	94.39%	97.67%
City-ranking	80.19%	73.74%	75.88%	82.17%	75.46%	78.25%	82.68%	78.31%	80.19%	82.68%	78.31%	80.19%
Winequality-red	74.52%	71.48%	80.88%	76.38%	74.21%	81.77%	77.64%	75.46%	83.23%	78.95%	76.20%	83.90%
Quake	72.97%	79.95%	58.52%	72.97%	79.95%	58.52%	76.75%	83.45%	64.82%	76.75%	83.45%	64.82%
Winequality-white	72.50%	73.56%	82.85%	74.50%	74.91%	83.35%	76.50%	75.29%	85.20%	77.69%	75.82%	86.24%

Dataset ID	conditions ⩽ 70%			conditions ⩽ 80%			conditions ⩽ 90%			conditions ⩽ 100%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	97.89%	97.06%	98.67%	98.11%	97.56%	98.83%	98.17%	97.78%	98.89%	98.17%	97.78%	98.89%
City-ranking	85.67%	78.82%	82.44%	85.67%	78.82%	82.44%	87.35%	79.61%	84.10%	87.98%	80.03%	84.55%
Winequality-red	79.96%	77.02%	84.64%	80.79%	77.82%	85.17%	81.52%	78.41%	85.58%	82.66%	79.54%	86.33%
Quake	76.75%	83.45%	64.82%	80.79%	84.50%	66.02%	80.79%	84.50%	66.02%	82.03%	85.29%	66.05%
Winequality-white	78.14%	76.02%	87.15%	78.56%	76.27%	87.51%	78.80%	76.47%	87.71%	79.11%	76.79%	87.99%

(b) With threshold $b = 0.2$.

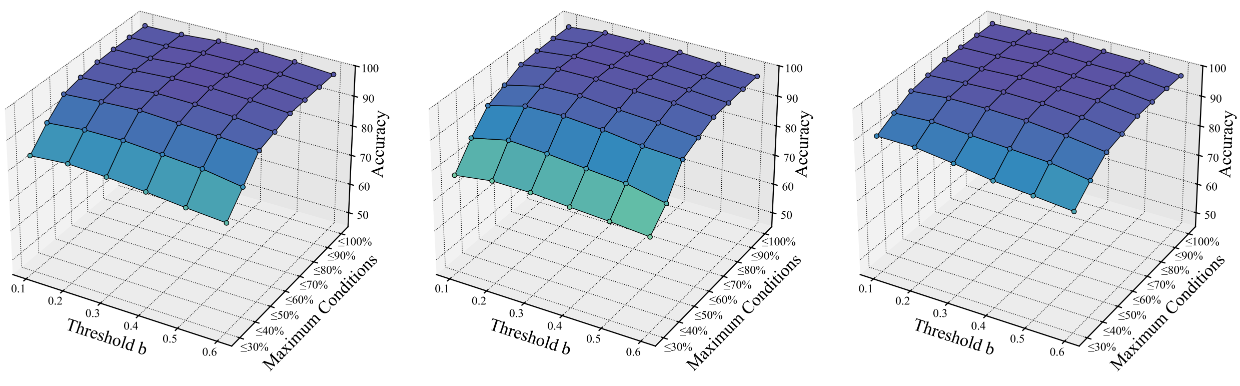
Dataset ID	conditions ⩽ 30%			conditions ⩽ 40%			conditions ⩽ 50%			conditions ⩽ 60%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	85.17%	79.56%	90.50%	91.72%	88.33%	94.44%	96.50%	95.11%	97.67%	97.50%	97.33%	98.39%
City-ranking	79.31%	73.01%	75.08%	82.88%	76.10%	79.40%	85.69%	78.61%	81.35%	85.69%	78.61%	81.35%
Winequality-red	75.34%	72.53%	82.42%	77.09%	74.97%	84.02%	78.63%	76.60%	84.77%	79.97%	77.42%	86.03%
Quake	72.41%	77.21%	58.36%	72.41%	77.21%	58.36%	78.99%	82.40%	68.60%	78.99%	82.40%	68.60%
Winequality-white	73.50%	74.62%	83.63%	75.97%	76.06%	84.69%	77.88%	76.58%	86.07%	79.18%	77.11%	87.47%

Dataset ID	conditions ⩽ 70%			conditions ⩽ 80%			conditions ⩽ 90%			conditions ⩽ 100%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	98.39%	98.83%	98.94%	98.39%	98.94%	99.06%	98.39%	98.94%	99.22%	98.39%	98.94%	99.22%
City-ranking	87.11%	79.48%	83.72%	87.11%	79.48%	83.72%	88.40%	80.15%	85.32%	89.22%	80.51%	85.86%
Winequality-red	81.39%	78.48%	86.67%	82.32%	79.28%	87.08%	83.24%	80.04%	87.50%	84.71%	81.45%	88.40%
Quake	78.99%	82.40%	68.60%	82.42%	84.40%	70.45%	82.42%	84.40%	70.45%	83.69%	85.42%	70.50%
Winequality-white	79.77%	77.34%	88.17%	80.26%	77.54%	88.68%	80.53%	77.77%	88.92%	80.88%	78.13%	89.24%

(c) With threshold $b = 0.3$.

Dataset ID	conditions ⩽ 30%			conditions ⩽ 40%			conditions ⩽ 50%			conditions ⩽ 60%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	84.56%	79.50%	88.94%	91.83%	87.56%	93.56%	97.17%	95.39%	97.72%	98.50%	97.67%	98.89%
City-ranking	78.32%	72.34%	74.23%	83.37%	76.66%	80.28%	86.06%	79.21%	82.52%	86.06%	79.21%	82.52%
Winequality-red	75.61%	73.41%	83.01%	77.46%	75.18%	84.83%	79.18%	76.91%	86.10%	80.66%	78.02%	87.34%
Quake	71.94%	74.89%	57.69%	71.94%	74.89%	57.69%	79.70%	81.04%	69.01%	79.70%	81.04%	69.01%
Winequality-white	74.23%	75.54%	84.11%	77.04%	76.91%	85.23%	79.00%	77.53%	86.72%	80.26%	77.94%	88.22%

Dataset ID	conditions ⩽ 70%			conditions ⩽ 80%			conditions ⩽ 90%			conditions ⩽ 100%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	99.17%	98.94%	99.50%	99.33%	99.00%	99.67%	99.33%	99.00%	99.67%	99.33%	99.00%	99.67%
City-ranking	87.79%	80.07%	84.04%	87.79%	80.07%	84.04%	88.94%	80.76%	86.32%	90.08%	81.10%	87.05%
Winequality-red	82.12%	79.07%	87.91%	83.25%	80.00%	88.37%	84.33%	80.76%	88.79%	85.71%	82.04%	89.61%
Quake	79.70%	81.04%	69.01%	83.32%	82.70%	71.15%	83.32%	82.70%	71.15%	85.10%	84.09%	71.16%
Winequality-white	80.98%	78.19%	89.07%	81.54%	78.44%	89.60%	81.83%	78.68%	89.86%	82.20%	79.05%	90.17%



(a) Explanations of TOPSIS. (b) Explanations of VIKOR. (c) Explanations of COPRAS.

Fig. 5. Visualizations of experimental results on Pollution.

Table 9
The average accuracy of generated rules from real-life datasets ($b \in \{0.4, 0.5, 0.6\}$).

(a) With threshold $b = 0.4$.

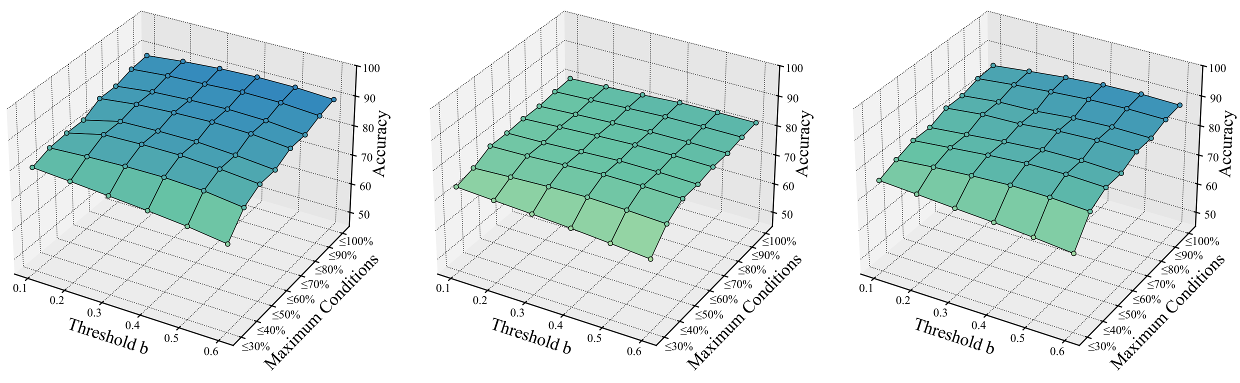
Dataset ID	conditions ⩽ 30%			conditions ⩽ 40%			conditions ⩽ 50%			conditions ⩽ 60%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	83.50%	78.61%	87.06%	90.94%	86.39%	92.78%	96.50%	94.44%	97.33%	98.33%	96.56%	98.61%
City-ranking	77.30%	71.56%	73.47%	83.47%	76.99%	80.90%	86.09%	79.78%	83.43%	86.09%	79.78%	83.43%
Winequality-red	75.18%	73.71%	83.12%	77.18%	75.18%	84.95%	79.18%	76.83%	86.28%	81.02%	78.08%	87.78%
Quake	71.39%	73.22%	57.47%	71.39%	73.22%	57.47%	79.68%	78.56%	68.32%	79.68%	78.56%	68.32%
Winequality-white	74.73%	76.04%	84.11%	77.68%	77.35%	85.32%	79.66%	77.96%	86.95%	80.98%	78.38%	88.53%
Dataset ID	**conditions ⩽ 70%**			**conditions ⩽ 80%**			**conditions ⩽ 90%**			**conditions ⩽ 100%**		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	99.39%	98.44%	99.11%	99.39%	98.67%	99.17%	99.39%	98.78%	99.17%	99.39%	98.83%	99.17%
City-ranking	88.17%	80.58%	85.20%	88.17%	80.58%	85.20%	89.62%	81.31%	87.15%	90.50%	81.84%	87.90%
Winequality-red	82.50%	79.17%	88.43%	83.70%	80.06%	88.92%	84.72%	80.83%	89.28%	86.13%	82.05%	90.01%
Quake	79.68%	78.56%	68.32%	83.30%	81.21%	70.52%	83.30%	81.21%	70.52%	85.55%	82.69%	70.57%
Winequality-white	81.71%	78.64%	89.44%	82.30%	78.91%	89.93%	82.62%	79.12%	90.18%	82.98%	79.49%	90.50%

(b) With threshold $b = 0.5$.

Dataset ID	conditions ⩽ 30%			conditions ⩽ 40%			conditions ⩽ 50%			conditions ⩽ 60%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	82.25%	77.83%	86.06%	90.00%	85.33%	92.00%	95.83%	92.78%	96.11%	97.67%	95.83%	97.56%
City-ranking	76.09%	70.55%	72.52%	82.87%	76.59%	80.74%	85.29%	79.74%	83.57%	85.29%	79.74%	83.57%
Winequality-red	75.33%	73.09%	82.60%	78.93%	74.68%	84.37%	80.90%	76.48%	85.95%	81.99%	77.69%	87.51%
Quake	71.06%	71.75%	56.97%	71.06%	71.75%	56.97%	79.95%	77.26%	66.88%	79.95%	77.26%	66.88%
Winequality-white	75.01%	76.40%	83.58%	78.21%	77.94%	85.04%	80.19%	78.50%	86.90%	81.50%	78.89%	88.48%
Dataset ID	**conditions ⩽ 70%**			**conditions ⩽ 80%**			**conditions ⩽ 90%**			**conditions ⩽ 100%**		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	98.89%	97.94%	98.67%	98.94%	98.00%	98.83%	98.94%	98.11%	98.89%	98.94%	98.11%	98.89%
City-ranking	87.82%	80.66%	85.71%	87.82%	80.66%	85.71%	89.47%	81.45%	87.50%	90.42%	81.95%	88.15%
Winequality-red	83.03%	78.89%	88.28%	84.11%	79.88%	88.91%	85.06%	80.63%	89.30%	86.19%	81.68%	89.87%
Quake	79.95%	77.26%	66.88%	84.05%	79.77%	69.13%	84.05%	79.77%	69.13%	86.17%	81.53%	69.18%
Winequality-white	82.27%	79.19%	89.41%	82.91%	79.42%	89.91%	83.24%	79.62%	90.12%	83.56%	79.94%	90.46%

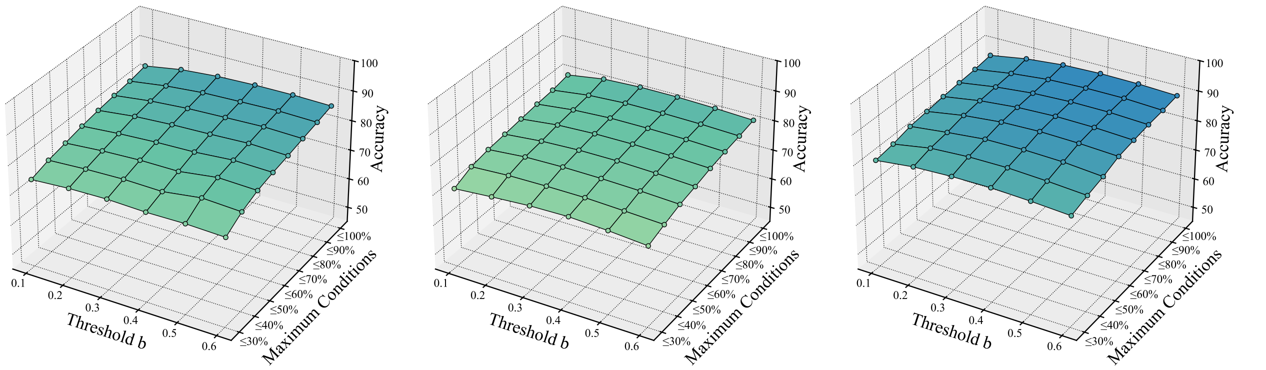
(c) With thresholds $b = 0.6$.

Dataset ID	conditions ⩽ 30%			conditions ⩽ 40%			conditions ⩽ 50%			conditions ⩽ 60%		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	81.25%	76.78%	84.89%	87.94%	82.72%	90.00%	94.94%	92.11%	94.44%	96.44%	94.61%	95.78%
City-ranking	74.34%	69.45%	71.29%	81.25%	75.96%	80.00%	84.11%	79.40%	83.04%	84.11%	79.40%	83.04%
Winequality-red	74.87%	72.10%	81.81%	78.25%	74.11%	83.52%	80.42%	75.89%	84.92%	81.65%	77.04%	86.86%
Quake	70.59%	70.54%	56.40%	70.59%	70.54%	56.40%	79.82%	76.25%	65.17%	79.82%	76.25%	65.17%
Winequality-white	74.89%	76.28%	82.53%	78.28%	78.10%	84.65%	80.32%	78.70%	86.47%	81.64%	79.12%	88.01%
Dataset ID	**conditions ⩽ 70%**			**conditions ⩽ 80%**			**conditions ⩽ 90%**			**conditions ⩽ 100%**		
	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS	TOPSIS	VIKOR	COPRAS
Pollution	98.00%	96.83%	97.17%	98.06%	97.11%	97.44%	98.06%	97.39%	97.61%	98.06%	97.44%	97.61%
City-ranking	86.64%	80.51%	85.53%	86.64%	80.51%	85.53%	88.55%	81.43%	87.32%	89.72%	81.95%	87.85%
Winequality-red	82.59%	78.39%	87.65%	83.73%	79.39%	88.32%	84.75%	80.17%	88.70%	85.90%	81.09%	89.33%
Quake	79.82%	76.25%	65.17%	84.43%	79.10%	67.75%	84.43%	79.10%	67.75%	86.72%	80.65%	67.86%
Winequality-white	82.38%	79.43%	88.98%	83.04%	79.64%	89.49%	83.38%	79.82%	89.73%	83.67%	80.08%	90.05%



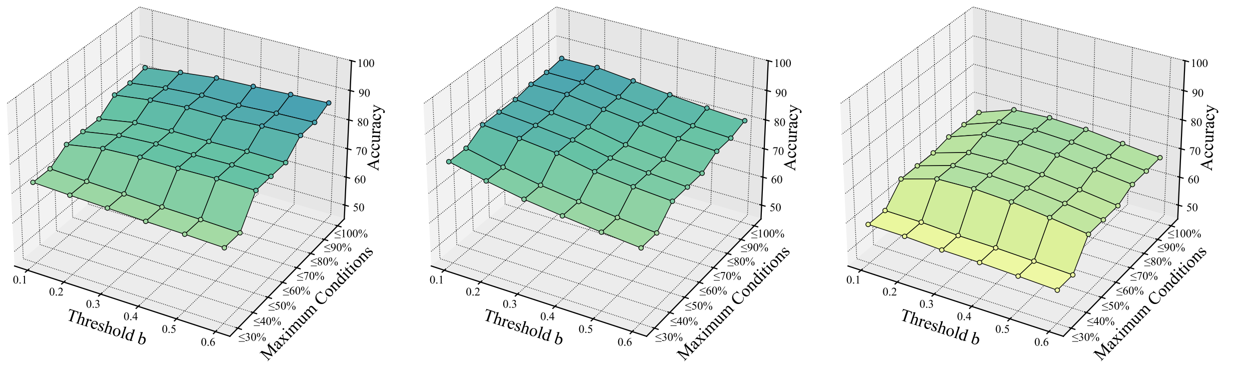
(a) Explanations of TOPSIS. (b) Explanations of VIKOR. (c) Explanations of COPRAS.

Fig. 6. Visualizations of experimental results on City-Ranking.



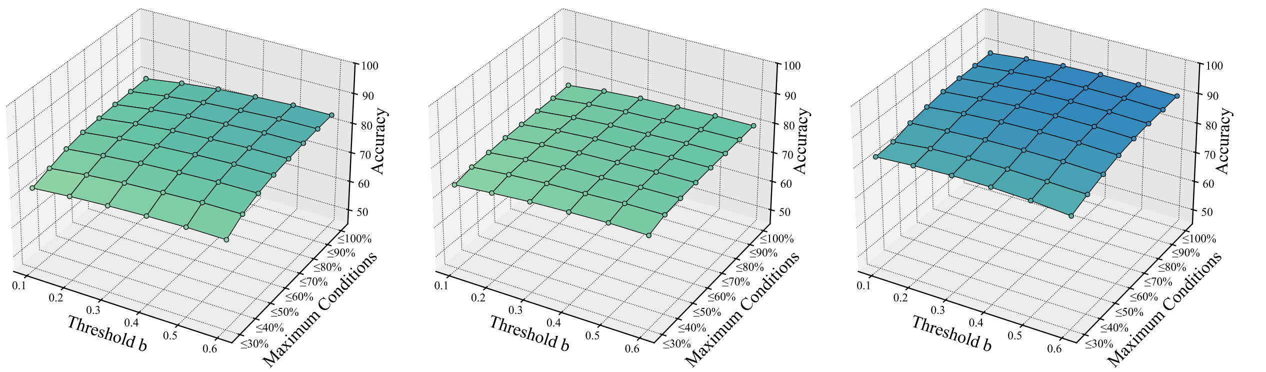
(a) Explanations of TOPSIS. (b) Explanations of VIKOR. (c) Explanations of COPRAS.

Fig. 7. Visualizations of experimental results on Winequality-red.



(a) Explanations of TOPSIS. (b) Explanations of VIKOR. (c) Explanations of COPRAS.

Fig. 8. Visualizations of experimental results on Quake.



(a) Explanations of TOPSIS. (b) Explanations of VIKOR. (c) Explanations of COPRAS.

Fig. 9. Visualizations of experimental results on Winequality-white.

The computational cost of Algorithm 3 varies depending on the size of the dataset, especially the number of criteria used to build the tree. In this case, we collect the accuracies and computational times for each dataset and then average them through different thresholds b and MCDM methods. The relationship between accuracy and computational cost across datasets reveals a trade-off between model performance and efficiency. As the maximum conditions increase, the accuracies and the computational times increase correspondingly. In some datasets, the accuracy may have small gains, but the computational time increases a lot. For instance, the "City-ranking" dataset achieves nearly peak accuracy at 70% and 80% conditions with a modest compute time, but further increases in conditions yield small additional accuracy but a large amount of runtime. Similarly, "Winequality-red" and "Winequality-white"

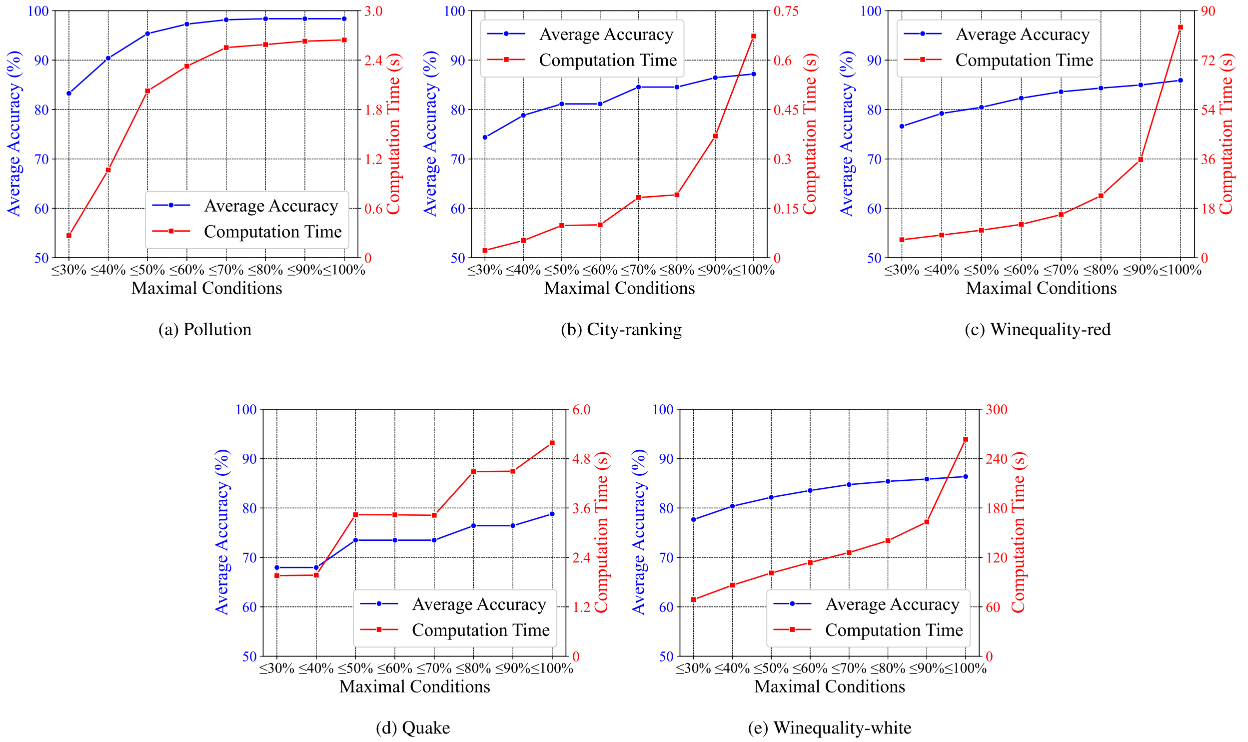


Fig. 10. The results of accuracies and computational costs.

show that accuracy improves only marginally as computing time rises a large amount. This highlights the importance of tailoring maximum conditions to balance performance and practicality.

6. Conclusions and future work

In summary, this paper presented an explainable multi-criteria decision-making (XMCDM) framework based on three-way decision and SMV spaces. Our framework enriches the explainability of existing MCDM methods by incorporating trilevel explanations with them, including explainable MC data preparation at symbols-level, explainable MC decision analysis at meaning-level, and explainable decision support at value-level. At the symbols level, we listed primary tasks as explaining problem definition and explaining data collection. At the meaning level, we explored the working principles of TOPSIS and introduced three-way rankings, which provide individual rankings of alternatives with respect to each single criterion. At the value level, we modified the ID3 algorithm to create a ternary decision tree and induce rules to explain the rankings generated by a certain MCDM method.

We conducted a series of experiments on five real-life datasets and tested our framework with three popular MCDM methods. The results showed that our approach is feasible and adaptable to various data characteristics. During the exploration and pre-processing of the datasets, we found that most data presented a normal distribution, while there was still a subset of data presenting a non-normal distribution. It is a very interesting direction to further investigate statistical three-way rankings regarding various distribution types. Jointly considering the types of distribution, it is much meaningful to create an adaptive method to fulfill the three-way ranking part in our proposed XMCDM framework. We also demonstrated that the explainable MC decision support is straightforward and easy to understand, and rule-based explanations can build decision makers' understanding of the reasons behind the suggested actions. The rules are also helpful in determining the ranking of novel decision alternatives.

The proposed XMCDM framework has several potential applications in various domains. For example, in product planning, the manager is able to understand the positions of both their product and competitive products. They may have a deep knowledge of product advantages and disadvantages through the explanations. Additionally, in financial management, the XMCDM framework can be used to identify the most suitable investment opportunities for customers by explaining the comparison between different investments. The rules generated within the framework can guide investors in making wise decisions.

While the experimental results demonstrate the feasibility and effectiveness of the proposed framework, there are still several areas for future research. One area is to explore the application of the framework to larger and more complex datasets, and to develop more efficient algorithms to handle the increased computational complexity. To achieve optimal performance, hyperparameter tuning and optimization can be tailored to individual datasets. Further processing can be done in the generation of decision trees, for example, we may abandon certain branches whose corresponding decision rule comes with low accuracy or low coverage. This pruning may lower the model's performance but increase the quality of decision rules. Apart from that, the framework can be extended to deal with other decision-making paradigms, such as group decision-making and fuzzy decision-making, to improve its flexibility and adaptability to various decision scenarios.

In conclusion, the proposed XMCDM framework contributes to the development of explainability of MCDM that provides trilevel explanations. It is a significant step towards addressing the need for more explainability in multi-criteria decision-making scenarios, especially providing explainable decision support. Our approach can enhance the decision-making process in various domains and pave the way for further research in this field. It should also be pointed out that the proposed trilevel framework is a high-level abstract construct. It serves the purposes of understanding the research problems, raising up the basic questions, and offering possible solutions. On the other hand, extensive efforts are needed to move from the conceptual model to actual methods.

CRediT authorship contribution statement

Chengjun Shi: Writing – original draft, Software, Methodology, Conceptualization. **Yiyu Yao:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors thank reviewers and editors for their constructive and valuable comments. This work was partially supported by a Discovery Grant from NSERC, Canada.

Data availability

Data will be made available on request.

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