$$\int \frac{dq_n dp_n}{2\pi} e^{-\frac{p_n^2}{2m}\tau - ip_n(q_{n+1}-q_n) - iJ_n p_n} =$$

$$\int \frac{dq_n dp_n}{2\pi} e^{-\frac{p_n^2}{2m}\tau - ip_n(q_{n+1}-q_n+J_n)} =$$

$$= \int dq_n \sqrt{\frac{m}{2\pi\tau}} e^{-\frac{m(q_{n+1}-q_n+J_n)^2}{2\tau^2}\tau}$$

$$\langle p_n \rangle = i\frac{\partial}{\partial J_n}\int dq_n \sqrt{\frac{m}{2\pi\tau}} e^{-\frac{m(q_{n+1}-q_n+J_n)^2}{2\tau^2}\tau}\Big/_{J_n=0} =$$

$$= i\sqrt{\frac{m}{2\pi\tau}}\int dq_n \left(-\frac{m\tau}{\cancel{2}\tau^2}\cancel{2}(q_{n+1}-q_n)\right) e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau^2}\tau} =$$

$$= \int dq_n \left(-i\frac{m(q_{n+1}-q_n)}{\tau}\right)\sqrt{\frac{m}{2\pi\tau}} e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau^2}\tau} =$$

$$= \left\langle -im\frac{q_{n+1}-q_n}{\tau}\right\rangle$$

$$\int \frac{dq_n\, dp_n}{2\pi}\, e^{-\sqrt{p_n^2+m^2}\,\tau - i p_n (q_{n+1}-q_n) - i J_n p_n} =$$

$$= \int \frac{dq_n\, dp_n}{2\pi}\, e^{-\sqrt{p_n^2+m^2} - i p_n (q_{n+1}-q_n+J_n)} =$$

$$= \int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}}\, K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}\right)$$

$$\langle p_n \rangle^* = i \frac{\partial}{\partial J_n} \int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}}\, K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}\right) =$$

$$= i \frac{\partial}{\partial J_n} \int dq_n \frac{m^2\tau}{\pi}\, \frac{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}\right)}{m\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}} =$$

$$= \frac{i m^2 \tau}{\pi} \int dq_n \frac{\partial}{\partial J_n} \frac{K_1(x)}{x}\, ; \quad x = m\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}$$

$$\frac{\partial}{\partial J_n} x = m \frac{\cancel{2}(q_{n+1}-q_n+J_n)}{\cancel{2}\sqrt{\tau^2+(q_{n+1}-q_n+J_n)^2}} = \frac{m^2(q_{n+1}-q_n+J_n)}{x}$$

$$\langle p_n \rangle^* = \frac{i m^2 \tau}{\pi} \int dq_n \left(\frac{K_1'(x)}{x}\, \frac{m^2(q_{n+1}-q_n+J_n)}{x} - \right.$$

$$\left. - \frac{K_1(x)}{x^2}\, \frac{m^2(q_{n+1}-q_n+J_n)}{x} \right) =$$

$$= \frac{i m^2 \tau}{\pi} \int dq_n \frac{m^2(q_{n+1}-q_n+J_n)}{x^3} \left(x K_1'(x) - K_1(x) \right) =$$

$$= \frac{i m^2 \tau}{\pi} \int dq_n \, \frac{m^2(q_{n+1} - q_n + J_n)}{x^3} \left(-x K_0(x) - 2 K_1(x) \right)$$

$$\langle p_n \rangle = \langle \check{p}_n \rangle \Big|_{J_n = 0} =$$

$$= \frac{i m^2 \tau}{\pi} \int dq_n \, \frac{m^2(q_{n+1} - q_n)}{m^3 \left(\tau^2 + (q_{n+1} - q_n)^2\right)^{3/2}} \left(-m\sqrt{\tau^2 + (q_{n+1} - q_n)^2} \, K_0\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right) \right.$$

$$\left. - 2 K_1\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right) \right) =$$

$$= -\int dq_n \, \frac{i m \tau (q_{n+1} - q_n)}{\pi \left(\tau^2 + (q_{n+1} - q_n)^2\right)^{3/2}} K_1\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right) \Bigg[2 +$$

$$+ m\sqrt{\tau^2 + (q_{n+1} - q_n)^2} \, \frac{K_0\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)}\right)}{K_1\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right)} \Bigg] =$$

$$= \int dq_n \, \frac{m \tau}{\pi \sqrt{\tau^2 + (q_{n+1} - q_n)^2}} K_1\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right) \times$$

$$\times \frac{(-i)(q_{n+1} - q_n)}{\tau^2 + (q_{n+1} - q_n)^2} \left[2 + m\sqrt{\tau^2 + (q_{n+1} - q_n)^2} \, \frac{K_0\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right)}{K_1\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right)} \right] =$$

$$= -\left\langle \frac{i (q_{n+1} - q_n)}{\tau^2 + (q_{n+1} - q_n)^2} \left[2 + m\sqrt{\tau^2 + (q_{n+1} - q_n)^2} \, \frac{K_0\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right)}{K_1\left(m\sqrt{\tau^2 + (q_{n+1} - q_n)^2}\right)} \right] \right\rangle$$