$$\int \frac{dq_n\, dp_n}{2\pi}\, e^{-\frac{p_n^2}{2m}\tau - i p_n (q_{n+1}-q_n)} =$$

$$= \int \frac{dq_n\, dp_n}{2\pi}\, e^{-\frac{\tau}{2m} p_n^2 - 2p_n \frac{i(q_{n+1}-q_n)}{2}} =$$

$$= \int \frac{dq_n\, dp_n}{2\pi}\, e^{-\frac{\tau}{2m}\left(p_n^2 - 2p_n \frac{i(q_{n+1}-q_n)}{\cancel{2}} \frac{\cancel{2}m}{\tau}\right)} =$$

$$= \int \frac{dq_n\, dp_n}{2\pi}\, e^{-\frac{\tau}{2m}\left(p_n - i\frac{m}{\tau}(q_{n+1}-q_n)\right)^2 - \frac{\tau}{2m}\frac{m^2}{\tau^2}(q_{n+1}-q_n)^2} =$$

$$= \int \frac{dq_n\, d\tilde{p}_n}{2\pi}\, e^{-\frac{\tau}{2m}\tilde{p}_n^2 - \frac{m(q_{n+1}-q_n)^2}{2\tau^2}\tau} =$$

$$= \int \frac{dq_n}{2\pi} \sqrt{\frac{\pi 2m}{\tau}}\, e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau^2}\tau} =$$

$$= \int dq_n \sqrt{\frac{m}{2\pi\tau}}\, e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau^2}\tau}$$

$$\left\langle \frac{p_n^2}{2m} \right\rangle = -\frac{\partial}{\partial \tau} \int \frac{dq_n\, dp_n}{2\pi}\, e^{-\frac{p_n^2}{2m}\tau - i p_n (q_{n+1}-q_n)}$$

$$\left\langle \frac{p_n^2}{2m} \right\rangle = -\frac{\partial}{\partial \tau} \int dq_n \sqrt{\frac{m}{2\pi\tau}}\, e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau}} =$$

$$= -\int dq_n \sqrt{\frac{m}{2\pi}} \left(-\frac{1}{2}\frac{1}{\tau^{3/2}} + \frac{m(q_{n+1}-q_n)^2}{2\tau^2}\frac{1}{\sqrt{\tau}}\right) e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau}} =$$

$$= \int dq_n \sqrt{\frac{m}{2\pi\tau}} \left(\frac{1}{2\tau} - \frac{m(q_{n+1}-q_n)^2}{2\tau^2}\right) e^{-\frac{m(q_{n+1}-q_n)^2}{2\tau}}$$

$$\Rightarrow \left\langle \frac{p_n^2}{2m} \right\rangle = \frac{1}{2\tau} - \left\langle \frac{m(q_{n+1}-q_n)^2}{2\tau^2} \right\rangle$$

$$\int \frac{dq_n\, dp_n}{2\pi}\, e^{-\sqrt{p_n^2+m^2}\,\tau - i p_n (q_{n+1}-q_n)} =$$

$$= \int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n)^2}} K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)$$

$$\left\langle \sqrt{p_n^2+m^2} \right\rangle = -\frac{\partial}{\partial\tau} \int \frac{dq_n\, dp_n}{2\pi}\, e^{-\sqrt{p_n^2+m^2}\,\tau - i p_n (q_{n+1}-q_n)} =$$

$$= -\frac{\partial}{\partial\tau} \int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n)^2}} K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) =$$

$$= -\int \frac{dq_n\, m}{\pi} \left[\frac{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{\sqrt{\tau^2+(q_{n+1}-q_n)^2}} + \tau K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)\left(-\frac{1}{2}\right)\frac{2\tau}{\left(\tau^2+(q_{n+1}-q_n)^2\right)^{3/2}} \right.$$

$$\left. + \frac{\tau}{\sqrt{\tau^2+(q_{n+1}-q_n)^2}} K_1'\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) m \frac{2\tau}{2\sqrt{\tau^2+(q_{n+1}-q_n)^2}} \right] =$$

$$= -\int \frac{dq_n\, m}{\pi} \left[\frac{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{\sqrt{\tau^2+(q_{n+1}-q_n)^2}} - \frac{\tau^2 K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{\left(\tau^2+(q_{n+1}-q_n)^2\right)^{3/2}} + \right.$$

$$\left. + \frac{m\tau^2}{\tau^2+(q_{n+1}-q_n)^2} K_1'\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) \right]$$

$$= -\int \frac{dq_n\, m}{\pi} \left[\frac{(q_{n+1}-q_n)^2 K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{\left(\tau^2+(q_{n+1}-q_n)^2\right)^{3/2}} + \right.$$

$$\left. + \frac{m\tau^2}{\tau^2+(q_{n+1}-q_n)^2} K_1'\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) \right] =$$

$$\left\{ \begin{aligned} &\left(x^\nu K_\nu(x)\right)' = -x^\nu K_{\nu-1}(x) \\ &\nu x^{\nu-1} K_\nu(x) + x^\nu K_\nu'(x) = -x^\nu K_{\nu-1}(x) \\ &\quad \Rightarrow K_\nu'(x) = -K_{\nu-1}(x) - \frac{\nu}{x} K_\nu(x) \\ &\Rightarrow K_1'(x) = -K_0(x) - \frac{1}{x} K_1(x) \end{aligned} \right.$$

$$= -\int \frac{dq_n\, m}{\pi} \left[\frac{(q_{n+1}-q_n)^2 K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{\left(\tau^2+(q_{n+1}-q_n)^2\right)^{3/2}} + \right.$$

$$\left. + \frac{m\tau^2}{\tau^2+(q_{n+1}-q_n)^2} \left(-K_0\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) - \frac{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{m\sqrt{\tau^2+(q_{n+1}-q_n)^2}} \right) \right] =$$

$$= -\int \frac{dq_n\, m}{\pi} \left(\frac{(q_{n+1}-q_n)^2 - \tau^2}{\left((q_{n+1}-q_n)^2+\tau^2\right)^{3/2}} K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) - \right.$$

$$\left. - \frac{m\tau^2}{\tau^2+(q_{n+1}-q_n)^2} K_0\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) \right) =$$

$$= \int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n)^2}} K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)\left(\frac{m\tau}{\sqrt{\tau^2+(q_{n+1}-q_n)^2}} \frac{K_0\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)} -\right.$$

$$\left. - \frac{(q_{n+1}-q_n)^2-\tau^2}{\tau\left((q_{n+1}-q_n)^2+\tau^2\right)}\right) =$$

$$= \int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n)^2}} K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)\left(\frac{m\tau}{\sqrt{\tau^2+(q_{n+1}-q_n)^2}} \frac{K_0\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)} +\right.$$

$$\left. + \frac{\tau^2-(q_{n+1}-q_n)^2}{\tau\left(\tau^2+(q_{n+1}-q_n)^2\right)}\right) =$$

$$= \left\langle \frac{m\tau}{\sqrt{\tau^2+(q_{n+1}-q_n)^2}} \cdot \frac{K_0\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)}{K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right)} + \frac{\tau^2-(q_{n+1}-q_n)^2}{\tau\left(\tau^2+(q_{n+1}-q_n)^2\right)} \right\rangle$$

$$\int dq_n \frac{m\tau}{\pi\sqrt{\tau^2+(q_{n+1}-q_n)^2}} K_1\left(m\sqrt{\tau^2+(q_{n+1}-q_n)^2}\right) =$$

$$= \int dq_n \frac{m\cancel{\tau}}{\pi\cancel{\tau}\left(1+\left(\frac{q_{n+1}-q_n}{\tau}\right)^2\right)^{1/2}} \sqrt{\frac{\pi}{2m\tau\sqrt{1+\left(\frac{q_{n+1}-q_n}{\tau}\right)^2}}}\, e^{-m\tau\sqrt{1+\left(\frac{q_{n+1}-q_n}{\tau}\right)^2}} =$$

$$= \int dq_n \left(\frac{m}{2\pi\tau}\right)^{1/2}\left(1+\left(\frac{q_{n+1}-q_n}{\tau}\right)^2\right)^{-3/4} e^{-m\tau\left(1+\left(\frac{q_{n+1}-q_n}{\tau}\right)^2\right)^{1/2}}$$

$$m\to\infty \qquad \left(\frac{q_{n+1}-q_n}{\tau}\right)\to 0 \;\Big/\; m\left(\frac{q_{n+1}-q_n}{\tau}\right)^2 = \text{const.}$$