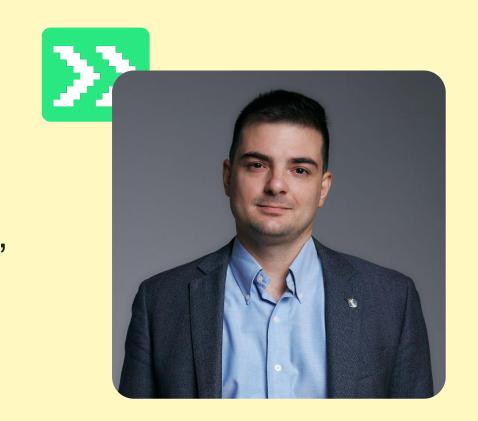
Value-based подходы в RL; Q-learning

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Содержание

- Reinforcement Learning problem recap
- Reward discounting
- State-value and action-value functions
- ApproximateQ-learning
- **1** \$&%__!\$%

References

These slides are almost the exact copy of Practical RL course week 2 slides by Shvechikov Pavel.

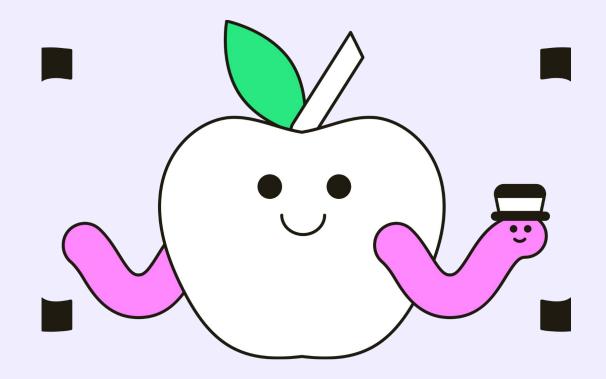
Special thanks to YSDA team for making them publicly available.

Original slides link: https://github.com/yandexdataschool/Practical_RL

Reinforcement Learning problem recap

01

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Reinforcement learning

- Given: Usually no reference answers
 - Objects and reference answers
 - imes Loss/objective function $\int\! L(\hat{y},y)$
 - \circ Model family $f \in \mathcal{F}, f: \mathcal{X} \longrightarrow \mathcal{Y}$
- Goal:
 - \circ $% \mathbf{r}_{0}$ Find optimal mapping $f^{*}=\arg\min_{f}L(f(x),y)$

 $x \in \mathcal{X}$ E.g. want the robot to walk

Usually even hard to

non-differentiable

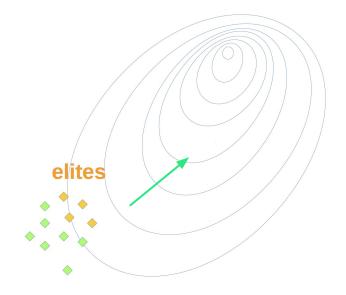
Previously

The MDP formalism

State, Action, Reward, next State

Cross-Entropy Method (CEM)

- easy to implement, good results
- rich theoretical background
- black box
 - no knowledge of environment
 - no knowledge of intermediate rewards



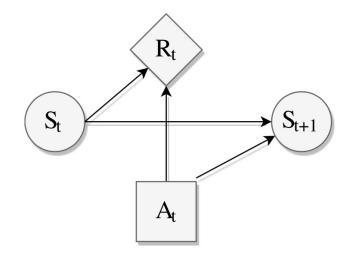
Improve on the CEM \rightarrow dive into the black box

Given dynamics, how to find an optimal policy?

Definition of Markov Decision Process

MDP is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$, where

- \circ S set of states of the world
- \bigcirc \mathcal{A} set of actions
- $\mathfrak{I}: \mathcal{S} \times \mathcal{A} \mapsto \triangle(\mathcal{S})$ state-transition function, giving us $p(s_{t+1} \mid s_t, a_t)$
- $\mathcal{R}: \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R} \text{reward function,}$ giving us $\mathbb{E}_R \left[R(s_t, a_t) \, | \, s_t, a_t \, \right].$



Markov property

$$p(r_t, s_{t+1} | s_0, a_0, r_0, ..., s_t, a_t) = p(r_t, s_{t+1} | s_t, a_t)$$

(next state, expected reward) depend on (previous state, action)

Goal: solve an MDP by finding an optimal policy

- 1. What is the objective?
 - a. Reward: discounting and design
 - b. Expected objective: state- and action-value function
- 2. How to evaluate the objective?
 - a. Bellman expectation equations
- 3. How to improve the objective?
 - a. Bellman optimality equations

Explaining goals to agent through reward

Reward hypothesis (R.Sutton)

Goals and purposes can be thought of as the maximization of the expected value of the cumulative sum of a received scalar signal

Explaining goals to agent through reward

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Cumulative reward is called a return:

$$G_t \triangleq R_t + R_{t+1} + R_{t+2} + \dots + R_T$$

E.g.: reward in **chess** – value of taken opponent's piece

Explaining goals to agent through reward

Reward hypothesis (R.Sutton)

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$$G_t \triangleq R_t + R_{t+1} + R_{t+2} + ... + R_T$$
immediate reward

E.g.: reward in **chess** – value of taken opponent's piece

E.g.: data center non-stop cooling system

States – temperature measurements
Actions – different fans speed
R = 0 for exceeding temperature thresholds
R = +1 for each second system is cool

What could go wrong with such a design?

E.g.: data center non-stop cooling system

States – temperature measurements

Actions – different fans speed

R = 0 for exceeding temperature thresholds

R = +1 for each second system is cool

What could go wrong with such a design?

Infinite return for non optimal behaviour!

$$G_t = 1 + 1 + 0 + 1 + 1 + 1 + 0 + \dots = \sum_{t=1}^{\infty} R_t = \infty$$

E.g.: moving to destination



State – position, velocities of joints

Actions – actuator forces to joints

$$R = \max(0, d(x, B) - d(x', B))$$

What could go wrong with such a design?

E.g.: moving to destination



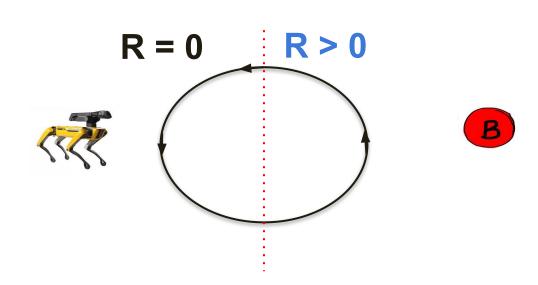
State – position, velocities of joints

Actions – actuator forces to joints

$$R = \max(0, d(x, B) - d(x', B))$$

What could go wrong with such a design?

Positive feedback loop!

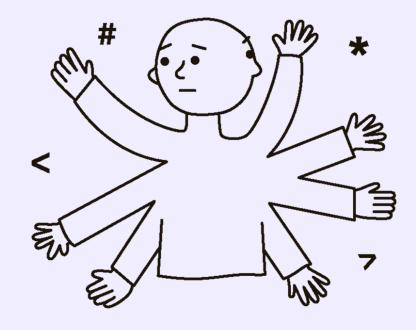


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Reward discounting



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Reward discounting

Get rid of infinite sum by discounting

$$0 \le \gamma < 1$$

$$G_t \triangleq R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + ... = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$
 discount factor

The same cake compared to today's one worth

- γ times less tomorrow
- γ^2 times less the day after tomorrow



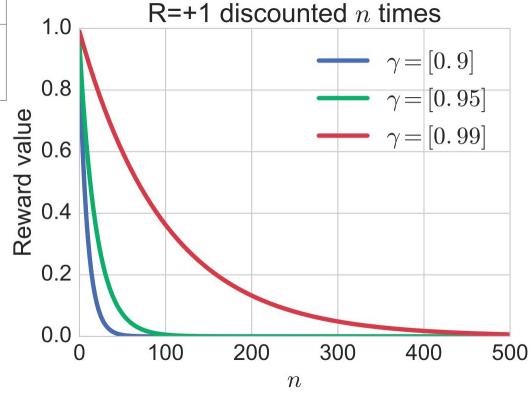
 γ will eat it day by day

Discounting makes sums finite

Maximal return for R = +1

γ	0.9	0.95	0.99
$\frac{1}{1-\gamma}$	10	20	100

$$G_0 = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$



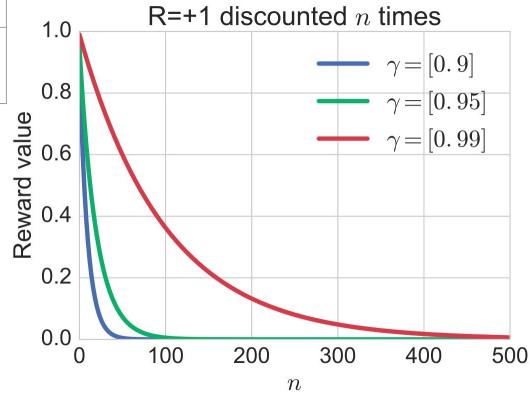
Discounting makes sums finite

Maximal return for R = +1

γ	0.9	0.95	0.99
$\frac{1}{1-\gamma}$	10	20	100

Any discounting changes optimisation task and its solution!

$$G_0 = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$



Discounting is inherent to humans

Quasi-hyperbolic

$$f(t) = \beta \gamma^t$$

Hyperbolic discounting

$$f(t) = \frac{1}{1 + \beta t}$$

Discounting is inherent to humans

Quasi-hyperbolic

$$f(t) = \beta \gamma^t$$

Hyperbolic discounting

$$f(t) = \frac{1}{1 + \beta t}$$

Mathematical convenience

$$G_t = R_t + \gamma (R_{t+1} + \gamma R_{t+2} + ...)$$

$$= R_t + \gamma G_{t+1}$$
Remember this one!
We will need it later

Discounting is a stationary end-of-effect model Any action affects (1) immediate reward (2) next state

Discounting is a stationary end-of-effect model

Any action affects (1) immediate reward (2) next state

Action indirectly affects future rewards

But how long does this effect lasts?

$$G_{0} = R_{0} + \gamma R_{1} + \gamma^{2} R_{2} + \dots + \gamma^{T} R_{T}$$

$$= (1 - \gamma) R_{0}$$

$$+ (1 - \gamma) \gamma (R_{0} + R_{1})$$

$$+ (1 - \gamma) \gamma^{2} (R_{0} + R_{1} + R_{2})$$

$$\dots$$

$$+ \gamma^{T} \cdot \sum_{t=0}^{T} R_{t}$$

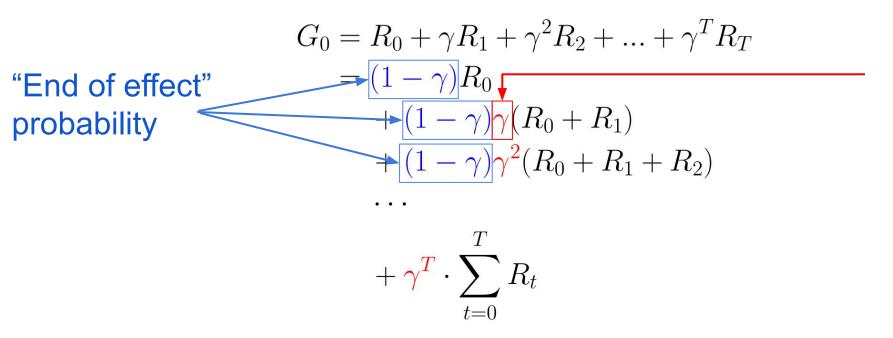
G is expected return under stationary end-of-effect model

Discounting is a stationary end-of-effect model

Any action affects (1) immediate reward (2) next state

Action indirectly affects future rewards

But how long does this effect lasts?



"Effect continuation" probability

G is expected return under stationary end-of-effect model

Reward design – don't shift, reward for WHAT

E.g.: chess – value of taken opponent's piece

Problem: agent will not have a desire to win!

E.g.: moving to destination

Problem: agent will not bother about the goal!

Reward design – don't shift, reward for WHAT

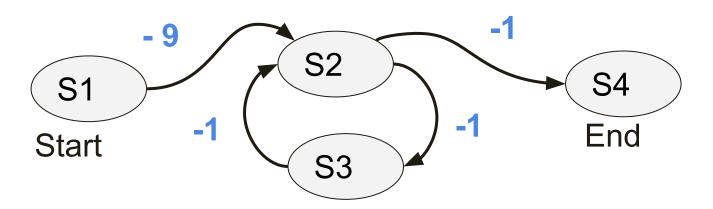
E.g.: chess – value of taken opponent's piece

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E.g.: moving to destination

Problem: agent will not bother about the goal!

Take away: reward only for WHAT, but never for HOW



Reward design – don't shift, reward for WHAT

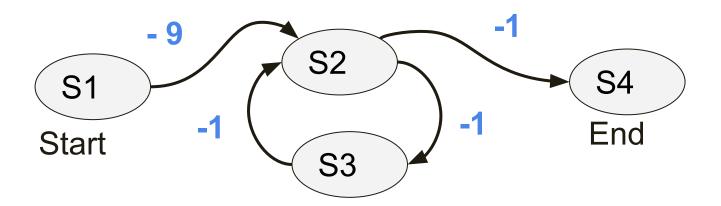
E.g.: chess – value of taken opponent's piece

Problem: agent will not have a desire to win!

E.g.: moving to destination

Problem: agent will not bother about the goal!

Take away: reward only for WHAT, but never for HOW



Take away: do not subtract mean from rewards

Faulty reward functions

- Reward for ball possession in soccer
 - Vibrating near the ball
- Cyclic behaviours



Reward design – scaling, shaping

What transformations do not change optimal policy?

Reward **scaling** – division by positive constant

May be useful in practise for approximate methods

Reward design - scaling, shaping

What transformations do not change optimal policy?

Reward scaling – division by positive constant

May be useful in practise for approximate methods

Powerd charing and a potential based charing function

Reward **shaping** – add a potential-based shaping function F(s, a, s'):

$$R'(s, a, s') = R(s, a, s') + F(s, a, s')$$

Intuition: when no discounting F adds as much as it subtracts from the total return

$$F(s, a, s') = \gamma \Phi(s') - \Phi(s)$$

State-value (V) and action-value (Q) functions





Optimal policy maximizes expected return

$$\mathbb{E}[G_{0}] = \mathbb{E}[R_{0} + \gamma R_{1} + \dots + \gamma^{T} R_{T}]$$

$$= \mathbb{E}_{E,\pi_{\theta}}[G_{0}]$$

$$= \mathbb{E}_{\pi_{\theta}}[G_{0}]$$

$$= \mathbb{E}[G_{0} \mid \pi_{\theta}]$$

$$= \mathbb{E}_{s_{0:T}}[G_{0}]$$

$$= \mathbb{E}_{s_{0:T}}[G_{0}]$$

$$= \mathbb{E}_{s_{0}}[\mathbb{E}_{a_{0}\mid s_{0}}[R_{0} + \mathbb{E}_{s_{1}\mid s_{0}, a_{0}}[\mathbb{E}_{a_{1}\mid s_{1}}[\gamma R_{1} + \dots]]]]]$$

$$= \sum_{t=0}^{T} \mathbb{E}_{(s_{t}, a_{t}) \sim p_{\theta}}[\gamma^{t} R_{t}]$$

$$= \mathbb{E}_{\tau \sim p_{\theta}(\tau)}[G(\tau)]$$

$$\tau \triangleq (s_{0}, a_{0}, s_{1}, \dots, a_{T-1}, s_{T})$$

$$p_{\theta}(\tau) = p(s_{0}) \prod_{t=0}^{T-1} \pi_{\theta}(a_{t}\mid s_{t}) p(s_{t+1}\mid s_{t}, a_{t})$$

v(s) is expected return conditional on state:

$$v_{\pi}(s) \triangleq \mathbb{E}_{\pi} [G_{t} | S_{t} = s]$$

$$= \mathbb{E}_{\pi} [R_{t} + \gamma G_{t+1} | S_{t} = s]$$

$$= \sum_{a} \pi(a | s) \sum_{r,s'} p(r, s' | s, a) [r + \gamma \mathbb{E}_{\pi} [G_{t+1} | S_{t+1} = s']]$$

$$= \sum_{a} \pi(a | s) \sum_{r,s'} p(r, s' | s, a) [r + \gamma v_{\pi}(s')]$$

Intuition: value of following policy π from state s

Slide source: Lecture 02 from Practical RL course by Shvechikov Pavel

v(s) is expected return conditional on state:

stochasticity in policy & environment
$$v_{\pi}(s) \triangleq \mathbb{E}_{\pi}[G_{t} \mid S_{t} = s]$$

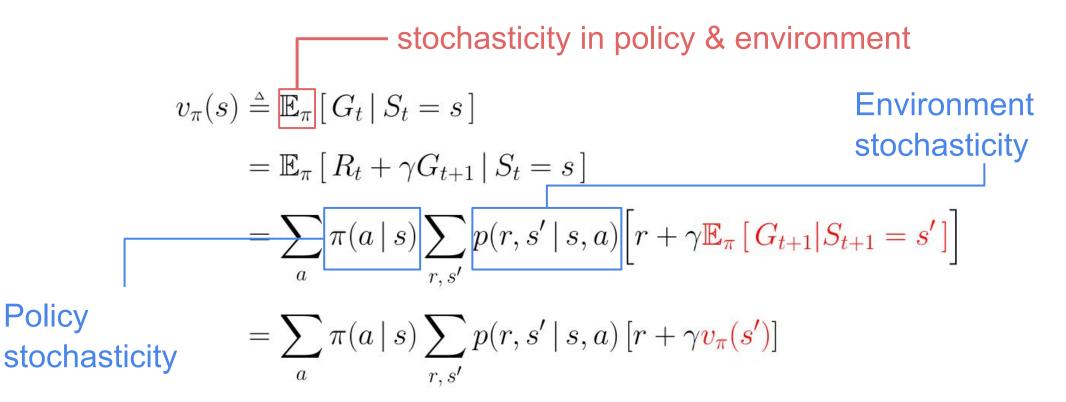
$$= \mathbb{E}_{\pi}[R_{t} + \gamma G_{t+1} \mid S_{t} = s]$$

$$= \sum_{a} \pi(a \mid s) \sum_{r,s'} p(r,s' \mid s,a) \left[r + \gamma \mathbb{E}_{\pi}[G_{t+1} \mid S_{t+1} = s']\right]$$

$$= \sum_{a} \pi(a \mid s) \sum_{r,s'} p(r,s' \mid s,a) \left[r + \gamma v_{\pi}(s')\right]$$

Intuition: value of following policy π from state s

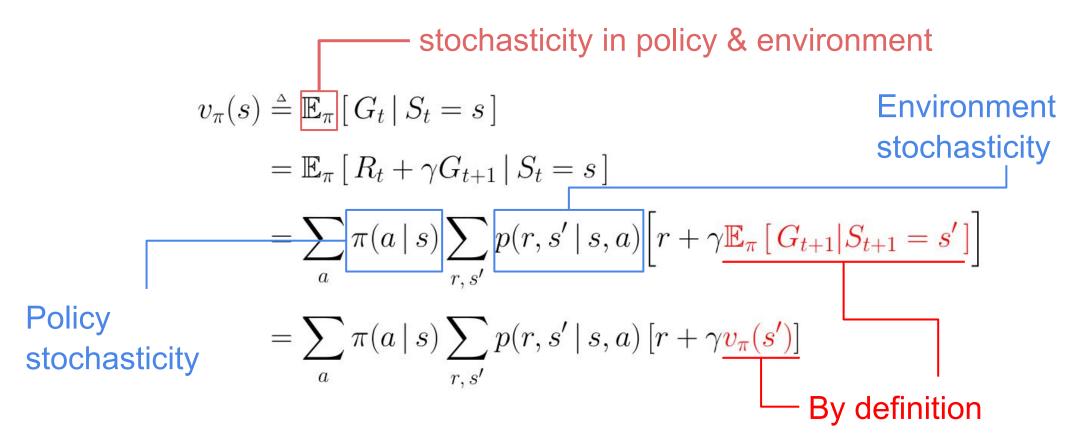
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v(s) is expected return conditional on state:



Intuition: value of following policy π from state s

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Action-value function q(s, a)

Is expected return conditional on state and action:

Intuition: value of following policy <u>after</u> commuting action **a** in state **s**

$$q_{\pi}(s, a) = \mathbb{E}_{\pi} [G_t | S_t = s, A_t = a]$$

$$= \mathbb{E}_{\pi} [R_t + \gamma G_{t+1} | S_t = s, A_t = a]$$

$$= \sum_{r, s'} p(r, s' | s, a) [r + \gamma \mathbb{E}_{\pi} [G_{t+1} | S_{t+1} = s']]$$

$$= \sum_{r, s'} p(r, s' | s, a) [r + \gamma v_{\pi}(s')]$$

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Action-value function q(s, a)

Is expected return conditional on state and action:

Intuition: value of following policy after commuting action **a** in state **s**

No policy stochasticity at first step

$$q_{\pi}(s, a) = \mathbb{E}_{\pi} [G_{t} | S_{t} = s, A_{t} = a]$$

$$= \mathbb{E}_{\pi} [R_{t} + \gamma G_{t+1} | S_{t} = s, A_{t} = a]$$

$$= \sum_{r, s'} p(r, s' | s, a) [r + \gamma \mathbb{E}_{\pi} [G_{t+1} | S_{t+1} = s']]$$

$$= \sum_{r, s'} p(r, s' | s, a) [r + \gamma v_{\pi}(s')]$$

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Relations between v(s) and q(s,a)

We already know how to write q(s,a) in terms of v(s)

$$q_\pi(s,a) = \sum_{r,s'} p(r,s'\,|\,s,a)\,[r + \gamma \pmb{v_\pi(s')}]$$
 What about v(s) in terms of q(s,a)?

Relations between v(s) and q(s,a)

We already know how to write q(s,a) in terms of v(s)

$$q_{\pi}(s, a) = \sum_{\mathbf{r}, \mathbf{r}'} p(r, s' \mid s, a) \left[r + \gamma \mathbf{v}_{\pi}(s') \right]$$

What about v(s) in terms of q(s,a)? r, s'

$$v_{\pi}(s) = \sum_{a} \pi(a \mid s) \sum_{r,s'} p(r,s' \mid s,a) [r + \gamma v_{\pi}(s')]$$
$$= \sum_{a} \pi(a \mid s) q_{\pi}(s,a)$$

So, we could now write q(s, a) in terms of q(s,a)!

$$q_{\pi}(s,a) = \sum_{s,s,t} p(r,s'\mid s,a) \Big[r + \gamma \sum_{s,t} \pi(a'\mid s') q_{\pi}(s',a') \Big]$$

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All in one slide

$$G_{t} = \sum_{t'=t}^{T} \gamma^{(t'-t)} r_{t'}$$

$$Q^{\pi}(s, a) = E_{\pi}[G_{t}|s_{t} = s, a_{t} = a]$$

$$V^{\pi}(s) = E_{\pi}[G_{t}|s_{t} = s]$$

Recurrent relations

$$Q^{\pi}(s, a) = E_{s_{t+1}}[r_t + \gamma V^{\pi}(s_{t+1})]$$

$$Q^{\pi}(s, a) = E_{s_{t+1}, a_{t+1} \sim \pi}[r_t + \gamma Q^{\pi}(s_{t+1}, a_{t+1})]$$

Optimal policy

For all
$$\pi, s, a$$
: $Q^{\pi^*}(s, a) \ge Q^{\pi}(s, a)$
$$\pi^*(s) = argmax_a Q^{\pi^*}(s, a)$$

Bellman optimality equation

$$Q^*(s_t, a) = E_{s_{t+1}}[r_t + \max_{a'} Q^*(s_{t+1}, a')]$$

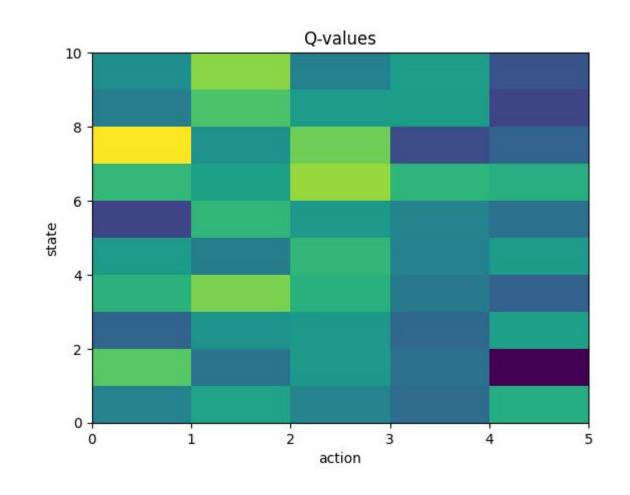
Optimal policy (for tabular case)

For all
$$\pi, s, a$$
: $Q^{\pi^*}(s, a) \ge Q^{\pi}(s, a)$

$$\pi^*(s) = argmax_a Q^{\pi^*}(s, a)$$

Bellman optimality equation

$$Q^*(s_t, a) = E_{s_{t+1}}[r_t + \max_{a'} Q^*(s_{t+1}, a')]$$



Q-learning

Training step

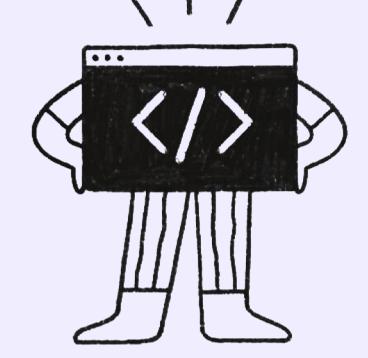
$$Q(s_t, a_t) \longleftarrow Q(s_t, a_t) + \alpha (r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t))$$

Q-learning as MSE minimization

$$L = (r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t))^2$$
Const

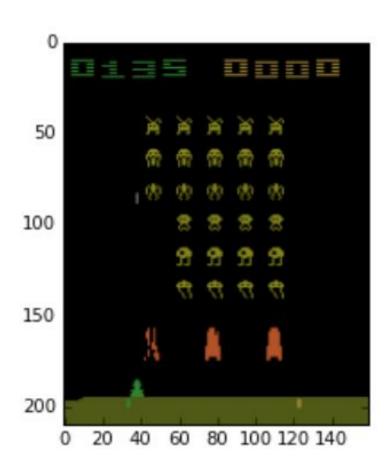
$$\nabla L = 2 \cdot \left(r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right)$$

Approximate Q-learning



04

Real world



How many states are there? Approximately

$$|S| = 2^{210 \cdot 160 \cdot 8 \cdot 3}$$

Problem

State space is usually large, sometimes continuous.

Two solutions:

- Binarize state space (obvious)
- Approximate agent with a function (crossentropy method)

Which one would you prefer for atari?

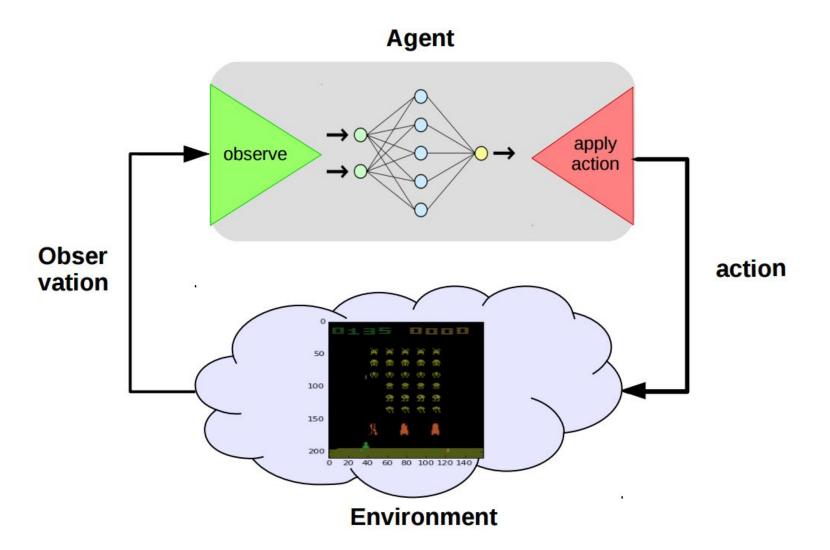
Problem

State space is usually large, sometimes continuous.

And so is action space;

Two solutions:

- Binarize state space Too many bins or handcrafted features
- Approximate agent with a function Let's pick this one



From tables to approximations

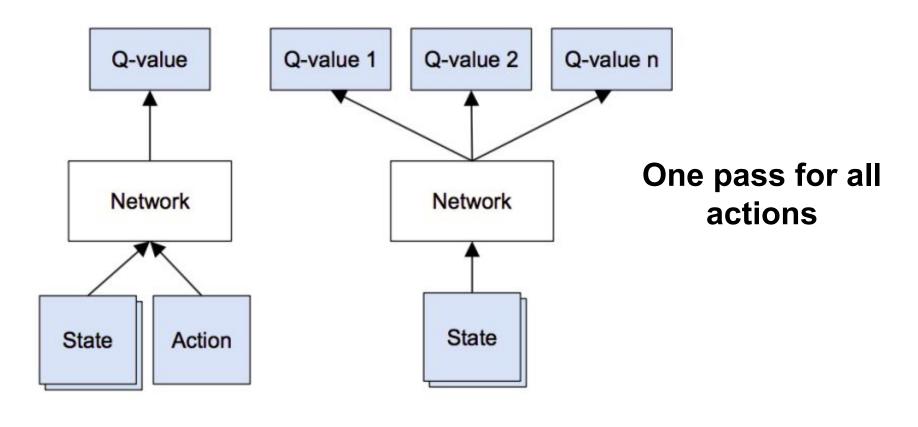
- Before:
 - For all states, for all actions, remember Q(s,a)
- Now:
 - Approximate Q(s,a) with some function
 - e.g. linear model over state features

$$argmin_{w,b}(Q(s_t,a_t)-[r_t+\gamma\cdot max_a,Q(s_{t+1},a')])^2$$

Question: should we use **classification** or **regression** model? (e.g. logistic regression Vs linear regression)

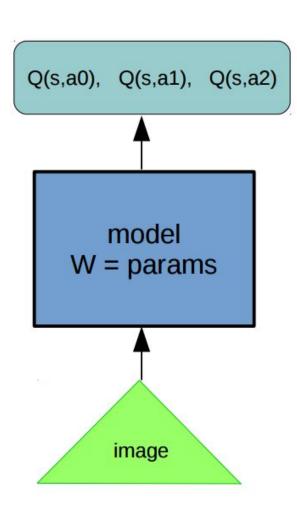
Possible architectures

Continuous control or large number of actions



Given **(s,a)** Predict Q(s,a) Given **s** predict all q-values Q(s,a0), Q(s,a1), Q(s,a2)

Approximate Q-learning



Q-values:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

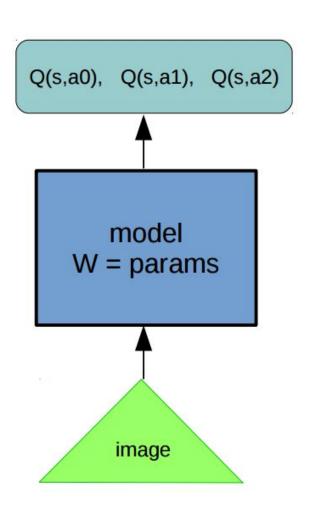
Objective:

$$L = (Q(s_t, a_t) - [r + \gamma \cdot max_{a'} Q(s_{t+1}, a')])^2$$
Const

Gradient step:

$$w_{t+1} = w_t - \alpha \cdot \frac{\delta L}{\delta w}$$

Approximate Q-learning



Objective:

$$L = (Q(s_t, a_t) - \hat{Q}(s_t, a_t))^2$$

Q-learning:

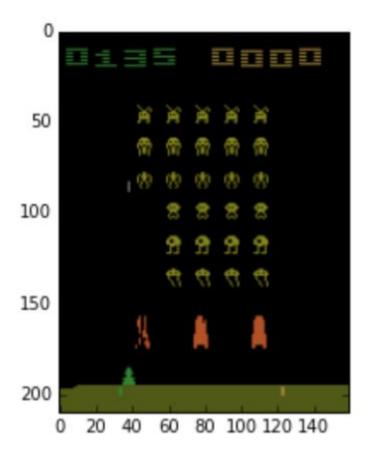
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot max_{a'} Q(s_{t+1}, a')$$

SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot Q(s_{t+1}, a_{t+1})$$

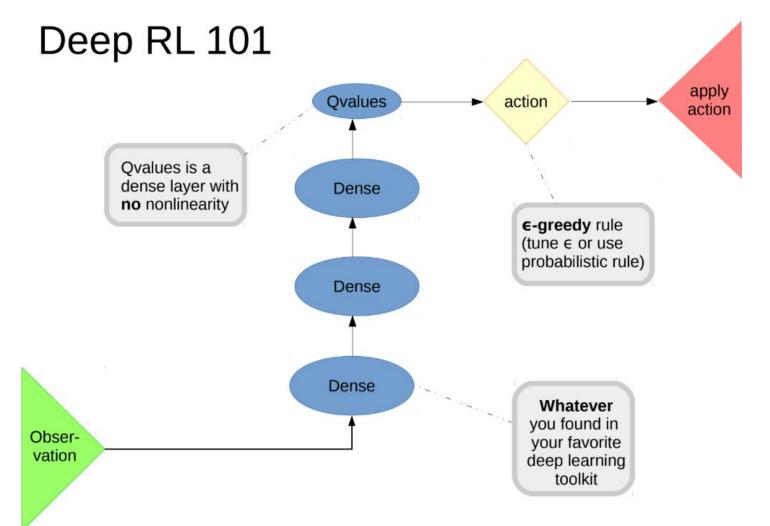
Expected Value SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \underset{a' \sim \pi(a|s)}{E} Q(s_{t+1}, a')$$



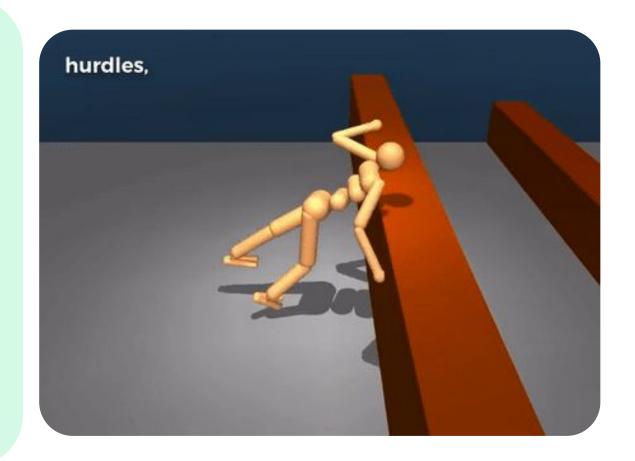
What kind of network digests images well?

Basic deep Q-learning



Outro

- We can approximate the reward function and exploit it!
- We just reformulate the RL problem and still use Supervised Learning approaches
- Remember the Markov assumptions (again!)

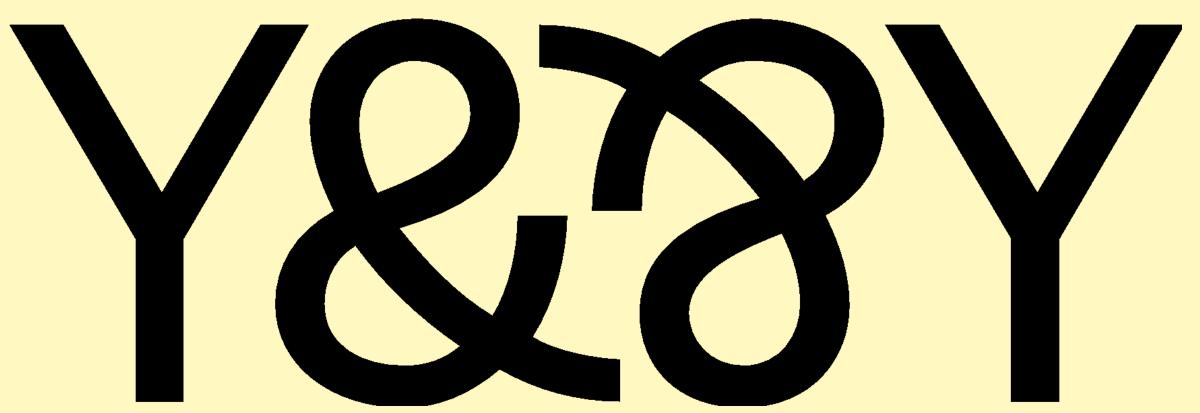




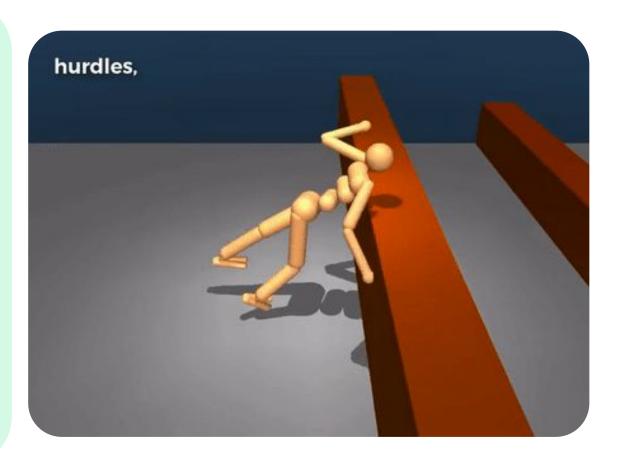
<u>@Rads_ai</u>
Канал Радослава
с текстовыми
разборами занятий

@Young and Yandex
Канал стажировок
Яндекса

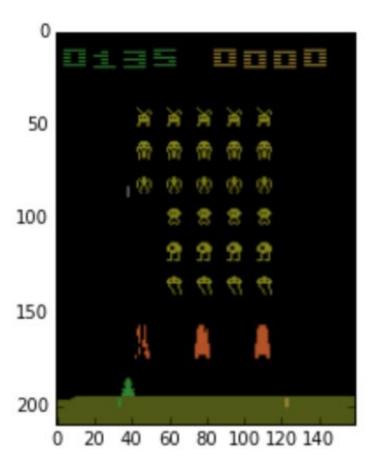




Outro..?



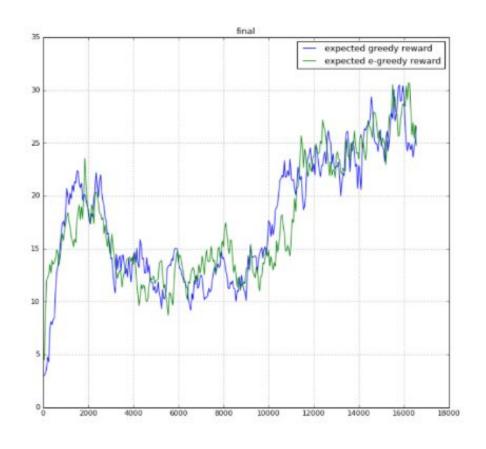




How bad it is if agent spends next 1000 ticks under the left rock? (while training)

Problem

- Training samples are not "i.i.d",
- Model forgets parts of environment it hasn't visited for some time
- Drops on learning curve
- Any ideas?



Multiple agents trick

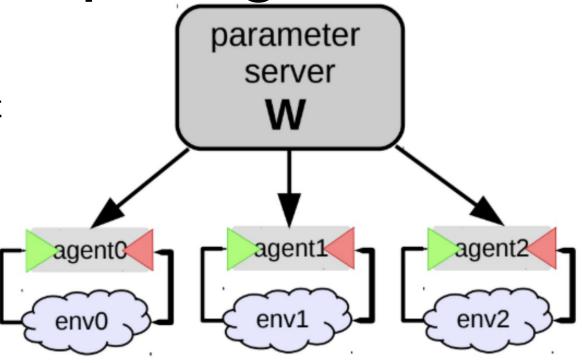
Idea: Throw in several agents with shared W.

 Chances are, they will be exploring different parts of the environment

More stable training

Requires a lot of interaction

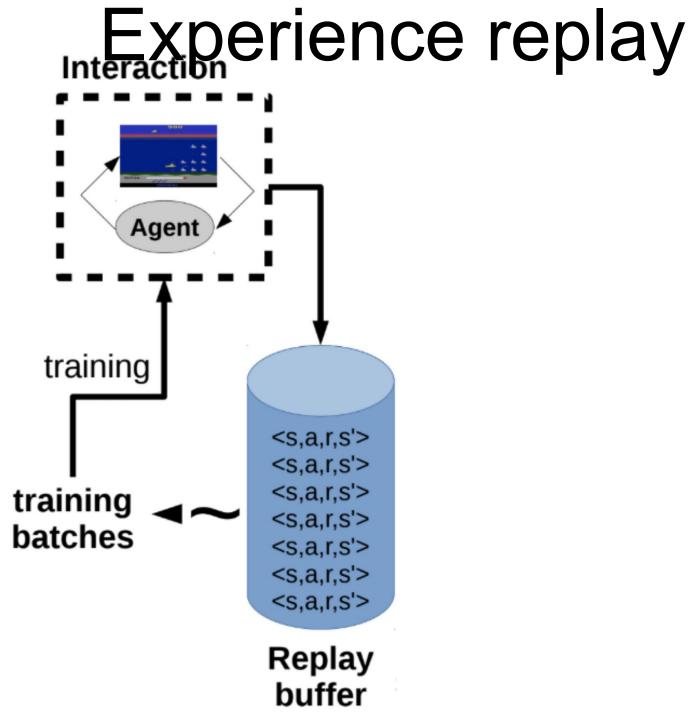
Question: your agent is a real robot car. Will there be any problems?



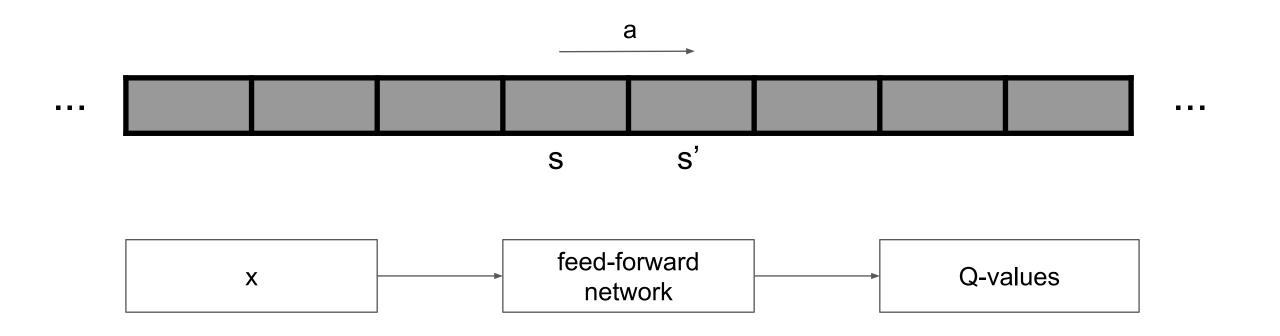


Idea: store several past interactions <*s*,*a*,*r*,*s*'> Train on random subsamples

- Atari DQN > 10⁵ interactions
- Closer to i.i.d.
 pool contains several sessions
- Older interactions were obtained under weaker policy



Autocorrelation



Target is based on prediction

Q(s, a) correlates with Q(s', a)

Target network

Idea: use network with frozen weights to compute the target

$$L(\Theta) = E_{s \sim S, a \sim A}[(Q(s, a, \Theta) - (r + \gamma \max_{a'} Q(s', a', \Theta^-)))^2]$$
 where Θ^- is the frozen weights Hard target network:

Update Θ^- every **n** steps and set its weights as Θ

Target network

Idea: use network with frozen weights to compute the target

$$L(\Theta) = E_{s \sim S, a \sim A}[(Q(s, a, \Theta) - (r + \gamma \max_{a'} Q(s', a', \Theta^{-}))^{2}]$$

$$\Theta^{-}$$
 is the frozen weights
arget network:

Const

where Θ^- is the frozen weights

Hard target network:

Update Θ^- every **n** steps and set its weights as Θ

Soft target network:

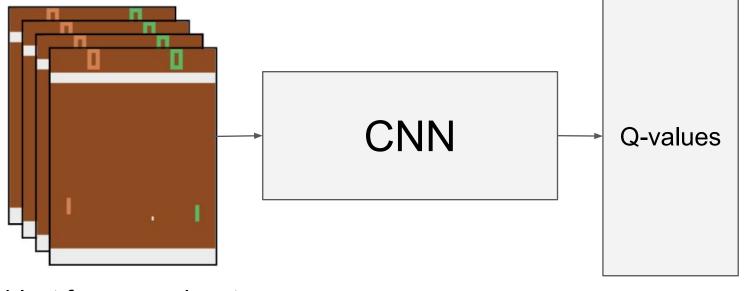
Update Θ^- every step:

$$\Theta^{-} = (1 - \alpha)\Theta^{-} + \alpha\Theta$$

Playing Atari with Deep Reinforcement Learning

(2013, Deepmind)

Experience replay

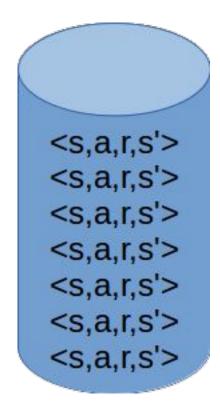


4 last frames as input

Update weights using:

$$L(\Theta) = E_{s \sim S, a \sim A}[(Q(s, a, \Theta) - (r + \gamma \max_{a'} Q(s', a', \Theta^{-})))^2]$$

Update Θ^- every 5000 train steps



10⁶ last transitions

We use "max" operator to compute the target

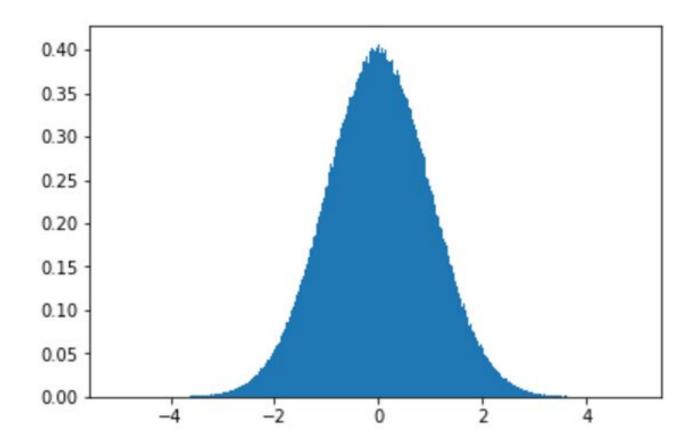
$$L(s, a) = (Q(s, a) - (r + \gamma \max_{a'} Q(s', a')))^{2}$$

We have a problem

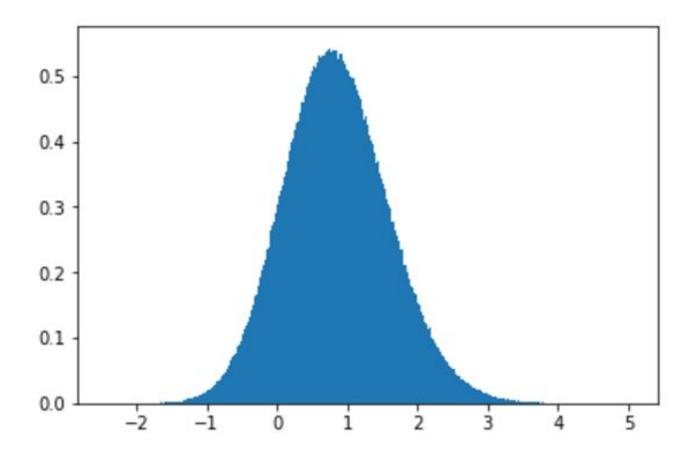
(although we want $E_{s \sim S, a \sim A}[L(s, a)]$ to be equal zero)

Normal distribution 3*10⁶ samples

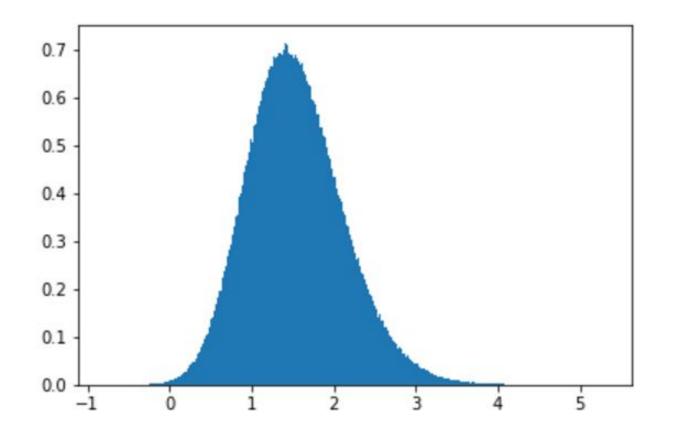
mean: ~0.0004

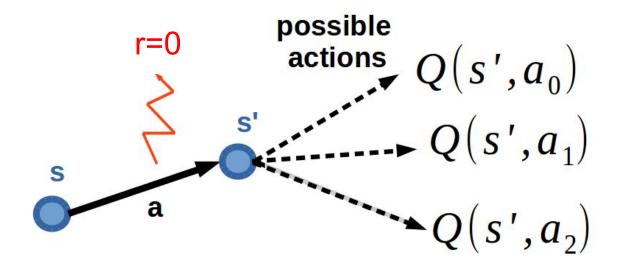


Normal distribution 3*10⁶ x 3 samples
Then take maximum of every tuple mean: ~0.8467



Normal distribution 3*10⁶ x 10 samples
Then take maximum of every tuple mean: ~1.538

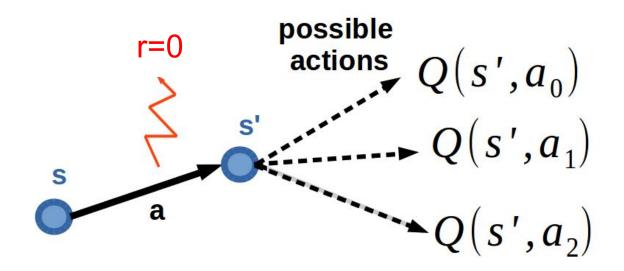




Suppose true Q(s', a') are equal to **0** for all a'

But we have an approximation (or other) error $\sim N(0,\sigma^2)$

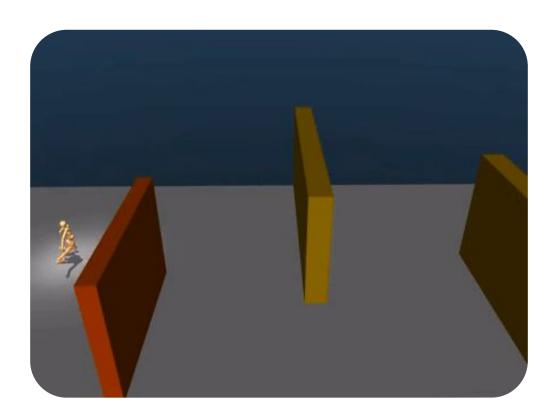
So Q(s,a) should be equal to **0**



But if we update Q(s,a) towards $r + \gamma \max_{a'} Q(s',a')$ we will have overestimated $Q(s,a) > \mathbf{0}$ because

$$E[\max_{a'} Q(s', a')] > = \max_{a'} E[Q(s', a')]$$

Вот теперь точно все





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