SRG Market microstructure

Report on my research

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Implementation of the Generalized OW Market Impact Model

The key recursive formula of an efficient implementation (from ”Handbook of Price Impact Modeling” – A.3) generalizes to arbitrary event times t_i:

$$I_{t_{i+1}} = \rho(t_{i+1}, t_i) I_{t_i} + \lambda \Delta_{i+1} Q, \tag{1}$$

where I_{t_k} – market impact, $\Delta_i Q$ – change of the position (order volume); and the following ρ types are considered:

$$\rho(t_{i+1}, t_i) = const,$$
$$\rho(t_{i+1}, t_i) = \rho^{t_{i+1} - t_i}.$$

We considered the square root model and AR(1) as a banchmarks.

Our ideas

Also, we tried the following conclusions from the formula (1) and our ideas inspired by it:

$$
\begin{aligned}
I_{t+1} &= \rho I_t + \lambda \sqrt{Q_{t+1}} \\
\frac{y_{i+1} - y_i}{\Delta t_{i+1}} &= \rho y_i + \lambda \\
\frac{I_{i+1} - I_i}{\Delta t_{i+1}} &= \rho I_i + \lambda \frac{Q_{i+1}}{\Delta t_{i+1}}.
\end{aligned}
$$

Model tests results

Model	MAE on all the data	MAE on all the data with window
$I_{t_{i+1}} = \rho^{t_{i+1}-t_i} I_{t_i} + \lambda Q_{t_{i+1}}$	1.57	1.36
$I_{t+1} = \rho I_t + \lambda Q_{t+1}$	1.57	1.39
$I_{t+1} = \rho I_t + \lambda \sqrt{Q_{t+1}}$	1.99	1.55
AR(1)	2.08	1.62
$I_t = C\sqrt{Q_t}$	2.12	1.54
$\frac{y_{i+1}-y_i}{\Delta t_{i+1}} = \rho y_i + \lambda$	4.62	4.04
$\frac{I_{i+1}-I_i}{\Delta t_{i+1}} = \rho I_i + \lambda \frac{Q_{i+1}}{\Delta t_{i+1}}$	10.92	12.21

How to find ρ and λ?

The OW model:

$$I_{t_{i+1}} = \rho(t_{i+1}, t_i) I_{t_i} + \lambda \Delta_{i+1} Q \tag{2}$$

looks like ARX model:

$$I(t+1) = a_1 I(t) + b_1 Q(t),$$

where $a_1 = \rho$ and $b_1 = \lambda$. So, we can use time series metodology to estimate them. Moreover, dividing data by parts and fitting the model for each part we can find the graph of $\rho(t_{i+1}, t_i)$.

What to do with that knowledge?

1. It is of great interest to determine the approximate type of trajectories of that coefficients.
2. One is able to find ρ and λ on real data just to predict market impact.
3. After, it is possible to use them to create a realistic OW market simulator.
4. The same ρ and λ are needed in continuos OW optimal execution strategy.

Another way: discrete OW model.

The article ”Optimal trading strategy and supply/demand dynamics” contains (Proposition 1, p. 14) an algorithm for optimal execution:

$$x_n = -\frac{1}{2}\delta_{n+1}[D_{t_n}(1 - \beta_{n+1}e^{-\rho\frac{T}{N}} + 2\kappa\gamma_{n+1}e^{-2\rho\frac{T}{N}}) - X_{t_n}(\lambda + 2\alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\frac{T}{N}})],$$

where D_t is a price; $\alpha_{n+1}, \beta_{n+1}, \gamma_{n+1}, \delta_{n+1}$ are determined recursively; κ and ρ are hyperparameters. Here and further, x_n – the volume of nth optimal order, T – total time to trade, N – total number of orders. These notations are simplified, details are in the article.

Another way: discrete OW model.

In my opinion, it is better to start with the simpler analogue from ”Algorithmic Trading and Quantitative Strategies” (p. 366, eq. 10.24):

$$x_1 = x_n = \frac{X}{\rho T + 2}$$
$$x_t = \frac{\rho X}{\rho T + 2}$$

where ρ is hyperparameter, that can be estimated (?) from:

$$A_t = \overline{p}_t + \frac{s}{2} + x_1 \kappa e^{-\rho t},$$

where A_t – ask price after execution, $\overline{p}_t + \frac{s}{2}$ defines steady state level (here $\overline{p}_t$ is a price and s is a spread), κ and ρ are hyperparameters.