SRG Market microstructure

Report on the article "The Market Impact Puzzle"

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Structure

The square root model of market impact

The square root model of market impact was proposed by Barra (1997), based on empirical regularities observed by Loeb (1983), as a practical way for asset managers to measure market impact empirically.

$$G = g(\sigma, P, V; Q) \sim \sigma \left(\frac{|Q|}{V} \right)^{1/2}, \tag{1}$$

here G denotes the percentage cost of executing a bet of Q shares of stock with price P.

The square root formula is dimensionally consistent: bet size Q is measured in shares, vollume V is measured in shares per day, and returns variance σ^2 is measured per day, the proportionality coefficient is dimensionless.

Disadvantages of the square root model

This square root model is elegantly simple and empirically realistic, but has two distinct disadvantages:

- There is still no consensus on whether market impact functions can indeed be described exactly by the square root function.
- Most theoretical models of market microstructure lead to a model of linear market impact, not a square root model. Models with non-linear market impact are usually analytically intractable and seem to allow for simple arbitrage strategies.

Example 1

Given market impact 1, one could make profits by executing over time ten buy trades of 100 shares each and then selling 1000 shares at once.

Restrictions Based on Volume and Volatility Equations

Suppose the percentage market impact G of executing a bet of size $|Q|$ is described by a power function:

$$G_\beta = \alpha|Q|^\beta. \tag{2}$$

The parameter α is a dimensional coefficient that may depend on some asset-specific characteristics such as volume and volatility, and β is the exponent of the market impact function.
Consider the system of three equations:

$$\begin{cases} V = \gamma E\{|Q|\}, \\ \sigma^2 = \gamma E\{G^2\}, \\ E\{G^2\} = \alpha^2 E\{|Q|^{2\beta}\}. \end{cases} \tag{3}$$

Here V denotes expected trading volume (shares per day), σ^2 denotes expected returns variance, γ denotes the expected number of bets (per day).

Market Impact Functions

The system implies the trading cost:

$$G_\beta = \frac{\sigma}{\sqrt{\gamma}} \frac{|Q|^\beta}{\sqrt{E\{|Q|^{2\beta}\}}} \tag{4}$$

Considering different cases of the function 4 for different values of the exponent β we can get new models:

$$G_0 = \alpha = \frac{\sigma}{\sqrt{\gamma}} = \sqrt{\frac{\sigma^2}{PV} PE\{|Q|\}}.$$

$$G_1 = \alpha|Q| = \sqrt{\frac{\sigma^2}{PV} PE\{|Q|\}} \frac{|Q|}{\sqrt{E\{|Q|^2\}}}.$$

$$G_{1/2} = \sigma \sqrt{\left(\frac{|Q|}{V}\right)}.$$

Restrictions Implied by Transaction Cost Invariance

Transaction costs invariance hypothesizes that the ex ante expected dollar cost $E\{|PQ|G\}$ of executing a bet, without conditioning on the size of a bet, is constant and equals C. From 2 we have the fourth equation (besides the system 3):

$$C = \alpha P E\{|Q|^{1+\beta}\}.$$

Introducing two dimensionless moment ratios m and m_β:

$$m := \frac{E\{|Q|\}\sqrt{E\{|Q|^{2\beta}\}}}{E\{|Q|^{\beta+1}\}}, \qquad m_\beta := \frac{(E\{|Q|\})^{\beta+1}}{E\{|Q|^{\beta+1}\}};$$

one can get:

$$G = \frac{m_\beta C^{(1-2\beta)/3}}{m^{2(1+\beta)/3}} \left(\frac{\sigma^2}{PV}\right)^{(\beta+1)/3} |PQ|^\beta.$$

Restrictions Implied by Bet Size Invariance

Bet size invariance hypothesizes that the dollar risk a bet transfers per unit of business time,

$$I := PQ\frac{\sigma}{\sqrt{\gamma}},$$

has an invariant mean $E|I|$ for all markets. It can be shown that the transaction cost and bet size invariance hypotheses are closely related to each other:

$$C = \frac{1}{m}E\{|PQ|\}\frac{\sigma}{\sqrt{\gamma}} = \frac{1}{m}E\{|I|\}.$$

A universal market impact formula

Let $1/L$ denote the illiquidity measure that is defined as follows:

$$\frac{1}{L} := \frac{C}{E\{|PQ|\}} = \left(\frac{\sigma^2 C}{m^2 PV}\right)^{1/3}.$$

The system 3 leads to:

$$G = \frac{1}{L} f(Z),$$

where Z scales bet size by its mean, in particular:

$$Z := \frac{Q}{E\{|Q|\}} = \frac{PQ}{CL}, \qquad f(Z) = m_\beta |Z|^\beta.$$

A Dimensional Analysis Approach with Leverage Neutrality

Leverage Neutrality: the economic costs of trading bundles of risky securities and a cash-equivalent asset are the same regardless of any positive or negative amount of cash-equivalent assets included in a bundle.

Example 2

Suppose an amount of cash equal to aP is added to each share of stock. This decrease in leverage raises the stock price to $(1+a)P$, lowers returns volatility to $\sigma/(1+a)$, but it does not change V, $E\{|Q|\}$, or γ.

Assuming the basis $\{V, P, \sigma^2, C\}$ one can apply dimensional analysis, leverage neutrality, and invariance to obtain the market impact formula:

$$G = g(\sigma, P, V; Q) \sim \left(\frac{\sigma^2}{PV}\right)^{1/3} f\left(\left(\frac{\sigma^2}{PV}\right)^{1/3} |PQ|\right).$$

Conclusions

- Both general approaches in this paper lead to a market impact function of the type:
$$G = \frac{1}{L} f(Z).$$
- Under both approaches, the square root model is a special knife-edged case of the general market impact formula. It is the only case for which the market impact function depends only on volume and volatility.
- To satisfy leverage neutrality, another market characteristic must be added to the basis $\{V, P, \sigma^2\}$.

Future Research

- The dimensional analysis approach leaves many unanswered questions.
- The author's approach implicitly relies on the assumption that information generates discrete trades of large sizes so that one can identify bets and their market impact in the data. How to deal with it?
- Are the variables $C, m,$ and m_β approximately constant across markets, countries, and time periods? If so, what are their values? Or if not, alternatively, can one identify a set of regimes in which these variables are relatively constant? Are there other similar variables that can be almost invariant?
- Is there a theory based on financial economics which leads to a square root model of market impact?
- Is it possible for the author's approach to generate quantitative predictions about the dynamic properties of market impact?

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