Теорема 1. *In the regression:*

$$\frac{\Delta A_{k+2}}{\Delta t_{k+2}} - \frac{\Delta A_{k+1}}{\Delta t_{k+1}} = -\rho\Delta A_{k+1} + \rho\lambda x_{t_{k+1}} + (\kappa+\lambda)(\frac{x_{t_{k+2}}}{\Delta t_{k+2}} - \frac{x_{t_{k+1}}}{\Delta t_{k+1}}).$$

the coefficients ρ, κ and λ the same as in OW model describes the market with dynamics describing by series $A_k, \Delta t_k, x_k$. Here:

- x_k *— the trade size at* t_k.
- A_k *— ask price at* t_k.

Доказательство. From the definitions of the model follows three equations:

$$A_{t_k} = V_{t_k} + \frac{s}{2} + \sum_{i=0}^{k-1} x_{t_i}\kappa e^{-\rho(t_k - t_i)} \tag{1}$$

$$V_{t_{k+1}} = V_{t_k} + \lambda x_{t_k} \to V_{t_{k+1}} - V_{t_k} = \lambda x_{t_k} \tag{2}$$

$$D_{t_k} = A_{t_k} - V_{t_k} - \frac{s}{2} \tag{3}$$

From (1) and (3):

$$\begin{aligned}
D_{t_k} &= \sum_{i=0}^{k-1} x_{t_i}\kappa e^{-\rho(t_k-t_i)} \\
\Delta D_{t_k} &= \sum_{i=0}^{k} x_{t_i}\kappa e^{-\rho(t_{k+1}-t_i)} - \sum_{i=0}^{k-1} x_{t_i}\kappa e^{-\rho(t_k-t_i)} = \sum_{i=0}^{k-1} x_{t_i}\kappa(e^{-\rho(t_{k+1}-t_i)} - e^{-\rho(t_k-t_i)}) + x_{t_i}\kappa e^{-\rho(t_{k+1}-t_k)} = \\
&= \sum_{i=0}^{k-1} x_{t_i}\kappa e^{-\rho(t_k-t_i)}(e^{-\rho(t_{k+1}-t_k)} - 1) + x_{t_i}\kappa e^{-\rho(t_{k+1}-t_k)} = [\text{considering } e^{-\rho(t_{k+1}-t_k)} = 1 - \rho(t_{k+1}-t_k)] = \\
&= \sum_{i=0}^{k-1} x_{t_i}\kappa e^{-\rho(t_k-t_i)}((1-\rho(t_{k+1}-t_k)) - 1) + x_{t_i}\kappa(1-\rho(t_{k+1}-t_k)) = -\rho\Delta t_k D_{t_k} + x_{t_i}\kappa(1-\rho\Delta t_k).
\end{aligned}$$

On the other hand, from (2) and (3) one has:

$$\Delta D_k = D_{k+1} - D_k = A_{k+1} + V_{k+1} - A_k - V_k = \Delta A_k - \Delta V_k \to \Delta A_k = \Delta D_k + \Delta V_k.$$

Thus, one has an equation:

$$\Delta A_k = -\rho\Delta t_k D_{t_k} + x_{t_i}\kappa(1-\rho\Delta t_k) + \lambda x_{t_k}.$$

But one does not want to deal with D_{t_k}, so he divides regression for Δt_k and considers a divided difference of $A_{t_{k+1}}$ and A_{t_k}:

$$\begin{aligned}
\frac{\Delta A_{k+1}}{\Delta t_{k+1}} - \frac{\Delta A_k}{\Delta t_k} &= -\rho D_{t_{k+1}} + x_{t_{k+1}}\kappa\left(\frac{1}{\Delta t_{k+1}} - \rho\right) + \lambda\frac{x_{t_{k+1}}}{\Delta t_{k+1}} + \rho D_{t_k} - x_{t_k}\kappa\left(\frac{1}{\Delta t_k} - \rho\right) - \lambda\frac{x_{t_k}}{\Delta t_k} = \\
&= -\rho(\Delta A_k - \Delta V_k) + (\lambda+\kappa)\left(\frac{x_{t_{k+1}}}{\Delta t_{k+1}} - \frac{x_{t_k}}{\Delta t_k}\right) - \rho\kappa(x_{t_{k+1}} - x_{t_k}) = \\
&= -\rho\Delta A_k + \rho(\lambda+\kappa)x_{t_k} - \rho\kappa x_{t_{k+1}} + (\lambda+\kappa)\left(\frac{x_{t_{k+1}}}{\Delta t_{k+1}} - \frac{x_{t_k}}{\Delta t_k}\right).
\end{aligned}$$

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