

SRG Market microstructure

# Report on my research

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### **Optimal execution problem**



Task: we want to buy or sell a big amount of asset.



Figure: The difference between market order and limit order

So, let's go to the exange and just buy as many assets as needed!



## The structure of LOB and problems of such approach

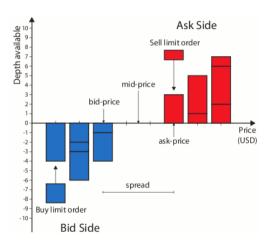


Figure: The difference between market order and limit order

### Formalization of the problem



$$J_0 = \min_{\{x_0 \cdots x_N\}} E_0 \left[ \sum_{n=0}^N [A_{t_n} + x_n/(2q)] x_n \right], \tag{1}$$

#### where:

- The trader has to buy  $X_0$  units of a security over a fixed time period [0, T];
- $x_n$  is the trade size at  $t_n = \tau n$ , where  $\tau = T/N$  and  $\sum x_n = X_0$ ;
- $A_{t_n}$  is an ask price at  $t_n$ ;
- q is a LOB density.



# The limit of the optimal execution strategy in OW framework

#### Theorem 1

As  $N \to \infty$ , the optimal execution strategy becomes:

$$\lim_{N \to \infty} x_0 = x_{t=0} = \frac{X_0}{\rho T + 2}, 
\lim_{N \to \infty} x_n / (T/N) = \dot{X}_t = \frac{\rho X_0}{\rho T + 2}, \quad t \in (0, T), 
\lim_{N \to \infty} x_n / (T/N) = x_{t=T} = \frac{X_0}{\rho T + 2},$$

where  $x_0$  is the trade at the beginning of trading period,  $x_N$  is the trade at the end of trading period, and  $\dot{X}_t$  is the speed of trading in between these trades.

### How to find $\rho$ ?



We provide our methodology to find  $\rho$ . We find it, considering the time series of elements of the model that can be calculated from market data. As an example, we are going to consider the regression:

$$\frac{\Delta A_{k+2}}{\Delta t_{k+2}} - \frac{\Delta A_{k+1}}{\Delta t_{k+1}} = -\rho \Delta A_{k+1} + \rho \lambda x_{k+1} + (\kappa + \lambda)(\frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}}).$$

Where all the information needed can be extracted from the l3 data:

- $\Delta A_k$  is an ask change after execution of the limit order with the depth  $x_k$ .
- $\Delta t_k$  is a time between k and k+1 orders of dataset.

### Research plan



- Develop methodology for fitting OWM paramete rs and use it to get optimal execution strategy.
- Help to parse l3 data.
- Compare different approaches of measuring resiliency on l3 data.
- Compare discrete and limit OW execution strategies.
- Propose a backtest procedure for the optimal execution algorithm, implement it, and compare the algorithm with TWAP on real market data.

