Optimal execution problem in Obizhaeva–Wang framework

VEGA INSTITUTE
Science Promotion Foundation

Zaostrovsky Vsevolod

Student Research Group «Market microstructure»
Supervisors: Anthon Belyakov, Ph. D.

Introduction

Issues related to the structure of the order book are very important for the industry, so in recent decades a new young and interesting science has been built around these issues. In our research, we are looking for a way to connect the latest advances in this science associated with various variations of the Obizhaeva–Wang model with the needs of industry.

Optimal execution problem

The idea of that problem is quite simple. If one wants to sell or buy an amount of an asset large enough to have a significant impact on the market, he, obviously should not do it by one order: it would be very expensive, since a large order would remove all the upper levels in the limit order book. Therefore, in practice, all large orders are split into a large number of small ones. For example, one can simply divide an order into N equal parts and sell them at regular intervals (this is called TWAP). To find a better solution, we consider the OW model, in which terms the problem has the following form:

$$J_0 = \min_{\{x_0 \cdots x_N\}} E_0 \left[\sum_{n=0}^{N} [A_{t_n} + x_n/(2q)] x_n \right], \tag{1}$$

$$A_{t_n} = F_{t_n} + \lambda(X_0 - X_{t_n}) + s/2 + \sum_{i=0}^{n-1} x_i \kappa e^{-\rho\tau(n-i)} \tag{2}$$

Here:

- The trader has to buy $\mathbf{X_0}$ units of a security over a fixed time period $[0, T]$. x_{t_n} – the trade size at $t_n = \tau n$, where $\tau = T/N$. $X_{t_n} := X_0 - \sum_{t_k < t_n} x_{t_k}$.
- B_{t_n} and A_{t_n} – bid and ask prices at t_n. $V_{t_n} = \frac{A_{t_n} + B_{t_n}}{2}$ – the mid-quote price; s – the bid–ask spread.
- F_t – the fundamental value of the security.
- Parameter λ captures the permanent price impact.
- Parameter q depends on LOB density.
- $\kappa = \frac{1}{q} - \lambda$
- Parameter ρ captures the resiliency.

Optimal execution strategies

Proposition 2 from ?? gives an optimal strategy for big N.

1

As $N \to \infty$, the optimal execution strategy becomes:

$$\lim_{N\to\infty} x_0 = \frac{X_0}{\rho T + 2}, \tag{3}$$

$$\lim_{N\to\infty} x_n/(T/N) = \frac{\rho X_0}{\rho T + 2}, \tag{4}$$

(5)

Conclusion

This recursive branching and self-similar pattern in the structure of broccoli are what make it an example of a fractal in nature. Fractals can be found in various natural phenomena, and broccoli serves as a visually appealing example of fractal geometry in plants.

In conclusion, broccoli exhibits fractal characteristics in its structure. The self-repeating patterns and recursive branching observed in broccoli's stalks and florets resemble the mathematical concept of fractals. This self-similarity at different scales is a defining characteristic of fractals. Therefore, broccoli can be considered an example of a fractal in nature.

Follow us in Telegram.

References

[Alm+05] Robert Almgren et al. "Direct estimation of equity market impact". In: *Risk* 18.7 (2005), pp. 58–62.

[Gre05] Robin Greenwood. "Short-and long-term demand curves for stocks: theory and evidence on the dynamics of arbitrage". In: *Journal of Financial Economics* 75.3 (2005), pp. 607–649.

[OW13] Anna A Obizhaeva and Jiang Wang. "Optimal trading strategy and supply/demand dynamics". In: *Journal of Financial markets* 16.1 (2013), pp. 1–32.

[KO18] Albert S Kyle and Anna A Obizhaeva. "The market impact puzzle". In: *Anna A., The Market Impact Puzzle (February 4, 2018)* (2018).

[Vel20] Raja Velu. *Algorithmic trading and quantitative strategies*. CRC Press, 2020.

[Web23] Kevin T Webster. *Handbook of Price Impact Modeling*. CRC Press, 2023.