SRG Market microstructure

Optimal execution problem in Obizhaeva–Wang framework

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Optimal execution problem

Task: we want to buy or sell a big amount of asset.

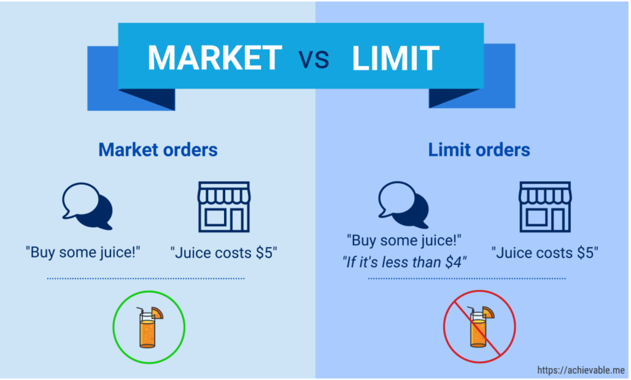

Figure: The difference between market order and limit order

So, let's go to the exchange and just buy as many assets as needed!

The structure of LOB and problems of such approach

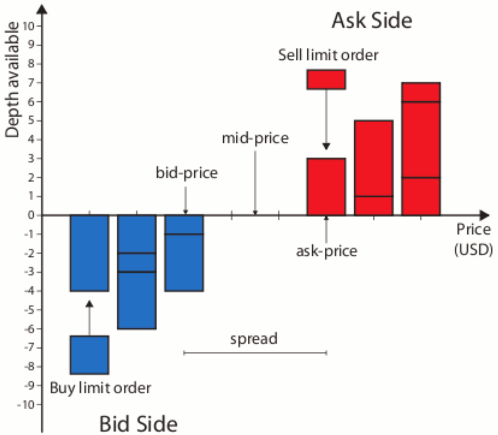

Figure: The structure of LOB

Time Weighted Average Price

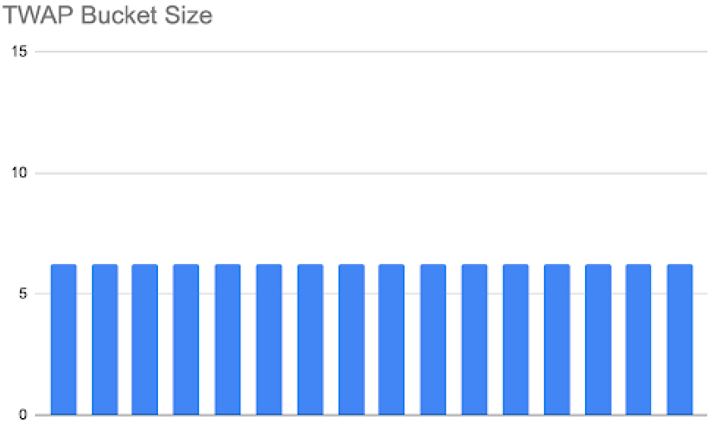

Figure: TWAP strategy

Resilency

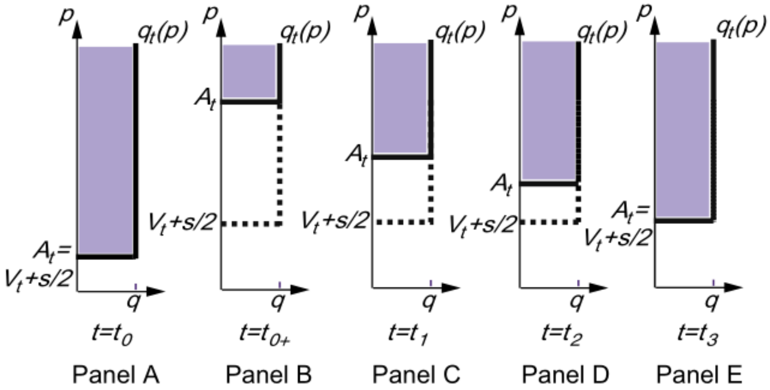

Figure: The demonstration of resiliency

Formalization of the problem in the discrete time

$$J_0 = \min_{\{x_0 \cdots x_N\}} E_0 \left[\sum_{n=0}^{N} [A_{t_n} + x_n/(2q)] x_n \right], \tag{1}$$

$$A_{t_n} = V_{t_n} + s/2 + \sum_{i=0}^{n-1} x_i \kappa e^{-\rho\tau(n-i)}, \tag{2}$$

where:

- $V_{t_n} = F_{t_n} + \lambda(X_0 - X_{t_n})$
- The trader has to buy X_0 units of a security over a fixed time period $[0, T]$;
- x_n is the market order size at $t_n = \tau n$, where $\tau = T/N$ and $\sum x_n = X_0$;
- $X_{t_n} := X_0 - \sum_{t_k < t_n} x_{t_k}$;
- A_{t_n} is an ask price at t_n;
- F_t is the fundamental value of the security;
- q is a LOB density. $\kappa = \frac{1}{q} - \lambda$.

The limit of the optimal execution strategy in OW framework

Theorem 1

As $N \to \infty$, the optimal execution strategy becomes:

$$\lim_{N\to\infty} x_0 = x_{t=0} = \frac{X_0}{\rho T + 2},$$
$$\lim_{N\to\infty} x_n/(T/N) = \dot{X}_t = \frac{\rho X_0}{\rho T + 2}, \qquad t \in (0, T),$$
$$\lim_{N\to\infty} x_n/(T/N) = x_{t=T} = \frac{X_0}{\rho T + 2},$$

where x_0 is the trade at the beginning of trading period, x_N is the trade at the end of trading period, and $\dot{X}_t$ is the speed of trading in between these trades.

Our methodology to fit parameters ρ, κ, q

We chose regression to find parameters:

$$\frac{\Delta A_{k+2}}{\Delta t_{k+2}} - \frac{\Delta A_{k+1}}{\Delta t_{k+1}} = -\rho \Delta A_{k+1} + \rho \lambda x_{k+1} + (\kappa + \lambda)(\frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}}).$$

Where all the information needed can be extracted from the l3 data:

- ΔA_k is an ask change after execution of the limit order with the depth x_k.
- Δt_k is a time between k and $k+1$ orders of dataset.

Research plan

- Develop methodology for fitting OWM parameters and use it to get optimal execution strategy.
- Compare different approaches of measuring resiliency on l3 data.
- Compare discrete and limit OW execution strategies.
- Propose a backtest procedure for the optimal execution algorithm, implement it, and compare the algorithm with TWAP on real market data.

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