Optimal execution problem in Obizhaeva–Wang framework

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Introduction

Issues related to the structure of the order book are very important for the industry, so in recent decades a new and interesting science has been built around these issues. In our research, we are looking for a way to connect the latest advances in this science associated with various variations of the Obizhaeva–Wang model with the needs of industry.

Optimal execution problem

The idea of that problem is quite simple. If one wants to sell or buy an amount of an asset large enough to have a significant impact on the market, he, obviously, should not do it by one order: it would be very expensive, since a large order would remove all the upper levels in the limit order book. Therefore, in practice, all large orders are split into a large number of small ones. For example, one can simply divide an order into N equal parts and sell them at regular intervals (this is called TWAP). To find a better solution, we consider the OW model, in which terms the problem has the following form:

$$J_0 = \min_{\{x_0 \cdots x_N\}} E_0 \left[\sum_{n=0}^{N} [A_{t_n} + x_n/(2q)] x_n \right],$$

$$A_{t_n} = F_{t_n} + \lambda(X_0 - X_{t_n}) + s/2 + \sum_{i=0}^{n-1} x_i \kappa e^{-\rho\tau(n-i)}.$$

Here:

- The trader has to buy $\mathbf{X_0}$ units of a security over a fixed time period $[0,T]$. x_{t_n} – the trade size at $t_n = \tau n$, where $\tau = T/N$. $X_{t_n} := X_0 - \sum_{t_k < t_n} x_{t_k}$.
- B_{t_n} and A_{t_n} – bid and ask prices at t_n. $V_{t_n} = \frac{A_{t_n} + B_{t_n}}{2}$ – the mid-quote price; s – the bid–ask spread.
- F_t – the fundamental value of the security.
- Parameter λ captures the permanent price impact.
- Parameter q depends on LOB density.
- $\kappa = \frac{1}{q} - \lambda$
- Parameter ρ captures the resiliency.

Optimal execution strategy

Proposition 2 from [OW13] gives an optimal strategy for big N.

Theorem 1

As $N \to \infty$, the optimal execution strategy becomes:

$$\lim_{N\to\infty} x_0 = x_{t=0} = \frac{X_0}{\rho T + 2},$$

$$\lim_{N\to\infty} x_n/(T/N) = \dot{X}_t = \frac{\rho X_0}{\rho T + 2}, \qquad t \in (0,T),$$

$$\lim_{N\to\infty} x_n/(T/N) = x_{t=T} = \frac{X_0}{\rho T + 2},$$

where x_0 is the trade at the beginning of trading period, x_N is the trade at the end of trading period, and $\dot{X}_t$ is the speed of trading in between these trades.

The key question here is:

How to find ρ?

We provide our methodology to find ρ. We find it, considering time series on elements of the model that can be calculated from market data. As an example, we are going to consider the regression:

Theorem 2

In regression:

$$\frac{\Delta A_{k+2}}{\Delta t_{k+2}} - \frac{\Delta A_{k+1}}{\Delta t_{k+1}} = -\rho \Delta A_{k+1} + \rho\lambda x_{k+1} + (\alpha+\lambda)\left(\frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}}\right).$$

ρ the same as in OW model.

Proof.

$$D_{k+1} - D_k = -\rho D_k \Delta t_{k+1} + \alpha x_{k+1}$$

$$\Delta t_{k+1} := t_{k+1} - t_k, \qquad D_k := D_{t_k}, \qquad x_k := x_{t_k}, \qquad \Delta D_{k+1} := D_{k+1} - D_k.$$

$$V_{k+1} - V_k = \lambda x_{k+1} \to \Delta D_{k+1} = \Delta A_{k+1} - \lambda x_k$$

$$\frac{\Delta D_{k+1}}{\Delta t_{k+1}} = -\rho D_k + \alpha \frac{x_{k+1}}{\Delta t_{k+1}}$$

$$\frac{\Delta D_{k+2}}{\Delta t_{k+2}} - \frac{\Delta D_{k+1}}{\Delta t_{k+1}} = -\rho \Delta D_{k+1} + \alpha\left(\frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}}\right)$$

□

Purposes

- Propose methodology for fitting OWM factors and use it to get optimal execution strategy.
- Propose a backtest procedure for the optimal execution algorithm, implement it and compare the algorithm with TWAP.

References

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