SRG Market microstructure

Report on my research

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Implementation of the Generalized OW Market Impact Model

The key recursive formula of an efficient implementation (from ”Handbook of Price Impact Modeling” — A.3) generalizes to arbitrary event times t_i:

$$I_{t_{i+1}} = \rho(t_{i+1}, t_i) I_{t_i} + \lambda \Delta_{i+1} Q, \tag{1}$$

where I_{t_k} — market impact, $\Delta_i Q$ — change of the position (order volume); and the following ρ types are considered:

$$\rho(t_{i+1}, t_i) = const,$$
$$\rho(t_{i+1}, t_i) = \rho^{t_{i+1} - t_i}.$$

We considered the square root model and AR(1) as a banchmarks.

Our ideas

Also, we tried the following conclusions from the formula (1) and our ideas inspired by it:

$$
\begin{aligned}
I_{t+1} &= \rho I_t + \lambda \sqrt{Q_{t+1}} \\
\frac{y_{i+1} - y_i}{\Delta t_{i+1}} &= \rho y_i + \lambda \\
\frac{I_{i+1} - I_i}{\Delta t_{i+1}} &= \rho I_i + \lambda \frac{Q_{i+1}}{\Delta t_{i+1}}.
\end{aligned}
$$

Model tests results

Model	MAE on all the data	MAE on all the data with window
$I_{t_{i+1}} = \rho^{t_{i+1}-t_i} I_{t_i} + \lambda Q_{t_{i+1}}$	1.57	1.36
$I_{t+1} = \rho I_t + \lambda Q_{t+1}$	1.57	1.39
$I_{t+1} = \rho I_t + \lambda \sqrt{Q_{t+1}}$	1.99	1.55
AR(1)	2.08	1.62
$I_t = C\sqrt{Q_t}$	2.12	1.54
$\frac{y_{i+1}-y_i}{\Delta t_{i+1}} = \rho y_i + \lambda$	4.62	4.04
$\frac{I_{i+1}-I_i}{\Delta t_{i+1}} = \rho I_i + \lambda \frac{Q_{i+1}}{\Delta t_{i+1}}$	10.92	12.21

How to find ρ and λ?

The OW model:

$$I_{t_{i+1}} = \rho(t_{i+1}, t_i) I_{t_i} + \lambda \Delta_{i+1} Q \quad (2)$$

looks like ARX model:

$$I(t+1) = a_1 I(t) + b_1 Q(t),$$

where $a_1 = \rho$ and $b_1 = \lambda$. So, we can use time series metodology to estimate them. Moreover, dividing data by parts and fitting the model for each part we can find the graph of $\rho(t_{i+1}, t_i)$.

Discrete OW model.

The article ”Optimal trading strategy and supply/demand dynamics” contains (Proposition 1, p. 14) an algorithm for optimal execution:

$$x_n = -\frac{1}{2}\delta_{n+1}[D_{t_n}(1 - \beta_{n+1}e^{-\rho\frac{T}{N}} + 2\kappa\gamma_{n+1}e^{-2\rho\frac{T}{N}}) - X_{t_n}(\lambda + 2\alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\frac{T}{N}})],$$

where D_t is a price; $\alpha_{n+1}, \beta_{n+1}, \gamma_{n+1}, \delta_{n+1}$ are determined recursively; κ and ρ are hyperparameters. Here and further, x_n – the volume of nth optimal order, T – total time to trade, N – total number of orders. These notations are simplified, details are in the article.

Limit of the discrete OW model.

In my opinion, it is better to start with the simpler analogue from "Algorithmic Trading and Quantitative Strategies" (p. 366, eq. 10.24):

$$x_1 = x_n = \frac{X}{\rho T + 2}$$
$$x_t = \frac{\rho X}{\rho T + 2}$$

where ρ is hyperparameter, that can be estimated (?) from:

$$A_t = \overline{p}_t + \frac{s}{2} + x_1 \kappa e^{-\rho t},$$

where A_t – ask price after execution, $\overline{p}_t + \frac{s}{2}$ defines steady state level (here $\overline{p}_t$ is a price and s is a spread), κ and ρ are hyperparameters.