

# Optimal execution problem in Obizhaeva–Wang framework

Vsevolod Zaostrovsky

Student Research Group «Market Microstructure»

Supervisors: Anton O. Belyakov, Alexey Savin



## Introduction

Issues related to the structure of the order book are very important for the industry, so in recent decades a new young and interesting science has been built around these issues. In our research, we are looking for a way to connect the latest advances in this science associated with various variations of the Obizhaeva–Wang model with the needs of industry.

## Optimal execution problem

The idea of that problem is quite simple. If one wants to sell or buy an amount of an asset large enough to have a significant impact on the market, he, obviously should not do it by one order: it would be very expensive, since a large order would remove all the upper levels in the limit order book. Therefore, in practice, all large orders are split into a large number of small ones. For example, one can simply divide an order into  $N$  equal parts and sell them at regular intervals (this is called TWAP). To find a better solution, we consider the OW model, in which terms the problem has the following form:

$$J_0 = \min_{\{x_0 \dots x_N\}} E_0 \left[ \sum_{n=0}^N [A_{t_n} + x_n/(2q)] x_n \right],$$

$$A_{t_n} = F_{t_n} + \lambda(X_0 - X_{t_n}) + s/2 + \sum_{i=0}^{n-1} x_i \kappa e^{-\rho \tau(n-i)}.$$

Here:

- The trader has to buy  $X_0$  units of a security over a fixed time period  $[0, T]$ .  $x_{t_n}$  – the trade size at  $t_n = \tau n$ , where  $\tau = T/N$ .  $X_{t_n} := X_0 - \sum_{t_k < t_n} x_{t_k}$ .
- $B_{t_n}$  and  $A_{t_n}$  – bid and ask prices at  $t_n$ .  $V_{t_n} = \frac{A_{t_n} + B_{t_n}}{2}$  – the mid-quote price;  $s$  – the bid–ask spread.
- $F_t$  – the fundamental value of the security.
- Parameter  $\lambda$  captures the permanent price impact.
- Parameter  $q$  depends on LOB density.
- $\kappa = \frac{1}{q} - \lambda$
- Parameter  $\rho$  captures the resiliency.

## Optimal execution strategy

Proposition 2 from [OW13] gives an optimal strategy for big  $N$ .

### Theorem 1

As  $N \rightarrow \infty$ , the optimal execution strategy becomes:

$$\lim_{N \rightarrow \infty} x_0 = x_{t=0} = \frac{X_0}{\rho T + 2},$$

$$\lim_{N \rightarrow \infty} x_n/(T/N) = \dot{X}_t = \frac{\rho X_0}{\rho T + 2}, \quad t \in (0, T),$$

$$\lim_{N \rightarrow \infty} x_n/(T/N) = x_{t=T} = \frac{X_0}{\rho T + 2},$$

where  $x_0$  is the trade at the beginning of trading period,  $x_N$  is the trade at the end of trading period, and  $\dot{X}_t$  is the speed of trading in between these trades.

The key question here is:

### How to find $\rho$ ?

We provide our methodology to find  $\rho$ . We find it, considering time series on elements of the model that can be calculated from market data. As an example, we are going to consider the regression:

### Theorem 2

In regression:

$$\frac{\Delta A_{k+2}}{\Delta t_{k+2}} - \frac{\Delta A_{k+1}}{\Delta t_{k+1}} = -\rho \Delta A_{k+1} + \rho \lambda x_{k+1} + (\alpha + \lambda) \left( \frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}} \right).$$

$\rho$  the same as in OW model.

### Proof.

$$D_{k+1} - D_k = -\rho D_k \Delta t_{k+1} + \alpha x_{k+1}$$

$$\Delta t_{k+1} := t_{k+1} - t_k, \quad D_k := D_{t_k}, \quad x_k := x_{t_k}, \quad \Delta D_{k+1} := D_{k+1} - D_k.$$

$$V_{k+1} - V_k = \lambda x_{k+1} \rightarrow \Delta D_{k+1} = \Delta A_{k+1} - \lambda x_k$$

$$\frac{\Delta D_{k+1}}{\Delta t_{k+1}} = -\rho D_k + \alpha \frac{x_{k+1}}{\Delta t_{k+1}}$$

$$\frac{\Delta D_{k+2}}{\Delta t_{k+2}} - \frac{\Delta D_{k+1}}{\Delta t_{k+1}} = -\rho \Delta D_{k+1} + \alpha \left( \frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}} \right)$$

□

## Purposes

- Propose methodology for fitting OWM factors and use it to get optimal execution strategy.
- Propose a backtest procedure for the optimal execution algorithm, implement it and compare the algorithm with TWAP.

## References

- [Alm+05] Robert Almgren et al. “Direct estimation of equity market impact”. In: *Risk* 18.7 (2005), pp. 58–62.
- [OW13] Anna A Obizhaeva and Jiang Wang. “Optimal trading strategy and supply/demand dynamics”. In: *Journal of Financial markets* 16.1 (2013), pp. 1–32.
- [KO18] Albert S Kyle and Anna A Obizhaeva. “The market impact puzzle”. In: *Anna A., The Market Impact Puzzle (February 4, 2018)* (2018).
- [Vel20] Raja Velu. *Algorithmic trading and quantitative strategies*. CRC Press, 2020.
- [Web23] Kevin T Webster. *Handbook of Price Impact Modeling*. CRC Press, 2023.