Optimal execution problem in Obizhaeva–Wang framework

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Introduction

The introduction of resiliency — the speed at which supply/demand recovers to its steady state after a trade — characterizes the beginning of a new stage in the development of optimal execution models. In our research we develope a practical way to utilize that object.

The supply/demand of financial securities is in general not perfectly elastic. This fact is true even for liquid European markets, if we talk about much less liquid Russian markets, neglecting this fact can be disastrous. The main difference between OW model and others is precisely that resiliency plays a key role in it.

Optimal execution problem

If one wants to sell or buy an amount of an asset large enough to have a significant impact on the market, he, obviously, should not do it by one order: it would be very expensive, since a large order would remove all the upper levels in the limit order book. Therefore, in practice, all large orders are split into a large number of small ones. For example, one can simply divide an order into N equal parts and sell them at regular intervals (this is called TWAP). To find a better solution, we consider the OW model, in which terms the problem has the following form:

$$J_0 = \min_{\{x_0 \cdots x_N\}} E_0 \left[\sum_{n=0}^{N} [A_{t_n} + x_n/(2q)] x_n \right],$$

$$A_{t_n} = F_{t_n} + \lambda(X_0 - X_{t_n}) + s/2 + \sum_{i=0}^{n-1} x_i \kappa e^{-\rho\tau(n-i)}.$$

Here:

- The trader has to buy $\mathbf{X_0}$ units of a security over a fixed time period $[0, T]$.
- x_{t_n} — the trade size at $t_n = \tau n$, where $\tau = T/N$.
- $X_{t_n} := X_0 - \sum_{t_k < t_n} x_{t_k}$.
- B_{t_n} and A_{t_n} — bid and ask prices at t_n.
- $V_{t_n} = \frac{A_{t_n} + B_{t_n}}{2}$ — the mid-quote price;
- s — the bid–ask spread.
- F_t — the fundamental price of the security.
- q, λ and ρ is a LOB density, the permanent price impact and the resiliency.
- $\kappa = \frac{1}{q} - \lambda$

Optimal execution strategy

Optimal execution strategy in OW framework

As $N \to \infty$, the optimal execution strategy becomes:

$$\lim_{N\to\infty} x_0 = x_{t=0} = \frac{X_0}{\rho T + 2},$$

$$\lim_{N\to\infty} x_n/(T/N) = \dot{X}_t = \frac{\rho X_0}{\rho T + 2}, \qquad t \in (0, T),$$

$$\lim_{N\to\infty} x_n/(T/N) = x_{t=T} = \frac{X_0}{\rho T + 2},$$

where x_0 is the trade at the beginning of trading period, x_N is the trade at the end of trading period, and $\dot{X}_t$ is the speed of trading in between these trades.

The key question: how to find ρ?

We provide our methodology to find ρ. We find it, considering time series on elements of the model that can be calculated from market data. As an example, we are going to consider the regression:

Our method to find ρ

$$\frac{\Delta A_{k+2}}{\Delta t_{k+2}} - \frac{\Delta A_{k+1}}{\Delta t_{k+1}} = -\rho \Delta A_{k+1} + \rho\lambda x_{k+1} + (\kappa + \lambda)\left(\frac{x_{k+2}}{\Delta t_{k+2}} - \frac{x_{k+1}}{\Delta t_{k+1}}\right).$$

Discrete optimal execution strategy

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he solution to the optimal execution problem (??) is

$$x_n = -\frac{1}{2}\delta_{n+1}[D_{t_n}(1 - \beta_{n+1}e^{-\rho\frac{T}{N}} + 2\kappa\gamma_{n+1}e^{-2\rho\frac{T}{N}}) - X_{t_n}(\lambda + 2\alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\frac{T}{N}})],$$

with $x_N = X_N$ and $Dt\mathrm{ij}A_t - V_t - s/2$. The expected cost for future trades under the optimal strategy is determined according to

$$J_{t_n} = (F_{t_n} + s/2)X_{t_n} + \lambda X_0 X_{t_n} + \alpha_n X_{t_n}^2 + \beta_n D_{t_n} X_{t_n} + \gamma_n D_{t_n}^2,$$

where the coefficients α_{n+1}, β_{n+1}, γ_{n+1}, and $\delta n + 1$ are determined recursively as follows:

with $\delta_{n+1} = [1/(2q) + \alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\tau} + \gamma_{n+1}\kappa^2 e^{-2\rho\tau}]^{-1}$ and terminal conditions

$$\alpha_N = 1/(2q) - \lambda, \qquad \beta_N = 1, \qquad \gamma_N = 0.$$

Problems

- It seems that the task formulated in the KPI is more indirectly related to the article [OW13] than directly. [OW13] and [Vel20] pose the problem significantly differently. Similar terminology we have found in [Web23], but we did not find the theory to work with in that framework.
- The data we previously had did not have a sufficient level of detail to extract accurate model values. It was necessary to make assumptions that significantly distorted the final result. New data will require significant time to parse and research. Anyway, data work is very complicated.
- This area is very rich and complicated. It is very hard to do even easy steps in the theoretical research, because we did not have courses on that theory.

Purposes

- Develope methodology for fitting OWM factors and use it to get optimal execution strategy.
- Compare different approaches to measuring resiliency on l3 data.
- Propose a backtest procedure for the optimal execution algorithm, implement it and compare the algorithm with TWAP.

References

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[Vel20] Raja Velu. *Algorithmic trading and quantitative strategies*. CRC Press, 2020.

[Web23] Kevin T Webster. *Handbook of Price Impact Modeling*. CRC Press, 2023.