SRG Market microstructure

Report on my research

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Implementation of the Generalized OW Market Impact Model

The key recursive formula of an efficient implementation (from ”Handbook of Price Impact Modeling” — A.3) generalizes to arbitrary event times t_i:

$$I_{t_{i+1}} = \rho(t_{i+1}, t_i) I_{t_i} + \lambda \Delta_{i+1} Q, \tag{1}$$

where I_{t_k} — market impact, $\Delta_i Q$ — change of the position (order volume); and the following ρ types are considered:

$$\rho(t_{i+1}, t_i) = const, \qquad \rho = (1 - \beta \Delta t)$$

$$\rho(t_{i+1}, t_i) = \rho^{t_{i+1} - t_i}, \qquad \rho = const$$

$$\rho(t_{i+1}, t_i) = \frac{\rho_{t_{i+1}}}{\rho_{t_i}}, \qquad \rho_t = \exp - \int_0^t \beta_s ds$$

How to find ρ and λ?

The OW model:

$$I_{t_{i+1}} = \rho(t_{i+1}, t_i) I_{t_i} + \lambda \Delta_{i+1} Q \tag{2}$$

looks like ARX model:

$$I(t+1) = a_1 I(t) + b_1 Q(t),$$

where $a_1 = \rho$ and $b_1 = \lambda$. So, we can use time series metodology to estimate them. Moreover, dividing data by parts and fitting the model for each part we can find the graph of $\rho(t_{i+1}, t_i)$.

What to do with that knowledge?

1. It is of great interest to determine the approximate type of trajectories of that coefficients.
2. One is able to find ρ and λ on real data just to predict market impact.
3. After, it is possible to use them to create a realistic OW market simulator.
4. The same ρ and λ are needed in continuos OW optimal execution strategy.

Another way: discrete OW model.

The article ”Optimal trading strategy and supply/demand dynamics” contains (Proposition 1, p. 14) an algorithm for optimal execution:

$$x_n = -\frac{1}{2}\delta_{n+1}[D_{t_n}(1-\beta_{n+1}e^{-\rho\frac{T}{N}}+2\kappa\gamma_{n+1}e^{-2\rho\frac{T}{N}})-X_{t_n}(\lambda+2\alpha_{n+1}-\beta_{n+1}\kappa e^{-\rho\frac{T}{N}})],$$

where D_t is a price; $\alpha_{n+1}, \beta_{n+1}, \gamma_{n+1}, \delta_{n+1}$ are determined recursively; κ and ρ are hyperparameters. Here and further, x_n – the volume of nth optimal order, T – total time to trade, N – total number of orders. These notations are simplified, details are in the article.

Another way: discrete OW model.

In my opinion, it is better to start with the simpler analogue from ”Algorithmic Trading and Quantitative Strategies” (p. 366, eq. 10.24):

$$x_1 = x_n = \frac{X}{\rho T + 2}$$
$$x_t = \frac{\rho X}{\rho T + 2}$$

where ρ is hyperparameter, that can be estimated (?) from:

$$A_t = \overline{p}_t + \frac{s}{2} + x_1 \kappa e^{-\rho t},$$

where A_t – ask price after execution, $\overline{p}_t + \frac{s}{2}$ defines steady state level (here $\overline{p}_t$ is a price and s is a spread), κ and ρ are hyperparameters.