

## Phân tích thuật toán

Exercises 1.2. - Anany's Book (page 43 - 44).

### 1. Old World Puzzle

A peasant finds himself on a riverbank with a wolf, a goat, and a head of cabbage. He needs to transport all three to the other side of the river in his boat.

Solution: The peasant can solve the problem using the following steps:

- 1, Take the goat across the river.
- 2, Return alone.
- 3, Take the wolf across the river.
- 4, Return with the goat.
- 5, Take the cabbage across the river.
- 6, Return alone.
- 7, Take the goat across the river.

### 2. New World Puzzle

Solution: The four people can cross the bridge in the following steps:

1. Person 1 and Person 2 cross the bridge (2 minutes).
2. Person 1 returns with the flashlight (1 minute).
3. Person 3 and Person 4 cross the bridge (10 minutes).



4. Person 2 returns with the flashlight (2 minutes).

5. Person 1 and Person 2 cross the bridge again (2 minutes).

Total time:  $2 + 1 + 10 + 2 + 2 = 17$  minutes.

3.

a.  $S = \sqrt{p(p-a)(p-b)(p-c)}$ , where  $p = (a+b+c)/2$

b.  $S = \frac{1}{2} b \cdot c \sin A$ , where  $A$  is the angle between sides  $b$  and  $c$ .

c.  $S = \frac{1}{2} a \cdot h_a$ , where  $h_a$  is the height to base  $a$ .

Solution: All these formulas can be considered algorithms for computing the area of a triangle, as they provide a step-by-step procedure for the calculation.

4. Algorithm design technique: A method or a set of guidelines used to create efficient and effective algorithms to solve problems.

pseudo code: A high-level description of an algorithm that uses the structural conventions of programming but omits detailed syntax, allowing the algorithm to be easily understood and translated into actual code.



Proving correctness of an algorithm: To prove an algorithm's correctness, you need to show that it produces the correct output for all possible valid inputs. This is typically done using techniques such as:

- +1 Mathematical induction.
- +1 Loop invariants.
- +1 Pre-condition and post-condition analysis.

5.

a, In English:

To convert a positive binary number to its decimal representation, follow these steps:

1. Start with the rightmost digit of the binary number.
2. Initialize a variable decimal to 0.
3. For each binary digit:
  - +1 Multiply the digit by  $2^{\text{position}}$
  - +1 Add the result to decimal.
4. The value of decimal after processing all digits is the decimal representation of the binary number.



b. In pseudo code:

Algorithm Binary To Decimal.

Input: binary

Output: Decimal.

decimal  $\leftarrow 0$

position  $\leftarrow 0$

for  $i \leftarrow \text{length}(\text{binary}) - 1$  downto 0 do

digit  $\leftarrow \text{binary}[i]$

decimal  $\leftarrow \text{decimal} + \text{digit} * 2^{\wedge} \text{position}$ .

position  $\leftarrow \text{position} + 1$

return decimal.

Bài 3:

a, Multiplication:  $a \times b$ .

$\text{multi}(a, 0) = 0 = P_0^1(a)$  is PRF since  $P$  is PRF. (1)

$\text{multi}(a, b+1) = \text{add}(a, \text{multi}(a, b))$  is PRF (2)

$\Rightarrow$   $=$

$\Rightarrow (1) \wedge (2)$  is PRF.

$b$ , Exponentiation:  $a^b$

$\text{Exp}(a, 0) = 1 = P_0^1(a)$  is PRF (1)

$\text{Exp}(a, b+1) = \text{multi}(a, \text{Exp}(a, b))$  is PRF. (2)

$\leq$

$\Rightarrow (1) \wedge (2) \Rightarrow$  PRF.



c, Factorial  $a!$ ;  $0! = 1$   
 $a! = a \cdot (a-1)!$

$\text{Fact}(0) = 1 = P_0^1(\text{Fact})$  is PRF

$\text{Fact}(a+1) = \text{mult}(a+1, \text{Fact}(a))$  is PRF.

$\Rightarrow$  PRP.

d,  $\text{pred}(a)$ : If  $a > 0$  then  $a-1$  else 0.

$\text{pred}(0) = 0 = P_0^1(0)$  is PRF.

$\text{pred}(a+1) = a = P_1^1(a)$  is PRF

$\Rightarrow$  PRF.

e, Proper subtraction  $a \perp b$ : If  $a \geq b$  then  $a-b$   
else 0.

$\text{sub}(a, 0) = a = P_1^2(a)$  is PRF.

$\text{sub}(a, b+1) = -1 = \text{sub}(\text{pred}(a), b)$  is PRF

$\Rightarrow$  PRF.