DERIVATION OF HIS-TO-RGB AND RGB-TO-HSI CONVERSION EQUATIONS. (From Gonzalez and Woods, Digital Image Processing, 1st ed. Addison-Wesley, 1992. See www.imageprocessingbook.com

The HSI color model

Recall from the discussion in Section 4.6.1 that hue is a color attribute that describes a pure color (pure yellow, orange, or red), whereas saturation gives a measure of the degree to which a pure color is diluted by white light. The HSI color model owes its usefulness to two principal facts. First, the intensity component, I, is decoupled from the color information in the image. Second, the hue and saturation components are intimately related to the way in which human beings perceive color. These features make the HSI model an ideal tool for developing image processing algorithms based on some of the color sensing properties of the human visual system.

Examples of the usefulness of the HSI model range from the design of imaging systems for automatically determining the ripeness of fruits and vegetables, to systems for matching color samples or inspecting the quality of finished color goods. In these and similar applications, the key is to base system operation on color properties the way a person might use those properties for performing the task in question.

The conversion formulas to go from RGB to HSI and back are considerably more complicated than in the preceding models. Rather than just stating these formulas, however, we take the time to derive them here in order to give the reader a deeper understanding of color manipulation.

Conversion from RGB to HSI. As discussed earlier, the RGB model is defined with respect to a unit cube. However, the color components of the HSI model (hue and saturation) are defined with respect to the color triangle shown in Fig. 4.45(a). (Recall from the discussion of the chromaticity diagram in Section 4.6.1 that all the colors obtainable by combining three given colors lie inside a triangle whose vertices are defined by the three initial colors.) In Fig. 4.45(a), note that the hue, H, of color point P is the angle of the vector shown with respect to the red axis. Thus when $H = 0°$, the color is red, when H is 60° the color is yellow, and so on. The saturation, S, of color point P is the degree to which the color is undiluted by white and is proportional to the distance from P to the center of the triangle. The farther P is from the center of the triangle, the more saturated its color is.

Intensity in the HSI model is measured with respect to a line perpendicular to the triangle and passing through its center. Intensities along this line lying below the triangle tend from dark down to black. Conversely, intensities above the triangle tend from light up to white.

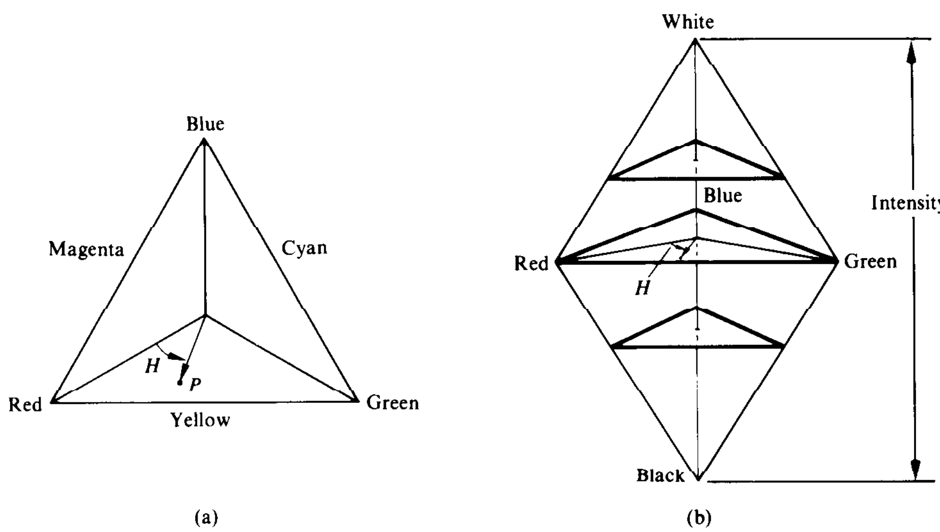

Figure 4.45 *(a) HSI color triangle; (b) HSI color solid.*

Combining hue, saturation, and intensity in a 3-D color space yields the three-sided, pyramidlike structure shown in Fig. 4.45(b). Any point on the surface of this structure represents a purely saturated color. The hue of that color is determined by its angle with respect to the red axis and its intensity by its perpendicular distance from the black point (that is, the greater the distance from black, the greater is the intensity of the color). Similar comments apply to points inside the structure, the only difference being that colors become less saturated as they approach the vertical axis.

Colors in the HSI model are defined with respect to normalized red, green, and blue values, given in terms of RGB primaries by

$$r = \frac{R}{(R + G + B)} \qquad [0,1] \tag{4.6-7}$$

$$g = \frac{G}{(R + G + B)} \tag{4.6-8}$$

and

$$b = \frac{B}{(R + G + B)} \tag{4.6-9}$$

where, as before, the assumption is that R, G, and B, have been normalized so that they are in the range [0, 1]. Equations (4.6-7)–(4.6-9) show that r, g, and b also are in the interval [0, 1] and that

$$r + g + b = 1. \tag{4.6-10}$$

Note that, whereas R, G, and B can all be 1 simultaneously, the normalized variables have to satisfy Eq. (4.6-10). In fact, this is the equation of the plane that contains the HSI triangle.

For any three R, G, and B color components, each in the range [0, 1], the intensity component in the HSI model is defined as

$$I = \frac{1}{3}(R + G + B) \tag{4.6-11}$$

which yields values in the range [0, 1].

The next step is to obtain H and S. To obtain H requires the geometric construction of the HSI triangle shown in Figs. 4.46(a), (b), and (c), from which we note the following conditions.

$$b_0 = \frac{3B}{(R+G+B)} = 3b$$

(a) The point W has coordinates (1/3, 1/3, 1/3).

(b) An arbitrary color point P has coordinates (r, g, b).

(c) The vector extending from the origin to W is denoted $\mathbf{w}$. Similarly, the vectors extending from the origin to P_R and to P are denoted $\mathbf{p}_R$ and $\mathbf{p}$, respectively.

(d) The lines P_iQ_i, $i = R, G, B$, intersect at W by construction.

(e) Letting $r_0 = R/I$, $g_0 = G/I$, and $b_0 = B/I$, where I is given in Eq. (4.6-11), we see from Fig. 4.46(a) that P_RQ_R is the locus of points (r_0, g_0, b_0) for which $g_0 = b_0$. Similarly, $r_0 = g_0$ along P_BQ_B, and $r_0 = b_0$ along P_GQ_G.

(f) Any point in the planar region bounded by triangle $P_RQ_RP_G$ has $g_0 \geq b_0$. Any point in the region bounded by triangle $P_RQ_RP_B$ has $b_0 \geq g_0$. Thus line P_RQ_R separates the $g_0 > b_0$ region from the $g_0 < b_0$ region. Similarly, P_GQ_G separates the $b_0 > r_0$ region from the $b_0 < r_0$ region, and P_BQ_B separates the $g_0 > r_0$ region from the $g_0 < r_0$ region.

(g) For $i = R, G$, or B, $|WQ_i|/|P_iQ_i| = 1/3$ and $|WP_i|/|P_iQ_i| = 2/3$, where $|\text{arg}|$ denotes the length of the argument.

(h) By definition the *RG sector* is the region bounded by WP_RP_G, the *GB sector* is the region bounded WP_GP_B, and the *BR sector* is the region bounded by WP_BP_R.

With reference to Fig. 4.46(a), the hue of an arbitrary color is defined by the angle between the line segments WP_R and WP or, in vector form (Fig. 4.46c), by the angle between vectors $(\mathbf{p}_R - \mathbf{w})$ and $(\mathbf{p} - \mathbf{w})$. For example, as

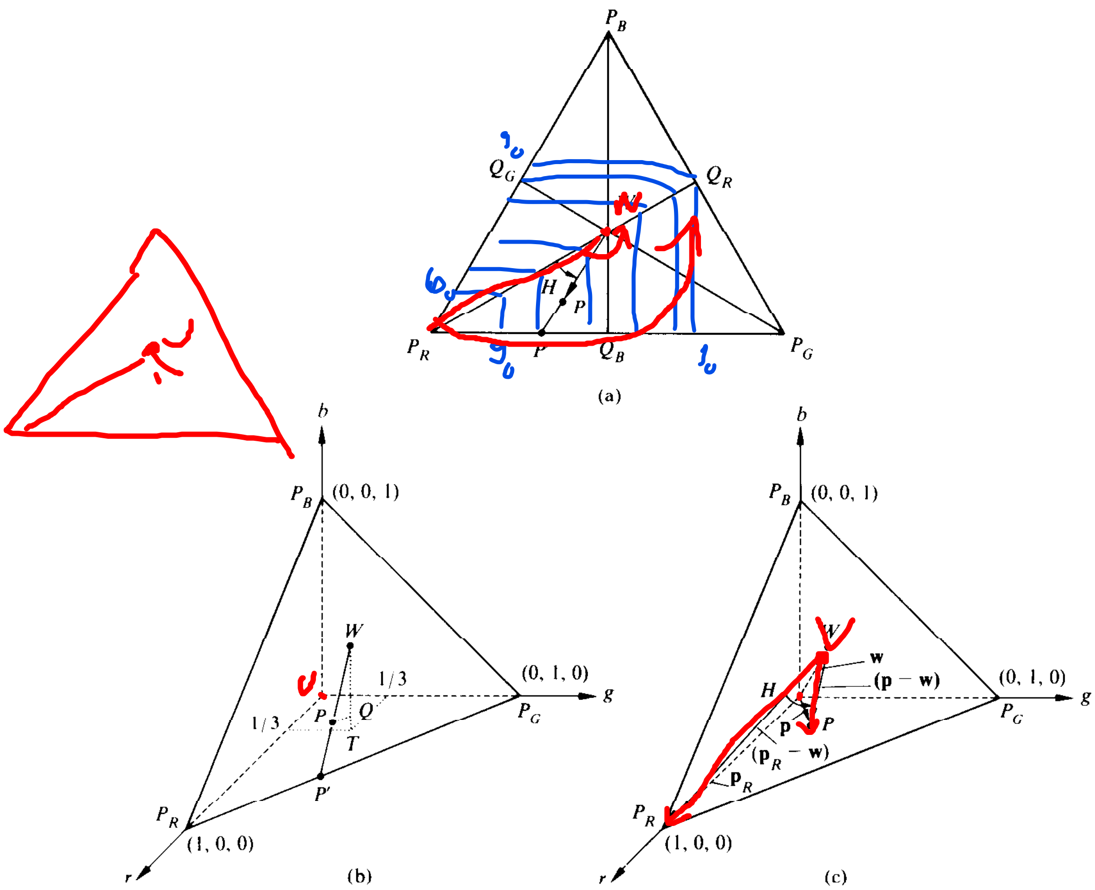

Figure 4.46 *Details of the HSI color triangle needed to obtain expressions for hue and saturation.*

stated earlier, $H = 0°$ corresponds to red, $H = 120°$ corresponds to green, and so on. Although the angle H could be measured with respect to any line passing through W, measuring hue with respect to red is a convention. In general, the following equation holds for $0° \leq H \leq 180°$:

$$(\mathbf{p} - \mathbf{w}) \cdot (\mathbf{p}_R - \mathbf{w}) = \|\mathbf{p} - \mathbf{w}\| \|\mathbf{p}_R - \mathbf{w}\| \cos H \tag{4.6-12}$$

where $(\mathbf{x}) \cdot (\mathbf{y}) = \mathbf{x}^T\mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\|\cos H$ denotes the dot or inner product of the two vectors, and the double bars denote the norm (length) of the vector argument. The problem now is to express this result in terms of a set of RGB primaries.

From conditions (a) and (b),

$$\|\mathbf{p} - \mathbf{w}\| = \left[\left(r - \frac{1}{3}\right)^2 + \left(g - \frac{1}{3}\right)^2 + \left(b - \frac{1}{3}\right)^2\right]^{1/2} \tag{4.6-13}$$

because the length of a vector $\mathbf{a}$ with components a_1, a_2, and a_3 is $\|\mathbf{a}\| = [a_1^2 + a_2^2 + a_3^2]^{1/2}$. Substituting Eqs. (4.6-7)–(4.6-9) for r, g, and b in Eq. (4.6-13) and simplifying yields

$$\|\mathbf{p} - \mathbf{w}\| = \left[\frac{9(R^2 + G^2 + B^2) - 3(R + G + B)^2}{9(R + G + B)^2}\right]^{1/2}. \tag{4.6-14}$$

As vectors $\mathbf{p}_R$ and $\mathbf{w}$ extend from the origin to points (1, 0, 0) and (1/3, 1/3, 1/3), respectively,

$$\|\mathbf{p}_R - \mathbf{w}\| = \left(\frac{2}{3}\right)^{1/2}. \tag{4.6-15}$$

Keep in mind that, for two vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a}\cdot\mathbf{b} = \mathbf{a}^T\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$. Then

$$\begin{aligned}(\mathbf{p} - \mathbf{w})\cdot(\mathbf{p}_R - \mathbf{w}) &= \frac{2}{3}\left(r - \frac{1}{3}\right) - \frac{1}{3}\left(g - \frac{1}{3}\right) + \frac{1}{3}\left(b - \frac{1}{3}\right)\\ &= \frac{2R - G - B}{3(R + G + B)}.\end{aligned} \tag{4.6-16}$$

From Eq. (4.6-12),

$$H = \cos^{-1}\left[\frac{(\mathbf{p} - \mathbf{w})\cdot(\mathbf{p}_R - \mathbf{w})}{\|\mathbf{p} - \mathbf{w}\|\|\mathbf{p}_R - \mathbf{w}\|}\right]. \tag{4.6-17}$$

Substituting Eqs. (4.6-14)–(4.6-16) into Eq. (4.6-17) and simplifying yields the following expression for H in terms of R, G, and B:

$$H = \cos^{-1}\left\{\frac{\frac{1}{2}[(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{1/2}}\right\}. \tag{4.6-18}$$

Equation (4.6-18) yields values of H in the interval $0° \le H \le 180°$. If $b_0 > g_0$, then H has to be greater than 180°. So, whenever $b_0 > g_0$, we simply let $H = 360° - H$. Sometimes the equation for hue is expressed in terms of the tangent by using the trigonometric identity $\cos^{-1}(x) = 90° - \tan^{-1}(x/\sqrt{1 - x^2})$. However, Eq. (4.6-18) not only is simpler to visualize, but it also is superior in terms of hardware implementation.

The next step is to derive an expression for S in terms of a set of RGB primary values. To do so again requires Figs. 4.46(a) and (b). Because the saturation of a color is the degree to which that color is undiluted by white, from Fig. 4.46(a) the saturation, S, of color point P is given by the ratio $|WP|/|WP'|$, where P' is obtained by extending line WP until it intersects the nearest side of the triangle.

With reference to Fig. 4.46(b), let T be the projection of W onto the rg plane, parallel to the b axis and let Q be the projection of P onto WT, parallel to the rg plane. Then

$$S = \frac{|WP|}{|WP'|} = \frac{|WQ|}{|WT|} = \frac{|WT| - |QT|}{|WT|} \tag{4.6-19}$$

where the second step follows from similar triangles. Since $|WT| = 1/3$ and $|QT| = b$ in the sector shown,

$r + g + b = 1$

$b = \frac{B}{(R+G+B)}$

$$\begin{aligned} S &= 3\left(\frac{1}{3} - b\right) \\ &= 1 - 3b \\ &= 1 - b_0 \end{aligned} \tag{4.6-20}$$

$3b = b_0$

where the last step follows from Eq. (4.6-10) and condition (e). Also, we note that $b_0 = \min(r_0, g_0, b_0)$ in the RG *sector*. In fact, an argument similar to the one just given would show that the relationship

$b_0 = \frac{B}{I}$

$$\begin{aligned} S &= 1 - \min(r_0, g_0, b_0) \\ &= 1 - \frac{3}{(R + G + B)}\boxed{[\min(R, G, B)]} \end{aligned} \tag{4.6-21}$$

$I = \frac{1}{3(R+G+B)}$

is true in general for any point lying on the HSI triangle.

The results obtained thus far give the following expressions for obtaining HSI values in the range [0, 1] from a set of RGB values in the same range:

$\Rightarrow 3b = b_0$

$$I = \frac{1}{3}(R + G + B) \tag{4.6-22}$$

$$S = 1 - \frac{3}{(R + G + B)}[\min(R, G, B)] \tag{4.6-23}$$

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and

$$H = \cos^{-1}\left\{\frac{\frac{1}{2}[(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{1/2}}\right\} \tag{4.6-24}$$

where, as indicated earlier, we let $H = 360° - H$, if $(B/I) > (G/I)$. In order to normalize hue to the range [0, 1], we let $H - H/360°$. Finally, if $S - 0$, it follows from Eq. (4.6-19) that $|WP|$ must be zero, which means that W and P have become the same point, making it meaningless to define angle H. Thus hue is not defined when the saturation is zero. Similarly, from Eqs. (4.6-22) and (4.6-23), saturation is undefined if $I = 0$.

Conversion from HSI to RGB. For values of HSI in [0, 1], we now want to find the corresponding RGB values in the same range. The analysis depends on which of the sectors defined in condition (h) contains the given value of H. We begin by letting $H = 360°(H)$, which returns the hue to the range [0°, 360°].

For the *RG Sector* ($0° < H \leq 120°$), from Eq. (4.6-20),

$$b = \frac{1}{3}(1 - S). \tag{4.6-25}$$

Next, we find r by noting from Fig. 4.46(a) that the value of r is the projection of P onto the red axis. Consider the triangle $P_R O Q_R$ shown in Fig. 4.47, where O is the origin of the *rgb* coordinate system. The hypotenuse of this triangle is the line segment $P_R Q_R$ in Fig. 4.46(a), and the line extending from O to P_R is the red axis containing r. The dashed line is the intersection of triangle $P_R O Q_R$ with the plane that contains P and is perpendicular to the red axis. These two conditions imply that the plane also contains r. Furthermore, the point at which $P_R Q_R$ intersects the plane contains the projection of P onto line $P_R Q_R$, which, from Fig. 4.46(a), is $|WP| \cos H$. From similar triangles,

$$\frac{|P_R Q_R|}{|P_R O|} = \frac{a}{d}. \tag{4.6-26}$$

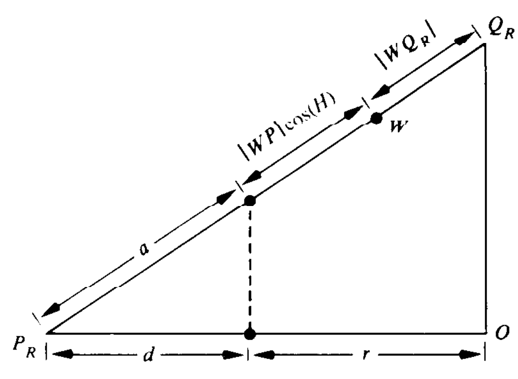

Figure 4.47 *Arrangement used to derive equations for converting from HSI to RGB*

But $|P_R O| = 1$, $d = 1 - r$, and $a = |P_R Q_R| - (|WP| \cos H + |WQ_R|)$. Substituting these results into Eq. (4.6-26) and simplifying yields

$$\begin{aligned} r &= \frac{|WQ_R|}{|P_R Q_R|} + \frac{|WP|}{|P_R Q_R|} \cos H \\ &= \frac{1}{3} + \frac{|WP|}{|P_R Q_R|} \cos H \end{aligned} \tag{4.6-27}$$

where we used $|P_R Q_R| = 3|WQ_R|$ from Fig. 4.46(a). The only unknown in this equation is $|WP|$, which, from Eq. (4.6-19), is $|WP| = S|WP'|$. In Fig. 4.46(a), the angle formed at W by line segments $P_R Q_R$ and WQ_B is 60°; therefore $|WQ_B| = |WP'| \cos(60° - H)$, or $|WP'| = |WQ_B| / \cos(60° - H)$. Noting that $|WQ_B| = |WQ_R|$ and substituting these results into Eq. (4.6-27) yields

$$\begin{aligned} r &= \frac{1}{3} + \frac{S|WQ_R| \cos H}{|P_R Q_R| \cos(60° - H)} \\ &= \frac{1}{3}\left[1 + \frac{S \cos H}{\cos(60° - H)}\right] \end{aligned} \tag{4.6-28}$$

where we used $|P_R Q_R| = 3|WQ_R|$ again. Finally, $g = 1 - (r + b)$ from Eq. (4.6-10). Hence the results for $0° < H \leq 120°$ are

$$b = \frac{1}{3}(1 - S) \tag{4.6-29}$$

$$r = \frac{1}{3}\left[1 + \frac{S \cos H}{\cos(60° - H)}\right] \tag{4.6-30}$$

and

$$g = 1 - (r + b). \tag{4.6-31}$$

The color components just obtained are normalized in the sense of Eq. (4.6-10). We recover the RGB components by noting from Eqs. (4.6-7)–(4.6-11) that $R = 3Ir$, $G = 3Ig$, and $B = 3Ib$.

For the *GB sector* ($120° < H < 240°$), a development similar to the one just completed yields

$$H = H - 120° \tag{4.6-32}$$

$$r = \frac{1}{3}(1 - S) \tag{4.6-33}$$

$$g = \frac{1}{3}\left[1 + \frac{S \cos H}{\cos(60° - H)}\right] \tag{4.6-34}$$

and

$$b = 1 - (r + g). \tag{4.6-35}$$

The values of R, G, and B, are obtained from r, g, and b in the manner previously described.

For the *BR sector* ($240° < H \leq 360°$),

$$H = H - 240° \tag{4.6-36}$$

$$g = \frac{1}{3}(1 - S) \tag{4.6-37}$$

$$b = \frac{1}{3}\left[1 + \frac{S \cos H}{\cos(60° - H)}\right] \tag{4.6-38}$$

and

$$r = 1 - (g + b). \tag{4.6-39}$$

The values of R, G, and B, are obtained from r, g, and b in the manner previously described.

Image processing examples using the HSI model are presented in Section 4.6.4.