approximate the given integrals using both the Trapezoidal rule and the Midpoint rule. The general formulas for these rules are:

Trapezoidal Rule

The Trapezoidal rule for approximating the integral $\int_a^b f(x)\,dx$ is given by:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}(f(a)+f(b))$$

Midpoint Rule

The Midpoint rule for approximating the integral $\int_a^b f(x)\,dx$ is given by:

$$\int_a^b f(x)\,dx \approx (b-a)f(\frac{a+b}{2})$$

Let's apply these rules to each of the given integrals.

a. $\int_{0.5}^{1} x^4\,dx$

Trapezoidal Rule:

$$\int_{0.5}^{1} x^4\,dx \approx \frac{1-0.5}{2}((0.5)^4+1^4) = \frac{0.5}{2}(0.0625+1) = 0.25\times 1.0625 = 0.265625$$

Midpoint Rule:

$$\int_{0.5}^{1} x^4\,dx \approx (1-0.5)((\frac{0.5+1}{2})^4) = 0.5(0.75^4) = 0.5\times 0.31640625 = 0.158203125$$

b. $\int_{0}^{0.5}(2x-4)\,dx$

Trapezoidal Rule:

$$\int_{0}^{0.5}(2x-4)\,dx \approx \frac{0.5-0}{2}((2\times 0-4)+(2\times 0.5-4)) = \frac{0.5}{2}(-4-3) = 0.25\times -7 = -1.75$$

Midpoint Rule:

$$\int_{0}^{0.5}(2x-4)\,dx \approx (0.5-0)(2(\frac{0+0.5}{2})-4) = 0.5(2\times 0.25-4) = 0.5\times -3.5 = -1.75$$

c. $\int_{1}^{1.5} x^2\ln x\,dx$

Trapezoidal Rule:

$$\int_{1}^{1.5} x^2\ln x\,dx \approx \frac{1.5-1}{2}(1^2\ln 1+1.5^2\ln 1.5) = \frac{0.5}{2}(0+2.25\ln 1.5) = 0.25\times 2.25\times 0.405465 = 0.228075$$

Midpoint Rule:

$$\int_{1}^{1.5} x^2 \ln x \, dx \approx (1.5-1)\left(\left(\frac{1+1.5}{2}\right)^2 \ln\left(\frac{1+1.5}{2}\right)\right) = 0.5\,(1.25^2 \ln 1.25) = 0.5 \times 1.5625 \times 0.223144 = 0.174017$$

d. $\int_0^1 x^2 e^{-x}\, dx$

Trapezoidal Rule:

$$\int_{0}^{1} x^2 e^{-x} \, dx \approx \frac{1-0}{2}\,(0^2 e^0 + 1^2 e^{-1}) = 0.5\,(0 + e^{-1}) = 0.5 \times 0.367879 = 0.183939$$

Midpoint Rule:

$$\int_{0}^{1} x^2 e^{-x} \, dx \approx (1-0)\left(\left(\frac{0+1}{2}\right)^2 e^{-\left(\frac{0+1}{2}\right)}\right) = 1\,(0.25 e^{-0.5}) = 0.25 \times 0.606531 = 0.151633$$

e. $\int_1^{1.6} \frac{2x}{x^2-4}\, dx$

Trapezoidal Rule:

$$\int_{1}^{1.6} \frac{2x}{x^2-4} \, dx \approx \frac{1.6-1}{2}\left(\frac{2\times 1}{1^2-4} + \frac{2 \times 1.6}{1.6^2-4}\right)$$

$$\int_{1}^{1.6} \frac{2x}{x^2-4} \, dx \approx \frac{0.6}{2}\left(\frac{2}{-3} + \frac{3.2}{-1.44}\right) = 0.3\left(-\frac{2}{3} - \frac{3.2}{1.44}\right)$$

$$\int_{1}^{1.6} \frac{2x}{x^2-4} \, dx \approx 0.3\,(-0.66667 - 2.22222) = 0.3 \times -2.88889 = -0.866667$$

Midpoint Rule:

$$\int_{1}^{1.6} \frac{2x}{x^2-4} \, dx \approx (1.6-1)\left(\frac{2\left(\frac{1+1.6}{2}\right)}{\left(\frac{1+1.6}{2}\right)^2 - 4}\right)$$

$$\int_{1}^{1.6} \frac{2x}{x^2-4} \, dx \approx 0.6\left(\frac{2 \times 1.3}{1.3^2-4}\right) = 0.6\left(\frac{2.6}{1.69-4}\right) = 0.6\left(\frac{2.6}{-2.31}\right) = 0.6 \times -1.12554 = -0.675324$$

f. $\int_0^{0.35} \frac{2}{x^2-4}\, dx$

Trapezoidal Rule:

$$\int_{0}^{0.35} \frac{2}{x^2-4} \, dx \approx \frac{0.35-0}{2}\left(\frac{2}{0^2-4} + \frac{2}{0.35^2-4}\right)$$

$$\int_{0}^{0.35} \frac{2}{x^2-4} \, dx \approx \frac{0.35}{2}\left(\frac{2}{-4} + \frac{2}{0.1225-4}\right) = 0.175\,(-0.5 - 0.520833) = 0.175 \times -1.02083 = -0.178646$$

Midpoint Rule:

$$\int_{0}^{0.35} \frac{2}{x^2-4} \, dx \approx (0.35-0)\left(\frac{2}{\left(\frac{0+0.35}{2}\right)^2 - 4}\right)$$

$$\int_0^{0.35} \frac{2}{x^2-4}\,dx \approx 0.35\left(\frac{2}{(0.175)^2-4}\right) = 0.35\left(\frac{2}{0.030625-4}\right) = 0.35\left(\frac{2}{-3.969375}\right) = 0.35 \times -0.504 < -0.1764$$

g. $\int_0^{\frac{\pi}{4}} x\sin x\,dx$

Trapezoidal Rule:

$$\int_0^{\frac{\pi}{4}} x\sin x\,dx \approx \frac{\frac{\pi}{4}-0}{2}\left(0\sin 0 + \frac{\pi}{4}\sin\frac{\pi}{4}\right)$$

$$\int_0^{\frac{\pi}{4}} x\sin x\,dx \approx \frac{\pi}{8}\left(0+\frac{\pi}{4}\times\frac{\sqrt{2}}{2}\right) = \frac{\pi}{8}\times\frac{\pi\sqrt{2}}{8} = \frac{\pi^2\sqrt{2}}{64}$$

Midpoint Rule:

$$\int_0^{\frac{\pi}{4}} x\sin x\,dx \approx \left(\frac{\pi}{4}-0\right)\left(\left(\frac{0+\frac{\pi}{4}}{2}\right)\sin\left(\frac{0+\frac{\pi}{4}}{2}\right)\right)$$

$$\int_0^{\frac{\pi}{4}} x\sin x\,dx \approx \frac{\pi}{4}\left(\frac{\pi}{8}\sin\frac{\pi}{8}\right) = \frac{\pi^2}{32}\sin\frac{\pi}{8}$$

h. $\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\,dx$

Trapezoidal Rule:

$$\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\,dx \approx \frac{\frac{\pi}{4}-0}{2}\left(e^0\sin 0 + e^{\frac{3\pi}{4}}\sin\frac{\pi}{2}\right)$$

$$\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\,dx \approx \frac{\pi}{8}\left(0+e^{\frac{3\pi}{4}}\right) = \frac{\pi e^{\frac{3\pi}{4}}}{8}$$

Midpoint Rule:

$$\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\,dx \approx \left(\frac{\pi}{4}-0\right)\left(e^{3\left(\frac{0+\frac{\pi}{4}}{2}\right)}\sin 2\left(\frac{0+\frac{\pi}{4}}{2}\right)\right)$$

$$\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\,dx \approx \frac{\pi}{4}\left(e^{3\left(\frac{\pi}{8}\right)}\sin\frac{\pi}{4}\right) = \frac{\pi}{4}\left(e^{\frac{3\pi}{8}}\times\frac{\sqrt{2}}{2}\right) = \frac{\pi e^{\frac{3\pi}{8}}\sqrt{2}}{8}$$