To find the value of y for which the coefficient of x^3 in the interpolating polynomial $P_3(x)$ is 6, we'll use the method of Lagrange interpolation.

Given the points:

$$(0, 0), (0.5, y), (1, 3), (2, 2)$$

The Lagrange interpolating polynomial $P_3(x)$ is given by:

$$P_3(x) = \sum_{i=0}^{3} y_i \ell_i(x)$$

where $\ell_i(x)$ are the Lagrange basis polynomials defined by:

$$\ell_i(x) = \prod_{j=0, j \neq i}^{3} \frac{x - x_j}{x_i - x_j}$$

Basis Polynomials Calculation

For $\ell_0(x)$:

$$\ell_0(x) = \frac{(x-0.5)(x-1)(x-2)}{(0-0.5)(0-1)(0-2)} = \frac{(x-0.5)(x-1)(x-2)}{(-0.5)(-1)(-2)} = \frac{(x-0.5)(x-1)(x-2)}{-1}$$

For $\ell_1(x)$:

$$\ell_1(x) = \frac{(x-0)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)} = \frac{(x)(x-1)(x-2)}{(0.5)(-0.5)(-1.5)} = \frac{(x)(x-1)(x-2)}{0.375}$$

For $\ell_2(x)$:

$$\ell_2(x) = \frac{(x-0)(x-0.5)(x-2)}{(1-0)(1-0.5)(1-2)} = \frac{(x)(x-0.5)(x-2)}{(1)(0.5)(-1)} = \frac{(x)(x-0.5)(x-2)}{-0.5}$$

For $\ell_3(x)$:

$$\ell_3(x) = \frac{(x-0)(x-0.5)(x-1)}{(2-0)(2-0.5)(2-1)} = \frac{(x)(x-0.5)(x-1)}{(2)(1.5)(1)} = \frac{(x)(x-0.5)(x-1)}{3}$$

Polynomial Construction

Now construct $P_3(x)$:

$$P_3(x) = 0 \cdot \ell_0(x) + y \cdot \ell_1(x) + 3 \cdot \ell_2(x) + 2 \cdot \ell_3(x)$$

We focus on the coefficient of x^3. Since x^3 term only comes from the products of x in all three terms of each $\ell_i(x)$, we only need the highest-degree term from each $\ell_i(x)$.

From $\ell_0(x)$:

$$\ell_0(x) = \frac{(x-0.5)(x-1)(x-2)}{-1} = \frac{-x^3+3.5x^2-3.5x+1}{-1} = x^3-3.5x^2+3.5x-1$$

From $\ell_1(x)$:

$$\ell_1(x) = \frac{(x)(x-1)(x-2)}{0.375} = \frac{x^3-3x^2+2x}{0.375} = \frac{x^3-3x^2+2x}{3/8} = 8/3(x^3-3x^2+2x)$$

From $\ell_2(x)$:

$$\ell_2(x) = \frac{(x)(x-0.5)(x-2)}{-0.5} = \frac{x^3-2.5x^2+x}{-0.5} = -2(x^3-2.5x^2+x)$$

From $\ell_3(x)$:

$$\ell_3(x) = \frac{(x)(x-0.5)(x-1)}{3} = \frac{x^3-1.5x^2+0.5x}{3} = \frac{1}{3}(x^3-1.5x^2+0.5x)$$

Combining Terms

$$P_3(x) = 0\cdot\ell_0(x) + y\cdot\frac{8}{3}(x^3-3x^2+2x) + 3\cdot -2(x^3-2.5x^2+x) + 2\cdot\frac{1}{3}(x^3-1.5x^2+0.5x)$$

$$= y\cdot\frac{8}{3}x^3 + 3\cdot -2x^3 + 2\cdot\frac{1}{3}x^3 = \frac{8}{3}yx^3 - 6x^3 + \frac{2}{3}x^3$$

$$= x^3\left(\frac{8}{3}y - 6 + \frac{2}{3}\right) = x^3\left(\frac{8}{3}y - \frac{16}{3}\right)$$

So the coefficient of x^3 is:

$$\frac{8}{3}y - \frac{16}{3} = 6$$

Solve for y:

$$\frac{8}{3}y - \frac{16}{3} = 6$$

Multiply both sides by 3:

$$8y - 16 = 18$$

Add 16 to both sides:

$$8y = 34$$

Divide both sides by 8:

$$y = \frac{34}{8} = \frac{17}{4} = 4.25$$

Vậy giá trị của y là 4.25.