Problem 6

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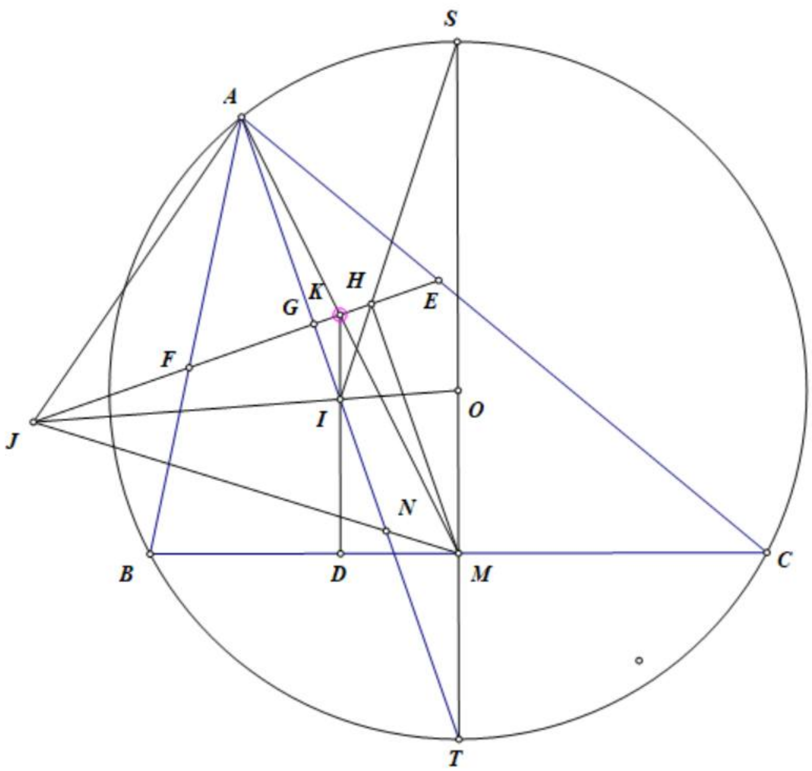

Consider an inversion about circle (I), it sends the Euler circle of (DEF) to (O) hence OI passes through the center of the Euler center of (DEF).
Therefore, OI is the Euler line of $\triangle DEF$ or O, I, J are collinear.
Let AI cut (O) and EF at T and G respectively, let TM cut (O) at S. It is well known that ID, EF, AM are concurrent at K.
Let IS cut EF at H, then

$$\frac{IH}{IS} = \frac{IG}{IA} = \frac{IF^2}{IA^2} = \frac{TB^2}{TS^2} = \frac{TM}{TS},$$

hence $MH \parallel IT$. Let MJ cut AI at N.
Since O is the midpoint of AT, we have:
$-1 = I(OD, ST) = I(JK, HG) = M(JK, HG) = M(NA, HG)$.
Combine this with the fact that $MH \parallel AN$, we get that G is the midpoint of AN, or $\angle AJG = \angle NJG = \angle MJG$, as desired.
Therefore, the problem is proven.