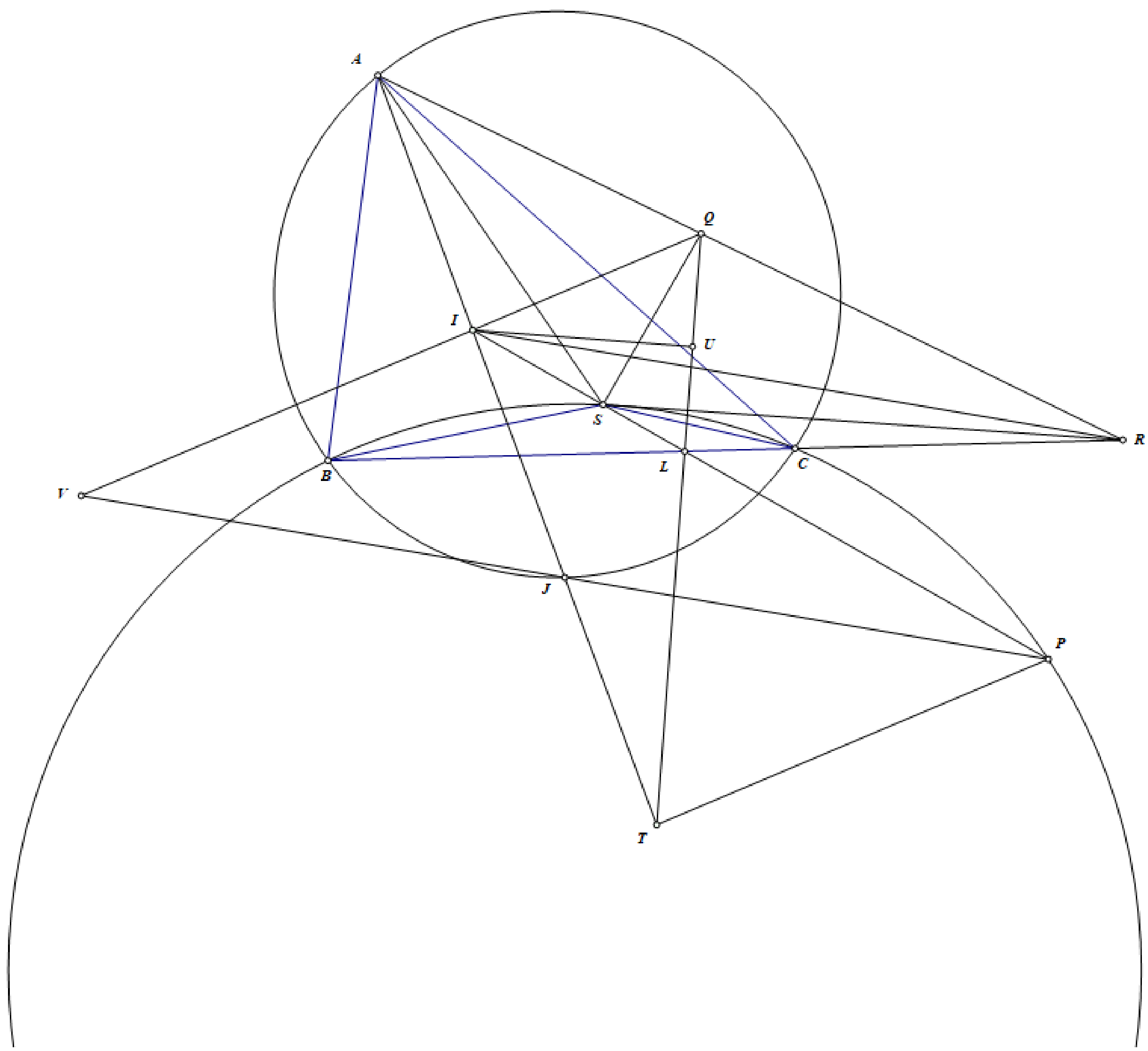
Problem 10

Ha Vu Anh



Redefine the problem: Take a point R on BC such that $\angle RSP = \angle ASI$
Prove that $IR \parallel JP$:

Let the line through S perpendicular to SI intersects AR at Q, then SQ is the bisector of $\angle ASR$
Let T be the A-excenter of triangle ABC, and let IP intersect BC at L, then $L(IT, AR) = -1 = L(IQ, AR)$, hence $\overline{T, L, Q}$
Let U be the projection of I onto TL, then B, I, U, C, T lie on a circle with diameter IT
Hence $\overline{LU} \cdot \overline{LT} = \overline{LB} \cdot \overline{LC} = \overline{LS} \cdot \overline{LP}$

It follows that $SUTP$ is cyclic, and $IUQS$ is cyclic with diameter IQ
By Reim's theorem, $IQ \parallel TP$
Let IQ intersect JP at V, since J is the midpoint of IT, J is also the midpoint of VP.
Moreover, $I(RJ, PV) = I(RA, SQ) = -1$, so $IR \parallel JP$, as desired.
Therefore, the problem is proven.