Problem 6

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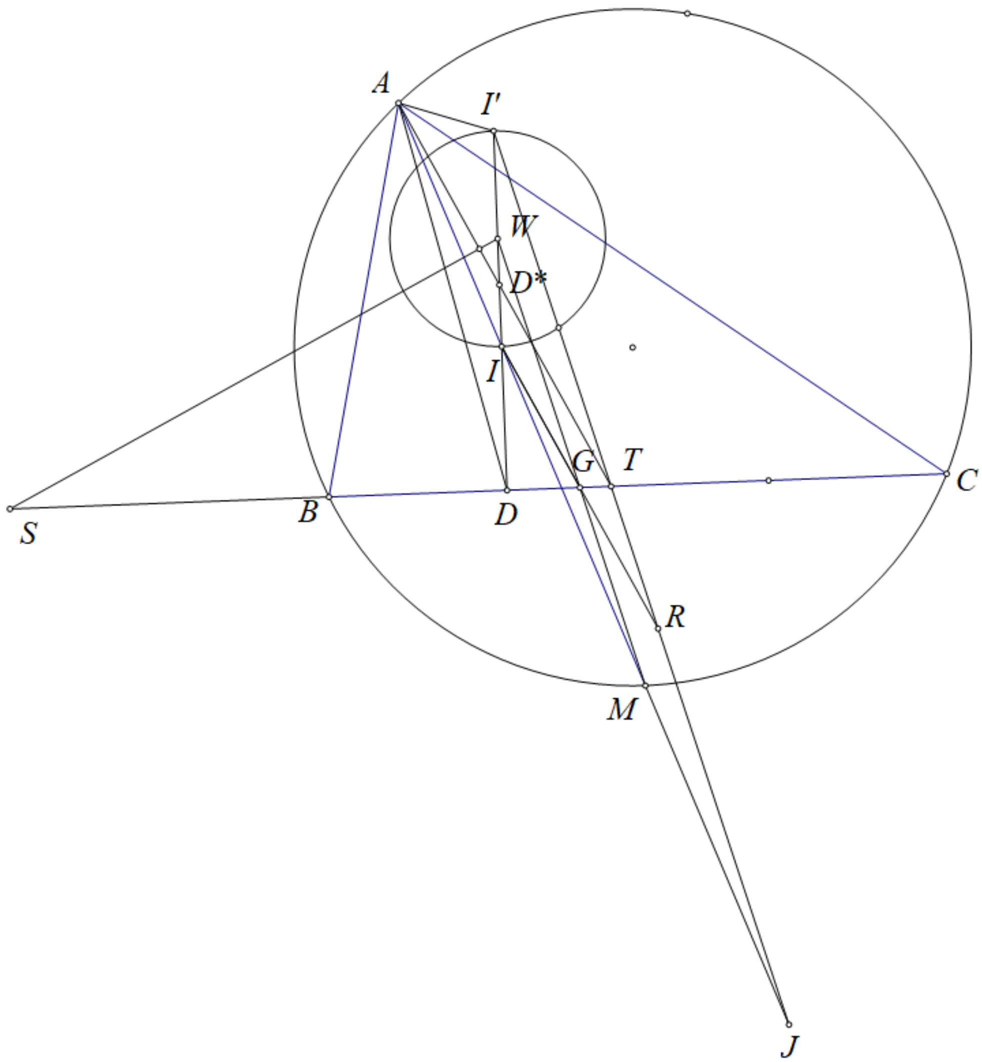

Denote the circumcenter of ω as W, Let M be the midpoint of arc BC of Ω not containing A, J be the $A-excenter$ of ABC, I' be the reflection of I through W, S be the pole of AD^* wrt ω. The problem is equivalent to proving S lies on the radical axis of ω and Ω.

We have $WI'^2 = WI^2 = WD^* \cdot WD$ therefore $(DD^*, II') = -1$ and S lies on the polar of D^* wrt ω which is BC.
Let JI' cut BC at T, WM cut BC at G.
We have:
$-1 = D(JI, GA) = D(JI', TA) = A(JI', TD)$ and $A(II', D^*D) = -1 = A(II', TD)$

therefore A, D^*, T are collinear and since $AD* \perp WS$ we get $TD^* \perp WS$.
Let IG cut $I'J$ at R then since M, W are the midpoints of IJ, II' respectively, G is the midpoint of IR.
Combine this with $-1 = T(DD^*, II') = T(GD^*, IR)$ we get $TD^* \parallel IG$
and therefore $IG \perp WS$ therefore I is the orthocenter of WSG therefore $SI \perp WM$.
Combine this with the fact that M is the center of (BIC) we get SI is the radical axis of ω and (BIC) therefore the power of S to ω equal to the power of S to (BIC) equal to the power of S to Ω therefore S also lies on the radical axis ω and Ω.

Hence the problem is done.