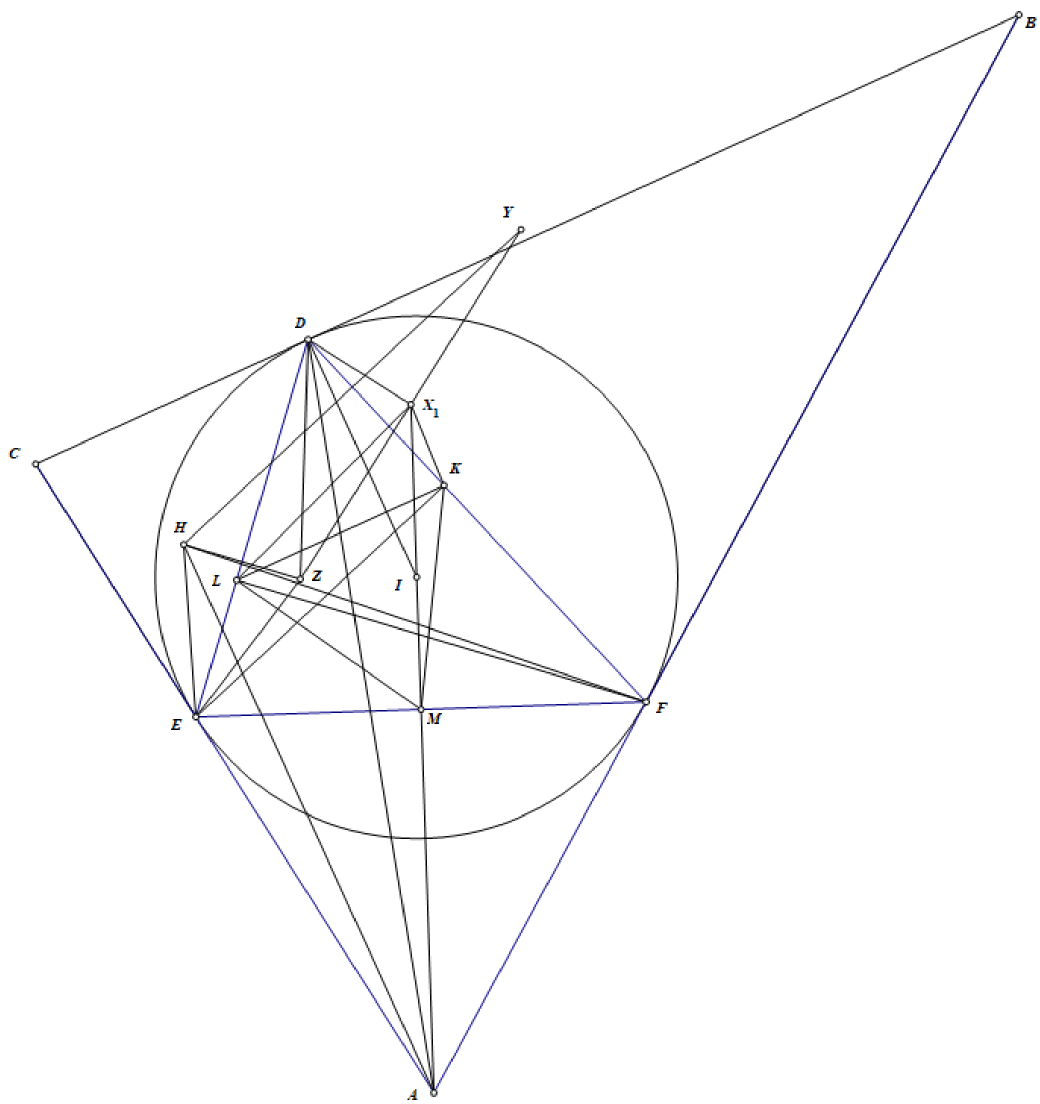
Problem 10

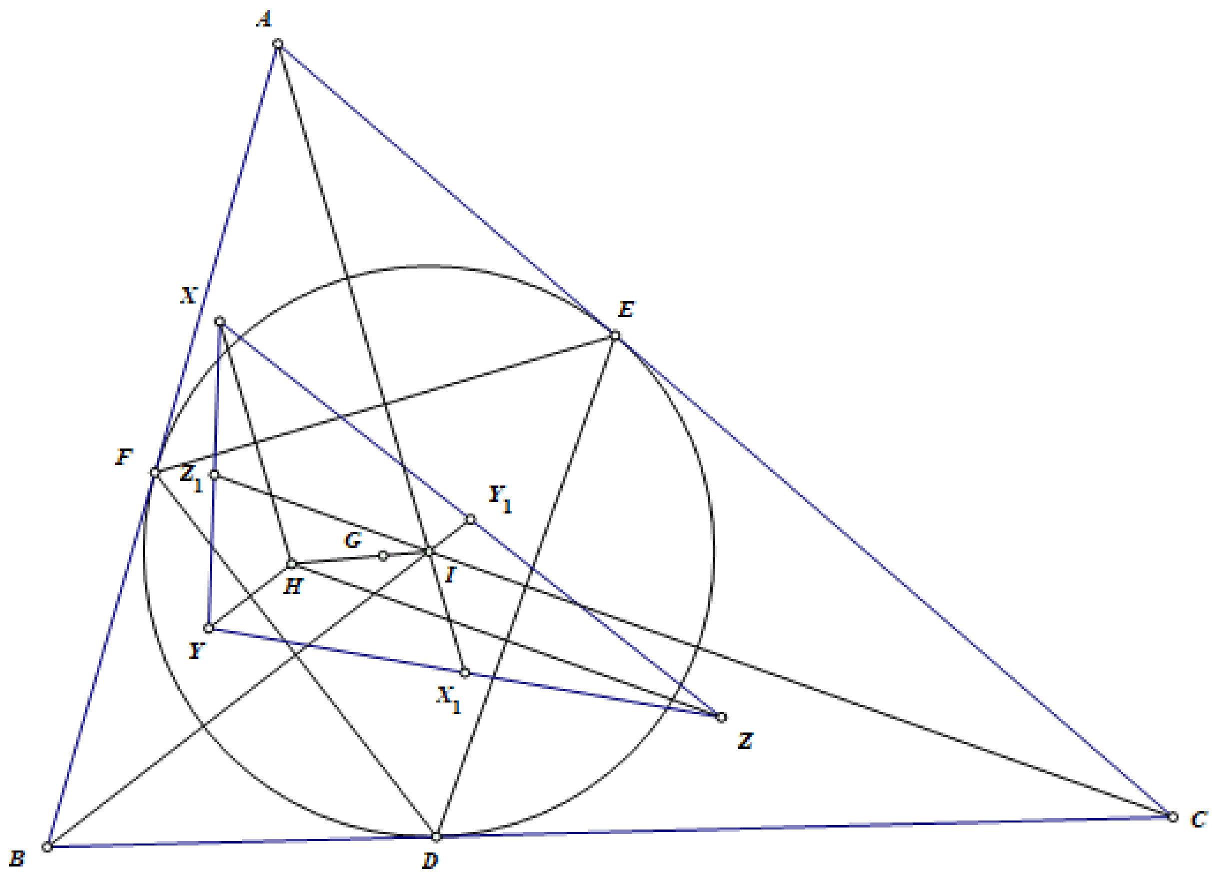
Ha Vu Anh

Claim: AI bisects YZ.



Proof: Let K, L be the projections of E, F on DF, DE respectively, X_1 be the midpoint of YZ. By simple angle chasing: $\triangle DX_1Z \sim \triangle DKE$ therefore $\triangle DX_1K \sim \triangle DZE \Rightarrow \angle X_1KL = \angle DKL + \angle X_1KD = \angle DEF + \angle ZED = \angle DEF + \angle DEH = \angle HEF$. Similiarly, $\angle X_1LK = \angle HFE$ therefore $\triangle HEF \sim \triangle X_1KL$. Let M be the midpoint of EF then $\triangle MKL \sim \triangle AFE$ therefore $\triangle DEF \cup \{A, H\} \sim \triangle DKL \cup \{M, X_1\} \Rightarrow \angle DMX_1 = \angle DAH = \angle ADI = \angle DMI$ therefore X_1 lies on MI hence the claim is proved.

Back to the main problem,



Applying the claim, we get AI bisects YZ and similarly, BI, CI bisects XZ, XY at Y_1, Z_1 respectively. Let G be the centroid of $\triangle XYZ$, H' be a point such that $\overrightarrow{H'G} = 2\overrightarrow{GI}$
we get $XH' \parallel IX_1 \Rightarrow XH' \perp EF$ and similiarly $YH' \perp DF, ZH' \perp DE$
therefore $H' \equiv H$ and hence G, which is the centroid of XYZ, lies on IH.