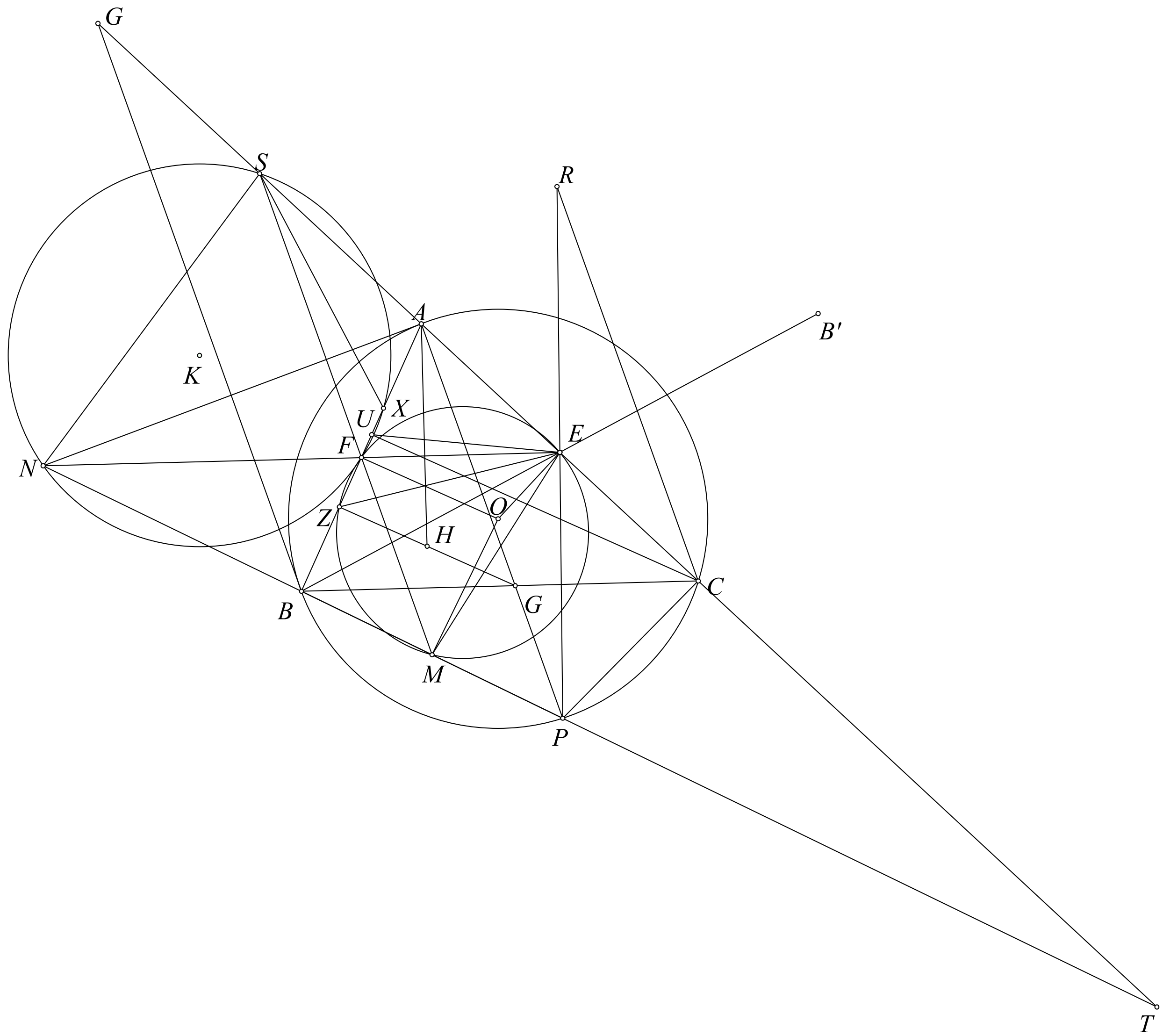
Notation: $\triangle ABC \cup (D)\ \triangle A_1B_1C_1 \cup (D_1)$ means that triangles ABC and $A_1B_1C_1$ are similar, with points D and D_1 have corresponding roles.



First, let AP intersect BC at G, (NSF) intersect AB at X, Z be the projection of G on AB. We will prove that F is the midpoint of XZ.

Let PE intersect the line through C parallel to AP at R. Since quadrilateral $AEPN$ is cyclic, we have $\angle SAF = \angle BPC$, $\angle PCB = \angle PAB = \angle SFA$, $\angle PRC = \angle APE = \angle ANF$, hence $\triangle PBC \sim \triangle ASF$, and $\triangle PRC \sim \triangle ANF$. Therefore, $\triangle PBC \cup (R) \sim \triangle ASF \cup (N)$.

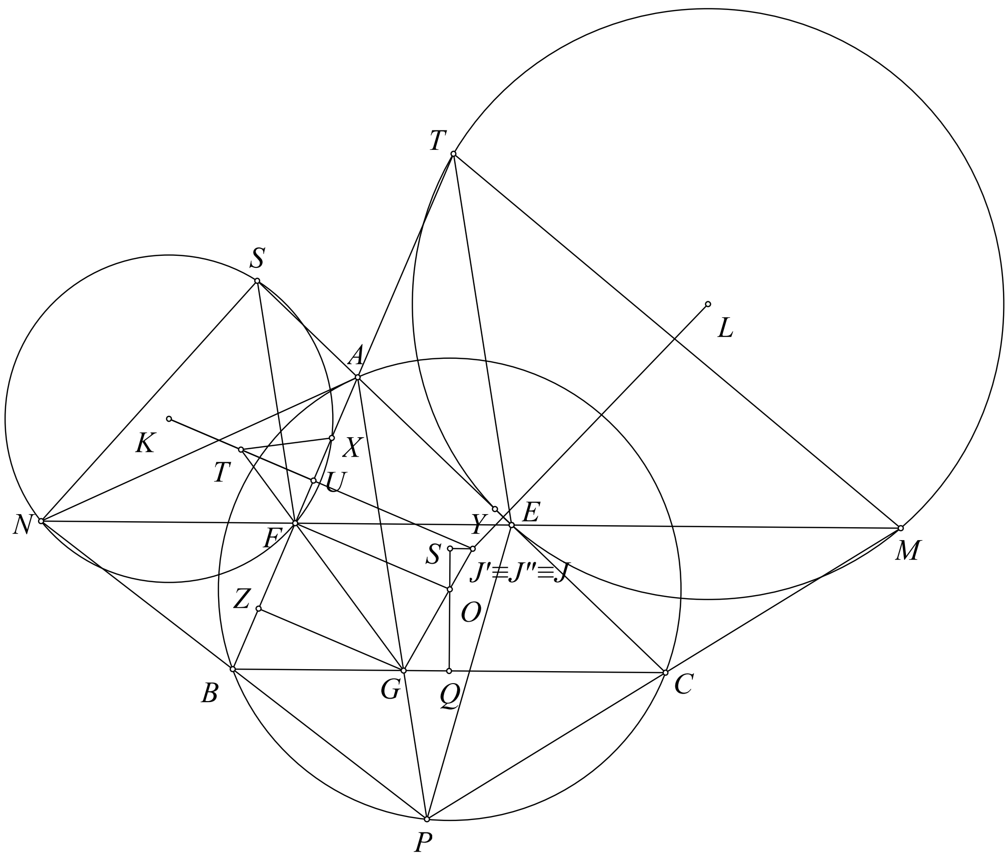
Let B' be the reflection of B across E. Then triangles BRC and $B'PA$ are symmetric with respect to E. The homothety centered at B with ratio $1/2$ maps triangle $B'PA$ to triangle EMF, hence $\triangle EMF \sim \triangle BRC \sim \triangle SNF$ (where M is the midpoint of BP).
Let H be the orthocenter of triangle ABC, and let AC intersect BP at T. Then H and O are isogonal conjugates in triangle ABT, so $MFEZ$ is cyclic.

Let U be the projection of C on AB. We have $EA = EU$, thus $\angle UEZ = \angle AUE - \angle UZE = \angle BAC - \angle FME = \angle BAC - \angle SNF = \angle BAC - \angle SXA = \angle XSA$. Hence

$\triangle SAX \sim \triangle EUZ$, so $\dfrac{AX}{AS} = \dfrac{UZ}{UE} = \dfrac{UZ}{AE}$ (1).

On the other hand, through B, draw a line parallel to AP intersecting AC at G. Then S is the midpoint of AG, hence $\dfrac{GC}{GB} = \dfrac{AC}{AG} = \dfrac{AS}{AE}$ (2).

Therefore, from (1) and (2) we get $\dfrac{AX}{BZ} = \dfrac{ZU}{ZB} \cdot \dfrac{AS}{AE} = \dfrac{GC}{GB} \cdot \dfrac{GB}{GC} = 1$, so $AX = BZ$, hence F is the midpoint of XZ.



Returning to the problem, let G be the intersection of AP and BC, and let J' be the intersection of the line through K perpendicular to AB with OG. Line GF intersects KJ' at T, and KJ' bisects FX at U. By Thales' theorem, we have $\dfrac{OJ'}{OG} = \dfrac{FT}{FG} = \dfrac{FU}{FZ} = \dfrac{1}{2}$.

Similarly, let the line through L perpendicular to AC intersect OG at J''. Then $\dfrac{OJ''}{OG} = \dfrac{1}{2}$, so J'' coincides with J', hence also with J. Thus, J lies on the ray opposite to OG and satisfies $\dfrac{OJ}{OG} = \dfrac{1}{2}$.

Let Q be the midpoint of BC, and S be the point on the ray opposite to OQ satisfying $\dfrac{OS}{OQ} = \dfrac{1}{2} = \dfrac{OJ}{OG}$. Then $JS \parallel BC$, and since S is fixed, J moves along a fixed line through S parallel to BC, as desired.
Therefore, the problem is proven