

Stress-Optical Effects with Generalized Plane Strain

The assumptions made for plane strain in the previous analysis of the waveguide structure (see the Application Library model Stress-Optical Effects in a Photonic Waveguide, the model name is stress optical) do not hold in a situation where the silicon-silica laminate is free to expand in the z direction. Instead, it is necessary to use a generalized plane strain model that allows for free expansion in the z direction. The boundary conditions in the xy-plane already allow the structure to expand freely in all directions in the plane. When the different materials in a laminate expand with different expansion coefficients, the laminate bends. In this model, the silica-silicon laminate bends in both the x and z directions. The Solid Mechanics interface configured for 2D plane strain does not cover the bending in the z direction, so you need to make modifications to the plane strain equations at the equation-system level.

Note: This application requires the Wave Optics Module and the Structural Mechanics Module.

Model Definition

GENERALIZED PLANE STRAIN

One possible extension of the plane strain formulation is to assume that the normal out-of -plane strain component has the form:

$$\varepsilon_z = e_0 + e_1 x + e_2 y$$

That is, the strain is linearly varying throughout the cross section. This approximation is expected to be good when the bending curvature is small with respect to the extents of the structure in the xy-plane and corresponds to a small rotation that is representative of each cross section of the structure along the z-axis. (A more general model would include second-order terms in x and y.)

Extension of the Plane Strain Equations

In COMSOL Multiphysics, the coefficients e_0 , e_1 , and e_2 in the expression for the ε_2 strain can be modeled as extra degrees of freedom that are constant throughout the model (global variables).

Start from the 3D stress-strain relation for linear isotropic material including thermal effects,

$$\begin{split} S_x &= S_{11} = D_{11}(\varepsilon_x - \varepsilon_{\text{th}}) + D_{12}(\varepsilon_y - \varepsilon_{\text{th}}) + D_{13}(\varepsilon_z - \varepsilon_{\text{th}}) \\ S_y &= S_{22} = D_{12}(\varepsilon_x - \varepsilon_{\text{th}}) + D_{22}(\varepsilon_y - \varepsilon_{\text{th}}) + D_{23}(\varepsilon_z - \varepsilon_{\text{th}}) \\ S_z &= S_{33} = D_{13}(\varepsilon_x - \varepsilon_{\text{th}}) + D_{23}(\varepsilon_y - \varepsilon_{\text{th}}) + D_{33}(\varepsilon_z - \varepsilon_{\text{th}}) \end{split}$$

where $\varepsilon_{\text{th}} = \alpha (T - T_{\text{ref}})$ and

$$\begin{split} \varepsilon_x &= \varepsilon_{11} = \frac{\partial u}{\partial x} \\ \varepsilon_y &= \varepsilon_{22} = \frac{\partial v}{\partial y} \\ \\ D_{11} &= D_{22} = D_{33} = \frac{E(1-v)}{(1+v)(1-2v)} \\ \\ D_{12} &= D_{23} = D_{31} = \frac{Ev}{(1+v)(1-2v)} \end{split}$$

where E is Young's modulus and v is Poisson's ratio.

The Solid Mechanics equations are implemented via the virtual work principle, which leads to the following weak contribution (see the *Structural Mechanics Module User's Guide* for more details):

$$\sum_{i,j} S_{ij} \operatorname{test}(\epsilon_{ij})$$

Comparison of the above expression with and without the assumption of $\varepsilon_z = 0$ shows that the following weak terms need to be added to extend the plane strain equations:

$$D_{13}\varepsilon_z \mathrm{test}(\varepsilon_x) + D_{23}\varepsilon_z \mathrm{test}(\varepsilon_y) + S_z \mathrm{test}(\varepsilon_z)$$

Note that

$$test(\varepsilon_z) = test(e_0) + test(e_1)x + test(e_2)y$$

which contributes to the equation for these extra three dependent variables.

Figure 1 shows the von Mises stress distribution together with the deformed shape of the waveguide.

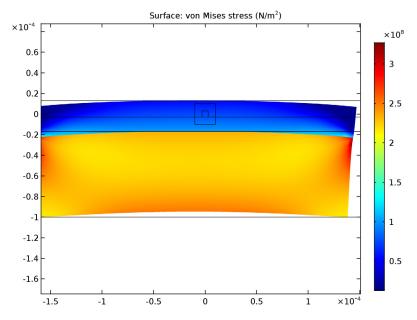


Figure 1: von Mises effective stress computed for the generalized plane strain.

For symmetry reasons, the strain components ε_x and ε_z should be equal. The plot in Figure 2 visualizes the area where the relative difference between ε_x and ε_z is within 5%. The model is most accurate in the regions close to the core, far from the boundaries on the far left and right.

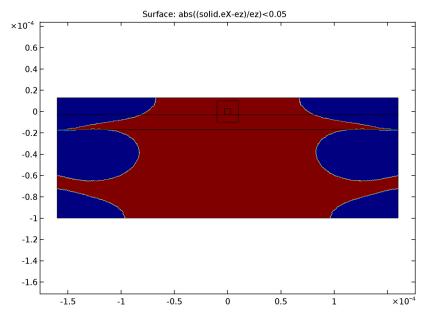


Figure 2: In the red-colored region, the relative difference between the x and z strain components is within 5%.

Figure 3 compares the effective mode indices for the first four propagating modes using the generalized plane strain equations with those obtained from the analysis in the previous model. As the plot shows, there is a systematic shift in the propagation constants when the strain in the z direction is taken into account.

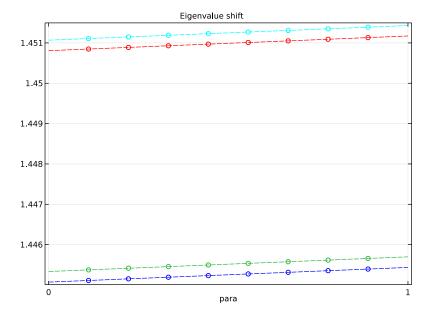


Figure 3: Effective mode indices assuming plane strain (para = 0) and generalized plane strain (para = 1).

Figure 4 shows the stress-induced birefringence along the symmetry line within the waveguide. Nonzero out-of-plane strain leads to an increase in the birefringence effect.

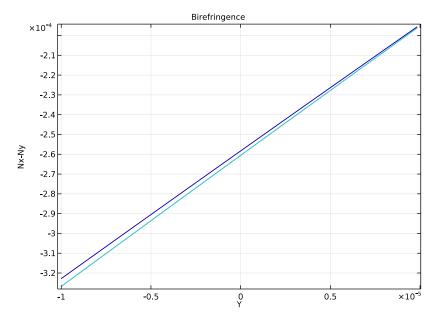


Figure 4: Birefringence along the vertical symmetry line within the waveguide for plane strain (lower curve) and generalized plane strain (upper curve).

Figure 5 gives the details of the eigenmode with the lowest effective mode index. It presents the visualization of the power flow, also called the optical intensity or the Poynting vector, in the out-of-plane direction.

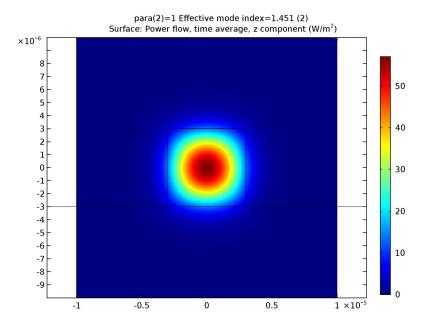


Figure 5: Eigenmode with lowest mode index, computed with the stress-optical effect under generalized plane strain assumption.

Notes About the COMSOL Implementation

The model includes an extension of the plane strain equation system. This implies that some of the results-processing variables available for the Solid Mechanics interface may no longer be valid or even give wrong results if used. Therefore, use the stress variables available under Definitions for any additional results processing.

Application Library path: Structural Mechanics Module/ Stress Optical Effects/stress optical generalized

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 2D.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 In the Select Physics tree, select Optics>Wave Optics>Electromagnetic Waves, Frequency Domain (ewfd).
- 5 Click Add.
- 6 Click Study.
- 7 In the Select Study tree, select Custom Studies>Preset Studies for Some Physics Interfaces> Stationary.
- 8 Click Done.

GLOBAL DEFINITIONS

Parameters

- I On the Home toolbar, click Parameters.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the application's Application Libraries folder and double-click the file stress_optical_parameters.txt.

Add a parameter to switch on and off the stress effects.

5 In the table, enter the following settings:

Name	Expression	Value	Description	
para	1	1	1: use generalized plane strain, 0: use plane strain	

GEOMETRY I

Rectangle I (rI)

- I On the Geometry toolbar, click Primitives and choose Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type **3.2E-4**.

- 4 In the Height text field, type 8.3E-5.
- **5** Locate the **Position** section. In the **x** text field, type -1.6E-4.
- **6** In the **y** text field, type -1E-4.

Rectangle 2 (r2)

- I On the Geometry toolbar, click Primitives and choose Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type **3.2E-4**.
- **4** In the **Height** text field, type 1.4E-5.
- 5 Locate the **Position** section. In the x text field, type -1.6E-4.
- 6 In the y text field, type -1.7E-5.

Rectangle 3 (r3)

- I On the Geometry toolbar, click Primitives and choose Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 3.2E-4.
- 4 In the Height text field, type 1.6E-5.
- **5** Locate the **Position** section. In the **x** text field, type -1.6E-4.
- 6 In the y text field, type -3E-6.

Rectangle 4 (r4)

- I On the Geometry toolbar, click Primitives and choose Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type 6E-6.
- 4 In the Height text field, type 6E-6.
- **5** Locate the **Position** section. In the **x** text field, type -3E-6.
- 6 In the y text field, type -3E-6.

Rectangle 5 (r5)

- I On the Geometry toolbar, click Primitives and choose Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 2E-5.
- 4 In the **Height** text field, type 2E-5.
- **5** Locate the **Position** section. In the **x** text field, type -1E-5.
- 6 In the y text field, type -1E-5.

7 Right-click Rectangle 5 (r5) and choose Build Selected.

The last rectangular region encloses the optical computational domain. It can be enlarged if needed for validating the results. The region should be chosen large enough so that the computed propagation constants do not change significantly if the region is enlarged.

SOLID MECHANICS (SOLID)

Linear Elastic Material I

In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Linear Elastic Material I.

Thermal Expansion 1

- I On the Physics toolbar, click Attributes and choose Thermal Expansion.
- **2** In the **Settings** window for Thermal Expansion, locate the **Model Inputs** section.
- **3** In the T text field, type T1.
- 4 Locate the Thermal Expansion Properties section. In the $T_{\rm ref}$ text field, type T0. You add the coefficients in the out-of-plane strain expression as dependent variables in a global equation.
- 5 In the Model Builder window's toolbar, click the Show button and select Advanced Physics Options in the menu. This will allow you to add a global equation and other advanced modeling features to the Solid Mechanics interface.

Global Equations 1

- I On the Physics toolbar, click Global and choose Global Equations.
- 2 In the Settings window for Global Equations, locate the Global Equations section.

3 In the table, enter the following settings:

Name	f(u,ut,utt, t) (l)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
e0	e0* (1-para)	0	0	
e1	e1* (1-para)	0	0	
e2	e2* (1-para)	0	0	

Setting para=0 will constrain e0, e1 and e2 to zero. For para=1, an extra equation for them will be set up via the weak contribution that you will add to all domains.

DEFINITIONS

Variables 1

- I On the Home toolbar, click Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
ez	para*(e0+(e1*X+e2*Y)/ 1[m])		Out-of-plane strain, z component
sx	solid.Sl11+solid.D13*ez	N/m²	Stress, x component
sy	solid.Sl22+solid.D23*ez	N/m²	Stress, y component
sz	solid.Sl33+solid.D33*ez	N/m²	Stress, z component
wc1	<pre>-(solid.D13*ez* test(solid.el11)+ solid.D23*ez* test(solid.el22)+sz* test(ez))*solid.d</pre>	N/m	Weak contribution due to out-of-plane strain

Variables 2

- I On the Home toolbar, click Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
N	nCore		Refractive index for stress-free material

- 4 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- **5** Select Domain 6 only.

Variables 3

- I On the Home toolbar, click Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
N	nSiO2		Refractive index for stress-free material

- 4 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- **5** Select Domains 4 and 5 only.

Variables 4

- I On the Home toolbar, click Variables and choose Local Variables.
- 2 In the **Settings** window for Variables, locate the **Variables** section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
Nx	N-(B1*sx+B2*(sy+sz))		Refractive index, x component
Ny	N-(B1*sy+B2*(sx+sz))		Refractive index, y component
Nz	N-(B1*sz+B2*(sx+sy))		Refractive index, z component

- 4 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- **5** Select Domains 4–6 only.

SOLID MECHANICS (SOLID)

Weak Contribution I

- I On the Physics toolbar, click Domains and choose Weak Contribution.
- **2** In the **Settings** window for Weak Contribution, locate the **Domain Selection** section.
- 3 From the Selection list, choose All domains.
- 4 Locate the Weak Contribution section. In the Weak expression text field, type wc1.

Fixed Constraint I

All regions have free boundaries, which also is the default boundary condition. However, these conditions will not be sufficient for creating a unique solution because the computational domain is allowed to move and rotate freely. The problem becomes well posed by adding constraints at two points to restrain such rigid body movements.

- I On the Physics toolbar, click Points and choose Fixed Constraint.
- 2 Select Point 1 only.

Prescribed Displacement I

- I On the Physics toolbar, click Points and choose Prescribed Displacement.
- 2 In the Settings window for Prescribed Displacement, locate the Prescribed Displacement section.
- 3 Select the Prescribed in y direction check box.
- **4** Select Point 15 only.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

The computational domain is reduced significantly for the optical mode analysis.

- I In the Model Builder window, under Component I (compl) click Electromagnetic Waves, Frequency Domain (ewfd).
- 2 Select Domains 4–6 only.

Wave Equation, Electric 1

- I In the Model Builder window, under Component I (compl)>Electromagnetic Waves, Frequency Domain (ewfd) click Wave Equation, Electric 1.
- 2 In the Settings window for Wave Equation, Electric, locate the Electric Displacement Field section.
- **3** From the *n* list, choose **User defined**. From the list, choose **Diagonal**.

4 In the *n* table, enter the following settings:

Nx	0	0
0	Ny	0
0	0	Nz

5 From the k list, choose **User defined**.

MATERIALS

In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.

Material I (mat I)

- I In the Settings window for Material, type Si in the Label text field.
- 2 Select Domain 1 only.
- **3** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Young's modulus	E	ESi	Pa	Basic
Poisson's ratio	nu	nuSi	I	Basic
Density	rho	rhoSi	kg/m³	Basic
Coefficient of thermal expansion	alpha	alphaSi	I/K	Basic

Material 2 (mat2)

- I Right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type SiO2 in the Label text field.
- **3** Select Domains 2–6 only.
- **4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Young's modulus	E	ESiO2	Pa	Basic
Poisson's ratio	nu	nuSiO2	I	Basic
Density	rho	rhoSiO2	kg/m³	Basic
Coefficient of thermal expansion	alpha	alphaSiO2	I/K	Basic

MESH I

In the Model Builder window, under Component I (compl) right-click Mesh I and choose Free Triangular.

Free Triangular I

In the Model Builder window, under Component I (compl)>Mesh I right-click Free Triangular I and choose Size.

Size 1

- I In the Settings window for Size, locate the Geometric Entity Selection section.
- 2 From the Geometric entity level list, choose Domain.
- **3** Select Domains 4–6 only.
- 4 Locate the Element Size section. Click the Custom button.
- 5 Locate the Element Size Parameters section. Select the Maximum element size check box.
- 6 In the associated text field, type 2E-7.

Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Extra fine.
- 4 Click Build All.

STUDY I

Mode Analysis

On the Study toolbar, click Study Steps and choose Other>Mode Analysis.

Step 2: Mode Analysis

- I In the Settings window for Mode Analysis, locate the Study Settings section.
- 2 Select the Search for modes around check box.
- 3 In the associated text field, type 1.46.
- 4 Select the Desired number of modes check box.
- 5 In the associated text field, type 4.
- 6 In the Mode analysis frequency text field, type c_const/lambda0_ewfd.

These settings make the eigenmode solver search for the 4 eigenmodes with effective mode indices closest to the value 1.46. This value is an estimate of the effective mode index for the fundamental mode.

Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for the Electromagnetic Waves, Frequency Domain interface.

Parametric Sweep

- I On the Study toolbar, click Parametric Sweep.
- **2** In the **Settings** window for Parametric Sweep, locate the **Study Settings** section.
- 3 Click Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
para	0 1	

5 On the Study toolbar, click Compute.

RESULTS

Stress (solid)

- I In the Model Builder window, under Results click Stress (solid).
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Data set list, choose Study I/Solution Store I (sol2).
- 4 On the Stress (solid) toolbar, click Plot.
- **5** Click the **Zoom Extents** button on the **Graphics** toolbar.

Electric Field (ewfd)

To visualize the area, where the relative difference between the x and z strain components is within 5%, follow these steps:

- I In the Model Builder window, under Results click Electric Field (ewfd).
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Data set list, choose Study I/Solution Store I (sol2).

Surface 1

- I In the Model Builder window, expand the Electric Field (ewfd) node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type abs((solid.eX-ez)/ez)<0.05.

- 4 Locate the Coloring and Style section. Clear the Color legend check box.
- 5 Click to expand the Quality section. From the Resolution list, choose Extra fine.
- **6** On the **Electric Field (ewfd)** toolbar, click **Plot**.
- 7 Click the **Zoom Extents** button on the **Graphics** toolbar.

Data Sets

Next, plot the stress-induced birefringence along the symmetry line within the waveguide.

Cut Line 2D L

- I On the Results toolbar, click Cut Line 2D.
- 2 In the Settings window for Cut Line 2D, locate the Data section.
- 3 From the Data set list, choose Study I/Parametric Solutions I (sol3).
- 4 Locate the Line Data section. In row Point 1, set Y to -1e-5.
- 5 In row Point 2, set X to 0 and Y to 1e-5.
- 6 Click Plot.

ID Plot Group 3

- I On the Results toolbar, click ID Plot Group.
- 2 In the Settings window for 1D Plot Group, locate the Data section.
- 3 From the Data set list, choose Cut Line 2D 1.

Line Graph 1

- I On the ID Plot Group 3 toolbar, click Line Graph.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type Nx-Ny.
- 4 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **5** In the **Expression** text field, type Y.
- 6 Click to expand the Coloring and style section.

ID Plot Group 3

- I In the Model Builder window, under Results click ID Plot Group 3.
- **2** In the **Settings** window for 1D Plot Group, click to expand the **Title** section.
- 3 From the Title type list, choose Manual.
- **4** In the **Title** text area, type Birefringence.
- 5 Locate the Plot Settings section. Select the x-axis label check box.
- **6** In the associated text field, type Y.

- 7 Select the y-axis label check box.
- 8 In the associated text field, type Nx-Ny.
- **9** On the **ID Plot Group 3** toolbar, click **Plot**.

To visualize the details of the eigenmode with the lowest effective mode index, you first set up a view that includes the optical computation domain only.

10 In the Model Builder window's toolbar, click the Show button and select Advanced Results **Options** in the menu.

View 2D 2

- I In the Model Builder window, under Results right-click Views and choose View 2D.
- 2 In the Settings window for View 2D, locate the View section.
- 3 Select the Lock axis check box.

Axis

- I In the Model Builder window, expand the View 2D 2 node, then click Axis.
- 2 In the Settings window for Axis, locate the Axis section.
- 3 In the x minimum text field, type -1e-5.
- 4 In the x maximum text field, type 1e-5.
- 5 In the y minimum text field, type -1e-5.
- 6 In the y maximum text field, type 1e-5.

2D Plot Group 4

- I On the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Data set list, choose Study I/Parametric Solutions I (sol3).
- 4 From the Effective mode index list, choose 1.451 (2).
- 5 Locate the Plot Settings section. From the View list, choose View 2D 2.

Surface I

- I Right-click 2D Plot Group 4 and choose Surface.
- 2 In the Settings window for Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I>Electromagnetic Waves, Frequency Domain>Energy and power>Power flow, time average (Spatial)>ewfd.Poavz -Power flow, time average, z component.

3 On the 2D Plot Group 4 toolbar, click Plot. This creates a visualization of the power flow, also called optical intensity or the Poynting vector, in the z direction (out-of-plane direction).

Derived Values

To collect all computed effective mode indices in a table, follow these steps:

Global Evaluation 1

- I On the Results toolbar, click Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Data section.
- 3 From the Data set list, choose Study I/Parametric Solutions I (sol3).
- **4** From the **Table columns** list, choose **Inner solutions**.
- 5 Click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I>Electromagnetic Waves, Frequency Domain>Global>ewfd.neff
 - Effective mode index.
- 6 Click Evaluate.

TABLE

I Go to the **Table** window.

If you see too few digits in the table, click the **Full Precision** toolbar button.

RESULTS

Finally, create a table plot to visualize the shift of the effective mode indices.

ID Plot Group 5

On the Results toolbar, click ID Plot Group.

Table Graph 1

- I On the ID Plot Group 5 toolbar, click Table Graph.
- 2 In the Settings window for Table Graph, locate the Coloring and Style section.
- 3 Find the Line style subsection. From the Line list, choose Dashed.
- 4 Find the Line markers subsection. From the Marker list, choose Circle.

ID Plot Group 5

- I In the Model Builder window, under Results click ID Plot Group 5.
- **2** In the **Settings** window for 1D Plot Group, click to expand the **Title** section.
- 3 From the Title type list, choose Manual.
- 4 In the Title text area, type Eigenvalue shift.

- 5 Click to expand the Grid section. Select the Manual spacing check box.
- 6 In the y spacing text field, type 1e-3.
- 7 On the ID Plot Group 5 toolbar, click Plot.

Electric Field (ewfd)

Note that the model includes an extension of the plane strain equation system. This implies that some of the results-processing variables available under the Solid Mechanics interface may no longer be valid or even give wrong results if used. Use the stress variables entered under the **Definition** for any additional postprocessing.