Linear Transformations

Review of Linear Transformations

A transformation *T* from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a rule that assigns to each vector *x* in $\mathbb{R}^n$ a vector

T(x) in $\mathbb{R}^m$. The set $\mathbb{R}^n$ is called the domain of *T* and $\mathbb{R}^m$ is called the codomain of *T*.

Definition of linear transformations

A transformation *T* is linear if:

(i) *T*(u + v) = *T*(u) + *T*(v) for all u; v in the domain of *T*;

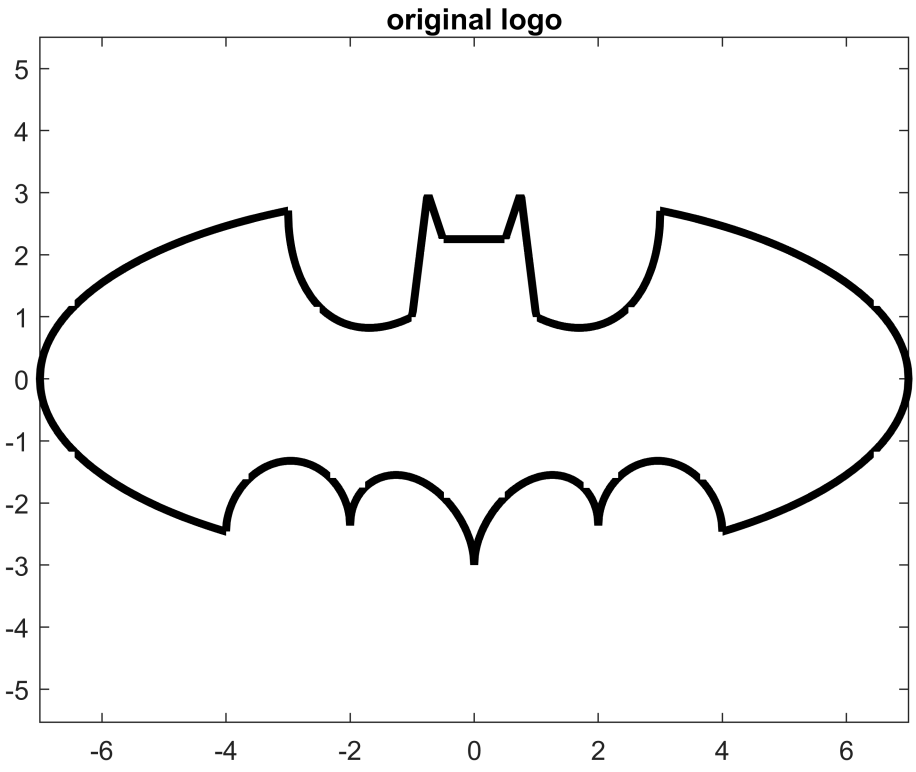
(ii) *T*(cu) = c*T* (u) for all scalars c and all u in the domain of *T*.

Note: Every matrix transformation is a linear transformation and satifies (i) and (ii).

Linear Transformations in MATLAB

Suppose we have a set of coordinates, X, and we want to transform it linearly in some fashion with a matrix A. Firstly, download the datasets `logox.txt' and `logoy.txt' and put them into your current MATLAB folder.

```
X = [ csvread('logox.txt') ; csvread('logoy.txt') ];
figure; plot(X(1, : ), X(2, : ), 'k', 'LineWidth', 3 ); title('original logo'), axis equal;
```



You should know how to perform the most used transformations, such as rotations, reflections, scaling, shearing, and projects. These transformation matrices are given below as a quick reference.

Rotation

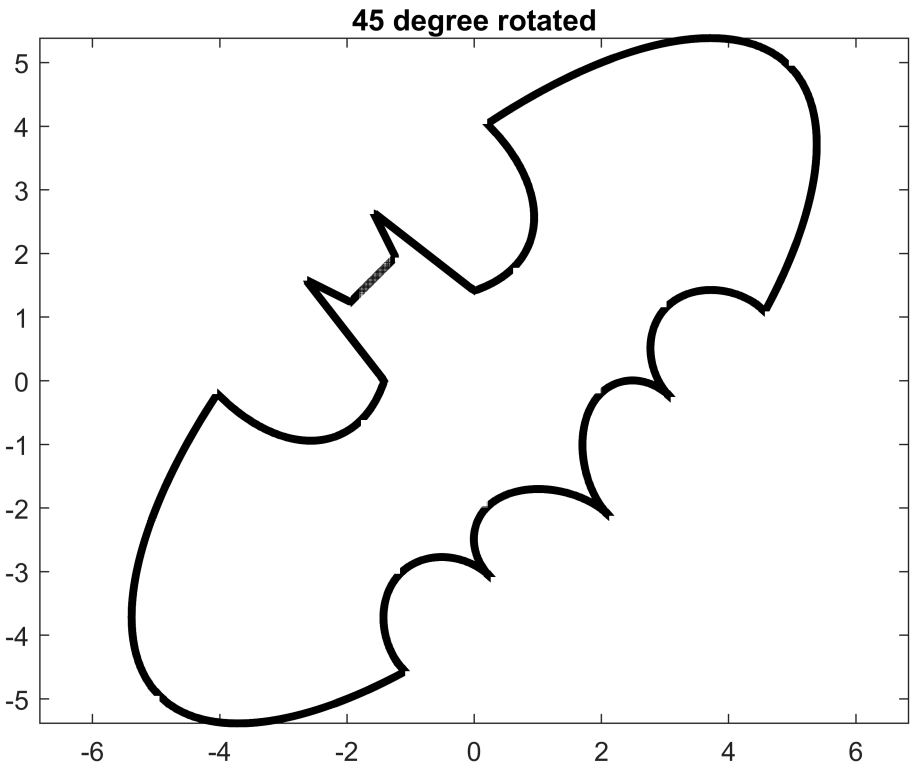
To rotate by angle θ counterclockwise, set your transformation matrix A as

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

```
theta = pi/4;
A = [cos(theta) -1*sin(theta); sin(theta) cos(theta)]
```

```
A = 2×2
    0.7071   -0.7071
    0.7071    0.7071
```

```
Y_rot = A*X;
figure; plot(Y_rot(1, : ), Y_rot(2, : ), 'k', 'LineWidth', 3 ), title('45 degree rotated'), axi
```



Reflection:

Reflection against the x-axis , set your transformation matrix A as:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Reflection against the y-axis, set your transformation matrix A as:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Reflection against the line $y = x$, set your transformation matrix A as:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

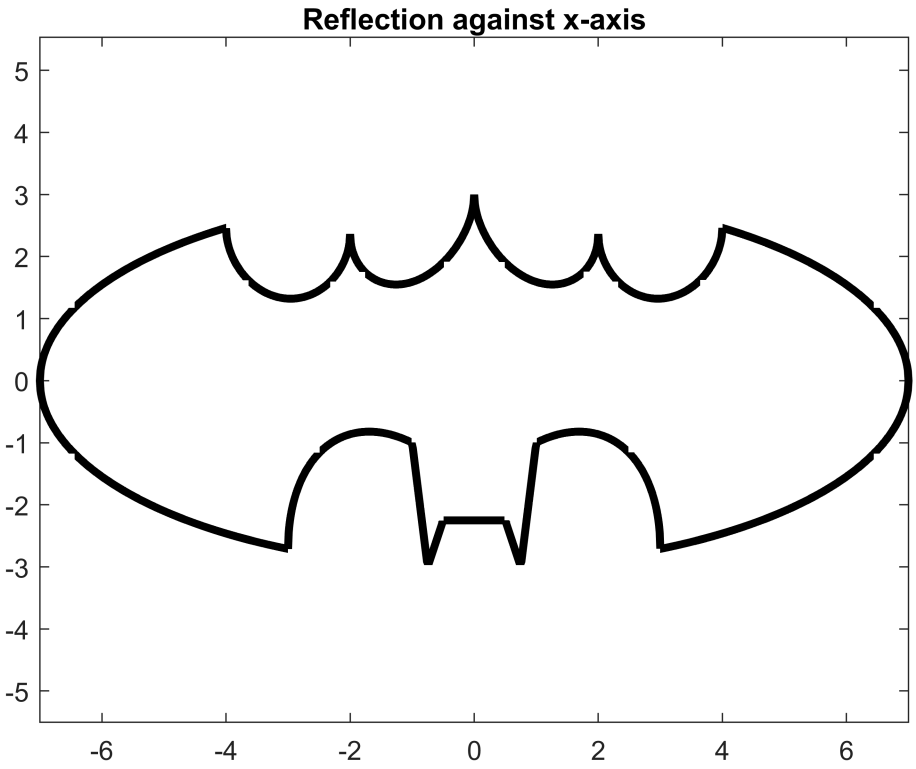
Reflection against the origin, set your transformation matrix A as:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

```
A = [1 0; 0 -1]
```

```
A = 2×2
     1     0
     0    -1
```

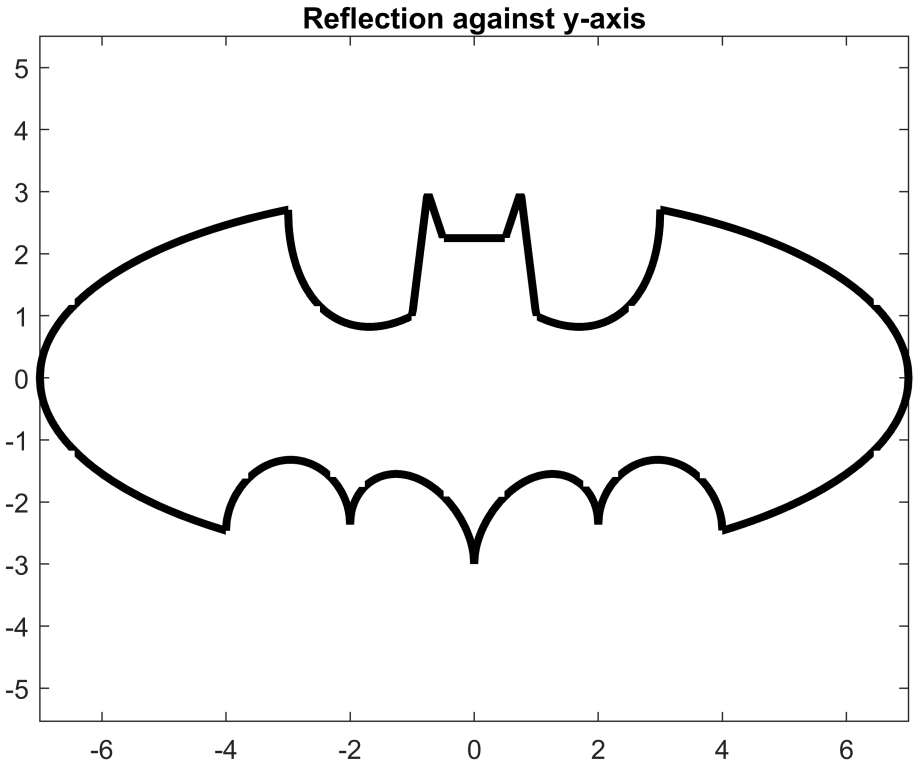
```
Y_refX = A*X;
figure; plot(Y_refX(1, : ), Y_refX(2, : ), 'k', 'LineWidth', 3 );title('Reflection against x-a
```




```
A = [-1 0; 0 1]
```

```
A = 2×2
    -1     0
     0     1
```

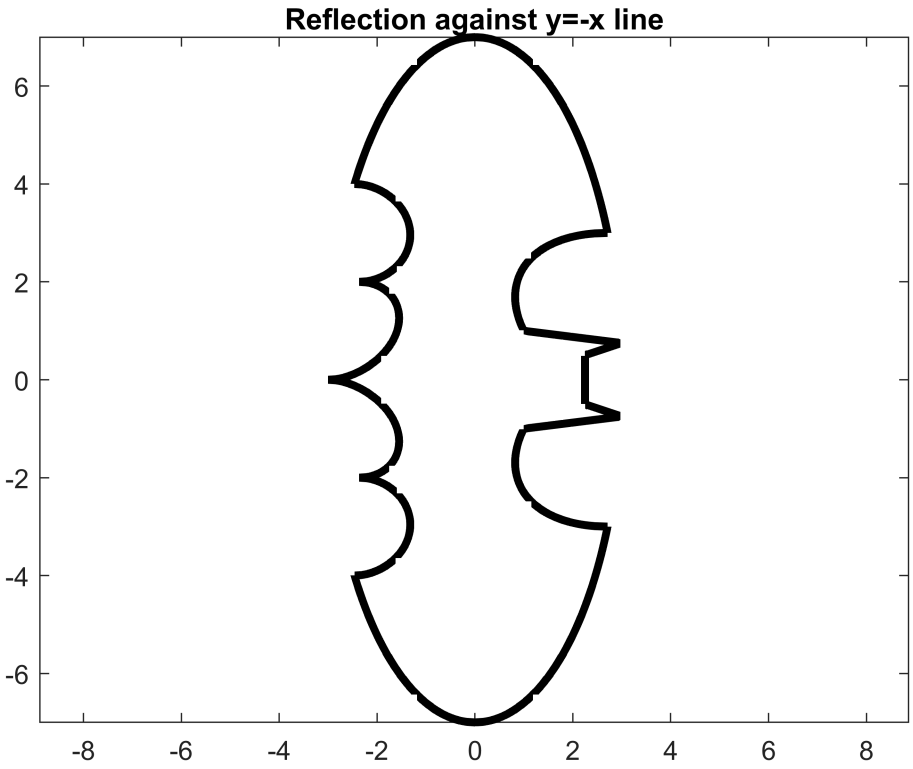
```
Y_refY = A*X;
figure; plot(Y_refY(1, : ), Y_refY(2, : ), 'k', 'LineWidth', 3 );title('Reflection against y-ax
```




```
A = [0 1; 1 0]
```

```
A = 2×2
     0     1
     1     0
```

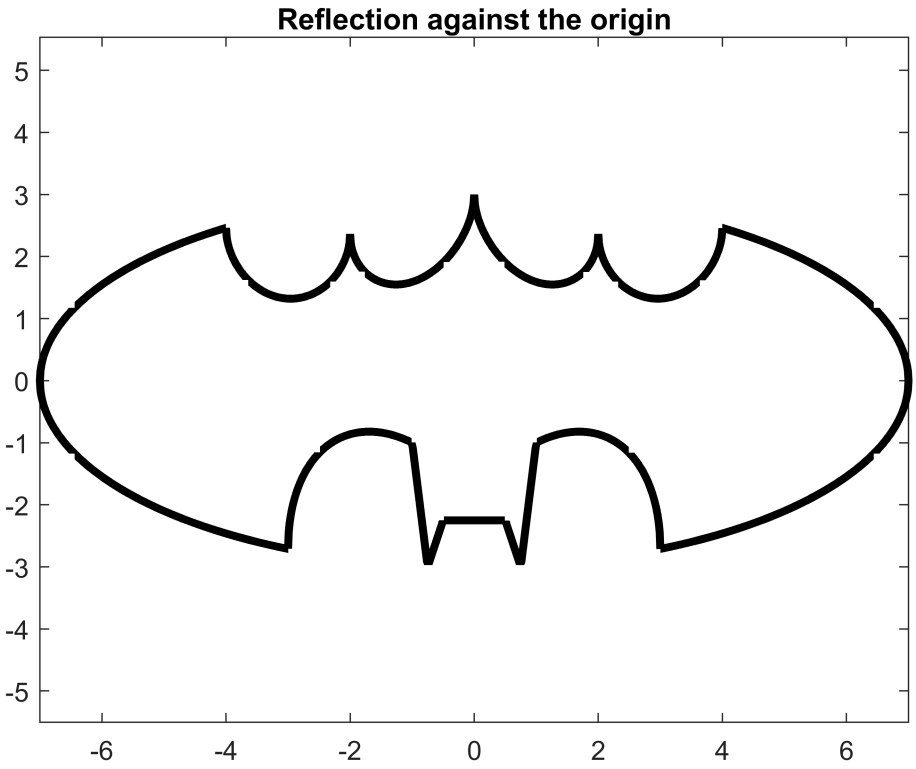
```
Y_refYX = A*X;
figure; plot(Y_refYX(1, : ), Y_refYX(2, : ), 'k', 'LineWidth', 3 );title('Reflection against y=
```




```
A = [-1 0 ; 0 -1]
```

```
A = 2×2
    -1     0
     0    -1
```

```
Y_ref0 = A*X;
figure; plot(Y_ref0(1, : ), Y_ref0(2, : ), 'k', 'LineWidth', 3 );title('Reflection against the
```



Scaling

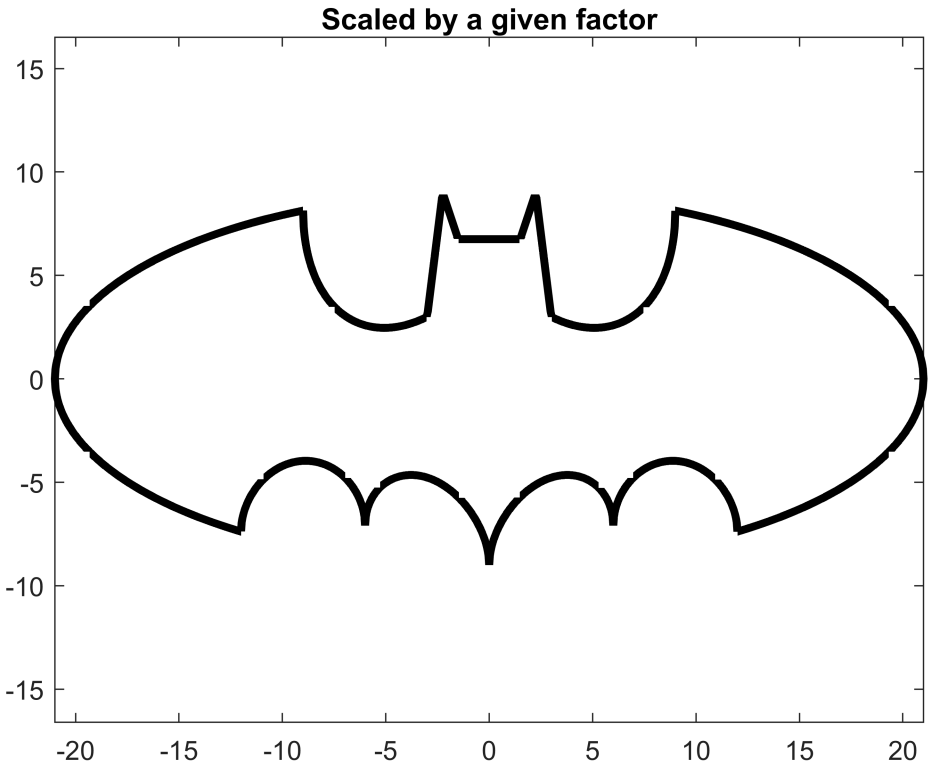
To scale by length a in all directions, set your transformation matrix A as:

$$A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$

```
sc = 3; % scale factor
A = [sc 0; 0 sc]
```

```
A = 2×2
     3     0
     0     3
```

```
Y_sc = A*X;
figure; plot(Y_sc(1, : ), Y_sc(2, : ), 'k', 'LineWidth', 3 );title('Scaled by a given factor'),
```



Shear

To shear by length b in the horizontal direction, set your transformation matrix A as:

$$A = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

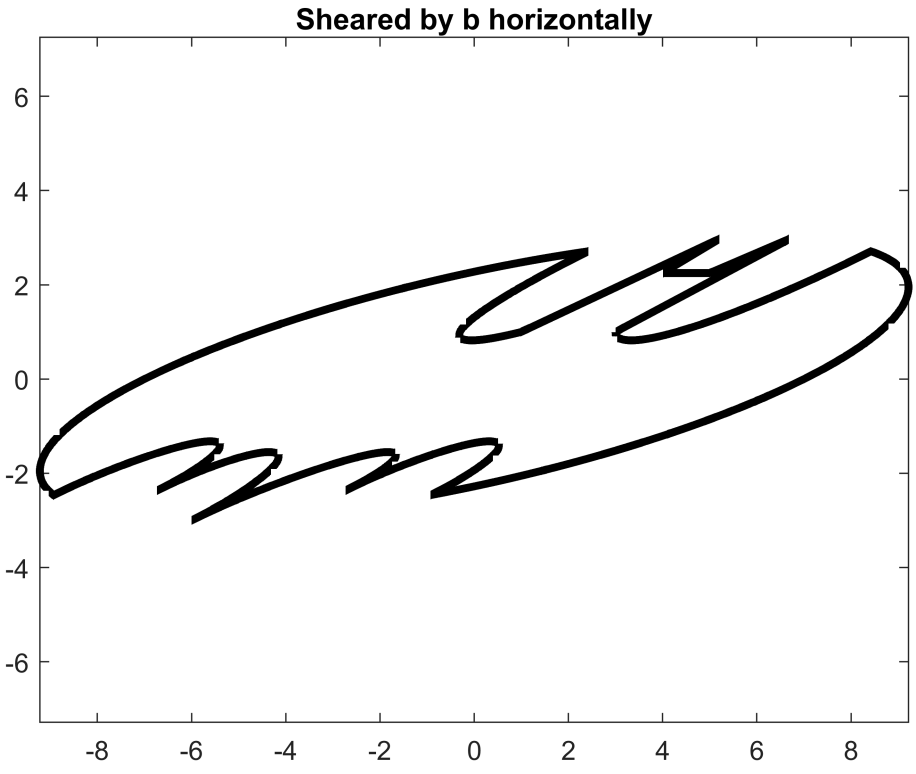
To shear by length b in the vertical direction, set your transformation matrix A as:

$$A = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

```
b = 2;
A = [1 b; 0 1]
```

```
A = 2×2
     1     2
     0     1
```

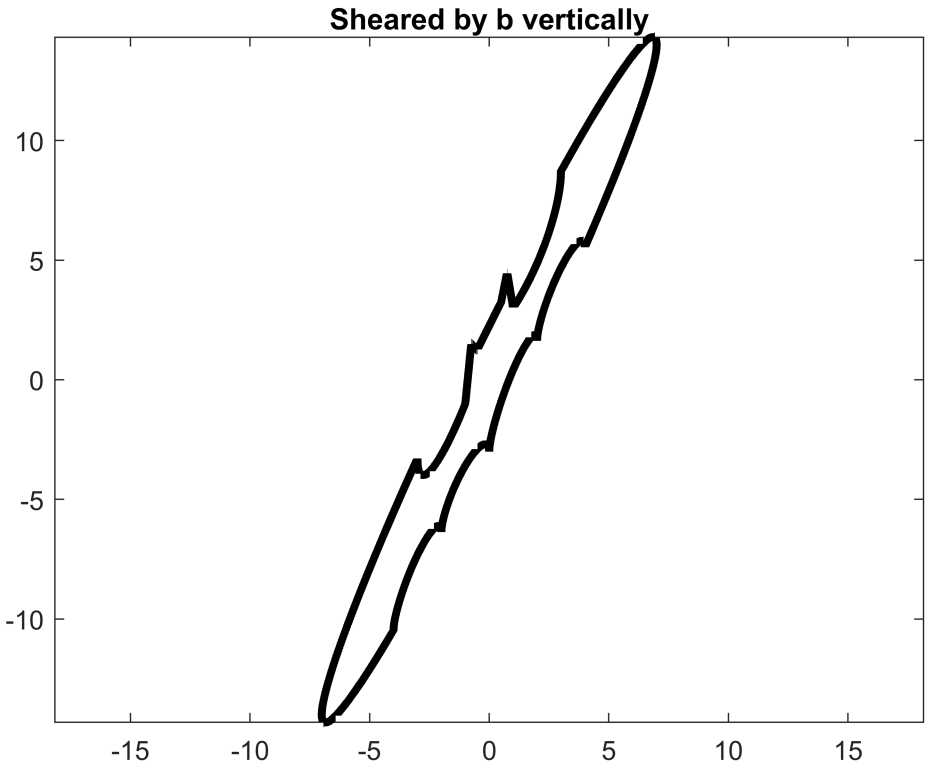
```
Y_shH = A*X;
figure; plot(Y_shH(1, : ), Y_shH(2, : ), 'k', 'LineWidth', 3 );title('Sheared by b horizontally
```




```
A = [1 0; b 1]
```

```
A = 2×2
     1     0
     2     1
```

```
Y_shV = A*X;
figure; plot(Y_shV(1, : ), Y_shV(2, : ), 'k', 'LineWidth', 3 );title('Sheared by b vertically')
```



Projection

To project to the x-axis, set your transformation matrix A as:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

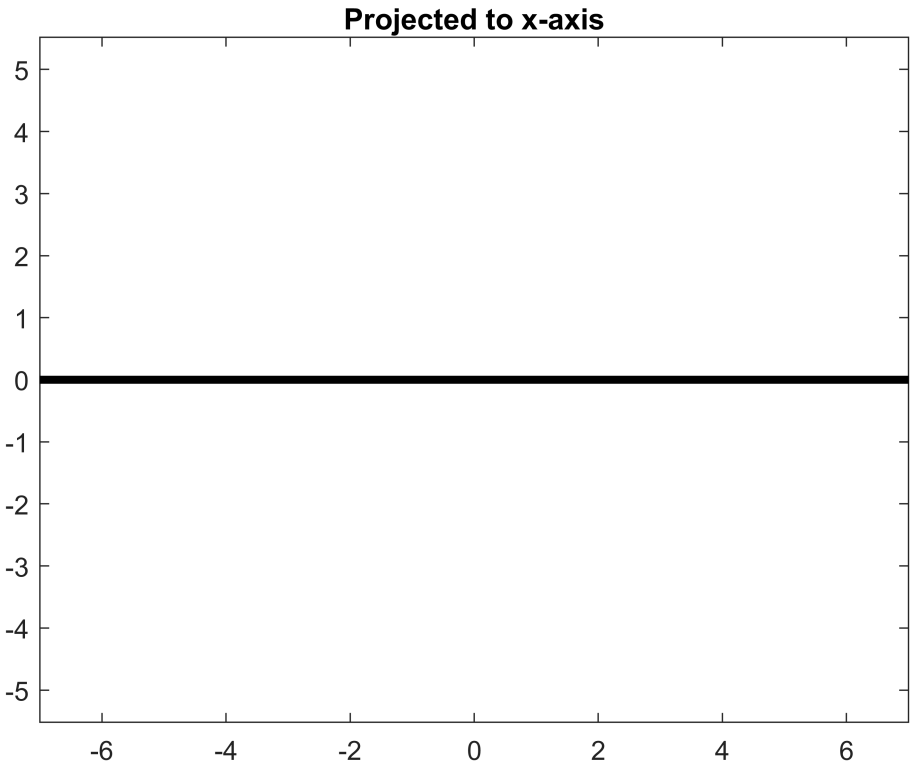
To project to the y-axis, set your transformation matrix A as:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

```
A = [1  0 ; 0 0]
```

```
A = 2×2
     1     0
     0     0
```

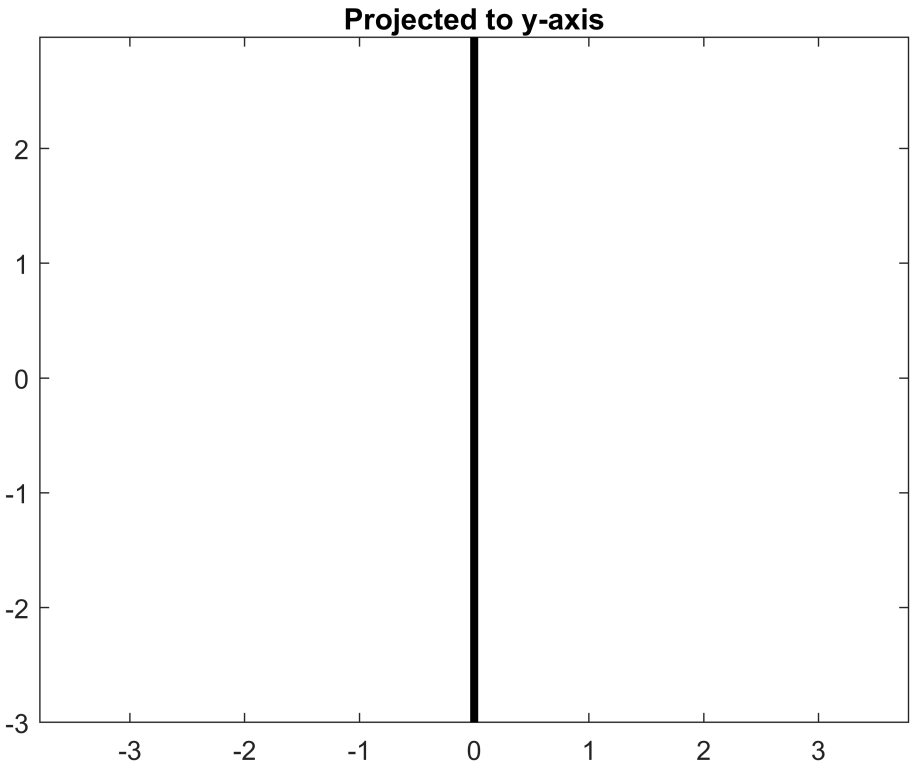
```
Y_prX = A*X;
figure; plot(Y_prX(1, : ), Y_prX(2, : ), 'k', 'LineWidth', 3 );title('Projected to x-axis'), ax
```




```
A = [0  0 ; 0 1]
```

```
A = 2×2
     0     0
     0     1
```

```
Y_prY = A*X;
figure; plot(Y_prY(1, : ), Y_prY(2, : ), 'k', 'LineWidth', 3 );title('Projected to y-axis'), ax
```



Note: You can find more about linear transformation in wikipedia page:

https://en.wikipedia.org/wiki/Linear_map#Examples_of_linear_transformation_matrices