

ASSESSMENT #2;

SECTION A

- Always keep your rough detailed working so that you can compare your solutions with those that will be sent to you. Also keep a copy of your answers/options. You may need it later.
- Show all the steps of your working and give your reasons clearly. Give a proper conclusion to your answers where applicable.

Question 1

State if the statement below is true, or false. If it is false, write the correct statement.

1.1 $\lim_{x \rightarrow a} f(x) = f(a).$ (2)

1.2 $\lim_{x \rightarrow a} (f(x) - g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x).$ (2)

1.3 $\lim_{x \rightarrow c} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} g(x)}.$ (2)

1.4 $\lim_{x \rightarrow a} f(x) \cdot g(x) = (\lim_{x \rightarrow a} f(x))(\lim_{x \rightarrow a} g(x)).$ (2)

1.5 $\lim_{x \rightarrow a} f(x)^n = (n - 1) \lim_{x \rightarrow a} f(x)^{(n-1)}.$ (2)

Question 2

Evaluate the following limits:

2.1 $\lim_{x \rightarrow 3} (x^3 + 2)(x^2 - 5x).$ (3)

2.2 $\lim_{x \rightarrow -4^-} \left(\frac{|x+4|}{x+4} \right).$ (3)

Question 3

Let

$$f(x) = \begin{cases} x - 1 & \text{if } x < 3 \\ x^2 - 4x + 6 & \text{if } x \geq 2. \end{cases}$$

Determine

3.1 $\lim_{x \rightarrow 2^-} f(x).$ (2)

3.2 $\lim_{x \rightarrow 2^+} f(x).$ (2)

3.3 Show that $\lim_{x \rightarrow 2} f(x)$ exist. (2)

Question 4

Let $f(x)$ be a function and $b \in R$. f is continuous at $x = b$ if and only if :

$$4.1 \dots\dots\dots (1)$$

$$4.2 \dots\dots\dots (1)$$

$$4.3 \dots\dots\dots (1)$$

Hint: 4.1, 4.2, 4.3 require you to state the conditions that must be satisfied for f to be continuous at $x = b$

Question 5

Let

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x > 1 \\ 3x^2 & \text{if } x < 1 \end{cases}$$

$$\text{Determine whether or not } f(x) \text{ is continuous at } x = 1. (5)$$

Question 6

Use the first principle to determine $f'(x)$ of the following functions:

$$6.1 \ f(x) = x^2. (3)$$

$$6.2 \ f(x) = x^2 - 4x + 7. (3)$$

Question 7

Use the appropriate differentiation techniques to determine the $f'(x)$ of the following functions (simplify your answer as far as possible):

$$7.1 \ f(x) = (x^3 - 2x^2 + 5)(x^4 - 3x^2 + 2). (4)$$

$$7.2 \ f(x) = \left(\frac{x-1}{x^4}\right)^{\frac{1}{2}}. (4)$$

$$7.3 \ f(x) = (x^2 - 1)\left(\frac{x^3 + 3x^2}{x^2 + 2}\right). (4)$$

Question 8

Differentiate the following with respect to the independent variables:

$$8.1 \ y = \ln|2t + 3| + \ln t^t. (4)$$

$$8.2 \ g(t) = 2^{\ln 2t} + \ln e^{2t}. (3)$$

[TOTAL: 55]

SECTION B

Question 1

A car moved distance S meters in a time t seconds, according to the formula $S = 3t^3 - 2t^2 + 4t - 1$. Determine:

1.1 velocity at (i) $t = 0$, (ii) $t = 1.5s$. (4)

1.2 Acceleration at (i) $t = 0$, (ii) $t = 1.5s$. (4)

Question 2

Let $f(x) = x^2 + 2x - 1$

2.1 Find the equation of the tangent at $x = 1$. (5)

2.2 Determine the equation of the normal to the tangent line at $x = 1$. (3)

Question 3

If $f(x) = \frac{x}{\sqrt{x^2+1}}$

3.1 Determine the equation of the tangent at $x = 1$. (6)

3.2 Determine the equation of the normal to the tangent line at $x = 1$. (3)

Question 4

Determine the derivative of the following functions with respect to the independent variables:

4.1 $y = x \sin x$. (3)

4.2 $y = \frac{\cos(2x)}{\sin(4x)}$. (3)

Question 5

Use logarithm differentiation techniques to determine the y' of the following functions:

5.1 $y = \frac{(\ln x)(\cos 3x)}{x^2 \sin 3x}$. (4)

5.2 $y = \frac{e^{3t} \sin(2t)}{5^t}$. (4)

Question 6

Determine the derivatives of the following inverse trigonometry functions

6.1 $f(x) = \tan^{-1} \sqrt{x}$. (5)

6.2 $h(t) = \sqrt{t} \cos^{-1}(e^t)$. (6)

[TOTAL: 50]