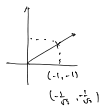


1a)



b)

μ_1	-4	-3	-2	-2
μ_2	0	1.666	3	3
x_1	1	1	1	1
x_2	2	1	1	1
x_3	2	2	2	2
x_4	2	2	2	2

13) Naive Bayes

$$P(x, y) = P(x|y)P(y)$$

$$P(x|y) = \frac{P(x, y)}{P(y)} = \frac{P(x, y)}{\sum_y P(x, y)}$$

$$= \frac{P(x, y)}{P(x, y=0) + P(x, y=1)}$$

Naive Bayes (All nodes discrete)



$$P(x|y) = P(x_1|y)P(x_2|y) \dots P(x_n|y)$$

$$P(x|y=0) = (2)(5)(4) = 40$$

$$P(x|y=1) = (2)(3)(20) = 120$$

$$P(y=1) = 0.4$$

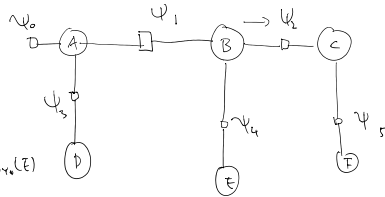
$$P(y=0) = 0.6$$

$$P(y=1|x) = \frac{40}{40+120} = \frac{1}{3}$$

$$y=0|x = 1 - P(y=1|x) = \frac{2}{3}$$

Q4

$$\mu_{B \rightarrow \psi_2} (B=0) = ?$$



$$\psi_{E \rightarrow \psi_4} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\psi_{A \rightarrow B} = \sum_E \psi_A(B, E) \mu_{E \rightarrow \psi_4}(E)$$

$$= \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\mu_{A \rightarrow B} = \sum_A \psi_A(A, B) \mu_{A \rightarrow \psi_1}(A)$$

$$\begin{bmatrix} 0.1 & 0.9 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$$

Since row sums = 1
 $P(B|A)$

$$\psi_{B \rightarrow \psi_3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$$

Obtain $\psi_{E \rightarrow \psi_4} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\mu_{A \rightarrow B} = \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mu_{B \rightarrow \psi_2} (B=0) = 0.1$$

$$\mu_{B \rightarrow \psi_2} (B=1) = 0$$

Q) $P(B|E=0) = \mu_{A \rightarrow B} \mu_{A \rightarrow \psi_1} / \mu_{A \rightarrow B}$

$$\mu_{A \rightarrow B} = \sum_C \psi_A(B, C) \mu_{C \rightarrow \psi_2}$$

$$\mu_{A \rightarrow B} = \begin{bmatrix} 0.1 & 0.9 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P(B|E=0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

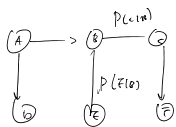
$$Z = 0.1 + 0 = 0.1$$

Since row sums = 1.

$$\begin{bmatrix} P(C=0|B=0) & P(C=1|B=0) \\ P(C=0|B=1) & P(C=1|B=1) \end{bmatrix} = P(C|B)$$

$$\psi_A = P(E|B)$$

ψ

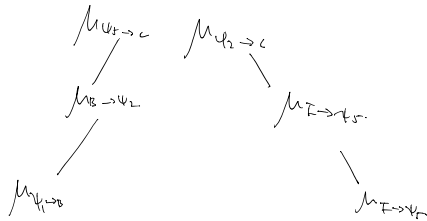
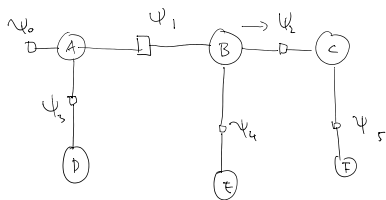


Qe) $P(C) = \mu_{A \rightarrow C} \mu_{A \rightarrow \psi_2}$

$$\psi_0 \rightarrow \psi_1 \rightarrow \psi_2$$

$$\mu_{A \rightarrow C} \mu_{A \rightarrow \psi_2}$$

1. 2. 3. 4. 5. 6.



Q5 (SAMPLING STRATES / ITERATIVE) MCMC

(Q2a)

$$\frac{0.2^{k-1} \cdot 0.32}{(0.2^{k-1}) (0.32) + 0.75^{k-1} \cdot 0.15}$$

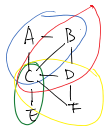
b) 61.7%

c) 75.5%

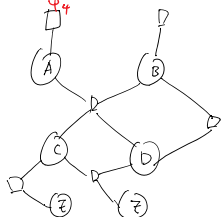
3a) $P(A)P(B)P(C|A,B)P(D|B)P(E|C,D)P(F|E)$

b) $T F T$

c)



d)



4c) $\mu_{\psi_1 \rightarrow B}(B) = \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$

$$\mu_{\psi_1 \rightarrow B}(B) = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$$

$$\mu_{\psi_2 \rightarrow B}(B) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P(B|E=0) = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

(correction)

6a) $\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{1}{3}$

b) $\frac{9}{20}, \frac{11}{20}, \frac{3}{8}, \frac{2}{5}$

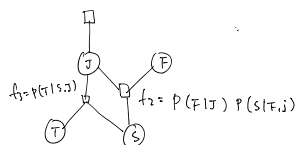
c) $\frac{2}{5}, \frac{3}{8}, \frac{4}{8}, \frac{1}{5}$

(Q23)

a) 70.8%

2a) "in the most likely joint outcome" *Question is changed!*

b) 74.7%



AND MESSAGING
PASSING ON JUNCTION TREE
OR LOOPY BELIEF

