① Introduction to Probability

Probability is about determining likelihood of an event or an experiment.

Examples of probability :-

ⓘ Probability of obtaining an head when tossing a fair coin.

Total number of outcomes possible = 2 {Head, Tail}

Number of favourable outcomes = 1 {Head}

$$\text{Probability (Head)} = \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}}$$

$$= \frac{1}{2} = 0.50$$

ⓘⓘ Probability of obtaining 1 when a fair die is rolled.

Total number of outcomes = 6 {1, 2, 3, 4, 5, 6}

Number of ~~possi~~ favorable outcomes = 1 {1}

$$\text{Probability (1)} = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}}$$

$$= \frac{1}{6} = 0.166$$

Similarly for obtaining 2,

$$\text{Probability } (X=2) = \frac{\text{Number of favorable outcomes } \{2\}}{\text{Total possible outcomes } \{1,2,3,4,5,6\}}$$

$$= \frac{1}{6} = 0.166$$

Mutual exclusive events:-

Two events are mutual exclusive if they cannot occur at the same time.

Examples:-

(i) Tossing a coin

$\boxed{H \quad T}$ Head and tail cannot be obtained at once.

So tossing a coin is a mutual exclusive event.

(ii) Rolling a die

$\boxed{\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix}}$ when a die is rolled, we cannot obtain two numbers at once.

So, rolling a die is a mutual exclusive event.

(a) Addition rule for probability:-

Addition rule for probability applies for mutual exclusive events. The addition rule for probability states that,

$$\boxed{Pr(X_1 \text{ or } X_2 \text{ or } \dots \text{ or } X_n) = Pr(X_1) + Pr(X_2) + \dots + Pr(X_n)}$$

Example for addition rule for probability:-

(i) Probability of obtaining tail or head when a coin is tossed.

$$Pr(\text{Tail or Head}) = Pr(\text{Tail}) + Pr(\text{Head})$$

$$= 1/2 + 1/2 = 1$$

This was applicable because tossing a coin is an mutual exclusive event.

(ii) Rolling a fair die and obtaining 1 or 5
since 1 and 5 cannot occur at once when a fair die is rolled, this is a mutual exclusive event.

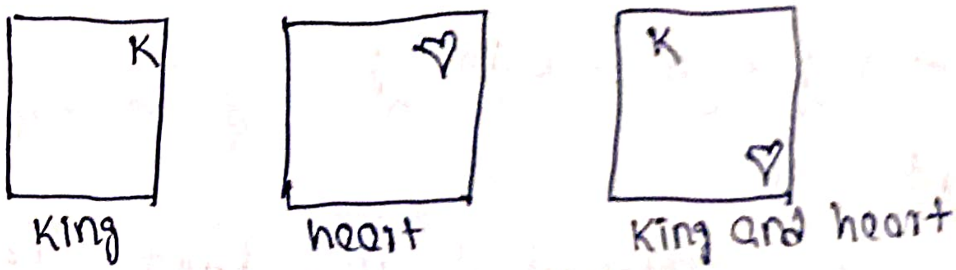
$$Pr(1 \text{ or } 5) = Pr(1) + Pr(5) = 1/6 + 1/6 = 1/3$$

Non-mutual exclusive event :-

Two events are non-mutual exclusive if they can occur at the same time.

Examples :-

(i) Taking a heart card from the deck or King.
The deck can contain King card, heart card and also a combination card containing King and heart.

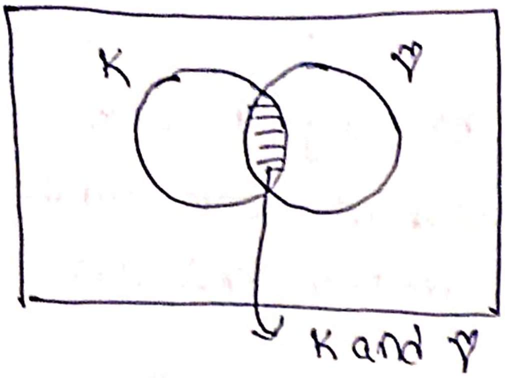


since King and heart can occur at once, this is a non-mutual exclusive event.

For a non-mutual exclusive event,

$$\boxed{Pr(K \text{ or } ♡) = Pr(K) + Pr(♡) - Pr(K \text{ and } ♡)}$$

when we draw a venn diagram for this case, we can find one overlap.



since there is an overlap in the venn diagram, this is a non-mutual exclusive event.

$Pr(K \text{ or } ♡) = Pr(K) + Pr(♡) - Pr(K \text{ and } ♡)$

$= (4/52) + (13/52) - (1/52)$

$= 16/52 = 4/13$

So, probability for obtaining K or ♡ follows addition rule of probability for non-mutual exclusive events.

ⓑ Multiplication rule in probability

Independent events:-

Two events are independent if they donot affect one another.

Example:- Tossing a coin and obtaining head first time and the tail second time.

when coin is tossed first time, probability of getting head is, $Pr(\text{head}) = 1/2$

After obtaining head first time, probability of getting tail next time is, $Pr(\text{Tail}) = 1/2$

obtaining head for the first time when the coin is tossed doesnot affect probability of occurence of tail next time. so, this is an independent event.

Another example:-

Tossing a fair die twice and obtaining 1 for the first time and obtaining 6 second time.

when the fair die is rolled first time,

$Pr(X=1) = 1/6$

After obtaining 1 for the first time, probability of obtaining 6 next time is

$Pr(X=6) = 1/6$

since obtaining 1 for the first time is not affecting obtaining 6 next time, probability of obtaining 1 and then 6 is an independent event.

Dependent events:-

Two events are dependent if they effect each other

Example:- Take a king card from the deck and then the queen card from the deck.

Taking king from the deck,

Total number of cards = 52

$Pr(King) = (4/52) = 1/13$

After obtaining the king card, we remove it from the deck.

Now, next time,

Total number of cards = 52 - 1 (The card removed in first move)

= 51

$Pr(Queen) = 4/51$

Since removing King from the deck first is effecting the probability of the queen, this is an example of dependent event.

Multiplication rule for independent events:-

Tossing a coin and obtaining head first and then obtaining a tail.

$Pr(\text{Head and Tail}) = Pr(\text{Head}) \times Pr(\text{Tail})$

$= (1/2)(1/2) = 1/4$

Multiplication rule for dependent events:-

Taking out Kings card from the deck and then the queen card.

$Pr(\text{King and Queen}) = Pr(\text{King}) \times Pr(\text{Queen} / \text{King})$

$Pr(\text{King}) = (4/52)$

$Pr(\text{Queen} / \text{King}) \rightarrow$ Probability of obtaining queen given that the King is removed

conditional probability

$(4/51)$

$Pr(\text{King and Queen}) = Pr(\text{King}) \times Pr(\text{Queen} / \text{King})$

$= (4/52) \times (4/51)$

So, addition rule in probability is used for mutual exclusive and non-mutual exclusive events and is always applicable for "OR" kind of probability events.

while for multiplication rule, it is important to determine whether the events are dependent or independent.