6. Standard Error Models

6.1. Static Error Model

$$\boxed{\varepsilon(t) = \tilde{\theta}^T(t)\omega(t),} \tag{6.1}$$

where $\varepsilon(t)$ is the output, $\tilde{\theta}(t) = \theta - \hat{\theta}(t) \in \mathbb{R}^m$ is the vector of parametric errors, $\omega(t) \in \mathbb{R}^m$ is the vector of measurable functions (regressor).

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Lyapunov function?

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$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

Time derivative:

$$\dot{V} = \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = -\frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = \quad ?$$

6.1. Static Error Model

$$\dot{V} = \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = -\frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = -\frac{1}{\gamma}\tilde{\theta}^T\gamma\omega\varepsilon =$$

$$\dot{\hat{\theta}} = \gamma\omega\varepsilon$$

6.1. Static Error Model

$$\dot{V} = \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = -\frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = -\frac{1}{\gamma}\tilde{\theta}^T\gamma\omega\varepsilon = -\varepsilon^2 < 0$$

$$\varepsilon = \tilde{\theta}^T\omega$$

6.1. Static Error Model

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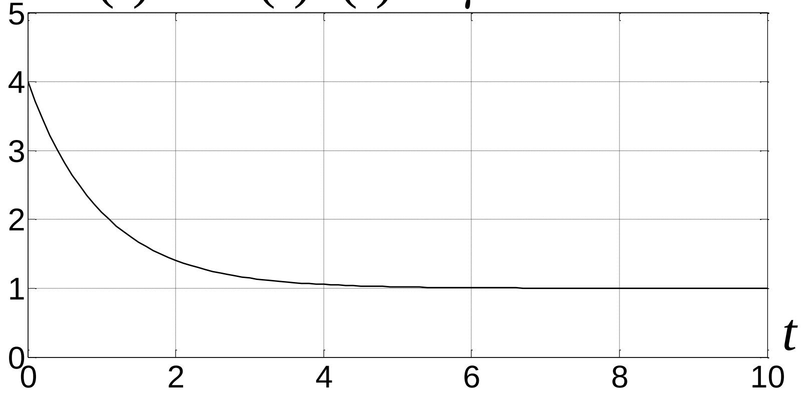
Summary and Discussion

Error Model	$\varepsilon = \tilde{\theta}^T\omega$,
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma\omega\varepsilon$
Lyapunov function	$V = \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} > 0$
Its time derivative	$\dot{V} = -\varepsilon^2 < 0$

What it means?

6.1. Static Error Model

1. $V(t) = \tilde{\theta}^T(t)\tilde{\theta}(t)/2\gamma$



$\Rightarrow$

$\varepsilon \to 0$ *asymptotically fast*

$\left\|\tilde{\theta}\right\|^2$ *is monotonically decreasing function*

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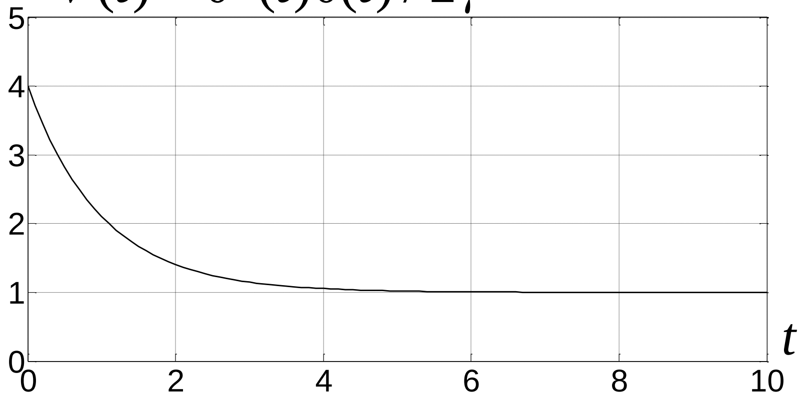

$\Rightarrow$

$\varepsilon(t)\to 0$ *asymptotically*

$\left\|\tilde{\theta}(t)\right\|^2$ *is a monotonically decreasing function*

2.

Does $\tilde{\theta}(t)$ always tend to zero or does the system has identification properties?

6.1. Static Error Model

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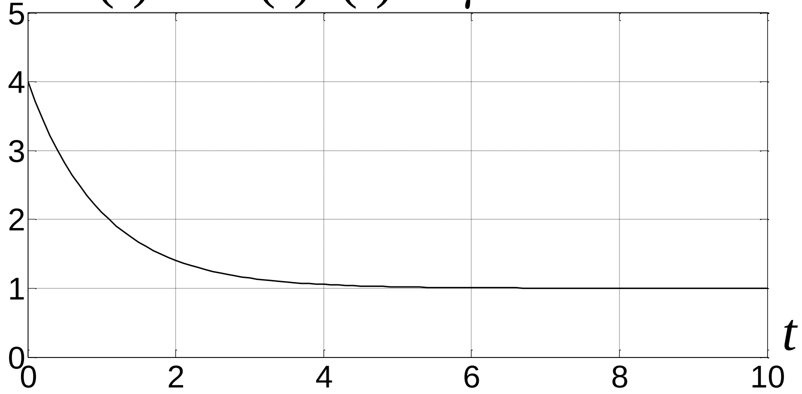

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2. Example 6.1

For a given error model $\varepsilon = \tilde{\theta}_1\omega_1 + \tilde{\theta}_2\omega_2$ there are the following scenarios:

a) $\omega_1 = 1,\ \omega_2 = 2$ *and* $\tilde{\theta}_1 \to 2,\ \tilde{\theta}_2 \to -1$

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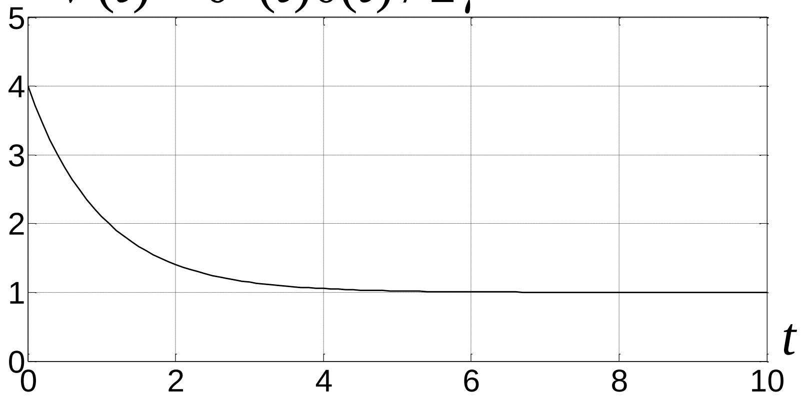

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b) $\omega_1 = 1,\ \omega_2 = 2$ *and* $\tilde{\theta}_1 \to 4,\ \tilde{\theta}_2 \to -2$

How many options for convergence?

6.1. Static Error Model

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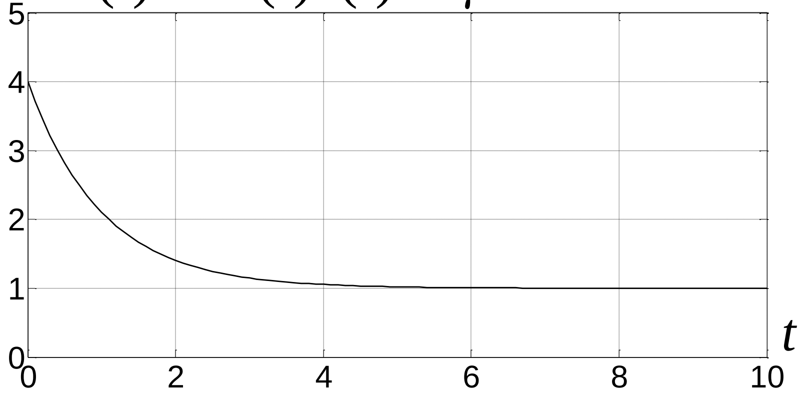

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c) $\omega_1 = \sin t,\ \omega_2 = 2\sin t\ \textit{and}\ \tilde{\theta}_1 \to 2,\ \tilde{\theta}_2 \to -1$

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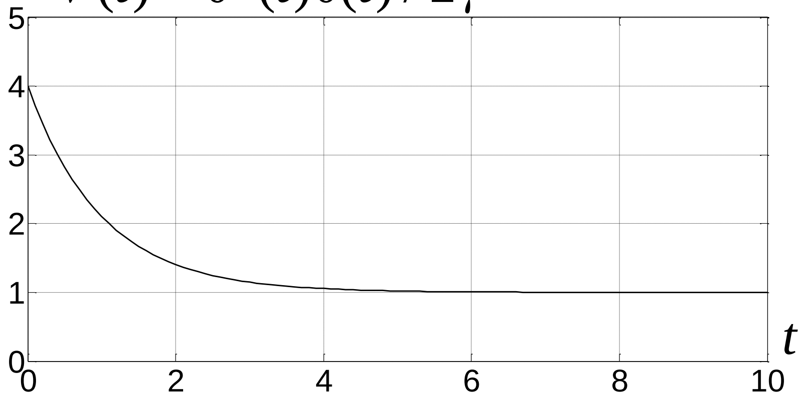

$\Rightarrow$ $\varepsilon(t)\to 0$ *asymptotically*

$\|\tilde{\theta}(t)\|^2$ *is a monotonically decreasing function*

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b) $\omega_1=1,\ \omega_2=2\ \textit{and}\ \tilde{\theta}_1\to 4,\ \tilde{\theta}_2\to -2$

c) $\omega_1=\sin t,\ \omega_2=2\sin t\ \textit{and}\ \tilde{\theta}_1\to 2,\ \tilde{\theta}_2\to -1$

d) $\omega_1=\sin t,\ \omega_2=2\sin 2t\ \textit{and}\ \tilde{\theta}_1\to ?,\ \tilde{\theta}_2\to ?$

How many options for convergence?

6.1. Static Error Model

Example 6.2

For the error model $\varepsilon = \tilde{\theta}_1\omega_1 + \tilde{\theta}_2\omega_2 + \tilde{\theta}_3\omega_3$ there are the following scenarios:

a) $\omega_1 = \sin t,\ \omega_2 = 2\sin t,\ \omega_3 = 3\sin t \ \textit{and}\ \tilde{\theta}_{1,2,3} \to ?$

b) $\omega_1 = \sin t,\ \omega_2 = 2\sin t,\ \omega_3 = 3\sin 2t \ \textit{and}\ \tilde{\theta}_{1,2,3} \to ?$

c) $\omega_1 = \sin t,\ \omega_2 = 2\sin 3t,\ \omega_3 = 3\sin 2t \ \textit{and}\ \tilde{\theta}_{1,2,3} \to ?$

d) $\omega_1 = \sin(t),\ \omega_2 = 2\sin(t+\pi),\ \omega_3 = 3\sin(t+\pi/2) \ \textit{and}\ \tilde{\theta}_{1,2,3} \to ?$

e) $\omega_1 = \sin(2t),\ \omega_2 = 2\sin(t+\pi),\ \omega_3 = 3\sin(t+\pi/2) \ \textit{and}\ \tilde{\theta}_{1,2,3} \to ?$

6.1. Static Error Model

Example 6.2

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a) $\omega_1 = \sin t,\ \omega_2 = 2\sin t,\ \omega_3 = 3\sin t\ \ and\ \ \tilde{\theta}_{1,2,3} \to$ *not ness. to zero*

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e) $\omega_1 = \sin(2t),\ \omega_2 = 2\sin(t+\pi),\ \omega_3 = 3\sin(t+\pi/2)\ \ and\ \ \tilde{\theta}_{1,2,3} \to 0$

Vector $\omega \in \mathbb{R}^m$ *has to contain at least* $m/2$ *different harmonics to provide identification properties*

6.1. Static Error Model

Summary

Properties of the closed-loop system:

1. If $\omega, \dot{\omega}$ are bounded, all the signals in the system are bounded;
2. Error $\varepsilon(t)$ approaches zero asymptotically;
3. The function $\left\|\tilde{\theta}(t)\right\|^2$ is nonincreasing;
4. $\left\|\tilde{\theta}(t)\right\|^2$ approaches zero exponentially if ω contains at least *m/2* harmonics and consists of linearly independent elements;

This property can be reformulated in terms of **Persistent Excitation Condition:**

$$\int_t^{t+T} \omega(\tau)\omega^T(\tau)d\tau \geq \alpha I \qquad (6.3)$$

for some positive α, T.

6.1. Static Error Model

Example 6.3. The problem of identification reduced to Static Error Model

Problem statement

Let a plant be described by

$$\ddot{y} + a_1\dot{y} + a_0 y = b_0 u \tag{6.4}$$

with unknown parameters a_0, a_1, b_0 and measurable input u and output y.

The objective is to design such estimates $\hat{a}_0, \hat{a}_1, \hat{b}_0$ that

$$\lim_{t\to\infty}\left(a_0 - \hat{a}_0(t)\right) = \lim_{t\to\infty}\left(a_1 - \hat{a}_1(t)\right) = \lim_{t\to\infty}\left(b_0 - \hat{b}_0(t)\right) = 0. \tag{6.5}$$

6.1. Static Error Model

Solution

Main idea of solution is to reduce the problem to the error model.

Then to get the adaptation algorithm generating the estimates.

6.1. Static Error Model

Solution

1. Apply transfer function

$$H(s) = \frac{1}{K(s)} = \frac{1}{s^2 + k_1 s + k_0}$$

with Hurwitz polynomial $K(s) = s^2 + k_1 s + k_0$ to the plant (6.4) assuming initial conditions $y(0), \dot{y}(0)$ equaled to zero:

$$\ddot{y} + a_1 \dot{y} + a_0 y = b_0 u$$

$$\Big\Downarrow H(s)[\square]$$

$$\frac{s^2}{K(s)}[y] + a_1 \frac{s}{K(s)}[y] + a_0 \frac{1}{K(s)}[y] = b_0 \frac{1}{K(s)}[u]$$

6.1. Static Error Model

Solution

$$\frac{s \pm k_1 s \pm k_0}{K(s)}[y]+a_1\frac{s}{K(s)}[y]+a_0\frac{1}{K(s)}[y]=b_0\frac{1}{K(s)}[u]$$

6.1. Static Error Model

Solution

$\pm k_1 s \pm k_0$

$$\frac{s}{K(s)}[y]+a_1\frac{s}{K(s)}[y]+a_0\frac{1}{K(s)}[y]=b_0\frac{1}{K(s)}[u]$$

$$y+(a_1-k_1)\frac{s}{K(s)}[y]+(a_0-k_0)\frac{1}{K(s)}[y]=b_0\frac{1}{K(s)}[u]$$

6.1. Static Error Model

Solution

$$\frac{s \pm k_1 s \pm k_0}{K(s)}[y] + a_1 \frac{s}{K(s)}[y] + a_0 \frac{1}{K(s)}[y] = b_0 \frac{1}{K(s)}[u]$$

$$y + (a_1 - k_1)\frac{s}{K(s)}[y] + (a_0 - k_0)\frac{1}{K(s)}[y] = b_0 \frac{1}{K(s)}[u]$$

$$y = \underbrace{(k_1 - a_1)}_{\theta_1}\underbrace{\frac{s}{K(s)}[y]}_{\omega_1} + \underbrace{(k_0 - a_0)}_{\theta_2}\underbrace{\frac{1}{K(s)}[y]}_{\omega_2} + \underbrace{b_0}_{\theta_3}\underbrace{\frac{1}{K(s)}[u]}_{\omega_3}$$

6.1. Static Error Model

Solution

$$\frac{s^{\cdot} \pm k_1 s \pm k_0}{K(s)}[y]+a_1\frac{s}{K(s)}[y]+a_0\frac{1}{K(s)}[y]=b_0\frac{1}{K(s)}[u]$$

$$y+(a_1-k_1)\frac{s}{K(s)}[y]+(a_0-k_0)\frac{1}{K(s)}[y]=b_0\frac{1}{K(s)}[u]$$

$$y=\underbrace{(k_1-a_1)}_{\theta_1}\underbrace{\frac{s}{K(s)}[y]}_{\omega_1}+\underbrace{(k_0-a_0)}_{\theta_2}\underbrace{\frac{1}{K(s)}[y]}_{\omega_2}+\underbrace{b_0}_{\theta_3}\underbrace{\frac{1}{K(s)}[u]}_{\omega_3}$$

Parameterized plant

$$y=\theta^T\omega \qquad (6.6)$$

$$\theta=col(\theta_1,\,\theta_2,\,\theta_3),\quad \omega=col(\omega_1,\,\omega_2,\,\omega_3)$$

6.1. Static Error Model

Solution

2. Design of error

$$\varepsilon = y - \hat{\theta}^T \omega \qquad \textbf{Why this?} \qquad (6.7)$$

where $\hat{\theta}$ is the estimate of θ.

6.1. Static Error Model

Solution

2. Design of error

$$\varepsilon = y - \hat{\theta}^T \omega \qquad (6.7)$$

where $\hat{\theta}$ is the estimate of θ.

$$y = \theta^T \omega$$

$$\Downarrow$$

Error model

$$\varepsilon = \tilde{\theta}^T \omega,$$

$\tilde{\theta} = \theta - \hat{\theta}$ is parametric error vector.

6.1. Static Error Model

<u>Solution</u>

2. Design of error

$$\varepsilon = y - \hat{\theta}^T \omega \qquad (6.7)$$

where $\hat{\theta}$ is the estimate of θ.

$$y = \theta^T \omega$$

$$\Downarrow$$

Error model $$\varepsilon = \tilde{\theta}^T \omega,$$

$\tilde{\theta} = \theta - \hat{\theta}$ is parametric error vector.

$$\Downarrow$$

3. Adaptation algorithm design.

Adaptation algorithm $$\dot{\hat{\theta}} = \gamma \omega \varepsilon \qquad (6.8)$$

6.1. Static Error Model

Solution Summary

Error

$$\varepsilon = y - \hat{\theta}^T\omega = y - (k_1 - \hat{a}_1)\frac{s}{K(s)}[y] - (k_0 - \hat{a}_0)\frac{1}{K(s)}[y] - \hat{b}_0\frac{1}{K(s)}[u]$$

Adaptation Algorithms

$$\dot{\hat{\theta}} = \gamma\omega\varepsilon$$

$$\dot{\hat{a}}_0 = -\gamma\frac{1}{K(s)}[y]\varepsilon$$

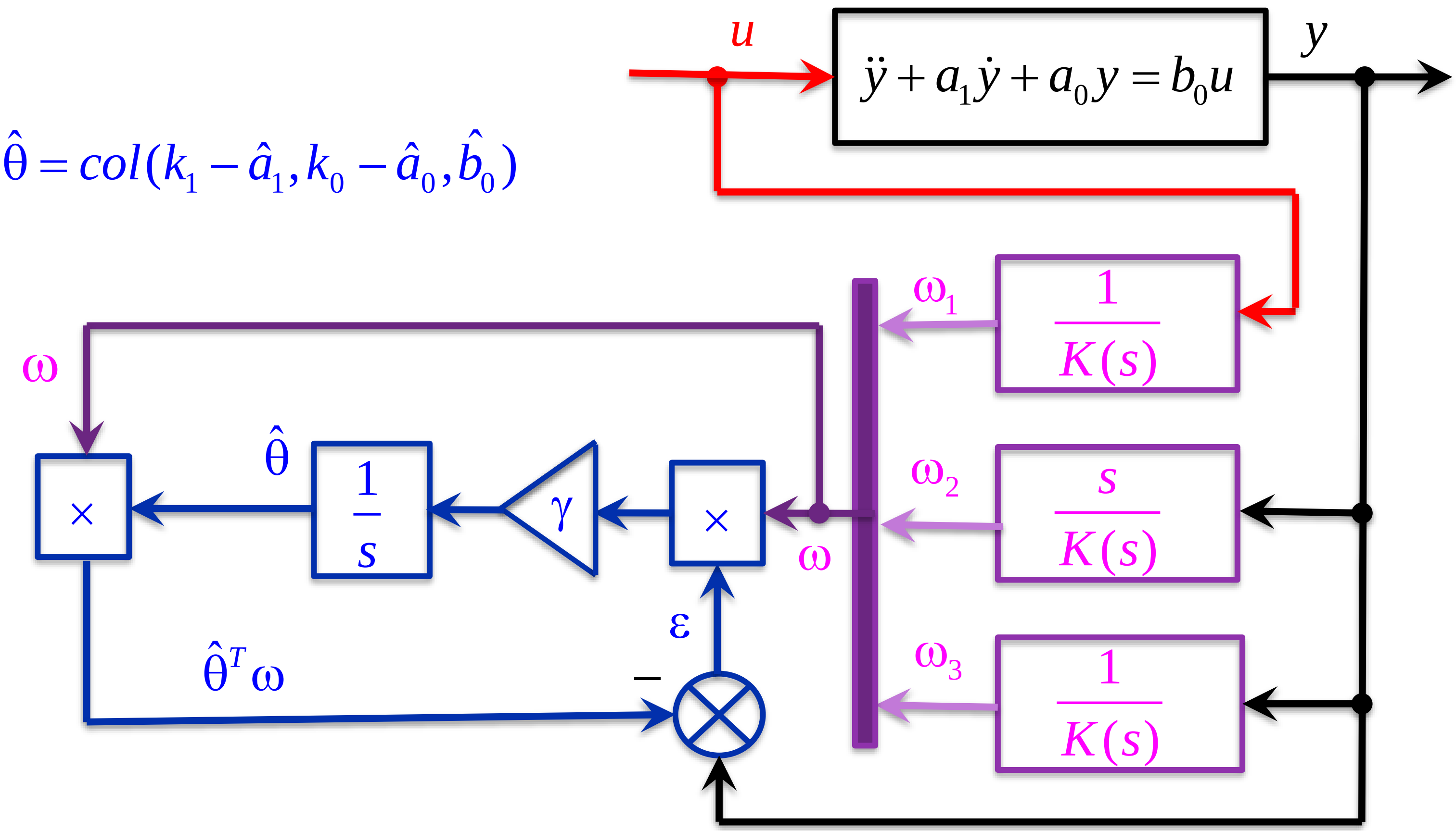
$$\dot{\hat{a}}_1 = -\gamma\frac{s}{K(s)}[y]\varepsilon$$

$$\dot{\hat{b}}_0 = \gamma\frac{1}{K(s)}[u]\varepsilon$$

6.1. Static Error Model

General Scheme

$$\hat{\theta} = col(k_1 - \hat{a}_1, k_0 - \hat{a}_0, \hat{b}_0)$$



6.1. Static Error Model

Simulation results

Plant $\ddot{y} + 2\dot{y} + y = 3u$

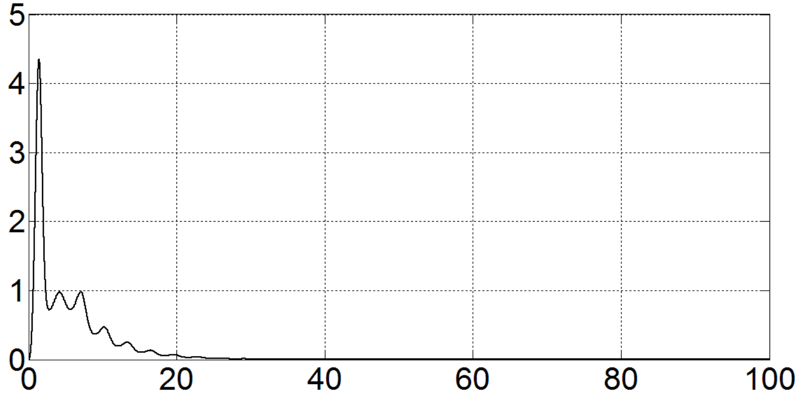
Filters polynomial $K(s) = s^2 + 5s + 6$

$\theta = col(3,\ 5,\ 3)$

Adaptation gain $\gamma = 1$

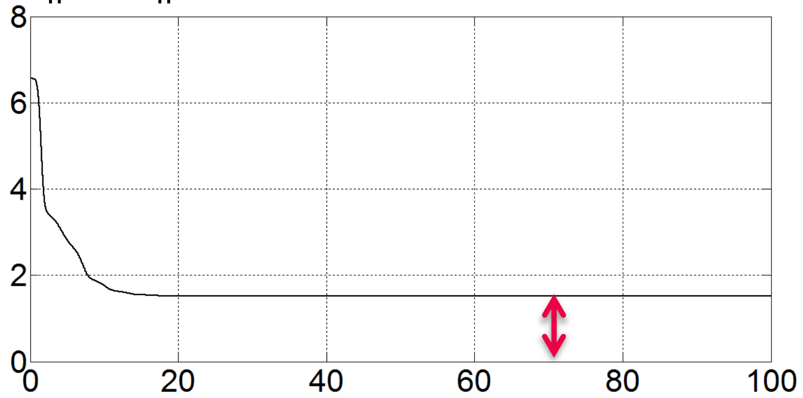
$u(t) = 10\sin t$

$\varepsilon(t)$



t

$\|\tilde{\theta}(t)\|$



t

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Plant $\ddot{y}+2\dot{y}+y=3u$

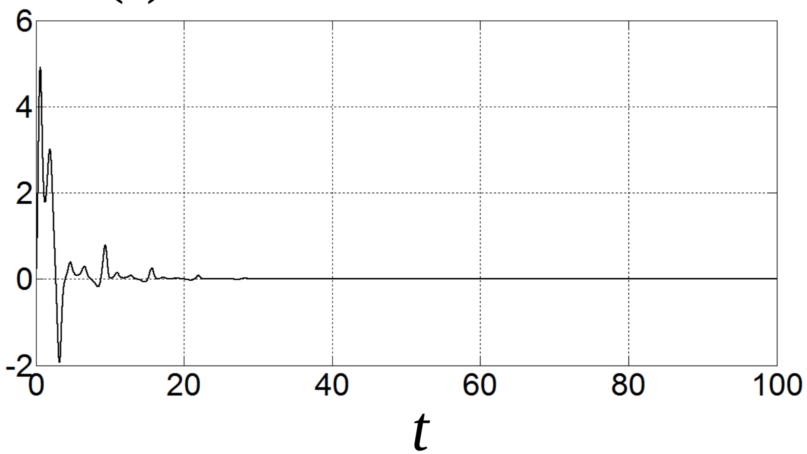
Filters polynomial $K(s)=s^2+5s+6$

$\theta = col(3,\ 5,\ 3)$

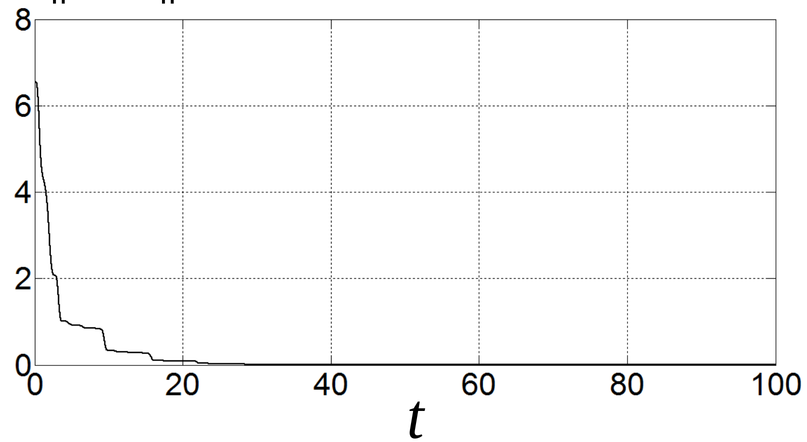
Adaptation gain $\gamma = 1$

$u(t)=10\sin t+20\cos 2t$

$\varepsilon(t)$



$\|\tilde{\theta}(t)\|$



6.1. Static Error Model

M.I.T. rule as an alternative methodology to Lyapunov functions

Error model $\varepsilon(t) = \tilde{\theta}^T(t)\omega(t),$

Lyapunov function	*M.I.T. rule*
Function	*Performance index*
$V = \dfrac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta}$	$J(\varepsilon) = \dfrac{1}{2}\varepsilon^2$
$\dot{V} = -\varepsilon^2$	$\dot{\hat{\theta}} = \gamma \underset{\tilde{\theta}}{grad}\ J(\varepsilon)$

Adaptation algorithm $\dot{\hat{\theta}} = \gamma\omega\varepsilon$

6.2. Dynamic error model with measurable state

$$\begin{aligned} \dot{e}(t) &= Ae(t) + b\tilde{\theta}^T(t)\omega(t), \\ \varepsilon(t) &= Ce(t) \end{aligned} \tag{6.9}$$

where $e \in \mathbb{R}^n$ is the state ε is the output, $\tilde{\theta} \in \mathbb{R}^m$ is the vector of parametric errors, $\omega \in \mathbb{R}^m$ is the vector of measurable functions (regressor).

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***Remark 6.2.* The model is widely used in the problems of state adaptive control (see example below).**

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The problem is to design an adaptation algorithm based on (6.9)

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Lyapunov function?

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***Remark 6.2.* The model is widely used in the problems of state adaptive control (see example below).**

$$V=\frac{1}{2}e^TPe+\frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} \qquad (6.10)$$

with a positive gain γ and positively defined symmetric matrix $P=P^T\succ 0$ defined later.

6.2. Dynamic error model with measurable state

Time derivative:

$$\dot{e} = Ae + b\tilde{\theta}^T\omega \qquad \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}}$$

6.2. Dynamic error model with measurable state

Time derivative:

$$\dot{e} = Ae + b\tilde{\theta}^T\omega \qquad \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = \frac{1}{2}\left(Ae + b\tilde{\theta}^T\omega\right)^T Pe + \frac{1}{2}e^T P\left(Ae + b\tilde{\theta}^T\omega\right) - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} =$$

6.2. Dynamic error model with measurable state

Time derivative:

$$\dot{e} = Ae + b\tilde{\theta}^T\omega \qquad \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = \frac{1}{2}\left(Ae + b\tilde{\theta}^T\omega\right)^T Pe + \frac{1}{2}e^T P\left(Ae + b\tilde{\theta}^T\omega\right) - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} =$$

$$\frac{1}{2}e^T A^T Pe + \frac{1}{2}e^T PAe + b^T\tilde{\theta}^T\omega Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}}$$

6.2. Dynamic error model with measurable state

Time derivative:

$$\dot{e} = Ae + b\tilde{\theta}^T\omega \qquad \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = \frac{1}{2}\left(Ae + b\tilde{\theta}^T\omega\right)^T Pe + \frac{1}{2}e^T P\left(Ae + b\tilde{\theta}^T\omega\right) - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} =$$

$$\frac{1}{2}e^T A^T Pe + \frac{1}{2}e^T PAe + b^T\tilde{\theta}^T\omega Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = \frac{1}{2}e^T\left(A^T P + PA\right)e + \tilde{\theta}^T\omega b^T Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}}$$

Since matrix A is Hurwitz, it is related to the matrix P via Lyapunov equation $A^T P + PA = -Q$ *with* $Q = Q^T \succ 0$

$$\dot{V} = -\frac{1}{2}e^T Qe + \tilde{\theta}^T\omega b^T Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}}$$

6.2. Dynamic error model with measurable state

Time derivative:

$$\dot{e} = Ae + b\tilde{\theta}^T\omega \qquad \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = \frac{1}{2}\left(Ae + b\tilde{\theta}^T\omega\right)^T Pe + \frac{1}{2}e^T P\left(Ae + b\tilde{\theta}^T\omega\right) - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} =$$

$$\frac{1}{2}e^T A^T Pe + \frac{1}{2}e^T PAe + b^T\tilde{\theta}^T\omega Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} = \frac{1}{2}e^T\left(A^T P + PA\right)e + \tilde{\theta}^T\omega b^T Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}}$$

Since matrix A is Hurwitz, it is related to the matrix P via Lyapunov equation $A^T P + PA = -Q$ *with* $Q = Q^T \succ 0$

$$\dot{V} = -\frac{1}{2}e^T Qe + \tilde{\theta}^T\omega b^T Pe - \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}}$$

Adaptation algorithm?

6.2. Dynamic error model with measurable state

$$\dot{V} = -\frac{1}{2}e^T Qe + \tilde{\theta}^T \omega b^T Pe - \frac{1}{\gamma}\tilde{\theta}^T \dot{\hat{\theta}}$$

If $\dot{\hat{\theta}} = \gamma\omega b^T Pe$,

$$\dot{V} = -\frac{1}{2}e^T Qe < 0 \tag{6.11}$$

6.2. Dynamic error model with measurable state

Summary and Discussion

Error Model	$\dot{e} = Ae + b\tilde{\theta}^T \omega$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma \omega b^T Pe$
Lyapunov function	$V = \frac{1}{2} e^T Pe + \frac{1}{2\gamma} \tilde{\theta}^T \tilde{\theta}$
Its time derivative	$\dot{V} = -\frac{1}{2} e^T Qe < 0$

What it means?

6.2. Dynamic error model with measurable state

Summary

Properties of the closed-loop system:

1. If ω is bounded, all the signals in the system are bounded;
2. Error $\|e(t)\|$ approaches zero asymptotically;
3. The function $V(t)$ is nonincreasing;
4. $\|\tilde{\theta}(t)\|^2$ approaches zero asymptotically if ω contains at least *m/2* harmonics and consists of linearly independent elements;

This property can be reformulated in terms of **Persistent Excitation Condition:**

$$\int_{t}^{t+T} \omega(\tau)\omega^T(\tau)d\tau \geq \alpha I$$

for some positive α, T .

6.2. Dynamic error model with measurable state

Example 6.4. The problem of state adaptive control

Problem statement

Let a plant be described by

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ b_0 \end{bmatrix}}_{b} u \qquad (6.12)$$

with **unknown** parameters a_0, a_1 , known b_0 and measurable state input u and output y.

The objective is to design a control u such that

$$\lim_{t\to\infty} \left\| x_M(t) - x(t) \right\| = 0 \qquad (6.13)$$

6.2. Dynamic error model with measurable state

x_M is the state of reference model

$$\underbrace{\begin{bmatrix} \dot{x}_{M1} \\ \dot{x}_{M2} \end{bmatrix}}_{\dot{x}_M} = \underbrace{\begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix}}_{A_M} \underbrace{\begin{bmatrix} x_{M1} \\ x_{M2} \end{bmatrix}}_{x_M} + \underbrace{\begin{bmatrix} 0 \\ b_{M0} \end{bmatrix}}_{b_M} g \tag{6.14}$$

with parameters a_{M0}, a_{M1}, b_{M0} responsible for transient performance of the closed-loop system and reference signal g .

6.2. Dynamic error model with measurable state

x_M is the state of reference model

$$\underbrace{\begin{bmatrix} \dot{x}_{M1} \\ \dot{x}_{M2} \end{bmatrix}}_{\dot{x}_M} = \underbrace{\begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix}}_{A_M} \underbrace{\begin{bmatrix} x_{M1} \\ x_{M2} \end{bmatrix}}_{x_M} + \underbrace{\begin{bmatrix} 0 \\ b_{M0} \end{bmatrix}}_{b_M} g \qquad (6.14)$$

with parameters a_{M0}, a_{M1}, b_{M0} responsible for transient performance of the closed-loop system and reference signal g .

Main idea of solution is to reduce the problem to the error model.

Then to get the adaptation algorithm.

6.2. Dynamic error model with measurable state

Solution

1. Let the parameters a_0, a_1 be known.

Form the error signal $e = x_M - x$ and take its derivative in view of the plant and reference model equations:

$$\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - Ax - bu$$

6.2. Dynamic error model with measurable state

Solution

1. Let the parameters a_0, a_1 be known.

Form the error signal $e = x_M - x$ and take its derivative in view of the plant and reference model equations:

$$\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - Ax - bu$$

Let $\dot{e} \Box A_M e \quad \left(e(t) = \exp(A_M t) e(0) \to 0 \;\; exponentially \;\; fast \right)$.

Then

$$A_M x_M + b_M g - Ax - bu \Box A_M e$$

$$A_M x_M + b_M g - Ax - bu \Box A_M x_M - A_M x$$

6.2. Dynamic error model with measurable state

Solution

1. Let the parameters a_0, a_1 be known.

Form the error signal $e = x_M - x$ and take its derivative view of the plant and reference model equations:

$$\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - Ax - bu$$

Let $\dot{e} \square A_M e \quad \left(e(t) = \exp(A_M t) e(0) \to 0 \ \textit{exponentially} \ \ \textit{fast} \right)$.

Then

$$A_M x_M + b_M g - Ax - bu \square A_M e$$

$$\cancel{A_M x_M} + b_M g - Ax - bu \square \cancel{A_M x_M} - A_M x$$

$$bu = \left(A_M - A \right) x + b_M g$$

6.2. Dynamic error model with measurable state

Solution

$$bu = \left(A_M - A\right)x + b_M g$$

$$\Downarrow$$

$$\begin{bmatrix} 0 \\ b_0 \end{bmatrix} u = \left(\begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \right) x + \begin{bmatrix} 0 \\ b_{M0} \end{bmatrix} g$$

6.2. Dynamic error model with measurable state

Solution

$$bu = \left(A_M - A\right)x + b_M g$$

$$\Downarrow$$

$$\begin{bmatrix} 0 \\ b_0 \end{bmatrix} u = \left(\begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \right) x + \begin{bmatrix} 0 \\ b_{M0} \end{bmatrix} g$$

$$\Downarrow$$

$$u = \frac{1}{b_0}\left[\left(a_0 - a_{M0}\right)x_1 + \left(a_1 - a_{M1}\right)x_2 + b_{M0} g \right]$$

6.2. Dynamic error model with measurable state

Solution

$$bu = (A_M - A)x + b_M g$$

$$\Downarrow$$

$$\begin{bmatrix} 0 \\ b_0 \end{bmatrix} u = \left(\begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \right) x + \begin{bmatrix} 0 \\ b_{M0} \end{bmatrix} g$$

$$\Downarrow$$

$$u = \frac{1}{b_0}\left[\underbrace{(a_0 - a_{M0})}_{\theta_1} x_1 + \underbrace{(a_1 - a_{M1})}_{\theta_2} x_2 + b_{M0} g \right]$$

$$\Downarrow$$

Nonadaptive control

$$u = \frac{1}{b_0}\left[\theta^T x + b_{M0} g \right] \qquad (6.15)$$

6.2. Dynamic error model with measurable state

Solution

2. Let the parameters a_0, a_1 be unknown. Control

$$u = \frac{1}{b_0}\left[\theta^T x + b_{M0} g\right]$$

is not implementable. Substitute estimate $\hat{\theta}$ for θ and obtain the implementable adjustable control:

Adjustable control

$$u = \frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right] \tag{6.16}$$

6.2. Dynamic error model with measurable state

Solution

2. Let the parameters a_0, a_1 be unknown. Control

$$u = \frac{1}{b_0}\left[\theta^T x + b_{M0} g\right]$$

is not implementable. Substitute estimate $\hat{\theta}$ for θ and obtain the implementable adjustable control:

Adjustable control

$$u = \frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right] \tag{6.16}$$

Replace (6.16) in the plant equation $\dot{x} = Ax + bu$:

$$\dot{x} = A\,x + b\frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right]$$

6.2. Dynamic error model with measurable state

Solution

Evaluate time derivative of error:

$$\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - Ax - b\frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right]$$

6.2. Dynamic error model with measurable state

Solution

Evaluate time derivative of error:

$$\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - Ax - b\frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right] \pm A_M x$$

$$\Downarrow$$

$$\dot{e} = A_M e + b_M g + \left(A_M - A\right)x - b\frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right]$$

6.2. Dynamic error model with measurable state

Solution

Evaluate time derivative of error:

$$\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - Ax - b\frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right] \pm A_M x$$

$$\Downarrow$$

$$\dot{e} = A_M e + \cancel{b_M g} + \left(A_M - A\right)x - b\frac{1}{b_0}\left[\hat{\theta}^T x + \cancel{b_{M0} g}\right]$$

$$\Downarrow$$

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} +$$

$$\left(\begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 0 \\ b_0 \end{bmatrix}\frac{1}{b_0}\hat{\theta}^T x$$

6.2. Dynamic error model with measurable state

Solution

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \underbrace{(a_0 - a_{M0})}_{\theta_1} + \underbrace{(a_1 - a_{M1})}_{\theta_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{k} \hat{\theta}^T x$$

6.2. Dynamic error model with measurable state

Solution

$$\begin{bmatrix} \dot e_1 \\ \dot e_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{M0} & -a_{M1} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \underbrace{(a_0 - a_{M0})}_{\theta_1} + \underbrace{(a_1 - a_{M1})}_{\theta_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{k} \hat\theta^T x$$

$$\Downarrow$$

$$\dot e = A_M e + k\theta^T x - k\hat\theta^T x$$

6.2. Dynamic error model with measurable state

Solution

$$\begin{bmatrix}\dot e_1\\ \dot e_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -a_{M0} & -a_{M1}\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix}+\begin{bmatrix}0\\ \underbrace{(a_0-a_{M0})}_{\theta_1}+\underbrace{(a_1-a_{M1})}_{\theta_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}-\underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{k}\hat\theta^T x$$

$$\Downarrow$$

$$\dot e = A_M\, e + k\theta^T x - k\hat\theta^T x$$

$$\Downarrow$$

$$\dot e = A_M\, e + k\tilde\theta^T x \qquad (6.17)$$

with parametric error $\tilde\theta = \theta - \hat\theta.$

6.2. Dynamic error model with measurable state

Solution

Error model $$\dot{e} = A_M e + k\tilde{\theta}^T x$$

$$\Downarrow$$

Adaptation Algorithm $$\dot{\hat{\theta}} = \gamma x k^T P e \qquad (6.18)$$

where γ is a positive gain, $P = P^T \succ 0$ is the solution of the Lyapunov equation

$$A_M^T P + P A_M = -Q \qquad (6.19)$$

with preliminary selected $Q = Q^T \succ 0$.

6.2. Dynamic error model with measurable state

Solution Summary

Adjustable control
$$u = \frac{1}{b_0}\left[\hat{\theta}^T x + b_{M0} g\right] \tag{6.16}$$

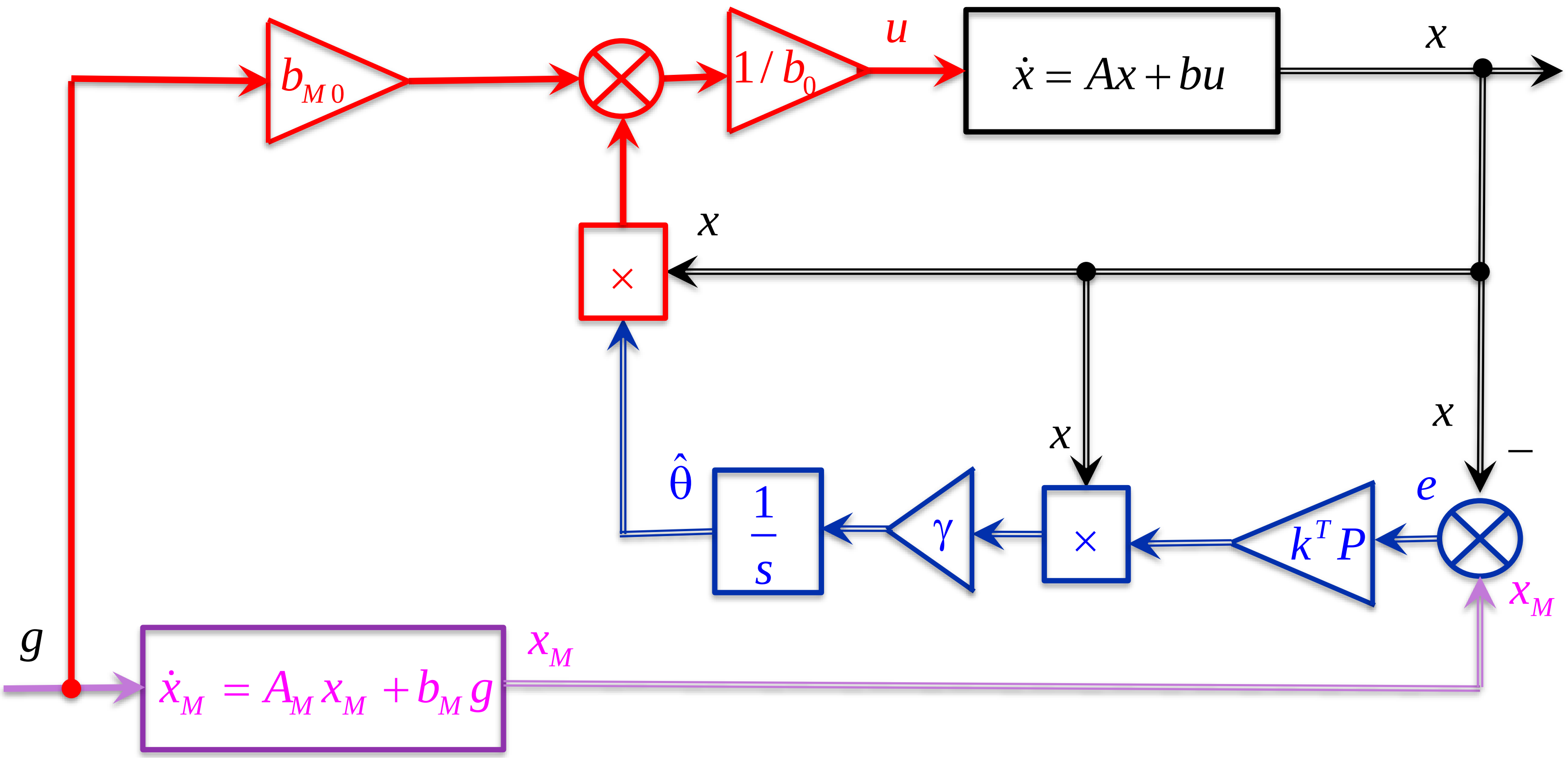
Adaptation Algorithm
$$\dot{\hat{\theta}} = \gamma\, x\, k^T P e \tag{6.18}$$

Error
$$e = x_M - x$$

Lyapunov equation
$$A_M^T P + P A_M = -Q \tag{6.19}$$

6.2. Dynamic error model with measurable state

General Scheme



6.2. Dynamic error model with measurable state

Simulation results

Plant (unstable)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Unknown parameters

$$a_0 = 1, \ a_1 = -2$$

Reference model

$$\begin{bmatrix} \dot{x}_{M1} \\ \dot{x}_{M2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_{M1} \\ x_{M2} \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \end{bmatrix} g$$

Adaptation gain

$$\gamma = 100$$

Matrix P

$$P = \begin{bmatrix} 1.1167 & 0.0833 \\ 0.0833 & 0.1167 \end{bmatrix}$$

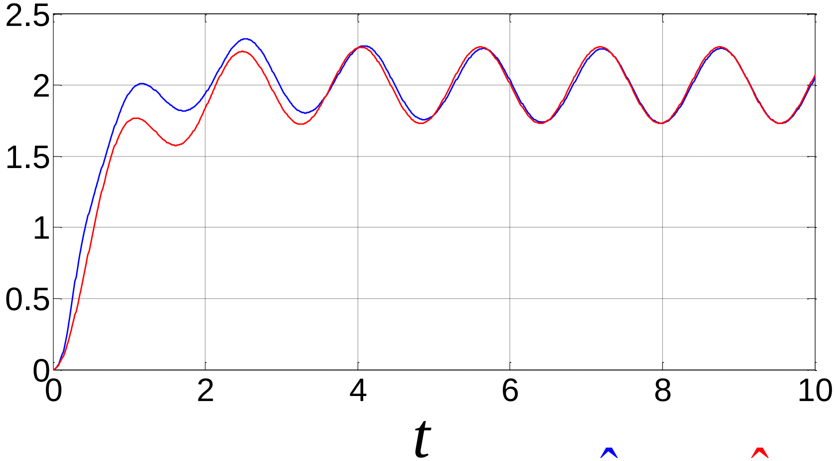
Reference

$$g(t) = \sin 4t + 2$$

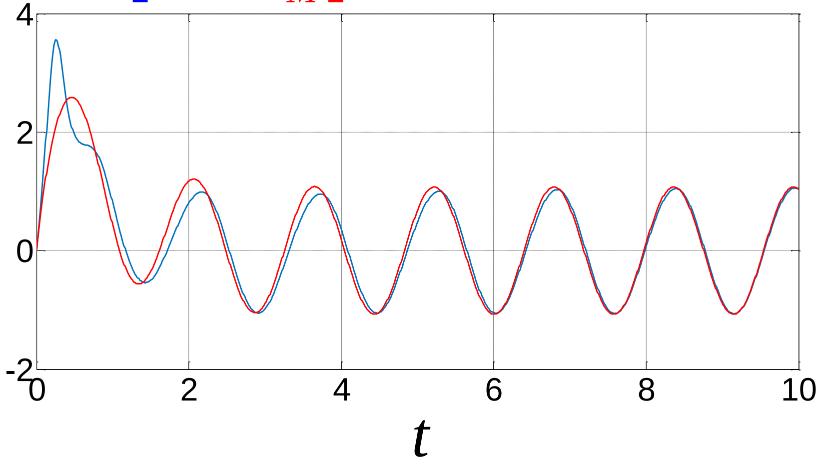
6.2. Dynamic error model with measurable state

Simulation results

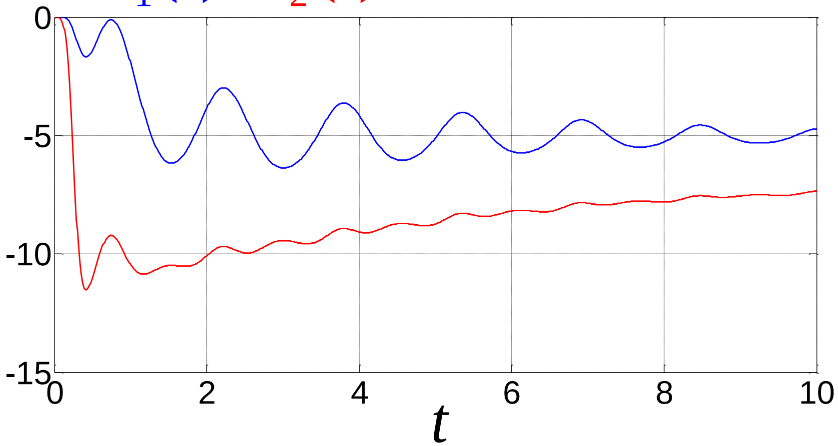
$x_1(t)$, $x_{M1}(t)$



$x_2(t)$, $x_{M2}(t)$



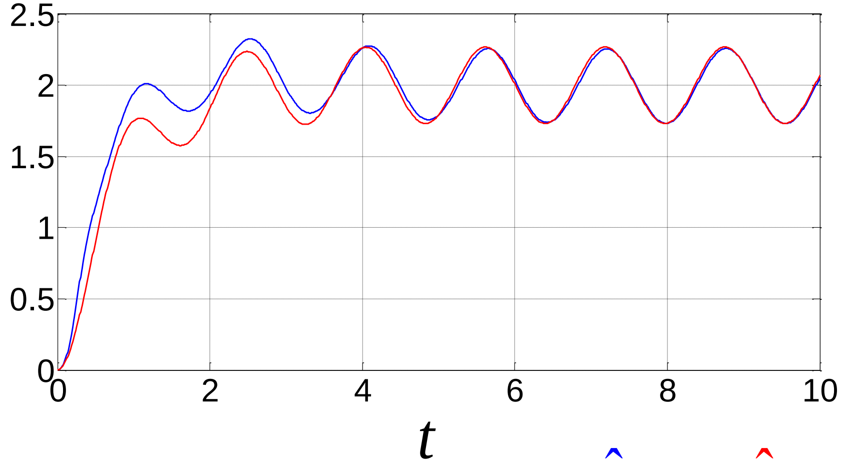
$\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$



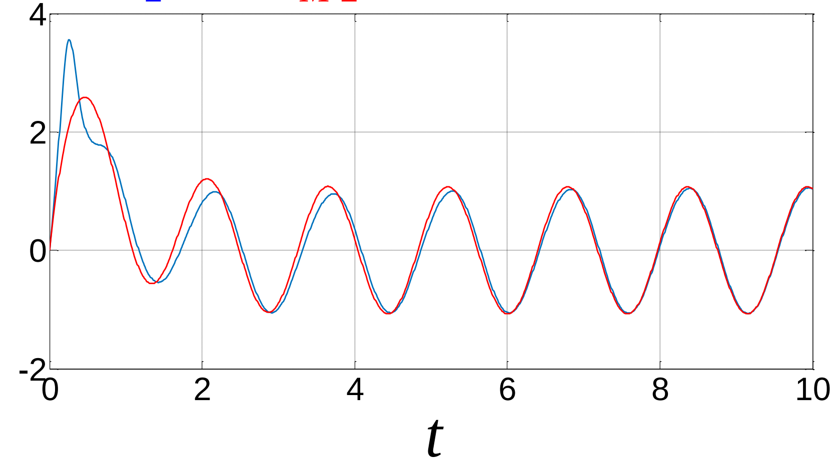
6.2. Dynamic error model with measurable state

Simulation results

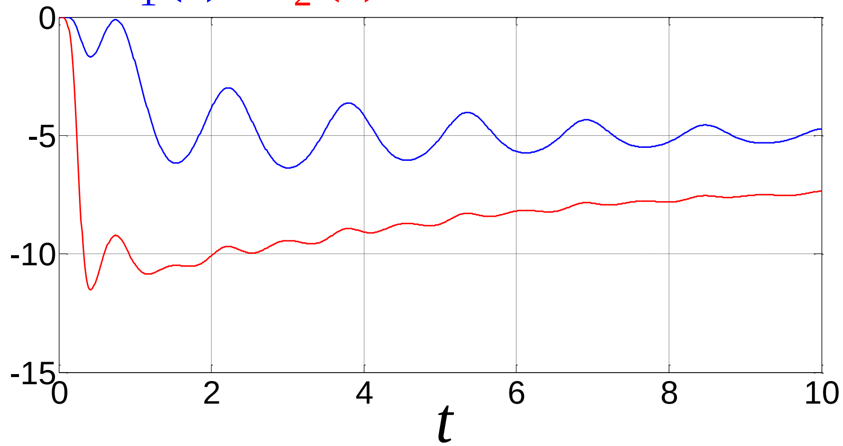
$x_1(t)$, $x_{M1}(t)$



$x_2(t)$, $x_{M2}(t)$



$\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$



Does AA provide identification properties?

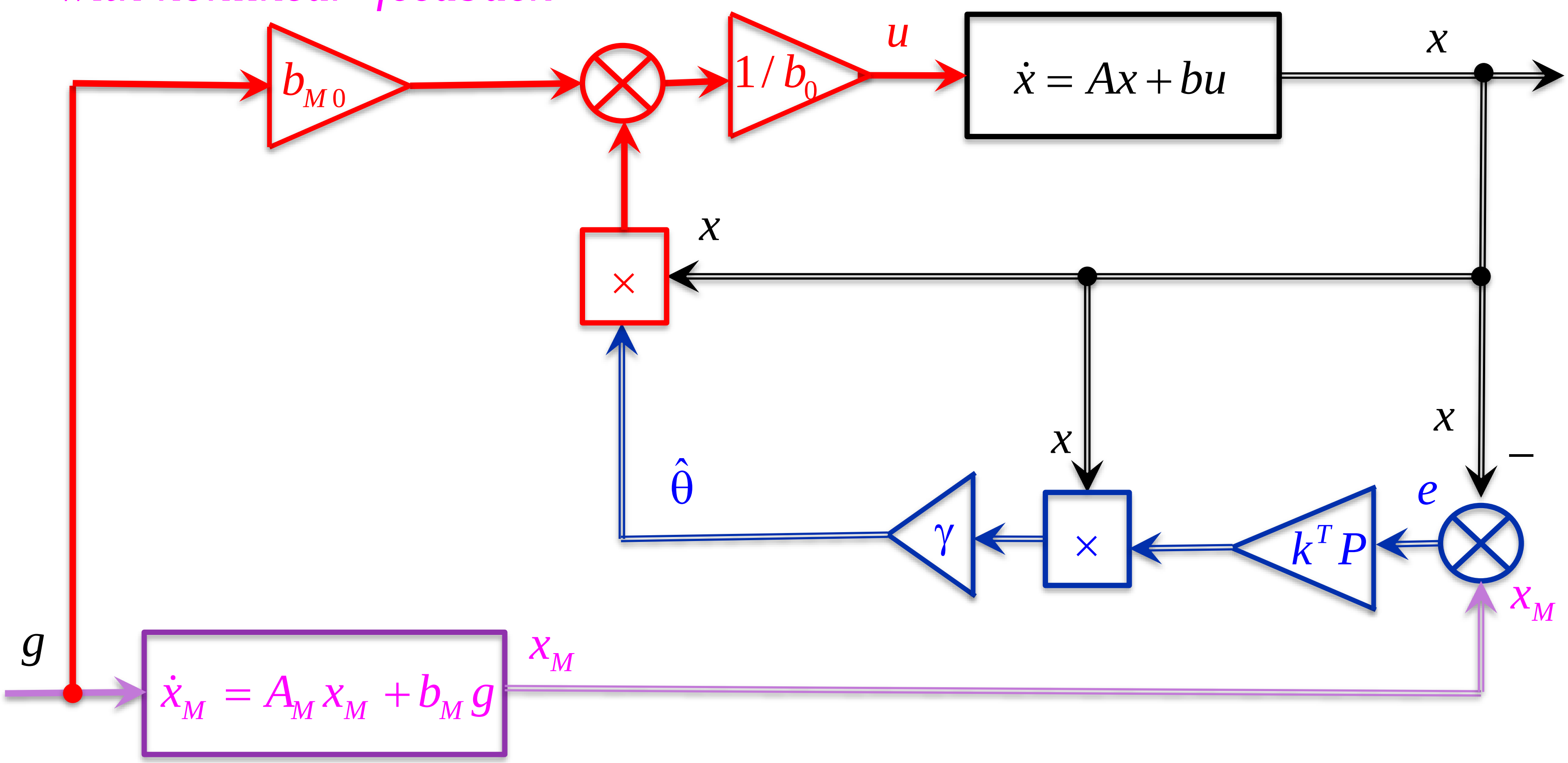
6.2. Dynamic error model with measurable state

Robust modifications of adaptation algorithm

Modification

with nonlinear feedback

$$\dot{\hat{\theta}} = \gamma \omega b^T P e$$



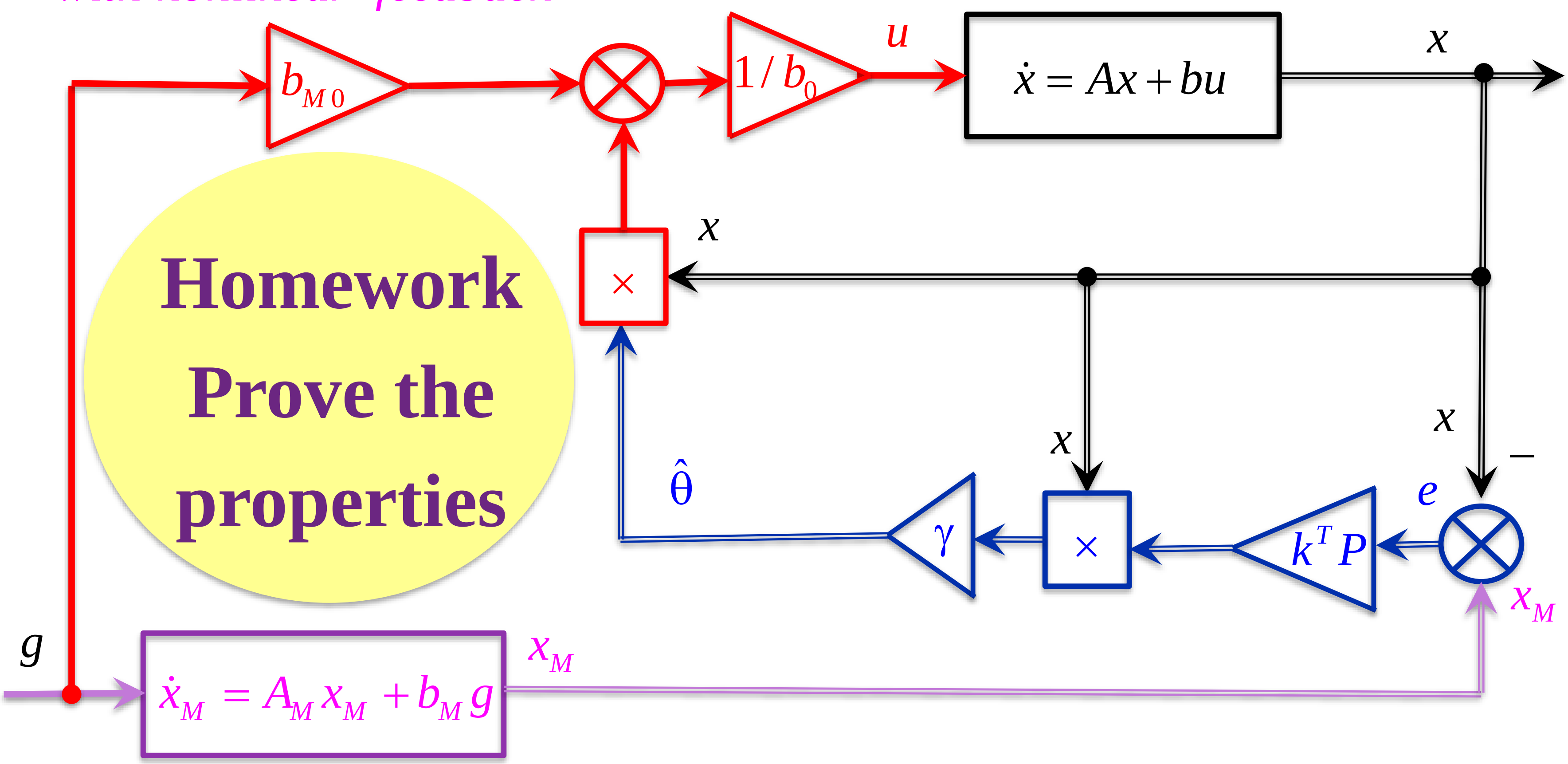
6.2. Dynamic error model with measurable state

Robust modifications of adaptation algorithm

Modification

with nonlinear feedback

$$\dot{\hat{\theta}} = \gamma\omega b^T P e$$

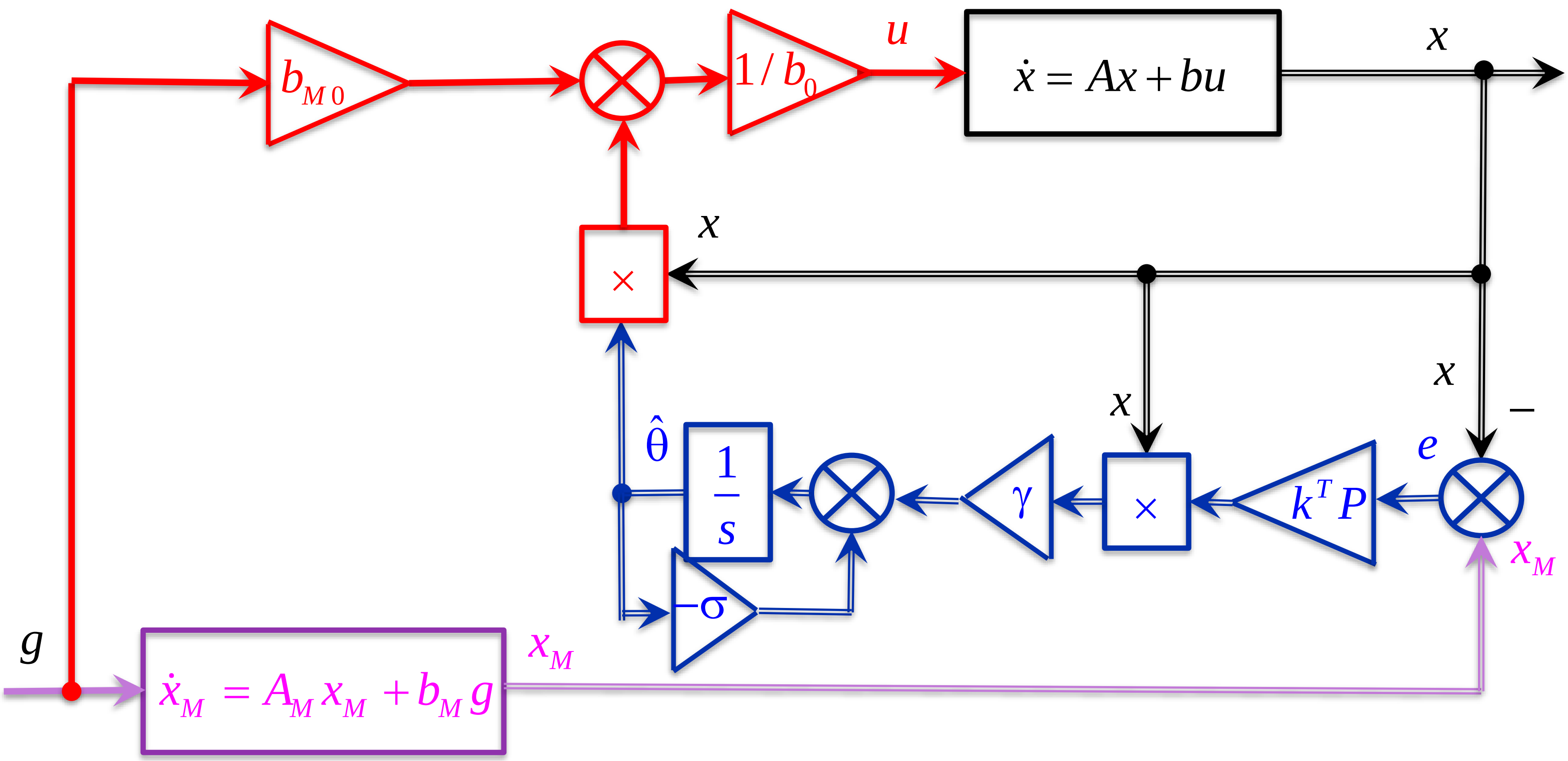


6.2. Dynamic error model with measurable state

Robust modifications of adaptation algorithm

σ – Modification

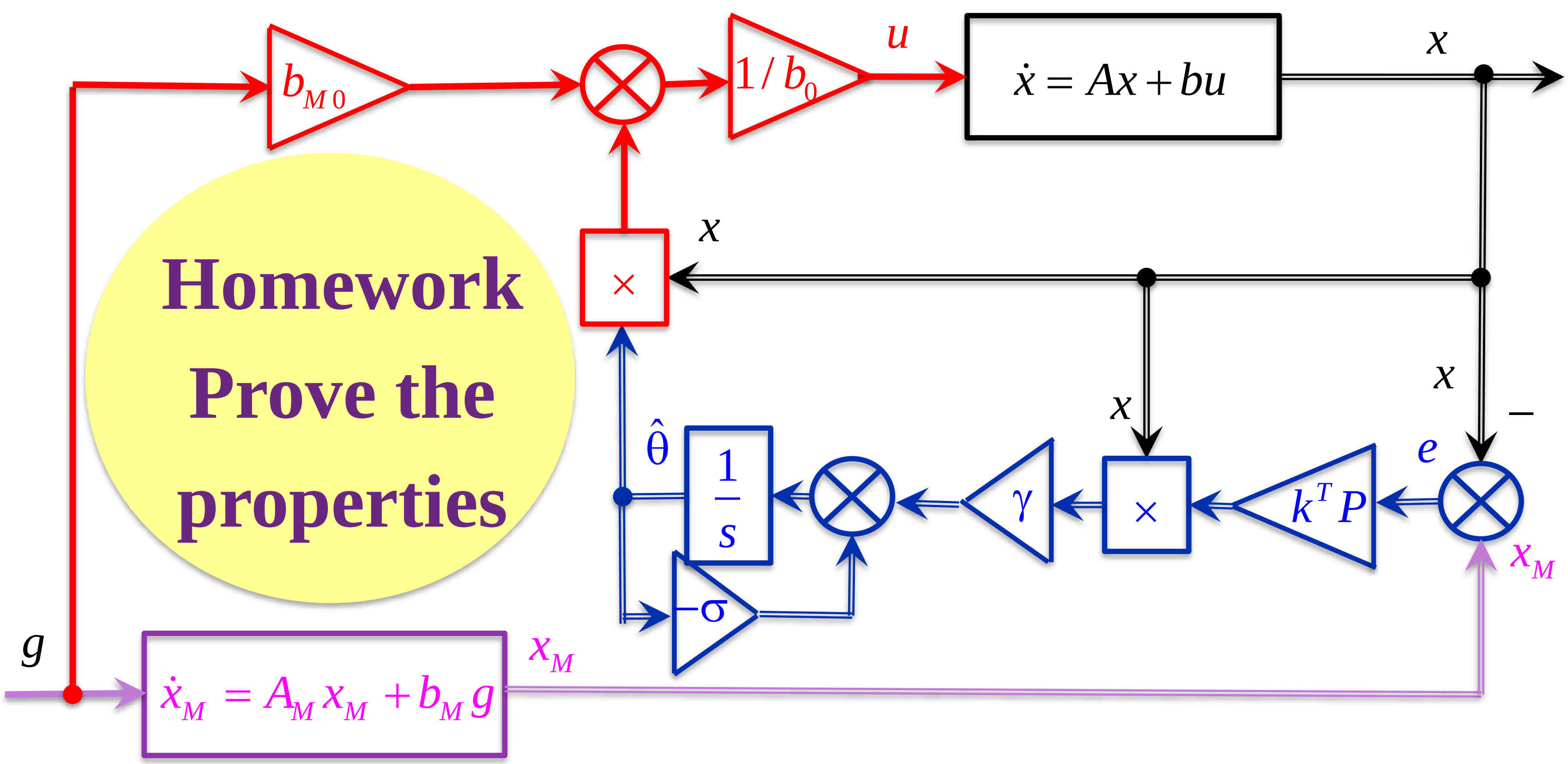
$$\dot{\hat{\theta}} = -\sigma\hat{\theta} + \gamma\omega b^T P e$$



6.2. Dynamic error model with measurable state

Robust modifications of adaptation algorithm

σ – *Modification* $$\dot{\hat{\theta}} = -\sigma\hat{\theta} + \gamma\omega b^T P e$$



Homework
Prove the properties

6.3. Dynamic error model with measurable output

$$\boxed{\begin{aligned}\dot{e}(t) &= Ae(t) + b\tilde{\theta}^T(t)\omega(t),\\ \varepsilon(t) &= c^T e(t)\end{aligned}} \qquad (6.20a)$$

where $e \in \mathbb{R}^n$ is the unmeasurable state ε is the output, $\tilde{\theta} \in \mathbb{R}^m$ is the vector of parametric errors, $\omega \in \mathbb{R}^m$ is the vector of measurable functions (regressor).

Remark 6.3. Since vector is not measurable, the model (6.20a) can be presented in the "Input-Output" form

$$\boxed{\varepsilon(t) = W(s)\left[\tilde{\theta}^T(t)\omega(t)\right]} \qquad (6.20b)$$

with transfer function $W(s) = c^T(Is - A)^{-1}b.$

6.3. Dynamic error model with measurable output

Remark 6.4. **The model is widely used in the problems of output adaptive control (see example below).**

The problem is to design an adaptation algorithm/algorithms based on (6.20)

6.3. Dynamic error model with measurable output

Solution #1

Can we just apply adaptation algorithm

$$\dot{\hat{\theta}} = \gamma\omega\varepsilon$$

used for static error model?

6.3. Dynamic error model with measurable output

Solution #1

Can we just apply adaptation algorithm

$$\dot{\hat{\theta}} = \gamma\omega\varepsilon$$

used for static error model?

IF YES, WHEN???

6.3. Dynamic error model with measurable output

Solution #1

$$\dot{e} = Ae + b\tilde{\theta}^T\omega,$$

$$\varepsilon = c^T e$$

$$\Downarrow$$

$$\dot{\hat{\theta}} = \gamma\,\omega b^T P e$$

"e" unmeasurable

6.3. Dynamic error model with measurable output

Solution #1

$$\dot{e} = Ae + b\tilde{\theta}^T\omega,$$

$$\varepsilon = c^T e$$

"*e*" *unmeasurable*

$$\Downarrow$$

$$\dot{\hat{\theta}} = \gamma\omega b^T P e$$

$$\Downarrow$$

If $b^T P = c^T$, adaptation algorithm becomes implementable since

$$\dot{\hat{\theta}} = \gamma\omega b^T P e = \gamma\omega\varepsilon \qquad (6.21)$$

6.3. Dynamic error model with measurable output

Solution #1

Lemma (Yakubovich-Kalman-Popov):

Matrix $P = P^T \succ 0$ satisfies both Lyapunov equation

$$A^T P + PA = -Q$$

and equation

$$b^T P = c^T$$

simultaniously iff transfer function

$$W(s) = c^T (Is - A)^{-1} b.$$

is Strictly Positive Real (SPR).

6.3. Dynamic error model with measurable output

Solution #1

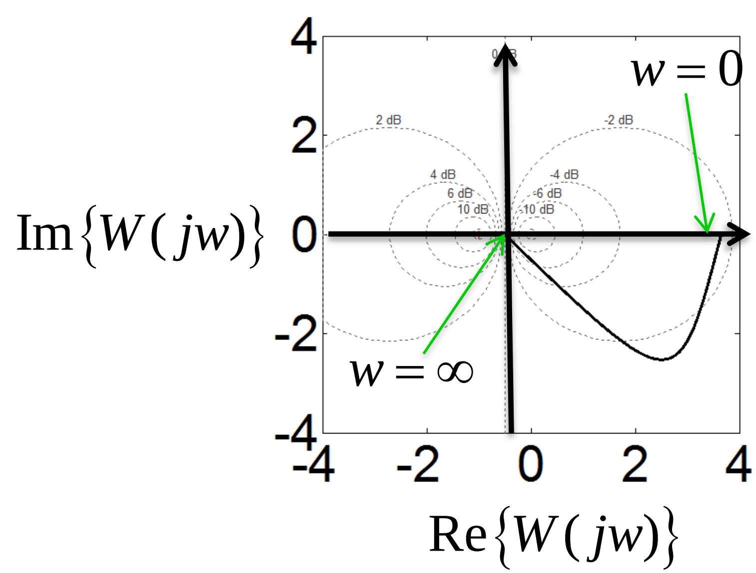
Definition 6.1. Transfer function $W(s) = c^T (Is - A)^{-1} b$ **is SPR if**

1. tt is stable, i.e. polynomial of its denominator is Hurwitz (has all the roots in the left half plane of root locus);
2. Nyiqust plot is placed in the right half plane of the diagram.

$$\mathrm{Re}\{W(jw)\} > 0, \quad \forall w \in [0, \infty).$$

3. the limit equality hold

$$\lim_{w \to \infty} w^2 \, \mathrm{Re}\{W(jw)\} > 0$$



6.3. Dynamic error model with measurable output

Example 6.5. SPR transfer function of first order block

$$W(s) = \frac{K}{Ts+1}$$

with some positive constant parameters K and T.

6.3. Dynamic error model with measurable output

Example 6.5. SPR transfer function of first order block

$$W(s) = \frac{K}{Ts+1}$$

with some positive constant parameters K and T .

Verification

1. Frequency transfer function

$$W(jw) = \frac{K}{Tjw+1} = \frac{K(-Tjw+1)}{(Tjw+1)(-Tjw+1)} = \underbrace{\frac{K}{T^2w^2+1}}_{\mathrm{Re}\{W(jw)\}} - j\underbrace{\frac{KTw}{T^2w^2+1}}_{-\mathrm{Im}\{W(jw)\}}$$

6.3. Dynamic error model with measurable output

Example 6.5. SPR transfer function of first order block

$$W(s) = \frac{K}{Ts+1}$$

with some positive constant parameters K and T .

Verification

1. Frequency transfer function

$$W(jw) = \frac{K}{Tjw+1} = \frac{K(-Tjw+1)}{(Tjw+1)(-Tjw+1)} = \frac{K}{T^2w^2+1} - j\frac{KTw}{T^2w^2+1}$$

2. The first condition: $Ts+1=0 \Rightarrow s_1 = -1/T \Rightarrow W(s)\ \text{is Hurwitz}$

6.3. Dynamic error model with measurable output

Example 6.5. SPR transfer function of first order block

$$W(s)=\frac{K}{Ts+1}$$

with some positive constant parameters K and T.

Verification

1. Frequency transfer function

$$W(jw)=\frac{K}{Tjw+1}=\frac{K(-Tjw+1)}{(Tjw+1)(-Tjw+1)}=\frac{K}{T^2w^2+1}-j\frac{KTw}{T^2w^2+1}$$

2. The first condition: $Ts+1=0 \Rightarrow s_1=-1/T \Rightarrow W(s)\ \text{is Hurwitz}$

3. The second condition:

$$\operatorname{Re}\{W(jw)\}=\frac{K}{T^2w^2+1}>0,\quad \forall w\in[0,\infty).$$

6.3. Dynamic error model with measurable output

4. The third condition:

$$\lim_{w\to\infty} w^2 \operatorname{Re}\{W(jw)\} = \lim_{w\to\infty} \frac{Kw^2}{T^2 w^2 + 1} = \frac{K}{T^2} > 0.$$

6.3. Dynamic error model with measurable output

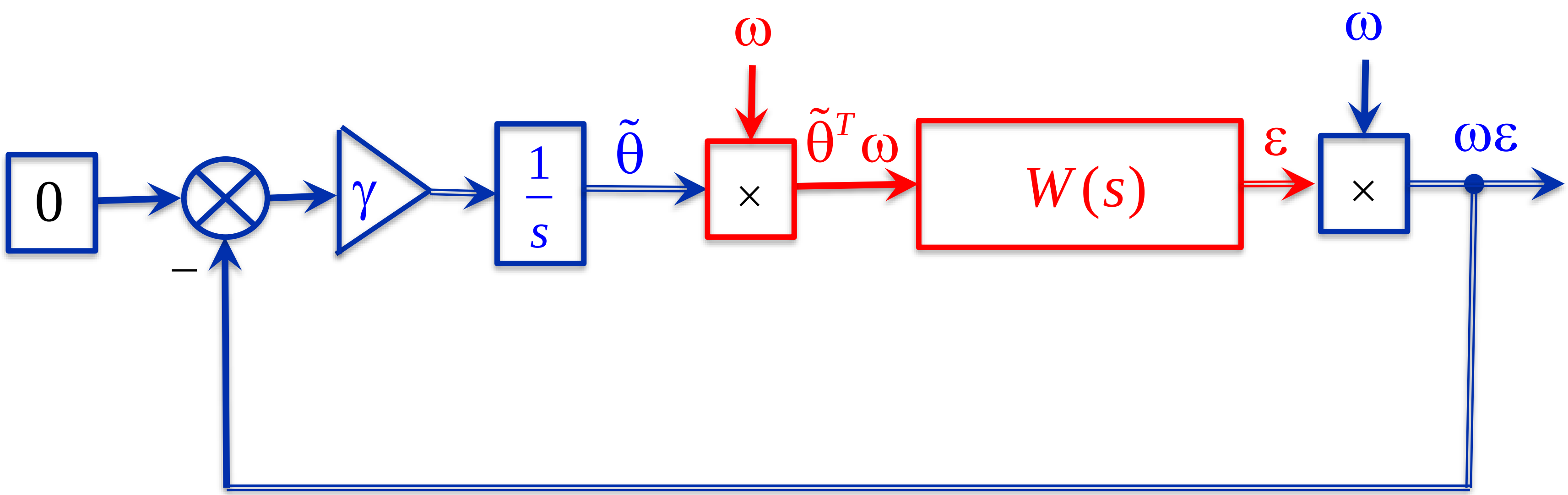
4. The third condition:

$$\lim_{w\to\infty} w^2 \operatorname{Re}\{W(jw)\} = \lim_{w\to\infty} \frac{Kw^2}{T^2w^2+1} = \frac{K}{T^2} > 0.$$

SPR transfer function is a function with property of the first order block, i.e. relative degree less than 2 (0 or 1)

6.3. Dynamic error model with measurable output

Remark 6.5. One-syllable words about adaptation algorithm and SPR transfer functions



Error model $\varepsilon(t) = W(s)\left[\tilde{\theta}^T(t)\omega(t)\right]$

Adaptation algorithm $\dot{\tilde{\theta}}(t) = -\dot{\hat{\theta}}(t) = -\gamma\,\omega(t)\varepsilon(t)$

6.3. Dynamic error model with measurable output

Solution #1

Summary and Discussion

Error Model	$\varepsilon = W(s)\left[\tilde{\theta}^T \omega\right],$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma\omega\varepsilon,$

where $W(s)$ is an SPR transfer function.

6.3. Dynamic error model with measurable output

Solution #1

Summary and Discussion

Error Model	$\varepsilon = W(s)\left[\tilde{\theta}^T\omega\right],$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma\omega\varepsilon,$

where $W(s)$ is an SPR transfer function.

SPR condition is quite restrictive and can narrow practical meaning of the problem

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Consider error model

$$\varepsilon = W(s)\left[\tilde{\theta}^T \omega\right]$$

and introduce augmentation signal

$$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T \omega\right]. \qquad (6.22)$$

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Consider error model

$$\varepsilon = W(s)\left[\tilde{\theta}^T \omega\right]$$

and introduce augmentation signal

$$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T \omega\right]. \tag{6.22}$$

Substitution of error model into (6.22) gives static error model **!!!**

$$\hat{\varepsilon} = \tilde{\theta}^T W(s)[\omega]. \tag{6.23}$$

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Consider error model

$$\varepsilon = W(s)\left[\tilde{\theta}^T \omega\right]$$

and introduce augmentation signal

$$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T \omega\right]. \qquad (6.22)$$

Substitution of error model into (6.22) gives static error model **!!!**

$$\hat{\varepsilon} = \tilde{\theta}^T W(s)[\omega]. \qquad (6.23)$$

Adaptation algorithm (see section **6.1. Static error model**)

$$\dot{\hat{\theta}} = \gamma W(s)[\omega]\hat{\varepsilon}. \qquad (6.24)$$

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary and Discussion

Error Model	$\varepsilon = W(s)\left[\tilde{\theta}^T \omega\right],$
Augmented error	$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T \omega\right],$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma W(s)[\omega]\hat{\varepsilon}.$

The solution relaxes restriction on the class of transfer functions

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary and Discussion

Error Model	$\varepsilon = W(s)\left[\tilde{\theta}^T\omega\right],$
Augmented error	$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T\omega\right],$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma W(s)[\omega]\hat{\varepsilon}.$

Proved using *the Swapping lemma*:

$$W(s)\left[\hat{\theta}^T\omega\right] = \hat{\theta}^T W(s)[\omega] - W_C(s)\left[W_b(s)\left[\omega^T\right]\dot{\hat{\theta}}\right]$$

where $W_C(s) = c^T(Is-A)^{-1}$, $W_b(s) = (Is-A)^{-1}b$ are the transfer matrices.

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary and Discussion

Error Model	$\varepsilon = W(s)\left[\tilde{\theta}^T\omega\right],$
Augmented error	$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T\omega\right],$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma W(s)[\omega]\hat{\varepsilon}.$

Augmented error simplified using the *Swapping lemma*:

$$\hat{\varepsilon} = \varepsilon - W_C(s)\left[W_b(s)\left[\omega^T\right]\dot{\hat{\theta}}\right],$$

where $W_C(s) = c^T\left(Is - A\right)^{-1}$, $W_b(s) = \left(Is - A\right)^{-1} b$ are the transfer matrices.

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary and Discussion

Error Model	$\varepsilon = W(s)\left[\tilde{\theta}^T \omega\right],$
Augmented error	$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W(s)[\omega] + W(s)\left[\hat{\theta}^T \omega\right],$
Adaptation Algorithm	$\dot{\hat{\theta}} = \gamma W(s)[\omega]\hat{\varepsilon}.$

Augmented error simplified using the *Swapping lemma*:

$$\hat{\varepsilon} = \varepsilon - \gamma W_C(s)\left[W_b(s)\left[\omega^T\right]W(s)[\omega]\hat{\varepsilon}\right],$$

where $W_C(s) = c^T(Is - A)^{-1}$, $W_b(s) = (Is - A)^{-1} b$ are the transfer matrices.

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary

Properties of the closed-loop system:

1. If ω is bounded, all the signals in the system are bounded;
2. Error $\hat{\varepsilon}(t)$ approaches zero asymptotically;
3. The norm $\|\tilde{\theta}(t)\|$ is nonincreasing;
4. The norm $\|\tilde{\theta}(t)\|$ approaches zero asymptotically, if ω satisfies the Persistent Excitation condition;
5.

$$\hat{\varepsilon} = \varepsilon - \gamma W_C(s)\left[W_b(s)\left[\omega^T\right]W(s)[\omega]\hat{\varepsilon}\right],$$

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary

Properties of the closed-loop system:

1. If ω is bounded, all the signals in the system are bounded;
2. Error $\hat{\varepsilon}(t)$ approaches zero asymptotically;
3. The norm $\|\tilde{\theta}(t)\|$ is nonincreasing;
4. The norm $\|\tilde{\theta}(t)\|$ approaches zero asymptotically, if ω satisfies the Persistent Excitation condition;
5.

$$\hat{\varepsilon} = \varepsilon - \gamma W_C(s)\Big[W_b(s)\big[\omega^T\big]W(s)[\omega]\hat{\varepsilon}\Big],$$

Does ε go to zero ???

6.3. Dynamic error model with measurable output

Solution #2 Augmented error algorithm (Monopoli, 1974)

Summary

Properties of the closed-loop system:

1. If ω is bounded, all the signals in the system are bounded;
2. Error $\hat{\varepsilon}(t)$ approaches zero asymptotically;
3. The norm $\|\tilde{\theta}(t)\|$ is nonincreasing;
4. The norm $\|\tilde{\theta}(t)\|$ approaches zero asymptotically, if ω satisfies the Persistent Excitation condition;
5. If ω is bounded, error $\varepsilon(t)$ approaches zero asymptotically.

6.3. Dynamic error model with measurable output

Example 6.6. The problem of output adaptive control

Problem statement

Let a plant be described by

$$\ddot{y} + a_1 \dot{y} + a_0 y = b_0 u \tag{6.25}$$

with **unknown** parameters a_0, a_1, known b_0 and unmeasurable state $\dot{y}$, known input u and output y.

The objective is to design a control u such that

$$\lim_{t\to\infty} \left\| y_M(t) - y(t) \right\| = 0, \tag{6.26}$$

where y_M is the output of reference model

$$\ddot{y}_M + a_{M1} \dot{y}_M + a_{M0} y_M = b_{M0} g \tag{6.27}$$

with the reference signal g.

6.3. Dynamic error model with measurable output

Solution

6.3. Dynamic error model with measurable output

Solution

Obstacles

Unmeasurable $\dot{y}$

Unknown parameters

6.3. Dynamic error model with measurable output

Solution

1. Obstacle of unmeasurable state.

Apply first order (*n-1*)th filter

$$\frac{1}{s+k}, \quad k>0$$

to the plant equation:

$$\frac{1}{s+k}\left[\ddot{y}+a_1\dot{y}+a_0 y\right]=b_0\frac{1}{s+k}\left[u\right]$$

6.3. Dynamic error model with measurable output

Solution

1. Obstacle of unmeasurable state.

Apply first order (*n-1*)th filter

$$\frac{1}{s+k},\quad k>0$$

to the plant equation:

$$\frac{1}{s+k}\left[\ddot{y}+a_1\dot{y}+a_0y\right]=b_0\frac{1}{s+k}[u]$$

$$\Downarrow$$

$$\frac{s\pm k}{s+k}[\dot{y}]+a_1\frac{s\pm k}{s+k}[y]+a_0\frac{1}{s+k}[y]=b_0\frac{1}{s+k}[u]$$

6.3. Dynamic error model with measurable output

Solution

1. Obstacle of unmeasurable state.

Apply first order (*n-1*)th filter

$$\frac{1}{s+k}, \quad k>0$$

to the plant equation:

$$\frac{1}{s+k}\left[\ddot{y}+a_1\dot{y}+a_0 y\right]=b_0\frac{1}{s+k}[u]$$

$$\Downarrow$$

$$\frac{s \pm k}{s+k}[\dot{y}]+a_1\frac{s \pm k}{s+k}[y]+a_0\frac{1}{s+k}[y]=b_0\frac{1}{s+k}[u]$$

$$\dot{y}=\left(k-a_1\right)y+\left(a_1k-k^2-a_0\right)\frac{1}{s+k}[y]+b_0\frac{1}{s+k}[u]$$

6.3. Dynamic error model with measurable output

Solution

$$\dot{y} = \underbrace{\left(k - a_1\right)}_{\theta_1^*}\underbrace{y}_{\omega_1} + \underbrace{\left(a_1 k - k^2 - a_0\right)}_{\theta_2^*}\underbrace{\frac{1}{s+k}[y]}_{\omega_2} + \underbrace{b_0}_{\theta_3^*}\underbrace{\frac{1}{s+k}[u]}_{\omega_3}$$

$$\Downarrow$$

$$\dot{y} = \theta^{*T}\omega$$

The derivative $\dot{y}$ is still not accessible, however presentable in the useful form of linear regression

6.3. Dynamic error model with measurable output

<u>Solution</u>

2. Obstacle of unknown parameters.

Main idea of solution is to reduce the problem to the error model.

Then to get the adaptation algorithm.

6.3. Dynamic error model with measurable output

Solution

2. Obstacle of unknown parameters.

Evaluate the **second** time derivative of error $\varepsilon = y_M - y$ in view of plant $\ddot{y} + a_1\dot{y} + a_0 y = b_0 u$

and reference model $\ddot{y}_M + a_{M1}\dot{y}_M + a_{M0}y_M = b_{M0}g$:

$$\ddot{\varepsilon} = \ddot{y}_M - \ddot{y} =$$

6.3. Dynamic error model with measurable output

Solution

2. Obstacle of unknown parameters.

Evaluate the **second** time derivative of error $\varepsilon = y_M - y$ in view of plant $\ddot{y} + a_1\dot{y} + a_0 y = b_0 u$

and reference model $\ddot{y}_M + a_{M1}\dot{y}_M + a_{M0}y_M = b_{M0}g$:

$$\ddot{\varepsilon} = \ddot{y}_M - \ddot{y} = -a_{M1}\dot{y}_M - a_{M0}y_M + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u =$$

6.3. Dynamic error model with measurable output

Solution

2. Obstacle of unknown parameters.

Evaluate the **second** time derivative of error $\varepsilon = y_M - y$ in view of plant $\ddot{y} + a_1 \dot{y} + a_0 y = b_0 u$

and reference model $\ddot{y}_M + a_{M1} \dot{y}_M + a_{M0} y_M = b_{M0} g$:

$$\ddot{\varepsilon} = \ddot{y}_M - \ddot{y} = -a_{M1} \dot{y}_M - a_{M0} y_M + b_{M0} g + a_1 \dot{y} + a_0 y - b_0 u$$

$$\ddot{\varepsilon} = -a_{M1} \left(\dot{y}_M \pm \dot{y} \right) - a_{M0} \left(y_M \pm y \right) + b_{M0} g + a_1 \dot{y} + a_0 y - b_0 u =$$

6.3. Dynamic error model with measurable output

Solution

2. Obstacle of unknown parameters.

Evaluate the **second** time derivative of error $\varepsilon = y_M - y$ in view of plant $\ddot{y} + a_1\dot{y} + a_0 y = b_0 u$

and reference model $\ddot{y}_M + a_{M1}\dot{y}_M + a_{M0}y_M = b_{M0}g$:

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$$\ddot{\varepsilon} = -a_{M1}\left(\dot{y}_M \pm \dot{y}\right) - a_{M0}\left(y_M \pm y\right) + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u =$$

$$-a_{M1}\dot{\varepsilon} - a_{M1}\dot{y} - a_{M0}\varepsilon - a_{M0}y + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u$$

6.3. Dynamic error model with measurable output

Solution

2. Obstacle of unknown parameters.

Evaluate the **second** time derivative of error $\varepsilon = y_M - y$ in view of plant $\ddot{y} + a_1\dot{y} + a_0 y = b_0 u$

and reference model $\ddot{y}_M + a_{M1}\dot{y}_M + a_{M0}y_M = b_{M0}g$:

$$\ddot{\varepsilon} = \ddot{y}_M - \ddot{y} = -a_{M1}\dot{y}_M - a_{M0}y_M + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u$$

$$\ddot{\varepsilon} = -a_{M1}\left(\dot{y}_M \pm \dot{y}\right) - a_{M0}\left(y_M \pm y\right) + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u =$$

$$-a_{M1}\dot{\varepsilon} - a_{M1}\dot{y} - a_{M0}\varepsilon - a_{M0}y + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u$$

6.3. Dynamic error model with measurable output

Solution

2. Obstacle of unknown parameters.

Evaluate the **second** time derivative of error $\varepsilon = y_M - y$ in view of plant $\ddot{y} + a_1\dot{y} + a_0 y = b_0 u$

and reference model $\ddot{y}_M + a_{M1}\dot{y}_M + a_{M0}y_M = b_{M0}g$:

$$\ddot{\varepsilon} = \ddot{y}_M - \ddot{y} = -a_{M1}\dot{y}_M - a_{M0}y_M + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u$$

$$\ddot{\varepsilon} = -a_{M1}\left(\dot{y}_M \pm \dot{y}\right) - a_{M0}\left(y_M \pm y\right) + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u =$$

$$-a_{M1}\dot{\varepsilon} - a_{M1}\dot{y} - a_{M0}\varepsilon - a_{M0}y + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u$$

$$\Downarrow$$

$$\varepsilon = \frac{1}{s^2 + a_{M1}s + a_{M0}}\left[-a_{M1}\dot{y} - a_{M0}y + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u\right]$$

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = \underbrace{\frac{1}{s^2 + a_{M1}s + a_{M0}}}_{W_M(s)}\left[-a_{M1}\dot{y} - a_{M0}y + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u\right]$$

$$\Downarrow$$

$$\varepsilon = W_M(s)\left[\left(a_1 - a_{M1}\right)\dot{y} + \left(a_0 - a_{M0}\right)y + b_{M0}g - b_0 u\right]$$

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = \underbrace{\frac{1}{s^2 + a_{M1}s + a_{M0}}}_{W_M(s)}\left[-a_{M1}\dot{y} - a_{M0}y + b_{M0}g + a_1\dot{y} + a_0 y - b_0 u\right]$$

$$\Downarrow$$

$$\varepsilon = W_M(s)\left[\left(a_1 - a_{M1}\right)\dot{y} + \left(a_0 - a_{M0}\right)y + b_{M0}g - b_0 u\right]$$

$$\varepsilon = W_M(s)\left[\left(a_1 - a_{M1}\right)\dot{y} + \left(a_0 - a_{M0}\right)y + b_{M0}g - b_0 u\right]$$

$$\dot{y} = \theta^{*T}\omega$$

$$\Downarrow$$

$$\varepsilon = W_M(s)\left[\theta^T\omega + b_{M0}g - b_0 u\right]$$

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = W_M(s)\left[\theta^T\omega + b_{M0}g - b_0u\right]$$

where $\omega = col\left(y, \frac{1}{s+k}[y], \frac{1}{s+k}[u]\right)$

$$\theta = col\left(\left(a_1 - a_{M1}\right)\theta_1^* + \left(a_0 - a_{M0}\right), \left(a_1 - a_{M1}\right)\theta_2^*, \left(a_1 - a_{M1}\right)\theta_3^*\right)$$

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = W_M(s)\left[\theta^T\omega + b_{M0}g - b_0 u\right]$$

where $\omega = col\left(y, \frac{1}{s+k}[y], \frac{1}{s+k}[u]\right)$

$$\theta = col\Big((a_1 - a_{M1})(k - a_1) + a_0 - a_{M0},$$

$$(a_1 - a_{M1})\left(a_1 k - k^2 - a_0\right),\ a_1 b_0 - a_{M1} b_0\Big)$$

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = W_M(s)\left[\theta^T\omega + b_{M0}g - b_0 u\right]$$

where $\omega = col\left(y, \frac{1}{s+k}[y], \frac{1}{s+k}[u]\right)$

$$\theta = col\Big(\big(a_1 - a_{M1}\big)\big(k - a_1\big) + a_0 - a_{M0},$$

$$\big(a_1 - a_{M1}\big)\big(a_1 k - k^2 - a_0\big),\ a_1 b_0 - a_{M1} b_0\Big)$$

Adjustable control $\quad u = \frac{1}{b_0}\left[\hat{\theta}^T\omega + b_{M0}g\right]$ (6.28)

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = W_M(s)\left[\theta^T\omega + b_{M0}g - b_0u\right]$$

where $$\omega = col\left(y, \frac{1}{s+k}[y], \frac{1}{s+k}[u]\right)$$

$$\theta = col\Big(\big(a_1 - a_{M1}\big)\big(k - a_1\big) + a_0 - a_{M0},$$
$$\big(a_1 - a_{M1}\big)\big(a_1k - k^2 - a_0\big),\ a_1b_0 - a_{M1}b_0\Big)$$

Adjustable control $$u = \frac{1}{b_0}\left[\hat{\theta}^T\omega + b_{M0}g\right] \qquad (6.28)$$

Error model $$\varepsilon = W_M(s)\left[\tilde{\theta}^T\omega\right] \qquad (6.29)$$

with parametric errors $\tilde{\theta} = \theta - \hat{\theta}.$

6.3. Dynamic error model with measurable output

Solution

$$\varepsilon = W_M(s)\left[\tilde{\theta}^T \omega\right],$$

$$\Downarrow$$

where

Augmented error

$$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W_M(s)[\omega] + W_M(s)\left[\hat{\theta}^T \omega\right] \tag{6.30}$$

Adaptation Algorithm

$$\dot{\hat{\theta}} = \gamma W_M(s)[\omega]\hat{\varepsilon} \tag{6.31}$$

6.3. Dynamic error model with measurable output

Solution Summary

Adjustable control $$u = \frac{1}{b_0}\left[\hat{\theta}^T\omega + b_{M0}g\right] \quad (6.28)$$

Augmented error $$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T W_M(s)[\omega] + W_M(s)\left[\hat{\theta}^T\omega\right] \quad (6.30)$$

Adaptation Algorithm $$\dot{\hat{\theta}} = \gamma(t) W_M(s)[\omega]\hat{\varepsilon} \quad (6.31)$$

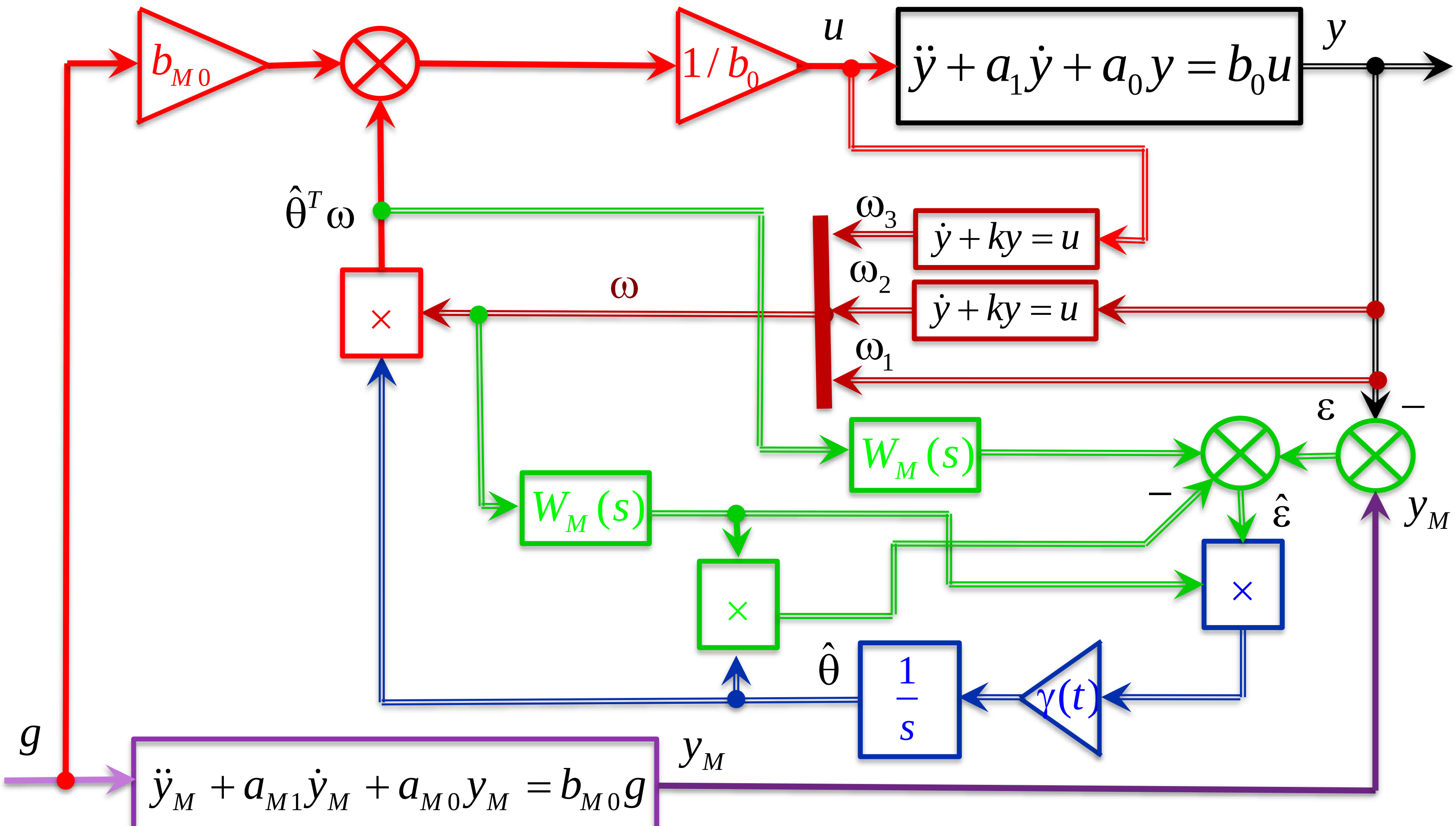
Error $$\varepsilon = y_M - y$$

Regressor with filters $$\omega = col\left(y, \frac{1}{s+k}[y], \frac{1}{s+k}[u]\right)$$

Normalization (ω is bounded?) $$\gamma(t) = \frac{\gamma_0}{1 + W_M(s)\left[\omega^T\right] W_M(s)[\omega]} \quad (6.32)$$

6.3. Dynamic error model with measurable output

General Scheme



6.3. Dynamic error model with measurable output

Simulation results

Plant

$$\ddot{y} + a_1\dot{y} + a_0 y = u$$

Unknown parameters

$$a_0 = 1,\ a_1 = 2$$

Reference model

$$\ddot{y}_M + 5\dot{y}_M + 6y_M = 6g$$

Adaptation gain

$$\gamma(t) = \frac{1000}{1 + W_M(s)\left[\omega^T\right]W_M(s)\left[\omega\right]}$$

Reference transfer function (with unity nominator)

$$W_M(s) = \frac{1}{s^2 + 5s + 6}$$

Reference

$$g(t) = \sin 4t$$

6.3. Dynamic error model with measurable output

Simulation results

Regressor

$$\omega = col\left(y, \frac{1}{s+8}[y], \frac{1}{s+8}[u] \right)$$

Augmented error

$$\hat{\varepsilon} = \varepsilon - \hat{\theta}^T \frac{1}{s^2+5s+6}[\omega] + \frac{1}{s^2+5s+6}\left[\hat{\theta}^T \omega\right],$$

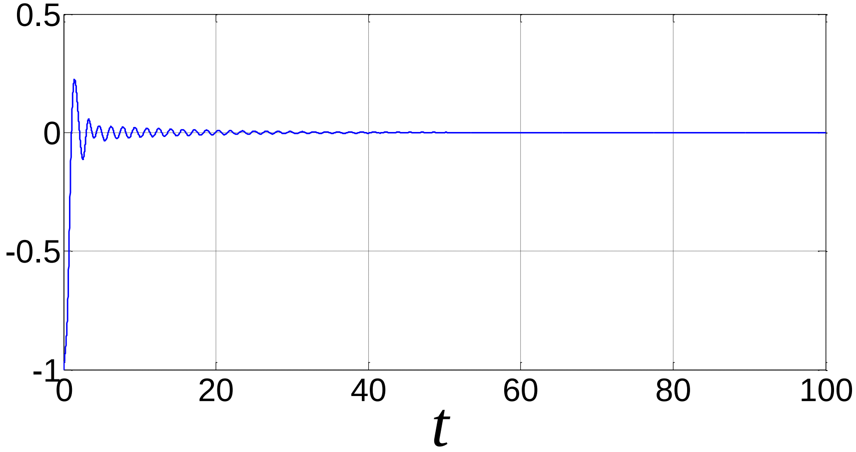
Error

$$\varepsilon = y_M - y$$

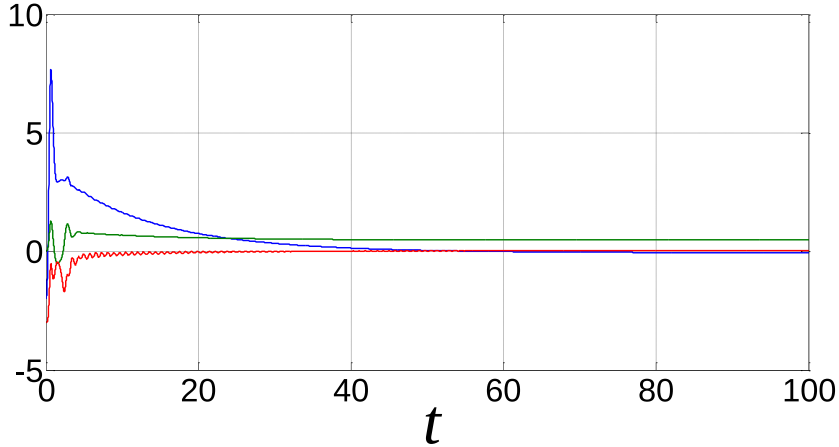
6.3. Dynamic error model with measurable output

Simulation results

$$\varepsilon(t) = y_M(t) - y(t)$$



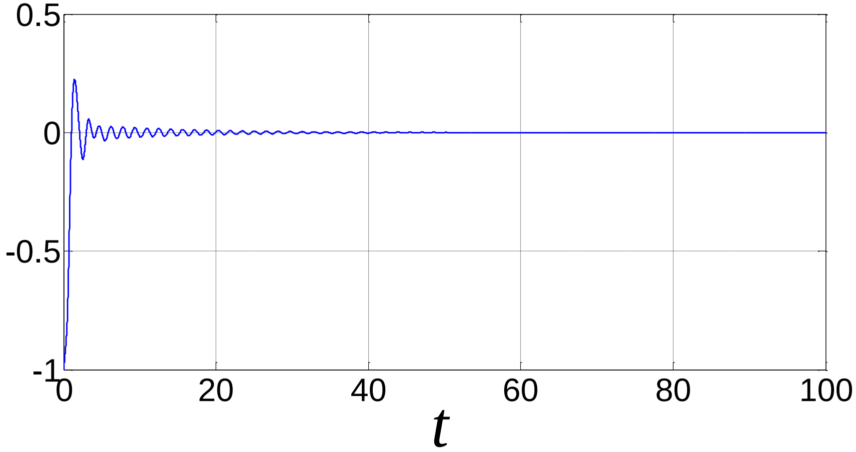
$$\tilde{\theta}_1(t),\ \tilde{\theta}_2(t),\ \tilde{\theta}_3(t)$$



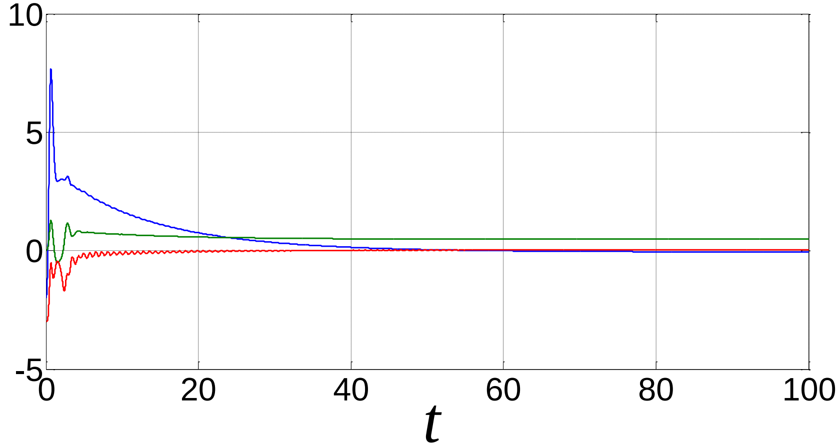
6.3. Dynamic error model with measurable output

Simulation results

$$\varepsilon(t) = y_M(t) - y(t)$$



$$\tilde{\theta}_1(t),\ \tilde{\theta}_2(t),\ \tilde{\theta}_3(t)$$



Does AA provide identification properties?