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Augmented error based adaptive control with improved parametric convergence.
Tutorial

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1. Motivation: MRAC and the Monopoli's scheme of augmented error

1.1. Problem statement and plant parameterization

Plant:

$$y = \frac{b(s)}{a(s)}[u], \tag{1}$$

where $a(s) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_0,$

$$b(s) = k\,s^m + b_{m-1}s^{m-1} + \cdots + b_0,$$

$a_i,\, b_j$ are unknown constant coefficients, $n > m,\; m$ are the known orders.

1.1. Problem statement and plant parameterization

Objective:

$$\lim_{t\to\infty} \varepsilon(t) = \lim_{t\to\infty} \left(y(t) - y_m(t) \right) = 0, \tag{2}$$

where y_m is the output of the reference model

$$y_m = \frac{a_m(0)}{a_m(s)}[g] \tag{3}$$

with Hurwitz polynomial $a_m(s)$ of degree $n-m$ with bounded PWC reference $g(t)$.

Assumptions:

1. The polynomial $b(s) = k\,s^m + b_{m-1}s^{m-1} + \cdots + b_0$ is Hurwitz;
2. The sign of the high frequency gain k is known (w.l.g. $k > 0$).

1.1. Problem statement and plant parameterization

Lemma (Monopoli): There exists a constant vector $\psi \in \mathrm{R}^{2n}$

$$\varepsilon \triangleq y - y_m = \frac{k}{a_m(s)}\left[\psi^T \phi - u\right] + \sigma, \tag{4}$$

where σ (here and hereafter) is the exponentially decaying term, $\phi = [y,\, v_1,\, v_2,\, g]^T$ is the regressor with the vectors $v_1, v_2 \in \mathrm{R}^{n-1}$ generated by the filters

$$\dot{v}_1 = \Lambda v_1 + \varsigma_{n-1} u, \qquad \Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 & 1 \\ -\lambda_0 & -\lambda_1 & \cdots & \cdots & -\lambda_{n-2} \end{bmatrix}, \; \varsigma_{n-1} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

$$\dot{v}_2 = \Lambda v_2 + \varsigma_{n-1} y,$$

Monopoli R. Model reference adaptive control with an augmented error, TAC, 19 (4), 474-484, 1974.

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Certainty equivalent controller:

$$u = \hat{\psi}^T \phi, \tag{5}$$

where $\hat{\psi}$ is the vector of adjustable parameters.

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where $\hat{\psi}$ is the vector of adjustable parameters.

Error model:

$$\varepsilon = \frac{k}{a_m(s)}\left[\tilde{\psi}^T \phi\right] + \sigma, \tag{6}$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

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1.2. The concept of augmented error

Error model:

$$\varepsilon = kW(s)\left[\tilde{\psi}^T \phi\right], \tag{7}$$

where $\varepsilon \in \mathrm{R}$ is the output error, $\tilde{\psi} = \psi - \hat{\psi} \in \mathrm{R}^q$ is vector of parametric errors, $\phi \in \mathrm{R}^q$ is the vector of measurable functions (regressor), $\hat{\psi}$ is the vector of adjustable parameters (or estimate), $k \in \mathrm{R}$ is a positive control gain, $W(s)$ is the asymptotically stable transfer function.

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k temporarily known

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1.2. The concept of augmented error

TRANSFER FUNCTION SWAPPING

Augmentation signal:

$$\varsigma_k = W(s)\left[\hat{\psi}^T \phi\right] - \hat{\psi}^T \phi_k, \tag{8}$$

where $\phi_k = kW(s)\left[\phi\right]$ is the filtered regressor.

Augmented error:

$$\tilde{\varepsilon}_k = \varepsilon + \varsigma_k. \tag{9}$$

Result of swapping:

$$\varepsilon = kW(s)\left[\tilde{\psi}^T \phi\right] \longrightarrow \tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \tag{10}$$

1.2. The concept of augmented error

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The term Swapping

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where $\phi_k = kW(s)\left[\phi\right]$ is the filtered regressor.

Augmented error:

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Integration by parts?

$$\int_0^t \phi^T(\tau)\tilde{\psi}(\tau)d\tau = \int_0^t \phi^T(\tau)d\tau\tilde{\psi} - \int_0^t \int_0^{\tau_2} \phi^T(\tau_1)d\tau_1\dot{\tilde{\psi}}(\tau_2)d\tau_2$$

Result of swapping:

$$\varepsilon = kW(s)\left[\tilde{\psi}^T \phi\right] \longrightarrow \tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \tag{10}$$

1.2. The concept of augmented error

Corollary of the results

Integration by parts!

$$\int_0^t \phi^T(\tau)\hat{\psi}(\tau)d\tau - \int_0^t \phi^T(\tau)d\tau\hat{\psi} = -\int_0^t\int_0^{\tau_2} \phi^T(\tau_1)d\tau_1\dot{\hat{\psi}}(\tau_2)d\tau_2$$

Augmentation signal:

$$\varsigma_k = -kW_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right], \tag{11}$$

where $W_C(s) = c^T\left(sI - A\right)^{-1}$, $W_B(s) = \left(sI - A\right)^{-1}b$ are the transfer function vectors.

Augmented error:

$$\underbrace{\tilde{\varepsilon}_k}_{\text{Augmented error}} = \underbrace{\varepsilon}_{\text{Control error}} - \underbrace{kW_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right]}_{\text{Proportional to the time derivative } \dot{\hat{\psi}}}. \tag{12}$$

1.2. The concept of augmented error

Main idea of the tutorial

is to combine the augmented error approach with modifications of Lion's (DRE) and Kreisselmeier's (MRE) adaptation algorithms [1,2,3] with fast parameters tuning to draw the augmentation term to zero AFAP

$$\tilde{\varepsilon}_k = \varepsilon - kW_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right],$$

Main contributions:

1. Improvement of output (tracking) error convergence for problems of adaptive control using augmented error: AE+DRE, AE+MRE;
2. Extension of the idea to systems with unknown steady-state coefficients (control gains) k : AE+DRE, AE+MRE;
3. *Ad hoc* extensions of augmented error concept with DRE and MRE to unstable LTI systems, delayed and MIMO LTI systems, nonlinear systems.

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2. Kreisselmeier G. Adaptive observers with exponential rate of convergence. TAC, 22(1), 2-8, 1977.

3. Ortega R., Nikiforov V., and Gerasimov D. On modified parameter estimators for identification and adaptive control: a unified framework and some new schemes, ARC, 50, 278-293., 2020.

1.2. The concept of augmented error

Dynamic Regressor Extension

Main idea of the tutorial

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$$\tilde{\varepsilon}_k = \varepsilon - kW_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right].$$

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Memory Regressor Extension

Main idea of the tutorial

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$$\tilde{\varepsilon}_k = \varepsilon - kW_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right],$$

Main contributions:

1. Improvement of output (tracking) error convergence for problems of adaptive control using augmented error: AE+DRE, AE+MRE;
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2. Augmented error and improved parametric convergence

TRANSFER FUNCTION SWAPPING

2.1. Schemes for known control gain k

Augmented error:

$$\tilde{\varepsilon}_k = \varepsilon + \varsigma_k. \tag{13}$$

Result of swapping:

$$\varepsilon = kW(s)\left[\tilde{\psi}^T \phi\right] \longrightarrow \tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \tag{14}$$

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (15)$$

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma \phi_k \tilde{\varepsilon}_k , \quad (16)$$

where $\gamma > 0$ is the adaptation gain.

Parametric error model:

$$\dot{\tilde{\psi}} = -\gamma \phi_k \phi_k^T \tilde{\psi} - \gamma \phi_k \sigma_k$$

Properties:

1. If $\phi_k \in PC$ and defined for all $t \geq 0$, then $\hat{\psi} \in L_\infty$ and $\tilde{\varepsilon}_k \in L_2$;
2. If $\phi_k , \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k , \hat{\psi} , \dot{\hat{\psi}} \in L_\infty$ and $\| \dot{\hat{\psi}}(t) \| , \tilde{\varepsilon}_k(t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_k \in PE$, then $\| \tilde{\psi}(t) \| \to 0$ exponentially fast and there exists an optimal γ , for which the rate of parametric convergence is maximum.

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \qquad (15)$$

Improved adaptation algorithm with DRE:

Dynamically extended error:

$$E_H = H(s)\left[\tilde{\varepsilon}_k + \phi_k^T \hat{\psi}\right] - \Phi_H^T \hat{\psi}, \qquad (17)$$

where $H(s) = \left[H_1(s), H_2(s), \ldots, H_{q-1}(s)\right]$ is the transfer function vector,

$\Phi_H^T = H(s)\left[\phi_k^T\right]$ is the dynamically extended regressor.

Dynamically extended error model:

$$E_H = \Phi_H^T \tilde{\psi} + \sigma_H . \qquad (18)$$

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \tag{15}$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma \Phi_H E_H , \tag{19}$$

where $\Phi_H^T = H(s)\left[\phi_k^T\right], \quad E_H = H(s)\left[\tilde{\varepsilon}_k + \phi_k^T \hat{\psi}\right] - \Phi_H^T \hat{\psi}$.

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2. If $\phi_k , \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k , \hat{\psi} , \dot{\hat{\psi}} \in L_\infty$ and $\| \dot{\hat{\psi}}(t) \| , \tilde{\varepsilon}_k (t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_k \in PE$, then $\| \tilde{\psi}(t) \| \to 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ ;
4. If $\lambda_H (t) \notin L_1$, where λ_H is the minimum eigenvalue of $\Phi_H \Phi_H^T$, then $\| \tilde{\psi}(t) \| \to 0$ asymptotically, and the rate of convergence can be increased by increasing γ .

2.1. Schemes for known control gain k

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Error model:

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Improved adaptation algorithm with DRE:

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2. If $\phi_k, \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \tilde{\varepsilon}_k(t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_k \in PE$, then $\|\tilde{\psi}(t)\| \to 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ;
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2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \qquad (15)$$

Improved adaptation algorithm with MRE:

Memory extended error:

$$E_L = L(s)\left[\phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi}\right] - \Phi_L \hat{\psi}, \qquad (20)$$

where $\Phi_L = L(s)\left[\phi_k \phi_k^T\right] \geq 0$ is the memory extended regressor,

$$L(s) = \prod_{i=1}^{p} \frac{1}{s+\beta_i} = \frac{1}{d(s)}, \quad \beta_i > 0. \qquad (21)$$

Memory extended error model:

$$E_L = \Phi_L^T \tilde{\psi} + \sigma_L. \qquad (22)$$

Nikiforov V. and Gerasimov D. , Adaptive regulation: reference tracking and disturbance rejection. Lecture Notes in CIS, Springer Nature, 2022

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Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \tag{15}$$

Improved adaptation algorithm with MRE:

$$\dot{\hat{\psi}} = \gamma E_L, \tag{23}$$

where $\Phi_L = L(s)\left[\phi_k \phi_k^T\right]$, $E_L = L(s)\left[\phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi}\right] - \Phi_L \hat{\psi}$.

Properties:

1. If $\phi_k \in PC$ and defined for all $t \geq 0$, then $\hat{\psi} \in L_\infty$ and $\tilde{\varepsilon}_k \in L_2$;
2. If $\phi_k, \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\| \dot{\hat{\psi}}(t) \|, \tilde{\varepsilon}_k(t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_k \in PE$, then $\| \tilde{\psi}(t) \| \to 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ;
4. If $\lambda_L(t) \notin L_1$, where λ_L is the minimum eigenvalue of Φ_L, then $\| \tilde{\psi}(t) \| \to 0$ asymptotically, and the rate of convergence can be increased by increasing γ.

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

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$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \tag{15}$$

Parametric error model:

$$\dot{\tilde{\psi}} = -\gamma \Phi_L \tilde{\psi} - \gamma \sigma_L \tag{23}$$

Improved adaptation algorithm with MRE:

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2.1. Schemes for known control gain k

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Properties:

5. High order tuner (Bonus Property)

$$\hat{\psi}^{(i+1)} = \gamma\left(Y_L^{(i)} - \sum_{j=0}^{p-1} C_j^i \Phi_L^{(i-j)} \hat{\psi}^{(j)}\right),\ i = 1, 2, \ldots, p, \qquad (24a)$$

$$Y_L^{(i)} = \frac{s^i}{d(s)}\left[\phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi}\right], \qquad \Phi_L^{(i-j)} = \frac{s^{i-j}}{d(s)}\left[\phi_k \phi_k^T\right].$$

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

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Improved adaptation algorithm with MRE:

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Properties:

5. High order tuner (Bonus Property)

$$\hat{\psi}^{(p+1)} + \left(d_{p-1} I_q + \gamma \Phi_L\right)\hat{\psi}^{(p)} + \left(d_{p-2} I_q + \gamma d_{p-1}\Phi_L + \gamma d_p C_{p-1}^{p} \dot{\Phi}_L\right)\hat{\psi}^{(p-1)} + \ldots + \left(d_0 I_q + \gamma \sum_{j=1}^{p} d_j C_1^j \Phi_L^{(j-1)}\right)\dot{\hat{\psi}} = \gamma \phi_k \tilde{\varepsilon}_k \tag{24b}$$

2.1. Schemes for known control gain k

STATE-SPACE SWAPPING

Error model (state-space):

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi, \quad e(0), \tag{24}$$

$$\varepsilon = c^T e, \tag{25}$$

where $e \in \mathrm{R}^n$ is the state error, (A, b, c) is the minimal realization of the transfer function, $W(s) = c^T (sI - A)^{-1} b,$ the matrix $A \in \mathrm{R}^{n \times n}$ is Hurwitz.

2.1. Schemes for known control gain k

STATE-SPACE SWAPPING

Error model (state-space):

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi \tag{26}$$

Filters:

$$\dot{\Omega} = A\Omega + kb\phi^T \tag{27}$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}} \tag{28}$$

Augmented error:

$$\tilde{e}_k = e + \omega \tag{29}$$

Result of swapping:

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi \longrightarrow \tilde{e}_k = \Omega\tilde{\psi} + \sigma \tag{30}$$

2.1. Schemes for known control gain k

STATE-SPACE SWAPPING

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma \Omega^T \tilde{e}_k \tag{31}$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma \Phi_H E_H , \tag{32}$$

where $\Phi_H^T = H(s)[\Omega]$, $E_H = H(s)\left[\tilde{e}_k + \Omega \hat{\psi}\right] - \Phi_H^T \hat{\psi}$.

Improved adaptation algorithm with MRE:

$$\dot{\hat{\psi}} = \gamma E_L , \tag{33}$$

where $\Phi_L = L(s)\left[\Omega^T \Omega\right]$, $E_L = L(s)\left[\Omega^T \tilde{e}_k + \Omega^T \Omega \hat{\psi}\right] - \Phi_L \hat{\psi}$.

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING

Augmentation signal:

$$\varsigma_u = W(s)\left[\hat{\psi}^T \phi\right] - \hat{\psi}^T \phi_u, \quad (34)$$

where $\phi_u = W(s)[\phi]$ is the filtered regressor.

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k}\varsigma_u, \quad (35)$$

where $\hat{k}$ is the additional adjustable parameter.

Result of swapping:

$$\varepsilon = kW(s)\left[\tilde{\psi}^T \phi\right] \longrightarrow \tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \quad (36)$$

where $\tilde{k} = k - \hat{k}$ is the additional parametric error.

Narendra K. and Annaswamy A. Stable Adaptive Systems. Prentice Hall, New Jersey, 1989.

2.2. Schemes for unknown control gain k

Corollary of the results

Augmentation signal:

$$\varsigma_u = -W_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right], \qquad (37)$$

where $W_C(s) = c^T\left(sI - A\right)^{-1}$, $W_B(s) = \left(sI - A\right)^{-1} b$ are the transfer function vectors.

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon - \hat{k}W_C(s)\left[W_B(s)\left[\phi^T\right]\dot{\hat{\psi}}\right].$$

Augmented error

Control error

Proportional to the time derivative $\dot{\hat{\psi}}$ and $\hat{k}$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \qquad (38)$$

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma_1 \phi_u \tilde{\varepsilon}_u, \qquad (39)$$

$$\dot{\hat{k}} = -\gamma_2 \varsigma_u \tilde{\varepsilon}_u, \qquad (40)$$

where $\gamma_1, \gamma_2 > 0$ are the adaptation gains.

Parametric error model:

$$\begin{bmatrix} \dot{\tilde{\psi}} \\ \dot{\tilde{k}} \end{bmatrix} = -\gamma \begin{bmatrix} \phi_u \phi_u^T & -\phi_u \varsigma_u \\ -\varsigma_u \phi_u^T & \varsigma_u^2 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} - \gamma \begin{bmatrix} \phi_u \\ \varsigma_u \end{bmatrix} \sigma_u$$

$$\gamma = diag\{\gamma_1, \gamma_2\}$$

Properties:

1. If $\phi_u \in PC$ and defined for all $t \ge 0$, then $\hat{\psi}, \hat{k} \in L_\infty$ and $\tilde{\varepsilon}_u \in L_2$;
2. If $\phi_u, \dot{\phi}_u \in L_\infty$, then $\tilde{\varepsilon}_u, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\| \dot{\hat{\psi}}(t) \|, \tilde{\varepsilon}_u(t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_u \in PE$, then $\| \tilde{\psi}(t) \| \to 0$ exponentially fast and there exists an optimal γ , for which the rate of parametric convergence is maximum.

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

Improved adaptation algorithm with DRE:

Dynamically extended error:

$$\bar{E}_H = \bar{H}(s)\left[\tilde{\varepsilon}_u - \hat{k}\varsigma_u\right] + \hat{k}\, Z_H, \tag{41}$$

where $\bar{H}(s) = \left[H_1(s), H_2(s), \ldots, H_q(s)\right]$ is the transfer function vector,

$$\varsigma_u = W(s)\left[\hat{\psi}^T \phi\right] - \hat{\psi}^T \phi_u,$$

$$Z_H = \bar{H}(s)\left[W(s)\left[\hat{\psi}^T \phi\right]\right] - \Phi_H^T \hat{\psi}, \qquad \Phi_H^T = \bar{H}(s)\left[\phi_u\right].$$

Dynamically extended error model:

$$\bar{E}_H = k\Phi_H^T \tilde{\psi} - \tilde{k} Z_H + \bar{\sigma}_H. \tag{42}$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T\phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \qquad \gamma_1 > 0 \tag{43}$$

$$\dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \qquad \gamma_2 > 0 \tag{44}$$

Parametric error model:

$$\begin{bmatrix} \dot{\tilde{\psi}} \\ \dot{\tilde{k}} \end{bmatrix} = -\gamma\Omega_H \begin{bmatrix} kI_q & O_q \\ O_{1\times q} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} - \gamma \begin{bmatrix} \Phi_H \\ Z_H^T \end{bmatrix} \bar{\sigma}_H,$$

$$\Omega_H = \begin{bmatrix} \Phi_H \Phi_H^T & -\Phi_H Z_H \\ -Z_H^T \Phi_H^T & Z_H^T Z_H \end{bmatrix}, \quad \gamma = diag\{\gamma_1, \gamma_2\}$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

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$$\dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \qquad \gamma_2 > 0 \tag{44}$$

Properties:

1. If $\phi_u \in PC$ and defined for all $t \geq 0$, then $\hat{\psi}, \hat{k} \in L_\infty$ and $\tilde{\varepsilon}_u \in L_2$;
2. If $\phi_u, \dot{\phi}_u \in L_\infty$, then $\tilde{\varepsilon}_u, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\| \dot{\hat{\psi}}(t) \|, \tilde{\varepsilon}_u(t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_u \in PE$, then $\| \tilde{\psi}(t) \| \to 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ_1, γ_2 ;
4. If $\bar{\lambda}_H(t) \notin L_1$, where $\bar{\lambda}_H$ is the minimum eigenvalue of Ω_H then $\tilde{\psi}(t), \tilde{k}(t) \to 0$ asymptotically, and the rate of convergence can be increased by increasing γ_1, γ_2.

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

Improved adaptation algorithm with MRE:

Filter:

$$L(s) = \prod_{i=1}^{p} \frac{1}{s+\beta_i} = \frac{1}{d(s)}, \quad \beta_i > 0. \tag{45}$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

Improved adaptation algorithm with MRE:

Memory extended error:

$$\bar{E}_L = \begin{bmatrix} L(s)\left[\phi_u\left(\tilde{\varepsilon}_u - \hat{k}\varsigma_u\right)\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T\phi\right]\left(\tilde{\varepsilon}_u - \hat{k}\varsigma_u\right)\right] \end{bmatrix} + \hat{k}\, Z_L, \tag{46}$$

where $\varsigma_u = W(s)\left[\hat{\psi}^T\phi\right] - \hat{\psi}^T\phi_u,$

$$Z_L = \begin{bmatrix} L(s)\left[\phi_u W(s)\left[\hat{\psi}^T\phi\right]\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T\phi\right]^2\right] \end{bmatrix} - \begin{bmatrix} L(s)\left[\phi_u\phi_u^T\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T\phi\right]\phi_u^T\right] \end{bmatrix}\hat{\psi}.$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

Improved adaptation algorithm with MRE:

Memory extended error model:

$$\bar{E}_L = \Omega_L \Psi \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} + \bar{\sigma}_L, \tag{47}$$

where

$$\Omega_L = \begin{bmatrix} L(s)\left[\phi_u \phi_u^T\right] & L(s)\left[\phi_u W(s)\left[\hat{\psi}^T \phi\right]\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T \phi\right]\phi_u^T\right] & L(s)\left[W(s)\left[\hat{\psi}^T \phi\right]^2\right] \end{bmatrix}, \quad \Psi = \begin{bmatrix} kI_q & \hat{\psi} \\ O_{1\times q} & -1 \end{bmatrix}.$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \qquad (38)$$

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Memory extended error model:

$$\bar{E}_L = \Omega_L \Psi \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} + \bar{\sigma}_L, \qquad (47)$$

where

$$\Omega_L = \begin{bmatrix} L(s)\left[\phi_u \phi_u^T\right] & L(s)\left[\phi_u W(s)\left[\hat{\psi}^T \phi\right]\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T \phi\right]\phi_u^T\right] & L(s)\left[W(s)\left[\hat{\psi}^T \phi\right]^2\right] \end{bmatrix} \geq 0, \quad \Psi = \begin{bmatrix} kI_q \geq 0 & \hat{\psi} \\ O_{1\times q} & -1 \end{bmatrix}.$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \qquad (38)$$

Improved adaptation algorithm with MRE:

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \hat{\Psi}^T \bar{E}_L, \qquad \gamma = diag\{\gamma_1, \gamma_2\} > 0, \qquad (48)$$

Parametric error model:

$$\begin{bmatrix} \dot{\tilde{\psi}} \\ \dot{\tilde{k}} \end{bmatrix} = -\gamma \hat{\Psi}^T \Omega_L \Psi \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} - \gamma \hat{\Psi}^T \bar{\sigma}_L,$$

$$\hat{\Psi} = \begin{bmatrix} I_q & \hat{\psi} \\ O_{1\times q} & -1 \end{bmatrix}, \; \Psi = \begin{bmatrix} kI_q & \hat{\psi} \\ O_{1\times q} & -1 \end{bmatrix}$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\varsigma_u + \sigma_u \tag{38}$$

Improved adaptation algorithm with MRE:

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Properties:

1. If $\phi_u \in PC$ and defined for all $t \geq 0$, then $\hat{\psi}, \hat{k} \in L_\infty$ and $\tilde{\varepsilon}_u \in L_2$;
2. If $\phi_u, \dot{\phi}_u \in L_\infty$, then $\tilde{\varepsilon}_u, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\| \dot{\hat{\psi}}(t) \|, \tilde{\varepsilon}_u(t), \varepsilon(t) \to 0$ as $t \to \infty$;
3. If $\phi_u \in PE$, then $\| \tilde{\psi}(t) \| \to 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ_1, γ_2 ;
4. If $\bar{\lambda}_\Omega(t) \notin L_1$, where $\bar{\lambda}_\Omega$ is the minimum eigenvalue of $\hat{\Psi}^T \Omega_L \hat{\Psi}$ then $\tilde{\psi}(t), \tilde{k}(t) \to 0$ asymptotically, and the rate of convergence can be increased by increasing γ_1, γ_2 .

2.2. Schemes for unknown control gain *k*

Gradient AA:

AA with DRE:

AA with MRE:

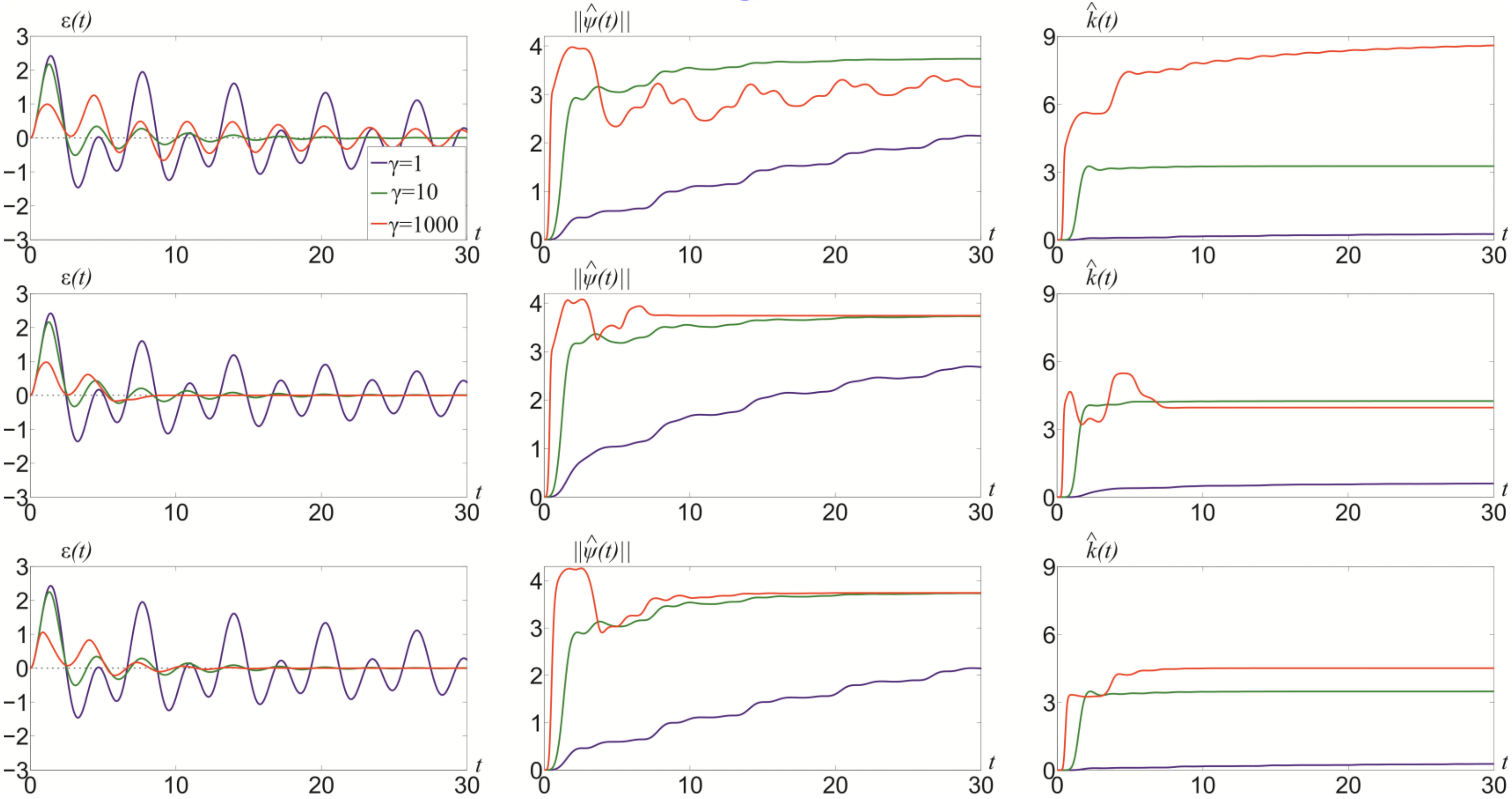


Fig. 1. Identification results in different adaptation algorithms with different gains $\gamma = \gamma_1 = \gamma_2$

2.2. Schemes for unknown control gain k

Gradient AA:

AA with DRE:

AA with MRE:

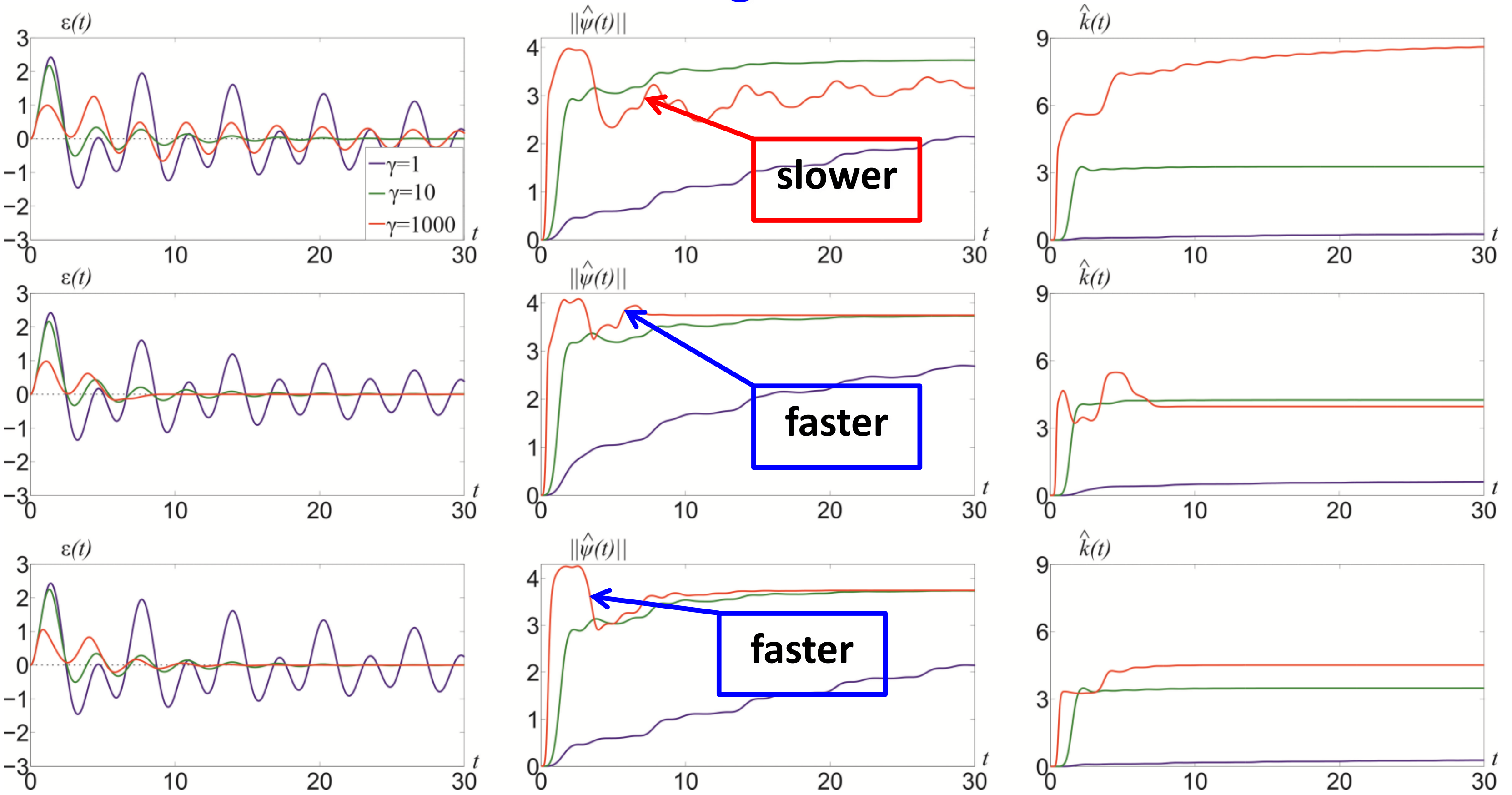


Fig. 1. Identification results in different adaptation algorithms with different gains $\gamma = \gamma_1 = \gamma_2$

2.2. Schemes for unknown control gain *k*

STATE-SPACE SWAPPING

Error model:

$$\dot{e} = Ae + kb\tilde{\psi}^T\phi \tag{49}$$

Filters:

$$\dot{\Omega} = A\Omega + b\phi^T\hat{\psi} \qquad \dot{\bar{\Omega}} = A\bar{\Omega} + b\phi^T, \tag{50}$$

$$\dot{w} = Aw - \hat{k}\bar{\Omega}\dot{\hat{\psi}} + \dot{\hat{k}}\bar{w}, \qquad \dot{\bar{w}} = A\bar{w} - \bar{\Omega}\dot{\hat{\psi}} \tag{51}$$

Augmented error:

$$\tilde{e}_u = e + w \tag{52}$$

Result of swapping:

$$\dot{e} = Ae + kb\tilde{\psi}^T\phi \longrightarrow \tilde{e}_u = k\bar{\Omega}\tilde{\psi} - \tilde{k}\bar{w} + \bar{\sigma}, \tag{53}$$

where $\tilde{k} = k - \hat{k}$ is the parametric error.

2.2. Schemes for unknown control gain k

STATE-SPACE SWAPPING

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma_1 \bar{\Omega}^T \tilde{e}_u, \qquad \dot{\hat{k}} = -\gamma_2 \bar{w} \tilde{e}_u \tag{54}$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \qquad \dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \tag{55}$$

where $\bar{E}_H = \bar{H}(s)\left[\tilde{e}_u - \hat{k}\bar{w}\right] + \hat{k} Z_H, \quad Z_H = \bar{H}(s)[\Omega] - \Phi_H^T \hat{\psi}, \quad \Phi_H^T = \bar{H}(s)\left[\bar{\Omega}^T\right],$

$\bar{H}(s) = \left[H_1(s), H_2(s), \ldots, H_q(s)\right]$

2.2. Schemes for unknown control gain k

STATE-SPACE SWAPPING

Improved adaptation algorithm with MRE:

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \hat{\Psi}^T \bar{E}_L, \tag{56}$$

where $\gamma = diag\left\{\gamma_1, \gamma_2\right\}, \quad \hat{\Psi} = \begin{bmatrix} I_q & \hat{\psi} \\ O_{1\times q} & -1 \end{bmatrix},$

$$\bar{E}_L = \begin{bmatrix} L(s)\left[\bar{\Omega}^T\left(\tilde{e}_u - \hat{k}\bar{w}\right)\right] \\ L(s)\left[\Omega^T\left(\tilde{e}_u - \hat{k}\bar{w}\right)\right] \end{bmatrix} + \hat{k}\, Z_L, \quad Z_L = \begin{bmatrix} L(s)\left[\bar{\Omega}^T \Omega\right] \\ L(s)\left[\Omega^T \Omega\right] \end{bmatrix} - \begin{bmatrix} L(s)\left[\bar{\Omega}^T \bar{\Omega}\right] \\ L(s)\left[\Omega^T \bar{\Omega}\right] \end{bmatrix} \hat{\psi}.$$

2.2. Schemes for unknown control gain (example)

Example 1. Model reference adaptive control

Plant:

$$y = \frac{k}{s^2 + a_1 s + a_0}[u],$$

where $k = 5,\ a_0 = 1,\ a_1 = 2$ are the unknown parameters,

Reference model:

$$y_m = \frac{20}{s^2 + 9s + 20}[g],$$

where $g(t) = \sin(t) + 2.$

Objective:

$$\lim_{t \to \infty} \varepsilon(t) = \lim_{t \to \infty} \left(y_m(t) - y(t) \right) = 0.$$

2.2. Schemes for unknown control gain (example)

Filters:

$$\dot{\nu}_1 = -5\nu_1 + u, \quad \dot{\nu}_2 = -5\nu_2 + y$$

forming $\phi = [y,\, \nu_1,\, \nu_2,\, g]^T$.

Control:

$$u = \hat{\psi}^T \phi$$

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k}\varsigma_u, \qquad \varsigma_u = W(s)\left[\hat{\psi}^T \phi\right] - \hat{\psi}^T \phi_u, \qquad \phi_u = W(s)[\phi]$$

Algorithm of adaptation (gradient):

$$\dot{\hat{\psi}} = \gamma_1 \frac{\phi_u}{r^2}\tilde{\varepsilon}_u, \quad \dot{\hat{k}} = -\gamma_2 \frac{\varsigma_u}{r^2}\tilde{\varepsilon}_u, \qquad \gamma_1, \gamma_2 > 0,$$

where $r = \sqrt{1 + \phi_u^T \phi_u}$ is the normalizing divider.

2.2. Schemes for unknown control gain (example)

Filters:

$$\dot{\nu}_1 = -5\nu_1 + u, \quad \dot{\nu}_2 = -5\nu_2 + y$$

forming $\phi = [y,\, \nu_1,\, \nu_2,\, g]^T$.

Control:

$$u = \hat{\psi}^T \phi$$

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k}\varsigma_u, \qquad \varsigma_u = W(s)\left[\hat{\psi}^T\phi\right] - \hat{\psi}^T\phi_u, \qquad \phi_u = W(s)[\phi]$$

Algorithm of adaptation (improved with DRE):

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \quad \dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \quad \gamma_1, \gamma_2 > 0,$$

$$\bar{E}_H = \bar{H}(s)\left[\frac{\tilde{\varepsilon}_u - \hat{k}\varsigma_u}{r}\right] + \hat{k} Z_H, \; Z_H = \bar{H}(s)\left[\frac{W(s)\left[\hat{\psi}^T\phi\right]}{r}\right] - \Phi_H^T\hat{\psi}, \; \Phi_H^T = \bar{H}(s)\left[\frac{\phi_u}{r}\right].$$

2.2. Schemes for unknown control gain (example)

Filters:

$$\dot{\nu}_1 = -5\nu_1 + u, \quad \dot{\nu}_2 = -5\nu_2 + y$$

forming $\phi = [y,\ \nu_1,\ \nu_2,\ g]^T$.

Control:

$$u = \hat{\psi}^T \phi$$

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k}\varsigma_u, \qquad \varsigma_u = W(s)\left[\hat{\psi}^T\phi\right] - \hat{\psi}^T\phi_u, \qquad \phi_u = W(s)[\phi]$$

Algorithm of adaptation (improved with MRE):

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \begin{bmatrix} I_q & O_q \\ \hat{\psi}^T & -1 \end{bmatrix} \bar{E}_L, \qquad \gamma = diag\{\gamma_1, \gamma_2\} > 0,$$

$$\bar{E}_L = \begin{bmatrix} L(s)\left[\phi_u\left(\tilde{\varepsilon}_u - \hat{k}\varsigma_u\right)/r^2\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T\phi\right]\left(\tilde{\varepsilon}_u - \hat{k}\varsigma_u\right)/r^2\right] \end{bmatrix} + \hat{k}Z_L, \quad Z_L = \begin{bmatrix} L(s)\left[\phi_u W(s)\left[\hat{\psi}^T\phi\right]/r^2\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T\phi\right]^2/r^2\right] \end{bmatrix} - \begin{bmatrix} L(s)\left[\phi_u\phi_u^T/r^2\right] \\ L(s)\left[W(s)\left[\hat{\psi}^T\phi\right]\phi_u^T/r^2\right] \end{bmatrix}\hat{\psi}.$$

2.2. Schemes for unknown control gain (example)

Simulation results:

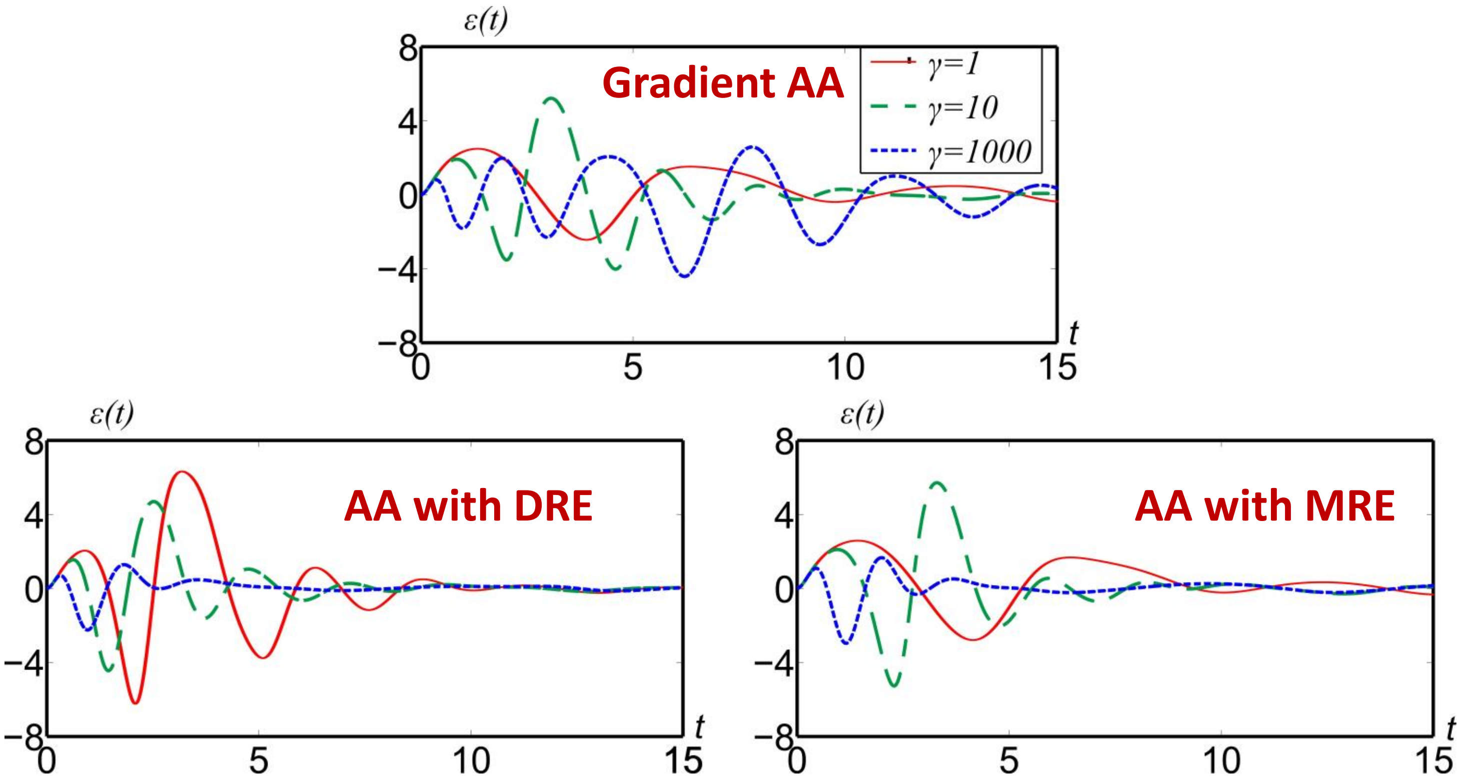
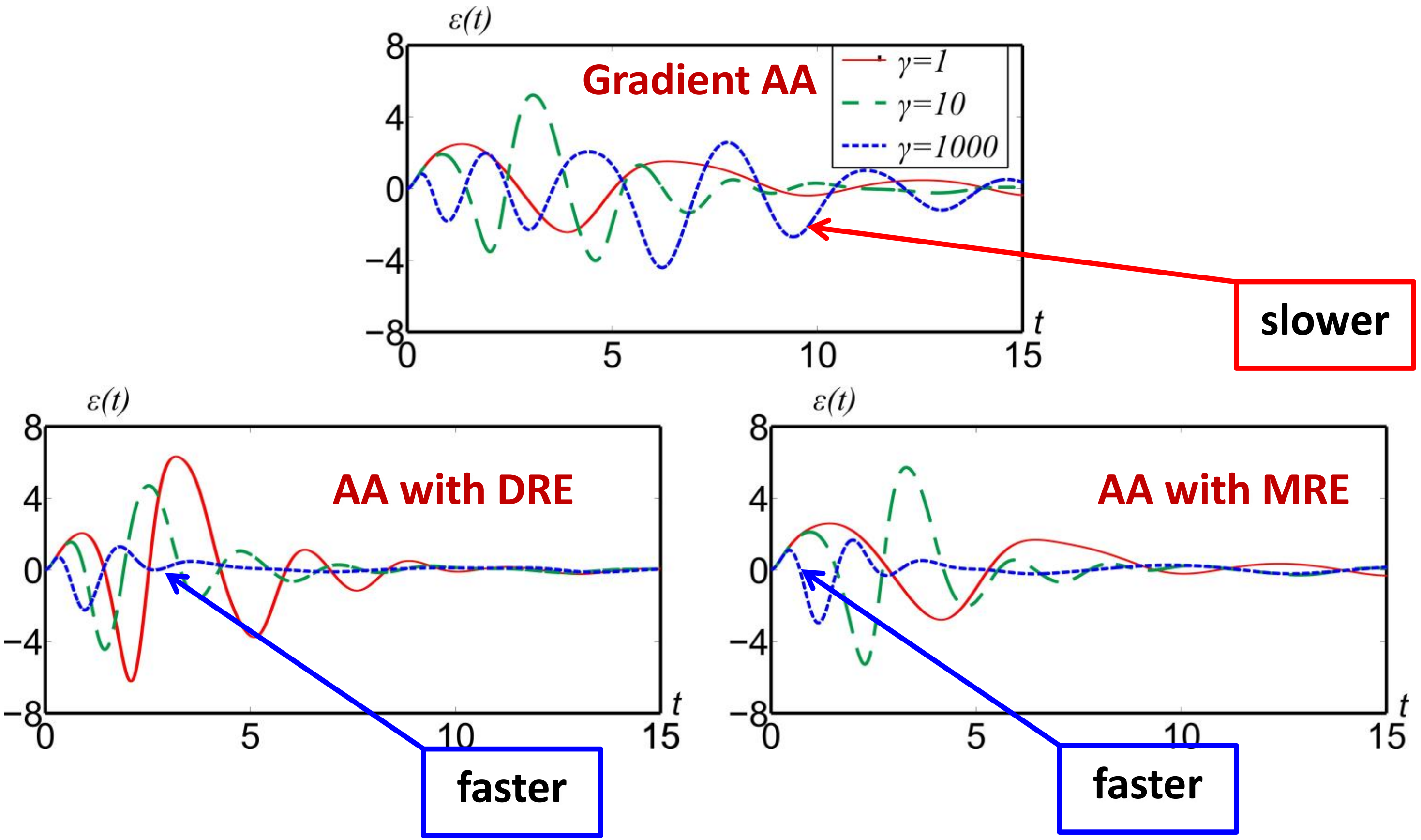


Fig. 2. Evolutions of tracking errors in the systems closed by different adaptation algorithms

2.2. Schemes for unknown control gain (example)

Simulation results:



3. Development of the concept and *adhoc* modifications of augmented error

3.1. Swapping for unstable systems

Error model (state-space):

$$\dot{e} = Ae + b\tilde{\psi}^T \phi, \quad e(0), \tag{57}$$

$$\varepsilon = c^T e, \tag{58}$$

where A is not Hurwitz, $A, b,$ and c are known.

Filters:

$$\dot{\Omega} = A_e \Omega + b\phi^T, \tag{59}$$

$$\dot{\omega} = A_e \omega - \Omega \dot{\hat{\psi}} - l_e c^T e \tag{60}$$

where the vector of feedback gains l_e is selected so that $A_e = A - l_e c^T$ is Hurwitz.

Nikiforov V.O., Paramonov A.V., Gerasimov D.N. Adaptive Control Algorithms in MIMO Linear Systems with Control Delay, Automation & Remote Control, 81 (6), 1091-1106, 2020.

3.1. Swapping for unstable systems

Augmented errors:

$$\tilde{e} = e + \omega, \qquad \tilde{\varepsilon} = \varepsilon + c^T \omega \tag{61}$$

Result of swapping:

$$\dot{e} = Ae + b\tilde{\psi}^T \phi, \qquad \tilde{e} = \Omega \tilde{\psi} + \sigma_e, \tag{62}$$

$$\varepsilon = c^T e \qquad \longrightarrow \qquad \tilde{\varepsilon} = c^T \Omega \tilde{\psi} + c^T \sigma_e \tag{63}$$

Nikiforov V.O., Paramonov A.V., Gerasimov D.N. Adaptive Control Algorithms in MIMO Linear Systems with Control Delay, Automation & Remote Control, 81 (6), 1091-1106, 2020.

3.2. Swapping for systems with input delay

Error model (state-space):

$$\dot{e} = Ae + b\tilde{\psi}^T(t-\tau)\phi, \tag{64}$$

$$\varepsilon = c^T e, \tag{65}$$

where A is Hurwitz, A, b, and c are known, τ is delay.

Filters:

$$\dot{\Omega} = A\Omega + b\phi^T, \tag{66}$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}} - b\phi^T\left(\hat{\psi}(t-\tau) - \hat{\psi}(t)\right) \tag{67}$$

Augmented errors:

$$\tilde{e} = e + \omega, \qquad \tilde{\varepsilon} = \varepsilon + c^T\omega \tag{68}$$

Gerasimov D., Paramonov A., and Nikiforov V., Algorithms of fast adaptive compensation of disturbance in linear systems with arbitrary input delay. IJC, 93(7), 1596-1604, 2020

3.2. Swapping for systems with input delay

Result of swapping:

$$\dot{e} = Ae + b\tilde{\psi}^T(t-\tau)\phi, \qquad \tilde{e} = \Omega\tilde{\psi}(t) + \sigma_e, \tag{69}$$

$$\varepsilon = c^T e \qquad\longrightarrow\qquad \tilde{\varepsilon} = c^T\Omega\tilde{\psi}(t) + c^T\sigma_\tau \tag{70}$$

Gerasimov D., Paramonov A., and Nikiforov V., Algorithms of fast adaptive compensation of disturbance in linear systems with arbitrary input delay. IJC, 93(7), 1596-1604, 2020

3.2. Swapping for systems with input delay (example)

Example 2. Disturbance compensation in delayed systems

Plant:

$$\dot{x} = Ax + b\big(u(t-\tau) + \delta\big), \tag{71}$$

$$y = c^T x, \tag{72}$$

where $A, b,$ and c are known, τ is delay. State x is measurable,

$$\delta(t) = a_0 + \sum_{i=1}^{p} a_i \sin(\omega_i t + \varphi_i)$$

is the inaccessible for measurement disturbance with unknown a_0 , a_i, ω_i , and φ_i but known maximum number of harmonics p .

Objective:

$$\lim_{t\to\infty} y(t) = 0. \tag{73}$$

Gerasimov D., Paramonov A., and Nikiforov V., Algorithms of fast adaptive compensation of disturbance in linear systems with arbitrary input delay. IJC, 93(7), 1596-1604, 2020

3.2. Swapping for systems with input delay (example)

Internal model of the disturbance:

$$\dot{z} = \Gamma z, \quad z(0), \tag{74}$$

$$\delta = h^T z, \tag{75}$$

where $\Gamma \in \mathrm{R}^{m\times m}$ is an unknown matrix with simple eigenvalues with zero real parts, h is an unknown m-dimensional vector, z is the inaccessible for measurement state.

Francis D. A. and Wonham W. N.,The internal model principle for linear multivariable regulators, AMO, 2(4), 170-194, 1975.

3.2. Swapping for systems with input delay (example)

Internal model of the disturbance:

$$\dot{z} = \Gamma z, \quad z(0), \tag{74}$$

$$\delta = h^T z, \tag{75}$$

where $\Gamma \in R^{m \times m}$ is an unknown matrix with simple eigenvalues with zero real parts, h is an unknown m-dimensional vector, z is the inaccessible for measurement state.

Assumptions:

- the plant is controllable;
- the pair (Γ, h) is observable;
- the dimension m (corresponding to the maximum number of harmonics) is known.

Francis D. A. and Wonham W. N.,The internal model principle for linear multivariable regulators, AMO, 2(4), 170-194, 1975.

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \tag{76}$$

$$\delta = \theta^T \xi, \tag{77}$$

where $G \in \mathrm{R}^{m\times m}$ is a Hurwitz matrix, l is a vector selected so that the pair (G, l) is controllable, θ is a vector of unknown parameters.

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \qquad (76)$$

$$\delta = \theta^T \xi, \qquad (77)$$

where $G \in \mathrm{R}^{m\times m}$ is a Hurwitz matrix, l is a vector selected so that the pair (G, l) is controllable, θ is a vector of unknown parameters.

Autonomous model:

$$\dot{\xi} = \left(G + l\theta^T\right)\xi \qquad (78)$$

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \tag{76}$$

$$\delta = \theta^T \xi, \tag{77}$$

where $G \in \mathrm{R}^{m\times m}$ is a Hurwitz matrix, l is a vector selected so that the pair (G, l) is controllable, θ is a vector of unknown parameters.

Autonomous model:

$$\dot{\xi} = \left(G + l\theta^T\right)\xi \tag{78}$$

$$\xi(t) = e^{(G+l\theta^T)t}\xi(t_0) \Rightarrow \xi(t+\tau) = e^{(G+l\theta^T)\tau}\xi(t)$$

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

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Autonomous model:

$$\dot{\xi} = \left(G + l\theta^T\right)\xi \tag{78}$$

$$\xi(t) = e^{(G+l\theta^T)t}\xi(t_0) \Rightarrow \xi(t) = e^{(G+l\theta^T)\tau}\xi(t-\tau)$$

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

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Canonical form of disturbance model:

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Autonomous model:

$$\dot{\xi} = \left(G + l\theta^T\right)\xi \tag{78}$$

New parameterization:

$$\delta(t) = \theta^T e^{(G+l\theta^T)\tau} \xi(t - \tau) \tag{79}$$

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \tag{76}$$

$$\delta = \theta^T \xi, \tag{77}$$

where $G \in \mathrm{R}^{m\times m}$ is a Hurwitz matrix, l is a vector selected so that the pair (G, l) is controllable, θ is a vector of unknown parameters.

Autonomous model:

$$\dot{\xi} = \left(G + l\theta^T\right)\xi \tag{78}$$

New parameterization:

$$\delta(t) = \psi^T \xi(t-\tau), \tag{79}$$

where $\psi = e^{(G^T + \theta l^T)\tau}\theta$ is the new vector of unknown parameters.

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

3.2. Swapping for systems with input delay (example)

Plant with parameterized disturbance:

$$\dot{x} = Ax + b\left(u(t-\tau) + \psi^T \xi(t-\tau)\right) \tag{80}$$

Exponentially stable disturbance observer:

$$\hat{\xi} = \eta + Nx, \tag{81}$$

$$\dot{\eta} = G\eta + (GN - NA)x - bu(t-\tau), \tag{82}$$

where $\hat{\xi}$ is the estimate of ξ , $N \in \mathrm{R}^{m\times n}$ is the matrix selected so that

$$Nb = l. \tag{83}$$

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

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Plant with parameterized disturbance:

$$\dot{x} = Ax + b\left(u(t-\tau) + \psi^T \hat{\xi}(t-\tau)\right) \tag{84}$$

Nikiforov V., Adaptive servocompensation of input disturbances. Proc. 13th IFAC World Congress, San-Francisco, 175-180, 1996.

3.2. Swapping for systems with input delay (example)

Certainty equivalent control:

$$\dot{x} = Ax + b\left(u(t-\tau) + \psi^T \hat{\xi}(t-\tau)\right) \qquad (85)$$

$$u = -\hat{\psi}^T \hat{\xi},$$

where $\hat{\psi} \in R^m$ is the vector of adjustable parameter.

3.2. Swapping for systems with input delay (example)

Certainty equivalent control:

$$\dot{x} = Ax + b\left(u(t-\tau) + \psi^T \hat{\xi}(t-\tau)\right)$$

$$u = -\hat{\psi}^T \hat{\xi}, \tag{85}$$

where $\hat{\psi} \in R^m$ is the vector of adjustable parameter.

Error model:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T (t-\tau)\hat{\xi}(t-\tau)\right), \tag{86}$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

3.2. Swapping for systems with input delay (example)

Certainty equivalent control:

$$u = -\hat{\psi}^T \hat{\xi}, \tag{85}$$

where $\hat{\psi} \in R^m$ is the vector of adjustable parameter.

Error model:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T (t-\tau)\hat{\xi}(t-\tau)\right), \tag{86}$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

$$\cancel{\dot{\hat{\psi}}(t-\tau) = \gamma\hat{\xi}(t-\tau)b^T Px}$$

3.2. Swapping for systems with input delay (example)

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right)$$

Filters:

$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t-\tau), \tag{87}$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}} - b\hat{\xi}^T(t-\tau)\left(\hat{\psi}(t-\tau) - \hat{\psi}\right) \tag{88}$$

Augmented error:

$$\tilde{x} = x + \omega \tag{89}$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \tag{90}$$

3.2. Swapping for systems with input delay (example)

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Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \tag{90}$$

Algorithm of adaptation (gradient):

$$\dot{\hat{\psi}} = \gamma\Omega^T\tilde{x}, \quad \gamma > 0 \tag{91}$$

3.2. Swapping for systems with input delay (example)

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right)$$

Filters:

$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t-\tau), \tag{87}$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}} - b\hat{\xi}^T(t-\tau)\left(\hat{\psi}(t-\tau) - \hat{\psi}\right) \tag{88}$$

Augmented error:

$$\tilde{x} = x + \omega \tag{89}$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \tag{90}$$

Algorithm of adaptation (improved with DRE):

$$\dot{\hat{\psi}} = \gamma\Phi_H E_H, \tag{92}$$

where $\Phi_H^T = H(s)[\Omega]$, $E_H = H(s)[\tilde{x} + \Omega\hat{\psi}] - \Phi_H^T\hat{\psi}$.

3.2. Swapping for systems with input delay (example)

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right)$$

Filters:

$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t-\tau), \tag{87}$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}} - b\hat{\xi}^T(t-\tau)\left(\hat{\psi}(t-\tau) - \hat{\psi}\right) \tag{88}$$

Augmented error:

$$\tilde{x} = x + \omega \tag{89}$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \tag{90}$$

Algorithm of adaptation (improved with MRE):

$$\dot{\hat{\psi}} = \gamma E_L, \tag{93}$$

where $\Phi_L = L(s)\left[\Omega^T\Omega\right]$, $E_L = L(s)\left[\Omega^T\tilde{x} + \Omega^T\Omega\hat{\psi}\right] - \Phi_L\hat{\psi}$.

3.2. Swapping for systems with input delay (example)

Simulation results:

$$\dot{x} = Ax + b\big(u(t-\tau) + \delta\big), \qquad A = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -5 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \; c = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix},$$

$$y = c^T x,$$

where $\delta(t) = a\sin(\omega t + \varphi)$ with unknown $a = 2,\ \omega = 1,\ \varphi = 1$; $\tau = 1\sec$ is known. For the observer design, the matrices are selected as follows:

$$G = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}, \; l = \begin{bmatrix} 0 \\ 20 \end{bmatrix}, \; N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 20 \end{bmatrix}.$$

3.2. Swapping for systems with input delay (example)

Simulation results:

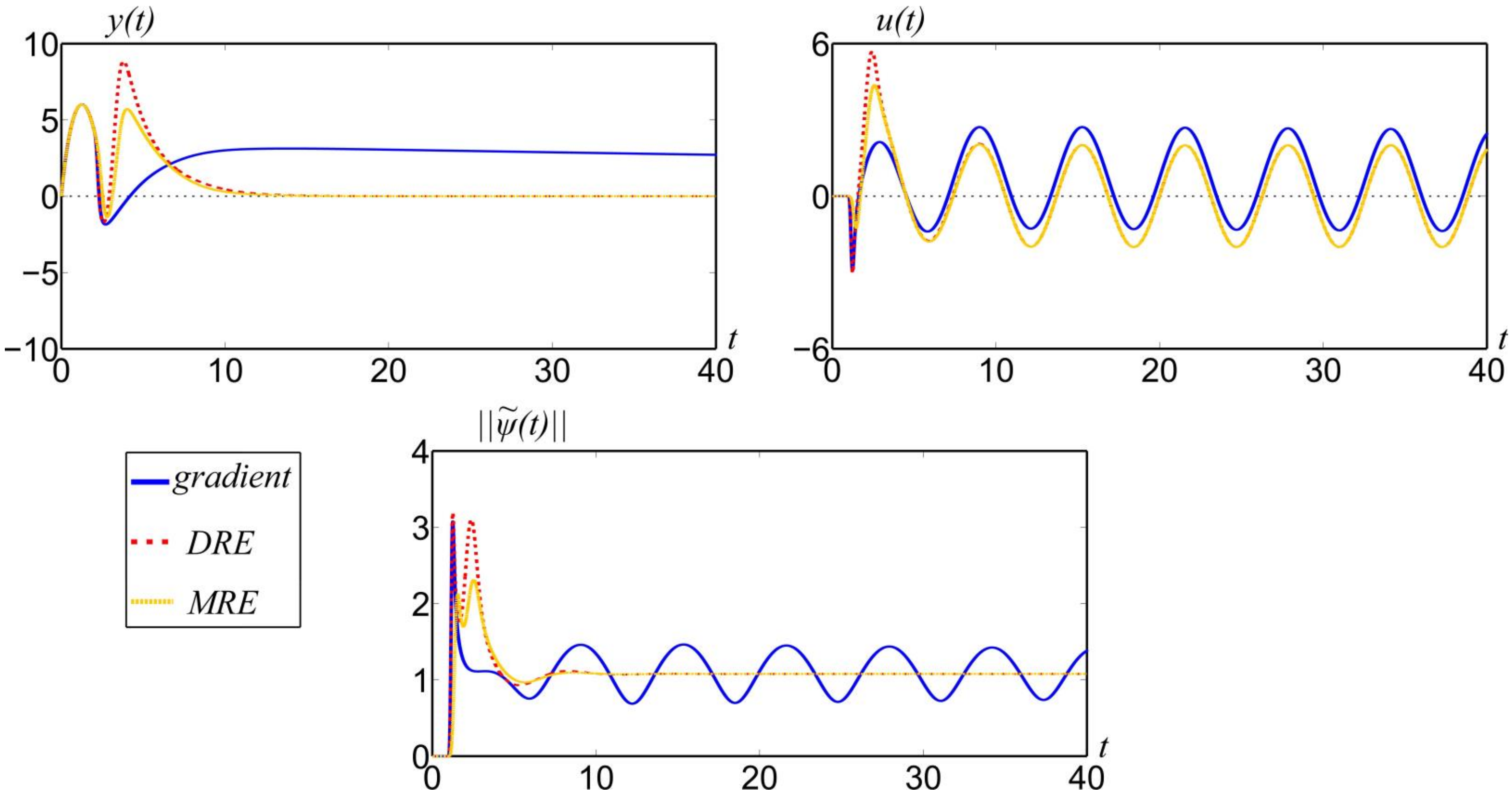

Fig. 3. Adaptive compensation of sinusoidal disturbance in delayed system with $\gamma = 5$.

3.3. Swapping for systems with state and input delays

Error model (state-space):

$$\dot{e} = A_0 e + \sum_{i=1}^{r} A_i e(t-\tau_{xi}) + \sum_{j=1}^{p} b_j \tilde{\psi}^T (t-\tau_{uj})\phi_j, \tag{94}$$

$$\varepsilon = c^T e, \tag{95}$$

where A_0 , A_i , b_j, and c are known, τ_{xi}, τ_{uj} are known delays. The plant is asymptotically stable .

Filters:

$$\dot{\Omega} = A_0 \Omega + \sum_{i=1}^{r} A_i \Omega(t-\tau_{xi}) + \sum_{j=1}^{p} b_j \phi_j^T, \tag{96}$$

$$\dot{\omega} = A_0 \omega - \Omega \dot{\hat{\psi}} - \sum_{j=1}^{p} b_j \phi_j^T \left(\hat{\psi}(t-\tau_{uj}) - \hat{\psi} \right) \tag{97}$$

Augmented errors:

$$\tilde{e} = e + \omega, \qquad \tilde{\varepsilon} = \varepsilon + c^T \omega \tag{98}$$

Gerasimov D.N., Paramonov A.V., Nikiforov V.O. , Output Adaptive Compensation of Multi-Sinusoidal Disturbance in Linear Systems with Input and State Delays , ECC2020, 65-70, 2020.

3.3. Swapping for systems with state and input delays

Result of swapping:

$$\dot{e} = A_0 e + \sum_{i=1}^{r} A_i e(t-\tau_{xi}) + \sum_{j=1}^{p} b_j \tilde{\psi}^T(t-\tau_{uj})\phi_j ,$$

$$\varepsilon = c^T e$$

$$\longrightarrow$$

$$\tilde{e} = \Omega \tilde{\psi}(t) + \sigma_e , \qquad (99)$$

$$\tilde{\varepsilon} = c^T \Omega \tilde{\psi}(t) + c^T \sigma_\tau \qquad (100)$$

Gerasimov D.N., Paramonov A.V., Nikiforov V.O. , Output Adaptive Compensation of Multi-Sinusoidal Disturbance in Linear Systems with Input and State Delays , ECC2020, 65-70, 2020.

3.4. Nonlinear swapping

Error model:

$$\dot{e} = A(e,t)e + b(e,t)\tilde{\psi}^T \phi, \quad e(0), \tag{101}$$

$$\varepsilon = c^T(e,t)e, \tag{102}$$

where $A(e,t) \in \mathrm{R}^{n\times n}, b(e,t) \in \mathrm{R}^n$, and $c(e,t) \in \mathrm{R}^n$ are the nonlinear mappings, locally Lipschitz in e and bounded in t. The matrix $A(e,t)$ is such that the system

$$\dot{x} = A(e,t)x \tag{103}$$

is exponentially stable.

M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, Nonlinear and adaptive control design. New York: Wiley, 1995.

3.4. Nonlinear swapping

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$$\dot{x} = A(e,t)x \tag{103}$$

is exponentially stable.

Filters:

$$\dot{\Omega} = A(e,t)\Omega + b(e,t)\phi^T, \tag{104}$$

$$\dot{\omega} = A(e,t)\omega - \Omega\dot{\hat{\psi}} - b(e,t)\phi^T\hat{\psi} \tag{105}$$

M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, Nonlinear and adaptive control design. New York: Wiley, 1995.

3.4. Nonlinear swapping

Augmented errors:

$$\tilde{e} = e + \omega, \qquad \tilde{\varepsilon} = \varepsilon + c^T(e,t)\omega \tag{106}$$

Result of swapping:

$$\begin{aligned} \dot{e} &= A(e,t)e + b(e,t)\tilde{\psi}^T \phi, \\ \varepsilon &= c^T(e,t)e, \end{aligned} \longrightarrow \begin{aligned} \tilde{e} &= \Omega\tilde{\psi} + \sigma_n, && (107) \\ \tilde{\varepsilon} &= c^T(e,t)\Omega\tilde{\psi} + c^T(e,t)\sigma_n && (108) \end{aligned}$$

where $\sigma_n(t)$ exponentially decays according to $\dot{\sigma}_n = A(e,t)\sigma_n$ with IC $\sigma_n(0) = e(0) + \omega(0) - \Omega(0)\tilde{\psi}(0)$.

M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, Nonlinear and adaptive control design. New York: Wiley, 1995.

3.4. Nonlinear swapping (example)

Example 3. Adaptive backstepping control with improved parameters tuning

Plant:

$$\dot{x}_i = x_{i+1} + \phi_i^T(x_1, \dots, x_i)\psi, \quad i = 1, 2, \dots, n-1, \tag{109}$$

$$\dot{x}_n = u + \phi_n^T(x)\theta, \tag{110}$$

$$y = x_1, \tag{111}$$

where $x \in \mathrm{R}^n$ is the measurable state vector with the elements $x_1, x_2, \dots, x_n$, ϕ_i, ϕ_n are the sufficiently smooth vector-functions, $\psi \in \mathrm{R}^q$ is the vector of unknown parameters.

Objective:

$$\lim_{t\to\infty} \varepsilon(t) = \lim_{t\to\infty} \left(y(t) - y_m(t) \right) = 0. \tag{112}$$

Nikiforov V., Gerasimov D., Pashenko A. Modular Adaptive Backstepping Design with a High-Order Tuner , TAC, 67 (5), p.2663-2668, 2022.

3.4. Nonlinear swapping (example)

Backstepping procedure (based on modular identifiers):

$$\alpha_1(x_1, \hat{\psi}, y_m) = -(c_1 + s_1)z_1 - \phi_1^T \hat{\psi}, \tag{113}$$

$$\alpha_i(x_1, \ldots x_i, \hat{\psi}, \ldots, \hat{\psi}^{(i-1)}, y_m, \ldots, y_m^{(i)}) = -z_{i-1} - (c_i + s_i)z_i \tag{114}$$

$$-\left(\phi_i^T - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j^T\right)\hat{\psi} + \sum_{j=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}^{(j-1)}} \hat{\psi}^{(j)} + \frac{\partial \alpha_{i-1}}{\partial y_m^{(j-1)}} y_m^{(j)}\right),$$

where $i = 2, 3, \ldots, n$, $z_1 = x_1 - y_m$, $z_i = x_i - \alpha_{i-1} - y_m^{(i-1)}$ are the new state errors,

$$s_1 = \mu_1 \left\|\phi_1\right\|^2, \qquad s_i = \mu_i \left\|\phi_i^T - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j^T\right\|^2$$

are the damping terms, $\hat{\psi}$ is the vector of adjustable parameters ,

$c_i, \mu_i > 0$ are constant design parameters.

Nikiforov V., Gerasimov D., Pashenko A. Modular Adaptive Backstepping Design with a High-Order Tuner , TAC, 67 (5), p.2663-2668, 2022.

3.4. Nonlinear swapping (example)

Actual control:

$$u = \alpha_n + y_m^{(n)} \qquad (115)$$

Nikiforov V., Gerasimov D., Pashenko A. Modular Adaptive Backstepping Design with a High-Order Tuner , TAC, 67 (5), p.2663-2668, 2022.

3.4. Nonlinear swapping (example)

Actual control:

$$u = \alpha_n + y_m^{(n)} \tag{115}$$

Closed-loop error model:

$$\dot{z} = A_z(z, \phi_i, \hat{\psi}, \hat{\psi}^{(j)})z + B_z(z, \phi_i, \hat{\psi}, \hat{\psi}^{(j)})\tilde{\psi}, \qquad \varepsilon = c_z z, \tag{116}$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors,

$$A_z = \begin{bmatrix} -c_1 - s_1 & 1 & \cdots & 0 \\ -1 & -c_2 - s_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots 0 & -1 & -c_n - s_n \end{bmatrix}, \quad B_z = \begin{bmatrix} \phi_1^T \\ \phi_2^T - \dfrac{\partial \alpha_1}{\partial x_1}\phi_1^T \\ \vdots \\ \phi_n^T - \displaystyle\sum_{j=1}^{n-1} \dfrac{\partial \alpha_{i-1}}{\partial x_j}\phi_j^T \end{bmatrix}, \quad c_z = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Nikiforov V., Gerasimov D., Pashenko A. Modular Adaptive Backstepping Design with a High-Order Tuner , TAC, 67 (5), p.2663-2668, 2022.

3.4. Nonlinear swapping (example)

$$\dot{z} = A_z z + B_z \tilde{\psi}$$

Swapping filters:

$$\dot{\Omega} = A_z \Omega + B_z , \tag{117}$$

$$\dot{\omega} = A_z \omega - \Omega \dot{\hat{\psi}} - B_z \hat{\psi} \tag{118}$$

Augmented error:

$$\tilde{z} = z + \omega \tag{119}$$

Result of swapping:

$$\dot{z} = A_z z + B_z \tilde{\psi} \longrightarrow \tilde{z} = \Omega \tilde{\psi} + \sigma_n$$

3.4. Nonlinear swapping (example)

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Swapping filters:

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Augmented errors:

$$\tilde{z} = z + \omega \tag{119}$$

Result of swapping:

$$\dot{z} = A_z z + B_z \tilde{\psi} \longrightarrow \tilde{z} = \Omega \tilde{\psi} + \sigma_n$$

Algorithm of adaptation (improved with MRE):

$$\dot{\hat{\psi}} = \gamma E_L , \tag{121}$$

where $\Phi_L = L(s)\left[\Omega^T \Omega\right], \; E_L = L(s)\left[\Omega^T \tilde{z} + \Omega^T \Omega \hat{\psi}\right] - \Phi_L \hat{\psi}.$

Nikiforov V., Gerasimov D., Pashenko A. Modular Adaptive Backstepping Design with a High-Order Tuner , TAC, 67 (5), p.2663-2668, 2022.

3.4. Nonlinear swapping (example)

Simulation results:

$$\dot{x}_1 = x_2 + \phi_1^T(x_1)\psi,$$
$$\dot{x}_2 = x_3,$$
$$\dot{x}_3 = u,$$
$$y = x_1,$$

$$\phi_1(x_1) = \begin{bmatrix} x_1^2 \\ \cos(x_1) \end{bmatrix}, \quad \psi = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

For controller design we use $y_m(t) = \sin(t) + 1$ and derive the control law

$$\alpha_1 = -(c_1 + s_1)z_1 - \phi_1^T\hat{\psi},$$

$$\alpha_2 = -z_1 - (c_2 + s_2)z_2 + \frac{\partial\alpha_1}{\partial x_1}\phi_1^T\hat{\psi} + \frac{\partial\alpha_1}{\partial x_1}x_2 + \frac{\partial\alpha_{i-1}}{\partial\hat{\psi}}\dot{\hat{\psi}} + \frac{\partial\alpha_1}{\partial y_m}\dot{y}_m,$$

$$u = -z_2 - (c_3 + s_3)z_3 + \frac{\partial\alpha_2}{\partial x_1}\phi_1^T\hat{\psi} + \sum_{j=1}^{2}\left(\frac{\partial\alpha_2}{\partial x_j}x_{j+1} + \frac{\partial\alpha_2}{\partial\hat{\theta}^{(j-1)}}\hat{\psi}^{(j)} + \frac{\partial\alpha_2}{\partial y_m^{(j-1)}}y_m^{(j)}\right) + \dddot{y}_m.$$

3.4. Nonlinear swapping (example)

Simulation results:

$$\dot{x}_1 = x_2 + \phi_1^T(x_1)\psi,$$
$$\dot{x}_2 = x_3,$$
$$\dot{x}_3 = u,$$
$$y = x_1,$$

$$\phi_1(x_1) = \begin{bmatrix} x_1^2 \\ \cos(x_1) \end{bmatrix}, \quad \psi = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

with $z_1 = x_1 - y_m,\ z_2 = x_2 - \alpha_1 - \dot{y}_m,\ z_3 = x_3 - \alpha_2 - \ddot{y}_m$ the damping terms

$$s_1 = 10^{-3}\left\|\phi_1\right\|^2,\ s_2 = 10^{-3}\left\|\frac{\partial\alpha_1}{\partial x_1}\phi_1\right\|^2,\ s_3 = 10^{-3}\left\|\frac{\partial\alpha_2}{\partial x_1}\phi_1\right\|^2$$

the swapping filters

3.4. Nonlinear swapping (example)

Simulation results:

$$\dot{x}_1 = x_2 + \phi_1^T(x_1)\psi,$$
$$\dot{x}_2 = x_3,$$
$$\dot{x}_3 = u,$$
$$y = x_1,$$

$$\phi_1(x_1) = \begin{bmatrix} x_1^2 \\ \cos(x_1) \end{bmatrix}, \quad \psi = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

and the adaptation algorithm

$$\dot{\hat{\psi}} = \gamma E_L,$$

where $\Phi_L = L(s)\left[\Omega^T\Omega\right]$, $E_L = L(s)\left[\Omega^T\tilde{z} + \Omega^T\Omega\hat{\psi}\right] - \Phi_L\hat{\psi}$, $L(s) = \dfrac{1}{s+1}$

or

$$\ddot{\hat{\psi}} + \left(I_2 + \gamma\Phi_L\right)\dot{\hat{\psi}} = \gamma\,\Omega^T\tilde{z}\,. \qquad (122)$$

3.4. Nonlinear swapping (example)

Simulation results:

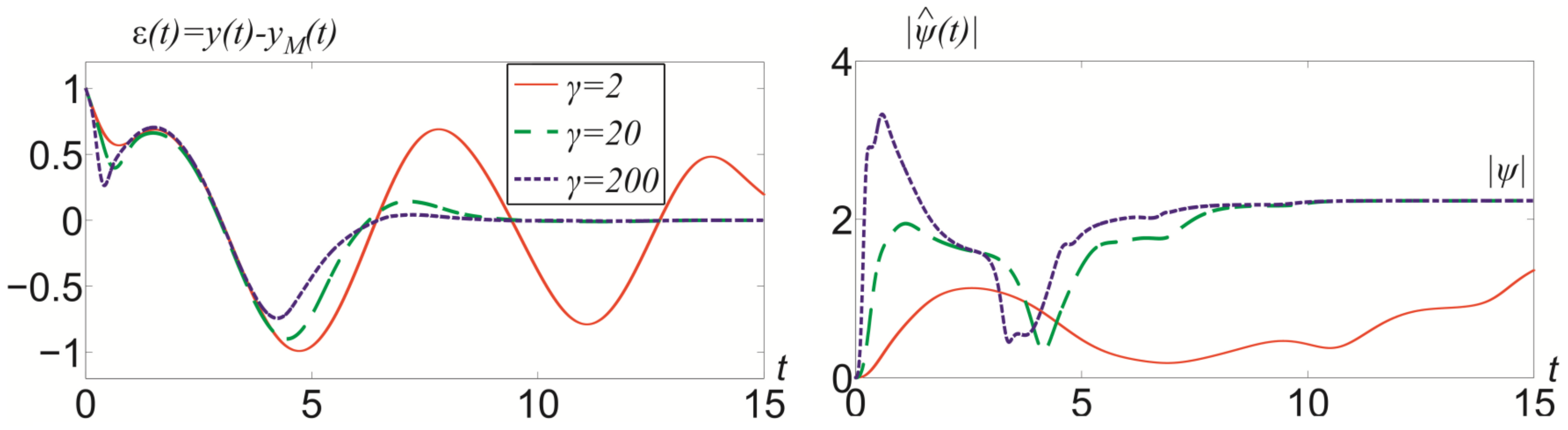

Fig. 4. Transients in the adaptive system closed by backstepping controller with adaptation algorithm with MRE for different γ.

3.5. Swapping for systems with unknown control gain

Error model:

$$\dot{e} = A(e,t)e + kb(e,t)\tilde{\psi}^T\phi, \quad e(0), \tag{123}$$

$$\varepsilon = c^T(e,t)e, \tag{124}$$

where $A(e,t) \in \mathrm{R}^{n\times n}, b(e,t) \in \mathrm{R}^n$, and $c(e,t) \in \mathrm{R}^n$ are the nonlinear mappings, locally Lipschitz in e and bounded in t. The matrix $A(e,t)$ is such that the system

$$\dot{x} = A(e,t)x$$

is exponentially stable.

Filters:

$$\dot{\Omega} = A(e,t)\Omega + b(e,t)\phi^T\hat{\psi} \qquad \dot{\bar{\Omega}} = A(e,t)\bar{\Omega} + b(e,t)\phi^T, \tag{125}$$

$$\dot{w} = A(e,t)w - \hat{k}\bar{\Omega}\dot{\hat{\psi}} + \dot{\hat{k}}\bar{w}, \qquad \dot{\bar{w}} = A(e,t)\bar{w} - \bar{\Omega}\dot{\hat{\psi}} \tag{126}$$

3.5. Swapping for systems with unknown control gain

Augmented error:

$$\tilde{e} = e + w \tag{127}$$

Result of swapping:

$$\dot{e} = A(e,t)e + kb(e,t)\tilde{\psi}^T \phi \longrightarrow \tilde{e} = k\bar{\Omega}\tilde{\psi} - \tilde{k}\bar{w} + \bar{\sigma}, \tag{128}$$

where $\tilde{k} = k - \hat{k}$ is the parametric error.

3.6. Swapping for MIMO linear systems

Error model:

$$\dot{e} = Ae + B\tilde{\Psi}\phi, \quad e(0), \tag{129}$$

$$\varepsilon = Ce, \tag{130}$$

where $A \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$, and $C \in \mathrm{R}^{m\times n}$ are the constant matrices, A is Hurwitz, $\tilde{\Psi} = \Psi - \hat{\Psi} \in \mathrm{R}^{m\times q}$ is the matrix of parametric errors, Ψ is a matrix of unknown parameters, $\hat{\Psi}$ is the matrix of adjustable parameters, $\phi \in \mathrm{R}^{q}$ is the regressor.

V.O. Nikiforov, A.V. Paramonov, D.N. Gerasimov, Adaptive Output Servocontroller for MIMO System with Input Delay, ACC, pp. 620-625, 2020.

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$$\dot{e} = Ae + B\Phi\tilde{\psi},$$

$$\Phi = \begin{bmatrix} \phi^T & O_{1\times q} & \cdots & O_{1\times q} \\ O_{1\times q} & \phi^T & \cdots & O_{1\times q} \\ \vdots & \vdots & \ddots & \vdots \\ O_{1\times q} & O_{1\times q} & \cdots & \phi^T \end{bmatrix} \in \mathbb{R}^{m\times q\cdot m}, \quad \tilde{\psi} = vec\{\tilde{\Psi}\} = vec\left\{\begin{bmatrix} \tilde{\psi}_1^T \\ \tilde{\psi}_2^T \\ \vdots \\ \tilde{\psi}_m^T \end{bmatrix}\right\} = \begin{bmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \\ \vdots \\ \tilde{\psi}_m \end{bmatrix} \in \mathbb{R}^{q\cdot m}.$$

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3.6. Swapping for MIMO linear systems

Filters:

$$\dot{\Omega} = A\Omega + B\Phi, \quad \Xi = C\Omega, \tag{131}$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}}, \quad \xi = C\omega \tag{132}$$

Augmented errors:

$$\tilde{e} = e + \omega, \quad \tilde{\varepsilon} = \varepsilon + \xi \tag{133}$$

Result of swapping:

$$\begin{aligned} \dot{e} &= Ae + B\tilde{\Psi}\phi, \\ \varepsilon &= Ce \end{aligned} \quad \longrightarrow \quad \begin{aligned} \tilde{e} &= \Omega\tilde{\psi} + \sigma, && (134) \\ \tilde{\varepsilon} &= \Xi\tilde{\psi} + C\sigma && (135) \end{aligned}$$

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Conclusion

1. It is shown in a systematic way that the concept of augmented error can be combined with adaptation algorithms with DRE and MRE used for improvement of transient performance of adaptive systems;
2. Special *ad hoc* extensions of the swapping schemes used in augmented errors are illustrated.

The extensions can be mixed depending on the control problem conditions.

Conclusion

1. It is shown in a systematic way that the concept of augmented error can be combined with adaptation algorithms with DRE and MRE used for improvement of transient performance of adaptive systems;
2. Special *ad hoc* extensions of the swapping schemes used in augmented errors are illustrated.
The extensions can be mixed depending on the control problem conditions.

Next steps:

1. Application of DREM [1] / MREM [2] for improvement of transient performance;
2. Involving of exponentially decaying terms into regressors;
3. Schemes with finite time convergence of parametric errors.

1. Aranovskiy, S., Bobtsov, A., Ortega, R., Pyrkin, A.: Performance enhancement of parameter estimator via dynamic regressor extension and mixing, TAC, 62(7), 3546–3550, 2017;
2. Ortega, R., Nikiforov, V., Gerasimov, D.: On modified parameter estimators for identification and adaptive control: a unified framework and some new schemes., Annu. Rev. Control , 50, 278–293, 2020.