1 Probability

1.1 INTRODUCTION :

The theory of probability owes its origins to the study of games of chance or gambling. For example the chance of winning a cricket match, the chance of getting pass in examination, the chance of getting railway ticket booking confirmed etc. Probability theory is designed to deal with uncertainties regarding happening of given phenomena. Thus when we throw a coin, a head is likely to occur but may not occur. When a product is manufactured may or may not defective. The cricket board refers numbers of names in the list who are likely to play for the country but are not certain to be included in the team.

1.2 RANDOM EXPERIMENTS :

Random experiments are those experiments whose results depend on chance. Examples of random experiments are : tossing a coin where head or tail can turn up in a single toss, throwing a die, selecting a card from a pack of playing cards, missile experiment depends on chance to target the location. In all these cases there are number of possible results which can occur but there is an uncertainty as to which one of them will actually occur. An experiment is also known as a **trial.**

The single performance of a random experiment is called an outcome. For example occurence of tail or head is the **outcome** of tossing a coin.

A set of all possible outcomes of an experiment is called a **sample space.** For example when we toss a coin there are two possible outcomes only head or tail. Thus the sample splace for this is {H, T}. Sample space of tossing two coins is {(H, H), (H, T), (T, H), (T, T)}. The sample space of tossing a coin and a die together is {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}.

Any subset of a sample space is called an **event.** event. For example for the sample space of tossing two coins the set {(H, H), (H, T)} is one of the events.

A sample space is said to be **discrete** if it contains finite or countably infinite elements. For continuum elements, the sample space is said to be **continuous.**

Equally likely events : The events are said to be equally likely when one event does not occur more often than the other. That is there is homogeneity in the occurance of events. For example in tossing an unbiased coin, the two events of getting a head and getting a tail are equally likely as both have the same chance of occurance.

Mutually Exclusive Events : If one of the events occurs, the other event cannot take place with the same subject at the same time then the events are said to be mutually exclusive events. In other words occurance of one event prevents the occurance of the other means they cannot occur simultaneously. For example if we toss a coin, head and tail are mutually exclusive events since we can get either head or tail but never both. As an another example an alive person can't be dead at the same time. In a single draw of a card from a pack of cards, we can get a red or a black card but not both. The town is in Gujarat and the town in India are not mutually exclusive because a town in Gujarat is in India. **(May 2015)**

Exhaustive Events : A set of events is said to exhaustive when it includes all possible outcomes of a trial. It is the case of sample space. **(May 2015)**

Simple Events : In this case we consider the probability of the happening or not happening of single events. For example in case of tossing only one coin at a time one may get head or tail is a simple event. That is, if it correspond to a single possible outcome of an experiment.

simultaneous possible outcomes are (H, H), (H, T), (T, H), (T, T). This a compound event.

Independent Events : Two events are said to be independent events if the occurence of one event in no way affects the occurence of the other. For example if we toss a coin two times, the result of the second throw shall in no way be affected by result of the first draw with the same coin. This case is relevent only when trials are consecutive and simultaneous. Suppose a bag contains 3 red and 4 green balls. Two draws are to be made. First draw gives us a green ball. In the second draw the event shall be independent only if the first ball drawn is replaced.

Dependent Events : In this case occurence of one event in any one trial affects the occurence of the other event in other trials. For example a bag contains 4 white and 3 red balls. If we draw a white ball in the first trial and if it is not replaced then it affects the draw of a white ball in the second trial.

Favourable Events : All those events which result in the happening or occurence of the event under consideration termed as favourable events. For example in a throw of a dice if the event is to get an even number then the appearance of numbers 2 or 4 or 6 are favourable events. Similarly if we toss a coin and the event is to get head, then the appearance of head on the coin is favourable event.

1.3 COUNTING PRINCIPLE :

If a certain operation is performed in m ways and second operation is performed in n ways then the number of ways of performing both the operations simultaneously in mn. It is called as **Fundamental Principle of Counting.**

Illustration 1 : Government provides instructions in 12 regional languages and 7 other projects. In how many ways can a person choose on regional language and one other project ?

Solution : By Fundamental Principle of counting, the number of ways of choice is $= 12 \times 7 = 84$.

Illustration 2 : How many numbers of different

because the number contains different digits. Similarly the third and fourth digits can be selected 2 and 1 ways respectively. Thus the number of the different digit number is $= 4 \times 3 \times 2 \times 1 = 24$

Illustration 3 : Three persons are to be designated among 8 positions. In how many ways can they be designated ?

Solution : First person can be disignated any one of the 8 positions.

$\therefore\ m = 8$

Second person can be designated any one of the remaining 7 positions.

$\therefore\ m = 7$

Third person can be designated any one of the remaining 6 positions.

$\therefore\ p = 6$

$\therefore$ Total ways of designation $= 8 \times 7 \times 6 = 336$.

Illustration 4 : In how many ways can a cricket eleven choose a captain, vice captain and a wicket keeper ?

Solution : Captain can be selected in 11 ways, Vice captain can be selected in 10 ways and a wicket keeper can be chosen in 11 ways as he can be a captain or vice captain.

$\therefore$ Total ways of selection $= 11 \times 10 \times 11 = 1210$.

1.4 PERMUTATIONS AND COMBINATIONS :

To understand and make convenient the theory of probability it requires to understand the basic concepts of the theory of permutations and combinations.

Permutation : The number of different arrangements of objects which are possible out of a given objects subject to the condition that no two arrangements are similar. In other words the number of ways of arranging r objects out of n objects is called permutation. It is denoted by,

$$n_{p_r} = \frac{n!}{(n-r)!} \text{ or } P(n, r) \text{ (Here the order of}$$

is done by : AB, BA, CA, AC, BC, CB. That is permutation = $3_{p_2} = 6$

Illustration 1 : There are 11 cards. Determine the arrangement if 4 cards are to be taken at a time.

Solution : Here $n = 11$, $r = 4$

$\therefore$ Total number of arrangements

$$= 11_{p_4} = \frac{11!}{7!} = 7920$$

Illustration 2 : Determine the number of ways of arrangement of 5 persons in a party (i) in a row of 5 chairs (ii) around a circular tables.

Solution :

(i) 5 person can arranded in a row in = $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

(ii) One person can seat on any place in the circular table. The other 4 persons can then arrange in $4 \times 3 \times 2 \times 1 = 24$ ways.

Illustration 3 : (i) In how many ways can 4 boys and 3 girls sit in a row ? (ii) In how many ways can they sit in a row if the boys and girls are each to sit together ? (iii) In how many ways can they sit in a row if just the girls are to seat together.

Solution :

(i) 4 boys and 3 girls can sit in a row in $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ ways.

(ii) They can be distributed in 2 ways according to sex :

BBBBGGG and GGGBBBB.

In each case boys can sit in $4 \times 3 \times 2 \times 1 = 24$ ways and girls can seat in $3 \times 2 \times 1 = 6$ ways.

Thus together they can sit in $2\ (24 \times 6) = 288$ ways.

(iii) We consider 3 girls a 1 group (person). So there are 5 persons (4 boys and 1 group of girls). Thus 5 persons can be arranged in $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. But in a group 3 girls can sit in $3 \times 2 \times 1 = 6$ ways.

$\therefore$ Total number of arrangement = $120 \times 6 = 720$

the number of ordered samples of size 2 with replacement and without replacement.

Solution :

(i) Each ball can be chosen in 4 ways. Thus total in $4 \times 4 = 16$ ways.

(ii) The first ball can be chosen in 4 ways, the second can be in 3 ways.

Thus in total $4 \times 3 = 12$ ways.

Combinations : The numbe of selection of r objects from n objects in which their arrangement or order is not considered is called combination. It is given by,

$$n_{C_r} = \frac{n!}{(n-r)!\,r!} = \frac{n_{p_r}}{r!} \text{ or}$$

$$C(n, r),\ n_{C_r} = n_{C_{n-r}}$$

e.g. The selection of two objects A and B from A, B and C give rise to only AB, BC and CA that is 3 combinations.

Illustration 1 : In how many ways can 13 cards be drawn from a pack of 52 cards.

Solution : The number of ways

$$= 52_{C_{13}} = \frac{52!}{(52-13)!\,13!}$$

Illustration 2 : Out of a pack of cards 5 card are to be drawn. Find the possible number of ways if,

(i) 3 cards are red and 2 cards are black

(ii) 2 cards of heart and others 3 are from the suits other than heart.

(iii) there are two kings and 3 queens.

(iv) there are two aces.

Solution :

(i) There are 26 red and 26 black cards in a pack.

$\therefore$ The number of ways = $26_{C_3} \times 26_{C_2}$

(ii) There are 13 cards of each suit. (Heart, spade, diamond, club)

$\therefore$ The number of ways

(iv) There are 4 aces.

$\therefore$ The number of ways $= 4_{C_2} \times 48_{C_3}$

Illustration 3 : There are 8 boys and 12 girls, a committee consisting of 3 boys and 4 girls is to be formed. In how many ways it is possible so that (i) a particular boy is in a committee (ii) two particular girls are not in a committee (iii) anybody can be included in the committee.

Solution :

(i) After selecting a particular boy, 2 boys can be selected out of remaining 7 boys in 7_{C_2} ways and 4 girls can be selected in 12_{C_4} ways.

$\therefore$ Total ways $= 7_{C_2} \times 12_{C_4}$

(ii) 3 boys can be selected in 8_{C_3} ways. After excluding 2 particular girls, 4 girls can be selected out of remaning 10 girls in 10_{C_4} ways.

$\therefore$ Total ways $= 8_{C_3} \times 10_{C_4}$

(iii) Total ways $= 8_{C_3} \times 12_{C_4}$

Illustration 4 : A student is to answer 5 out of 7 questions in an examination (i) how many choice has he ? (ii) how many if he answers the first 2 questions ? (iii) how many if he answer at least 3 out of first 5 questions ?

Solution :

(i) 5 questions can be answered in $7_{C_5} = \frac{7 \times 6}{2} = 21$ ways.

(ii) If he answers first 2 questions then he can choose 3 questions out of the remaining 5 questions in

$5_{C_3} = \frac{5 \times 4}{2} = 10$ ways.

remaining 2 questions out of the remaining two questions in $2_{C_2} = 1$ ways. Thus in total $10 \times 1 = 10$ ways.

If he answers 4 questions out of the first 5 questions, it can be done in $5_{C_4} = 5$ ways. The 1 question out of the remaining 2 questions in $2_{C_1} = 2$ ways. Thus in total $5 \times 2 = 10$ ways.

If he answers 5 questions out of the first 5 questions it can be done in $5_{C_5} = 1$ ways. There is no question left from the remaining 2. Thus in total 1 way.

Thus he can choose 5 questions in $10 + 10 + 1 = 21$ ways.

Illustration 5 : There are 5 black balls and 4 red balls. Find the number of ways in which 6 balls can be selected so that there are atleast 2 red balls in that selection.

Solution :

(a) If 2 red balls are selected then the number of ways $= 4_{C_2} \times 5_{C_4} = 6 \times 5 = 30$

(b) If 3 red balls are selected then the number of ways $= 4_{C_3} \times 5_{C_3} = 4 \times 10 = 40$

(c) If 4 red balls are selected then the number of ways $= 4_{C_4} \times 5_{C_2} = 1 \times 10 = 10$

$\therefore$ Total number of ways $= 30 + 40 + 10 = 80$

EXERCISE 1.1

1. Three coins are tossed simultaneously. Find out the possible number of associations. **Ans. :** 8
2. Three candidates are to apply for 3 posts. In how many ways it is possible that all of them apply for the same post. **Ans. :** 3
3. In how many ways can captain, vice captain and

2 books are to be taken at a time. **Ans. :** 20

5. In how many ways can 9 persons be arranged around a circular table. **Ans. :** 8!

6. How many 5 digit telephone numbers be formed from the digits 1, 2, 3, 4, 5, 6, 7 if no digit is repeated. **Ans. :** $^7P_5$

7. There are three different rings to be worn on four fingers with atmost one on each finger. In how many ways can this be done. **Ans. :** 24

8. It is required to seat 4 men and 5 women in a row so that the men occupy the even places. How many such arrangements are possible ?

Ans. : $^4P_4 \times {}^5P_5 = 2880$

9. An electric network contains 14 switches such that each switch may have three possible positions. How many different switchings are there ? **Ans. :** 3^{14}

10. The Indian Cricket team consists of 17 players. It includes 2 wicket keepers and 4 bowlers. In how many ways can a cricket eleven be selected if we have to select 1 wicket keeper and atleast 3 bowlers ? **Ans. :** 2640 + 924 = 3564

11. Out of 5 boys and 3 girls a committee of 3 is to be formed. In how many ways can it be formed selecting (i) exactly 1 woman (ii) atleast 1 woman ? **Ans. :** (i) 30 (ii) 30 + 15 + 1 = 46

12. In an election a voter may vote for any number of condidates but not greater than the number to be chosen. There are seven candidates and four are to be chosen. In how many possible ways can a person vote ? **Ans. :** $^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4$

13. For the post of 5 teachers there are 25 applicants, 2 posts are reserved for S.C. and 1 post is reserved for O.B.C. candidates. There are 7 S.C. and 8 O.B.C. candidates among the applicants. In how many ways can the selection be made ?

Ans. : $^7C_2 + {}^8C_1 + {}^{10}C_2$

14. In a bag there are 5 green, 6 yellow and 4 white

there.

Ans. : (i) $^5C_1 \times {}^6C_1 \times {}^4C_1$ (ii) $^6C_2 \times {}^9C_1$

(iii) $^{11}C_3 \times {}^4C_0$

1.5 PROBABILITY :

The chance of happening of an event when expressed quantitatively is called **probability.**

Consider a random experiment with possible results as cases. Let S be the selected sample space. Let n be the number of sample points in S, where we assume n to be finite. Let all the simple events in S are equally likely to occur as an outcome. Let m sample points of them are favourable to an event E. Then the probability of happening of E is defined as

$$p = P(E) = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

$$= \frac{m}{n} = \frac{n(E)}{n(S)}$$

This is the classical definition of probability.

Illustration 1 : What is the probability of getting an odd number while tossing a die ?

Solution : Here E = {1, 3, 5} and $n(S) = 6$

$$\therefore \quad P(E) = \frac{3}{6} = \frac{1}{2}$$

Illustration 2 : What is the chance of getting king in a draw from the pack of 52 cards ?

Solution : Here $n(S) = 52$

$n(E)$ = number of getting king = 4

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}.$$

Illustration 3 : An unbiased coin is tossed 3 times. What is the probability of obtaining two heads ? (May 2016, May 2017)

Solution : We have E = {HHT, HTH, THH} and n(s) = 8

$\therefore \quad P(E) = \frac{3}{\ldots}$

two outcomes is equal to 6 ? **(May 2016)**

Solution : We have E = {(2, 4), (3, 3), (4, 2)} and n(s) = 16.

$$\therefore\ P(E) = \frac{3}{16}$$

AXIOMS OF PROBABILITY :

(i) If E is an impossible event (e.g. for the toss of die S = {1, 2, 3, 4, 5, 6}, and the event to get a number greater the 6 is called impossible event) then E = { } = ϕ.

$\therefore$ P(E) = 0

(ii) P(S) = 1

(iii) $0 \le P(E) \le 1$

Here P(E) = 0 means that event will not occur and P(A) = 1 means that the event is certain.

(iv) If p is the probability of occurrence and q is the probability of non–occurrence of that event then $p + q = 1$.

or $P(E) + P(\overline{E}) = 1$. Where $\overline{E}$ is the complement of E.

It is also known as **complementation rule.**

(v) If A and B are two events then the probability of occurrence of at least one of the two events is given by

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(vi) If A and B are mutually exclusive events then occurence of either A.

A or B is given by

$P(A \cup B) = P(A) + P(B)$

(vii) If A and B are two events then

$P(\overline{A} \cap B) = P(B) - P(A \cap B)$

(viii) Boole's inequality : $P(A \cup B) \le P(A) + P(B)$

(ix) Bonferroni's inequality :

$P(A \cap B) \ge P(A) + P(B) - 1$

(x) **Multiplication Law of Probability :** This law is used to find the probability of the combined events of the two or more independent events

of which 3 are defective. If 4 objects are chosen at random, what is the probability that none of them is defective ?

Solution : 4 objects can be selected in 10_{C_4} = 210 ways.

If none of them is defective then it must come from 7 non–defective objects. So the number of favourable cases is $7_{C_4} = 35$

$$\therefore\ \text{The required probability} = \frac{35}{210} = \frac{1}{6}$$

Illustration 2 : From a pack of 52 cards three are drawn at random. Find the chance that they are a king, a queen and a jack.

Solution : 3 cards can be drawn in 52_{C_3} ways.

There are 4 kings, 4 queens and 4 jacks. Thus the number of favourable cases

$$= 4_{C_1} \times 4_{C_1} \times 4_{C_1} = 4^3 = 64$$

$$\therefore\ \text{The required probability} = \frac{64}{52_{C_3}}$$

Illustration 3 : There are 3 statisticians, 2 economists and 4 engineers. A committee of 4 is to be formed in such a way that

(i) there are 2 statisticians and 2 engineers

(ii) engineer is not in the committee.

Find the probabilities.

Solution : Total possible ways = $9_{C_4} = 126$

(i) Number of favourable cases = $3_{C_2} \times 4_{C_2} \times 2_{C_0}$

$= 3 \times 6 \times 1 = 18$

$$\therefore\ \text{Required Probability} = \frac{18}{126}$$

(ii) Number of favourable cases = $5_{C_4} = 5$

Illustration 4 : A room has three lamp sockets. From a collection of 10 light bulbs of which only 6 are good. A person selects 3 at random and puts them in the socket. What is the probability that the room will have light ? (Nov. 2017)

Solution : Number of ways of selecting 3 bulbs

$= {}^{10}C_3 = \frac{10 \times 9 \times 8}{6} = 120$

Event is to have a light in the room. Means to find of putting at least good bulb in the socket.

$\therefore$ Number of favourable cases

$= {}^6C_1 {}^4C_2 + {}^6C_2 {}^4C_1 + {}^6C_3 {}^4C_0$

$= 6 \times 6 + 15 \times 4 + 20 \times 1 = 116$

$\therefore$ Required probability $= \frac{116}{120} = \frac{29}{30}$

OR

P(Room will not have light)

$= \frac{{}^4C_3}{120} = \frac{4}{120} = \frac{1}{30}$

$\therefore$ P(Room will have light) $= 1 - \frac{1}{30} = \frac{29}{30}$

Illustration 5 : The following table gives a distribution of monthly wages of 100 employees of a firm.

Wages (Rs.)	< 280	280–320	320–360	360–400	400–440	440–480	> 480
No. of workers	5	12	18	25	22	7	11

An individual is selected from the above group. What is the probability that his wages are (i) under Rs. 320 (ii) above Rs. 400 (iii) between Rs. 320 and 400.

Solution : Total number of employees = 100

(i) The number of favourable cases for the wages under Rs. 320 is 17.

$\therefore$ Required probability $= \frac{17}{100}$

(ii) The number of favourable cases for the wages ...

$\therefore$ Required probability $= \frac{40}{100}$

(iii) The number of favourable cases = 43.

$\therefore$ Required probability $= \frac{43}{100}$

Illustration 6 : (a) A person hits a target with rifle shot in 4 out of 5 times. Another person can hit the same target with the same rifle in 3 out of 4 times. Find the probability of the target being hit when both try or by atleast one hits the target.

Solution : The probability of first person hits the target $P(A) = \frac{4}{5}$

The probability that second person hits the target is $P(B) = \frac{3}{4}$

$\therefore$ Required probability

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{5} + \frac{3}{4} - \frac{4}{5} \times \frac{3}{4} = \frac{31}{20} - \frac{3}{5} = \frac{19}{20}$

(b) A person is known to hit the target in 3 out of 4 shots, where as another person is known to hit the target in 2 out of 3 shots. Find the probability of the target being hit at all when they both try. (May, 2015)

Solution : The probability of the first person hit the target $P(A) = \frac{3}{4}$.

$\therefore$ The probability that second person hit the target is $P(B) = \frac{2}{3}$.

$\therefore$ Required probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{2}{3} - \frac{3}{4} \cdot \frac{2}{3} = \frac{11}{12}$

Illustration 7 : The probability that a

2

one contract is $\frac{4}{5}$, **what is the probability that he will get both ?**

Solution : Let A : The event of getting plumbing contract

B : The event of getting electric contract.

$\therefore \quad P(A) = \frac{2}{3}, \quad P(B) = 1 - \frac{5}{9} = \frac{4}{9},$

$P(A \cup B) = \frac{4}{5}$

$\therefore \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{14}{25}$

Illustration 8 : Copying in exam is banned. Chances that a student will be caught by junior supervisor is 0.4, by the senior supervisor is 0.3 and by the observer is 0.1. Find the probability that you will be caught by any of them (or by at least one of them).

Solution : Let A be the event of a student caught by junior supervisor, B by senior supervisor and C by an observer.

$\therefore \quad P(A) = 0.4, \quad P(B) = 0.3, \quad P(C) = 0.1$

$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$- P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$= 0.4 + 0.3 + 0.1 - (0.4 \times 0.3) - (0.4 \times 0.1)$

$- (0.3)(0.1) + (0.4 \times 0.3 \times 0.1)$

$= 0.622$

or P(All fail to happen)

$= 0.6 \times 0.7 \times 0.9 = 0.378$

$\therefore$ P(atleast one of them) $= 1 - 0.378 = 0.622$

Illustration 9 : Three cars are moving from Ahmedabad to Mumbai. Odds in favour of their safe running are 10 : 5, 9 : 6 and 8 : 12. Find the probability that they run safely.

Solution : Let A, B, C be the events of safe

$P(C) = \frac{8}{20} = \frac{2}{5}$

All the events are independent.

$\therefore$ Probability that all cars run safely

$= P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

$= \frac{2}{3} \times \frac{3}{5} \times \frac{2}{5} = \frac{4}{25}$

Illustration 10 : The probability that machine A will be performing well in 5 years time is $\frac{1}{4}$ and that of machine B is $\frac{1}{3}$. Determine the probability that in 5 years time : (i) both machines performing well (ii) neither will be operating (iii) only machine B will be performing (iv) at least one of the machines will be operating.

Solution : Here $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$

(i) $P(A \cap B) = P(A) \times P(B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

(ii) P(neither operates) $= (1 - P(A))(1 - P(B))$

$= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

(iii) P(only B operates) $= (1 - P(A))\, P(B)$

$= \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{1}{2}$

Illustration 11 : Two students x and y work independently on a problem. The probability that x will solve it is $\frac{3}{4}$ and the probability that y will

solve it is $\frac{2}{3}$. What is the probability that problem will be solved ? **(Dec. 2015)**

Solution : Let,

A = Probability that x will solve problem.

B = Probability that y will solve problem.

$\therefore \quad P(A) = \frac{3}{4}, \; P(B) = \frac{2}{3}.$

Also both are independent.

$\therefore \quad P(A \cap B) = P(A) \cdot P(B) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$

$\therefore$ The probability that problem will be solved is :

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{4} + \frac{2}{3} - \frac{1}{2} = \frac{11}{12}$

Illustration 12 : Four cards are drawn from a pack of cards. Find the probability that (i) all are diamond (ii) there is one card of each suit (iii) there are two spades and two hearts. **(Nov. 2016)**

Solution : Four cards can be drawn in

${}^{52}C_4 = 270725$ ways.

(i) There are 13 diamond cards in a pack.

$\therefore$ Probability of getting all diamond cards

$$= \frac{{}^{13}C_4}{270725} = \frac{13.12.11.10}{52.51.50.49} = \frac{11}{4165}$$

(ii) Each suit have 13 cards.

$\therefore$ Required probability

$$= \frac{{}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1}{{}^{52}C_4} = \frac{13.13.13.13.24}{52.51.50.49}$$

$$= \frac{2197}{20825}$$

(iii) There are 13 spades and 13 hearts in a pack.

$\therefore$ Required probability

1.6 COMPOUND EVENTS :

When two or more events occur in connection with each other, their simultaneous occurence is called a **compound event.**

The multiplication law is not applicable when events are not independent. Let A and B be any two events. Then the probability of happening of the event A, knowing that the event B has already happened, is called the **conditional probability** of the event A. It is denoted by P(A/B).

Similarly P(B/A) denotes the conditional proability of happening of event B on the condition that the event A has already happened.

Thus, Probability of A given B has occured is given by,

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{or}$$

$$P(A \cap B) = P(B)\, P(A/B)$$

$$\text{Similarly, } P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{or}$$

$$P(A \cap B) = P(A)\, P(B/A)$$

Note :

1. In case of more than two events
$P(A \cap B \cap C) = P(A)\, P(B/A)\, P(C/A \cap B)$
2. If A and B are independent then P(A/B) = P(A) and P(B/A) = P(B).

Illustration 1 : Find the probability of drawing a king, queen in this order from a pack of 52 cards in two consecutive draws and the cards are not replaced.

Solution : The probability of drawing a king is

$$P(A) = \frac{4}{52}$$

After this draw, the card is not replaced.

Thus the probability of drawing a queen is

$$P(B/A) = \frac{4}{51}$$

working. The probability of A's failure during one year is 5 % that of B's failure is 15 % and that of C's failure is 10 %. What is the probability that the equipment will fail before the end of that year ?

Solution : The equipment will fail if A or B or C or any combination of these three will fail.

$\therefore$ The proability of equipment will fail during the year

= 1 – probability that the equipment will function throughout the year.

$\therefore \quad P(\bar{A}) = 1 - 0.05 = 0.95, \quad P(\bar{B}) = 1 - 0.15 = 0.85, \quad P(\bar{C}) = 1 - 0.10 = 0.90.$

$\therefore$ The probability that equipment does not fail $= 0.95 \times 0.85 \times 0.90 = 0.7267$

$\therefore$ The probability that equipment will fail $= 1 - 0.7267 = 0.2733$

Illustration 3 : An equipment consists of two parts A and B. In the process of manufacturing of part A, 9 out of 100 are likely to be defective and that of B 5 out of 100 are likely to be defective. Find the probability that the assembled article will not be defective.

Solution : Probability of part A to be defective is $P(A) = 0.09$

$\therefore \quad P(\bar{A}) = 1 - 0.09 = 0.91$

Probability of part B to be defective is $P(B) = 0.05$

$\therefore \quad P(\bar{B}) = 1 - 0.05 = 0.95$

$\therefore$ Probability that the equipment is not defective

$= P(\bar{A})\, P(\bar{B})$

$= (0.91)\,(0.95) = 0.8645$

Illustration 4 : A bag contains 5 white and 8 red balls. Two drawings of three balls are made such that (i) balls are replaced before the second draw. (ii) balls are not replaced before the second draw. Find the chance that the first draw will give three white and second three red balls in each case.

Total ways of drawing 3 balls = 13_{C_3}

The ways of drawing 3 white balls = 5_{C_3}

The ways of drawing 3 red balls = 8_{C_3}

$$\therefore \quad P(W) = \frac{5_{C_3}}{13_{C_3}}, \quad P(R) = \frac{8_{C_3}}{13_{C_3}}$$

$$\therefore \quad P(W \cap R) = \frac{5_{C_3}}{13_{C_3}} \times \frac{8_{C_3}}{13_{C_3}}$$

(ii) When the balls are not-replaced

The events are dependent

$$\therefore \quad P(W) = \frac{5_{C_3}}{13_{C_3}}$$

$$P(R/W) = \frac{8_{C_3}}{10_{C_3}}$$

$$\therefore \quad P(W \cap R) = P(W)\,P(R/W) = \frac{5_{C_3}}{13_{C_3}} \times \ldots$$

Illustration 5 : Compute P(A/B), if P(A) = ... P(B) = 0.7 and P(A ∩ B) = 0.3. (May 2...

Solution : We have

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.7} = \frac{3}{7}$$

Illustration 6 : In producing screws, le... mean "screw too slim" and B "screw too sm... Let P(A) = 0.1 and let the conditional probab... that a slim screw is also too small be P(B/A) = ... What is the probability that the screw that we p... randomly from a lot produced will be both too s... and too short ? (May 20...

Solution : We have $P(B/A) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore$ Required probability $P(A \cap B)$

To understand Bayes' theorem we consider an example. Suppose a probability of manufacturing a product by machine 1 is 0.7 and that of by machine 2 is 0.8. The probability of getting defective piece through machine 1 is 0.5 and that of by machine 2 is 0.9. If one product is selected at random then the question arise is to find the probability of the selected defective piece is manufactured by machine 1. Here we have to suppose A as the event of getting defective piece. Then we have the following information.

$P(M_1) = 0.7$, $P(M_2) = 0.8$, $P(A/M_1) = 0.5$,
$P(A/M_2) = 0.9$

The question is to find $P(M_1/A)$.

That means it is a case of inverse probability which is to be determined with the help of Bayes' theorem.

Statement of Bayes' Theorem :

Let $B_1, B_2, \ldots B_n$ be mutually exclusive and exhaustive events of the sample space S. Let the event A occur in conjunction with only one of the events $B_1, B_2, \ldots, B_n$. If the probabilities $P(B), P(B_2), \ldots, P(B_n)$ and $P(A/B_1), P(A/B_2), \ldots, P(A/B_n)$ are know then

$$P(B_j/A) = \frac{P(B_j)\, P(A/B_j)}{P(B_1)\, P(A/B_1) + P(B_2)\, P(A/B_2) + \ldots + P(B_n)\, P(A/B_n)}$$

Where $j = 1, 2, \ldots, n$.

Here $P(A) = P(B_1)\, P(A/B_1) + \ldots + P(B_n)\, P(A/B_n)$ is called the **rule of elimination** or the **rule of total probability.**

Illustration 1 : Three machines A, B and C produce 50 %, 30 % and 20 % of the total number of items. The production of defective item is 3 %, 4 %, 5 % respectively on each machine. If an item selected at random and is found to be defective, find the probability that the item was produced by machine A.

Solution : Let D be the event of getting defective item.

$P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

... selected defective piece came from machine A is :

$$P(A/D) = \frac{P(A)\, P(D/A)}{P(A)\, P(D/A) + P(B)\, P(D/B) + P(C)\, P(D/C)}$$

$$= \frac{(0.5)\,(0.03)}{(0.5)\,(0.03) + (0.3)\,(0.04) + (0.2)\,(0.05)}$$

$= 0.405$

Illustration 2 : Four boxes B_1, B_2, B_3 and B_4 contains some gold and copper coins. The percentage of coins in these boxes are 20, 30, 10 and 40 respectively. The fractions of gold coins in the boxes are 0.3, 0.1, 0.2 and 0.5 respectively. (i) If a coin is taken out at random, what is the probability that it is a gold coin ? (ii) If a coin taken out at random is found to be golden, what is the probability that it is taken from box B_2 ?

Solution : Let A be the event of getting a gold coin.

$P(B_1) = 0.2, P(B_2) = 0.3, P(B_3) = 0.1, P(B_4) = 0.4$
$P(A/B_1) = 0.3$, $P(A/B_2) = 0.1$, $P(A/B_3) = 0.2$,
$P(A/B_4) = 0.5$

(i) $P(A) = \sum_{i=1}^{4} P(B_i)\, P(A/B_i)$

$= (0.2)\,(0.3) + (0.3)\,(0.1) + (0.1)\,(0.2) + (0.4)\,(0.5) = 0.31$

(ii) $P(B_2/A) = \dfrac{P(B_2)\, P(A/B_2)}{P(A)}$

$= \dfrac{(0.3)\,(0.1)}{0.31} = 0.097$

Illustration 3 : Three bags contains 10%, 20% and 30 % defective items. An item is selected at random which is defective. Determine the probability that it came from 3rd bag, 2nd bag, 1st bag. (May 2017)

Solution : Let A be the event of selecting

$\therefore$ $P(A/B_1) = 0.1$, $P(A/B_2) = 0.2$, $P(A/B_3) = 0.3$

$\therefore$ Probability that it came from 3rd bag is :

$$P(B_3/A) = \frac{P(B_3)\,P(A/B_3)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

$$= \frac{\frac{1}{3}(0.3)}{\frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.3)}$$

$$= 0.5$$

Probability that it came from 2nd bag is :

$$P(B_2/A) = \frac{P(B_2)\,P(A/B_2)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

$$= \frac{\frac{1}{3}(0.2)}{\frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.3)} = 0.34$$

Probability that it came from 1st bag is :

$$P(B_1/A) = \frac{P(B_1)\,P(A/B_1)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

$$= \frac{\frac{1}{3}(0.1)}{\frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.3)} = 0.17$$

Illustration 4 : It came to know that the marks given by a certain examiner is correct in 90 % of the cases. Suppose that 40 % of the answer books are given to the examiner which can be given full marks in actual. What is the probability of the actual answer book given to the examiner have been corrected actual with full marks ?

$\therefore$ $P(A) = 0.4$, $P(\bar{A}) = 0.6$, $P(B/A) = 0.9$, $P(B/\bar{A}) = 0.1$

$$\therefore \quad P(A/B) = \frac{P(A)\,P(B/A)}{P(A)\,P(B/A) + P(\bar{A})\,P(B/\bar{A})}$$

$$= \frac{(0.4)\,(0.9)}{(0.4)\,(0.9) + (0.6)\,(0.1)} = 0.857$$

Illustration 5 : A Company has two plants to manufacture hydraulic machines. Plant I manufactures 70% of the hydraulic machines and plant II manufactures 30%. At plant I, 80% of hydraulic machines are rated standard quality and at plant II, 90% of hydraulic machines are rated standard quality. A machine is picked up at random and is found to be of standard quality. What is the chance that it has come from plant I ?

(May, 2015, Nov. 2017)

Solution : Let A be the event of getting standard quality machine. B_1, B_2 are the events of manufacturing machines at plant I & II respectively.

$\therefore$ $P(B_1) = 0.70$, $P(B_2) = 0.30$

Also $P(A/B_1) = 0.8$, $P(A/B_2) = 0.9$

$\therefore$ Using Bayes' theorem, the probability that the selected machine has come from plant I is :

$$P(B_1/A) = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)\,P(A/B_2)}$$

$$= \frac{(0.70)\,(0.8)}{(0.7)\,(0.8) + (0.3)\,(0.9)} = 0.6747$$

Illustration 6 : State Bayes' theorem. In a bolt factory, three machines A, B and C manufacture 25%, 35% and 40% of the total product respectively. Of these outputs 5%, 4% and 2% respectively, are defective bolts. A bolt is picked up at random and found to be defective. What are the probabilities that it was manufactured by machines A, B and C ? **(Dec. 2015)**

Solution : Let D be the event of getting defective

$\therefore$ $P(A/B_1) = 0.1$, $P(A/B_2) = 0.2$, $P(A/B_3) = 0.3$

$\therefore$ Probability that it came from 3rd bag is :

$$P(B_3/A) = \frac{P(B_3)\,P(A/B_3)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

$$= \frac{\frac{1}{3}(0.3)}{\frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.3)}$$

$$= 0.5$$

Probability that it came from 2nd bag is :

$$P(B_2/A) = \frac{P(B_2)\,P(A/B_2)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

$$= \frac{\frac{1}{3}(0.2)}{\frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.3)} = 0.34$$

Probability that it came from 1st bag is :

$$P(B_1/A) = \frac{P(B_1)\,P(A/B_1)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

$$= \frac{\frac{1}{3}(0.1)}{\frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.3)} = 0.17$$

Illustration 4 : It came to know that the marks given by a certain examiner is correct in 90 % of the cases. Suppose that 40 % of the answer books are given to the examiner which can be given full marks in actual. What is the probability of the actual answer book given to the examiner have been corrected actual with full marks ?

$\therefore$ $P(A) = 0.4$, $P(\bar{A}) = 0.6$, $P(B/A) = 0.9$, $P(B/\bar{A}) = 0.1$

$$\therefore \quad P(A/B) = \frac{P(A)\,P(B/A)}{P(A)\,P(B/A) + P(\bar{A})\,P(B/\bar{A})}$$

$$= \frac{(0.4)(0.9)}{(0.4)(0.9) + (0.6)(0.1)} = 0.857$$

Illustration 5 : A Company has two plants to manufacture hydraulic machines. Plant I manufactures 70% of the hydraulic machines and plant II manufactures 30%. At plant I, 80% of hydraulic machines are rated standard quality and at plant II, 90% of hydraulic machines are rated standard quality. A machine is picked up at random and is found to be of standard quality. What is the chance that it has come from plant I ?

(May, 2015, Nov. 2017)

Solution : Let A be the event of getting standard quality machine. B_1, B_2 are the events of manufacturing machines at plant I & II respectively.

$\therefore$ $P(B_1) = 0.70$, $P(B_2) = 0.30$

Also $P(A/B_1) = 0.8$, $P(A/B_2) = 0.9$

$\therefore$ Using Bayes' theorem, the probability that the selected machine has come from plant I is :

$$P(B_1/A) = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)\,P(A/B_2)}$$

$$= \frac{(0.70)(0.8)}{(0.7)(0.8) + (0.3)(0.9)} = 0.6747$$

Illustration 6 : State Bayes' theorem. In a bolt factory, three machines A, B and C manufacture 25%, 35% and 40% of the total product respectively. Of these outputs 5%, 4% and 2% respectively, are defective bolts. A bolt is picked up at random and found to be defective. What are the probabilities that it was manufactured by machines A, B and C ? **(Dec. 2015)**

Solution : Let D be the event of getting defective

$P(D/A) = 0.05$, $P(D/B) = 0.04$,

$P(D/C) = 0.02$

$\therefore$ Using Bayes' theorem the probability that it was manufactured by machines A, B, C is :

$$P(A/D) = \frac{P(A)\,P(D/A)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$\text{Here } P(D) = P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)$$
$$= (0.25)\,(0.05) + (0.35)\,(0.04) + (0.4)\,(0.02) = 0.0345$$

$$\therefore \quad P(A/D) = \frac{(0.25)\,(0.05)}{0.0345} = 0.3623$$

$$P(B/D) = \frac{P(B)\,P(D/B)}{P(D)} = \frac{(0.35)\,(0.04)}{0.0345}$$
$$= 0.4058$$

$$P(C/D) = \frac{P(C)\,P(D/C)}{P(D)} = \frac{(0.4)\,(0.02)}{0.0345}$$
$$= 0.2319$$

Illustration 7 : In a certain assembly plant, three machines, B_1, B_2 and B_3 make 30%, 45% and 25%, respectively, of the products. It is known from past experience that 2%, 3% and 2% of the products made by each machine, respectivley, are defective. Now, suppose that a finished product is randomly selected, what is probability that it is defective ?

Solution : Let A be the event of getting defective product.

$P(B_1) = 0.3$, $P(B_2) = 0.45$, $P(B_3) = 0.25$

Also $P(A/B_1) = 0.02$, $P(A/B_2) = 0.03$,

$P(A/B_3) = 0.02$

$$\therefore \quad P(A) = \sum_{i=1}^{3} P(B_i)\,P(A/B_i)$$
$$= (0.3)\,(0.02) + (0.45)\,(0.03)$$

microchip company has two machines that produce the chips. Machine I produces 65% of the chips, but 5% of its chips are defective. Machine II produces 35% of the chips and 15% of its chips are defective. A chip is selected at random and found to be defective. What is the probability that it came from Machine I ? (Nov. 2016)

Solution : Let A be the event of getting defective chip. B_1, B_2 are the events of production of chips at Machine I and II respectively.

$\therefore$ $P(B_1) = 0.65$, $P(A/B_1) = 0.05$, $P(B_2) = 0.35$,

$P(A/B_2) = 0.15$

Using Bayes' theorem, the probability that the selected chip has come from Machine I is :

$$P(B_1/A) = \frac{P(B_1)\,P(A/B_1)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2)}$$

$$= \frac{(0.65)\,(0.05)}{(0.65)\,(0.05) + (0.35)\,(0.15)} = 0.3824$$

Illustration 9 : An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the probability that it is a white ball ? (Nov. 2016)

Solution : Two balls drawn from the first urn gives : A = both white, B = both black, C = one white and one black.

$$\therefore \quad P(A) = \frac{{}^{10}C_2}{{}^{13}C_2} = \frac{10.9}{13.12} = \frac{15}{26}.$$

$$P(B) = \frac{{}^{3}C_2}{{}^{13}C_2} = \frac{1}{26},$$

$$P(C) = \frac{{}^{10}C_1 \cdot {}^{3}C_1}{{}^{13}C_2} = \frac{10}{26}$$

Now substitution of these two balls in second urn will contain.

$\therefore \quad P(W/A) = \frac{{}^{5}C_1}{{}^{10}C_1} = \frac{5}{10}, \; P(W/B) = \frac{3}{10},$

$P(W/C) = \frac{4}{10}$

$\therefore \quad P(W) = P(A)\,P(W/A) + P(B)\,P(W/B) + P(C)\,P(W/C)$

$$= \frac{15}{26}\cdot\frac{5}{10} + \frac{1}{26}\cdot\frac{3}{10} + \frac{10}{26}\cdot\frac{4}{10} = \frac{118}{260} = \frac{59}{130}$$

EXERCISE 1.2

1. Find a probability of throwing 10 with two dices.

Ans. : $\frac{1}{12}$

2. What is the probability of getting a black ball if a bag contains 10 white and 30 black balls ?

Ans. : $\frac{3}{4}$

3. A committee consists of 9 students two of which are from first year, three from 2nd year and four from 3rd year. Three students are to be removed at random. What is the porbability that
   (i) the three students belong to different classes.
   (ii) two belong to the same class and third to different class
   (iii) the three belong to the same class.

Ans. : $\frac{2}{7}, \frac{55}{84}, \frac{5}{84}$

4. A bag contains 10 red and 90 green balls. A person makes a draw. What are the chances of getting (i) red ball (ii) not getting a red ball ?

Ans. : 0.1, 0.90

5. Following is the distribution relating to number of cars in different houses :

Cars	0	1	2	3	4	5
Houses	10	20	15	12	10	2

than 2 cars (iii) 4 or more than 4 cars.

Ans. : $\frac{12}{69}, \frac{45}{69}, \frac{12}{69}$

6. Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{6}$.

Find the values of $P(A/B)$, $P(\bar{A} \cap B)$, $P(\bar{A} \cap \bar{B})$ and $P(\bar{A} \cup \bar{B})$.

Ans. : $\frac{2}{3}, \frac{1}{12}, \frac{7}{12}, \frac{3}{4}$

7. Six men in a company of 15 are engineers. If 3 men are selected out of the 15 at random. What is the probability of at least one engineer ?

Ans. : $\frac{371}{455}$

8. A can hit a target 4 times in 5 shots, B 3 times in 4 shots, C twice in 3 shots. They fire a volley. What is the probability that two shots at least heat.

Ans. : $\frac{5}{6}$

9. A probability that a team will score 0 goal is 0.40, 1 goal is 0.34, 2 goals is 0.26 and 3 and more goals is 0.06 in a hockey match. Find out the probability of the team scoring at least one goal.

Ans. : 0.60

10. The probability that a student get passed in calculus is $\frac{2}{3}$, that of passed in physics is $\frac{4}{9}$. If the probability of passing at least one subject is $\frac{4}{5}$ what is the probability that he will pass both subjects ?

Ans. : $\frac{14}{45}$

11. An investment consultant predicts that the odds against the price of a certain stock will go up during the next week are 2 : 1 and the odds in favour of price remaining the same are 1 : 3. What is the probability that the price of the stock will go down during next week.

Ans. : $\frac{5}{12}$

12. An urn contains 6 red and 4 green balls. Two balls are drawn at random one after the other but without replacement. Find the probability that both the balls are green.

Ans. : $\frac{2}{15}$ $P(A \cap B) = P(A)\,P(B/A)$

13. Three machines M1, M2 and M3 manufacture 20, 45 and 35 percent respectively of the total output. Of their outputs 3, 5 and 4 percent respectively are defective. One product is drawn at random from the total output and is found to be defective. Find the probability that it was manufactured by the machine M1, M2 or M3.

Ans. : 0.14, 0.53, 0.33

14. Box A contains 3 red and 5 white balls, Box B contains 2 red and 1 white balls and Box C contains 2 red and 3 white balls. A ball is drawn at random and it is red. What is the probability that it came from Box A ? **Ans. :** 0.26

15. Three suppliers A, B and C supply items in the proportion of $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$. Of these items 5 %, 6 % and 8 % respectively are defective. An item selected at random found to be defective. What is the probability that it was supplied by A, B and C ? **Ans. :** $\frac{15}{35}, \frac{12}{35}, \frac{8}{35}$

16. By examining the X–ray probability that T.B. is detected when a person is actually suffering is 0.99. The probability that the doctor diagnosed incorrectly that a person has T.B. is 0.001. In a certain city 1000 persons suffers from T.B. A person selected at random is dignosed to have T.B. what is the chance that he actually has T.B.

[**Hint :** A = Person has T.B., B = Person has no T.B., E = Person diagnosed to have T.B.,

$P(A) = \frac{1}{1000}$, $P(B) = \frac{999}{1000}$, $P(E/A) = 0.99$, $P(E/B) = 0.001$] **Ans. :** 0.498

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