# Normal Distribution:

- In statistics, normal distribution arises as a limiting case of several discrete and conti. probability distribution. It has application in sampling theory.

- By central limit theorem, it follows that the sampling distribution of the sample mean is appro. normal even if the distribution of the population from which the sample is drawn ~~is~~ is not normal.

- It has large applications in statistical quality control in Industry for setting control limits.

**Def$^n$:** A continuous r.v. $X$ with parameter $\mu$ and $\sigma$ is said to follow normal distribution (Gaussian) if its PDF is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

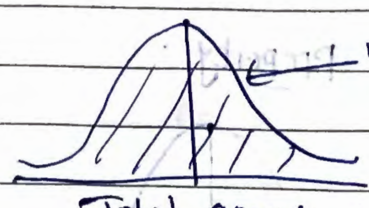
where $-\infty < x < \infty$
$-\infty < \mu < \infty$, $\sigma > 0$

If $X$ is normal r.v. with parameters $\mu$ and $\sigma$ we denote it by $X \sim N(\mu, \sigma^2)$

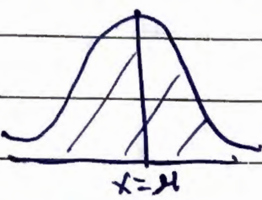
## Some Important Point about Normal Distribution

(1) $f(x) \geq 0 \quad \forall x$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$




3) Mean, Median and mode all are equal for normal distri.




The curve is symmetrical about the line $x = \mu$

4) When Variable X is normally distributed with mean $\mu$ & std deviation $\sigma$ it is expressed symbolically as

$$X \sim N(\mu, \sigma^2)$$

- If we introduced new variable say $Z = \frac{x-\mu}{\sigma}$

then this new vari. Z is also normally distributed with mean 0 and std. devi. one.

$$\text{i.e } Z \sim N(0, 1)$$

Z is called std normal variable.

& PDF

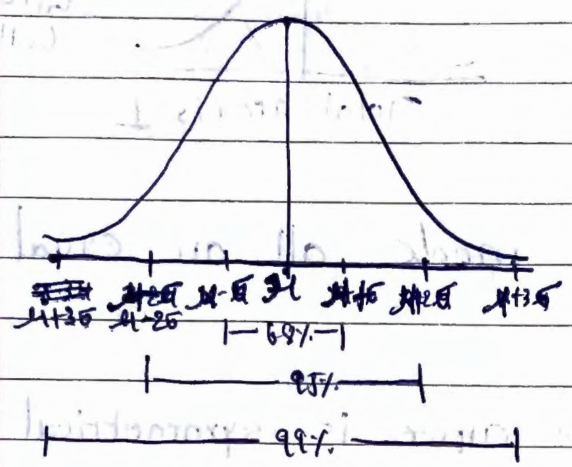
$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \quad \text{where } -\infty < z < \infty$$

5) X axis is the Asymptote (11^{el}) to the normal curve

6) No portion of the curve lies bellow x axis as f is the prob. fun which is not -ve.

7) MGF of normal distribution is $M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

Area property :




$P(\mu - \sigma < X < \mu + \sigma) = 0.6826$

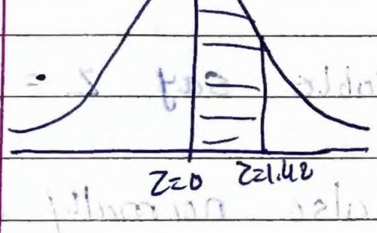
$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

compute the values of the following probabilites :

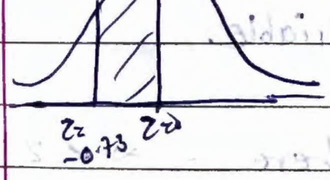
1) $P(0 \leq Z \leq 1.42)$

$= 0.4222$ (from Z table)



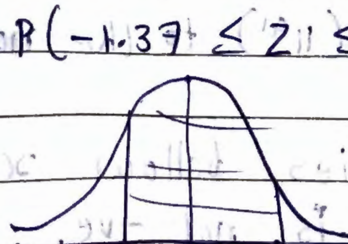

2) $P(-0.73 < Z < 0) = P(0 < Z < 0.73)$ ($\because$ symmetry so area equals on both sides)

$= 0.2673$ (from table)




3) $P(-1.37 \leq Z \leq 2.01) = P(-1.37 \leq Z \leq 0) + P(0 \leq Z \leq 2.01)$

$= P(0 \leq Z \leq 1.37) + P(0 \leq Z \leq 2.01)$

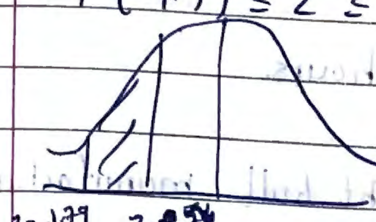



$= 0.4147 + 0.4778 = 0.8925$

4 $P(0.65 \le Z \le 1.26) = P(0 \le Z \le 1.26) - P(0 \le Z \le 0.65)$


$= 0.3962 - 0.2422$

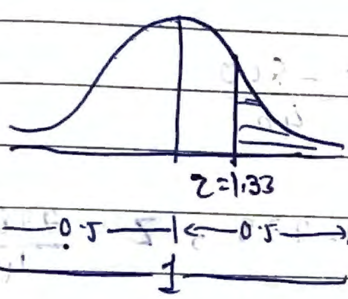
5 $P(-1.79 \le Z \le -0.54) = P(0.54 \le Z \le 1.79)$




$= P(0 \le Z \le 1.79) - P(0 \le Z \le 0.54)$

$= 0.4633 - 0.2054$

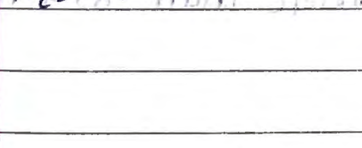
6 $P(Z \ge 1.33) = 0.5 - P(0 \le Z \le 1.33)$




$= 0.5 - 0.4082$

7 $P(|Z| \le 0.5) = P(-0.5 \le Z \le 0.5)$

$= 2P(0 \le Z \le 0.5)$




$= 2(0.1915)$

$= 0.3836$

## Examples:

1 An electrical firm manufactures light bulbs that have life before burn out, that is normally distributed with mean equal to 800 hours and a std. deviation of 40 hours. Find the prob. that a bulb burns-

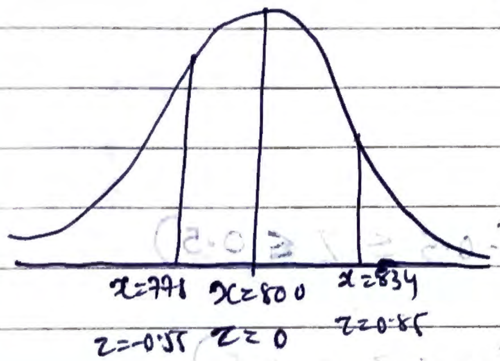
(i) More than 834 hours

(ii) between 778 and 834 hours.

→ Let $X$ be the life of the light bulb manufactured by the electrical firm.

$X$ is normally distributed with mean $\mu = 800$ hours and std. devi $\sigma = 40$ hours.

$$Z = \frac{X-\mu}{\sigma} = \frac{X-800}{40}$$




For $x = 778$, $Z = \frac{778-800}{40}$

$= \frac{-22}{40}$

$= -0.55$

For $X = 834$, $Z = \frac{834-800}{40}$

$= 0.85$

(i) The prob. that a bulb burns more than 834 hours is $P(X > 834)$

$= P(Z > 0.85)$

$= 0.5 - P(0 < Z < 0.85)$

$= 0.5 - 0.3023 = 0.1977$

(ii) $P(778 < X < 834) = P(-0.55 < Z < 0.85)$

$= P(-0.55 < Z < 0) + P(0 < Z < 0.85)$

$= P(0 < Z < 0.55) + P(0 < Z < 0.85)$

$= 0.2088 + 0.3023$

$= 0.5111$

2. The marks obtained by a no. of students in a certain subject are approximately normally distributed with mean 65 and std. deviation 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75?

So→

$X$ – marks obtained by a student in a subject.

$X \sim N(65, 5)$

so,

$$Z = \frac{X - 65}{5}$$

$$P(X \geq 75) = P(Z > 2) = 0.5 - P(0 < Z < 2)$$
$$= 0.5 - 0.4772$$
$$= 0.0228.$$

Since 3 students are selected at random.

we have $n = 3$.

If we consider the event of a student's scoring above 75 as a "success", then it is clear that $X$ follows binomial distribution with parameter $n = 3$ & $p = 0.0228$

ie $P(X = x) = {}^3C_x\, p^x q^{3-x}$, where $x = 0, 1, 2, 3$

At least one student would have scored above 75 is

$$P(X \geq 1) = 1 - P(X = 0)$$
$$= 1 - q^3$$
$$= 1 - (0.9772)^3$$
$$= 0.0669$$

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3. The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 500 with a Std. deviation of Rs. 50. Estimate the no. of workers whose weekly wages will be:

(i) between Rs. 400 and Rs. 600.

(ii) less than Rs. 400.

(iii) More than Rs. 600.

→

$$X \sim N(500, 50)$$

$$\therefore Z = \frac{X - 500}{50}$$

(i) $P(400 < X < 600)$

when $X = 400$ ; $Z = \frac{400-500}{50} = -2$

when $X = 600$ , $Z = \frac{600-500}{50} = 2$

Hence

$$P(400 < X < 600) = P(-2 < Z < 2)$$

$$= 2P(0 < Z < 2)$$

$$= 2(0.4772)$$

$$= 0.9544$$

Out of 1000 workmen, the expected no. of workers whose weekly wages will be betn Rs 400 & 600Rs is

$$N \times P(400 < X < 600) = 1000 \times 0.9544$$

$$= 954.4 \approx 954$$

(ii) $P(X < 400) = P(Z < -2) = P(Z > 2) = 0.5 - P(0 < Z < 2)$

$$= 0.0228$$

$\therefore NP(X < 400) = 1000 \times 0.0228 = 22.8 \approx 23$

(iii) $P(X > 600) = P(Z > 2)$

$\approx 23$

4. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and std. deviation of the distribution

Sol → Let $X \sim N(\mu, \sigma^2)$

(i) $P(X > 64) = 0.08$

(ii) $P(X < 45) = 0.31$

We define the std normal variate as

$$Z = \frac{X - \mu}{\sigma}$$

When $X = 64$, let $Z = Z_1$.

Then

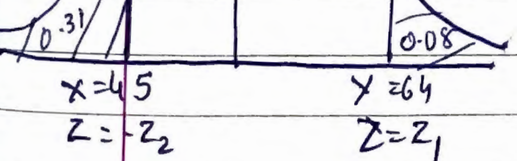
$$\frac{64 - \mu}{\sigma} = Z_1 \quad \text{or} \quad 64 - \mu = Z_1 \sigma$$

When $X = 45$, let $Z = -Z_2$ (say)

(Since $P(X < 45) = 0.31 < 0.5$, it is clear that the corresponding value of $Z$ is negative.)

Then

$$\frac{45 - \mu}{\sigma} = -Z_2 \quad \text{or} \quad 45 - \mu = -Z_2 \sigma$$




It is clear from fig that

$$P(0 < Z < Z_1) = 0.42$$

$$\Rightarrow \boxed{Z_1 = 1.4}$$

$$P(X > 64) = \frac{8}{100} = 0.08$$

$$P\left(\frac{X-\mu}{\sigma} > \frac{64-\mu}{\sigma}\right) = 0.08$$

$$P(Z > Z_1) = 0.08$$

ie $P(Z_1 < Z < \infty) = 0.08$

ie $0.5 - P(0 < Z < Z_1) = 0.08$

$\therefore P(0 < Z < Z_1) = 0.5 - 0.08 = 0.42$

& $P(X \le 45) = 0.31$

$\therefore P\left(\frac{X-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$

let $Z_2 = -y_1$

ie $P(Z < Z_2) = 0.31$ ie $P(Z > y_1) = 0.31$

ie $0.5 - P(0 < Z < y_1) = 0.31$

$\therefore P(0 < Z < y_1) = 0.5 - 0.31$

$= 0.19$

$\therefore y_1 = 0.5$

$\therefore -y_1 = -0.5$

$\therefore Z_2 = -0.5$

Now $64 - \mu = 1.4\sigma$

$45 - \mu = -0.5\sigma$

$\therefore$ solving

$\sigma = 10$

$\therefore \mu = 64 - 14 = 50$