Sardar Vallabhbhai Patel Institute of Technology, Vasad
B. E. First Sem (Mathematics 1)

Tutorial: 09

1 Find the volume of the solid obtained by rotating the region bounded by given curves about the specified line:

(i) $y = \sqrt{x}$, x=4, y=0, aboutX-axis (ii) y^2=x, x=2y, aboutY-axis

(iv) y^2=x, x^2=y, aboutx= -1.

2 Using cylindrical shell method find the volume of the solid obtained by rotating the region bounded by given curves about the specified line:

(i) y=2x-1, $y=\sqrt{x}$, x=0, about Y-axis

(ii) $y = x - x^2$ and y=0, aboutthe line x=2.

3 (i) Find the area of the region that lies inside the cardioid $r = 1+\cos\theta$ and outside the circler=1.

(ii) Find the area of the region bounded by the parabola $y = 2 - x^2$ and the line $y = -x$.

4 Show that the volume of a sphere of radiusris $V = \frac{4}{3}\pi r^3$ by slicingmethod.

5 Using L' Hospital's Rule, evaluate following Limits:

(i) $\lim_{x\to 0} \frac{x-\sin x}{x^3}$ (ii) $\lim_{x\to 0} \frac{3^x-2^x}{2x}$ (iii) $\lim_{x\to \frac{\pi}{2}} \frac{\log(x-\frac{\pi}{2})}{\tan x}$ (iv) $\lim_{x\to 1}(x^2-1)\tan(\frac{\pi x}{2})$

(v) $\lim_{x\to 0}[\frac{1}{x^2} - \frac{1}{\sin^2 x}]$ (vi) $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ (vii) $\lim_{x\to 0}(\sin x)^{\tan x}$ (viii) $\lim_{x\to 0}(\cos x)^{\frac{\pi}{2}-x}$

6 Evaluate the integrals.

(i) $\int_0^\infty \frac{1}{x^2+1}\,dx$ (ii) $\int_{-1}^1 \frac{1}{\frac{2}{x^3}}\,dx$ (iii) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

7 Discuss Type I and Type II improper integrals with examples of each. Evaluate

$$\int_2^\infty \frac{x+3}{(x-1)(x^2+1)}\,dx$$