AHIR SIR ENGINEERING MATHS ACADEMY
ALL MATHS OF DEGREE ENGINEERING (GTU)
"We Don't Speak, Our Result Speak..."
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Ami M-I : AA LJ
Krishna M-I : AA LD
Monika M-II : AA AIT
Hemal M-II : AA Indus
Vidhi M-II : AA AIT
Dhwani M-III : AA AIT
Amir M-III : AA AIT
Pooja M-III : AA Indus
Sunny M-III : AA AIT
Chandani M-IV : AA AIT
Harshil M-IV : AA VGEC
Sagar M-IV : AA LD
Sheela M-IV : AA AIT
Anuradha M-IV : AA AIT
Rachana M-IV : AA AIT

"We don't Speak, Our Result Speaks"

Ahir Sir
(Engineering Maths Academy)
(All Maths of Degree Engineering)

Toppers Of 4th Sem with AA Grade

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SPI : 9.7

Harshayu Desai
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SPI : 9.4

Aditya Tilve
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SPI : 9.0

Dheer Varyani
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SPI : 8.6

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Ajay Chavda
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Payal Mistry
SAL
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Shivam Rao
LJIT
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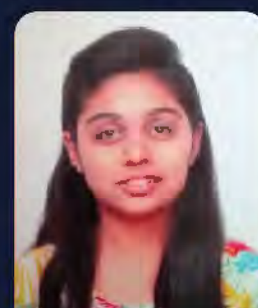

Pooja Desai
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Achievers Academy, T-17 Raspan Arcade, Opp. Gokul Party Plot, Nikol, Ahmedabad.

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"We Don't Speak, Our Result Speaks.."

ALL MATHS OF DEGREE ENGINEERING (GTU)

Maths-1 (CALCULUS)

Maths-2 (VCLA)

Maths-3 (AEM)

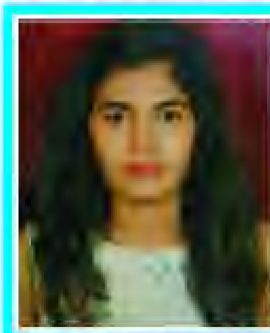
Maths-4 (CVNM)

RESULT OF 3rd SEM
AA GRADE STUDENTS

9.18 SPI

DHRUVI MODI
LJ

9.09 SPI



SOLANKI KIJALKUVARBA
SVBIT

9.00 SPI

KISHAN
INDUS

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HARSH
INDUS

- The only institute which focuses on Engineering Maths in Ahmedabad since last 14 years.
- We provide the Best coaching for one of the toughest subjects of Engineering.
- All topics are covered and explained by subject experts.
- Special revision of tough and important topics..
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Rajhans Cinema, Nikol,
Ahmedabad.

Gamma Function

- Defination :-

The Gamma function of m for improper integral defined as,

$$\boxed{\Gamma n = \int_0^{\infty} e^{-x} x^{n-1}\, dx \quad (n>0)}$$

- Properties :→

1) $\Gamma m+1 = m\,\Gamma m$

2) $\Gamma m+1 = m!$ (If m is positive integer)

3) $\Gamma \frac{1}{2} = \sqrt{\pi}$

- Simplify :→

$$\Gamma \frac{11}{2} = \frac{9\cdot 7\cdot 5\cdot 3\cdot 1}{2\cdot 2\cdot 2\cdot 2\cdot 2}\Gamma \frac{1}{2}$$

$$\Gamma \frac{14}{3} = \frac{11}{3}\cdot\frac{8}{3}\cdot\frac{5}{3}\cdot\frac{2}{3}\Gamma \frac{2}{3}$$

$\Rightarrow$ $\boxed{\text{Type} = 1}$

- $\int_0^\infty e^{-x^2}\,dx \Rightarrow$ let $x^2 = t$
- $\int_0^\infty e^{-\sqrt{x}}\, x\,dx \Rightarrow$ let $\sqrt{x} = t$
- $\int_0^\infty e^{-x^4}\, x^2\,dx \Rightarrow$ let $x^4 = t$
- $\int_0^\infty e^{-x^3}\,dx \Rightarrow$ let $x^3 = t$

• Examples :⟶

1. Evaluate $\int_0^\infty e^{-x^2}\,dx$

Solⁿ let $x^2 = t$

$\therefore x = t^{1/2}$

$\therefore dx = \frac{1}{2} t^{-1/2}$

$x \to 0 \Rightarrow t \to 0$	
$x \to \infty \Rightarrow t \to \infty$	

$$= \frac{1}{2} \int_{t=0}^{\infty} e^{-t}\, t^{-1/2}\, dt$$

$$= \frac{1}{2} \Gamma\left(-\frac{1}{2} + 1\right)$$

$$= \frac{1}{2} \Gamma\left(\frac{1}{2}\right)$$

$$\boxed{= \sqrt{\pi}}$$

Evaluate $\int_{0}^{\infty} e^{-x^3}\,dx$

Solⁿ

Let $x^3 = t$

$\therefore x = t^{1/3}$

$\therefore dx = \frac{1}{3} t^{-2/3}\,dt$

$x \to 0 \Rightarrow t \to 0$	
$x \to \infty \Rightarrow t \to \infty$	

$= \frac{1}{3} \int_{t=0}^{\infty} e^{-t}\, t^{-2/3}\,dt$

$= \frac{1}{3} \Gamma\left(\frac{-2}{3}+1\right)$

$= \boxed{\frac{1}{3}\Gamma\left(\frac{1}{3}\right)}$

3. Evaluate $\int_{0}^{\infty} e^{-x^4}\, x^2\,dx$

Solⁿ

Let $x^4 = t$

$\therefore x^2 = t^{1/2}$

$\therefore x = t^{1/4}$

$\therefore dx = \frac{1}{4} t^{-3/4}\,dt$

$\therefore$

$x \to 0 \Rightarrow t \to 0$	
$x \to \infty \Rightarrow t \to \infty$	

$= \frac{1}{4} \int_{t=0}^{\infty} e^{-t}\, t^{1/2}\, t^{-3/4}\,dt$

$= \frac{1}{4} \int_{t=0}^{\infty} e^{-t}\, t^{\left(\frac{2-3}{4}\right)}\,dt$

$= \frac{1}{4} \int_{0}^{\infty} e^{-t}\, t^{-1/4}\,dt$

$$= \frac{1}{4}\,\Gamma\left(\frac{-1}{4}+1\right)$$

$$= \boxed{\frac{1}{4}\,\Gamma\left(\frac{3}{4}\right)}$$

4. Prove that, $\int_0^\infty e^{-x^2}\sqrt{x}\,dx \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\,dx = \frac{\pi}{2\sqrt{2}}$

<u>Solⁿ</u> Let $x^2 = t$

$$\therefore x = t^{1/2}$$

$$\therefore dx = \frac{1}{2}\,t^{-1/2}\,dt$$

$$\boxed{\begin{array}{lcl} x \to 0 & \Rightarrow & t \to 0 \\ x \to \infty & \Rightarrow & t \to \infty \end{array}}$$

$$= \frac{1}{2}\int_{t=0}^{\infty} e^{-t}\, t^{1/4}\, t^{-1/2}\,dt \;\; \frac{1}{2}\int_0^\infty e^{-t}\,\frac{1}{t^{1/4}}\, t^{-1/2}\,dt$$

$$= \frac{1}{4}\int_0^\infty e^{-t}\, t^{-1/4}\,dt \int_0^\infty e^{-t}\, t^{-3/4}\,dt$$

$$= \frac{1}{4}\left[\int_0^\infty e^{-t}\, t^{-1/4}\,dt \int_0^\infty e^{-t}\, t^{-3/4}\,dt\right]$$

$$= \frac{1}{4}\left[\Gamma\left(\frac{-1}{4}+1\right)\right]\left[\Gamma\left(\frac{-3}{4}+1\right)\right]$$

$$= \frac{1}{4}\,\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)$$

$$= \frac{1}{4}\,\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)$$

Euler's formula :-

$$\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

By Euler's $\because n = 1/4$

$$= \frac{1}{4}\left[\frac{\pi}{\sin \pi/4}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{1/\sqrt{2}}\right]$$

$$= \frac{1}{4}\left[\sqrt{2}\pi\right]$$

$$= \boxed{\frac{\pi}{2\sqrt{2}}}$$

5. $\int_0^\infty e^{-ax} x^{n-1}\, dx$

Sol^n Let $ax = t$

$\therefore x = t/a$

$\therefore dx = dt/a$

$x \to 0$	$\Rightarrow t \to 0$
$x \to \infty$	$\Rightarrow t \to \infty$

$$= \int_{t=0}^{\infty} e^{-t}\left(\frac{t}{a}\right)^{n-1}\frac{dt}{a}$$

$$= \int_0^\infty e^{-t}\,\frac{t^{n-1}}{a^{n-1}}\,\frac{dt}{a}$$

$$= \frac{1}{a^n}\int_0^\infty e^{-t}\, t^{n-1}\, dt$$

$$= \boxed{\frac{1}{a^n}\,\Gamma(n)}$$

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$\boxed{\text{Type} = 2}$

- $\int_0^\infty \frac{x^p}{p^x}\,dx$ let $p^x = e^t$
- $\int_0^\infty \frac{x^3}{3^x}\,dx$ let $3^x = e^t$
- $\int_0^\infty \frac{x^m}{m^x}\,dx$ let $m^x = e^t$
- $\int_0^\infty \frac{x^{100}}{100^x}\,dx$ let $100^x = e^t$

1. Evaluate $\int_0^\infty \frac{x^c}{c^x}\,dx$

Solⁿ let $c^x = e^t$

$\therefore x \log c = t$

$\therefore x = \frac{t}{\log c}$

$\therefore dx = \frac{dt}{\log c}$

$x \to 0 \Rightarrow t \to 0$
$x \to \infty \Rightarrow t \to \infty$

$$-\int_{t=0}^\infty \frac{1}{e^t}\left(\frac{t}{\log c}\right)^c \frac{dt}{\log c}$$

$$= \int_{t=0}^\infty e^{-t}\,\frac{t^c}{(\log c)^c}\,\frac{dt}{(\log c)^1}$$

$$= \frac{1}{(\log c)^{c+1}}\int_0^\infty e^{-t}\,t^c\,dt$$

$$= \boxed{\frac{1}{(\log c)^{c+1}}\,\overline{|c+1}}$$

Evaluate $\int_0^\infty \frac{x^4}{4^x}\,dx$

Solⁿ Let $4^x = e^t$

$\therefore x \log 4 = t$

$\therefore x = t/\log 4$

$\therefore dx = dt/\log 4$

$x \to 0$	$\Rightarrow$	$t \to 0$
$x \to \infty$	$\Rightarrow$	$t \to \infty$

$$= \int_{t=0}^{\infty} \frac{1}{e^t}\left(\frac{t}{\log 4}\right)^4 \frac{dt}{\log 4}$$

$$= \int_{t=0}^{\infty} e^{-t}\, \frac{t^4}{(\log 4)^4}\, \frac{dt}{(\log 4)}$$

$$= \frac{1}{(\log 4)^5}\int_{t=0}^{\infty} e^{-t}\, t^4\, dt$$

$$= \frac{1}{(\log 4)^5}\,\Gamma 5$$

$$= \boxed{\frac{24}{(\log 4)^5}}$$

$\Rightarrow$ $\boxed{\text{Type} = 3}$

- $\int_0^\infty a^{-bx^2}\,dx \rightarrow$ let, $a^{-bx^2} = e^{-t}$
- $\int_0^\infty 7^{-4x^2}\,dx \Rightarrow 7^{-4x^2} = e^{-t}$
- $\int_0^\infty 5^{-3x^2}\,dx \Rightarrow 5^{-3x^2} = e^{-t}$
- $\int_0^\infty p^{-qx^2} \Rightarrow p^{-qx^2} = e^{t}$

• Examples :$\longrightarrow$

1. $\int_0^\infty a^{-bx^2}\,dx$

Sol^n

Let $a^{-bx^2} = e^{-t}$

$x \to 0$	$\Rightarrow t = 0$
$x \to \infty$	$\Rightarrow t \to \infty$

$\therefore -bx^2 \log a = -t$

$\therefore x^2 = \dfrac{t}{b\log a}$

$\therefore x = \dfrac{1}{\sqrt{b\log a}}\, t^{1/2}$

$\therefore dx = \dfrac{1}{\sqrt{b\log a}}\, \dfrac{1}{2}\, t^{-1/2}\,dt$

$\therefore dx = \dfrac{1}{2\sqrt{b\log a}}\, t^{-1/2}\,dt$

$\rightarrow \dfrac{1}{2\sqrt{b\log a}} \int_{t=0}^{\infty} e^{-t}\, t^{-1/2}\,dt$

$= \dfrac{1}{2\sqrt{b\log a}} \int_0^\infty e^{-t}\, t^{-1/2}\,dt$

$$= \frac{1}{2\sqrt{b\log a}} \Gamma\left(-\frac{1}{2}+1\right)$$

$$= \frac{1}{2\sqrt{b\log a}} \Gamma\left(\frac{1}{2}\right)$$

$$= \boxed{\frac{\sqrt{\pi}}{2\sqrt{b\log a}}}$$

2. $\int_0^\infty 7^{-4x^2}\, dx$

Sol^n Let $7^{-4x^2} = e^{-t}$

$$\boxed{\begin{array}{l} x \to 0 \;\rightarrow\; t \to 0 \\ x \to \infty \;\rightarrow\; t \to \infty \end{array}}$$

$$\therefore 4x^2 \log 7 = t$$

$$\therefore x^2 = t/4\log 7$$

$$\therefore x = \frac{t^{1/2}}{\sqrt{4\log 7}}$$

$$\therefore dx = \frac{\frac{1}{2}\, t^{-1/2}\, dt}{2\sqrt{\log 7}}$$

$$\therefore dx = \frac{1}{4}\, \frac{t^{-1/2}\, dt}{\sqrt{\log 7}}$$

$$= \frac{1}{4}\int_{t=0}^{\infty} e^{-t}\, \frac{t^{-1/2}}{\sqrt{\log 7}}\, dt$$

$$= \frac{1}{4\sqrt{\log 7}} \int_0^\infty e^{-t}\, t^{-1/2}\, dt$$

$$= \frac{1}{4\sqrt{\log 7}} \Gamma\left(-\frac{1}{2}+1\right)$$

$$= \frac{\Gamma(1/2)}{4\sqrt{\log 7}} = \boxed{\frac{\sqrt{\pi}}{4\sqrt{\log 7}}}$$

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Type = 4

- $\int_0^1 \log\left(\frac{1}{x}\right) dx \Rightarrow$ let, $\log\left(\frac{1}{x}\right) = t$
- $\int_0^1 \left[\log\left(\frac{1}{x}\right)\right]^{n-1} dx \qquad \log\left(\frac{1}{x}\right) = t$
- $\int_0^1 \frac{dx}{\sqrt{-\log x}} \qquad \log x = t$

- Examples :→

1. $\int_0^1 \left[\log\left(\frac{1}{x}\right)\right]^{(p-1)} dx$

Solⁿ Let $\log\left(\frac{1}{x}\right) = t$

$\therefore \frac{1}{x} = e^t$

$\therefore x = e^{-t}$

$\therefore dx = (-e^{-t})\, dt$

$\boxed{\begin{array}{lll} x \to 0 & \Rightarrow & t \to \infty \\ x \to 1 & \Rightarrow & t \to 0 \end{array}}$

$= \int_{t=\infty}^{0} (t)^{p-1} (-e^{-t})\, dt$

$= -\int_{t=\infty}^{0} e^{-t}\, t^{p-1}\, dt$

$\Rightarrow \int_{t=0}^{\infty} e^{-t}\, t^{p-1}\, dt = \boxed{\sqrt{p}}$

2. $\int_0^1 \left(\log\frac{1}{x}\right)^n x^m\, dx$

Solⁿ Let $\log\left(\frac{1}{x}\right) = t$

$\therefore \frac{1}{x} = e^t$

$$\therefore x = e^{-t}$$
$$\therefore dx = -e^{-t}\,dt$$

$x \to 0$	$\Rightarrow$	$t \to \infty$
$x \to 1$	$\Rightarrow$	$t \to 0$

$$= \int_{t=\infty}^{0} t^n \left(e^{-t}\right)^m \left(-e^{-t}\right) dt$$

$$= -\int_{0}^{\infty} e^{-(1+m)t}\, t^n\, dt$$

$$= \int_{0}^{\infty} e^{-(1+m)t}\, t^n\, dt$$

Let $(1+m)\,t = y$

$$\therefore t = y/(1+m)$$

$t \to 0$	$\Rightarrow$	$y \to 0$
$t \to \infty$	$\Rightarrow$	$y \to \infty$

$$\therefore dt = \frac{dy}{1+m}$$

$$= \int_{y=0}^{\infty} e^{-y} \left(\frac{y}{1+m}\right)^n \frac{dy}{(1+m)}$$

$$= \frac{1}{(1+m)^{n+1}} \int_{y=0}^{\infty} e^{-y}\, y^n\, dy$$

$$= \frac{\Gamma(n+1)}{(1+m)^{n+1}}$$

$$= \boxed{\frac{n!}{(1+m)^{n+1}}}$$

3. Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{-\log x}}$

Soln Let $-\log x = t$

$$\therefore x = e^{-t}$$
$$\therefore dx = -e^{-t}\,dt$$

$x \to 0$	$\Rightarrow$	$t \to \infty$
$x \to 1$	$\Rightarrow$	$t \to 0$

$$= \int_{t=\infty}^{0} \frac{-e^{-t}}{\sqrt{t}}\, dt$$

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$$= -\int_{t=\infty}^{0} e^{-t}\, t^{-1/2}\, dt$$

$$= \int_{t=0}^{\infty} e^{-t}\, t^{-1/2}\, dt$$

$$= \Gamma\left(-\frac{1}{2}+1\right)$$

$$= \boxed{\sqrt{\pi}}$$

4. Evaluate $\displaystyle\int_0^1 x^3 (\log x)^4\, dx$

Solⁿ

Let $\log x = -t$

$\therefore x = e^{-t}$

$\therefore dx = -e^{-t}\, dt$

$$\boxed{\begin{array}{l} x \to 0 \Rightarrow t \to \infty \\ x \to 1 \Rightarrow t \to 0 \end{array}}$$

$$= \int_{t=\infty}^{0} (e^{-t})^3 (-t)^4 (-e^{-t})\, dt$$

$$= -\int_{t=\infty}^{0} e^{-3t}\, t^4\, e^{-t}\, dt$$

$$= \int_{t=0}^{\infty} e^{-4t}\, t^4\, dt$$

Let, $4t = u \Rightarrow t = u/4$

$\therefore dt = du/4$

$$\boxed{\begin{array}{l} t \to 0 \Rightarrow u \to 0 \\ t \to \infty \Rightarrow u \to \infty \end{array}}$$

$$= \int_{u=0}^{\infty} e^{-u} \left(\frac{u}{4}\right)^4 \frac{du}{4}$$

$$= \frac{1}{(4)^5} \int_{u=0}^{\infty} e^{-u}\, u^4\, du$$

$$= \frac{1}{(4)^5} \Gamma 5 = \frac{4!}{(4)^5} = \boxed{\frac{3}{128}}$$

Beta functions

⇒ Defination :-

The improper Integral is known as Beta function is defined as,

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx \quad (m>0,\ n>0)$$

• Example :→

1. $\beta(4,6) = \int_0^1 x^3 (1-x)^5\,dx$

2. $\beta\left(\frac{3}{2},\frac{7}{2}\right) = \int_0^1 x^{1/2}(1-x)^{5/2}\,dx$

3. $\beta\left(\frac{1}{2},\frac{5}{2}\right) = \int_0^1 u^{-1/2}(1-u)^{3/2}\,du$

4. $\beta\left(\frac{5}{2},\frac{12}{5}\right) = \int_0^1 t^{+3/2}(1-t)^{7/5}\,dt$

⇒ Properties :-

1. $\beta(n,m) = \beta(m,n)$

2. $\beta(n,m) = 2\int_0^{\pi/2} \sin\theta^{2m-1}\cos\theta^{2n-1}\,d\theta$

3. $\beta(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

⟹ Relationship Between Beta & Gamma :⟶

$$\boxed{\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}}$$

• Examples :—

1. $\beta(2,3) = \dfrac{\Gamma 2 \, \Gamma 3}{\Gamma 5}$

2. $\beta\left(\frac{1}{2}, \frac{5}{2}\right) = \dfrac{\Gamma \frac{1}{2} \, \Gamma \frac{5}{2}}{\Gamma \frac{1}{2}+\frac{5}{2}}$

3. $\beta(3,7) = \dfrac{\Gamma 3 \, \Gamma 7}{\Gamma 10}$

$\boxed{\text{Type} = 1}$

→ Use of Equations,

$$\boxed{\begin{aligned} \beta(m,n) &= \frac{\Gamma m \, \Gamma n}{\Gamma m+n} \\ \Gamma m+1 &= m\,\Gamma m \end{aligned}}$$

* Examples :→

1. P.T $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

sol^n R.H.S $= \beta(m+1,n) + \beta(m,n+1)$

$$= \frac{\Gamma m+1 \;\Gamma n}{\Gamma m+n+1} + \frac{\Gamma m\;\Gamma n+1}{\Gamma m+n+1}$$

$$= \frac{m\,\Gamma m\;\Gamma n}{\Gamma (m+n)+1} + \frac{\Gamma m\;(n\,\Gamma n)}{\Gamma (m+n)+1}$$

$$= \frac{\Gamma m\;\Gamma n\;(m+n)}{(m+n)\;\Gamma m+n}$$

$$= \frac{\Gamma m\;\Gamma n}{\Gamma m+n}$$

$$= \beta(m,n)$$

$$= \text{L.H.S}$$

2. $\dfrac{\beta(m,n+1)}{n} = \dfrac{\beta(m,n)}{m+n}$

Sol^n

L.H.S $\dfrac{\beta(m,n+1)}{n} = \dfrac{1}{n}\,\dfrac{\Gamma m\;\Gamma n+1}{\Gamma (m+n)+1}$

$$= \frac{1}{n} \cdot \frac{\Gamma m \; n\Gamma n}{(m+n)\,\Gamma(m+n)}$$

$$= \frac{n}{n(m+n)} \cdot \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$$

$$= \frac{1}{m+n}\,\beta(m,n)$$

$$= \frac{\beta(m,n)}{m+n}$$

$= R.H.S$

3. $$\frac{\beta(m+1,n)}{\beta(m,n)} = \frac{m}{m+n}$$

Sol^n $$L.H.S = \frac{\beta(m+1,n)}{\beta(m,n)}$$

$$= \frac{\Gamma(m+1)\,\Gamma n}{\Gamma(m+n+1)} \cdot \frac{1}{\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}}$$

$$= \frac{\Gamma(m+1)\,\Gamma n}{\Gamma(m+n+1)} \cdot \frac{\Gamma(m+n)}{\Gamma m\,\Gamma n}$$

$$= \frac{m\,\cancel{\Gamma m}\,\cancel{\Gamma n}}{(m+n)\,\cancel{\Gamma(m+n)}} \cdot \frac{\cancel{\Gamma(m+n)}}{\cancel{\Gamma m}\,\cancel{\Gamma n}}$$

$$= \frac{m}{m+n}$$

$= R.H.S$

4. $$\beta(m,n)\cdot\beta(m+n,p)\cdot\beta(m+n+p,q) = \frac{\Gamma m\,\Gamma n\,\Gamma p\,\Gamma q}{\Gamma(m+n+p+q)}$$

L.H.S

$$= \beta(m,n) \cdot \beta(m+n, p) \cdot \beta(m+n+p, q)$$

$$= \frac{\Gamma m \, \Gamma n}{\Gamma m+n} \cdot \frac{\Gamma m+n \; \Gamma p}{\Gamma m+n+p} \cdot \frac{\Gamma m+n+p \cdot \Gamma q}{\Gamma m+n+p+q}$$

$$= \frac{\Gamma m \, \Gamma n \, \Gamma p \, \Gamma q}{\Gamma m+n+p+q}$$

$= \text{R.H.S}$

→ $\boxed{\text{Type} = 2}$

- $\int_0^1 x^2 (1-x^4)^3 \, dx \qquad x^4 = t$
- $\int_0^3 x^3 (3-x^2)^5 \, dx \qquad x^2 = 3t$
- $\int_0^5 x^5 (5-x^7)^2 \, dx \qquad x^7 = 5t$
- $\int_0^2 x^{1/2} (2-\sqrt{x})^5 \, dx \qquad \sqrt{x} = 2t$

• <u>Example</u> :→

1. $\int_0^1 \sqrt{x} \sqrt{1-x^2} \, dx$

<u>Sol^n</u>

Let $x^2 = t$

$x = t^{1/2}$

$\therefore dx = \frac{1}{2} t^{-1/2} dt$

$x \to 0$	$\Rightarrow$	$t \to 0$
$x \to 1$	$\Rightarrow$	$t \to 1$

- $\int_{t=0}^{1} (t^{1/2})^{1/2} (1-t)^{1/2} \frac{1}{2} t^{-1/2} dt$

$= \frac{1}{2} \int_{t=0}^{1} t^{1/4 - 1/2} (1-t)^{1/2} dt$

$= \frac{1}{2} \int_{t=0}^{1} t^{-1/4} (1-t)^{1/2} dt$

$= \frac{1}{2} \beta \left(-1/4 + 1 , 1/2 + 1\right)$

$= \boxed{\frac{1}{2} \beta \left(\frac{3}{4}, \frac{3}{2}\right)}$

Evaluate $\int_0^2 x^4 (8-x^3)^{-1/3}\,dx$

Sol^n

Let $x^3 = 8t$

$\therefore x = (8t)^{1/3}$

$x \to 0 \Rightarrow t \to 0$	
$x \to 2 \Rightarrow t \to 1$	

$\therefore x = 2t^{1/3}$

$\therefore dx = 2\,\frac{1}{3}\,t^{-2/3}\,dt$

$\therefore dx = \frac{2}{3}\,t^{-2/3}\,dt$

$$= \int_{t=0}^{1} (2t^{1/3})^4 (8-8t)^{-1/3}\,\frac{2}{3}\,t^{-2/3}\,dt$$

$$= (2)^4 (8)^{-1/3}\,\frac{2}{3}\int_{t=0}^{1} t^{4/3}\,t^{-2/3}\,(1-t)^{-1/3}\,dt$$

$$= \frac{16}{(8)^{1/3}}\,\frac{2}{3}\int_{t=0}^{1} t^{2/3}(1-t)^{-1/3}\,dt$$

$$= \frac{16}{3}\,\beta\left(\frac{2}{3}+1,\ -\frac{1}{3}+1\right)$$

$$= \boxed{\frac{16}{3}\,\beta\left(\frac{5}{3},\frac{2}{3}\right)}$$

5. P.T $\int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}} = \frac{128}{35}$

Sol^n

Let $x^{1/4} = t$

$\therefore x = t^4$

$\therefore dx = 4t^3\,dt$

$x \to 0 \Rightarrow t \to 0$	
$x \to 1 \Rightarrow t \to 1$	

$$= \int_{t=0}^{1} \frac{4t^3\,dt}{\sqrt{1-t}}$$

$$= 4\int_{t=0}^{1} t^3 (1-t)^{-1/2}\,dt$$

$$= 4\,\beta\,(4, 1/2)$$

$$= \frac{4\,\Gamma 4\,\Gamma \tfrac{1}{2}}{\Gamma\left(4+\frac{1}{2}\right)}$$

$$= \frac{4\,(3!)\,(\Gamma \tfrac{1}{2})}{\Gamma \tfrac{9}{2}}$$

$$= \frac{24\,\Gamma \tfrac{1}{2}}{\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\,\Gamma\frac{1}{2}}$$

$$= \boxed{\frac{128}{35}}$$

$$= \text{R.H.S}$$

4. $$\int_0^2 x^7 \left(16 - x^4\right)^{10} dx = \frac{(16)^{11}}{33}$$

Sol^n

Let $x^4 = 16t$

$\therefore x = (16t)^{1/4}$

$= 2\,t^{1/4}$

$$\boxed{\begin{array}{l} x \to 0 \Rightarrow t \to 0 \\ x \to 2 \Rightarrow t \to 1 \end{array}}$$

$$\therefore dx = 2\left(\frac{1}{4}\,t^{-3/4}\,dt\right)$$

$$\therefore dx = \frac{1}{2}\,t^{-3/4}\,dt$$

$$= \int_{t=0}^{1} \left[\left(2\,t^{1/4}\right)^7 (16 - 16t)^{10}\, \frac{1}{2}\, t^{-3/4}\, dt \right.$$

$$= 2^7\,(16)^{10}\,\frac{1}{2} \int_{t=0}^{1} t^{7/4 - 3/4}\, (1-t)^{10}\, dt$$

$$= 2^6\,(16)^{10} \int_{t=0}^{1} t^1\,(1-t)^{10}\, dt$$

$$= 2^2 \cdot 2^4 \cdot (16)^{10} \int_{t=0}^{1} t \, (1-t)^{10} \, dt$$

$$= 4 \, (16)^{11} \int_{t=0}^{1} t^1 \, (1-t)^{10} \, dt$$

$$= 4 \, (16)^{11} \quad \beta(2,11)$$

$$= 4 \, (16)^{11} \quad \frac{\Gamma 2 \, \Gamma 11}{\Gamma 13}$$

$$= 4 \, (16)^{11} \left[\frac{(1!)(10!)}{12!} \right]$$

$$= 4 \, (16)^{11} \left[\frac{10!}{12 \times 11 \times 10!} \right]$$

$$= \boxed{\frac{(16)^{11}}{33}}$$

$= R.H.S$

Type = 3

$$\boxed{\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{2\,\Gamma\frac{p+q+1}{2}}}$$

Proof :→

We know that according to,

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

$$\frac{\Gamma m\,\Gamma n}{\Gamma m+n} = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

Let, $2m-1 = p$	$2n-1 = q$
$\therefore 2m = p+1$	$\therefore 2n = q+1$
$\therefore \boxed{m = \frac{p+1}{2}}$	$\therefore \boxed{n = \frac{q+1}{2}}$

$$\frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}} = 2\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta$$

$$\frac{1}{2}\,\frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}} = \int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta$$

$$\therefore \int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2}\,\frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}}$$

Ex :- $\int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta = \frac{\Gamma 2\,\Gamma 3/2}{2\,\Gamma 7/2}$

$p = 3,\ q = 2$

Euler's Formula

$\Rightarrow$ $$\Gamma n \, \Gamma 1-n = \frac{\pi}{\sin n\pi}$$

- Examples :-

1. $\Gamma \frac{1}{2} \, \Gamma \frac{1}{2}$

Sol^n

We can written,

$$\Gamma \frac{1}{2} \, \Gamma 1-\frac{1}{2}$$

$$\Gamma \frac{1}{2} \, \Gamma 1-\frac{1}{2} = \frac{\pi}{\sin \pi/2} = \boxed{\pi}$$

2. $\Gamma \frac{1}{4} \, \Gamma \frac{3}{4}$

Sol^n

$$\Gamma \frac{1}{4} \, \Gamma 1-\frac{1}{4} \qquad (n = 1/4)$$

$$\Gamma \frac{1}{4} \, \Gamma 1-\frac{1}{4} = \frac{\pi}{\sin \pi/4} = \frac{\pi}{1/\sqrt{2}} = \boxed{\sqrt{2}\pi}$$

3. Prove that, $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$

Sol^n

$$\int_0^{\pi/2} \sin^{1/2}\theta \, d\theta \int_0^{\pi/2} \sin^{-1/2}\theta \, d\theta$$

$$= \int_0^{\pi/2} \sin^{1/2}\theta \cos^0\theta \, d\theta \int_0^{\pi/2} \sin^{-1/2}\theta \cos^0\theta \, d\theta$$

$p = 1/2, \; q = 0 \qquad\qquad p = -1/2, \; q = 0$

$$\frac{\Gamma\left(\frac{1/2+1}{2}\right)\Gamma\left(\frac{0+1}{2}\right)}{2\,\Gamma\left(\frac{3}{4}+\frac{1}{2}\right)} \cdot \frac{\Gamma\left(\frac{-1/2+1}{2}\right)\Gamma\left(\frac{0+1}{2}\right)}{2\,\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}$$

$$= \frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(1/2\right)}{2\,\Gamma\left(\frac{5}{4}\right)} \quad \frac{\Gamma\left(1/4\right)\Gamma\left(1/2\right)}{2\,\Gamma\left(3/4\right)}$$

$$= \frac{1}{4}\,\frac{\Gamma(1/2)\,\Gamma(1/4)\,\Gamma(1/2)}{\frac{1}{4}\,\Gamma(1/4)}$$

$$= \Gamma(1/2)\,\Gamma(1/2)$$

$$= \sqrt{\pi}\,\sqrt{\pi}$$

$$= \boxed{\pi}$$

$= \underline{R.H.S}$

4. $$4\int_0^1 \frac{x^2}{(1-x^4)^{1/2}}\,dx \int_0^1 \frac{dx}{(1-x^4)^{1/2}}\,dx$$

$\underline{\underline{sol^n}}$ let $x^4 = \sin^2\theta$

$\therefore x^2 = \sin\theta$

$\therefore x = \sin^{1/2}\theta$

$\therefore dx = \frac{1}{2}\sin^{-1/2}\theta\,\cos\theta\,d\theta$

$x \to 0$	$\Rightarrow$	$\theta \to 0$
$x \to 1$	$\Rightarrow$	$\theta \to \pi/2$

$$= 4\int_{\theta=0}^{\pi/2} \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\;\frac{1}{2}\sin^{-1/2}\theta\,\cos\theta\,d\theta$$

$$= \int_0^{\pi/2} \frac{1/2\,\sin^{-1/2}\theta\,\cos\theta\,d\theta}{(1-\sin^2\theta)^{1/2}}$$

$$4\left[\frac{1}{2}\int_0^{\pi/2}\frac{\sin\theta^{(1-1/2)}}{\cos\theta}\cos\theta\,d\theta \times \frac{1}{2}\int_0^{\pi/2}\frac{\sin^{-1/2}\theta\cos\theta}{\cos\theta}\,d\theta\right.$$

$$= 4\cdot\frac{1}{4}\left[\int_0^{\pi/2}\sin^{1/2}\theta\,d\theta\int_0^{\pi/2}\sin^{-1/2}\theta\,d\theta\right.$$

As per previous Example,

$$= \boxed{\pi}$$

5. P.T $\int_0^{\pi/2}\sin^p x\,dx\int_0^{\pi/2}\sin^{p+1}x\,dx = \frac{\pi}{2(p+1)}$

Sol^n $\int_0^{\pi/2}\sin^p x\cos^0 x\,dx\int_0^{\pi/2}\sin^{p+1}x\cos^0 x\,dx$

$$= \frac{\Gamma\frac{p+1}{2}\Gamma\frac{0+1}{2}}{2\Gamma\frac{p+2}{2}}\cdot\frac{\Gamma\frac{(p+1)+1}{2}\Gamma\frac{0+1}{2}}{2\Gamma\frac{p+3}{2}}$$

$$= \frac{\Gamma\frac{p+1}{2}\Gamma 1/2\,\Gamma 1/2\,\Gamma\frac{p+2}{2}}{4\Gamma\frac{p+2}{2}\Gamma\left(\frac{p+1}{2}\right)+1}$$

$$= \frac{\pi}{4}\frac{\Gamma\frac{p+1}{2}}{\Gamma\frac{p+1}{2}\left(\frac{p+1}{2}\right)}$$

$$= \frac{\pi}{2}\left(\frac{1}{p+1}\right)$$

$$= \boxed{\frac{\pi}{2(p+1)}}$$

$= R.H.S$

6. P.T $\displaystyle\int_0^{\pi/2} \frac{dx}{\tan^p x} = \frac{\pi}{2} \sec \frac{p\pi}{2}$

Sol^n L.H.S $= \displaystyle\int_0^{\pi/2} \frac{\cos^p x}{\sin^p x}\, dx$

$$= \int_0^{\pi/2} \sin^{-p} x \,\cos^p x \, dx$$

$$= \frac{\Gamma\left(\frac{-p+1}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{2\,\Gamma\left(\frac{(1-p)+(1+p)}{2}\right)}$$

$$= \frac{\Gamma\left(\frac{1-p}{2}\right)\Gamma\left(\frac{1+p}{2}\right)}{2\,\Gamma 1}$$

$$= \frac{1}{2}\, \Gamma\left(\frac{1+p}{2}\right)$$

$$\Gamma\left(1-\left(\frac{1+p}{2}\right)\right)$$

Using Euler's formula, $\left(n = \frac{1+p}{2}\right)$

$$= \frac{1}{2}\left[\frac{\pi}{\sin\left(\left[\frac{p+1}{2}\right]\pi\right)}\right]$$

$$= \frac{\pi}{2}\left[\frac{1}{\sin\left(\frac{\pi}{2}+\frac{p\pi}{2}\right)}\right]$$

$$= \frac{\pi}{2}\left[\frac{1}{\cos \pi/2}\right]$$

$$= \boxed{\frac{\pi}{2} \sec\left(\frac{p\pi}{2}\right)}$$

$= R.H.S$

P.T $\int_0^\infty \frac{dx}{x^n + a^n} = \frac{\pi}{n a^{n-1} \sin\left(\frac{\pi}{n}\right)}$

Sol^n

Let $x^n = a^n \tan^2\theta$

$\therefore x = a \tan^{2/n}\theta$

$\therefore dx = \frac{2a}{n} \tan^{2/n - 1}\theta \sec^2\theta \, d\theta$

$x \to 0 \Rightarrow \theta \to 0$

$x \to \infty \Rightarrow \theta \to \pi/2$

$$\frac{2a}{n} \int_{\theta=0}^{\pi/2} \frac{\tan^{2/n-1}\theta \sec^2\theta \, d\theta}{a^n \tan^2\theta + a^n}$$

$$= \frac{2a}{n} \int_0^{\pi/2} \frac{\tan^{2/n-1}\theta \sec^2\theta \, d\theta}{a^n (\tan^2\theta + 1)}$$

$$= \frac{2a}{a^n \cdot n} \int_0^{\pi/2} \frac{\tan^{2/n-1}\theta \cancel{\sec^2\theta} \, d\theta}{\cancel{\sec^2\theta}}$$

$$= \frac{2}{n a^{n-1}} \int_0^{\pi/2} \tan^{2/n-1}\theta \, d\theta$$

$$= \frac{2}{n a^{n-1}} \int_0^{\pi/2} \frac{\sin^{2/n-1}\theta}{\cos^{2/n-1}\theta} \, d\theta$$

$$= \frac{2}{n a^{n-1}} \int_0^{\pi/2} \sin^{2/n-1}\theta \cos^{1-2/n}\theta \, d\theta$$

$$= \frac{2}{n a^{n-1}} \int_0^{\pi/2} \sin^{2/n-1}\theta \cos^{1-2/n}\theta \, d\theta$$

$$= \frac{2}{n a^{n-1}} \int_0^{\pi/2} \sin^{2/n-1}\theta \cos^{1-2/n}\theta \, d\theta$$

$$= \frac{2}{n\,a^{n-1}} \cdot \frac{\Gamma\left(\frac{\left(\frac{2}{n}-1\right)+1}{2}\right)\Gamma\left(\frac{\left(1-\frac{2}{n}\right)+1}{2}\right)}{2\,\Gamma\left(\frac{1}{n}+1-\frac{1}{n}\right)}$$

$$= \frac{2}{n\,a^{n-1}} \cdot \frac{\Gamma(1/n)\,\Gamma(1-1/n)}{2\,\Gamma(1)}$$

$$= \frac{1}{n\,a^{n-1}}\,\Gamma\left(\frac{1}{n}\right)\Gamma\left(1-\frac{1}{n}\right)$$

$$= \boxed{\frac{1}{n\,a^{n-1}} \cdot \frac{\pi}{\sin(\pi/n)}}$$

$$= \frac{\pi}{n\,a^{n-1}}\,\operatorname{cosec}\left(\frac{\pi}{n}\right)$$

$$= \text{R.H.S}$$

→ **Formula :-** $\boxed{\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \beta(m,n)}$

Ex $\displaystyle\int_0^\infty \frac{x^2}{(1+x)^5}\,dx = \int_0^\infty \frac{x^{3-1}}{(1+x)^{3+2}}$

$$= \boxed{\beta(3,2)}$$

Ex :- $\displaystyle\int_0^\infty \frac{x^{3/2}}{(1+x)^{9/2}}\,dx$

$$= \int_0^\infty \frac{x^{5/2-1}}{(1+x)^{5/2+2}}$$

$$= \boxed{\beta\left(\frac{5}{2}, 2\right)}$$

P.T $\int_0^\infty \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}} = 2\beta(p,q)$

Sol^n

$$L.H.S = \int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}}\,dx + \int_0^\infty \frac{x^{q-1}}{(1+x)^{p+q}}\,dx$$

$$= \beta(p,q) + \beta(q,p)$$

$$= \beta(p,q) + \beta(p,q) \quad \left(\because \beta(q,p) = \beta(p,q)\right)$$

$$= \boxed{2\beta(p,q)}$$

$$= \underline{R.H.S}$$

2. P.T $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}\,dx$

Sol^n

$$= \int_0^\infty \frac{x^4}{(1+x)^{15}}\,dx + \int_0^\infty \frac{x^9}{(1+x)^{15}}\,dx$$

$$= \int_0^\infty \frac{x^{5-1}}{(1+x)^{5+10}}\,dx + \int_0^\infty \frac{x^{10-1}}{(x+1)}\,dx$$

$$= \beta(5,10) + \beta(10,5)$$

$$= 2\beta(5,10) \qquad \left[\beta(10,5) = \beta(5,10)\right]$$

$$= \frac{2\,\Gamma 5\,\Gamma 10}{\Gamma 15}$$

$$= \frac{2\,(4!)(9!)}{14!}$$

$$= \frac{\cancel{2}\,(\overset{2}{\cancel{24}})(\cancel{9!})}{\underset{7}{\cancel{14}} \times 13 \times \cancel{12} \times 11 \times \underset{5}{\cancel{10}} \times \cancel{9!}}$$

$$= \boxed{\frac{1}{5005}}$$

Ahir_Sir

- **Special Example :→**

$$\boxed{\begin{array}{l} \int_a^b f(x)\,dx \\ \quad \text{let} \quad t = \dfrac{x - L.L}{U.L - L.L} \\ \quad\quad t = \dfrac{x-a}{b-a} \end{array}}$$

- **Examples :→**

1. P.T $\int_3^7 \sqrt[4]{(x-3)(7-x)}\,dx = \dfrac{2\left(\sqrt{1/4}\right)^2}{3\sqrt{\pi}}$

Sol^n let $t = \dfrac{x-3}{7-3}$

$\therefore t = \dfrac{x-3}{4}$

$\therefore 4t = x-3$

$\therefore x = 3+4t$

$\therefore dx = 4dt$

$$\boxed{\begin{array}{lll} x \to 3 & \Rightarrow & t \to 0 \\ x \to 7 & \Rightarrow & t \to 1 \end{array}}$$

$$= \int_{t=0}^{1} \left[(4t)\left[7-(3+4t)\right] \right]^{1/4} (4dt)$$

$$= \int_{t=0}^{1} \left[4t\ (4-4t) \right]^{1/4} 4dt$$

$$= 4\int_{t=0}^{1} \left[16t\ (1-t) \right]^{1/4} dt$$

$$= 4\ (16)^{1/4} \int_{t=0}^{1} t^{1/4}\ (1-t)^{1/4}\ dt$$

$$= 8 \int_0^1 t^{1/4} (1-t)^{1/4}\, dt$$

$$= 8\, \beta\left(\frac{5}{4}, 5/4\right)$$

$$= 8 \frac{\Gamma\frac{5}{4}\,\Gamma\frac{5}{4}}{\Gamma\left(\frac{5}{4}+\frac{5}{4}\right)}$$

$$= 8 \frac{\left(\frac{1}{4}\Gamma\frac{1}{4}\right)\left(\frac{1}{4}\Gamma\frac{1}{4}\right)}{\Gamma\frac{5}{2}}$$

$$= \frac{1}{2} \frac{\left(\Gamma\frac{1}{4}\right)^2}{\frac{3}{2}\,\frac{1}{2}\,\Gamma\frac{1}{2}}$$

$$= \boxed{\frac{2}{3} \frac{\left(\Gamma\frac{1}{4}\right)^2}{\sqrt{\pi}}}$$

2. P.T $\int_{-1}^{1} (1+x)^m (1-x)^n\, dx = 2^{m+n+1} \frac{m!\, n!}{(m+n+1)!}$

Sol^n Let $t = \frac{x - L.L}{U.L - L.L}$

$\therefore t = \frac{x-(-1)}{1-(-1)}$ $\qquad \therefore t = \frac{x+1}{2}$

$\therefore x = 2t - 1$

$\therefore dx = 2\,dt$

$x \to -1 \Rightarrow t \to 0$	
$x \to 1 \to t \to 1$	

$$2 \int_0^1 (2t)^m \left[1 - (2t-1)\right]^n dt$$

$$= 2 \cdot 2^{m} \int_0^1 t^m \left[(2-2t)\right]^n dt$$

$$= 2^{m+n+1} \int_0^1 t^m (1-t)^n \, dt$$

$$= 2^{m+n+1} \beta(m+1, n+1)$$

$$= 2^{m+n+1} \frac{\Gamma m+1 \; \Gamma n+1}{\Gamma (m+n+1)+1}$$

$$= 2^{m+n+1} \frac{m! \, n!}{(m+n+1)!}$$

Duplication Formula (legendre's formula)

$$\boxed{\Gamma n \,\Gamma\!\left(n+\tfrac{1}{2}\right) = \frac{\sqrt{\pi}\,\Gamma 2n}{2^{2n-1}}}$$

1. P.T $\beta(m,m)\,\beta\left(m+\tfrac{1}{2}, m+\tfrac{1}{2}\right) = \frac{\pi}{m}\, 2^{1-4m}$

Sol^n

$$L.H.S = \frac{\Gamma m\,\Gamma m}{\Gamma 2m}\;\frac{\Gamma\!\left(m+\tfrac{1}{2}\right)\Gamma\!\left(m+\tfrac{1}{2}\right)}{\Gamma(2m+1)}$$

$$= \frac{\Gamma m\,\Gamma m}{\Gamma 2m}\;\frac{\left(\Gamma\!\left(m+\tfrac{1}{2}\right)\right)^2}{2m\,\Gamma 2m}$$

$$= \frac{1}{2m}\left[\frac{\Gamma m\,\Gamma\!\left(m+\tfrac{1}{2}\right)}{\Gamma 2m}\right]^2$$

By Duplication formula

$$= \frac{1}{2m}\left[\frac{\sqrt{\pi}}{2^{2m-1}}\right]^2$$

$$= \frac{1}{2m}\left[\frac{\pi}{2^{4m-2}}\right]$$

$$= \frac{\pi}{m}\;\frac{1}{2^{4m-1}}$$

$$= \boxed{\frac{\pi}{m}\,2^{1-4m}}$$

$= R.H.S$

* **Proofs :→**

1. $\Gamma\frac{1}{2} = \sqrt{\pi}$

sol^n

We know that,

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2} \, \Gamma\frac{q+1}{2}}{2 \, \Gamma\frac{p+q+2}{2}}$$

Let, $p = q = 0$

$$\therefore \int_0^{\pi/2} d\theta = \frac{\Gamma\frac{0+1}{2} \, \Gamma\frac{0+1}{2}}{2 \, \Gamma 1}$$

$$\therefore \frac{\pi}{2} = \frac{1}{2}\left(\Gamma\frac{1}{2}\right)^2$$

$$\therefore \pi = \left(\Gamma\frac{1}{2}\right)^2$$

$$\therefore \boxed{\Gamma\frac{1}{2} = \sqrt{\pi}}$$

2. $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$

sol^n property of Gamma function,

$$\frac{\Gamma m}{t^m} = \int_0^\infty e^{-tx} x^{m-1} \, dx$$

let $m = m+n$

$t = 1+y$

$$\frac{\Gamma m+n}{(1+y)^{m+n}} = \int_0^\infty e^{-x(1+y)} x^{m+n-1} \, dx$$

$$= \int_0^{\infty} e^{-x} e^{-xy} x^{m+n-1} \, dx$$

multiply both side by $\int_0^{\infty} y^{n-1} \, dy$

$$\Gamma(m+n) \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} \, dy = \int_0^{\infty} e^{-x} x^{m+n-1} \, dx \int_0^{\infty} e^{-xy} y^{n-1} \, dy$$

$$\Gamma(m+n) \, \beta(n,m) = \int_0^{\infty} e^{-x} x^{m-1} \cancel{x^{n}} \, dx \, \frac{\Gamma(n)}{\cancel{x^{n}}}$$

$$= \int_0^{\infty} e^{-x} x^{m-1} \, dx \, \Gamma(n)$$

$$\Gamma(m+n) \, \beta(m,n) = \Gamma(n) \int_0^{\infty} e^{-x} x^{m-1} \, dx$$

$$\Gamma(m+n) \, \beta(m,n) = \Gamma(n) \, \Gamma(m)$$

$$\therefore \boxed{\beta(m,n) = \frac{\Gamma(m) \, \Gamma(n)}{\Gamma(m+n)}}$$

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