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B. E. First Sem (Mathematics 1)

Tutorial: 01

1 For given ε, find $\delta > 0$ such that for all (x, y),

$\sqrt{x^2+y^2} < \delta \;\Rightarrow\; f(x,y) - f(0,0) < \varepsilon$

$\quad$ *where* $\; f(x,y) = (x+y)/(x^2+1), \qquad \varepsilon = 0.01$

2 Find limit for the following functions as $(x,y) \to (0,0)$, if exists.

(1) $f(x,y) = \dfrac{x+y}{x+2y}$ $\qquad$ (2) $f(x,y) = \dfrac{x^4-y^2}{x^4+y^2}$ $\qquad$ (3) $f(x,y) = \dfrac{x^2-y^2}{x^2+y^2}$

3 Show that

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & : (x,y) \neq 0 \\ 0 & : (x,y) = 0 \end{cases}$$ is continuous at origin.

4 Find all the second order partial derivatives

(i) $g(x,y) = x^2y - \cos y + y\sin x$

(ii) $h(x,y) = \tan^{-1}\left(\dfrac{y}{x}\right)$

5 If $u = x^2y + y^2z + z^2x$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = (x+y+z)^2$

6 If $u = x^3y + e^{xy^2}$ then prove that $\dfrac{\partial^2 u}{\partial x\partial y} = \dfrac{\partial^2 u}{\partial y\partial x}$

7 If $u = \log(x^2+y^2) + \tan^{-1}\left(\dfrac{y}{x}\right)$ then prove that u satisfies Laplace's equation

$u_{xx} + u_{yy} = 0$

8 If $u = \log(x^3+y^3+z^3-3xyz)$ then prove that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 u = -\dfrac{9}{(x+y+z)^2}$

9 Let $w = f(x, y, z)$ be a function of three independent variables, write the formal definition of the partial derivative for $\frac{\partial f}{\partial z}$ at (1,2,3) for $f(x,y,z) = x^2yz^2$.

10 Write chain rule for dw/dt where w=f(x, y, z), x=g_1(t), y=g_2(t), z=g_3(t) & Find dw/dt at given value of t.

(i) $w = \frac{x}{z} + \frac{y}{z}, x = \cos^2 t, y = \sin^2 t, z = 1/t; t = 3.$

(ii) $w = 2ye^x - \ln z,\ x = \ln(t^2+1),\ y = \tan^{-1} t,\ z = e^t;$

11 Find the value of $\frac{\partial z}{\partial x}$ at the point $(1,1,1,)$ if the equation $xy + z^3x - 2yz = 0$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

12 If z= f(x,y) , x= $e^u + e^{-v}$, y = $e^{-u} - e^v$ then prove that $\frac{\partial f}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

13 By using partial derivatives find the value of $\frac{dy}{dx}$ for $xe^y + \sin(xy) + y - \log 2 = 0$ at (0,log2).