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Subject: Probability and Statistics   Subject Code: 310006
Tutorial 6: PDF PMF CDF Joint PMF joint PDF Expected value

Q.1	The probability distribution of a random variable X is given below.

X:	-2	-1	0	1	2
P(x=X)	0.2	0.1	0.3	0.3	0.1

Find (i) E(X), (ii) Var(X)

Q.2	If two random variables X and Y have the joint density

$$f(x,y) = \begin{cases} k(x^2 + y), & for\ 0 < x < 1. 0 < y < 10 \\ 0, & otherwise \end{cases}$$

find k and the mean of the conditional density $f_1(x \mid 0.5)$ where $f_1(x)$ is the marginal probability density of X.

Q.3	Three balanced coins are tossed. Let X denote the number of heads on the first two coins and Y denote the number of tails on the last two coins. Find the joint distribution of X and Y.

Q.4	The number of page requests that arrive at a Web server is a Poisson random variable. Its probability distribution is as follows:

Number of requests/sec.	x	0	1	2	3	4	5	6
Probability	$f(x)$	0.368	0.368	0.184	0.061	0.015	0.003	0.001

Q.5	In a certain district, the proportion of highway sections requiring repairs in any given year is a random variable having the probability density

$$f(x) = \begin{cases} 12x^2(1-x), for\ 0 < x < 10 \\ 0, & otherwise \end{cases}$$

Find the distribution function and use it to determine the probability that at least half of the highways sections will require repairs in any given year.

Q.6	The joint probability density of two random variables $X1$ and $X2$ is given by

$$f(x,y) = \begin{cases} \frac{1}{5}(x + 2y), for\ 0 < x < 1 and\ 0 < y < 2 \\ 0, & otherwise \end{cases}$$

$f(x1,x2)=\{15(x1+2x2),\ for\ 0<x1<1,0<x2<20,\ elsewhere.$
Find the marginal densities of both the random variables and check whether the two random variables are independent.

Q.7	The joint probability density of two random variables is given by

$$f(x,y) = \begin{cases} 6e^{-2x-3y}, for\ x > 0\ and\ y > 0 \\ 0, & otherwise \end{cases}$$

Find the marginal densities of both the random variables and hence show that the two random variables are independent.

Q.8	A random variable X has the following probability distribution

x:	-2	-1	0	1	2	3
P(x):	0.1	K	0.2	2K	0.3	3K

	Find K Ans: 1/15 Evaluate P (X < 2) and P(-2<x<2) Ans: ½, 2/5 Find the cdf of X Ans: 1/10,1/6,11/30,1/2,4/5,1 Evaluate the mean of X Ans: 16/15
Q.9	A random variable X has the following probability distribution | X:	0	1	2	3	4	5	6	7
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P(x):	0	K	2K	2K	3K	K2	2K2	7K2+K	 (1) Find the value of K Ans: 1/10 (2) P(1.5<X<4.5)/X>2) Ans: 5/7 [Hint: use P(A∩B)/P(B), Take A = 1.5<X<4.5 and B = X>2] (3) The smallest value of λ for which P(X≤λ)>1/2. Ans: 4
Q.10	If P(x) = $\begin{cases} xe^{-\frac{x^2}{2}} & x \geq 0 \\ 0 & x = 0 \end{cases}$ Show that P(x) is a PDF Find its distribution function P(x) [Hint: distribution function F(x) = P(X≤x) = $\int_0^x f(X)dx$]
Q.11	The function defines by f(x) = $\begin{cases} |x| & -1 \leq x \leq 1 \\ 0 & otherwise \end{cases}$ Check whether the given function is a probability density function? If it is PDF find mean and variance of the distribution. Ans: yes it is PDF, 0, ½
Q.12	The probability that a man fishing at a place will catch 1, 2, 3, 4 fish are 0.4, 0.3, 0.2 and 0.1 respectively. What is the expected number of fishes caught? Ans: 2
Q.13	A and B enter a bet according to which A will get Rs. 200 if it rains on that day and will lose Rs. 100 if it does not rain. The probability of raining on that day is 0.7. What is the mathematical expectation of A? Ans: 110Rs. (Calculate E(X) = 200(0.7) − 100(0.3))
Q.14	The distribution function of a random variable X is given by F(x) = 1 − (1+x)e^{-x}, x≥0. Find the density function, mean and variance of X. [Hint: density function f(x) = d/dx(F(x)] Ans: f(x) = xe-x, mean = 2, variance = 2