

## Light-semiconductor interaction

\* Syllabus - Optical transition in bulk semiconductors - absorption, spontaneous emission and stimulated emission.

- Joint density of states.
- Density of states for photons.
- transition rate (Fermi's golden rule)
- optical loss and gain, photovoltaic effect - Exciton
- Drude model.

① Optical transition in bulk semiconductors, absorption, Spontaneous emission and stimulated emission.

According to Einstein whenever radiation and matter interact, there can be main three processes occurs. Absorption, spontaneous emission, stimulated-emission.

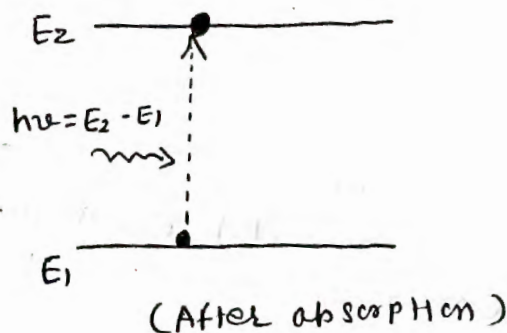
Let us consider two energy levels and an atom to describe that three processes.

\* Induced Absorption :-

(high energy level)  
 $E_2$  \_\_\_\_\_

$E_1$  \_\_\_\_\_  
(lower energy level)

(before absorption)



here the electron is situated in lower energy state  $E_1$ . if we apply a light energy (photon) having energy  $h\nu(E_2 - E_1)$  on that atom of lower energy state, the atom will absorb the incident energy and excited to higher energy level  $E_2$ .

This process is called induced Absorption.

The rate of induced absorption ( $R_{12}$ ) is directly proportional to the radiation ( $S$ ) and population of lower energy state ( $N_1$ ).

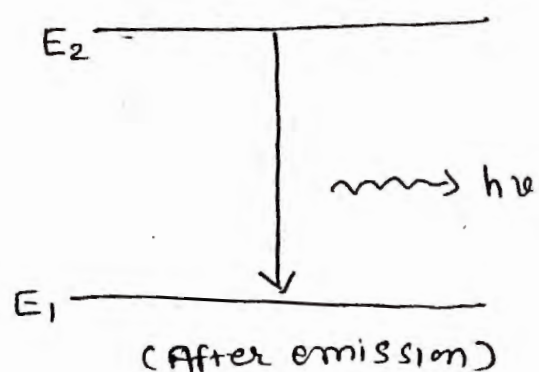
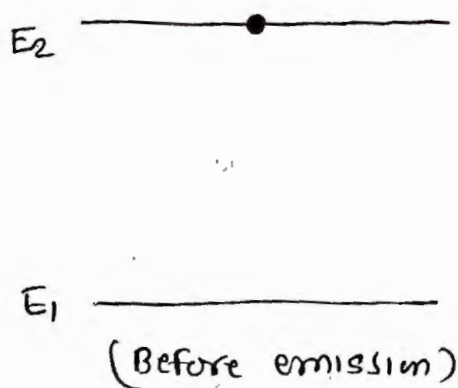
$$\therefore R_{12} \propto N_1 \cdot S.$$

$$\therefore R_{12} = B_{12} N_1 \cdot S$$

where  $B_{12}$  = proportionality const. for absorption per unit time.

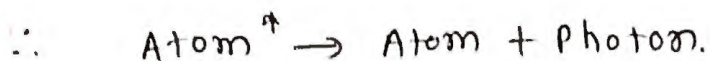
### \* Spontaneous emission :

it is a process in which there is an emission of a photon whenever an atom transits from higher energy state to lower energy state without help of external energy.



Suppose an atom is at higher energy level  $E_2$ . At this level it will be unstable so it will return to the lower energy state on its own.

while returning it will emit energy equal to (3) difference of two energy levels. ( $h\nu = E_2 - E_1$ )



The rate of Spontaneous emission  $R_{21}(\text{sp})$  is proportional to the population of higher level ( $N_2$ ).

$$\therefore R_{21}(\text{sp}) \propto N_2$$

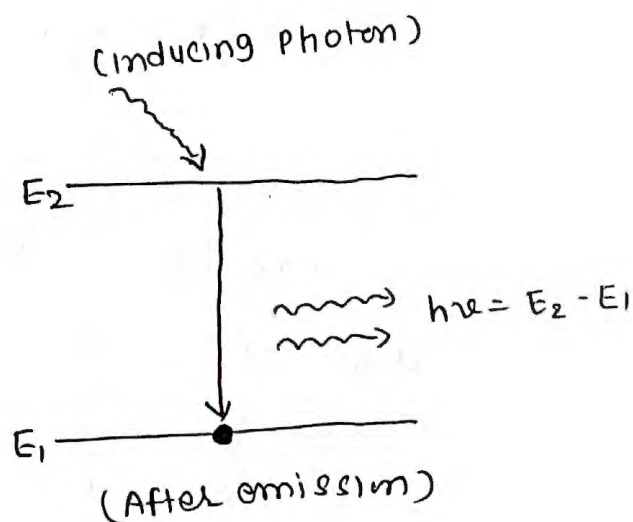
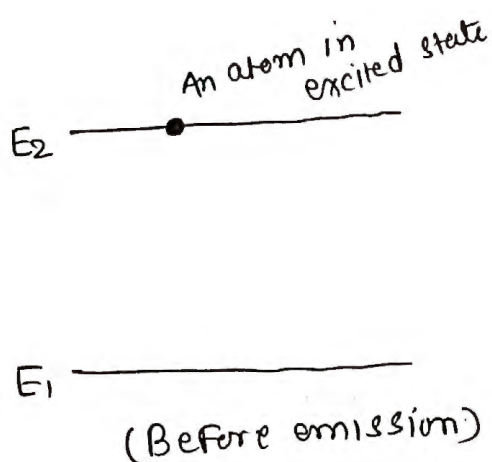
$$\therefore R_{21}(\text{sp}) = A_{21} \cdot N_2$$

where  $A_{21}$  = proportionality const for spontaneous emission per Unit time.

### \* Stimulated Emission :

it is a process in which there is an emission of a photon whenever an atom transits from a higher energy state to lower energy state with help of external energy.

for this process the atom has to be in higher energy state.

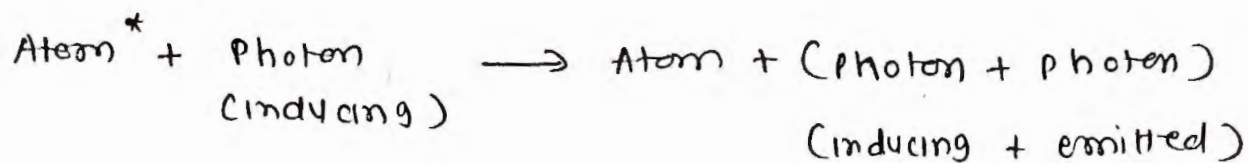


Suppose a photon having energy ( $E_2 - E_1$ ) is incident on this atom. ~~Because~~ Because of that the atom is transit to lower energy state  $E_1$  and while returning ~~and~~ produce release energy.



Both the inducing and emitted photons will have same phase, energy and direction. So they will be coherent.

This is the process responsible for Laser action.



The rate of stimulated emission  $R_{21}(st)$  is directly proportional to radiation ( $\rho$ ) and population in higher state ( $N_2$ ).

$$\therefore R_{21}(st) \propto N_2 \cdot \rho$$

$$\therefore R_{21}(st) = B_{21} N_2 \cdot \rho.$$

where

$B_{21}$  = proportionality const. for stimulated emission per unit time.

\* **metastable state :**

This is the energy level in atomic system where atom stays for longer time than the excited state. ( $10^{-3} - 10^{-2}$  sec).

\* **Life time :**

The maximum time period for which an atom stays in excited state is called Life time.

generally it is  $10^{-9} - 10^{-8}$  sec.

# Difference between Spontaneous & Stimulated Emission. ⑤

## Spontaneous emission

- ① This is a random process.
- ② Instant emission, direction, phase, polarisation state all are random quantities can't be controlled from outside.

- ③ output photons are non-directional.

- ④ Light is not monochromatic.

- ⑤ Light is non coherent.

- ⑥ multiplication of photons does not take place.  
No amplification of light

- ⑦ Light is ~~un~~ polarised.  
unpolarised.

- ⑧ The net intensity of light is  $I_T = N \cdot I$

where  $N$  = number of atoms emitting photons

$I$  = intensity of each photon.

## Stimulated Emission.

- ① Not a random process

- ② The stimulating photon imposes its characteristics on photon emitted, and can be controlled from outside.

- ③ The output photons are directional.

- ④ Light is nearly monochromatic.

- ⑤ Light is coherent.

- ⑥ multiplication of light occurs, and also light amplification occurs.

- ⑦ Light is polarised.

- ⑧ The net intensity of light is  $I_T = N^2 I$

where  $N$  = Number of atoms emitting photons

$I$  = intensity of each photon.

## ② Density of states for photons.

We know that density of states defines number of allowed energy states per unit volume.

To find number of emission (transitions) we need to know the emissions per unit volume.

This can also give an idea of power emitted.

The total number of emission per unit volume can be found by multiplying density of states and probability occupation.

$$\therefore \text{No. of emission/unit volume} = \text{density of states}$$

$\times$   
probability of  
occupation.

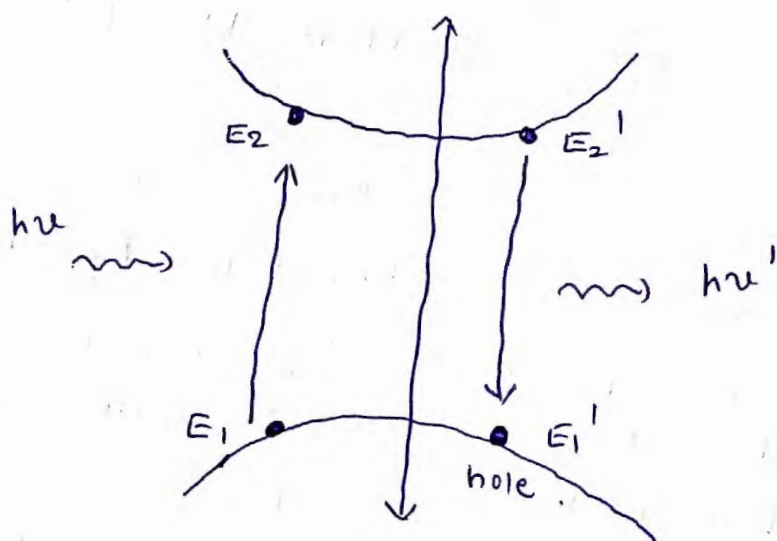
Now,

concentration of ele. in  $C_b$

$$\text{is } n_e = \int Z_c(E) \cdot d(E) f(E)$$

concentration of holes in  $V_b$

$$n_h = \int Z_v(E) \cdot dE [1 - f(E)]$$



Suppose a radiation having energy  $h\nu$  is incident on ele in  $V_b (E_1)$ . Because of radiation it makes transition in  $C_b (E_2)$ .



means absorption involved here. Similarly if there is an electron in energy level  $E_2'$  in  $C_b$  and it makes transition in vacant state at a level  $E_1'$  in  $V_b$ , a photon with energy  $h\nu$  is emitted.

Thus we can say absorption and emission involve a state in  $V_b$  and  $C_b$ .

For a photon interaction we have one state in  $V_b$  and one state in  $C_b$ . So we define "Joint density of states", that take care of number of states for a given energy  $h\nu$ .

for a fix incident energy  $h\nu$  if  $E_2$  is fixed then  $E_1$  also fixed.

Since absorption or emission take place from top of  $V_b$  and bottom of  $C_b$  we can have a parabolic approximation as,

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c} \quad \longrightarrow (1)$$

(for any level in  $C_b$  with lowest level as  $E_c$ )

$$E_1 = E_v + \frac{\hbar^2 k^2}{2m_v} \quad \longrightarrow (2)$$

(for any level in  $V_b$  with max level as  $E_v$ )

The energy of photons

$$h\nu = E_2 - E_1$$

$$h\nu = E_c + \frac{\hbar^2 k^2}{2m_c} - E_v - \frac{\hbar^2 k^2}{2m_v}$$

$$h\nu = (E_c - E_v) + \frac{\hbar^2 k^2}{2} \left( \frac{1}{m_c} - \frac{1}{m_v} \right)$$

$$\therefore h\nu = E_g + \frac{\hbar^2 k^2}{2} \left( \frac{1}{m_r} \right)$$

where  $E_g = E_c - E_v = \text{forbidden gap}$

$\frac{1}{m_r} = \frac{1}{m_c} - \frac{1}{m_v} = \text{reduced mass}$

$$\therefore h\nu = E_g + \frac{\hbar^2 k^2}{2m_r}$$

$$\therefore k^2 = \frac{(h\nu - E_g) \cdot 2m_r}{\hbar^2} \rightarrow (3)$$

Substituting the value of  $k^2$  in eqn (1) & (2)

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c}$$

$$= E_c + \frac{\hbar^2}{2m_c} \cdot (h\nu - E_g) \cdot \left( \frac{2m_r}{\hbar^2} \right)$$

$$E_2 = E_c + \frac{m_r}{m_c} (h\nu - E_g) \rightarrow (4)$$

~~Now~~  $E_2 = \sqrt{E_c^2 + \frac{\hbar^2 k^2}{2m_c^2}}$  ~~in~~

In above eqn  $E_c$ ,  $m_r$ ,  $m_c$ ,  $E_g$  are constants  
only  $E_2$  and  $\nu$  are variables.

so there is 1-1 correspondence bet<sup>n</sup>  $\nu$  and  $E_2$

Similarly,

$$E_1 = E_v + \frac{\hbar^2 k^2}{2m_v}$$

$$\therefore = E_v + \frac{\hbar^2}{2m_v} \cdot (h\nu - E_g) \cdot \left( \frac{2m_r}{\hbar^2} \right)$$

$$\therefore E_1 = E_v + \left( \frac{m_r}{m_v} \right) (h\nu - E_g) \rightarrow (5)$$

There is 1-1 correspond between  
 $E_1$  and  $\nu$



So we have

$$Z_c(E_2) dE_2 = Z(v) dv \rightarrow (6)$$

where  $Z(v) dv$  is no. of states per unit volume available for photons of energy between  $h\nu$  and  $h(\nu + d\nu)$

once we get  $Z(v) dv$  (density of states) then by multiplying by probability of emission or absorption, we will get joint density of states for photons.

From eqn (6)  $Z(v) = Z_c(E_2) \frac{dE_2}{dv} \rightarrow (7)$

from eqn (4)  $E_2 = E_c + \frac{m_r}{m_c} (h\nu - E_g)$

$$\therefore (E_2 - E_c)^{1/2} = \left(\frac{m_r}{m_c}\right)^{1/2} (h\nu - E_g)^{1/2} \rightarrow (8)$$

from eqn (7) we have

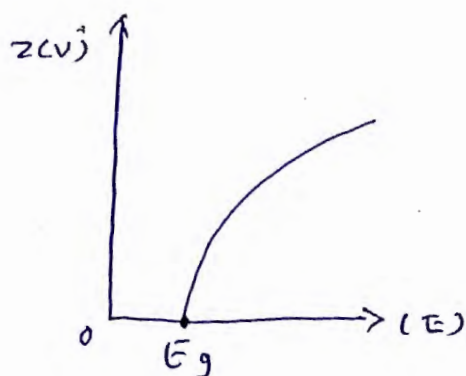
$$Z(v) = \frac{1}{2\pi^2} \cdot \left(\frac{2m_c}{\hbar^2}\right)^{3/2} \cdot (E_2 - E_c)^{1/2} \cdot h \cdot \frac{m_r}{m_c}$$

Substituting the value of  $(E_2 - E_c)^{1/2}$  from eqn (8)

$$\therefore Z(v) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2}\right)^{3/2} \cdot h \cdot \frac{m_r}{m_c} \left(\frac{m_r}{m_c}\right)^{1/2} (h\nu - E_g)^{1/2}$$

$$\therefore \boxed{Z(v) = \frac{1}{\pi \hbar^2} (2m_r)^{3/2} \cdot (h\nu - E_g)^{1/2}}$$

↓  
eqn for joint optical density  
eqn for optical joint density of states.



from the graph, for every  $h\nu > E_g$ , we will have emission or absorption.

### ③ Transition Rates (Fermi's Golden Rule)

in quantum physics Fermi's Golden Rule is used to calculate transition rates.

The transition rate depends upon the strength of coupling between the initial and final state of a system and upon the number of ways the transition can happen. (Joint density of states).

The transition probability is given by

$$\lambda_{if} = \frac{2\pi}{h} |M_{if}|^2 Z_f$$

↓  
Fermi's Golden Rule

where

$\lambda_{if}$  = transition probability

$|M_{if}|$  = matrix element for interaction.

$Z_f$  = joint density of final state.

The transition probability  $\lambda$  is also called the decay probability and it is related to mean lifetime  $\tau$  of the state.

$$\lambda = \frac{1}{\tau}$$

The general form of Fermi's Golden Rule can be applied to atomic transition, nuclear decay and scattering.

The transition can be more speedy if coupling bet<sup>n</sup> final & first state is stronger.

This coupling term is traditionally called the 11 matrix element for the transition.

This matrix element can be placed in the form of an integral, where interaction is expressed as a potential 'v' that operates on initial state wave-function.

The transition probability is proportional to the square of integral of interaction over all of the space appropriate to the problem.

$$M_{if} = \int \psi_f^* \cdot v \cdot \psi_i \cdot dv$$

where  $v$  = operator for physical interaction that couples initial & final states.

$\psi_f^*$  = wave function for final state

$\psi_i$  = wave function for initial state.

This transition probability is also proportional to the joint density of states.

#### ④④ photovoltaic effect and photovoltaic cell (solar cell).

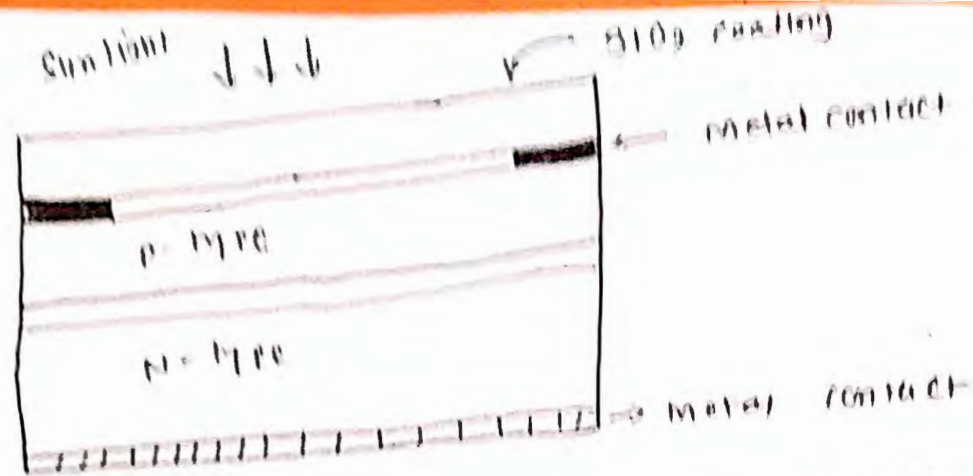
The effect due to which light energy is converted into electrical energy is called photovoltaic cell.

The photovoltaic (solar) cell is a device that converts solar light into electrical energy.

#### solar cell (photovoltaic cell) -

construction: solar cell are made of high purity single crystal like silicon with some impurities like Boron and Phosphorus.





A thin layer of p-type material is diffused with n-type material forming a simple p-n junction.

The thickness of p-layer is very small ( $\sim 0.2 - 0.5 \text{ mm}$ ). The upper surface of solar cell is made by a thin layer of p-type material for light to enter easily.

To increase the absorption of light by the material there is an anti-reflection coating on top of p-type material.

Metal contacts are made on p- and n-type material which act as + and - terminal respectively.

Working : - when sunlight is incident on a solar cell the electrons in the  $V_b$  gain energy and travel to  $C_b$  so produce e-h pair.

If incident photon is having energy greater than band gap the e-h pair is generated.

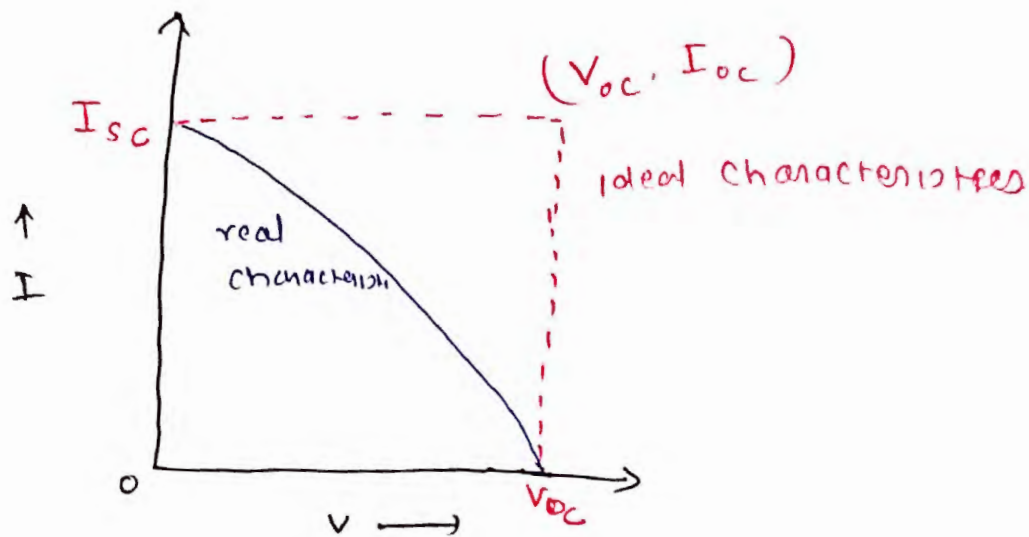
The minority charge carriers exist for a short time before recombination. If the carriers recombine e-h pairs are lost and no current produced.

The p-n junction prevents this recombination due to internal e.f. field.

When these light generated e.h. pair (13) reach to the junction they swept across junction and now become majority charge carriers.

If the solar cell is short circuited current starts flowing.

Thus solar cell behave as a ~~better~~ battery with N side as -ve and P side as +ve terminal.



$$\text{Ideal power o/p} = P_{\text{ideal}} = V_{oc} \times I_{sc}$$

$V_{oc}$  = open circuit voltage

$I_{sc}$  = short circuit current

$$\text{Actual power o/p} = P_{\text{max}} = I_m \times V_m$$

The Fill factor (FF) of solar cell is defined as the maximum useful power wrt ideal power.

$$\therefore FF = \frac{P_{\text{max}}}{P_{\text{ideal}}} = \frac{V_m \times I_m}{I_{sc} \times V_{oc}}$$

$$\text{The efficiency of Solar cell } \eta = \frac{I_m \times V_m}{\text{Incident light-power}}$$

It is around 15-20% for silicon cells.

\* Advantages :- Input energy (solar energy) is free.  
- free from pollution.

\* Disadvantages :- Input (solar) is not always constant  
so o/p is not constant

- low efficiency,

- requires large area to be installed.

\* Application :- used in power plants

- used for domestic lighting, water pump TV etc.

- used for fire alarms, etc

- used in battery recharge stations

- used in for power back-up in medical applications,

- used in power of aircrafts.

- in street lights

- in water drip irrigation.

- in domestic applications.

\* Optical losses :-

The optical loss in solar cell, mainly affect the power output by lowering the short circuit current ( $I_{sc}$ ), thus reducing efficiency.

It is due to below mentioned reasons

- mismatch of the bandgap of material with solar spectrum
- Total spectrum of solar energy is not absorbed.
- Reflection loss of light
- less intensity of light



## \* Remedies :-

(15)

- Proper selection of material can cause increases the efficiency of solar cell.
- Thicker solar cell can be made to absorb more light.
- Anti Reflection coating can be used.
- optical path length can be increased by light trapping.

## \* Exciton :-

Due to Absorption of a photon in semiconductor there is ~~or~~ a ~~ele~~ ~~er~~ and hole created in  $C_b$  and  $V_b$  respectively.

~~These~~ these opposite charge particles attract each other by coulomb interaction and there may be probability of formation of a neutral electron-hole pair called 'Exciton'.

" An exciton is a bound state of an electron and hole which are attracted to each other by an electrostatic coulomb force.

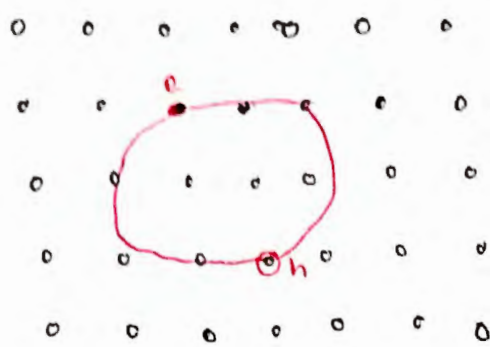
excitons are of two types.

### (1) Frenkel exciton :



" when there is a strong electron-hole attraction, like in ionic crystals, the ~~ele~~-hole are tightly bound to each other this type of exciton is called Frenkel exciton."

## (2) Wannier - Mott exciton.



" When ele-hole separation is much large to the lattice constant the exciton is known as Wannier - Mott exciton.

- In this exciton ele-hole pair is weakly bound.

## (5) Drude model :-

This model was proposed in 1900 by Paul Drude to explain transport properties of electrons in materials.

### \* Assumptions of Drude model :-

- (1) Between the collisions, the electron move in a straight line. (In absence of em field)
- (2) The effect of ele-ele interaction is ignored. Also ele-ion interaction ignored.
- (3) Mean free path time between collisions is  $\tau$ .  $\tau$  is independent of ele. position & velocity.
- (4) Ele. achieve thermal equilibrium by collisions with lattice.
- (5) After collisions they move in random directions with a speed depend on the temp. of region where collision occurred.

### \* Applications of Drude model ;

- (1) DC ele. conductivity of metal.

we know  $V = IR$ ,  $E = \rho I$   
and  $J = -nev \rightarrow (1)$

as the ele. randomly move after collisions  
we will take average velocity  $v$   
in equilibrium average velocity vanish so  
no net electric current.

$$\therefore v = v_0 - eEt$$

$$v = \frac{v_0 e \cdot E(t)}{m}$$

( $t$  = time since collisions  
 $v_0$  = velocity after collision)

$$\therefore v = -\frac{eEt}{m} \rightarrow (2)$$

by substituting the values of  $v$  from (2) in eqn (1)

$$J = -ne \left( -\frac{eEt}{m} \right)$$

$$\therefore J = \frac{ne^2 \cdot E t}{m}$$

we know that  $\sigma = \frac{J}{E}$

$$\therefore \boxed{\sigma = \frac{n \cdot e^2 \tau}{m}}$$

eqn for DC conductivity in drude model.

## ② Hall effect and magnetoresistance:

magnetoresistance  $\rho(H) = \frac{E_x}{J_{yc}}$

Hall coefficient  $R_H = \frac{E_y}{J_x \cdot B_z}$



### ③ calculation of Hall coefficient & magneto resistance

In the presence of ele. field & mag. field we have

$$\frac{ds}{dt} + \frac{s(t)}{\tau} = -e \left( E + \frac{sB}{m} \right)$$

In steady state current is time independent

$$\frac{s(t)}{\tau} = -e \left( E + \frac{sB}{m} \right)$$

$$\therefore \frac{s_x}{\tau} = -e E_x - \frac{e s_y \cdot B}{m}$$

$$\therefore \frac{s_y}{\tau} = -e E_y - \frac{e s_x \cdot B}{m}$$

$$\therefore \sigma_0 E_x = J_x + \omega_c \cdot \tau j_y$$

$$\therefore \sigma_0 E_y = J_y - \omega_c \tau j_x$$

$$\therefore E = \frac{1}{\sigma_0} \begin{bmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{bmatrix} j$$

For hall samples  $j_y = 0$

$$\text{So } R_H = \frac{E_y}{j_x \cdot B_z}$$

$$\therefore R_H = \frac{-1}{ne}$$

- Actually carrier density are not always given correctly by drude formula.

- the sign was also not explained till quantum theory came  
15

(4) Ac Electrical conductivity of metals

19

Time dependent electrical field

$$E(t) = E_0 \cdot e^{-i\omega t} \quad \text{eqn of motion is}$$

$$\frac{de}{dt} = -\frac{e(t)}{\tau} - eE$$

$$\therefore -e(t) = s_0 e^{-i\omega t}$$

$$\therefore -i\omega s_0 = -\frac{s_0}{\tau} - eE_0$$

$$J = -\frac{ne s}{m}$$

$$= \frac{\sigma_0}{1 - i\omega\tau} \cdot E$$

$$\therefore \frac{J}{E} = \frac{\sigma_0}{1 - i\omega\tau}$$

$$\therefore \boxed{\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}}$$

↓  
AC conductivity.