Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2021

Subject Code:3110014 Date:19/03/2022

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

MARKS

Q.1 (a) If $u = \log(tanx + tany + tanz)$ then show that **03**

$$sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z} = 2$$

(b) Evaluate **04**

$$\lim_{x\to 0}\frac{e^x - e^{-x} - 2log(1+x)}{xsinx}$$

(c) Find the extreme values of the function **07**

$$f(x,y) = x^3 + y^3 - 3x - 12y + 20$$

Q.2 (a) Use Ratio test to check the convergence of the series **03**

$$\sum_{n=1}^{\infty}\frac{2^n + 1}{3^n + 1}$$

(b) Find the Maclaurin's series of $cosx$ and use it to find the series of sin^2x. **04**

(c) Find the Fourier series of $f(x) = x^2$ in the interval $(0,2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$ **07**

OR

(c) Find the Fourier series of $f(x) = 2x - x^2$ in the interval (0,3). **07**

Q.3 (a) Find the directional derivative of $f(x,y,z) = xyz$ at the point $P(-1,1,3)$ in the direction of the vector $\bar{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$. **03**

(b) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to row echelon form. **04**

(c) Find the eigenvalues and corresponding eigenvectors of the matrix **07**

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

OR

Q.3 (a) If $u = f(x-y, y-z, z-x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **03**

(b) Find the inverse of the following matrix by Gauss-Jordan method: **04**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(c) Verify Cayley-Hamilton theorem for the following matrix and use it to find A^{-1} **07**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Q.4 **(a)** If 1 is an eigenvalue of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then find its corresponding eigen vector. **03**

(b) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$ **04**

(c) Solve following system by using Gauss Jordan method **07**

$$x + 2y + z - w = -2$$
$$2x + 3y - z + 2w = 7$$
$$x + y + 3z - 2w = -6$$
$$x + y + z + w = 2$$

OR

Q.4 **(a)** Use integral test to show that the following infinite series is convergent **03**

$$\sum_{n=1}^{\infty} \frac{1}{n(1 + \log^2 n)}$$

(b) For the odd periodic function defined below, find the Fourier series **04**

$$f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$

(c) Determine the radius and interval of convergence of the following infinite series **07**

$$x - \frac{x^2}{4} + \frac{x^3}{9} - \frac{x^4}{16} + \cdots$$

Q.5 **(a)** Show the following limit does not exist using different path approach **03**

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y^2}{x^4 + y^4}$$

(b) Evaluate the following integral along the region R **04**

$$\iint_R (x + y)\,dydx$$

where R is the region bounded by $x = 0, x = 2, y = x, y = x + 2$. Also, sketch the region.

(c) Change the order of integration and hence evaluate the same. Do sketch the region. **07**

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\,dydx$$

OR

Q.5 **(a)** The following integral is an improper integral of which type? Evaluate **03**

$$\int_0^{\infty} \frac{dx}{x^2 + 1}$$

(b) If $x = rsin\theta cos\varphi, y = rsin\theta sin\varphi, z = rcos\theta$, then find the jacobian **04**

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)}$$

(c) Find the volume of the solid generated by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$. **07**

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