30/8/19

Q) ~~The period of~~

* Taylor's series (one variable)

→ If $f(x)$ is an infinitely differentiable f^n of $f(x)$ x throughout some interval containing 'a' as an interior point then $f(x)$ can be expanded as a power series in $(x-a)$. This series is called Taylor series of $(x-a)$ form.

→ $f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \ldots$ — ①

*Case 1: If we put $(x-a) = h \Rightarrow x = a+h$

$\therefore f(a+h) = f(a) + h(f'(a)) + \frac{(h)^2}{2!} f''(a) + \ldots$

*Case 2 If we put $a = 0$ in ① then

$$f(x) = f(0) + (x) f'(0) + \frac{(x)^2}{2!} f''(0) + \ldots \quad \text{— ②}$$

This series ② is called Maclaurin's series.
This series is a special case of Taylor series.

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Q.1) Find the Taylor series of $f(x)=e^x$ in terms of x

Ans: By using Taylor series:

$f(x)=e^x$ and $a=0$

$$\Rightarrow f(x)=f(a)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\dots \quad (1)$$

$f(x)=e^x$ $\quad$ $f(0)=1$

$f'(x)=e^x$ $\quad$ $f'(0)=1$

$f''(x)=e^x$ $\quad$ $f''(0)=1$

$f'''(x)=e^x$ $\quad$ $f'''(0)=1$

Sub value in (1).

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$$

Q.2) Expand $f(x)=\log(1+x)$ in terms of x.

Sol^n By using taylor series.

$f(x)=\log(1+x)$ & $a=0$

$f(x)=\log(1+x)$ $\quad$ $f(0)=0$

$f'(x)=\frac{1}{1+x}(1)$ $\quad$ $f'(0)=1$

$f''(x)=\frac{-(1)}{(1+x)^2}$ $\quad$ $f''(0)=-1$

$f'''(x)=\frac{2}{(1+x)^3}$ $\quad$ $f'''(0)=2$

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$\therefore$ ~~f(x)~~

$$\log(1+x) = 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \cdots$$

Q.3) Expand $\log x$ in power (terms) of $(x-1)$ also evaluate $f(x)$ $f(1.1) = \log(1.1)$

Sol^n $f(x) = \log x$.

$a = 1$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots$$

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$f(1) = 0$

$f'(x) = \frac{1}{x}$ $\qquad f'(1) = 1$

$f''(x) = \frac{-1}{x^2}$ $\qquad f''(1) = -1$

$$\therefore \log x = (x-1) - \frac{(x-1)^2}{2!} + \cdots$$

$$\Rightarrow \log(1.1) \simeq (1.1-1) - \frac{(1.1-1)^2}{2!} + \cdots$$

$$\simeq 0.1 - \frac{(0.1)^2}{2!}$$

$$\simeq 0.1 - \frac{0.01}{2}$$

$$\simeq 0.1 - 0.005$$

$$\simeq 0.095$$

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Q.4) $f(x) = \sin x$ in terms of $(x-1)$

$f(x) = \sin x$ $\quad a = 1$

$f(1) = \sin 1$

$f'(x) = \cos x$ $\quad f'(1) = \cos 1$

$f''(x) = -\sin x$ $\quad f''(1) = -\sin 1$

$$\therefore \sin x = \sin 1 + (x-1)(\cos 1) + \frac{(x-1)^2}{2!}(-\sin 1) + \cdots$$

Q.5) Expand $f(x) = \sin\left(\frac{\pi}{4} + x\right)$ in terms of x.

~~f(x) = sin~~ Also evaluate $\sin 44^\circ$

Sol^n $f(x) = \sin\left(\frac{\pi}{4} + x\right)$ $\quad a = 0$

$f(0) = \frac{1}{\sqrt{2}}$

$f'(x) = \cos\left(\frac{\pi}{4} + x\right)$

$f'(0) = \frac{1}{\sqrt{2}}$

$f''(x) = -\sin\left(\frac{\pi}{4} + x\right)$

$f''(0) = \frac{-1}{\sqrt{2}}$

$f'''(x) = -\cos\left(\frac{\pi}{4} + x\right)$

$f'''(0) = -\frac{1}{\sqrt{2}}$

$$\sin\left(\frac{\pi}{4} + x\right) = \frac{1}{\sqrt{2}} + x \cdot \frac{1}{\sqrt{2}} + \frac{x^2}{2!} \cdot \left(\frac{-1}{\sqrt{2}}\right) + \frac{x^3}{3!}\left(\frac{-1}{\sqrt{2}}\right)$$

$$\therefore \sin\left(\frac{\pi}{4} + x\right) = \frac{1}{\sqrt{2}} + \frac{x}{\sqrt{2}} - \frac{x^2}{2! \cdot (\sqrt{2})} - \frac{x^3}{3!(\sqrt{2})}$$

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$\sin 44^\circ = \sin(45^\circ - 1^\circ)$

$= \sin\left(\frac{\pi}{4} - \frac{\pi}{180}\right) \Rightarrow \sin\left[\frac{\pi}{4} + \left(\frac{-\pi}{180}\right)\right]$

$\Rightarrow x = -\frac{\pi}{180}$

$= \sin\left(\frac{\pi}{4} + \left(\frac{-\pi}{180}\right)\right)$

$= \sin 44^\circ$

$\therefore \sin\left(\frac{\pi}{4} - \frac{\pi}{180}\right) = \frac{1}{\sqrt{2}}\left[1 - \frac{\pi}{180} - \frac{(-\pi/180)^2}{2!} - \frac{(-\pi/180)^3}{3!} + \ldots\right.$

Q.6) express $(x-1)^4 + 2(x-1)^3 + 5(x-1) + 2$ in terms of x.

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Taylor series interms of $f(x,y)$

in term of $(x-a)$, $(y-b)$

$$f(x,y) = f(a,b) + \left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right]$$
$$+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] + \dots$$

in terms of x & y

$\Rightarrow a = b = 0$

$$f(x,y) = f(0,0) + \left[x f_x(0,0) + y f_y(0,0)\right]$$
$$+ \frac{1}{2!}\left[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$
$$+ \frac{1}{3!}\left[(x-a)^3 f_{xxx}(a,b) + (y-b)^3 f_{yyy}(a,b) + 3(x-a)^2(y-b) f_{xxy}(a,b) + 3(x-a)(y-b)^2 f_{xyy}(a,b)\right]$$

Example:

1) Expand $xy^2 + xy + 3$ in powers of $(x-1)$ & $(y+2)$

Sol^n $a = 1$, $b = -2$

$$f(x,y) = f(a,b) + \left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] + \frac{1}{2!}$$
$$\frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$

$$= 5 + \left[(x-1)2 + (y+2)(-3)\right] + \frac{1}{2!}\left[(x-1)^2 0 + 2(x-1)(y+2)(-3) + (y+2)^2(2)\right]$$

$$= 5 + \left[2(x-1) - 3(y+2)\right] + \frac{1}{2!}\left[-6(x-1)(y+2) + 2(y+2)^2\right]$$

2) Expand $f(x,y) = \sin x . \cos y$ ~~in~~ at origin.

Sol^n. $\quad a = b = 0$

$f(x,y) = f(0,0) + [x f_x(0,0) + y f_y(0,0)] + \frac{1}{2!}[x^2 f_{xx}(0,0) +$

$2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)] + \dots$

$= 0 + [x(1) + y(0)] + \frac{1}{2!}[x^2(0) + 2xy(0) + y^2(0)] + \dots$

$= 0 + [x] + \frac{1}{2!}(0) \qquad = x$

3) Expand $f(x,y) = \tan^{-1}\frac{y}{x}$ in terms of $(x-1)$ & $(y-1)$
Also compute $f(1.1, 0.9)$ approx.

Sol^n $\quad a = 1, \quad b = 1$

$f(x,y) = f(a,b) + [(x-a)f_x(a,b) + (y-b)f_y(a,b)] +$

$\frac{1}{2!}[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)] + \dots$

$= \frac{\pi}{4} + \left[(x-1)\left(\frac{-1}{2}\right) + (y-1)\left(\frac{1}{2}\right)\right] +$

$\frac{1}{2!}\left[(x-1)^2\left(\frac{1}{2}\right) + 2(x-1)(y-1)(0) + (y-1)^2\left(\frac{-1}{2}\right)\right]$

$f(x,y) = \frac{\pi}{4} + \left[-\frac{1}{2}(x-1) + \left(\frac{1}{2}\right)(y-1)\right] + \frac{1}{2}\left[\frac{1}{2}(x-1)^2 - \frac{1}{2}(y-1)^2\right]$

$$f(1.1, 0.9) \simeq \frac{\pi}{4} + \left[\frac{-1}{2}((1.1\cancel{8}-1) + \frac{1}{2}(0.9-1)\right]$$

$$+\frac{1}{2}\left[\frac{1}{2}(1.1-1)^2 - \frac{1}{2}(0.9-1)^2\right]$$

$$\simeq \frac{\pi}{4} + \left[\frac{-1}{2}(0.1) + \frac{1}{2}(-0.1)\right] + \frac{1}{2}\left[\frac{1}{2}(0.1)^2 - \frac{1}{2}(-0.1)^2\right]$$

$$\simeq \frac{\pi}{4} + \left[-0.05 - 0.05\right] + \frac{1}{4}\left[0.01 - 0.01\right]$$

$$\simeq \frac{\pi}{4} + \left[-0.1\right] + 0.$$

$$\simeq \frac{\overset{\to 3.14}{\pi}}{4} - 0.1$$

$$\simeq 0.685$$

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Friday

Q) Find the maclurin series of fn : $f(x) = x^3 + 2x^2 + x + 1$

Soln $a = 0.$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$$

$$= 1 + x(1) + \frac{x^2}{2}(4) + \frac{x^3}{6} \times 6 + \ldots$$

$$= 1 + x + 2x^2 + x^3$$

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Q) Find expansion of x^3+2x^2+x+1 at $x=1$

Sol^n $(x-1)$: $a=1$

$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots$

$= 5 + (x-1)8 + \frac{(x-1)^2}{2} 10 + \frac{(x-1)^3}{6} 6$

$= 5 + 8(x-1) + 5(x-1)^2 + (x-1)^3 + \ldots$

$= 5 + 8x - 8 + 5(x^2-2x+1) + (x-1)^3 + \ldots$

$= 5 + 8x - 8 + 5x^2 - 10x + 5 + x^3 - 1 - 3x^2 + 3x$

$= \cancel{2}\cancel{5x^2}\cancel{-2x}\cancel{+2}\ x^3+2x^2+x+1$

Q) i) Use Taylor series to find quadratic approximation to $f(x,y) = 1 + \sin x . \sin y$ near origin

Sol^n $(a,b) = (0,0)$.

$f(x,y) = f(0,0) + [y f_y(0,0) + x f_x(0,0)] + \frac{1}{2!}[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)] + \ldots$

$= 1 + [y(0) + x.(0)] + \frac{1}{2!}[x^2.(0) + 2.xy.1 + y^2.(0)]$

$= 1 + \frac{1}{2}(2xy) \quad = \underline{1 + xy}$

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Q) 2) $f(x,y) = (x^2 y + \sin y + e^x)$ if power of $(x-1)$ & $(y-\pi)$

Sol^n $(a,b) = (1,\pi)$

$$f(x,y) = f(a,b) + [(x-a) f_x(a,b) + (y-b) f_y(a,b)] + \frac{1}{2!}[(x-a)^2 \cdot f_{xx}(a,b) + 2(x-a)(y-b) f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)] + \dots$$

$$= (\pi+e) + [(x-1)(2\pi+e) + 2(x-1)(y-\pi)\,0]$$

$$+ \frac{1}{2}[(x-1)^2 (2\pi+e) + 2(x-1)(y-\pi)2 + (y-\pi)^2 (0)]$$

$$= (\pi+e) + [2\pi x - 2\pi + ex - e + \cancel{2xy - 2x\pi - 2y + 2\pi} + 0]$$

$$+ \frac{1}{2}[(x^2 - 2x + 1)(2\pi+e) + (4x-4)(y-\pi) + \cancel{y^2 - 2\pi y + \pi}\,0]$$

$$= (\pi+e) + [2\pi x - 2\pi + ex - e] + \frac{1}{2}[2\pi x^2 - 4\pi x + 2\pi + ex^2 - 2xe + e + 4xy - 4\pi x - 4y + 4\pi]$$

$$= \pi + \cancel{e} + \cancel{2\pi x} - 2\pi + \cancel{ex} - \cancel{e} + \pi x^2 - \cancel{2\pi x} + \pi + \frac{1}{2}ex^2 - \cancel{x}e + \frac{e}{2}$$

$$= \pi + \pi x^2 - 2\pi + \frac{1}{2}ex^2 + e + 2xy - 2\pi x - 2y + 2\pi + \frac{1}{2}e$$

$$= \pi + \pi x^2 + \pi + \frac{1}{2}ex^2 + 2xy - 2\pi x - 2y + \frac{5e}{2}$$

$$= (\pi + e/2)x^2 + 2xy - 2\pi x - 2y + \left(\pi + \frac{5}{2}e\right)$$

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e^{ax}

$[0, 2\pi]$

$= \frac{1}{\pi}\int_0^{2\pi} e^{ax}\, dx$

$\frac{1}{\pi}\left[\frac{e^{ax}}{a}\right]_0^{2\pi} = \frac{1}{\pi a}\left[e^{2\pi a} - 1\right]$

$= \frac{1}{\pi}$