

Properties of Gases + Numericals +

3.14 3 kg of ethane gas is compressed according to the law
' $pV^{1.3} = \text{constant}$ ' from 1.013 bar, 27°C to 8 bar pressure.

Determine (i) heat transferred (ii) work done,

(iii) change in internal energy.

Molecular weight of ethane is 30 kg/kg.mol & $C_p = 1.75 \text{ kJ/kg.K}$ for ethane. Assume ethane is a perfect gas.

[$R_0 = 8314.4 \text{ J/kg.mol.K}$].

soln:

Given data:

$m = 3 \text{ kg}$ polytropic index $n = 1.3$

$p_1 = 1.013 \times 10^5 \text{ N/m}^2$, $p_2 = 8 \times 10^5 \text{ N/m}^2$

$T_1 = 27^\circ\text{C} \rightarrow 273 + 27 = 300 \text{ K}$

M (molecular weight) = 30 kg/kg.mol , $C_p = 1.75 \text{ kJ/kg.K}$ &

$R_0 = 8314.4 \text{ J/kg.mol.K}$.

4 $Q = ?$, $W = ?$, $\Delta U = ?$

* Work done for polytropic process:-

$W = \frac{p_1 V_1 - p_2 V_2}{n-1}$ or $W = \frac{m R (T_1 - T_2)}{n-1}$

→ For T_2 ,

$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$

$\therefore \frac{T_2}{300} = \left(\frac{8 \times 10^5}{1.013 \times 10^5} \right)^{\frac{1.3-1}{1.3}}$

$\therefore T_2 = 300 (7.897)^{0.231}$

$\therefore T_2 = 300 \times 1.6118$

$\therefore T_2 = 483.54 \text{ K}$

$R = \frac{R_0}{M}$
Univex
 $R_0 =$

→ For Characteristic Gas Constant:

$R = \frac{R_0}{M} = \frac{8314.4}{30} = 277.14 \text{ J/kg.K}$
 $= 0.277 \text{ kJ/kg.K}$

$$\therefore W = \frac{mR(T_1 - T_2)}{n-1}$$

$$\begin{aligned}\therefore W &= \frac{3 \times 0.287 (300 - 483.54)}{1.3-1} \\ &= \frac{0.831(-183.54)}{0.3} \\ &= -508.40 \text{ kJ}\end{aligned}$$

• Also, change in internal energy;

$$\begin{aligned}\Delta U &= mC_v \Delta T \\ &= mC_v (T_2 - T_1) \\ &= 3C_v (483.54 - 300) \\ &= 550.62 \text{ kJ}\end{aligned}$$

→ For C_v :- we have $C_p - C_v = R$

$$\begin{aligned}\therefore C_v &= C_p - R \\ &= 1.75 - 0.287 = 1.463 \text{ kJ/kg}\cdot\text{K}\end{aligned}$$

$$\begin{aligned}\therefore \Delta U &= 550.62 \times 1.463 \\ &= \underline{\underline{811.06 \text{ kJ}}}\end{aligned}$$

★ Heat transferred $\therefore Q = \Delta U + W$

$$\begin{aligned}&= 811.06 - 508.40 \\ &= 302.66 \text{ kJ}\end{aligned}$$

Q3.15 One cubic metre of air at pressure 1 bar & 60°C is compressed final pressure 6 bar & volume 0.25 m³. Determine (i) mass of (ii) index 'n' for compression, (iii) change in internal energy, (iv) heat transferred. [Take $\gamma = 1.4$ & $R = 287 \text{ J/kg}$]

Soln.

$$V_1 = 1 \text{ m}^3, V_2 = 0.25 \text{ m}^3$$

$$P_1 = 1 \times 10^5 \text{ N/m}^2, P_2 = 6 \times 10^5 \text{ N/m}^2, T_1 = 60 + 273 = 333 \text{ K}$$

$$\uparrow \quad m = ?, \quad n = ?, \quad \Delta U = ?, \quad Q = ?$$

* For 'm': we have eqⁿ $P_1 V_1 = m R T_1$

$$\therefore m = P_1 V_1 / R T_1$$

$$= 105 / 95571$$

$$\therefore m = 1.046 \text{ kg}$$

* Index 'n':

$$P_1 V_1^n = P_2 V_2^n$$

$$\therefore \left(\frac{P_1}{P_2} \right) = \left(\frac{V_2}{V_1} \right)^n$$

$$\therefore 0.166 = (0.25)^n$$

$$\therefore \log(0.166) = n \log(0.25)$$

$$-0.779 = n(-0.602) \quad \therefore n = 1.29$$

* For ΔU : $\Delta U = m C_V (T_2 - T_1)$

$$C_V = \frac{R}{\gamma - 1}$$

$$= \frac{287}{0.4} = 717.5 \text{ J/kg K}$$

& for T_2 : $\left(\frac{T_2}{T_1} \right) = \left(\frac{V_1}{V_2} \right)^{n-1}$

$$\therefore T_2 = T_1 \cdot (1/0.25)^{0.29}$$

$$\therefore T_2 = 496.17 \text{ K}$$

$$\therefore \Delta U = m C_V (T_2 - T_1)$$

$$= 1.046 (0.717) (163.17)$$

$$= 122.37 \text{ kJ}$$

* Work done: $W = \frac{m R (T_1 - T_2)}{n - 1}$

$$= \frac{1.046 (287) (-163.17)}{0.29} \rightarrow W = -168910 \text{ J}$$

$$= -168.91 \text{ kJ}$$

* Heat transferred:

$$Q = \Delta U + W \text{ (Acc' to First law of Thermodynamics)}$$

$$\therefore Q = 122.37 + (-168.91)$$

$$= -46.54 \text{ kJ}$$

Q. 3.28

A steel cylinder contains O_2 at a pressure of 25 bar & temperature $27^\circ C$. After using some quantity of gas the pressure was found to be 5 bar at $20^\circ C$. 700 litres of O_2 was originally put into cylinder at N.T.P. Density of O_2 at NTP is 1.43 g/litre . Find mass of O_2 used.

Solⁿ: Given data:-

$$p_1 = 25 \text{ bar}, p_2 = 5 \text{ bar}$$

$$T_1 = 27 + 273 = 300 \text{ K}, T_2 = 20 + 273 = 293 \text{ K}$$

$$\text{Initial volume of } O_2 = 700 \text{ litres} = 0.7 \text{ m}^3$$

$$\text{Pressure at NTP: } p = 101.325 \text{ kPa}$$

$$\text{Temperature at NTP, } T = 0^\circ C = 273 \text{ K}$$

$$\text{We have, } pV = mRT$$

$$\therefore p \cdot \frac{V}{m} = RT, \text{ as } \frac{m}{V} = \rho$$

$$\therefore \frac{p}{\rho} = RT$$

$$\therefore R = \frac{p}{\rho \cdot T} = \frac{101.325}{1.43 \times 273} = 0.26 \text{ kJ/kg.K}$$

$$\therefore m_1 = \frac{pV}{RT} = \frac{101.325 \times 0.7}{0.26 \times 273} = 0.999 \approx 1 \text{ kg.}$$

$$\rightarrow \text{Volume of tank, } V_1 = \frac{m_1 RT_1}{p_1} = \frac{1 \times 0.26 \times 300}{2500} = 0.0312 \text{ m}^3.$$

$$\rightarrow \text{Final mass, } m_2 = \frac{p_2 V_1}{RT_2} = \frac{500 \times 0.0312}{0.26 \times 293}$$

$$\therefore m_2 = 0.205 \text{ kg.}$$

\therefore Mass of O_2

$$\text{which is } = m_1 - m_2$$

$$\text{used } = 1 - 0.205 = 0.795 \text{ kg}$$

J. C. Nishu Patel

THERMODYNAMIC PROCESSES

— Jenish Patel

Types of process	Governing equations	Heat interaction Q	Work done during state change from 1 to 2. $\therefore W = \int p dV$	change in Internal energy ΔU	change in Enthalpy ΔH
1) Isobaric process $P = \text{constant}$	$\frac{T_2}{T_1} = \frac{V_2}{V_1}$ index $n = 0$	$m C_p (T_2 - T_1)$	$W = p(V_2 - V_1)$	$\Delta U = m C_v (T_2 - T_1)$	$\Delta H = m C_p (T_2 - T_1)$
2) Isochoric process (Constant volume process)	$\frac{T_1}{T_2} = \frac{P_1}{P_2}$ index $n = \infty$	$m C_v (T_2 - T_1)$	$W = 0$	$\Delta U = m C_v (T_2 - T_1)$	$\Delta H = m C_p (T_2 - T_1)$
3) Isothermal process (Constant temperature process)	$P_1 V_1 = P_2 V_2$ index $n = 1$	$= P_1 V_1 \log_e \left(\frac{V_2}{V_1} \right)$ $= P_1 V_1 \log_e \left(\frac{P_1}{P_2} \right)$	$W = P_1 V_1 \log_e \left(\frac{V_2}{V_1} \right)$ $= P_1 V_1 \log_e \left(\frac{P_1}{P_2} \right)$	$\Delta U = 0$	$\Delta H = 0$

Types of process	Governing equations	Heat interaction 'Q'	Work done during state change from 1 to 2 $W = \int_1^2 p dV$	Change in Internal energy ΔU	change in Enthalpy ΔH
4) ADIABATIC PROCESS	<ul style="list-style-type: none"> $p_1 V_1^\gamma = p_2 V_2^\gamma$ $\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ index $n = \gamma$ 	0	$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$ also, $W = \frac{mR(T_1 - T_2)}{\gamma - 1}$	$\Delta U = mC_v(T_2 - T_1)$	$\Delta H = mC_p(T_2 - T_1)$
5) POLYTROPIC PROCESS	<ul style="list-style-type: none"> $p_1 V_1^n = p_2 V_2^n$ $\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{n-1}$ $\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ 	$\left(\frac{\gamma - n}{\gamma - 1}\right) \times \text{work}$	$W = \frac{p_1 V_1 - p_2 V_2}{n - 1}$	$\Delta U = mC_v(T_2 - T_1)$	$\Delta H = mC_p(T_2 - T_1)$