5 Superconducting Materials

Learning Objectives

After studying this chapter, the student will be able to:

- Define superconductor and critical temperature
- Understand the basic properties of superconductors
- Distinguish between Type I and Type II superconductors
- Describe the structure and composition of high temeperature superconductors
- Enlist the different applications of superconductors such as Maglev train and SQUID.

5.1 INTRODUCTION

***Superconductivity** is a phenomenon in which certain metals, alloys and ceramics conduct electricity without resistance when it is cooled below a certain temperature called the critical temperature.*

Superconductivity was discovered by a Dutch physicist, Heike Kammerlingh Onnes, in 1911 and it is still an exciting field of discovery and technological applications. This new state was first discovered in mercury when cooled below 4.2 K. Since then, a large number and wide variety of metals, alloys, binary and ternary chemical compounds have been found to show superconductivity at various temperatures.

In the following sections fundamental terms and phenomena of superconductors, its properties, types and its applications are discussed in brief.

5.2 SUPERCONDUCTOR

A superconductor is a material that loses all its resistance (offers zero resistance) to the flow of electric current when it is cooled below a certain temperature called the critical temperature or transition temperature T_c.

Examples: Mercury (Hg), Zinc (Zn), Vanadium (V), Tin (Sn) and Niobium (Nb).

5.3 CRITICAL TEMPERATURE T_C (TRANSITION TEMPERATURE)

The temperature at which a material's electrical resistivity drops to absolute zero is called the critical temperature or transition temperature T_c.

At and below T_c, the material is said to be in the superconducting state and above this temperature, the material is said to be in the normal state.

Figure 5.1 shows the variation of electrical resistivity of a normal metal silver (Ag) and a superconducting metal mercury (Hg) versus temperature.

From Figure 5.1, it can be seen that the electrical resistivity of normal metal decreases steadily as the temperature is decreased and reaches a low value at 0 K *called the residual resistivity* ρ_0. But in contrast, the electrical resistivity of mercury suddenly drops to zero at critical temperature T_c and is 4.2 K for Hg.

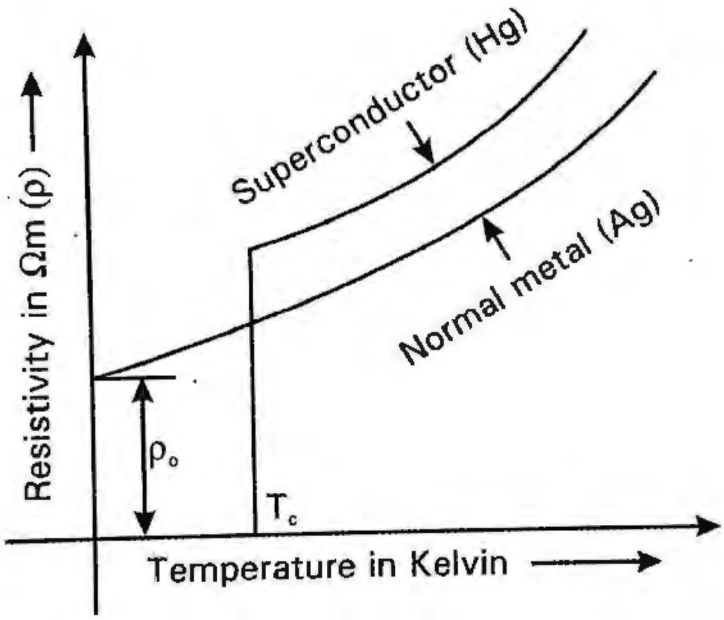


Figure 5.1: Electrical resistivity vs temperature plots for a superconductor and a normal metal

Note:

- Good electrical conductors such as silver (Ag), gold (Au) and copper (Cu) are not good superconductors because the resistivity of these conductors at low temperatures is limited to low resistivity ρ_0 (residual resistivity) value due to scattering of electrons from crystal defects and impurities.
- Similarly, good superconducting materials like Zn and Pb are not good electrical conductors.

Below the critical temperature, not only does the superconductor suddenly achieves zero resistance, it also exhibits a variety of several astonishing magnetic and electrical properties.

The T_c values for some selected metals, intermetallic and ceramic superconductors are given in Table 5.1.

Table 5.1

Metals	T_c in K	Intermetallic compounds	T_c in K	Ceramic compounds	T_c in K
Tin (Sn)	3.72	NbTi	9.5	$Y_1 Ba_2 Cu_3 O_{7-x}$	93
Mercury (Hg)	4.2	Nb_3Sn	21	Tl – Ba – Ca – Cu	125
Vanadium (V)	5.3	Nb_3Ge	23.2	HgBaCuO	133

5.4 PROPERTIES OF SUPERCONDUCTORS

Few important properties of superconductors are explained in brief in this section.

(i) *Electrical resistance*

The electrical resistance of a superconducting material is very low and is of the order of 10^{-7} Ωm.

(ii) *Effect of impurities*

When impurities are added to superconducting elements, the superconducting property is not lost, but the T_c value is lowered.

(iii) *Effect of pressure and stress*

Certain materials are found to exhibit the superconductivity phenomena on increasing the pressure over them. For example, cesium is found to exhibit superconductivity phenomena at $T_c = 1.5$ K on applying a pressure of 110 Kbar.

In superconductors, the increase in stress results in increase of the T_c value.

(iv) *Isotope effects*

The critical or transition temperature T_c value of a superconductor is found to vary with its isotopic mass. This variation in T_c with its isotopic mass is called the isotopic effect.

The relation between T_c and the isotopic mass is given by

$$T_c \propto \frac{1}{\sqrt{M}} \text{ where M is the isotopic mass,}$$

i.e., the transition temperature is inversely proportional to the square root of the isotopic mass of a single superconductor.

(v) *Magnetic field effect*

If a sufficiently strong magnetic field is applied to a superconductor at any temperature below its critical temperature T_c, the superconductor is found to undergo a transition from the superconducting state to the normal state.

This minimum magnetic field required to destroy the superconducting state is called the ***critical magnetic field H_c.***

The critical magnetic field of a superconductor is a function of temperature. The variation of H_c with temperature is given by

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right] \quad \text{...(1)}$$

where H_0 is the critical field at $T=0$ K. The critical field decreases with increasing temperature and becoming zero at $T=T_c$.

Figure 5.2 shows the variation of critical field H_c as a function of temperature. The material is said to be in the superconducting state within the curve and is non-superconducting (i.e., normal state) in the region outside the curve.

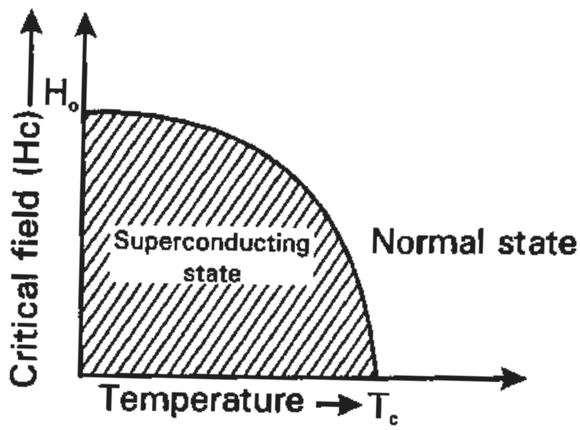


Figure 5.2: Dependence of H_c on T

(vi) Critical current density J_c and critical current I_c

The critical current density is another important characteristic feature of the superconducting state.

When the current density through a superconducting sample exceeds a critical value J_c, the superconducting state is found to disappear in the sample. This happens because the current through the superconductor itself generates a magnetic field, and at a sufficiently high current density the magnetic field will start exceeding the critical magnetic field H_c, thereby making the superconducting state to disappear in the material.

Hence, the critical current density can be defined as the maximum current that can be permitted in a superconducting material without destroying its superconductivity state. The critical current density is a function of temperature, i.e., colder the temperature for a superconductor the more is the current it can carry.

For a thin long cylindrical superconducting wire of radius r, the relation between critical current I_c and critical magnetic field H_c is given by

$$I_c = 2\pi r H_c$$

Similarly, the relation between critical current density J_c and critical current I_c is given by

$$J_c = \frac{I_c}{A}$$

where A is the superconducting specimen's cross-sectional area.

(vii) Persistent current

When current is made to flow through a superconducting ring (say a loop of lead wire), which is at a temperature either equal to its T_c value or less than its T_c value, it was observed that the current was flowing through the material without any significant loss in its value.

This steady flow of current in a superconducting ring without any potential deriving it is called the persistent current.

(viii) Meissner effect (Diamagnetic property)

The complete expulsion of all the magnetic field by a superconducting material is called the ***'Meissner effect'.***

When a superconducting material is placed in a magnetic field ($H > H_c$) at room temperature, the magnetic field is found to penetrate normally throughout the material (Figure 5.3(a)).

However, if the temperature is lowered below T_c and with $H < H_c$ the material is found to reject all the magnetic field penetrating through it, as shown in Figure 5.3(b).

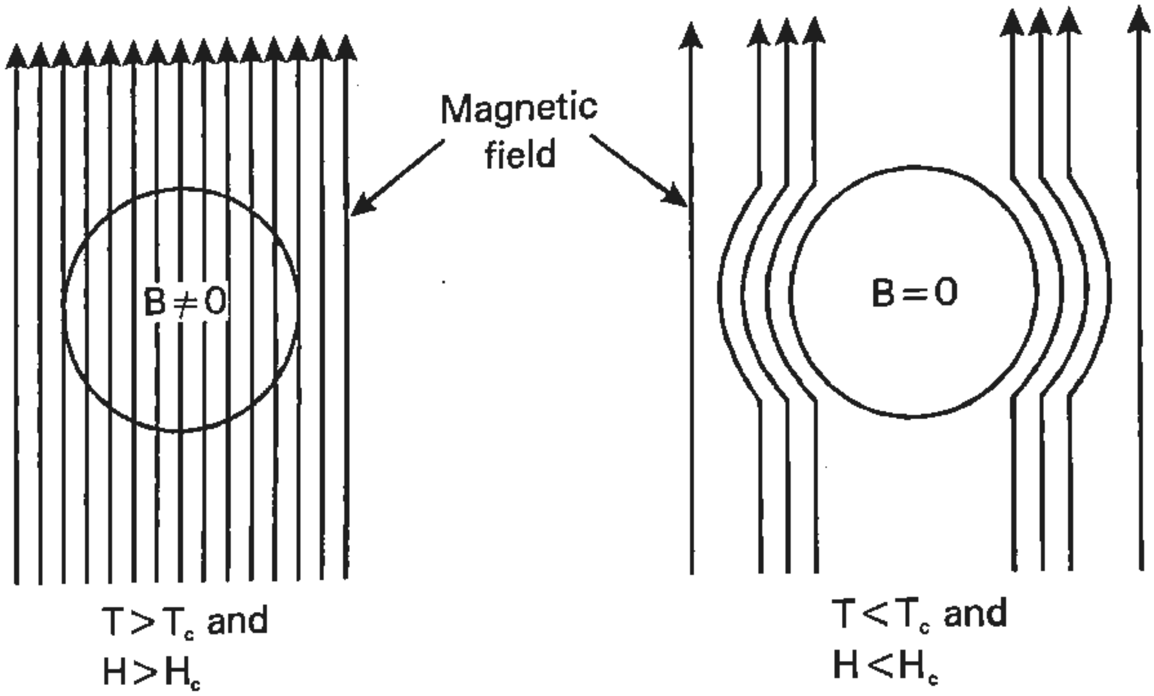


Figure 5.3a: Normal state

Figure 5.3b: Superconducting state

The above process occurs due to the development of surface current, which in turn results in the development of magnetization M within the superconducting material. Hence, as the developed magnitation and the applied field are equal in magnitude but opposite in direction, they cancel each other everywhere inside the material. Thus, below T_c a superconductor is a perfectly diamagnetic substance ($\chi_m = -1$).

The Meissner effect is a distinct characteristic of a superconductor from a normal perfect conductor. In addition, this effect is exhibited by the superconducting materials only when the applied field is less than the critical field H_c.

5.4.1 To Prove $\chi_m = -1$ for Superconductors

We know that for a magnetic material the magnetic induction or magnetic flux density B is given by

$$B = \mu_0 (M + H) \qquad \text{...(1)}$$

where μ_0 is the permeability of free space

M is the intensity of magnetisation

and H is the applied magnetic field.

But, we know that for a superconductor $B = 0$

Therefore, equation (1) can be written as

$$0 = \mu_0 (M + H)$$

$$\because \quad \mu_0 \neq 0$$

$$M + H = 0$$

or

$$M = -H$$

or

$$\frac{M}{H} = -1$$

Hence, $\chi_m = -1$ *where* $\chi_m = \dfrac{M}{H}$ *is called the magnetic susceptibility.* Thus this means that for a superconductor the susceptibility is negative and maximum, i.e., a *superconductor exhibits perfect diamagnetism.*

5.4.2 Three Important Factors to Define a Superconducting State

In general, the superconducting state is defined by three important factors:

(i) Critical temperature T_c

(ii) Critical current density J_c

(iii) Critical magnetic field H_c

Each of the above three parameters is very dependent on the other two properties. To sustain superconducting state in a material, it is required to have both the current density and magnetic field, as well as the temperature, to remain below their critical values; and all of these would depend on the material.

The relationship between T_c, J_c and H_c is shown in the phase diagram (Figure 5.4).

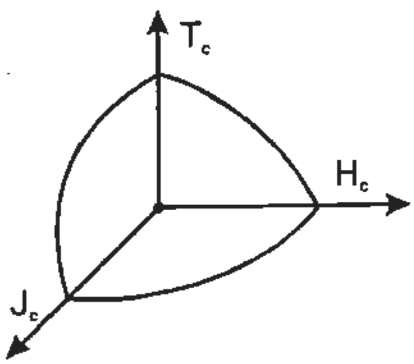


Figure 5.4: Critical surface phase diagram

The highest values for H_c and J_c occur at 0 K, while the highest value for T_c occurs when H and J are zero. Thus, the plot of all these three parameters represents a critical surface.

Within the surface the material is superconducting and outside the surface the material is said to be in the normal state.

Example 5.1

For mercury of mass number 202, the α value is 0.50 and T_c is 4.2 K. Find the transition temperature for the isotope of mercury of mass number 200.

Solution:

Given,

$$\text{Mass number } M_1 = 202$$
$$\alpha = 0.5$$
$$T_{c1} = 4.2 \text{ K}$$
$$\text{Mass number } M_2 = 200$$
$$T_{c2} = ?$$

We know, $M^\alpha T_{c1} = \text{constant}$

Using this,

$$M_1^\alpha T_{c1} = M_2^\alpha T_{c2}$$

or $$T_{c2} = \left(\frac{M_1}{M_2}\right)^\alpha T_{c1} = \left(\frac{202}{200}\right)^{0.5} \times 4.2 \qquad \left[\because \alpha = \frac{1}{2}\right]$$

$$= 1.004987 \times 4.2$$

$$\mathbf{T_{c2} = 4.2209\ K.}$$

Example 5.2

The critical temperature of Nb is 9.15 K. At zero kelvin the critical field is 0.196 T. Calculate the critical field at 6 K.

Solution:

Given, $T_c = 9.15$ K; $T = 6$ K; $H_o = 0.196$ T; $H_c = ?$

$$H_c = H_o\left(1-\left(\frac{T}{T_c}\right)^2\right) = 0.196\left[\left(1-\left(\frac{6}{9.15}\right)^2\right)\right]$$

$$= 0.196\,[1 - 0.4299]$$

$$= 0.196\,[0.5701]$$

$$\therefore \quad \mathbf{H_c = 0.1117\ T.}$$

Example 5.3

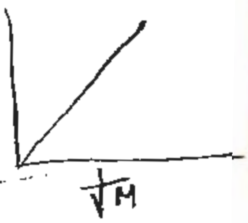
The critical temperature for a metal with isotopic mass 199.5 is 4.185 K. Calculate the isotopic mass if the critical temperature falls to 4.133 K.

Solution:

Given, $M_1 = 199.5$; $T_{c1} = 4.185$ K; $T_{c2} = 4.133$ K; $M_2 = ?$

$T_c \propto \frac{1}{\sqrt{M}}$

$T_c \cdot M^{\alpha} =$



Formula:

$$M_1^{\alpha}\, T_{c1} = M_2^{\alpha}\, T_{c2}$$

$$\therefore \quad M_2^{\alpha} = (199.5)^{\alpha}\, \frac{4.185}{4.133}$$

$$M_2^{0.5} = (199.5)^{0.5} \times 1.01258 \qquad \left[\because \alpha = \frac{1}{2}\right]$$

$$\sqrt{M_2} = \sqrt{199.5} \times 1.01258 = 14.124 \times 1.01258$$

$$M_2 = (14.301)^2$$

$$\therefore \quad \mathbf{M_2 = 204.55.}$$

Example 5.4

Calculate the critical current through a long thin superconducting wire of radius 0.5 mm. The critical magnetic field is 7.2 kA/m.

Solution:

Given, $H_c = 7.2 \times 10^3$ A/m; $r = 0.5 \times 10^{-3}$ m; $I_c = ?$

Formula:

$$I_c = 2\pi r H_c$$

$$= 2 \times 3.14 \times 0.5 \times 10^{-3} \times 7.2 \times 10^3$$

$$\therefore \quad \mathbf{I_c = 22.608\ A.}$$

Example 5.5

Superconducting Sn has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 T at 0 K. Find the critical field at 2 K.

Solution:

Given, $T_c = 3.7$ K; $H_o = 0.0306$ T; $H_c = ?$; $T = 2$ K

Formula:

$$H_c = H_o\left[1 - \left(\frac{T}{T_c}\right)^2\right] = 0.0306\left[1 - \left(\frac{2.0}{3.7}\right)^2\right]$$

$$= 0.0306\,(1 - 0.29218) = 0.0306 \times 0.70782$$

$$\therefore \quad \mathbf{H_c = 0.021659\ tesla.}$$

Example 5.6

Calculate the critical current for a superconducting wire of lead having a diameter of 1 mm at 4.2 K. Critical temperature for lead is 7.18 K and $H_c(0) = 6.5 \times 10^4$ A/m.

Solution:

Given, $H_o = 6.5 \times 10^4$ A/m; $T_c = 7.18$ K; $r = 0.5 \times 10^{-3}$ m; $T = 4.2$ K; $I_c = ?$; $H_c = ?$

Formula:

$$H_c = H_o\left[1-\left(\frac{T}{T_c}\right)^2\right] = 6.5 \times 10^4\left[1-\left(\frac{4.2}{7.18}\right)^2\right]$$

$$= 6.5 \times 10^4 (1 - 0.34217) = 6.5 \times 10^4 \times 0.65783$$

$$\therefore \quad \mathbf{H_c = 42.758\ kA/m.}$$

$$I_c = 2\pi r H_c = 2 \times 3.14 \times 0.5 \times 10^{-3} \times 42.758 \times 10^3$$

$$\therefore \quad \mathbf{I_c = 134.26\ A.}$$

Example 5.7

The critical field for vanadium is 10^5 Am^{-1} at 8.58 K and 2×10^5 Am^{-1} at 0 K. Determine the T_c value.

Solution:

Given, $H_c = 10^5$ Am^{-1}; $H_o = 2 \times 10^5$ Am^{-1}; $T = 8.58$ K; $T_c = ?$

Formula:

$$H_c = H_o\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$\therefore \quad \left(\frac{H_c}{H_o}\right) = 1-\left(\frac{T}{T_c}\right)^2$$

$$\left(\frac{T}{T_c}\right)^2 = 1-\left(\frac{H_c}{H_o}\right)$$

$$\therefore \quad T_c = \frac{T}{\sqrt{1-\left(\frac{H_c}{H_o}\right)}} = \frac{8.58}{\sqrt{1-\frac{10^5}{2\times 10^5}}} = \frac{8.58}{\sqrt{1-0.5}} = \frac{8.58}{\sqrt{0.5}} = \frac{8.58}{0.7071}$$

$$\therefore \quad \mathbf{T_c = 12.1334\ K.}$$

5.5 TYPES OF SUPERCONDUCTORS

Based on the behaviour of superconducting materials in an applied magnetic field, the superconductors are classified into type I and II superconductors.

5.5.1 Type I Superconductors

Type I superconductors exhibit complete Meissner effect, i.e., they are completely diamagnetic. The magnetization curve for type I superconductor is shown in Figure 5.5(a). The values of H_c for type of I superconducting materials are always too low.

The magnetization curve shows that the transition at H_c is reversible. This means that if the magnetic field is reduced below H_c, the material again acquires superconducting property and the field is expelled.

Type I superconductors are also called as ***soft superconductors*** because of their tendency to allow the field penetration even for a lower applied field. Many pure elements, alloys and some compound superconductors exhibit type I behaviour.

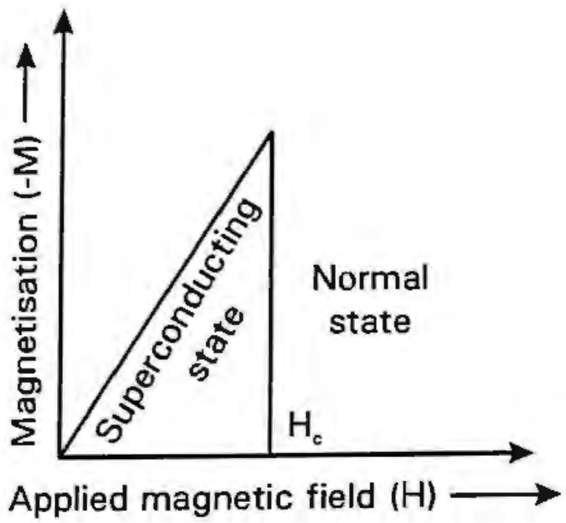


Figure 5.5a: Type I superconductor

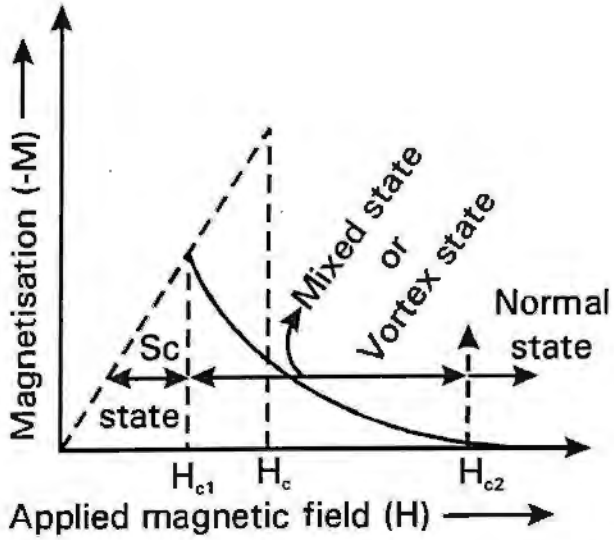


Figure 5.5b: Type II superconductor

5.5.2 Type II Superconductors

Type II superconductors behave differently in an increasing field, as shown in Figure 5.5(b). For an applied field below H_{c1}, the material is perfectly diamagnetic and hence the field is completely excluded. *H_{c1} is called as lower critical field.* At H_{c1}, the field starts to thread the specimen and this penetration increases until H_{c2} is reached at which the magnetization vanishes and the specimen becomes normal. *H_{c2} is called the upper critical field.*

In the region between H_{c1} and H_{c2} the material is in the mixed state or the vortex state. The value of H_{c2} for type II may be 100 times more or even higher than that of type I superconducting material. As H_{c2} and T_c of type II superconducting materials are higher than that of type I superconductors, the type II superconducting materials are most widely used in all engineering applications.

Type I superconducting materials are also called as ***hard superconductors*** because of relatively large magnetic field requirement to bring them back to their normal state.

Examples for type I and type II superconducting materials with their H_c values are listed in Table 5.2.

Table 5.2: Examples of type I and type II superconducting materials

Type I		Type II	
Material	*H_c in tesla*	*Material*	*H_c in tesla*
Ta	0.083	$Y_1 Ba_2 Cu_3 O_7$	300
Pb	0.08	$Ba_{2-x} B_x Cu O_4$	150
Hg	0.014	Nb_3Sn	24.5
Sn	0.030	$Nb_3 Ge$	38

5.6 COMPARISON BETWEEN TYPE I AND TYPE II SUPERCONDUCTORS

Type I superconductor	Type II superconductor
1. These superconductors are called as soft superconductors.	1. These superconductors are called as hard superconductors.
2. Only one critical field exists for these superconductors.	2. Two critical fields H_{c1} (lower critical field) and H_{c2} (upper critical field) exist for these superconductors.
3. The critical field value is very low.	3. The critical field value is very high.
4. These superconductors exhibit perfect and complete Meissner effect.	4. These do not exhibit a perfect and complete Meissner effect.
5. These materials have limited technical applications because of very low field strength value. ***Examples:*** Pb, Hg, Zn, etc.	5. These materials have wider technological applications because of very high field strength value. ***Examples:*** Nb_3Ge, Nb_3Si, $Y_1Ba_2Cu_3O_7$, etc.

5.7 HIGH T_C SUPERCONDUCTORS

Based on the coolants to achieve superconductivity phenomena in materials, the superconductors fall in two categories:

1. Low-temperature superconductors
2. High-temperature superconductors

5.7.1 Low-Temperature Superconductors

Superconductors that require liquid helium as coolant are called low-temperature superconductors (LTS or Low $-T_c$). Liquid helium temperature is 4.2 K above absolute zero.

High Temperature Super conductors: Superconductors having their T_c Values above the temperature of liquid nitrogen (77 K. or −196°c) are called High temp. Superconductors

* Penetration depth : Magnetic field

→ In 1935, F. London and H. London obtained an expression for penetration of applied mag. field into superconducting material from the surface.

→ According to them, the applied magnetic field does not drop to zero at the surface of superconductor but decreses exponentially as given by the relation

$$H = H_0 \, e^{-x/\lambda}$$

where H = intensity of magnetic field at depth x from the surface.

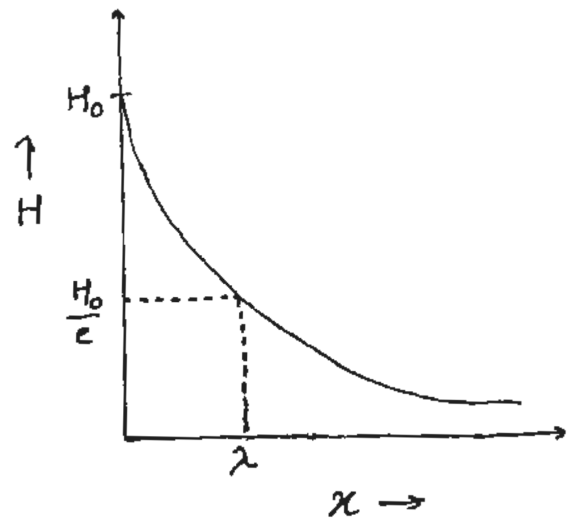
H_0 = intencity of magnetic field at the surface

λ = London penetration depth.

→ "London penetration depth is defined as the distance from the surface of the superconductor to a point inside the material at which the intencity of magnetic field falls to $1/e$ times the magnetic field at the surface [i.e. H_0/e]".

→ The order of penetration of magnetic field is about 10 - 100 nm.

→ The variation of intensity of magnetic field with distance from the surface into the material is shown in fig. →



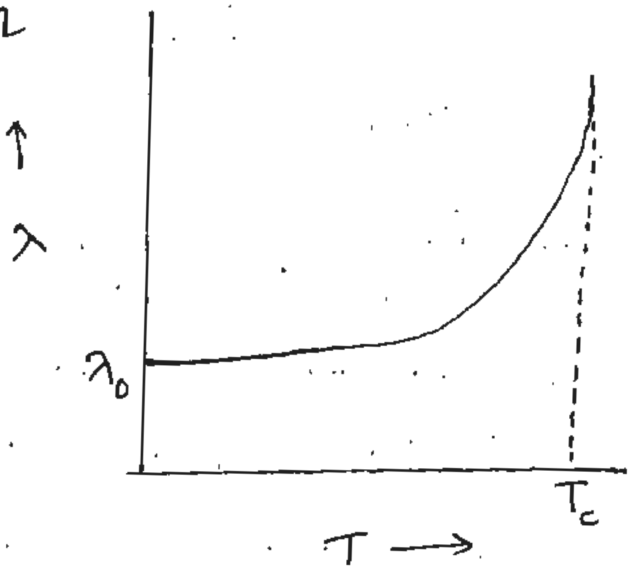
→ The penetration depth is not constant but Varies with temperature is shown in fig. (below) and Variation may be expressed as

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$$

Where $\lambda(T)$ = Value of depth at temp. T.

$\lambda(0)$ is the Value at $T=0\,K$.

→ From the fig. it can be seen, the penetration depth increases rapidly and approaches infinity as the temp approaches the transition temp (T_c) of the material.

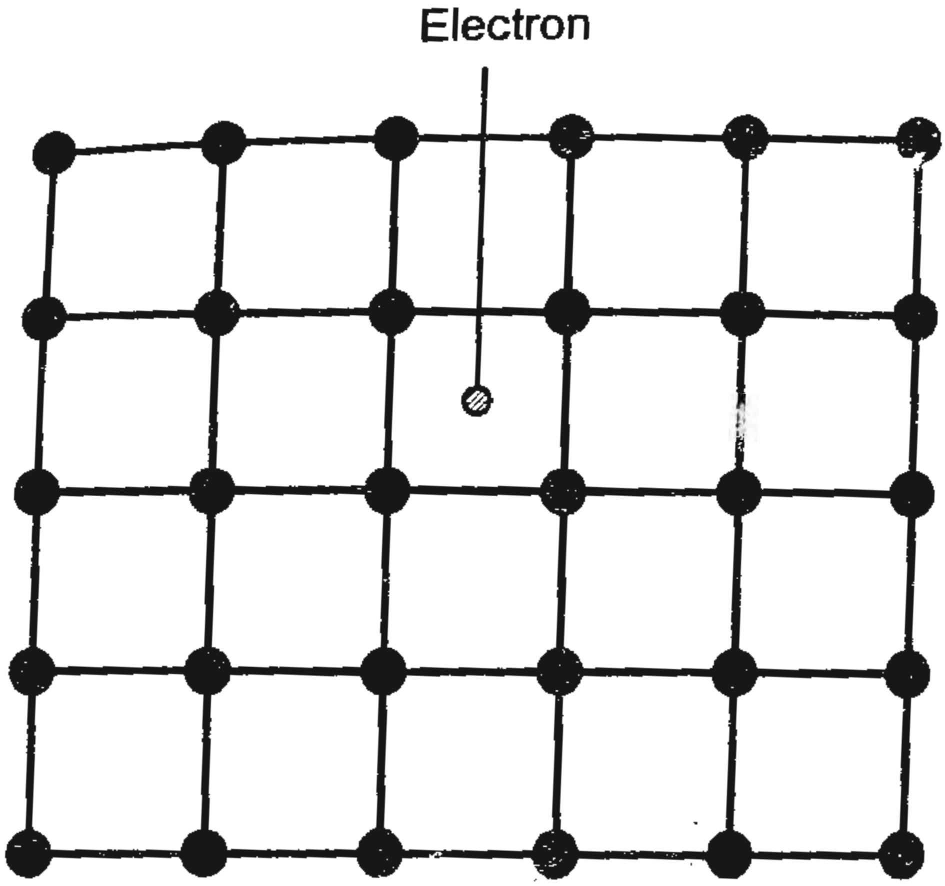


4.3 Mechanism of Superconductivity: BCS Theory

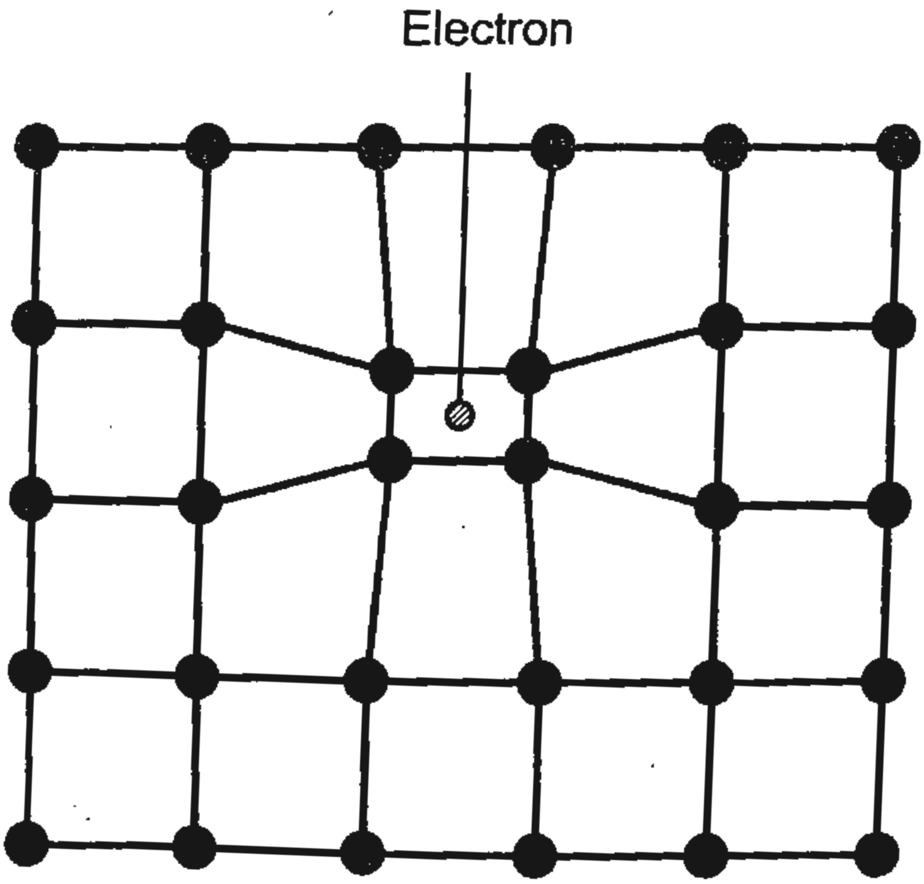
Though invention of superconductivity was done in 1911 by *Onnes*, well – defined mechanism explaining the superconductivity was developed by three scientists namely **Bardeen**, **Cooper** and Schrieffer in 1957 after research of number of years. This theory is named as BCS theory based on first letter of their names. To understand this theory, first of all behaviour of electron in normal condition is required to be understand as explained below:

- At normal condition (room temperature) in an ordinary conductor, as per the basis of quantum mechanics, electrons with same energy having same energy level will have opposite momenta, *i.e.*, one electron is moving towards left then other electron moves towards right. Therefore, when no electrical current is flowing, the energy levels are filled bottom upwards with the pair of electrons of opposite momentum in each energy level. Also, the crystal lattice is in (almost) stationary condition.

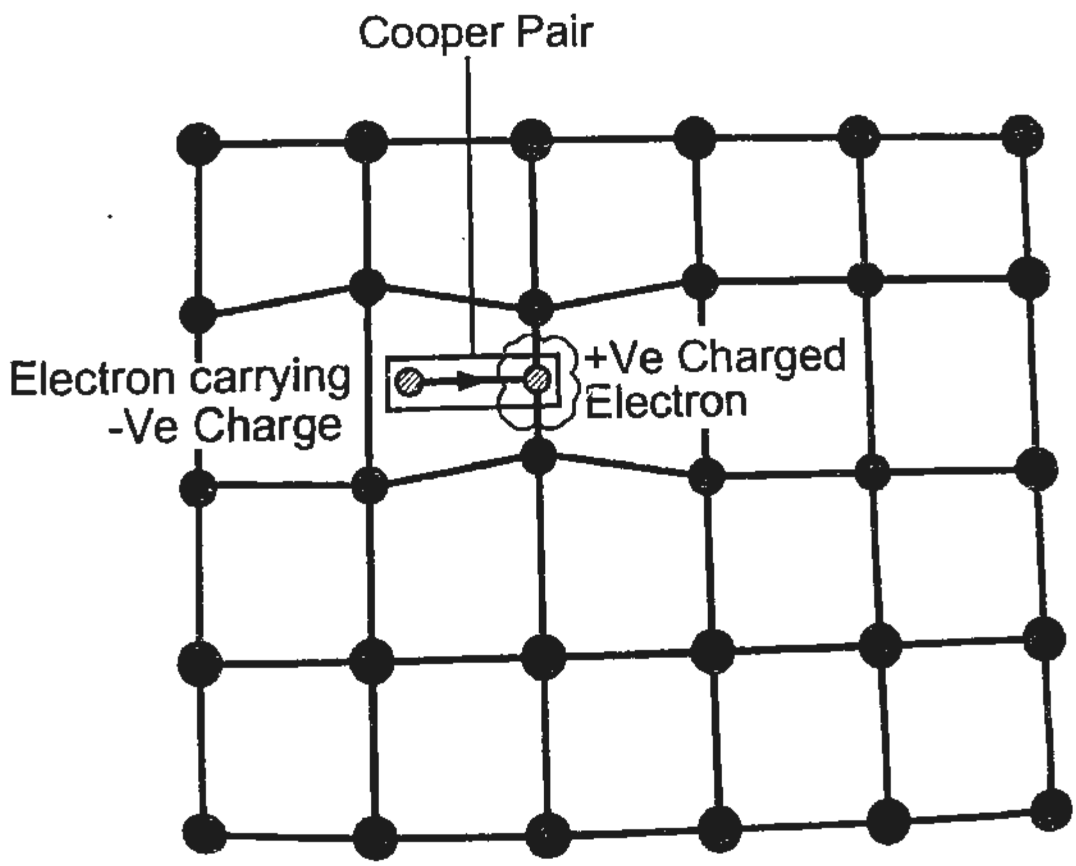
- When electrical field is applied, the crystal lattice starts to vibrate and electrons also move from one position to other. During this motion of electrons, they collide with crystal lattice continuously and exchange of energy takes place between them which develops heat. Due to this collision, electrons are facing resistance to their motion which is termed as electrical resistance. As the temperature increases (due to collision) the electrical resistance also increase.
- Now, when the temperature of material is reduced below to its critical temperature, crystal lattice vibrates very slowly in comparison to normal temperature. Assume an electron which is passing through a crystal lattice as shown in fig.



Since an electron carries negative charge, whereas ions have positive charge, all ions are attracted towards negatively charged electrons and hence the crystal lattice gets distorted as shown in fig.



- Due to this distorted lattice, a cloud of positive charge is being developed around the electron which is known as **phonon** and due to this phonon electron interaction, negatively charged electron now carries a positive charge with it.
- When this positively charged electron moves ahead, it attracts the other negatively charged electron and interacts with each other and form a weak bonding due to presence of phonon.
- This indicates that for superconducting materials, the interaction of electrons and phonons is modified by the fact that the electrons may no longer be independent, but may be bound together in pairs, known as **Cooper pairs**, which drifts in the lattice when current flows.
- Cooper pair is the new particle made up of two electrons having same momentum but opposite spin. It has twice the mass and twice the charge of an electron. Motion of such cooper pairs is only responsible for electrical current flow in the superconductor.



- Now, if any of the electron is to be scattered by a collision in the direction of current flow, the phonon must provide sufficient energy to break the cooper pair bonding in addition to the ordinary criteria pertaining to allowed energy and momentum before collision. But at temperature below critical temperature, the number of phonons having sufficient energy to break the cooper pair bonding is very few, and hence such pairs may pass through the lattice without breaking of the bonds and hence they experience zero resistance.
- If the temperature is raised above critical temperature, the energy of breaking the cooper pairs increases and hence above its critical temperature, the superconductivity ends.

This theory critically postulates the mechanism of current flow in the superconductivity state.

5.8.2 Josephson Effect and Its Application

Josephson junction: *Two superconductors separated by a very thin strip of an insulator (Figure 5.8) forms a Josephson junction.*

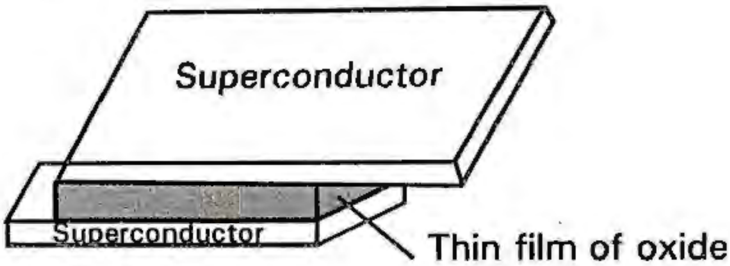


Figure 5.8: Josephson junction

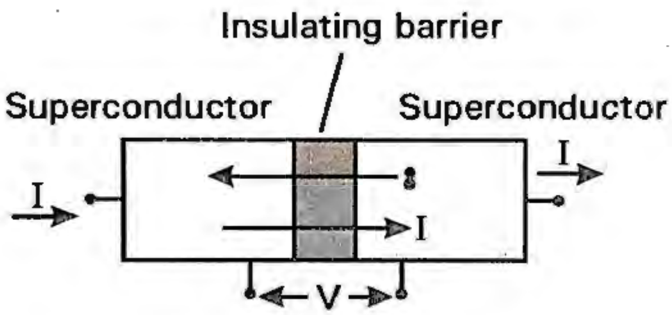


Figure 5.9: Josephson junction with an applied external voltage (V)

Josephson effect

The wave nature of moving particles makes the electrons to tunnel through the barrier (insulator), i.e., the electrons can tunnel from one superconductor to the other. As a consequence of the tunneling of electrons (cooper pairs) across the insulator, there is a net current across the junction. This is called as ***d.c. Josephson effect.*** The current flows even in the absence of a potential difference.

The magnitude of the current depends on the thickness of the insulator, the nature of the material naturė and the temperature.

On the other hand, when a potential difference V is applied between the two sides of the junction (Figure 5.9), there will be an oscillation of the tunneling current with angular frequency $v = \frac{2eV}{h}$. This is called the ***a.c. Josephson effect.*** Thus, according to a.c. Josephson effect, the junction generates an a.c. current at a frequency of $\frac{2eV}{h}$ Hz per volt.

Note:

- Cooper pair is a bound pair of electrons formed by the interaction between the electrons with opposite momenta and spin in a phonon field.

Application of Josephson junction

Josephson junctions are used in sensitive magnetometers called SQUID-Superconducting Quantum Interference Device.

A SQUID is formed by connecting two Josephson junctions in parallel (Figure 5.10).

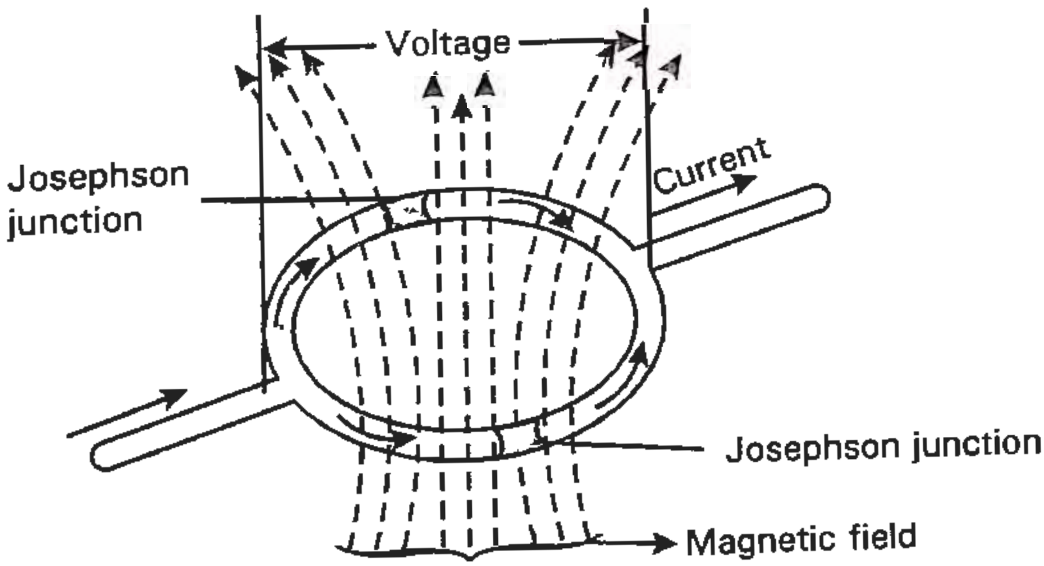


Figure 5.10: SQUID

When current is passed into this arrangement, it splits flowing across the two opposite arc. The current through the circuit will have a periodicity which is very sensitive to the magnetic flux passing normally through the closed circuit. As a result, extremely small magnetic flux can be detected with this device.

This device can also be used to detect voltages as small as 10^{-15} V.

Magnetic field changes as small as 10^{-21} T can be detected.

Weak magnetic fields produced by biological currents such as those in the brain can also be detected using SQUIDs.

SQUID detectors are used to measure the levels of iron in liver-so that iron built-up can be treated before much harm is done to the body.

5.8.3 Other Applications of Superconductors

(1) Superconductors can be used to transmit electrical power over very long distances without any power loss or any voltage drop.

(2) Superconducting generators has the benefit of small size and low energy consumption than the conventional generators.

(3) Superconducting coils are used in N.M.R. (nuclear magnetic resonance) imaging equipments which are used in hospitals for scanning the whole body to diagnose medical problems.

(4) Very strong magnetic fields can be generated with coils made of high T_c superconducting materials.

(5) Superconductors can act as relay or switching system in a computer. They can also be used as a memory or storage element in computers.

Cryotron: It is a relay or switch made of superconductors whose size can be made very small. In addition, these switches consume very less current.

The cryotron consists of two superconducting materials A and B. Let the material A be inside the coil of wire B, as shown in Figure 5.11.

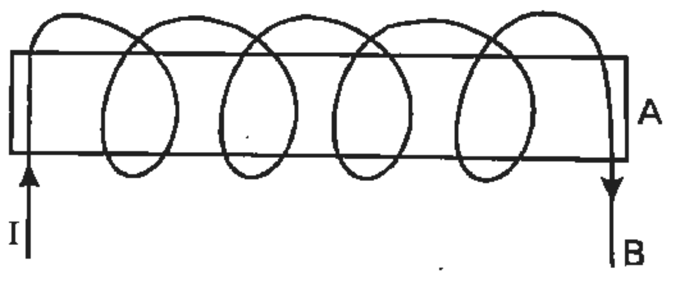


Figure 5.11: Cryotron

Let the critical field of the material A be H_{cA} and that of B be H_{cB}, respectively, and also let $H_{cA} < H_{cB}$. If a current I is passed through the material B, the current induces a magnetic field H. If this induced field H happens to be greater than H_{cA} then the superconducting property of the material A gets destroyed.

Hence the resistivity increases and the contact is broken. Thus, the current in A can be controlled by the current in B, and hence this system can act as a relay or switch element.

(6) Very fast and accurate computers can be constructed using superconductors and the power consumption is also very low.

(7) Ore separation can be done efficiently using superconducting magnets.

Example 5.8

A voltage of 5.9 μV is applied across a Josephson junction. What is the frequency of the radiation emitted by the junction?

Solution:

Given, $V = 5.9 \times 10^{-6}$ V; $\nu = ?$

Formula:

$$\nu = \frac{2eV}{h} = \frac{2 \times 1.6 \times 10^{-19} \times 5.9 \times 10^{-6}}{6.62 \times 10^{-34}}$$

$$\nu = \mathbf{2.851 \times 10^{9}\ Hz.}$$

SOLVED EXAMPLES

Example 1: *A lead superconductor with T_C=7.2 K has a critical magnetic field of $6.5\times10^3\ Am^{-1}$ at absolute zero. Calculate the magnitude of critical magnetic field at 5 K temperature.*

Solution : For a superconductor, we have

$$H_c = H_o\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Given that, $T_C = 7.2\ K,\quad T = 5K$ and $H_0 = 6.5\times10^3\ Am^{-1}$.

Therefore,
$$H_c = 6.5\times10^3\left[1-\left(\frac{5}{7.2}\right)^2\right]$$
$$= 6.5\times10^3\left[1-\frac{25}{51.84}\right]$$
$$= 6.5\times10^3\,[0.5177]$$
$$= 3.365\times10^3\ Am^{-1}$$

Example 2: *In a superconductor ring of radius 0.02m, critical magnetic field is $2\times10^3\ Am^{-1}$ at 5 K. What is its critical value of current?*

Sulution : The critical current flowing through a given wire is given by

$$I_c = 2\pi r H_c$$

Here H_c=$2\times10^3\ Am^{-1}$, r=0.02m

Therefore,
$$I_c = 2\times3.14\times0.02\times10^3$$
$$= 251.2 \text{ Ampere}$$

Example 3: *For a superconducting specimen, the values of critical fields are 1.4×10^5 and $4.2\times10^5\ Am^{-1}$ for 14 K and 13 K, respectively. Calculate the transition temperature and critical fields at 0 K and 4.2 K.*

Solution : Let H_{C_1} and H_{C_2} be the critical fields at temperatures T_1 and T_2, respectively Then

$$H_{C_1} = H_o\left[1-\left(\frac{T_1}{T_c}\right)^2\right] \quad ...(i)$$

and

$$H_{c_2} = H_o\left[1-\left(\frac{T_2}{T_c}\right)^2\right] \quad ...(ii)$$

$$\therefore \qquad \frac{H_{c_1}}{H_{c_2}} = \frac{T_c^2 - T_2^2}{T_c^2 - T_1^2}$$

Here, $\quad H_{c_1} = 1.4\times10^5\ Am^{-1},\ \ H_{c_2} = 4.2\times10^5\ Am^{-1}$

and $\quad T_1 = 14K,\ T_2 = 13K.$

$$\therefore \qquad \frac{1.4}{4.2} = \frac{T_c^2 - (14)^2}{T_c^2 - (13)^2}$$

or $\quad 1.4T_c^2 - 1.4\times169 = 4.2T_c^2 \times 4.2\times196$

$\therefore \quad T_c^2(4.2-1.4) = (4.2\times196)-(1.4\times196)$

or $\quad T_c = 14.5\ K$.

Putting 14.5 for T_C and 1.4×10^5 for H_C in equation (i), we have

$$H_0 = \frac{1.4\times10^5}{\left[1-\left(\frac{14}{14.5}\right)^2\right]}$$

$$= \frac{1.4\times10^5}{1-0.9322} = 20.7\times10^5\ Am^{-1}$$

The critical fields at 0 K and 4.2 K may be calculated as follows :

At 0 K, $\quad H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right] = H_0(1-0) = H_0$

$\therefore \quad H_c = 20.7\times10^5\ \ Am^{-1}$

At 4.2 K, $\quad H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$

$$= 20.7\times10^5\left[1-\left(\frac{4.2}{14.5}\right)^2\right]$$

$18.9\times10^5\ Am^{-1}$.

Example 4: *Calculate the critical current density for 1mm diameter of wire of lead at (i) 4.2 K and (ii) 7 K. A parabolic dependence of H_C on T may be assumed.*

Given that, for lead T_C =7.18 K and H_0=$6.5\times10^4\ \ Am^{-1}$

Solution : We know that

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Here, $H_0 = 6.5\times10^4\ Am^{-1},\ r = \dfrac{d}{2} = 0.5\ mm = 0.5\times10^{-3}m$ and $T_c = 7.18\,K$

Case (i) For $T = 4.2\,K,\ H_c = 6.5\times10^4\left[1-\left(\dfrac{4.2}{7.18}\right)^2\right]$

$$= 4.28\times10^4\ Am^{-1}.$$

Critical current density is given by

$$J_c = \frac{I_c}{A} = \frac{2\pi r H_c}{\pi r^2} = \frac{2H_c}{r}$$

$$= \frac{2\times4.28\times10^4}{0.5\times10^{-3}} = 1.71\times10^8\ Am^{-2}$$

Case (ii), $H_c = 6.5\times10^4\left[1-\left(\dfrac{7}{7.18}\right)^2\right]$

$$= 3.25\times10^3\ Am^{-2}$$

$\therefore$ $$J_c = \frac{3.25\times10^3}{0.5\times10^{-3}} = 6.5\times10^6\ Am^{-2}.$$

Example 5: *The critical temperature of a superconductor in the absence of magnetic field is T_c. Calculate the temperature at which the critical field becomes half of its value at 0 K.*

Solution : We know that

$$H_C = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

In this case, $H_C = \dfrac{H_0}{2}$

$\therefore$ $$\frac{H_0}{2} = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

or $$\left(\frac{T}{T_c}\right)^2 = \frac{1}{2}$$

or $$T = \frac{T_C}{\sqrt{2}} = 0.24\ T_C$$

Example 6: *Experiments on a sample show that a critical fields of 15 T and 18 T exist at 12 K and 10 K, respectively. Calculate the transition temperature and critical field at 0 K.*

Solution : We know that

$$H_c = H_o\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

At $T = 12\ K$, $H_C = 15\ T$

At $T = 10\ K$, $H_C = 18\ T$

$\therefore$ Above relation may be written as

$$H_c(12) = H_o\left[1-\left(\frac{12}{T_c}\right)^2\right] = 15$$

and
$$H_c(10) = H_o\left[1-\left(\frac{10}{T_c}\right)^2\right] = 18$$

$\therefore$
$$\frac{H_c(12)}{H_c(10)} = \frac{\left[1-\left(\frac{12}{T_c}\right)^2\right]}{\left[1-\left(\frac{10}{T_c}\right)^2\right]} = \frac{15}{18}$$

or
$$\frac{T_c^2 - 144}{T_c^2 - 100} = \frac{15}{18} = \frac{5}{6}$$

or
$$6T_c^2 - 864 = 5T_c^2 - 500$$

or
$$T_c^2 = 364$$

or
$$T_c = 19.08 \text{ (approx).}$$

Example. 7: *In a superconducting material isotropic mass is* 199.5 *a.m.u. and critical temperature is* 5 *K. Calculate isotopic mass at* 5.1 *K*

Solution : Isotopic relation is given by

$$T_c.\sqrt{M} = \text{constant}$$

i.e.
$$T_1\sqrt{M_1} = T_2\sqrt{M_2}$$

or
$$M_2 = \left(\frac{T_1}{T_2}\right)^2 M_1$$

In this case, $T_1 = 5K, T_2 = 5.1K$ and $M_1 = 199.5\ a.m.u$

$\therefore$
$$M_2 = \left(\frac{5}{5.1}\right)^2 \times 199.5$$
$$= 191.67\ a.m.u$$

Example 8: *Critical temperature of sample with isotopic mass of 204.87 a.m.u. is 19.2 K. Calculate T_C when isotopic mass changes to 218.87 a.m.u.*

Solution : Isotopic relation is

$$T_c.\sqrt{M} = \text{Constant}$$

$$\frac{T_{c_1}\cdot\sqrt{M_1}}{T_{c_2}\cdot\sqrt{M_2}} = \text{Constant}$$

Here $\quad T_{c_1} = 19.2\ K, M_1 = 204.87\ a.m.u$ and $M_2 = 218.87\ a.m.u$

$$\therefore \quad T_{c_2} = \frac{19.2\times(204.87)^{1/2}}{(218.87)^{1/2}} = 18.58\,K.$$

Example 9: *The critical temperature T_C for Hg with isotopic mass 199.5 a.m.u. is 4.185 K. Calculate the critical temperature for an isotopic mass of 202.3 a.m.u.*

Solution : We know that $T_C\cdot\sqrt{M}$ = Constant

$$\therefore \quad \frac{T_{c_1}}{T_{c_2}} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{202.3}{199.5}}$$

or

$$T_{c_2} = \sqrt{\frac{199.5}{202.3}}\times 4.185$$

$$= 4.156\,K.$$

Example 10: *London penetration depth for a sample at 5 K and 7 K are 41.2 nm and 180.3 nm, respectively. Calculate its tansition temperature and pentration depth at 0 K.*

Solution : We know that $\lambda(T) = \dfrac{\lambda(0)}{\left[1-\left(\dfrac{T}{T_c}\right)^4\right]^{1/2}}$ ...(*i*)

$$\lambda(5) = 41.2\,nm$$

$$\lambda(7) = 180.3\,nm$$

$$\therefore \quad \frac{\lambda(5)}{\lambda(7)} = \frac{\left[1-\left(\dfrac{5}{T_c}\right)^4\right]^{1/2}}{\left[1-\left(\dfrac{7}{T_c}\right)^4\right]^{1/2}}$$

or

$$\left(\frac{42.1}{180.3}\right)^2 = \frac{T_c^4-(5)^4}{T_c^4-(7)^4}$$

which gives $T_C = 5.92\,K$.

Putting in equation (i), we get

$$\lambda(5) = \frac{\lambda(0)}{\left[1-\left(\frac{5}{T_c}\right)^4\right]^{1/2}}$$

$$\therefore \qquad \lambda(0) = 41.1\,nm \times \left[1-\left(\frac{5}{T_c}\right)^4\right]^{1/2}$$

$$= 29.5\,nm$$