15. Simplify the function T1 and T2 to a minimum number of literal. Prove that $T1 = T2'$.

A	B	C	T1	T2
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$\Rightarrow$ $T1 = AB'C + ABC' + ABC$

$T2 = A'B'C' + A'B'C + A'BC' + A'BC + AB'C'$

$T1 = AB'C + ABC' + ABC \cancel{+ AB'C}$

$= AB'C + ABC' + ABC + AB'C$

$= \cancel{AB'(B+}$

$= AB'C + ABC + ABC' + ABC$

$= AC(B'+B) + AB(C'+C)$

$= AC + AB$

$= A(C+B)$

$T_2 = A'B'C' + A'B'C + A'BC' + A'BC + AB'C'$

$= A'B'(C'+C) + A'B(C'+C) + AB'C'$

$= A'B' + A'B + AB'C'$

$= A'(B'+B) + AB'C'$

$= A' + AB'C'$

$\cancel{AA' + AB' + AC'}$

$= (A'+A)(A'+B')(A'+C')$

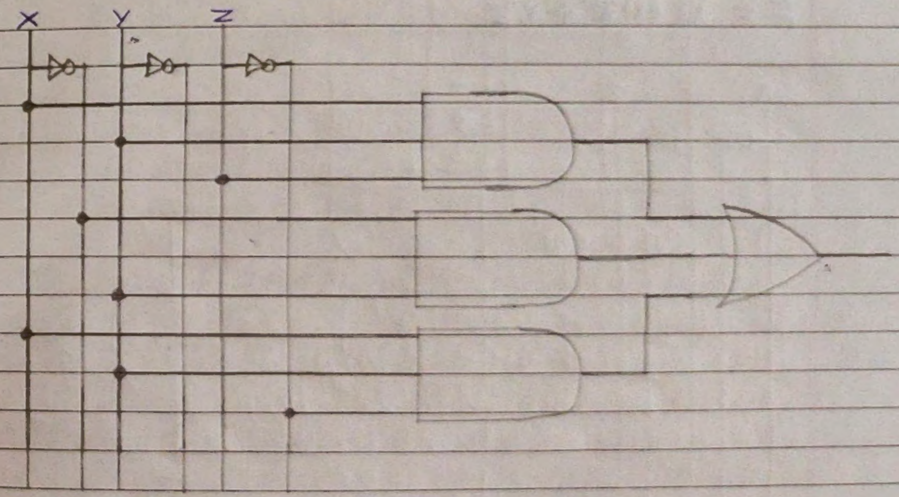
$= (A'+B')(A'+C')$

ASSIGNMENT 3

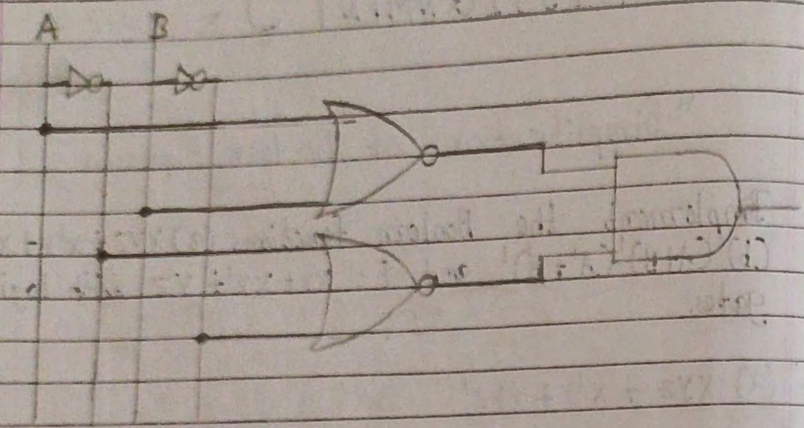
"Simplification of Boolean Expression"

1. Implement the Boolean functions (a) $xyz + x'y + xyz'$ (b) $(A+B)'(A'+B')'$ and $F = xy + xy' + y'z$ with logic gates.

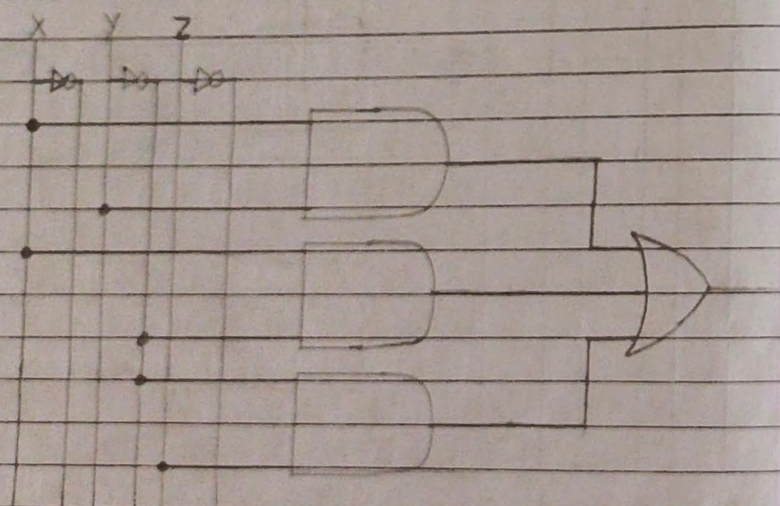
$\Rightarrow$ (a) $xyz + x'y + xyz'$



(b) $(A+B)'(A'+B')'$



$F = XY + XY' + Y'Z$



2. Obtained the simplified expression in sum of product for the following boolean functions.

(a) Implement the Boolean
(a) $F = \Sigma(0,1,4,5,10,11,12,14)$ and
(b) $F = \Sigma(11,12,13,14,15)$

=> (a) $F = \Sigma(0,1,4,5,10,11,12,14)$

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11	1			1
10			1	1

$F = A'C' + AB'C + ABD'$

(b) $F = \Sigma(11,12,13,14,15)$

AB \ CD	00	01	11	10
00				
01				
11	1	1	1	1
10			1	

$F = AB + ACD$

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3. Implement the function $F = \Sigma(1,3,7,11,15)$ with don't care conditions $d = \Sigma(0,2,5)$. Discuss the effect of don't care conditions.

$\Rightarrow$ $F = \Sigma(1,3,7,11,15)$
$d = \Sigma(0,2,5)$

AB \ CD	00	01	11	10
00	X	1	1	X
01		X	1	
11			1	
10			1	

$F = A'D + CD$

The "Don't Care" conditions allows us to replace the empty cell of a K-map to form a grouping of the variables which is larger then that of forming groups without don't care.

While forming groups of cells, we consider a "Don't Care" cell as 1 or 0 or we can also ignore that cell.

Therefore, the "Don't care" condition can help us to form a larger group of cells.

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4. Find the complement of the following Boolean function and reduce to a minimum number of literals.

$B'D + A'BC' + ACD + A'BC$

$\Rightarrow$ $F = B'D + A'BC' + ACD + A'BC$

$F' = (B'D + A'BC' + ACD + A'BC)'$
$= (B'D)'(A'BC')'(ACD)'(A'BC)'$
$= (B + D')(A + B' + C)(A' + C' + D')(A + B' + C')$
$= (AA' + B + CC' + D')(A + B' + C + DD')(A' + BB' + C' + D')$
$(A + B' + C' + DD')$
$= (A + B + C + D')(A' + B + C' + D')(A + B' + C + D)(A + B' + C + D')$
$(A' + B + C' + D')(A' + B' + C' + D')(A + B' + C' + D)(A + B' + C' + D')$
$= (A + B + C + D')(A' + B + C' + D')(A + B' + C + D)(A + B' + C + D')$
$(A' + B' + C' + D')(A + B' + C' + D)$

5. Obtain the simplified expressions in sum of products using k-map:

$x'z + w'xy' + w(x'y + xy')$

$\Rightarrow$ $F = x'z + w'xy' + w(x'y + xy')$
$= x'z(w + w')(y + y') + w'xy'(z + z') + wx'y + wxy'$
$= wx'yz + w'x'y'z + w'xy'z + w'xy'z' + wx'y(z + z') + wxy'(z + z')$
$= wx'yz + w'x'y'z + w'xy'z + w'xy'z' + wx'yz + wx'yz' + wxy'z + wxy'z'$
$= wx'yz + w'x'y'z + w'xy'z + w'xy'z' + wx'yz' + wxy'z + wxy'z'$
$= \Sigma(11, 1, 5, 4, 10, 13, 12)$
$= \Sigma(1, 4, 5, 10, 11, 12, 13)$

wx \ yz	00	01	11	10
00	0	1 1	3	2
01	4 1	5 1	7	6
10	12 1	13 1	15	14
11	8	9	11 1	10 1

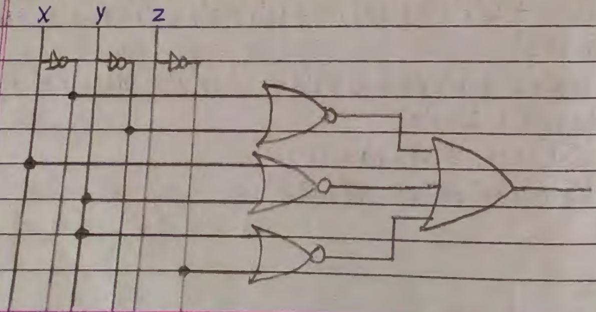
$F = xy' + w'y'z + wx'y$

6. Given Boolean function $F = xy + x'y' + y'z$

i. Implement with only OR & NOT gates
ii. Implement with only AND & NOT gates

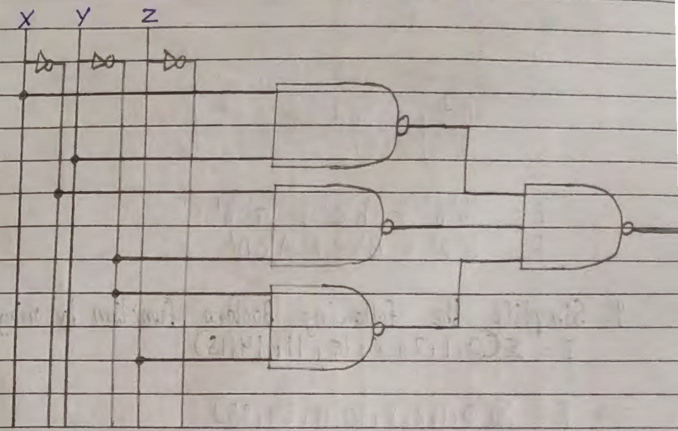
$\Rightarrow$ i. Implement with only OR & NOT gates:

$$F = xy + x'y' + y'z$$
$$= ((xy + x'y' + y'z)')'$$
$$= ((xy)'(x'y')'(y'z)')'$$
$$= ((x'+y')(x+y)(y+z'))'$$
$$(F')' = (x'+y)' + (x+y)' + (y+z')'$$



ii. Implement with only AND & NOT gates:

$$F = (xy + x'y' + y'z)$$
$$= ((xy + x'y' + y'z)')'$$
$$(F')' = ((xy)'(x'y')'(y'z)')'$$



8. Simplify the following booleen function using K-Map.
$F = A'B'C' + B'CD' + A'BCD' + AB'C'$

$\Rightarrow$ $F = A'B'C' + B'CD' + A'BCD' + AB'C'$
$$= A'B'C'(D+D') + (A+A')B'CD' + A'BCD' + AB'C'(D+D')$$
$$= A'B'C'D + A'B'C'D' + AB'CD' + A'B'CD' + A'BCD' + AB'C'D + AB'C'D'$$
$$= A'B'C'D + A'B'C'D' + AB'CD' + A'B'CD' + A'BCD' + AB'C'D + AB'C'D'$$

AB \ CD	$C'D'$ 00	$C'D$ 01	CD 11	CD' 10
$A'B'$ 00	1	1		1
$A'B$ 01				1
AB 11				
AB' 10	1	1		1

$F = b'd' + b'c' + a'cd'$

$F = B'D' + B'C' + A'CD'$

9. Simplify the following Boolean function by using k-map.
$F = \Sigma(0,1,2,8,10,11,14,15)$

$\Rightarrow F = \Sigma(0,1,2,8,10,11,14,15)$

AB \ CD	00	01	11	10
00	1	1		1
01				
11			1	1
10	1		1	1

$F = AC + B'D' + A'B'C'$

10. Simplify Boolean function $F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$ using k-map and implement it using (i) NAND gates only (ii) NOR gates only.

$\Rightarrow F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$

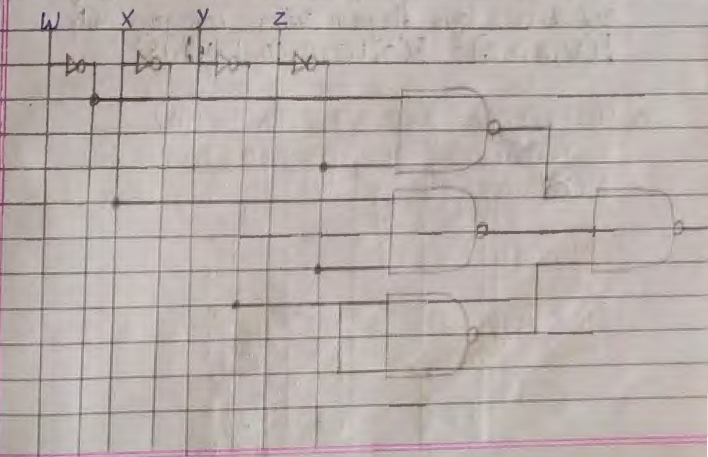
wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$F = y' + w'z' + xz'$

$F = w'z' + xz' + y'$

$= ((w'z') + (xz') + (y'))')'$

$(F')' = ((w'z')'(xz')'(y')')'$



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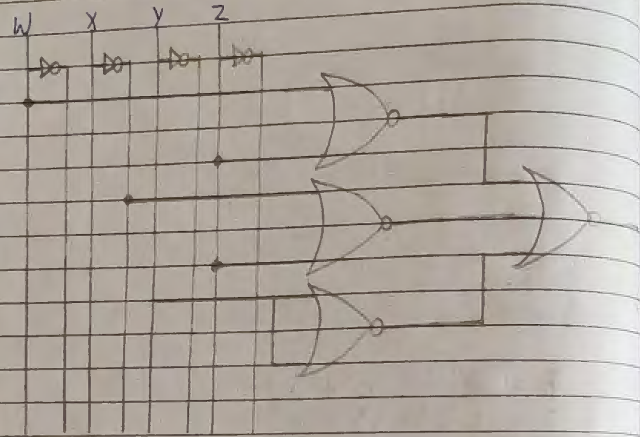
$\therefore F = w'z' + xz' + y'$

$= ((((w'z') + (xz') + (x'))')')'$

$= (((w'z')'(xz')'(y')')')'$

$= (((w'' + z'')(x' + z'')(y''))')'$

$((F')')' = ((w+z)' + (x'+z)' + (y)')'$



11. Simplify the following Boolean function using k-map and draw logic diagram using NOR gates only.
$F(w,x,y,z) = \Sigma(0,1,2,8,10,11,14,15)$

$\Rightarrow F(w,x,y,z) = \Sigma(0,1,2,8,10,11,14,15)$

wx \ yz	00	01	11	10
00	1	1		1
01				
11			1	1
10	1		1	1

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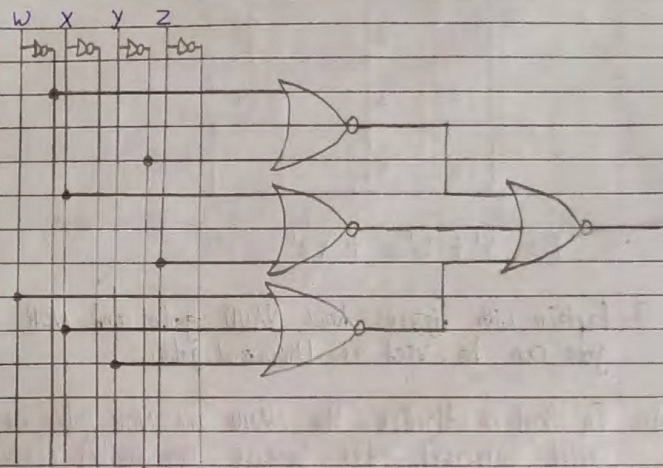
$F = wy + x'z' + w'x'y'$

$= (((wy + x'z' + w'x'y')')')'$

$= (((wy)'(x'z')'(w'x'y')')')'$

$= (((w'+y')(x+z)(w+x+y))')'$

$= ((w'+y')' + (x+z)' + (w+x+y)')'$



12. Simplify the Boolean function:
(1) $F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$
(2) $F(w,x,y) = \Sigma(0,1,3,4,5,7)$

$\Rightarrow$ (2) $F(w,x,y) = \Sigma(0,1,3,4,5,7)$

w \ xy	00	01	11	10
0	1	1	1	
1	1	1	1	

$F = x' + y$

(1) $F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$F = y' + w'z' + xz'$

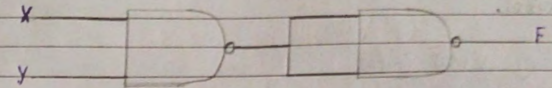
13. Explain with figures how NAND gate and NOR gate can be used as Universal gate.

Ans In Boolean Algebra, the NAND and NOR gates are called universal gates because any digital circuit can be implemented by using any one of these two i.e. any logic gate can be created using NAND or NOR gates only.

i. Implementation of AND Gate using Universal gates.

a) Using NAND Gates

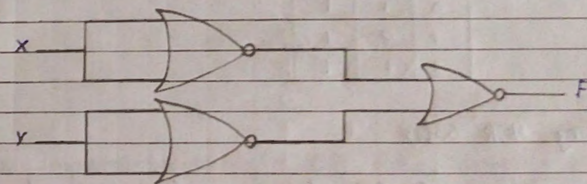
The NAND gate can be implemented by using two NAND gates.



x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

b) Using NOR Gates

Implementation of AND gate using only NOR gates.



x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

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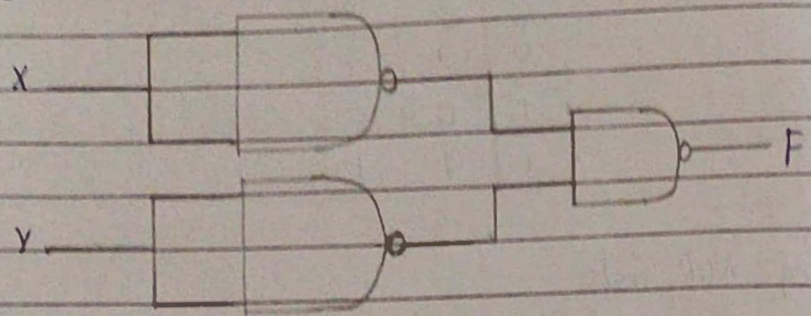
ii. Implementation of OR Gate using Universal Gates.

a) Using NAND Gates

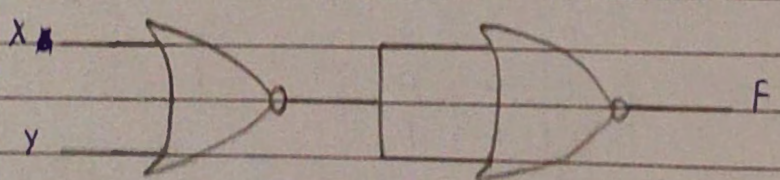
The OR gate can be implemented using the NAND gate.



X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

b) Using NOR Gates

Implementation of OR gate using two NOR gates. ~~as~~ shown



X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

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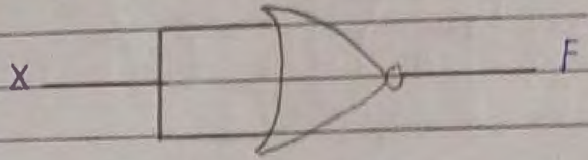
iii. Implementation of NOT Gate using Universal gates.

a) Using NAND Gates

X	F
0	1
1	0

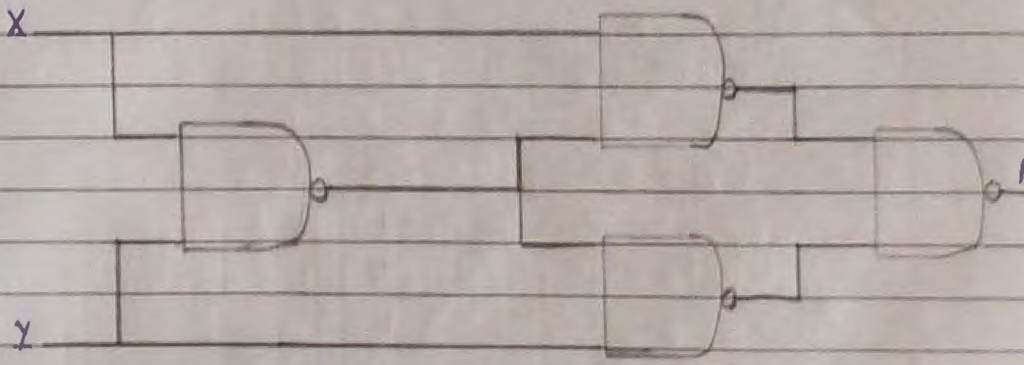
b) Using NOR Gates



X	F
0	1
1	0

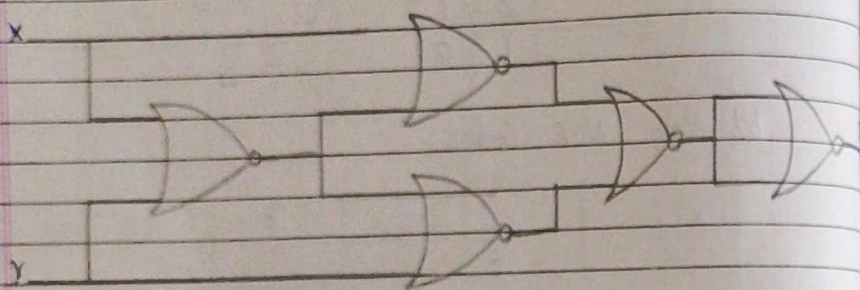
iv. Implementation of XOR Gate using Universal gates.

a) Using NAND Gates



X	Y	F
0	0	0
0	1	1
1	0	1
1	1	0

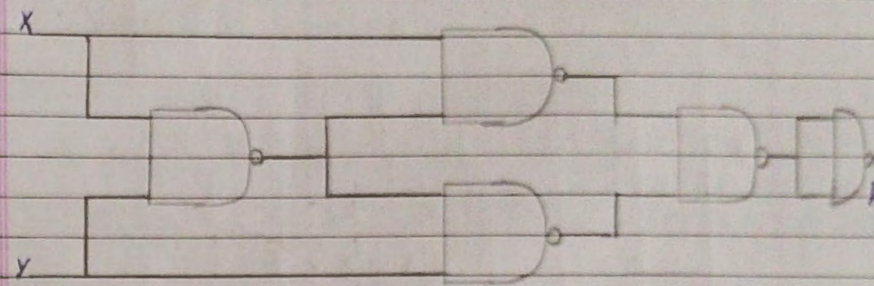
b) Using NOR Gates



X	Y	F
0	0	0
0	1	1
1	0	1
1	1	0

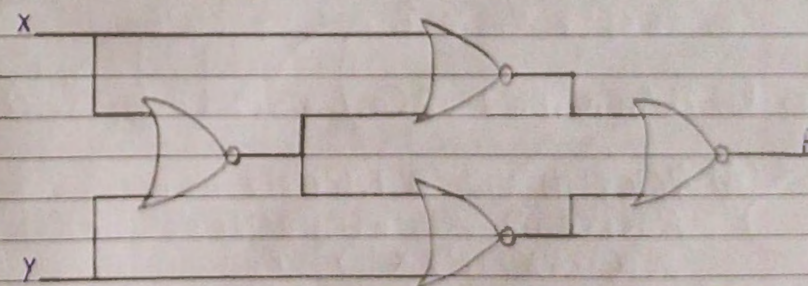
V. Implementation of XNOR Gate using Universal gates.

a) Using NAND Gate



X	Y	F
0	0	1
0	1	0
1	0	0
1	1	1

b) Using NOR Gate



X	Y	F
0	0	1
0	1	0
1	0	0
1	1	1

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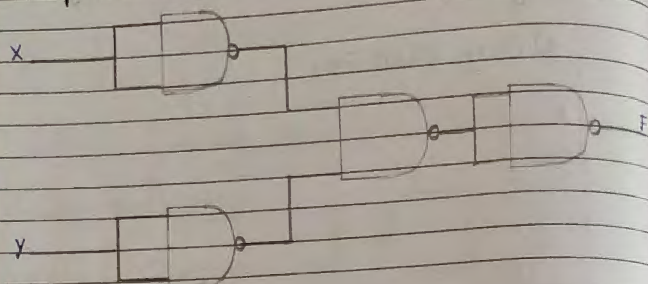
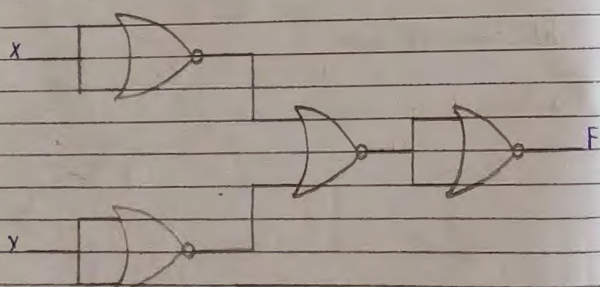
Vi. Implementation of NOR Gate using NAND Gates



X	Y	F
0	0	1
0	1	0
1	0	0
1	1	0

Vii. Implementation of NAND Gate using NOR Gates



X	Y	F
0	0	1
0	1	1
1	0	1
1	1	0

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14. Simplify the Boolean function:

(1) $F = A'B'C' + B'CD' + A'BCD' + AB'C'$

(2) $F = A'B'D' + A'CD + A'BC$

$d = A'BC'D + ACD + AB'D'$ where "d" indicates Don't Care Conditions.

=> (1) $F = A'B'C' + B'CD' + A'BCD' + AB'C'$

$= A'B'C'(D+D') + (A+A')B'CD' + A'BCD' + AB'C'(D+D')$

$= A'B'C'D + A'B'C'D' + AB'CD' + A'B'CD' + AB'C'D + AB'C'D' + A'BCD'$

$= A'B'C'D + A'B'C'D' + AB'CD' + A'B'CD' + AB'C'D + AB'C'D' + A'BCD'$

$= \Sigma(1, 0, 10, 2, 9, 8, 6)$

$= \Sigma(0, 1, 2, 6, 8, 9, 10)$

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$F = B'C' + B'D' + A'CD$

(2) $F = A'B'D' + A'CD + A'BC$

$= A'B'D'(C+C') + A'CD(B+B') + A'BC(D+D')$

$= A'B'CD' + A'B'C'D' + A'BCD + A'B'CD + A'BCD + A'BCD'$

$= A'B'CD' + A'B'C'D' + A'BCD + A'B'CD + A'BCD'$

$= \Sigma(2, 0, 7, 3, 6)$

$= \Sigma(0, 2, 3, 6, 7)$

$d = A'BC'D + ACD + AB'D'$

$= A'BC'D + ACD(B+B') + AB'D'(C+C')$

$= A'BC'D + ABCD + AB'CD + AB'CD' + AB'C'D'$

$= \Sigma(5, 15, 11, 10, 8)$

$= \Sigma(5, 8, 10, 11, 15)$

AB \ CD	00	01	11	10
00	1		1	1
01		X	1	1
11			X	
10	X		X	X

$F = B'D' + A'C$

15. Obtained the simplified expressions in sum of products for the following Boolean functions:

(i) $F(A,B,C,D,E) = \Sigma(0,1,4,5,16,17,21,25,29)$

(ii) $A'B'CE' + A'B'C'D' + B'D'E' + B'CD$

$\Rightarrow$ (i) $F(A,B,C,D,E) = \Sigma(0,1,4,5,16,17,21,25,29)$

(ii) $A'B'CE' + A'B'C'D' + B'D'E' + B'CD$

$= A'B'CE'(D+D') + A'B'C'D'(E+E') + AB'D'E'(A+A')(C+C') + B'CD(A+A')(E+E')$

$= A'B'CDE' + A'B'CD'E' + A'B'C'D'E + A'B'C'D'E' + AB'CD'E' + A'B'C'D'E' + AB'CDE + A'B'CDE'$

$= A'B'CDE' + A'B'CD'E' + A'B'C'D'E + A'B'C'D'E' + AB'CD'E + AB'CDE$

$= \Sigma(6,4,1,0,21,23)$ or $\Sigma(0,1,4,6,21,23)$

AB \ CDE	000	001	011	010	110	111	101	100
00	1	1			1			1
01								
11								
10						1	1	

$F = A'B'C'D' + A'B'CE' + AB'CE$

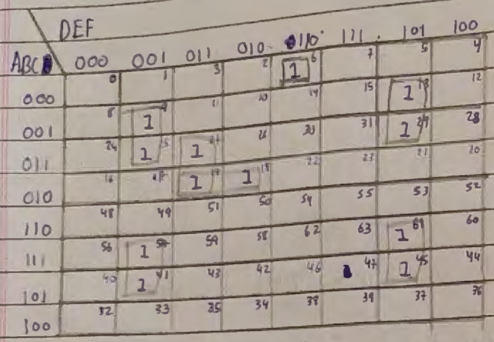
(i) $F(A,B,C,D,E) = \Sigma(0,1,4,5,16,17,21,25,29)$

AB \ CDE	000	001	011	010	110	111	101	100
00	1	1					1	1
01								
11		1					1	
10	1	1					1	

$F = B'C'D' + A'B'D' + AD'E$

17. Simplify the Boolean functions using k-map

$\Rightarrow F(A,B,C,D,E,F) = \Sigma(6,9,13,18,19,25,27,29,41,45,57,61)$



$F = A'B'C'DEF' + A'BC'D'E + A'BD'EF + CE'F$

$F(A,B,C,D,E,F,G) = \Sigma(20,28,52,60)$

$F = \Sigma(20,28,52,60)$

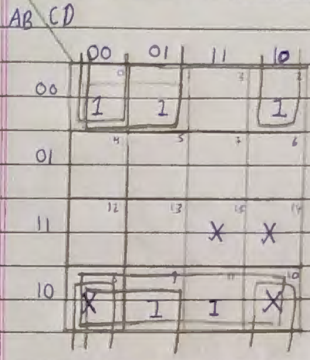
variable = A,B,C,D,E,F,G $N=7$ which is > 6, is not allowed.

18. Implement the following Boolean function using don't care conditions.

a) $F(A,B,C,D) = \Sigma(0,1,2,9,11)$
$d(A,B,C,D) = \Sigma(8,10,14,15)$

b) $F = B'D + B'C + ABCD$
$d = A'BD + AB'C'D'$

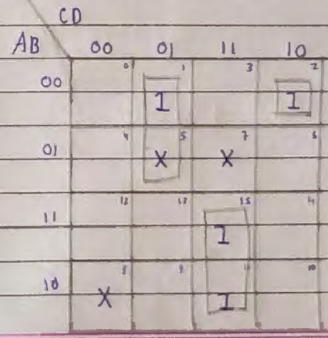
$\Rightarrow$ a) $F(A,B,C,D) = \Sigma(0,1,2,9,11)$
$d(A,B,C,D) = \Sigma(8,10,14,15)$



$F = B'D' + AB' + B'C'$

b) $F = B'D + B'C + ABCD$
$= (A+A')(C+C')B'D + (A+A')(D+D')B'C + ABCD$
$= AB'CD + A'B'C'D + AB'CD + A'B'CD' + ABCD$
$= AB'CD + A'B'C'D + A'B'CD' + ABCD$
$f = \Sigma(11,1,2,15)$
$f = \Sigma(1,2,11,15)$

$d = A'BD + AB'C'D'$
$= A'BD(C+C') + AB'C'D'$
$= A'BCD + A'BC'D + AB'C'D'$
~~$d = \Sigma(5,7,8)$~~ $d = \Sigma(7,5,8)$
$d = \Sigma(5,7,8)$



$F = A'C'D + ACD + A'B'CD'$