

L - 5 SUPERCONDUCTIVITY

DELUXE
PAGE NO.:
DATE:

(17-1)

- Introduction to superconductivity
- Properties of superconductors
 - (i) Effect of magnetic field
 - (ii) Pressure effect
 - (iii) Meissner effect
 - (iv) Impurity effect
 - (v) Isotropic mass effect
- Mechanism of superconductivity : BCS theory
- Penetration depth: Magnetic field
- Josephson Junction and its application
- Application of superconductors.

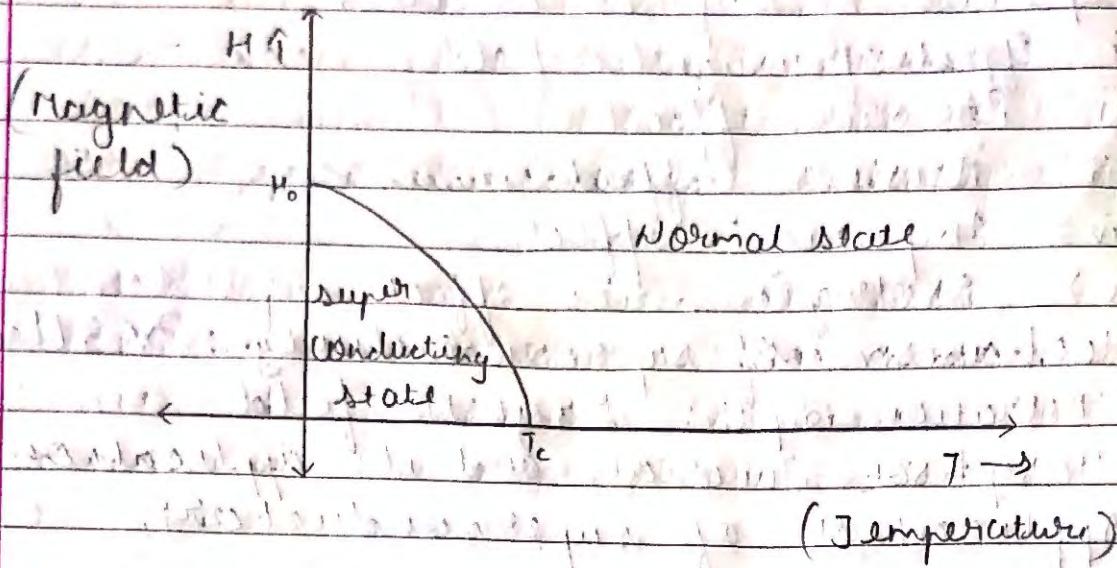
* Superconductivity means the electrical resistance of the material suddenly drops to zero, when it is cooled below at the certain temperature called critical temperature.

→ Critical or Transition temperature (T_c) :-
The temperature at which internal resistance of material becomes zero is known as critical temperature.
OR

The temperature at which a normal metal is converted into super conductor and vice-versa ($T_c = 4.2\text{ K}$ for Hg)

* Properties of Superconductors:-

1. Effect of magnetic field



- When a large magnetic field is applied to the superconductor, the superconductivity is destroyed. The minimum magnetic field required to destroy the superconductivity is known as critical magnetic field (H_c).
- The variation of magnetic field and temperature is given in the above graph.
- H_c is depended on temperature.
- The formula for H_c is given by,

$$\sqrt{H_c} \propto H_0 \left(1 - \left(\frac{T}{T_c} \right)^2 \right)$$

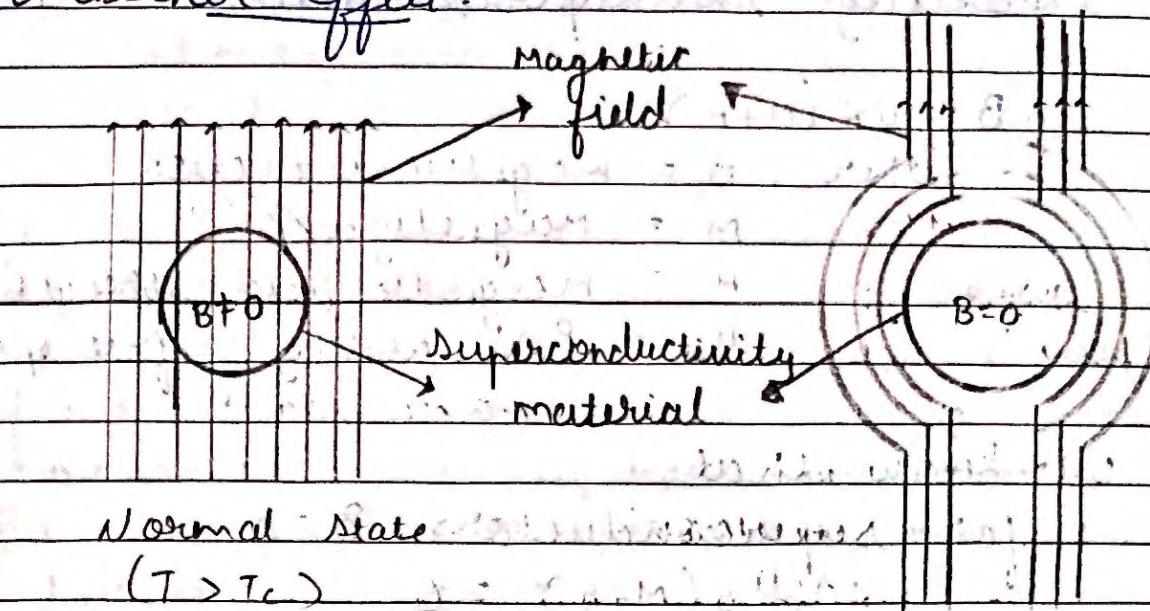
where, H_c = critical magnetic field

H_0 = magnetic field at $T = 0\text{K}$

T_c = critical temperature

or H_0 = the value of H_c at $T = 0\text{K}$

2. Meissner Effect:-



Normal state

$(T > T_c)$

$(H > H_c)$

Superconducting state
 $(T < T_c)$ $(H < H_c)$

- The complete diversion (expulsion) of all the magnetic field from a superconducting material is known as meissner effect.
- When a material is in normal state ($T > T_c$, $H > H_c$) and we apply magnetic flux on it, the magnetic flux pass through (penetrates) the material.
- When a material is in superconducting state ($T < T_c$, $H < H_c$) and we apply magnetic flux on it, the magnetic flux diverge from the superconducting material. (Repelled away from the superconductor).
- It is known as meissner effect.
- It is because of the development of surface current and magnetization in the material.
- The magnetization (m_s) and applied field are equal and opposite.

→ Providing the superconductors are diamagnetic

$$B = \mu_0 (M+H)$$

where, B = magnetic induction

M = magnetization

H = magnetic field strength

μ_0 = Permeability of free space.

We know that,

for superconductors $B = 0$

$$\therefore \mu_0 (M+H) = 0$$

$$\mu_0 = 0 \quad \text{or} \quad (H+M) = 0$$

But $\mu_0 \neq 0$

$$M+H=0$$

$$M \approx -H$$

$$\frac{M}{H} = -1$$

χ_m is also known as magnetic susceptibility

$$\chi_m = \frac{M}{H}$$

$$\boxed{\chi_m = -1}$$

$\chi_m < 0$, so we can say superconductors are perfect diamagnetic materials.

3. Pressure effect :-

Some of the materials shows the superconduct when we increase pressure on that, is called pressure effect.

Eg. Cesium is a normal conductor at normal atmospheric pressure, but if the pressure is 330 K bar at 1.5 K then it can be converted into superconductor.

28-1-19

4. Impurity Effect :-

- If we add impurities in superconducting materials its superconductivity is not lost but the critical temperature value lowers. It is known as impurity effect.

5. Isotopic mass effect:-

(Isotopes \rightarrow same atomic no. and different atomic mass)

Maxwell found that critical temperature (T_c) is inversely proportional to the square root of isotopic mass effect.

$$\text{i.e., } T_c \propto \frac{1}{\sqrt{M}}$$

$$T_c / \sqrt{M} = \alpha$$

where $\alpha = \text{constant}$.

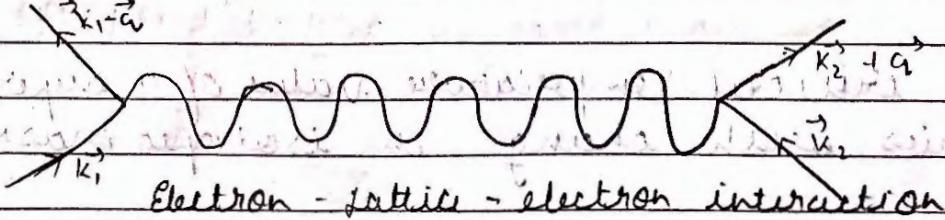
- The critical temperature value of a super conductor varies with change in isotopic mass (M).

Mechanism of superconductivity :-

(Hansmaur)

[BCS theory]

- This theory was introduced by Bardeen, Cooper and Schrieffer in 1957. It explains the superconductivity at lower temperature.
- Generally superconductivity occurs at low temperatures because the resistance decreases which is given by damping of interior magnetic field and absence of electrical resistance.
- According to BCS theory, the superconducting is because of attractive interaction between electrons at lower temperature. Due to that electrons start moving with pair and they don't collide with ions.
- If two electrons are bonded to each other and behave like a single particle known as cooper pair. The two electrons in cooper pair exchange phonon (energy), through lattice ion.



- For example an electron of wave vector k_1 emits a virtual phonon which is absorbed by an electron k_2 .

30-1-19

DELUXE
PAGE NO.:
DATE:

- Now, \mathbf{k} is scattered as $\mathbf{k}_1 - \mathbf{q}$ and $\mathbf{k}_2 + \mathbf{q}$ as shown in the figure. Thus the attractive interaction and the wave vector are represented by ~~wave~~ \mathbf{k}_1 and \mathbf{k}_2 .
- Coherence Length:- The maximum distance upto which the state of pair electron are subjected to produce superconductivity is known as coherence length. It is denoted by ξ_0 . Generally $\xi_0 = 10^{-6} \text{ m}$.
- The ratio of London penetration depth (λ) to the coherence length (ξ_0) is given by k .

$$k = \frac{\lambda}{\xi_0}$$

For type-I superconductor $\rightarrow k < \frac{1}{\sqrt{2}}$

For type-II superconductor $\rightarrow k > \frac{1}{\sqrt{2}}$

* Penetration depth (London's equation): magnetic field.

According to this theory the applied magnetic field (H) does not drop to zero at the surface of the superconductor. But decreases exponentially.

$$H = H_0 \exp\left(-\frac{x}{\lambda}\right)$$

where, H = applied magnetic field ^{at depth x} from surface
 H_0 = intensity of magnetic field at surface

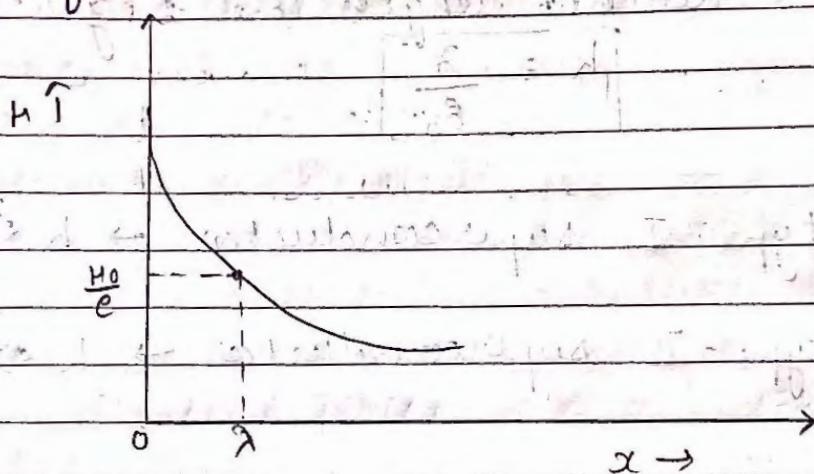
λ = London's penetration depth
 x = depth from surface

31-1-19

*

London penetration depth (λ):-

It is defined as the distance from the surface of superconductor to a point inside the material at which the magnetic field is $\frac{1}{e}$ of the magnetic field at surface.



Generally magnetic field penetrates upto 100 nm from the surface.

λ is not constant but changes with the slope of the graph or changes with temperature.

λ at $T < T_c$ can be given by,

$$\lambda = \lambda_0 \left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{1/2}$$

where, λ_0 = penetration depth at surface
at $T=0K$

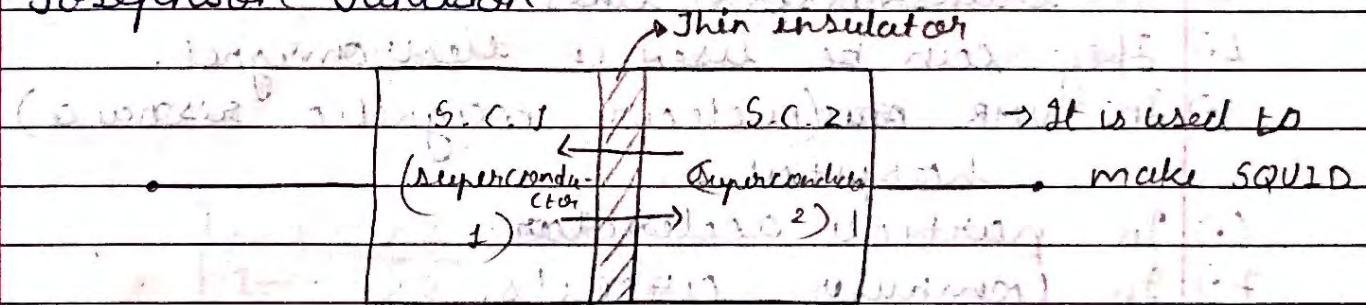
31-1-19

* Application of Superconductor:-

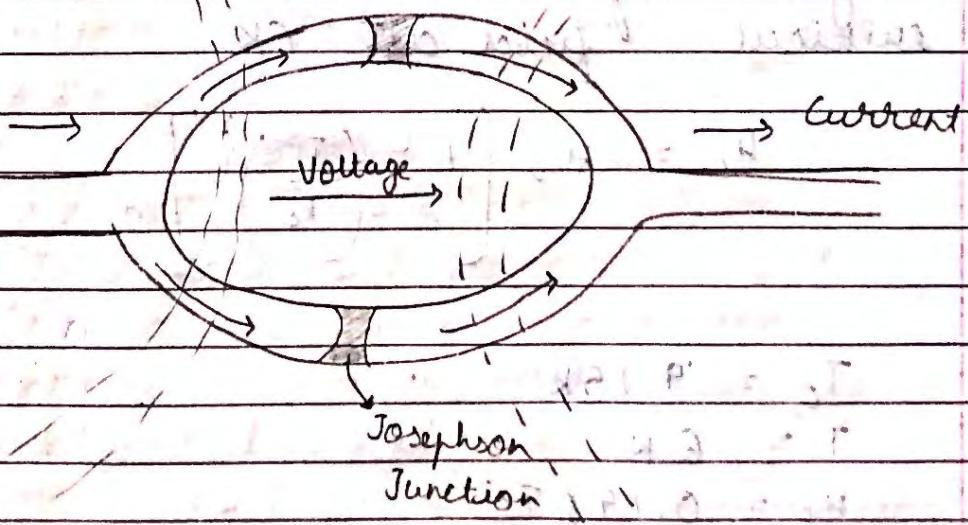
AND

Josephson Junction and its application:-

1. Josephson Junction and its applications



→ SQUID [superconducting Quantum Interference Device]



- SQUID can be made by arranging two Josephson junctions in parallel combination.

- SQUID can be useful to detect a small range of current, voltage and magnetic flux
- SQUID can measure voltage upto 10^{-15} V
- SQUID can measure magnetic flux upto 10^{-21}

② MAGLEV Train (meissner effect)

- Superconductor can be used in electrical transmission lines.
- They can be used in electromagnet.
- In NMR (nuclear magnetic resonance) in hospital
- In particle accelerator.
- In computer circuits.

4-2-19

- The T_c of a superconductor is 9.15K. At 0K magnetic field is 0.196T. Calculate the critical field at 6K.

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$T_c = 9.15 \text{ K}$$

$$T = 6 \text{ K}$$

$$H_0 = 0.196 \text{ T}$$

$$H_c = 0.196 \left[1 - \left(\frac{6}{9.15} \right)^2 \right]$$

$$H_c = 0.196 \text{ (0.57)}$$

$$H_c = 0.111727$$

- * Q. Calculate the critical current through long superconducting wire having radius of 0.5 mm, the critical magnetic field 7.2 $\frac{\text{KA}}{\text{m}}$.

$$I_c = 2\pi r H_c \quad I_c = \text{critical current}$$

$$J_c = \frac{I_c}{A} \quad J_c = \text{critical current density}$$

$$I_c = 2\pi \times 0.5 \times 10^{-3} \times 7.2 \times 10^3$$

$$I_c = 22.619 \text{ A}$$

- Q. The critical temperature of a metal with isotopic mass 199.5 is 4.185 K. Calculate the isotopic mass. If the critical temperature falls to 4.133 K.

$$T_{c_1} = 4.185 \text{ K}$$

$$m_1 = 199.5$$

$$T_{c_1} \propto \sqrt{m_1}$$

$$a = 59.1108$$

$$T_{c_2} = 4.133$$

$$m = \left(\frac{a}{T_{c_2}} \right)^2$$

$$m = 204.55$$

Q. Calculate the critical current for a superconducting wire having diameter 1mm at 4.2 K. If T_c of the material is 7.18 K and critical magnetic field at 0K (H_0) is $6.5 \times 10^4 \frac{A}{m}$

$$d = 0.5 \text{ mm}$$

$$T_c = 7.18 \text{ K}$$

$$T = 4.2 \text{ K}$$

$$H_0 = 6.5 \times 10^4 \frac{A}{m}$$

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$= 6.5 \times 10^4 \left[1 - \left(\frac{4.2}{7.18} \right)^2 \right]$$

$$H_c = 4.2758 \times 10^4 \frac{A}{m}$$

$$I_c = 2\pi d H_c$$

$$= 2\pi \times 0.5 \times (4.2758 \times 10^4)$$

$$= 1.34 \times 10^5$$

$$\boxed{I_c = 134.3282 \text{ A}}$$

critical magnetic field

Q. The $\frac{\partial I}{\partial T}$ for Vanadium is $10^5 \frac{A}{K}$ at 8.58K and $2 \times 10^5 \frac{A}{m}$ at 0K. Determine the T_c value.

$$H_C = 10^5 \text{ A/m} \quad T_{c, \text{me}} = 8.58 \text{ K}$$

$$H_0 = 2 \times 10^5 \frac{\text{A}}{\text{m}}$$

$$H_C = \left[H_0 \left(1 - \left(\frac{T}{T_c} \right)^2 \right) \right]$$

$$1 - \frac{H_C}{H_0} = \left(\frac{T}{T_c} \right)^2$$

$$1 - \frac{10^5}{2 \times 10^5} = \left(\frac{T}{T_c} \right)^2$$

$$0.707 = \frac{T}{T_c}$$

$$T_c = \frac{8.58}{0.707} \approx 12.135 \text{ K}$$

$$T_c = 12.135 \text{ K}$$

Q. Prove that the superconductors are perfect diamagnetic materials.

OR

$$\text{Prove } \mu \times m = M$$

$$\rightarrow B = \mu_0(M + H)$$

where, B = magnetic induction

m = magnetization

H = magnetic field strength

μ_0 = Permeability of free space.

for superconductor, $B = 0$

$$\mu_0 (M+H) = 0$$

$$\mu_0 = 0 \text{ or } M+H = 0$$

But $\mu_0 \neq 0$

$$\therefore M+H=0$$

$$M = -H$$

$$\frac{M}{H} = -1$$

$\frac{M}{H}$ ~~is~~ = magnetic susceptibility (χ_m)

$$\chi_m = \frac{M}{H}$$

$$\boxed{\chi_m = -1}$$

The isotopic mass of superconductor is 0.90 times the isotopic mass of the isotope. Calculate the ratio of T_c .

$$T_c \propto \frac{1}{\sqrt{m}} \quad m_1 = (0.9)m$$

$$\frac{T_{c1}}{T_{c2}} = \frac{\sqrt{m_2}}{\sqrt{m_1}} = \frac{1}{0.9}$$

For $m_1 = 0.9m_2$, $\frac{1}{\sqrt{m_1}} = \frac{1}{\sqrt{0.9m_2}} = \frac{1}{\sqrt{0.9}} \approx 0.33$

so $T_{c1}/T_{c2} = 0.33$

or $T_{c1} = 0.33 T_{c2}$

-1 ELECTRONIC MATERIALS

Free electron theory

Density of states and energy band diagram.

Kronring - Penny model (to introduce origin of band gap)

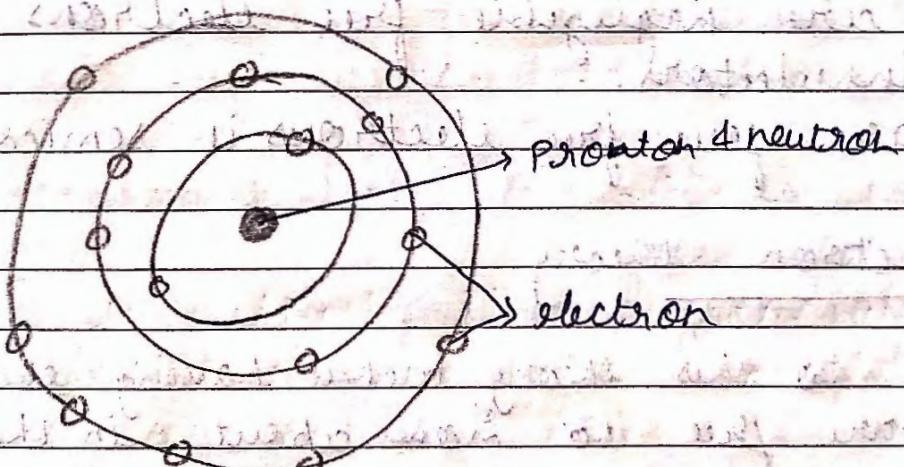
E - K diagram, direct and indirect band gap.

Types of electronic material; metal, insulators and semiconductors.

Density of states and occupation probability.

Fermi level, effective mass, Phonon

Atomic structure :- Atom is generally made up of 3 particles, electron, proton and neutron.



Charges :- electron \rightarrow -ve (e^-) $\rightarrow 1.602 \times 10^{-19}$ C

proton \rightarrow +ve (p^+) $\rightarrow 1.602 \times 10^{-19}$ C

neutron \rightarrow neutral

mass :- electron $\rightarrow 9.107 \times 10^{-31}$ kg

proton $\rightarrow 1.672 \times 10^{-27}$ kg

neutron $\rightarrow 1.975 \times 10^{-27}$ kg

- * Electrons in orbit:- Maximum two electrons in first shell

~~2nd~~ → 8

~~3rd~~ → 18

The electrons which are present in outermost orbit or shell is called valence electron. This electron decide physical properties of a material and having highest energy level.

- Free Electrons:- These are the electrons which are not attached to nucleus of an atom and free to move when external energy is applied.
 - A conductor is having large amount of free electrons.
 - There are negligible free electrons in insulators.
 - There are few free electrons in semiconductors.

11-2-19

* Free Electron theory

- According to this theory metal having electrons which are free to move about in the crystals like gas atoms in a container. Free electrons are available in the metal even in absence of electrical field.
- The free electrons collide with other free electron ion ^{core} or wall of the container and is known as elastic collision.
- Here the total energy is assumed as kinetic energy.
- Suppose if we apply electrical field by external

power source, free electron are directed towards higher potential and electron having constant velocity known as drift velocity (v_d)

- Drift Velocity :- It is defined as the average velocity acquired by free electron in a direction when external field is applied.
- Mean free path (λ) :- The average distance travelled by an electron between two successive collisions in a metal when external field is applied is known as mean free path (λ).

where, $\lambda = v \cdot \tau$

where τ = relaxation time

v = total velocity of electrons.

- Relaxation time (τ) :- It is time taken by a free electron to reach equilibrium position from its ~~any~~ disturbed position when field is applied is known as relaxation time (τ).

$$\tau = \frac{I}{V_d}$$

where, I = distance travelled by electron
 V_d = drift velocity

- Mean collision time (τ_c) :- The average time between two consecutive ~~relaxation~~ collision of an electron is

known as mean collision time (τ_c).

This theory (free electron theory) is also known as ~~Born~~ Drude Lorentz's theory.

Assumptions:-

- (i) Large number of free electrons are available inside a metal.
- (ii) Free electrons are situated like gas molecules in a container.
- (iii) Energy of an atom is given by,

$$E = \frac{3}{2} k_B T$$

k_B = Boltzmann's constant

T = temperature

- (iv) Force of attraction and repulsion between similar and opposite charge is ignored.
- (v) Field because of positive ion core is neglected.
- (vi) Electrical conductivity in a metal is because of drift velocity.

Advantages and Disadvantages of Free electron theory:-

Advantages:-

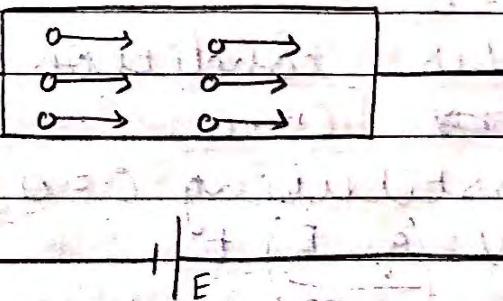
Ohm's law can be verified by this theory.
It explains thermal and electrical conductivity of metal.

Few optical properties can be explain.

→ Drawbacks:-

- Specific heat cannot be explained by this theory.
- Superconductivity cannot be explained.
- Photoelectric effect cannot be explained.
- Wrong prediction of temperature dependence of electrical conductivity (σ).
- It fails to provide mathematical formula of thermal conductivity.
- Ferromagnetism cannot be explained by this theory.

* Mobility (μ)



• It is defined as the average velocity because of a unit electrical field of an atom.

• Consider a conducting material having electrical field (E), n = no. of electrons, m = mass of electron and q = charge of electron. applying newton's second law of motion ($F = qE$). where, F = force experienced by an electron,

q = charge of electron and

E = electrical field.

$$F = q \cdot E \rightarrow (1)$$

$$\therefore a = \frac{F}{m} \quad (\alpha = \text{acceleration})$$

$$a = \frac{q \cdot E}{m} \rightarrow (2)$$

Now,

$$\int q dt = \int \frac{q \cdot E}{m} \cdot dt$$

$$V = \frac{q \cdot E t + c}{m} \rightarrow (3)$$

where, V = velocity of electron

c = integration constant

at $t = 0$

$$V = 0.$$

Putting this condition in equation (3), we get $\boxed{c = 0}$

Now substituting $c = 0$ in equation (3),

$$V = \left(\frac{q \cdot E}{m} \right) t$$

$$\therefore V = \left(\frac{q t}{m} \right) E$$

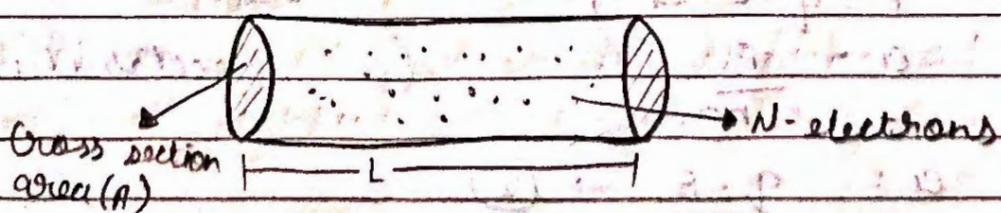
Here, In $V \propto E$

$$V = \mu E$$

where μ = mobility $\text{m}^2/\text{V}\cdot\text{s}$

$$\text{Unit} \rightarrow \frac{\text{m}^2}{\text{V}\cdot\text{s}}$$

* Electrical Current in a conductor:-



- Consider a conducting material having cross section area (A), no. of free electrons (n) and length of conductor (L).
- The average velocity of electron ($v = \frac{L}{T}$).
- No. of electron passing through any area per second is N/T .

\therefore Current (I) = Charge \times no. of electron passing through any area.

$$I = q \cdot \frac{N}{T}$$

$$I = q \cdot \frac{N}{T} \cdot \frac{L}{L}$$

$$\boxed{I = \frac{q N v}{L}}$$

- So the current in a conductor is given by

$$I = \frac{q \cdot N \cdot v}{L}$$

$$\text{Current density } (J) = \frac{I}{A} = \frac{q \cdot N \cdot v}{L \cdot A}$$

$$J = \frac{q \cdot v \cdot N}{L \cdot A}$$

Here $\frac{N}{L \cdot A} = n$ = concentration of electron per unit volume.

$$\therefore \boxed{J = q \cdot v \cdot n}$$

* Electrical Conductivity (σ)

- The current density $J = q \cdot n \cdot v \rightarrow (1)$
- 'J' can be explained by $J = \sigma \cdot E \rightarrow (2)$
where σ is conductivity and E is applied electrical field.
- Now comparing $sq^n (1) \& (2)$.

$$q \cdot n \cdot v = \sigma \cdot E$$

$$q \cdot n (v \cdot E) = \sigma \cdot E$$

$$\sigma = \infty q \cdot n \cdot v$$

$$\therefore \sigma = \frac{1}{\rho} = q \cdot n \cdot v$$

where, ρ = resistivity.

* Thermal Conductivity (K) :-

$$Q \propto \frac{dT}{dx}$$

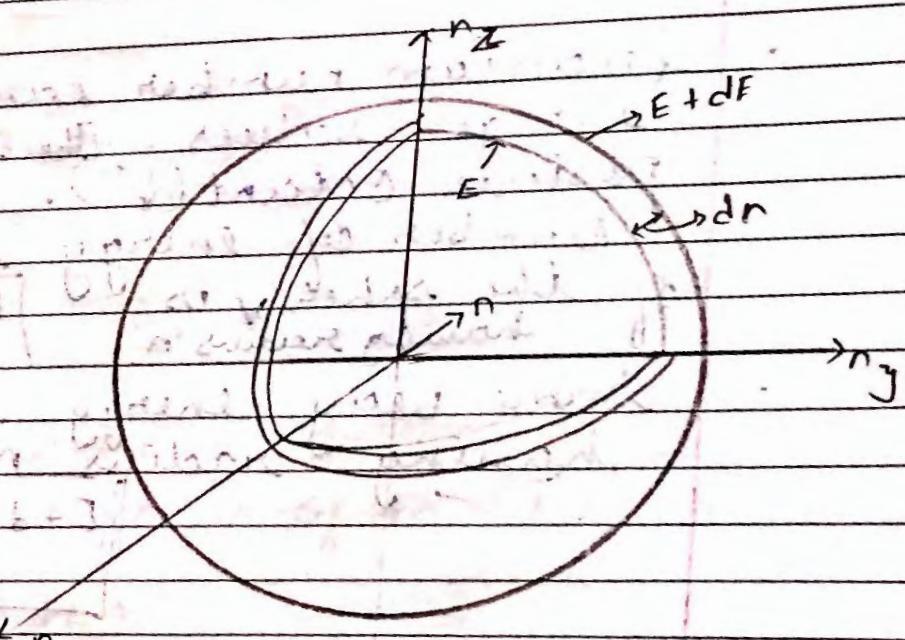
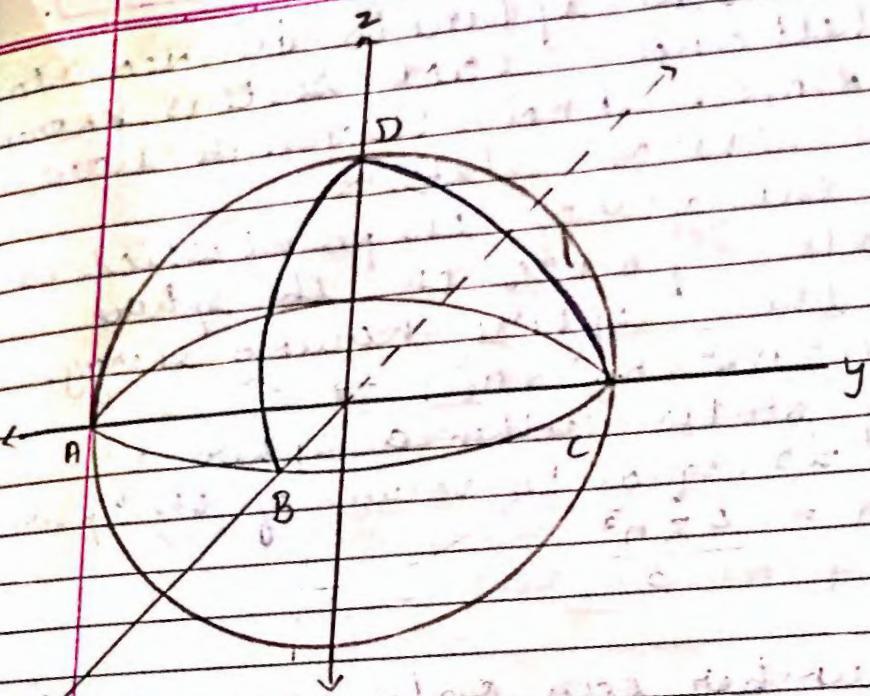
Where, Q = heat energy

$\frac{dT}{dx}$ = temperature gradient.

$$Q = K \frac{dT}{dx}$$

* Density of States :- (7m)

- It is defined as the number of available electron state per unit volume per unit energy at a certain energy level (E).



Density of states is given by $\mathcal{Z}(E) dE$.

Number of energy state is $N(E) dE$

The density of states is given by,

$$\mathcal{Z}(E) dE = \frac{\text{no. of energy state } N(E) dE}{\text{unit volume of the sample.}} \rightarrow ①$$

- Suppose that the sphere is divided into number of shell and each shell is known as n_x , n_y and n_z . And they will have their own energy shell.
 - 'E' is the energy of the point which is same for all points in the sphere.
 - Radius of the sphere having energy,
- $\Rightarrow n = n_x^2 + n_y^2 + n_z^2 \rightarrow ②$
- No. of energy states within a sphere of radius 'n' is equal to volume of the sphere
- $$n = \frac{4\pi n^3}{3}$$

- Quantum number can only have positive integer values. The n value can only be positive octant ($\frac{1}{8}$).
 - Number of energy state within one octant of the sphere is $E = \frac{1}{8} \left(\frac{4\pi n^3}{3} \right) \rightarrow ③$
 - Same way energy state within one octant having radius $n+dn$ is
- $$E+dn = \frac{1}{8} \left[\frac{4\pi (n+dn)^3}{3} \right] \rightarrow ④$$

- The number of energy state having energy value between E and $E+dn$ is given by
- $$NE(dE) = (E+dn) - E$$
- From eqⁿ ③ & ④.

$$NE(dE) = \frac{1}{8} \left[\frac{4\pi (n+dn)^3}{3} \right] - \frac{1}{8} \left[\frac{4\pi n^3}{3} \right]$$

$$= \frac{1}{8} \left(\frac{4}{3} \times [(n+dn)^3 - n^3] \right)$$

$$= \frac{\pi}{6} [(n+dn)^3 - n^3]$$

$$\left(\cancel{(n+dn)^3 = a^3}} \right)$$

\therefore Expanding $(n+dn)^3 - n^3$ by $a^3 + b^3 + 3ab(a+b) = (a+b)^3$

$$= \frac{\pi}{6} (n^3 + dn^3 + 3n^2dn + 3ndn^2 - n^3)$$

$$NE(dE) = \frac{\pi}{6} [dn^3 + 3ndn(n+dn)]$$

~~Now,~~

~~$NE(dE) = \frac{4}{8} \times \frac{3}{3}$~~

20-2-15

Neglecting the higher power of dn like $dn^2, dn^3, \text{ etc}$

$$NE(dE) = \frac{\pi}{6} (3n^2dn)$$

$$NE(dE) = \frac{3\pi}{2} n^2 dn \rightarrow (5)$$

$$NE(dE) = \frac{\pi}{2} n (ndn) \rightarrow (6)$$

Assume that a cubic metal with cubic age
then energy $E = \frac{n^2 h^2}{8ma^2} \rightarrow (7)$

so from above eqⁿ:

$$n^2 = \frac{8ma^2 E}{h^2} \rightarrow (8)$$

So,

$$n = \left(\frac{8ma^2 E}{h^2} \right)^{\frac{1}{2}} \rightarrow (9)$$

Taking differentiation of eqⁿ (9).

$$\therefore 2n dn = \frac{8ma^2 dE}{h^2} \rightarrow (10)$$

$$\therefore n dn = \frac{4ma^2}{h^2} dE \rightarrow (11)$$

Putting the value of 'n' and $n dn$ from eqⁿ (9)(11)
in eqⁿ (7).

$$NE(dE) = \frac{\lambda}{2} \sqrt{\frac{8ma^2 E}{h^2}} \cdot \frac{(8ma^2 (dE))}{(2h^2)}$$

~~$$NE(dE) = \frac{\lambda}{2} \sqrt{\frac{(8ma^2)}{h^2}} \cdot \sqrt{E} \cdot dE$$~~

Here a^3 is the volume of metal. From pauli's loop exclusion principle we can say that maximum two electrons of opposite spin can stay in a shell.

$$NEDE = 2 \times \frac{\pi}{4} \left(\frac{8ma^2}{h^2} \right)^{3/2} \sqrt{E} dE \rightarrow (12)$$

$$\text{Let } a^3 = 1$$

$$NEDE = 2 \times \frac{\pi}{4} \left(\frac{8m}{h^2} \right)^{3/2} \sqrt{E} dE \rightarrow (13)$$

Now, density of the state.

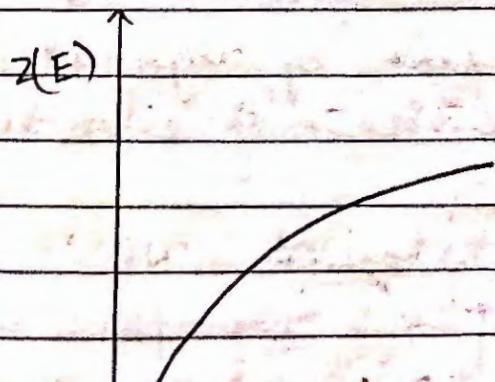
$$ZE(dE) = \frac{NEDE}{\text{Volume}}$$

$$\text{Volume } V = a^3 = 1$$

$$\therefore ZE(NE) = NEDE$$

$$ZE(dE) = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{3/2} \sqrt{E} dE \rightarrow (14)$$

Above equation represents density of energy states.



- The slope of graph will be exponential.

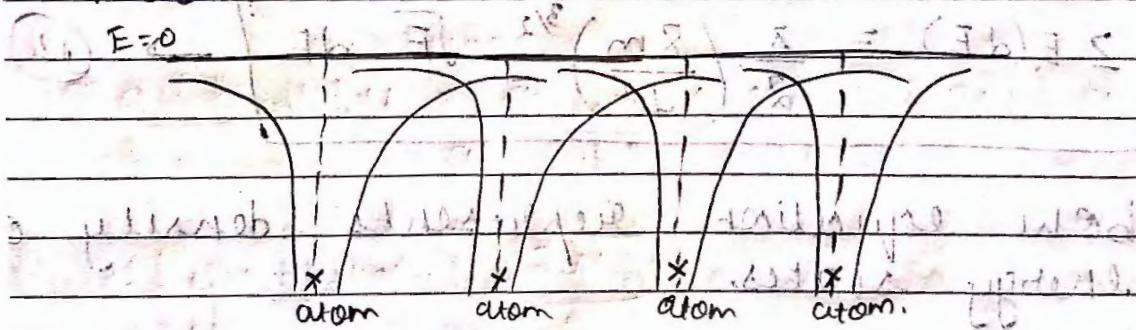
Kronning - Penny Model. (5-7 m)

In order to find out the allowed energy of electron in solid we have to apply Schrödinger equation in lattice.

It is assumed that electron is having zero potential energy near the nucleus of positive ion, and maximum when it is half way between adjacent nuclei.

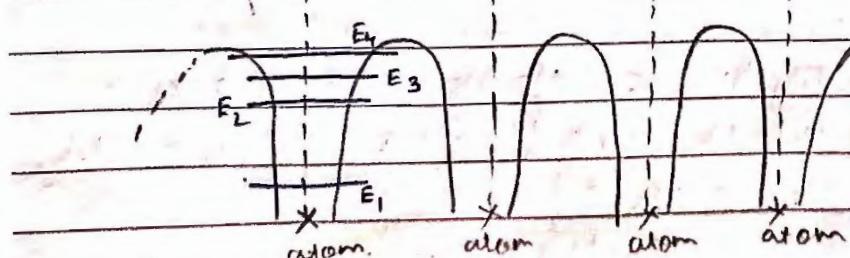
Kroning - Penny model suggested an infinite row of rectangular potential wells, separate by barrier width.

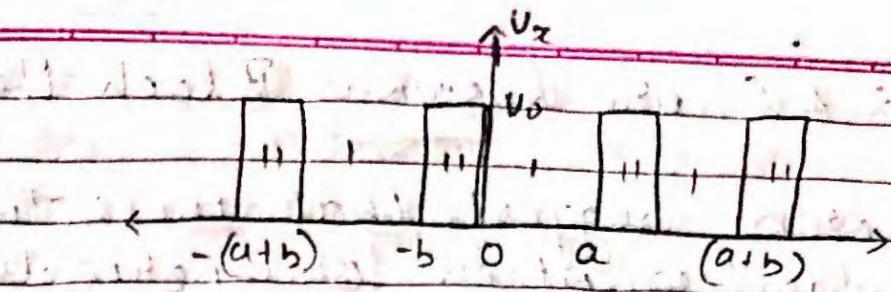
→ One dimensional representation of periodic lattice is known as Kroning - Penny model.



(a) overlapping energy functions of adjacent atoms

Indirect gap





Periodic potential function of Kronning-Perry model.

- Each well has width b and depth V_0 and the period of the potential is $(a+b)$.
- In the region where $0 < x < a$, the potential energy is assumed to be zero.
- And in the region $-b < x < 0$, the potential energy is assumed to be $-V_0$.
- By this model the Schrödinger equation can be solved.

The Schrödinger equation for two region can be given as,

$$\text{for } 0 < x < a \quad \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m E}{\hbar^2} \psi = 0 \rightarrow (1).$$

$$\text{for } -b < x < 0 \quad \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m (E - V_0)}{\hbar^2} \psi = 0 \rightarrow (2).$$

$$\text{Now taking } \alpha^2 = \frac{8\pi^2 m E}{\hbar^2}$$

Equation (3) & (4) are known as Bloch theorem

- Bloch theorem suggest that the solution of Schrödinger equation for a periodic potential is form a plane wave, modulated with periodicity. Which means the solution can be written as product of two functions: (1.) A free particle wave function and (2) Periodic function $U(x)$.

$$\Psi(x) = U(x) e^{ikx} \rightarrow (5)$$

$$U(x) = U(x+a) \rightarrow (6)$$

→ Formation of Energy bands:-

If we put the above wave function in Schrödinger equation and solve it in usual way by applying boundary condition we get,

$$\frac{mav_0^2}{\hbar^2} \sin \alpha a + \cos \alpha a = \cos k a \rightarrow (7)$$

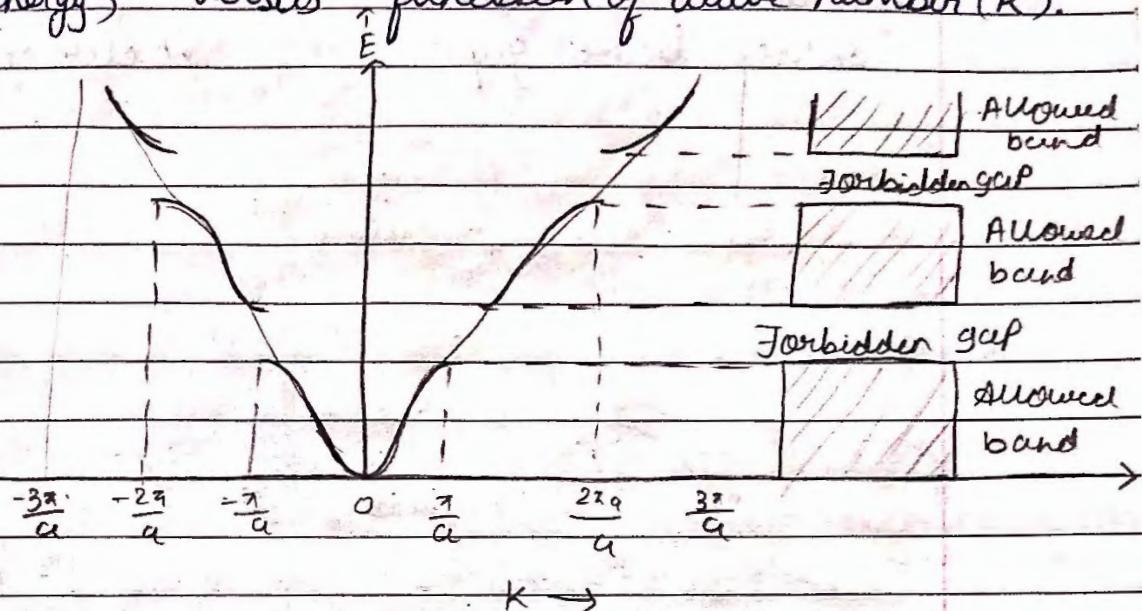
$$\text{where, } \alpha = \sqrt{\frac{2ME}{\hbar^2}}$$

- Equation (7) gives the solution of Schrödinger equation.
- Here trigonometric functions are involved so α can have certain values only
- RHS of eq (7) having cosine function will have values between -1 to 1. In fact it

restricted between these two limits.

So we can say that only certain values of ' k ' are allowed i.e. energy (E) is restricted to lie within certain ranges.

The above concept can be understood by graph of E (energy) versus function of wave number (k).



The above graph shows that there are discontinuities in the value of E and these discontinuities occur at $ka = \pm n\pi$.
 $\therefore k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$

So the origin of energy band gap (forbidden gap) and energy bands are clear from the graph.

\rightarrow Now $\rightarrow k\text{-vector} = \text{max of } V_b \text{ & min of } C_b$

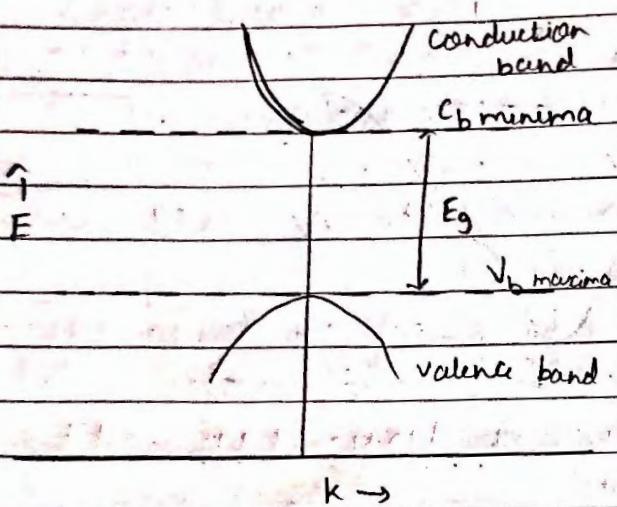
DELUXE
PAGE NO.:
DATE:

28-2-19

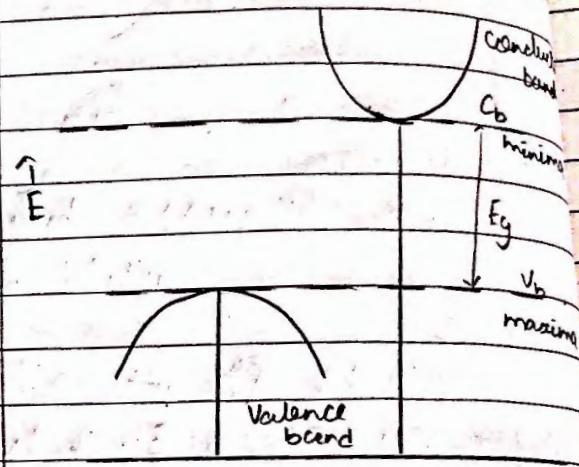
* Direct and Indirect band gap.

Band gap:- It can be defined as the minimum energy difference between top of valence band and bottom of conduction band.

Direct band gap



Indirect band gap



$k \rightarrow$

$k \rightarrow$

1. Here the maximum energy level of V_b aligns with minimum energy level of C_b w.r.t momentum.

2. Here the maximum energy level of V_b is not aligned with minimum energy level of C_b w.r.t. momentum.

2. $k\text{-vector}$ are same for V_b and C_b . $k\text{-vectors}$ are different for V_b and C_b .

3. When an electron from C_b recombines with hole from V_b , the energy & momentum are conserving.

• Here difference in momentum & recombination is only possible after momentum alignment.

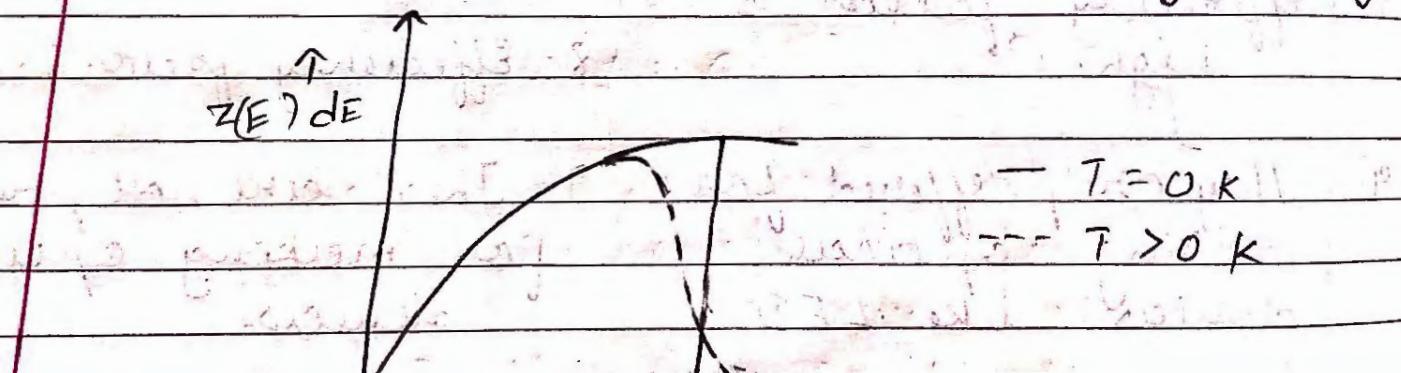
- | | |
|--|---|
| 4. The energy difference between V_b & C_b released in form of photon (light). | 4. Energy emitted as heat no recombination. |
| 5. k-vector of electron and hole are similar so momentum is conserved. | 5. Due to crystal effect electron in C_b loses Electron pass through energy and momentum at same time. |
| 6. Energy is conserved by emitting photons | 6. Electron pass through intermediate state and transfer momentum. |
| 7. The probability of radiative recombination is very high | 7. The probability of radiative recombination is negligible. |
| 8. Efficiency factor is high. | 8. Efficiency factor is low. |
| 9. They are preferred for making optical devices like LED. | 9. They are not preferable for making optical devices. |
| 10. Eg: Ga, As, P, etc. | 10. Eg. Si, Ge, etc. |

Energy band diagram

- Valence band
- Conduction band
- Forbiden energy gap.

* Density of states and occupation probability.

We know that the density of states in the interval E and $(E+dE)$ is given by



At $T = 0$ $Z(E)$ versus E graph suddenly drops to zero at $E = E_F$ where E_F = fermi level. The highest energy occupied is E_F .

Occupation Probability:-

We need to know how the electrons are distributed in various energy level at given temperature. Here we can't apply Maxwell - Boltzmann distribution law to electron.

Because; (i) Electron obey exclusion principle and (ii) They are indistinguishable particles.

Here if we apply fermi dirac distribution to the electron which is given by,

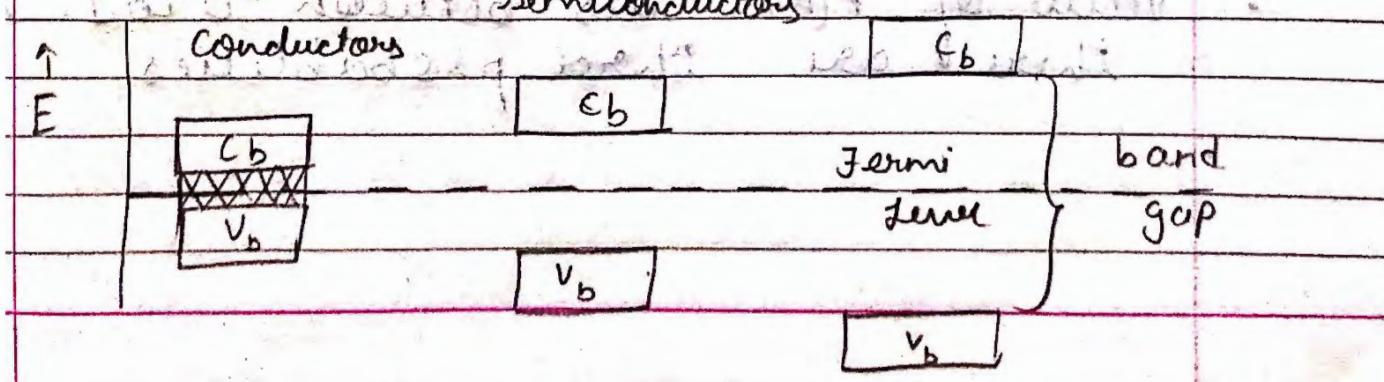
$$f(E) = \frac{e}{1 + e^{\frac{(E-E_F)}{kT}}}$$

$f(E)$ = fermi function.

The probability of electron to occupy in energy state E increases with temperature.

Fermi Level

Insulators



Fermi level is the highest energy level which an electron can occupy at $T = 0\text{K}$. It is between V_b and C_b .

In conductor, fermi level is inside C_b .

In semiconductor, fermi level is between V_b and C_b .

In insulator, fermi level is in very large energy gap between V_b & C_b so electrons don't cross it.

The Fermi function is

$$f(E) = \frac{1}{1 + e^{\frac{(E - E_F)}{kT}}}$$

where, E_F = fermi level for crystal
 E = energy level for allowed energy state

k = Boltzmann constant

T = temperature

$f(E)$ = Fermi Dirac function

Value of $f(E)$ lies between 0 to 1. So there are three probabilities