Assignment - 1

Q:1 Find the limit, if it exists.

(i) $\displaystyle\lim_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2}$

(ii) $\displaystyle\lim_{(x,y)\to(6,3)} xy\cos(x-2y)$

(iii) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2}$

(iv) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$

(v) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y e^{y}}{x^4+4y^2}$

(vi) $\displaystyle\lim_{(x,y,z)\to(3,0,1)} e^{-xy}\sin\left(\frac{\pi z}{2}\right)$

Q:2 Determine the set of points at which the function is continuous.

(i) $f(x,y,z) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ 1 & \text{if } (x,y)=(0,0) \end{cases}$

(ii) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$

(iii) $f(x,y) = \begin{cases} \dfrac{\sin xy}{xy} & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$

Q.3 Find first partial derivatives of the f's.

(i) $f(r,s) = r \ln(r^2 + s^2)$

(ii) ~~f(x,~~ $u = xy \sin^{-1}(yx)$

(iii) $w = z e^{xyz}$

(iv) $w = e^{v} / (u + v^2)$

(v) $u = x^{y}$

Q.4 Find all the second order partial derivatives:

(i) $f(x,y) = x^3 y^5 + 2x^4 y$

(ii) $f(x,y) = \sin^2(mx + ny)$

(iii) $w = \sqrt{u^2 + v^2}$

(iv) $v = \dfrac{xy}{x - y}$

(v) $v = e^{xe^{y}}$

(vi) $f(x, y)$

Q:5 Find $\partial z/\partial x$ & $\partial z/\partial y$

(i) $x^2 + y^2 + z^2 = 3xyz$

(ii) $yz = \ln(x+z)$

(iii) $\sin(xyz) = x + 2y + 3z$

Q:6 Use the chain rule to find $\partial z/\partial s$ & $\partial z/\partial t$.

(i) $z = x^2 y^3$, $x = s\cos t$, $y = s\sin t$

(ii) $z = \sin\theta\cos\phi$, $\theta = st^2$, $\phi = s^2 t$

(iii) $z = e^r \cos\theta$, $r = st$, $\theta = \sqrt{s^2 + t^2}$

(iv) $z = e^{x+2y}$, $x = s/t$, $y = t/s$

(v) $z = \tan\left(\frac{u}{v}\right)$, $u = 2s + 3t$, $v = 3s - 2t$

Q:7 Use the chain rule to find the indicated partial derivatives.

(i) $z = x^2 + xy^3$, $x = uv^2 + w^3$, $y = u + ve^w$;
find $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$ & $\frac{\partial z}{\partial w}$

(ii) if $u = \sqrt{r^2 + s^2}$, $r = y + x\cos t$, $s = x + y\sin t$;
find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial t}$ at Pt. $(1, 2, 0)$:

(iii) if $X = w\tan^{-1}(uv)$, $u = r + s$, $v = s + t$,
$w = t + r$; find $\frac{\partial X}{\partial r}$, $\frac{\partial X}{\partial s}$, $\frac{\partial X}{\partial t}$ at $(1, 0, 1)$

Date
Page

Q:8 The Voltage V in a simple electric circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's law, $V = IR$, to find how the current I is changing at the moment when $R = 400\,\Omega$, $I = 0.08\,A$, $\frac{dV}{dt} = -0.01\,V/s$ and $dR/dt = 0.03\,\Omega/s$.

Q:9 The radius of a right circular cone is increasing at a rate of 4.6 cm/s while its height is decreasing at a rate of 6.5 cm/s. At what rate is the Volume of the cone changing when the radius is 300 cm and the height is 350 cm ?

Q:10 Find the extrem values of following functions.

(i) $f(x, y) = x^2 + y^2 + x^2y + 4$

(ii) $f(x, y) = x^3y + 12x^2 - 8y$

(iii) $f(x, y) = xy - 2x - y$

(iv) $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$

Q:11 Find the shortest distance from the point $(2, 1, -1)$ to the plane $x + y - z = 1$

Q:12 Find the dimensions of the box with Volume $1000\,cm^3$ that has minimal surface area.

Q:13 Find the dimensions of the rectangular box of ~~p~~ with largest volume if the total surface area is given as 64 cm^2.

Q:14 A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of carboard used.