Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 3110014 **Date: 06/06/2019**

Subject Name: Mathematics – I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Use L'Hospital's rule to find the limit of $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{lnx}\right)$.	03
	(b)	Define Gamma function and evaluate $\int_0^\infty e^{x^2}dx$.	04
	(c)	Evaluate $\int_0^3\int_{\frac{\sqrt{x}}{3}}^1 e^{y^2}dydx$.	07
Q.2	**(a)**	Define the convergence of a sequence (a_n) and verify whether the sequence whose n^{th} term is $a_n=\left(\frac{n+1}{n-1}\right)^n$ converges or not.	03
	(b)	Sketch the region of integration and evaluate the integral $\iint_R (y-2x^2)dA$ where R is the region inside the square $\lvert x\rvert+\lvert y\rvert=1$.	04
	(c)	**(i)** Find the sum of the series $\sum_{n\geq 2}\frac{1}{4^n}$ and $\sum_{n\geq 1}\frac{4}{(4n-3)(4n+1)}$. **(ii)** Use Taylor's series to estimate $sin38°$.	07
		OR	
	(c)	Evaluate the integrals $\int_0^\infty\int_0^\infty \frac{1}{(1+x^2+y^2)^2}dxdy$ and $\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{ln3}\frac{\pi e^{2x}sin\pi y^2}{y^2}dxdydz$.	07
Q.3	**(a)**	If an electrostatic field E acts on a liquid or a gaseous polar dielectric, the net dipode moment P per unit volume is $P(E)=\frac{e^E+e^{-E}}{e^E-e^{-E}}-\frac{1}{E}$. Show that $\lim_{E\to 0^+}P(E)=0$.	03
	(b)	For what values of the constant k does the second derivative test guarantee that $f(x,y)=x^2+kxy+y^2$ will have a saddle point at (0,0)? A local minimum at (0,0)?	04
	(c)	Find the series radius and interval of convergence for $\sum_{n=0}^\infty\frac{(3x-2)^n}{n}$. For what values of x does the series converge absolutely?	07
		OR	
Q.3	**(a)**	Determine whether the integral $\int_0^3\frac{dx}{x-1}$ converges or diverges.	03
	(b)	Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1, x=4$ about the line $y=1$.	04

(c) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{(lnn)^3}{n^3}$ and $\sum_{n=0}^{\infty}(-1)^n(\sqrt{n+\sqrt{n}}-\sqrt{n})$. **07**

Q.4 **(a)** Show that the function $f(x,y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x,y) approaches to (0,0). **03**

(b) Suppose f is a differentiable function of x and y and $g(u,v) = f(e^u + sinv, e^u + cosv)$. Use the following table to calculate $g_u(0,0), g_v(0,0), g_u(1,2)$ and $g_v(1,2)$. **04**

	f	g	f_x	f_y
(0,0)	3	6	4	8
(1,2)	6	3	2	5

(c) Find the Fourier series of 2π −periodic function $f(x) = x^2, 0 < x < 2\pi$ and hence deduce that $\frac{\pi^2}{6} = \sum_{n=0}^{\infty} \frac{1}{n^2}$. **07**

OR

Q.4 **(a)** Verify that the function $u = e^{-\alpha^2 k^2 t} \cdot sinkx$ is a solution f the heat conduction euation $u_t = \alpha^2 u_{xx}$. **03**

(b) Find the half-range cosine series of the function **04**

$f(x) = \begin{cases} 2, & -2 < x < 0 \\ 0, & 0 < x < 2 \end{cases}$.

(c) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3,1,-1)$. **07**

Q.5 **(a)** Find the directional derivative $D_u f(x,y)$ if $f(x,y) = x^3 - 3xy + 4y^2$and u is the unit vector given by angle $\theta = \frac{\pi}{6}$. What is $D_u f(1,2)$? **03**

(b) Find the area of the region bounded y the curves $y = sinx, y = cosx$ and the lines $x = 0$ and $x = \frac{\pi}{4}$. **04**

(c) Prove that $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ is diagonalizable and use it to find A^{13}. **07**

OR

Q.5 **(a)** Define the rank of a matrix and find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$. **03**

(b) Use Gauss-Jordan algorithm to solve the system of linear equations $2x_1 + 2x_2 - x_3 + x_5 = 0$ **04**

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

(c) State Cayley-Hamilton theorem and verify if for the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$. **07**