3 Measures of Central Tendency

3.1 INTRODUCTION :

Sometimes we cannot remember the whole set of data and the analysis of sub data is impossible. Thus to reduce the complexity and to make the data comparable, we resort to averaging, which is the representative of the whole data. For quantitative data there is a tendency of the data to be distributed about a central value which is a typical value and is called a measure of central tendency.

Average is the value of the variable round which they centre, or is generally located in the middle of the distribution.

Any statistical measure which gives an idea about the position of the point round which other observations cluster, is called a **measure of central tendency.**

There are three commonly used averages called as measures of central tendency : Mean, Media and Mode. The mean again may be of three types : Arithmetic Mean (A.M.), Geometric Mean (G.M.) and Harmonic Mean (H.M.). We shall discuss all the measures one by one.

3.2 ARITHMETIC MEAN (A.M.) :

The arithmetic mean is simply called an "average". It is defined as an average obtained by adding the values of items and dividing the sum by the number of items.

If $x_1, x_2, ..., x_n$ are n observations, then the arithmetic mean is defined and denoted by

$$\bar{x} = \text{A.M.} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + ... + x_n}{n}$$

(Direct Method)

$$\text{or} \quad \bar{x} = \text{A} + \frac{\sum_{i=1}^{n} d_i}{n} \qquad \text{(Short cut Method)}$$

(Definition, Nov. 2016)

where A is an assumed mean or arbitrary number assumed from x_i; $d_i = x_i - \text{A}$ called as deviation.

For A.M. of grouped data we assume the frequencies in each class are centered at its class mark (mid value of the class is called its class mark, i.e. class mark of 0–5 is 2.5, 10–20 is 15). Then if x_i denotes the class mark and f_i the corresponding class frequency, then

$$\bar{x} = \text{A.M.} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \qquad \text{(Direct Method)}$$

$$\text{or} \quad \bar{x} = \text{A.M.} = \text{A} + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

(Short cut Method)

where A is an assumed mean or one of the assumed class mark and $d_i = x_i - \text{A}$ called the **deviation.** This method is also known as **deviation method.**

We can make our calculations more simple by taking the quotient of deviation from assumed mean and the width or size of the class. Suppose A is an assumed mean and h the width of the class, we have

$$\bar{x} = \text{A.M.} = \text{A} + \frac{\sum_{i=1}^{n} f_i d_i'}{\sum_{i=1}^{n} f_i} \times h$$

where $d_i' = \frac{x_i - A}{h}$.

This method is known as **Step Deviation Method** and is applicable only when the size of each class is same.

✱ Weighted Arithmetic Mean :

If $w_1, w_2, ..., w_n$ are the weights assigned to the variable $x_1, x_2, ..., x_n$ respectively, then the weighted arithmetic mean is defined as :

Weighted arithmetic mean

$$= \frac{w_1x_1 + w_2x_2 + ... + w_nx_n}{w_1 + w_2 + ... + w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

✱ Combined Arithmetic Mean :

If we know the mean (average) value of different groups and the number of items falling in that group, we can find out the combined average by adopting a particular formula as :

$$\bar{x}_{12} = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$$

where $\bar{x}_1$ and $\bar{x}_2$ are averge of first and second groups; n_1 and n_2 are the number of items in first and second groups.

Illustration 1 : Find the mean of the temperature recorded in degrees centigrade during a week in April 2014.

38.2, 40.9, 39, 44, 39.6, 40.5, 39.5

Solution : We have, mean temperature is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$= \frac{38.2 + 40.9 + 39 + 44 + 39.6 + 40.5 + 39.5}{7}$$

$$= \frac{281.3}{7} = 40.2°C$$

or by the short cut method :

Let A = 39, Thus

$d_i = x_i - A = -0.8, 1.9, 0, 5, .6, 1.5, .5$

$$\therefore \quad \bar{x} = A + \frac{\sum_{i=1}^{n} d_i}{n}$$

$$= 39 + \frac{-0.8 + 1.9 + 0 + 5 + 0.6 + 1.5 + 0.5}{7}$$

$$= 39 + \frac{8.7}{7} = 39 + 1.242 = 40.242$$

Illustration 2 : Find the average wages for the construction of the building from the wages paid to different workers. (Nov. 2017)

Wages	100	200	300	400	500
No.of workers	3	5	6	9	2

Solution : We have

x_i	f_i	$x_i f_i$
100	3	300
200	5	1000
300	6	1800
400	9	3600
500	2	1000
	25	7700

$$\therefore \quad \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{7700}{25} = 308$$

Illustration 3 : Find the A.M. for the following data :

x_i	35	45	55	60	75	80
f	12	18	10	6	3	11

Solution : Let A = 55. We have

x_i	f_i	$d_i = x_i - A$	$f_i d_i$
35	12	–20	–240
45	18	–10	–180
55	10	0	0
60	6	5	30
75	3	20	60
80	11	25	275
	60		–55

$$\therefore \quad \bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} = 55 + \frac{(-55)}{60}$$

$$= 55 - 0.92 = 54.08$$

Illustration 4 : Following is the number of visitors in 180 days to a Zoo.

No. of visitors	1–10	11–20	21–30	31–40	41–50	51–60
No. of days	22	28	35	45	30	20

Find the average number of visitors per day.

Solution : Let A = 35.5. We have $h = 10$

No. of visitors	No. of days f_i	Mid value x_i	$d_i' = \frac{x_i - A}{h}$	$f_i d_i$
1–10	22	5.5	–3	–66
11–20	28	15.5	–2	–56
21–30	35	25.5	–1	–35
31–40	45	35.5	0	0
41–50	30	45.5	1	30
51–60	20	55.5	2	40
	180			– 87

$$\therefore \quad \bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i'}{\sum_{i=1}^{n} f_i} \times h$$

$$= 35.5 + \frac{(-87)}{180} \times 10 = 35.5 - 4.83 = 30.67$$

Illustration 5 : Consider the number of students in different subjects in B.E. first year and the average marks obtained by them as shown in the following table. Find the average marks (weighted) obtained per student.

Subject	Average marks obtained	No.of students
Calculus	40	25
Physics	70	30
EG	63	15
EME	78	42
C.S.	55	13

Solution : Here the values of x are average marks and w are the number of students.

$$\therefore \quad \text{Weighted Arithmetic Mean} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

$$= \frac{(40)(25) + (70)(30) + (63)(15) + (78)(42) + (55)(13)}{25 + 30 + 15 + 42 + 13}$$

$$= \frac{8036}{125} = 64.29$$

Illustration 6 : Let 12 tires have been run on a group of test truck and have shown an average of 14560 miles. Similarly 20 other tires have shown an average of 13425 miles. What is the mean mileage of all the tires ?

Solution : We have,

$$\text{Combined average } \bar{x}_{12} = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$$

$$= \frac{(14560)(12) + (13425)(20)}{12 + 20}$$

$$= 13850.625 \text{ miles}$$

Property : The algebraic sum of the deviations of a series from their mean is always equal to zero.

Proof : Let $x_1, x_2, ..., x_n$ be the values of a variable and $d_1, d_2, ..., d_n$ be the deviation from the mean.

$$\therefore \quad d_i = x_i - \bar{x},$$

$$\text{where } \bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$$

$$\therefore \quad d_1 + d_2 + ... + d_n$$
$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + ... + (x_n - \bar{x})$$
$$= (x_1 + x_2 + ... + x_n) - n\bar{x}$$
$$= n\bar{x} - n\bar{x} = 0$$

3.3 GEOMETRIC MEAN (G.M.) :

The geometric mean is the n^{th} root of the product of all observations comprising a group or items of a series. That is if $x_1, x_2, ..., x_n$ are the observations then

$$\text{Geometric mean} = (x_1 \cdot x_2 ..., x_n)^{1/n}$$

For simple frequency distribution

$$\text{G.M.} = (x_1^{f_1} \cdot x_2^{f_2} \ldots x_n^{f_n})^{1/N}, \ N = \sum_{i=1}^{n} f_i$$

For grouped frequency distribution x_i is taken as class mark.

Geometric mean is very much useful in the calculations of index numbers, dealing with ratios.

Remark :

1. If one of the x_i' s is zero, the G.M. is also zero.
2. G.M. is not used if any x_i's is negative.
3. G.M. cannot be calculate when any one x_i' s is missing.
4. This process can be applied when the number of items are less.
5. If the number of items is large and their size is big, calculations are done with the help of logarithm, that is $\log(\text{G.M.}) = \frac{1}{n}\{\log x_1 + \log x_2 + \ldots + \log x_n\}$

   $\therefore$ G.M. = Antilog

   $$\left[\frac{1}{n}\{\log x_1 + \log x_2 + \ldots + \log x_n\}\right]$$

   $\therefore$ Geometric mean is the anti–log of the mean of the logs.
6. The value of geometric mean will always be less than the arithmetic mean, but when all items in a series are of the same size, the geometric and arithmetic mean of the series have the same value.

Illustration 1 : The marks obtaind by a student are : 10, 20, 30, 5, 15, 25 and 35 in different seven subjects. Find G.M.

Solution :

$$\text{G.M.} = (10 \times 20 \times 30 \times 5 \times 15 \times 25 \times 37)^{1/7}$$

$$= \text{Antilog } \frac{1}{7} (\log 10 + \log 20 + \log 30 + \log 5 + \log 15 + \log 25 + \log 35)$$

$$= \text{Antilog } \frac{1}{7} (1 + 1.3010 + 1.4771 + 0.6990 + 1.1761 + 1.3979 + 1.5441)$$

$$= \text{Antilog } \frac{1}{7} (8.5952) = \text{Antilog } (1.2279)$$

$$= 16.9.$$

Illustration 2 : Find G.M. of the following distribution :

Wages paid	60	62	64	68	70
No. of persons	3	2	4	2	4

Solution : We have,

x	f	$\log x$	$f \log x$
60	3	1.77815	5.33445
62	2	1.79239	3.58478
64	4	1.80618	7.22472
68	2	1.83251	3.66502
70	4	1.84510	7.38040
	15		27.18937

$$\therefore \quad \text{G.M.} = \text{Antilog } \frac{1}{N}\left\{\sum_{i=1}^{n} f_i \log x_i\right\}$$

$$= \text{Antilog } \frac{1}{15} (27.18937)$$

$$= \text{Antilog } (1.81262) = 64.9561$$

Illustration 3 : Find G.M. for the following data of football players.

Goals	0–10	10–20	20–30	30–40	40–50
No. of players	5	9	10	16	4

Solution : We have

Mid value (x_i)	f_i	$\log x_i$	$f_i \log x_i$
5	5	0.6990	3.4950
15	9	1.1761	10.5849
25	10	1.3979	13.9790
35	16	1.5441	24.7056
45	4	1.6532	6.6128
	44		59.3773

$$\therefore \quad \text{G.M.} = \text{Antilog } \left\{\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right\}$$

$$= \text{Antilog } \left\{\frac{1}{44}(59.3773)\right\}$$

$$= \text{Antilog } (1.3495) = 22.37$$

Illustration 4 : The price of cereals increased by 5% from 2005 to 2006, 8% from 2006 to 2007 and 77% from 2007 to 2008. The average increase from 2005 to 2008 is quoted as 26.2% and not 30%. Explain and verify the result.

Solution : We have

increase in %	Price at the end of the year (Base price 100) x	log x
5	105	2.0212
8	108	2.0334
77	177	2.2480
		6.3026

$$\therefore \text{ G.M. } = \text{Antilog}\left\{\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right\}$$

$$= \text{Antilog}\left\{\frac{1}{3}(6.3026)\right\}$$

$$= \text{Antilog } (2.1009) = 126.2$$

$\therefore$ Average increase = 126.2 – 100 = 26.2 %

Here the arithmetic mean is = $\frac{1}{3}$ (5 + 8 + 77) = 30 is not a correct measure as G.M. is a suitable measure for averaging rates of increase or decrease.

Illustration 5 : Find out the average rate of decrease in visitors of a museum if it came down by 15% on Monday, 20% on Tuesday and 30% on Wednesday.

Solution : Average rate of decrease in visitors :

decrease in %	No.of visitors (Base 100) x_i	log x_i
15	85	1.9294
20	80	1.9031
30	70	1.8451
		5.6776

$$\therefore \text{ G.M.} = \text{Antilog}\left\{\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right\}$$

$$= \text{Antilog}\left\{\frac{5.6776}{3}\right\} = 78.07$$

$\Rightarrow$ The decrease is 21.93 %

* **Properties of G.M. :**

(i) The product of the ratio of observations to their G.M. is always equal to unity.

Proof : Let $x_1, x_2, ..., x_n$ are observations and g is the G.M.

$$\therefore \quad g = (x_1, x_2, ... x_n)^{1/n} \Rightarrow g^n = x_1 \cdot x_2 ..., x_n$$

$$\therefore \quad \text{Ration are : } \frac{x_1}{g}, \frac{x_2}{g}, ..., \frac{x_n}{g}$$

$$\therefore \quad \text{Product} = \frac{x_1}{g}, \frac{x_2}{g}, ..., \frac{x_n}{g}$$

$$= \frac{x_1 \cdot x_2 ... x_n}{g^n} = \frac{g^n}{g^n} = 1$$

(ii) The sum of deviations of logs of the values from the log of G.M. is always equal to zero.

Proof : We have deviations $d_i = \log x_i - \log$ (G.M.)

$$\therefore \quad \sum_{i=1}^{n} d_i = \sum_{i=1}^{n} (\log x_i - \log (\text{G.M.}))$$

$$= \sum_{i=1}^{n} \log x_i - n \log (\text{G.M.})$$

$$= \sum_{i=1}^{n} \log x_i - n\left\{\frac{1}{n}\sum_{i=1}^{n} \log x_i\right\} = 0$$

(iii) If each item of a series is replaced by the G.M., the product of each item remains same.

(iv) G.M. is reversible i.e. a change in reverse direction does not affect G.M. It is of great help in measuring index numbers.

3.4 HARMONIC MEAN (H.M.) :

The reciprocal of the arithmetic mean of the reciprocal of variates is called the **harmonic mean** of the variates.

Thus for the observations $x_1, x_2, ..., x_n$

$$\text{H.M.} = \frac{1}{\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right)} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_i}}$$

For simple frequency distribution,

$$\text{H.M.} = \frac{N}{\sum_{i=1}^{n} (f_i / x_i)}, \quad \text{where } N = \sum_{i=1}^{n} f_i$$

or grouped frequency distribution x_i is taken as the class mark.

Remark :

1. A.M. ≥ G.M. ≥ H.M.
2. H.M. is very useful for the calculation of average speed. When rates are involved, H.M. of the speed is an appropriate measure.
3. $(\text{G.M.})^2 = (\text{A.M.})\,(\text{H.M.})$ or

   $\text{G.M.} = \sqrt{(\text{A.M.})\,(\text{H.M.})}$

Illustration 1 : Find the Harmonic mean of the data : 5, 6, 7, 8, 9

Solution :

$$\therefore \quad \frac{1}{H} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}$$

$$= \frac{1}{5}\left\{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9}\right\}$$

$$= \frac{1}{5}\{0.2 + 0.167 + 0.143 + 0.125 + 0.111\} = 0.149$$

$$\therefore \quad H = \frac{1}{0.149} = 6.711$$

Illustration 2 : Suppose a car moves 50 km with a speed of 60 km/hr., then 55 km with a speed of 70 km/hr. and next 70 km at a speed of 80 km/hr. Find the average speed.

Solution : To find average speed, we require harmonic mean of 50, 55 and 70 with 60, 70 and 80 as the respective frequency.

$$\therefore \quad \frac{1}{H} = \frac{1}{N}\sum_{i=1}^{n}\frac{f_i}{x_i} = \frac{\frac{60}{50} + \frac{70}{55} + \frac{80}{70}}{60 + 70 + 80}$$

$$= \frac{3.616}{210}$$

$$\therefore \quad H = \frac{210}{3.616} = 58.075$$

$\therefore$ The average speed is 58.075 km per hour.

Illustration 3 : Find H.M. for the following distribution :

Marks	40–50	50–60	60–70	70–80	80–90	90–100
No. of students	12	10	15	17	8	3

Solution : We have

Class mark (x_i)	f_i	f_i/x_i
45	12	0.2664
55	10	0.1820
65	15	0.2310
75	17	0.2261
85	8	0.0944
95	3	0.0315
	65	1.0314

$$\therefore \quad \frac{1}{H} = \frac{1}{N}\sum_{i=1}^{n}\frac{f_i}{x_i} = \frac{1}{65}(1.0314)$$

$$\Rightarrow H = \frac{65}{1.0314} = 63.021$$

3.5 MEDIAN :

The median is defined as the measure of the central item when they are arranged in ascending or descending order of magnitude. **(Definition, Nov. 2016)**

(i) For ungrouped data : When the number of observations is odd and equal to, say, n then the $\left(\frac{n+1}{2}\right)^{\text{th}}$ value gives the median after arranging them in ascending or descending order.

When the total number of observations is even, say, 2n then the arithmetic mean of n^{th} and $(n+1)^{\text{th}}$ values is the median after arranging them in ascending or descending order.

Illustration 1 : Consider the data : 5, 15, 10, 20, 12. Find median.

Solution : Rearrange the data in ascending order :

5, 10, 12, 15, 20

The number of observations is = 5 and is odd.

$\therefore$ Median = The value of $\left(\frac{5+1}{2}\right)^{th}$ place (i.e. at 3rd place) = 12

Illustration 2 : Consider the data 2, 8, 4, 6, 10, 12, 4, 8, 14, 16. Find median.

Solution : Rearrange the data in descending order.

16, 14, 12, 10, 8, 8, 6, 4, 4, 2

The number of observations is = 10 and is even.

$\therefore$ Median = The A.M. of values of 5^{th} and 6^{th} places = $\frac{1}{2}$ (8 + 8) = 8

(ii) For grouped data :

(a) For simple frequency distribution the median is obtained by using less than cumulative frequency distribution. The median is given by the value of the variable (x) for which the cumulative frequency is just greater than $\frac{N}{2}$ where N = total frequency.

Illustration 3 : Consider the following table :

x	5	10	15	20	25
f	3	4	5	6	2

Find the median.

Solution : We have

x	f	Cumulative frequency
5	3	3
10	4	7
15	5	12
20	6	18
25	2	20

where $N = 20 \Rightarrow \frac{N}{2} = 10$.

Thus the cumulative frequency just greater than 10 is 12 and the corresponding variable is 15.

$\therefore$ The median is = 15.

(b) For grouped frequency distribution, the median is obtained as :

$$\text{Median} = L + \frac{h\left(\frac{N}{2} - F\right)}{f}$$

Where, L = lower limit or boundary of the median class

h = width of the class interval

f = frequency of the median class

N = total frequency

F = cumulative frequency of the class preceding the median class

Illustration 4 : Find the median of the following data :

Marks	Less than 20	21–30	31–40	41–50	51–60	61–70
No. of students	5	15	20	6	6	8

Solution : We have **(Nov. 2017)**

Marks	f	Cumulative frequency
< 20	5	5
21–30	15	20
31–40	20	40
41–50	6	46
51–60	6	52
61–70	8	60

Here N = 60. Thus the cumulative frequency just greater than $\frac{N}{2} = 30$ is 40 and hence the median class is : 31–40.

$\therefore$ L = 31, h = 9, f = 20, F = 20

$$\therefore \text{Median} = 31 + \frac{9(30-20)}{20} = 31 + 4.5 = 35.5$$

Illustration 5 : Find the missing frequency when median is 24.

Marks	0–10	10–20	20–30	30–40	40–50
Students	15	20	x	14	16

Solution : We have

Marks	Frequency	Cum. freq.
0–10	15	15
10–20	20	35
20–30	x	$35 + x$
30–40	14	$49 + x$
40–50	16	$65 + x$

Here, $N = 65 + x$. Given that median = 24. Thus median class : 20–30.

$$\therefore \text{ By formula, Median} = L + \frac{h\left(\frac{N}{2} - F\right)}{f},$$

we have

$$24 = 20 + \frac{10\left(\frac{65 + x}{2} - 35\right)}{x}$$

$$\therefore \quad 4 = \frac{10}{x}\left(\frac{65 + x - 70}{2}\right) \Rightarrow 8x = 10x - 50$$

$$\therefore \quad 2x = 50 \Rightarrow x = 25.$$

Illustration 6 : The following table shows the marks obtained by 10 students in a class test of calculus and physics, out of 30.

Roll No.	1	2	3	4	5	6	7	8	9	10
Marks–Calculus	7	8	11	14	17	20	21	22	25	30
Marks–Physics	9	10	11	13	16	18	20	24	28	29

Find the subject in which students are intelligent.

Solution : Here $n = 10$

$\therefore$ Median (Calculus) = Average of the values at 5^{th} and 6^{th} places

$$= \frac{17 + 20}{2} = 18.5$$

$$\text{and Median (Physics)} = \frac{16 + 18}{2} = 17$$

$\therefore$ The level of knowledge in calculus is higher than physics.

Illustration 7 : From the following data, calculate median. (May 2015)

Marks	< 5	< 10	< 15	< 20	< 25	< 30	< 35	< 40	< 45
No. of Students	29	224	465	582	634	644	650	653	655

Solution : From the data

Marks	f_i	C.F.
0 - 5	29	29
5 - 10	195	224
10 - 15	241	465
15 - 20	117	582
20 - 25	52	634
25 - 30	10	644
30 - 35	6	650
35 - 40	3	653
40 - 45	2	655

$$\therefore \quad \text{Median} = L + \frac{h}{f}\left(\frac{N}{2} - F\right)$$

Here N = 655. Thus the cumulative frequency just greater than $\frac{N}{2} = 327.5$ is 465. Thus the median class is 10 – 15.

$\therefore$ $L = 10$, $h = 5$, $f = 241$, $F = 224$.

$$\therefore \quad \text{Median} = 10 + \frac{5}{241}(327.5 - 224) = 12.1473$$

3.6 MODE :

Mode is defined as the value of the variable which occurs most frequently in the set of observations.

(Definition, May 2016, Nov. 2016)

That is the value of the variable with maximum frequency for a simple frequency distribution. The mode is of great importance to the large scale manufacturer of consumption goods. If one goes to shoes store then he may generally find shoes for his size. But for some persons the size may not be available.

For grouped frequency distribution the mode is obtained as :

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}h$$

Where, L = lower limit of boundary of the modal class

h = Width of the modal class

f_1 = frequency of the modal class

f_0 = frequency of the class preceeding the modal class

f_2 = frequency of the class succeeding the modal class

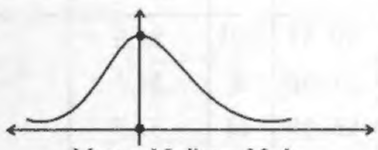
Modal class means the class having maximum frequency.

Remark :

1. If $2f_1 - f_0 - f_2 = 0$, then Mode = $L + \dfrac{f_1 - f_0}{|f_1 - f_0| + |f_1 - f_2|} h$
2. The above technique is not advisable to use if the maximum frequency is repeated.
3. For symmetric distribution

   Mean = Median = Mode

   That is for normal distribution.

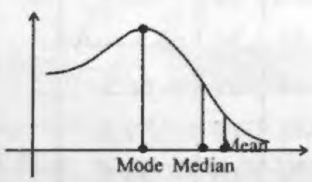


Mean = Median = Mode

But for skewd distribution, there is an empirical relationship due to Karl Pearson as follows :

Mode = 3 Median – 2 Mean

(May 2016)



There may be one or more modes in a frequency curve. Curves having a single mode are called as **unimodal,** those having two modes are called as **di–modal** and those having more than two modes are called as **multi–modal.**

Illustration 1 : Find the mode from the daily wage of 20 workers : 70, 80, 90, 60, 85, 100, 80, 60, 65, 80, 75, 90, 95, 80, 90, 100, 70, 80, 60, 75.

Solution : Since 80 occurs 5 times which is the maximum occurrence of others, the mode of the wages is 80.

Illustration 2 : Find the mode of the following data :

Height in cm	140	145	150	155	160	165	170
No. of students	25	40	58	32	28	16	8

Solution : Since the maximum frequency is 58, the corresponding variable value 150 is the mode of the data.

Illustration 3 : Find the mode for the distribution of total runs scored by 100 players in the tournament. (Nov. 2017)

Runs	0–50	50–100	100–150	150–200	200–250	250–300
No. of players	10	5	25	30	10	20

Solution : Here maximum frequency is 30, so the modal class is 150–200.

$\therefore$ L = 150, h = 50, $f_1 = 30$, $f_0 = 25$, $f_2 = 10$

$$\therefore \quad \text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} h$$

$$= 150 + \frac{30 - 25}{2(30) - 25 - 10} \times 50$$

$$= 150 + \frac{5}{25} \times 50 = 160$$

Illustration 4 : For the following data of a test of 50 marks find mean, median and mode.

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No.of students	5	2	3	4	7	3	4	2	2	1

Solution : We have

Class mark x_i	f_i	$d_i = x_i - A$	$d_i' = \frac{x_i - A}{h}$	$f_i d_i'$	Cum.freq.
2.5	5	–20	–4	–20	5
7.5	2	–15	–3	–6	7
12.5	3	–10	–2	–6	10
17.5	4	–58	–1	–4	14
22.5 = A	7	0	0	0	21
27.5	3	5	1	3	24
32.5	4	10	2	8	28
37.5	2	15	3	6	30
42.5	2	20	4	8	32
47.5	1	25	5	5	33
	33			– 6	

$$\therefore \quad \text{Mean} = A + \frac{\sum_{i=1}^{n} f_i d_i'}{\sum_{i=1}^{n} f_i} \times h$$

($\because h = 5$, the class size)

$$= 22.5 + \frac{(-6)}{33} \times 5 = 22.5 - 0.09$$

$$= 21.591 \text{ marks.}$$

For Median : Here $N = 33 \Rightarrow \frac{N}{2} = 16.5$ which is just less than the cumulative frequency 21. Thus median class is : 20–25.

$\therefore \quad L = 20, h = 5, f = 7, F = 14$

$$\therefore \quad \text{Median} = L + \frac{h\left(\frac{N}{2} - F\right)}{f} = 20 + \frac{5}{7}$$

$$(16.5 - 14) = 21.786 \text{ marks.}$$

Now, Mode $= 3$ Median $- 2$ Mean

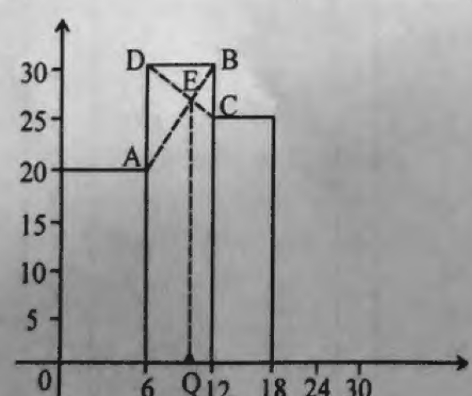
$= 3\,(21.786) - 2(21.591)$

$= 65.358 - 43.182 = 22.176$ marks.

Illustration 5 : Obtain mode by using histogram for the data.

Class	0–6	6–12	12–18	18–24	24–30
Frequency	20	30	25	16	12

Solution : The maximum frequency is 30. Hence modal class is : 6–12. Thus the histogram for the modal class, its preceeding and succeeding classes are as follows :



Join the points A and B; and C and D by lines as shown in figure. Their intersection is at E and its corresponding point on x–axis is Q which is be... 6 and 22.

Thus Q = 9.5 approximately

Note : The actual value is obtained as :

Modal class : 6–12. Thus $L = 6$, $h = 6$, $f_1 =$ ... $f_0 = 20$, $f_2 = 25$

$$\therefore \quad \text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 6 + \frac{30 - 20}{2(30) - 20 - 25} \times 6$$

$$= 6 + \frac{10}{15} \times 6 = 10$$

Illustration 6 : Find the mean, median and m... for the following data : (Dec. 2...

Class	50-53	53-56	56-59	59-62	62-65	65-68	68-71	71-74
Frequency	3	8	14	30	36	28	16	10

Solution : (i) Let A = 63.5, We have $h =$ ...

Class	f_i	x_i	$d_i' = \frac{x_i - A}{h}$	$f_i d_i'$
50–53	003	51.5	- 4	- 12
53–56	8	54.5	- 3	- 24
56–59	14	57.7	- 2	- 28
59–62	30	60.5	- 1	- 30
62–65	36	63.5 = A	0	0
65–68	28	66.5	1	28
68–71	16	69.5	2	32
71–74	10	72.5	3	30
74–77	5	75.5	4	20
	150			16

$$\therefore \quad \text{Mean} = A + \frac{\sum f_i d_i'}{\sum f_i} \times h = 63.5$$

$$+ \frac{16}{150}(3) = ...$$

(ii) For Mode :

Here maximum frequency is 36. So the ... class is 62 - 65.

$\therefore \quad L = 62, h = 3, f_1 = 36, f_0 = 30, f_2 =$ ...

$$\therefore \quad \text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} h = 62$$

$$+ \frac{36 - 30}{72 - 30 - 28} \times 3 = 63.29$$

(iii) We have

Mode = 3 Median - 2 Mean

$$\therefore \quad \text{Median} = \frac{\text{Mode} + 2\text{ Mean}}{3}$$

$$= \frac{63.29 + 2\,(63.82)}{3} = 63.64$$

or

For Median

Here N = 150. Thus the c.f. just greater than $\frac{N}{2} = 75$ is 91 and hence the median class is : 62 – 65.

$\therefore$ L = 62, h = 3, f = 36, F = 55.

$$\therefore \quad \text{Median} = L + \frac{h}{f}\left(\frac{N}{2} - F\right)$$

$$= 62 + \frac{3}{36}(75 - 55) = 63.67$$

EXERCISE 3.1

1. Find A.M. by constructing a frequency table of the data obtained from sugar–cane production from 19 different farms each of 10 acres.

   94, 96, 97, 94, 99, 102, 98, 99, 99, 100, 101, 99, 100, 99, 98, 98, 97, 101, 99. **Ans. : 98.42**

2. The following data is the daily wages of the 20 workers :

   110, 100, 103, 98, 151, 145, 85, 132, 95, 125, 130, 80, 76, 62, 71, 122, 65, 142, 118, 116

   Find A.M. by constructing a grouped distribution with class mark 19.

   Hint : First class is 61–80.

   Ans. : 39.50

3. Find A.M. for the grades of workers in a factory as shown below :

Grade	72.0-73.9	74.0-75.9	76.0-77.9	78.0-79.9	80.0-81.9	82.0-83.9	84.0-85.9	86.0-87.9	88-89.9
No. of workers	7	31	42	54	33	24	22	8	4

   Ans. : 79.61

4. Fifty students appeared in a test. The result of those who passed the test is given below :

Marks	4	5	6	7	8	9
Students	8	10	9	6	4	3

   If the average for all the fifty students was 5.16 marks, find the average marks those who failed.

   (**Hint :** $\bar{x}_{12} = 5.16,\ n_1 = 40,\ n_2 = 10,$

   $\bar{x}_1 = 237/40,\ \bar{x}_2 = ?$) **Ans. : 2.1 marks**

5. The mean of 200 items was 60. Later on it was discovered that two items were misread as 75 and 6 instead of 175 and 16. Find the correct mean. **Ans. : 60.55**

   [**Hint :** $\sum_{i=1}^{n} x_i = 200 \times 60 = 12000$

   $\Rightarrow$ Actual $\Sigma\, x = 12000 - 75 - 6 + 175 + 16$]

6. Find the G.M. of the monthly income of a batch of families :

   180, 250, 490, 120, 1400, 7000, 1050, 150, 360, 100, 80, 200, 500, 240. **Ans. : 34.06**

7. Find the G.M. of the following distribution :

Humidity reading	60	62	64	68	70
No.of days	3	2	4	2	4

   Ans. : 64.96

8. Find G.M. of the following data :

Marks	0–10	10–20	20–30	30–40	40–50
Students	5	9	10	16	4

   Ans. : 22.37 marks

9. A machine was purchased in 2001. Its value appreciated at the rate of 5 % per annum for the first 4 years and then at the rate of 8 % per annum for 6 years. Find out average rate (G.M.) of increase. **Ans. : 6.8 % p.a.**

10. Find H.M. for the data : 11, 12, 13, 14, 15.

    Ans. : 12.85