Multiple Integration, Indeterminate forms

Q.1 Evaluate after changing order of integration

1) $\int_0^2 \int_{y/2}^1 e^{x^2} \, dx \, dy$ $(e-1)$

2) $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx$ (1)

3) $\int_0^\infty \int_0^x x \, e^{-\frac{x^2}{y}} \, dx \, dy$ $\left(\frac{1}{2}\right)$

4) $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy \, dx$ $\left(\frac{16a^2}{3}\right)$

Q.2 Evaluate after changing into polar coordinates

1) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$ $\left(\frac{\pi}{4}\right)$

2) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} \, dy \, dx$ $\left(\frac{\pi}{2}\right)$

3) $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2 \sqrt{x^2+y^2} \, dx \, dy$ $\left(\frac{a^2 \pi}{20}\right)$

4) $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} \, dy \, dx$ $\left(\frac{\pi}{4}(1-e^{-1})\right)$

Q.3 Evaluate $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2} \, dx \, dy$ by applying transformation

$u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and integrating over appropriate region in uv plane (2)

Q.4 Evaluate $\iint_R (x+y)^2 \, dx \, dy$ where R is parallelogram

in xy plane with vertices $(1,0)$ $(3,1)$ $(2,2)$ $(0,1)$

using transformation $u = x+y$ and $v = x-2y$ (21)

Q.5 Evaluate $\iint\limits_{R} (x^2-y^2)^2 \, dA$ over area bounded by

lines $|x|+|y|=1$ using transformation $x+y=u, x-y=v$

(4/9)

Q.6 Evaluate

1) $\int_0^1 \int_0^x (x^2+y^2) \, dA = \left(\dfrac{1}{3}\right)$

2) $\int_1^4 \int_{\frac{3}{2}}^{\sqrt{x}} e^{y/\sqrt{x}} \, dy \, dx \quad \left[7(e-1)\right]$

3) $\int_0^2 \int_0^{x^2} e^{y/x} \, dy \, dx \quad (e^2-1)$

Q.7 Evaluate $\iint\limits_{S} \sqrt{xy-y^2} \, dx \, dy$ where S is triangle

whose vertices are $(0,0)$ $(10,1)$ and $(1,1)$ ⑥

Q.8 Evaluate $\iint\limits_{R} r\sqrt{a^2-r^2} \, dr \, d\theta$ over upper half of circle

$x = a \cos\theta$ $\qquad \left[\dfrac{3\pi-4}{18} a^3\right]$

Q.9 Evaluate

1) $\int_0^2 \int_1^z \int_0^{yz} xyz \, dx \, dy \, dz \qquad \left(\dfrac{7}{2}\right)$

2) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx \qquad \left(\dfrac{1}{48}\right)$

3) $\int_0^1 \int_0^1 \int_{z=\sqrt{x^2+y^2}}^{2} xyz \, dz \, dy \, dx \qquad \left(\dfrac{3}{8}\right)$

4) $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{\left(\frac{a^2-r^2}{a^2}\right)} r \, dz \, dr \, d\theta \qquad \left(\dfrac{5}{64}\pi a^3\right)$

Q.10 a) Find the area lying inside circle $r=a\sin\theta$

and outside cardioid $r=a(1-\cos\theta)$ $\left[a^2\left(1-\frac{\pi}{4}\right)\right]$

b) Find area bounded by $y^2=4ax$ and $x^2=4ay$ $\left(\dfrac{16a^2}{3}\right)$

c) " " " " $y=x^2$ and $y=x$ $(1/6)$

d) " " " " $r=2\sin\theta$ and $r=4\sin\theta$ (3π)

Q.11 Evaluate using cylindrical coordinates

4) Evaluate $\iiint\limits_{D} \sqrt{x^2+y^2}\, dv$, D is solid bounded by
   surfaces $x^2+y^2=z^2$, $z=0$, $z=1$ $\left(\dfrac{\pi}{6}\right)$

b) Find volume bounded by cone $x^2+y^2=z^2$
   and paraboloid $x^2+y^2=z$ $\left(\dfrac{\pi}{6}\right)$

c) Evaluate $\displaystyle\int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx$ $\left(\dfrac{1024\pi}{15}\right)$

Q.12 Evaluate using spherical coordinates

4) Evaluate $\iiint\limits_{D} \dfrac{dv}{(x^2+y^2+z^2)^{3/2}}$, where D is region
   bounded by spheres $x^2+y^2+z^2=a^2$ & $x^2+y^2+z^2=b^2$
   $(a>b>0)$ $\left[4\pi \log\left(\dfrac{a}{b}\right)\right]$

b) $\iiint\limits_{D} \sqrt{x^2+y^2+z^2}\, dv$ where D is region above
   xy plane bounded by cone $z^2=3(x^2+y^2)$ and
   by sphere $x^2+y^2+z^2=1$ $\left[\left(1-\dfrac{\sqrt{3}}{2}\right)\dfrac{\pi}{2}\right]$

Indeterminate form

A) Evaluate following Indeterminate forms

1) $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{\sin^3 x}$ $\left(\dfrac{1}{2}\right)$

2) $\lim\limits_{x \to 0} \dfrac{2^x - 3^x}{x}$ $\left(\dfrac{1}{3}\right)$

3) $\lim\limits_{x \to \infty} \dfrac{1^2 + 2^2 - \ldots + x^2}{x^3}$ $\left(\dfrac{1}{3}\right)$

4) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{3 \sec x}{1 + \tan x}$ (3)

5) $\lim\limits_{x \to 1} \dfrac{\log(1 - x^2)}{\log(1 - x)}$ (1)

6) $\lim\limits_{x \to 1} \left[\dfrac{x}{x-1} - \dfrac{1}{\log x}\right]$ $\left(\dfrac{1}{2}\right)$

7) $\lim\limits_{x \to \frac{\pi}{4}} (1 - \tan x) \sec 2x$ (1)

8) $\lim\limits_{x \to 1} (x^2 - 1) \tan \dfrac{\pi x}{2}$ $\left(-\dfrac{4}{\pi}\right)$

9) $\lim\limits_{x \to 0} \left(\dfrac{1}{x}\right)^{1 - \cos x}$ (1)

10) $\lim\limits_{x \to 1} (2 - x)^{\tan \frac{\pi x}{2}}$ $\left(e^{\frac{2}{\pi}}\right)$

11) $\lim\limits_{x \to \infty} x^{1/x}$ (1)

12) $\lim\limits_{x \to \infty} \left[\dfrac{1^{1/x} + 2^{1/x} + 3^{1/x}}{3}\right]^{3x}$ (6)

13) $\lim\limits_{x \to 0} \left[\dfrac{1}{\sin x} - \dfrac{1}{x}\right]$ (0)

14) $\lim\limits_{x \to 0} \left[\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right]$ $\left(\dfrac{1}{2}\right)$

15) $\lim\limits_{x \to \frac{\pi}{2}} (1 - \sin x) \tan x$ \hspace{2em} (0)

16) $\lim\limits_{x \to 0} \left(\dfrac{1 - x \cot x}{x} \right)$ \hspace{2em} (0)

17) $\lim\limits_{x \to 1} \left(\dfrac{x^x - 1}{x - 1 - \log x} \right)$ \hspace{2em} (2)

18) $\lim\limits_{x \to 0} \left(\dfrac{\cosh x - \cos x}{x \sin x} \right)$ \hspace{2em} (1)

19) $\lim\limits_{x \to 0} \left(\dfrac{e^x - 1 - x}{x^2} \right)$ \hspace{2em} $\left(\dfrac{1}{2} \right)$

20) $\lim\limits_{x \to 0} \dfrac{\log(1 + x^3)}{\sin^3 x}$ \hspace{2em} (1)

21) $\lim\limits_{x \to y} \dfrac{x^y - y^x}{x^x - y^y}$ \hspace{2em} $\left(\dfrac{1 - \log y}{1 + \log y} \right)$

22) $\lim\limits_{x \to 0} \log_x \dfrac{\tan x}{x}$ \hspace{2em} (1)

23) $\lim\limits_{x \to \infty} \left[\cosh^{-1} x - \log x \right]$ \hspace{2em} $(\log 2)$

24) $\lim\limits_{x \to \infty} 2^x \sin\left(\dfrac{a}{2^x} \right)$ \hspace{2em} (a)

25) $\lim\limits_{x \to \infty} \left(\dfrac{1}{x} \right)^{1/x}$ \hspace{2em} (1)

26) $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x - x}{x^3}$ \hspace{2em} $\left(\dfrac{1}{6} \right)$ (By expans.)

27) $\lim\limits_{x \to 0} \dfrac{\sin x - \tan^{-1} x}{x^2 \log(1 + x)}$ \hspace{2em} $\left(\dfrac{1}{6} \right)$ (By exp.)

28) $\lim\limits_{x \to 0} \dfrac{e^{2x} - (1 + x)^2}{x \log(1 + x)}$ \hspace{2em} (1) (By exp.)