Numericals

(1) **A conductor has an electron concentration of $5.9\times10^{28}/m^3$. What current dencity in the conductor corresponds to a drift velocity of 0.625 m/sec. Calculate the mobility of charge carriers. Given $\sigma = 6.22\times10^7 \mho m^{-1}$**

$n = 5.9\times10^{28}/m^3$ $\quad\quad$ $\sigma = 6.22\times10^7 \mho m^{-1}$

$V_d = 0.625\, m/\sec$ $\quad\quad$ $J = ?,\quad \mu = ?$

We know, $J = neV_d$

$$= (5.9\times10^{28})(1.6\times10^{-19})(0.625)$$

$\therefore\ J = 5.9\times10^9\ A/m^2$ ***Answer***

$\sigma = ne\mu$

$$\therefore \mu = \frac{\sigma}{ne}$$

$$= \frac{6.22\times10^7}{5.9\times10^{28}\times1.6\times10^{-19}}$$

$\therefore\ \mu = 6.588\times10^{-3} m^2/V.\sec$ ***Answer***

(2) **Calculate the drift velocity of free electrons with a mobility of $3.5\times10^{-3} m^2/V.\sec$ in copper for an electric field strenght of 0.5 V/m.**

$\mu = 3.5\times10^{-3} m^2/V.\sec$

E = 0.5 V/m

$V_d = ?$

$$V_d = \mu E$$

$$= 3.5\times10^{-3}\times0.5$$

$\therefore\ V_d = 1.75\times10^{-3}\ m/\sec$ ***Answer***

(3) **Find the drift velocity of free electrons in a copper wire whose cross-sectional area is A = 1.05 mm^2 when the wire carries a current of 1A. Assume that each copper atom contributes one electron to the electron gas. Density of free electrons in copper is $8.5\times10^{28}/m^3$.**

$n = 8.5\times10^{28}\ /\ m^3$

$A = 1.05\ mm^2 = 1.05\times10^{-6} m^2$, $I = 1A$, $V_d = ?$

$$J = neV_d$$

$$\therefore V_d = \frac{J}{ne} = \frac{I}{Ane}$$

$$= \frac{1}{1.05\times10^{-6}\times8.5\times10^{28}\times1.6\times10^{-19}}$$

$$\therefore V_d = 7.002\times10^{-5}\ m/\sec$$ ***Answer***

(4) For a metal having 6.5×10^{28} conduction electrons per m^3, find the relaxation time of conduction electrons if the metal resistivity is $1.435\times10^{-8}\ \Omega m$.

$n = 6.5\times10^{28}\ /\ m^3$ $\rho = 1.435\times10^{-8}\ \Omega m$

$\tau = ?$

$$\rho = \frac{ne^2\tau}{m}$$

$$\therefore \tau = \frac{m\sigma}{ne^2}$$ [But $\sigma = 1/\rho$]

$$\therefore \tau = \frac{m}{\rho ne^2}$$

$$= \frac{9.11\times10^{-31}}{1.435\times10^{-8}\times6.5\times10^{28}\times(1.6\times10^{-19})^2}$$

$$\therefore \tau = 3.8\times10^{-14}\ \sec$$ ***Answer***

(5) Calculate the mean free path between the collisions of free electrons in copper at $20^\circ C$. Resistivity of copper is $1.72\times10^{-8}\ \Omega m$ at $20^\circ C$ and density of free electrons is $8.48\times10^{28}\ /\ m^3$.

$$\rho = 1.72\times10^{-8}\ \Omega m$$

$$T = 20^\circ C = 293K$$

$n = 8.48 \times 10^{28} / m^3 \qquad \lambda = ?$

$$\sigma = \frac{ne^2\tau}{m}$$

$$\therefore \tau = \frac{m\sigma}{ne^2}$$

$$\therefore \tau = \frac{m}{\rho ne^2}$$

$$= \frac{9.11 \times 10^{-31}}{(1.72 \times 10^{-8})(8.48 \times 10^{28})(1.6 \times 10^{-19})^2}$$

$$\text{But } \tau = \frac{\lambda}{V}$$

and thermal velocity V,

$$V = \sqrt{\frac{3k_BT}{m}}$$

$$= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293}{9.11 \times 10^{-31}}}$$

$\therefore \; V = 1.15 \times 10^5 \; m/\text{sec}$

$\lambda = \tau V$

$= 2.43 \times 10^{-14} \times 1.15 \times 10^5$

$\therefore \; \lambda = 2.8 \times 10^{-9} \; m$ ***Answer***

(6) Calculate the drift velocity of electrone in a metal of thickness 1 mm across which a potential difference of 1 V is applied - Compare this value with thermal velocity at 300 k. Given that mobility is 0.04 $m^2/V.\text{sec}$ ·

$t = 1 \text{ mm} = 10^{-3} m$

$V = 1 \text{ V}, \; T = 300 \text{ k},$

$\mu = 0.04 m^2/V.\text{sec}$

$V = ?,\quad V_d = ?$

Thermal velocity

$$V = \sqrt{\frac{3k_BT}{m}}$$

$$= \sqrt{\frac{3\times1.38\times10^{-23}\times300}{9.11\times10^{-31}}}$$

$$\therefore\ V = 1.16\times10^{5}\ m/\sec \qquad ...(1)$$

Drift velocity $V_d = \mu E$

But $E = \dfrac{V}{t} = \dfrac{1}{10^{-3}} = 1000\,V/m$

$$\therefore\ V_d = 0.04\times(1000)$$

$$\therefore\ V_d = 40\,m/\sec \qquad ...(2)$$

From (1) and (2), we can say that drift velocity is very small as compared to thermal velocity. **Answer**

(7) **Calculate the drift velocity of electrons in copper and current density in a wire of diameter $0.16\times10^{-2}m$ which carries a current of 10A. Given that $n = 8.48\times10^{28}/m^3$.**

$I = 10A,\quad d = 0.16\times10^{-2}m$

$n = 8.48\times10^{28}/m^3$

$J = ?,\ V_d = ?$

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{10}{(3.14)(0.08\times10^{-2})^2}$$

$$\therefore\ J = 4.97\times10^{6}\ A/m^2$$

Drift velocity (V_d)

$$V_d = \frac{J}{ne} \qquad (\because\ J = neV_d)$$

$$\therefore\ V_d = \frac{4.97\times10^4}{(8.48\times10^{28})(1.6\times10^{-19})}$$

Answer

$$\therefore\ V_d = 3.66\times10^{-4}\ m/\sec$$

(8) A uniform silver wire has a resistivity of $1.54\times10^{-8}\,\Omega m$ at room temperature, for an electric field of 1 V/cm along the wire. Find the drift velocity of electron, assuming that there are 5.8×10^{28} conduction electrons $/m^3$. Also calculate the mobility.

$\rho = 1.54\times10^{-8}\,\Omega m$, $\qquad E = 1\ \text{V/cm} = 100\ \text{V/m}$

$n = 5.8\times10^{28}\ /m^3$, $\qquad V_d = ?$

$$V_d = \mu E$$

But, $\sigma = ne\mu$

$$\therefore\ \mu = \frac{\sigma}{ne} = \frac{1}{\rho ne}$$

$$\therefore\ \mu = \frac{1}{(1.54\times10^{-8})(5.8\times10^{28})(1.6\times10^{-19})}$$

$$\therefore\ \mu = 6.99\times10^{-3}\ m^2/V.\sec$$

Som $V_d = \mu E$

$$= (6.99\times10^{-3})(100)$$

$$\therefore\ V_d = 0.699\ m/\sec$$

Answer

(9) The following data is given for copper

(i) Density = $8.92\times10^3\ kg/m^3$

(ii) Resistivity = $1.73\times10^{-8}\,\Omega m$

(iii) Atomic weight = 63.5

Calculate the mobility and average collision time of electrons.

Density = $8.92\times10^3\ kg/m^3$

$\rho = 1.73\times10^{-8}\,\Omega m$

At wt = 63.5

$\mu = ?\,, \quad \tau = ?$

$$n = \frac{\text{Density} \times \text{Avogadro number}}{At \cdot \omega t}$$

$$\therefore n = \frac{8.92 \times 10^3 \times 6.023 \times 10^{26}}{63.5}$$

$$\therefore n = 8.45 \times 10^{28} / m^3$$

$$\tau = \frac{m}{ne^2 \rho}$$

$$= \frac{9.11 \times 10^{-31}}{(8.45 \times 10^{28})(1.6 \times 10^{-19})^2 (1.73 \times 10^{-8})}$$

Answer

$$\therefore \tau = 2.43 \times 10^{-14} \text{ sec}$$

$$\mu = \frac{\sigma}{ne} = \frac{1}{\rho ne}$$

$$= \frac{1}{(1.73 \times 10^{-8})(8.45 \times 10^{28})(1.6 \times 10^{-19})}$$

Answer

$$\therefore \mu = 0.427 \times 10^{-2} m^2 / V.\text{sec}$$

Numericals

(10) Find the thermal conductivity of copper at $20^{0}C$ with a free electron density of $8.48\times10^{28}/m^3$. The thermal velocity of copper is 1.1536×10^{5} m/sec at $20^{0}C$, with a mean free path of 2.813 nm.

$n = 8.48\times10^{28}/m^3$

$\lambda = 2.813\,nm = 2.813\times10^{-9}\,m$

$v = 1.1536\times10^{5}\,m/\sec$

K = ?

We know,

$$K = \frac{1}{2} nvk_B\lambda$$

$$= \frac{1}{2}(8.48\times10^{28})(1.1536\times10^{5})(1.38\times10^{-23})(2.813\times10^{-9})$$

$$\therefore\ K = 189.92\ W/m.K$$

Answer

(11) A brass disc of elecrical resistivity $50\times10^{-8}\,\Omega m$ conducts heat from a heat source to a heat sink at a rate of 10W. If its diameter is 26 mm and thickness 35 mm, find thermal conductivity and thermal resistance at 300 K.

$\rho = 50\times10^{-8}\,\Omega m$

$T = 300$ K, $\quad d = 26\,mm = 26\times10^{-3}\,m$

$t = 35$ mm $= 35\times10^{-3}\,m$

Thermal resistance is given by

$$R_T = \frac{l}{KA}$$

But $K = \sigma LT$ (Weidemann Franz law)

$$\therefore K = \frac{LT}{\sigma} = \frac{2.44\times10^{-8}\times300}{50\times10^{-8}}$$

$$\therefore K = 14.64\,W/mK$$

Answer

Now, $R_T = \dfrac{l}{KA}$

$$\therefore R_T = \frac{35\times10^{-3}}{14.64\times3.14\times(13\times10^{-3})^2}$$

$$\therefore R_T = 4.505\ K/W$$

Answer

Note :

(1) Probability of electron occupying an energy level is given by $f(E)=1$.

(2) Probability of electron not occupying an energy level is given by $1-f(E)$.

(3) Relation between fermi energy E_F, fermi velocity V_F and temperature T_F, is given by

$$V_F=\sqrt{\frac{2E_F}{m}} \qquad T_F=\frac{E_F}{K_B}$$

Numericals

(12) Evaluiate fermi function for an energy k_BT above fermi energy.

$$E-E_F=k_BT, \quad f(E)=?$$

$$f(E)=\frac{1}{1+e^{(E-E_F)/K_BT}}$$

$$=\frac{1}{1+e^{K_BT/K_BT}}$$

$$=\frac{1}{1+e}=\frac{1}{1+2.78}$$

$$\therefore\ f(E)=0.269$$ *Answer*

(13) Use fermi function to obtain the value of $f(E)$ for $E-E_F=0.010$ eV at 200K.

$$E-E_F=0.01\text{ eV}=0.01\times1.6\times10^{-19}\,J$$

$$T=200\text{ K}, \quad f(E)=?$$

$$f(E)=\frac{1}{1+e^{(E-E_F)/k_BT}}$$

$$=\frac{1}{1+e^{\left(\frac{0.01\times1.6\times10^{-19}}{1.38\times10^{-23}\times200}\right)}}$$

$$=\frac{1}{1+e^{0.5791}}$$

$$\therefore\ f(E) = \frac{1}{1+1.784} = \frac{1}{2.784}$$

$$\therefore\ f(E) = 0.35913$$

Answer

(14) Calculate the fermi velocity and mean free path for conduction electrons, given that its fermi energy is 11.63 eV and relaxation time for electrons is 7.3×10^{-15} sec.

$E_F = 11.63$ eV $= 11.63\times1.6\times10^{-19}\,J$

$T = 7.3\times10^{-15}$ sec

$V_F = ?,\quad \lambda = ?$

Fermi velocity

$$V_F = \sqrt{\frac{2E_F}{m}}$$

$$\therefore\ V_F = \sqrt{\frac{2\times11.63\times1.6\times10^{-19}}{9.11\times10^{-31}}}$$

$$= \sqrt{4.085\times10^{12}}$$

$$\therefore\ V_F = 2.02\times10^{6}\ m/\text{sec}$$

Answer

The mean free path

$$\lambda = \tau V_F$$

$$= (7.3\times10^{-15})(2.02\times10^{6})$$

$$\therefore\ \lambda = 1.47\times10^{-8}\,m$$

$$\therefore\ \lambda = 14.75\,nm$$

Answer

(15) Calculate the fermi energy and fermi temperature in a metal. The fermi veocity of electrons in the metal is 0.86×10^{6} m/sec.

$V_F = 0.86\times10^{6}\ m/\text{sec}$

$E_F = ?,\quad T_F = ?$

We know, $E_F = \frac{1}{2} m {V_F}^2$

$$\therefore\ E_F = \frac{1}{2}(9.11\times10^{-31})(0.86\times10^{6})^2$$

$$\therefore\ E_F = 3.36\times10^{-19}\,J$$

or

$$E_F = 2.105\,eV$$

Answer

Fermi temperature,

$$T_F = \frac{E_F}{k_B}$$

$$= \frac{3.36\times10^{-19}}{1.38\times10^{-23}}$$

$$\therefore\ T_F = 24.41\times10^{3}\,K$$

Answer

(16) Calculate the fermi temperature and fermi velocity for sodium whose fermi level is 3.2 eV.

$$E_F = 3.2\,eV = 3.2\times1.6\times10^{-19}\,J$$

$$T_F = ?,\quad V_F = ?$$

$$V_F = \sqrt{\frac{2E_F}{m}}$$

$$= \sqrt{\frac{2\times3.2\times1.6\times10^{-19}}{9.11\times10^{-31}}}$$

$$\therefore\ V_F = \sqrt{1.12\times10^{12}}$$

$$\therefore\ V_F = 1.06\times10^{6}\,m/\sec$$

Answer

Fermi temperature

$$T_F = \frac{E_F}{k_B}$$

$$= \frac{3.2\times1.6\times10^{-19}}{1.38\times10^{-23}}$$

$$\therefore\ T_F = 3.7\times10^{4}\ K$$

Answer

(17) **Using fermi function, evaluate the temperature at which there is 1% probability that an electron in a solid will have an energy 0.5 eV above E_F of 5 eV.**

$E = 5.5$ eV, $E_F = 5$ ev

$$\therefore\ E-E_F = 5.5-5 = 0.5\ \text{eV}$$

$$\therefore\ E-E_F = 0.5\times1.6\times10^{-19}\ J$$

$f(E) = 1\% = 0.01$

$T = ?$

We know,

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{k_BT}\right)}}$$

$$\therefore\ f(E) = \left[1+e^{\left(\frac{E-E_F}{k_BT}\right)}\right] = 1$$

$$\therefore\ f(E) + f(E)\cdot e^{\left(\frac{E-E_F}{k_BT}\right)} = 1$$

$$\therefore\ f(E)e^{\left(\frac{E-E_F}{k_BT}\right)} = 1-f(E)$$

$$\therefore\ e^{\left(\frac{E-E_F}{k_BT}\right)} = \frac{1-f(E)}{f(E)}$$

Taking logarithm on both sides

$$\frac{E-E_F}{k_BT} = \ln\left[\frac{1-f(E)}{f(E)}\right]$$

$$= \ln[1-f(E)] - \ln[f(E)]$$

$$\therefore\ \frac{1}{k_BT} = \frac{\ln[1-f(E)]-\ln[f(E)]}{(E-E_F)}$$

$$\therefore\ k_BT = \frac{(E-E_F)}{\ln[1-f(E)]-\ln[f(E)]}$$

$$\therefore T = \frac{(E - E_F)}{k_B \ln[1 - f(E)] - \ln[f(E)]}$$

$$= \frac{0.5 \times 1.6 \times 10^{-19}}{(1.38 \times 10^{-23})[\ln(1 - 0.01) - \ln(0.01)]}$$

$$= \frac{8 \times 10^{-20}}{(1.38 \times 10^{-23})[-0.01005 - (-4.6051)]}$$

$$= \frac{8 \times 10^{-20}}{(1.38 \times 10^{-23})(4.595)}$$

$$= \frac{8 \times 10^{-20}}{6.341 \times 10^{-23}}$$

$$\therefore T = 1261.597$$

$$\therefore T = 1.261 \times 10^3\ K$$ ***Answer***

(18) The fermi level in potassium is 2.1 eV. What are the energies for which the probabilities of occupancy at 300K are 0.99, 0.01 and 0.5 ?

$E_F = 2.1\text{ eV} = 2.1 \times 1.6 \times 10^{-19}\ J$

$T = 300\text{ K}$

$f(E_1) = 0.99,\ E_1 = ?$

$f(E_2) = 0.01,\ E_2 = ?$

$f(E_3) = 0.5,\ E_3 = ?$

We know,

$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{k_B T}\right)}}$$

$$\therefore f(E)\left(e^{\left(\frac{E - E_F}{k_B T}\right)} + 1\right) = 1$$

Taking Logarithm on both sides,

$$\frac{E-E_F}{k_BT} = \ln[1-f(E)] - \ln[f(E)]$$

$$\therefore\ E-E_F = k_BT[\ln(1-f(E)) - \ln(f(E))]$$

$$\therefore\ E = E_F + k_BT[\ln(1-f(E)) - \ln(f(E))]$$

To find E_1, E_2, E_3

(1) At $f(E_1) = 0.99$

$$\therefore E_1 = E_F + k_BT[\ln(1-f(E)) - \ln(f(E))]$$

$$= (2.1\times1.6\times10^{-19})[(1.38\times10^{-23})300[\ln(1-0.99) - \ln(0.99)]$$

$$\therefore E_1 = 3.36\times10^{-19} + 4.14\times10^{-21}[-4.6051 - (-0.01005)]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}(-4.595)$$

$$\therefore E_1 = 3.36\times10^{-19} - 1.902\times10^{-20}$$

$$\therefore E_1 = 3.16\times10^{-19}\,J$$

or

Answer

$$E_1 = 1.98eV$$

(2) For $f(E) = 0.01$

$$E_2 = E_F + k_BT\,[\ln(1-f(E)) - \ln(f(E))]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}[\ln(0.99) - \ln(0.01)]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}[-0.01005 - (-4.605)]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}(4.595)$$

$$= 3.36\times10^{-19} + 1.902\times10^{-20}$$

$$\therefore E_2 = 3.55\times10^{-19}\,J$$

Answer

or

$$\therefore E_2 = 2.21eV$$

(3) for $f(E) = 0.5$

$$E_3 = E_F + k_B T[\ln(1 - f(E)) - \ln(f(E))]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}[\ln 0.5 - \ln 0.5]$$

$$= 3.36\times10^{-19}\,J$$

or

$$\therefore E_3 = 2.1\ eV$$

Answer

(19) In a solid, consider an energy level lying 0.1 eV above fermi level. What is the probability of this level not being occupied by an electron at room temperature.

$$E - E_F = 0.1\text{ eV} = 0.1\times1.6\times10^{-19}\,J$$

$$T = 300\text{ K}$$

The probability of unoccupancy is given by

$$1 - f(E) = 1 - \left[\frac{1}{1+e^{\left(\frac{E-E_F}{k_B T}\right)}}\right]$$

$$= 1 - \left[\frac{1}{1+e^{\left(\frac{0.1\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}\right)}}\right] = 1 - \left(\frac{1}{1+e^{3.864}}\right)$$

$$= 1 - \left(\frac{1}{1+47.69}\right) = 1 - \frac{1}{48.69}$$

$$\therefore 1 - f(E) = 1 - 0.0205$$

$$\therefore 1 - f(E) = 0.9794$$

Answer

(20) Find the probability with which an energy level 0.02 eV above fermi level will be occupied at room temperature of 300 K and at 1000 K.

$$E - E_F = 0.02eV = 0.02\times1.6\times10^{-19}\,J$$

Probability of occupancy at 300 K

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{K_B T}\right)}}$$

$$= \frac{1}{1+e^{\left(\frac{0.02\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}\right)}}$$

$$= \frac{1}{1 + e^{0.7729}}$$

$$= \frac{1}{1 + 2.166}$$

$\therefore \quad f(E) = 0.315$ ***Answer***

Probability of occupancy at 1000 K

$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{k_B T}\right)}}$$

$$= \frac{1}{1 + e^{\left(\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 1000}\right)}}$$

$$= \frac{1}{1 + e^{0.2318}} = \frac{1}{1 + 1.2609}$$

$\therefore f(E) = 0.442$ ***Answer***