Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II (NEW) EXAMINATION – WINTER 2020

Subject Code:3110014 Date:16/03/2021

Subject Name:Mathematics – I

Time:10:30 AM TO 12:30 PM Total Marks:47

Instructions:

1. **Attempt any THREE questions from Q1 to Q6.**
2. **Q7 is compulsory.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Expand $\sin x$ in powers of $(x-\pi/2)$.	**03**
	(b)	Evaluate $\lim\limits_{x\to 0}\dfrac{\tan^2 x - x^2}{x^2\tan^2 x}$.	**04**
	(c)	**(i)** Check the convergence of $\int_4^\infty \dfrac{3x+5}{x^4+7}dx$.	**03**
		(ii) The region between the curve $y=\sqrt{x}$, $0\le x\le 4$ and the line $x = 4$ is revolved about the x – axis to generate a solid. Find its volume.	**04**
Q.2	**(a)**	If $u=\operatorname{cosec}^{-1}\left(\dfrac{x+y}{x^2+y^2}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$.	**03**
	(b)	Check the convergence of the series $\sum\limits_{n=1}^{\infty}\dfrac{2^n+5}{3^n}$.	**04**
	(c)	**(i)** Test the convergence of the series $\sum\limits_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.	**03**
		(ii) Test the convergence of $\sum\limits_{n=1}^{\infty}\dfrac{2^n}{n^3+1}$.	**04**
Q.3	**(a)**	Solve the following equations by Gauss' elimination method: $x+y+z=6, x+2y+3z=14, 2x+4y+7z=30$.	**03**
	(b)	If $u=f(x-y, y-z, z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$.	**04**
	(c)	**(i)** Find the equation of the tangent plane and normal line to the surface $x^2+2y^2+3z^2=12$ at (1, 2, -1).	**03**
		(ii) For $f(x,y)=x^3+y^3-3xy$, find the maximum and minimum values.	**04**
Q.4	**(a)**	Find the rank of the matrix $\begin{bmatrix} 8 & 0 & 0 & 16 \\ 0 & 0 & 0 & 6 \\ 0 & 9 & 9 & 9 \end{bmatrix}$.	**03**
	(b)	If $u=f(x+at)+g(x-at)$, prove that $\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2}$.	**04**

(c) **(i)** Show that the function $f(x,y)=\begin{cases}\dfrac{2x^2y}{x^3+y^3}, (x,y)\neq(0,0)\\ 0, (x,y)=(0,0)\end{cases}$ is not continuous at the origin. **03**

(ii) Find the shortest distance from the point (1,2,2) to the sphere $x^2+y^2+z^2=16$. **04**

Q.5 **(a)** Use Gauss-Jordan method to find A^{-1}, if $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$. **03**

(b) Using Caley-Hamilton theorem find A^2, if $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$. Also find A^{-1}. **04**

(c) Find the Fourier cosine series for $f(x)=x^2, 0<x<\pi$. Hence show that $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n^2}=\dfrac{\pi^2}{12}$. **07**

Q.6 **(a)** Evaluate $\iint_R e^{2x+3y}\,dA$, where R is the triangle bounded by $x = 0$, $y = 0$, $x + y = 1$. **03**

(b) Find the eigen values and eigen vectors for the matrix $\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$. **04**

(c) Evaluate $\int_0^{\infty}\int_x^{\infty}\dfrac{e^{-y}}{y}\,dA$ by changing the order of integration. **07**

Q.7 Evaluate $\int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a} r^2\,dr\,d\theta$. **05**

OR

Q.7 Find the area enclosed within the curves $y = 2 - x$ and $y^2 = 2(2-x)$. **05**