SARDAR VALLABHBHAI PATEL INSTITUTE OF TECHNOLOGY, VASAD

B. E. Semester-1

Subject: Mathematics-1(3110014)

Tutorial-2

Q:1 (i) If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

(ii) If $u = x^2 sin^{-1}\frac{y}{x} - y^2 cos^{-1}\frac{x}{y}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u.$

Q:2 Find the equation of Tangent Plane and Normal Line of

(i) The ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3.$ at the point (-2,1,-3)

(ii) $x^2yz + 3y^2 = 2xz^2 - 8z.$ at the point (1,2,-1)

Q:3 Find the extreme value of

(i) $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$

(ii) $f(x, y) = x + y + \frac{1}{x} + \frac{1}{y}$

Q:4 Find the absolute maximum and minimum values of $f(x, y) = 4xy - x^2y - xy^2$ on the closed square plate with vertices at (0,0), (3,0), (3,3), and (0,3).

Q:5 Find the minimum values of x^2yz^3, subject to the condition $2x + y + 3z = a$

Q:6 A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point (x, y, z) on the surface of the probe $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface.

Q:7 A rectangular box without lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.

Q:8 (i) Find the expansion of $\tan\left(x + \frac{\pi}{4}\right)$ in the ascending powers of x up to terms in x^3 and find approximately the value of $tan 43^\circ$.

(ii) Prove that $\ln\left(x + \sqrt{1 + x^2}\right) = x - \frac{x^3}{6} + \frac{3}{40}x^5 - \cdots$

Q:9 Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for each of the following functions:

(i) $u = x^2 - y^2, v = 2xy$

(ii) $u = x\ siny, v = y\ sinx.$