

GUJARAT TECHNOLOGICAL UNIVERSITY
BE –SEMESTER 1&2(NEW SYLLABUS)EXAMINATION- WINTER 2018

Subject Code: 3110014**Date: 07-01-2019****Subject Name: Mathematics - I****Time: 10:30 am to 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		Marks
Q.1	(a) State Cayley– Hamilton theorem. Find eigen values of A and A^{-1} , where $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$	03
	(b) State L' Hospital's Rule. Use it to evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$	04
	(c) Investigate convergence of the following integrals:	07
	(i) $\int_5^{\infty} \frac{5x}{(1+x^2)^3} dx$	
	(ii) $\int_0^{\infty} \frac{x^{10}(1+x^5)}{(1+x)^{27}} dx$	
Q.2	(a) Test the convergence of series $\sum_{n=1}^{\infty} \frac{4^n + 5^n}{6^n}$	03
	(b) State the p-series test. Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{n+1}{n^3 - 3n + 2}$	04
	(c) State D'Alembert's ratio test and Cauchy's root test. Discuss the convergence of the following series:	07
	(i) $\sum_{n=1}^{\infty} \frac{4^n (n+1)!}{n^{n+1}}$	
	(ii) $\sum_{n=2}^{\infty} \frac{n}{(\log n)^n}$	
	OR	
	(c) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \frac{x^3}{10 \cdot 11 \cdot 12} + \dots$; $x \geq 0$	07
Q.3	(a) Reduce matrix $A = \begin{bmatrix} 1 & 5 & 3 & -2 \\ 2 & 0 & 4 & 1 \\ 4 & 8 & 9 & -1 \end{bmatrix}$ to row echelon form and find its rank.	03
	(b) Derive half range sine series of $f(x) = \pi - x$, $0 \leq x \leq \pi$	04
	(c) Find the eigen values and corresponding eigen vectors for the matrix A	07
	where $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$	

OR

- Q.3** (a) Expand $e^{x \sin(x)}$ in power of x up to the terms containing x^6 . **03**
(b) Solve system of linear equation by Gauss Elimination method, if solution exists. **04**
 $x + y + 2z = 9; 2x + 4y - 3z = 1; 3x + 6y - 5z = 0$
(c) Find Fourier series of $f(x) = \begin{cases} x, & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$ **07**
- Q.4** (a) Discuss the continuity of the function f defined as **03**
$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(b) Define gradient of a function. Use it to find directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P(1, 1, 0)$ in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. **04**
(c) Find the shortest and largest distance from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. **07**

OR

- Q.4** (a) Find the extreme values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **03**
(b) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx$ by changing into polar coordinates. **04**
(c) (i) If $u = x^2y + y^2z + z^2x$ then find out $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ **07**
(ii) If $x^3 + y^3 = 6xy$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
- Q.5** (a) Evaluate $\iint_R y \sin(xy) dA$, where R is the region bounded by $x=1$, $x=2$, $y=0$ and $y=\frac{\pi}{2}$. **03**
(b) By changing the order of integration, evaluate $\int_0^3 \int_y^3 \frac{xdxdy}{x^2 + y^2}$ **04**
(c) Find the volume below the surface $z = x^2 + y^2$, above the plane $z=0$, and inside the cylinder $x^2 + y^2 = 2y$. **07**

OR

- Q.5** (a) Evaluate integral $\iint_R \frac{rdrd\theta}{\sqrt{a^2 + r^2}}$ over the region R which is one loop of $r^2 = a^2 \cos 2\theta$ **03**
(b) Evaluate the integral $\int_1^e \int_1^{\log y} \int_1^{e^x} (x^2 + y^2) dz dx dy$. **04**
(c) Find the volume of the solid obtained by rotating the region R enclosed by the curves $y=x$ and $y=x^2$ about the line $y=2$. **07**