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B. E. First Sem (Mathematics I)

Tutorial-3

1 **Which of the following matrices are in row echelon form?**

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad \begin{bmatrix}1 & 2 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix} \qquad \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 2 & 0\end{bmatrix} \qquad \begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

2 **Which of the following matrices are in reduced row echelon form?**

$$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix} \qquad \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \qquad \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{bmatrix}$$

3 **Solve each of the following systems by Gaussian elimination.**

$$x_1 + x_2 + x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

$$\frac{-1}{x} + \frac{3}{y} + \frac{4}{z} = 30$$
$$\frac{3}{x} + \frac{2}{y} - \frac{1}{z} = 9$$
$$\frac{2}{x} - \frac{1}{y} + \frac{2}{z} = 10$$

$$x - y + z = 3$$
$$2x - 3y + 5z = 10$$
$$x + y + 4z = 4$$

4 **Solve each of the following systems by Gauss-Jordan elimination.**

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

$$-2y + 3z = 1$$
$$3x + 6y - 3z = -2$$
$$6x + 6y + 3z = 5$$

5 **Solve the following homogeneous systems of linear equations by any method.**

$$x_3 + x_4 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$2x_1 + 2x_2 - x_3 + x_5 = 0$$

$$2x - y - 3z = 0$$
$$-x + 2y - 3z = 0$$
$$x + y + 4z = 0$$

6 **For which values of a will the following system have no solutions? Exactly one solution? Infinitely many solutions?**

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

7 For which value(s) of λ does the system of

$$(\lambda - 3)x + y = 0$$
$$x + (\lambda - 3)y = 0$$

equations have nontrivial solutions?

8 Find the rank of the following matrices.

$$\begin{bmatrix} 1 & 5 & 3 & -2 \\ 2 & 0 & 4 & 1 \\ 4 & 8 & 9 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 8 \\ 3 & 4 & 5 \end{bmatrix}$$

9 Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ -6 & 3 & -8 \end{bmatrix}$ by row echelon form.