Sardar Vallabhbhai Patel Institute of Technology,Vasad

B. E. First Semester (Mathematics-1 -3110014)

Tutorial: 5

1 Discuss the convergence of

(a) $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{1}{n^2+n+1}\right)$ (b) $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$ (c) $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$

2 Discuss the convergence of

(a) $\sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}}{2^n}$ (b) $\sum_{n=1}^{\infty} n!e^{-n}$ (c) $\sum_{n=1}^{\infty} \frac{n}{(\ln n)^n}$ (d) $\sum_{n=1}^{\infty} \frac{4^n+5^n}{6^n}$ (e) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n+2}\right)^n x^n$

3 Discuss the convergence of

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2(n+1)}\right)$

4 Which of the following series is converges absolutely

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{tan^{-1} n}{n^2+1}$

5 Do as directed.

(i) Dose the sequence whose nth term is $a_n = \left(\frac{n+1}{n+2}\right)^n$ cgt? If so, find $\lim_{n\to\infty} a_n$.

(ii) State Sandwich theorem on Sequences and using it show that, if $x \in R$ with $|x| < 1$, then $x^n \to 0 \; as \; n \to \infty$

(iii) Is the sequence $a_n = \frac{n}{n^2+1}$ convergent?

6 For the following series

(a) Find the radius of convergence of the series.
(b) For what values of x, the series converges absolutely?
(c) For what values of x, the series converges conditionally?

(i) $\sum_{n=0}^{\infty} \frac{(-1)^{n-1}(3x-1)^n}{n^2}$ (ii) $\sum_{n=0}^{\infty} \frac{(2x+3)^{2n+1}}{n!}$

7 Test the convergence for the following series.

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n+1)^n}{(2n)^n}$ (ii) $\sum_{n=1}^{\infty} \frac{1}{n(1+\log^2 n)}$ (iii) $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$ (iv) $\sum_{n=0}^{\infty} n!(x-4)^n$

(v) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{(2n-1)}, 0 < x < 1$ (vi) $\sum_{n=1}^{\infty} ne^{-n^2}$ (vii) $\sum_{n=1}^{\infty} \frac{n^3+2}{2^n+2}$ (viii) $\sum_{n=0}^{\infty} \frac{n^p}{\sqrt{n+1}+\sqrt{n}}$

8 Test for convergence of the following series.

(i) $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+.....$ (ii) $1-2x+3x^2-4x^3+....0<x<1$

(iii) $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+.....$