Important sequences and their limits :

1. $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

2. $\lim_{n\to\infty} \sqrt[n]{n} = 1$

3. $\lim_{n\to\infty} x^{1/n} = 1 \ (x > 0)$

4. $\lim_{n\to\infty} x^n = 0 \ (|x| < 1)$

5. $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \ (\text{any } x)$

6. $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \ (\text{any } x)$

Polar Co-ordinates

$x = r \cos \theta$ $\qquad r = \sqrt{x^2 + y^2}$

$y = r \sin \theta$ $\qquad \theta = \tan^{-1} \frac{y}{x}$

$dxdy = r\, drd\theta$

Exponential function e^x

$e = 2.71828\ 18284$

$e^x \cdot e^y = e^{x+y}$, $\frac{e^x}{e^y} = e^{x-y}$, $(e^x)^y = e^{xy}$

Natural logarithm $/nx$ or $\log_e x$.

$/n(xy) = /n\ x + /n\ y$, $\qquad /n\left(\frac{x}{y}\right) = /n\ x - /ny$, $/n\ (x^n) = n \log x$

$e^{/nx} = x$, $\qquad e^{-/nx} = e^{/n\left(\frac{1}{x}\right)} = \frac{1}{x}$

Logarithm of base ten $\log_{10} x$

$\log_{10} x = \log e\ /nx$, $\qquad \log e = 0.43429$

$/nx = \frac{1}{\log e} \log_{10} x$, $\qquad \frac{1}{\log e} = 2.30258$

$10^{\log_{10} x} = x$, $\qquad 10^{-\log_{10} x} = \frac{1}{x}$

Sine and Cosine functions

$\sin x$ and $\cos x$ have period 2π.

$\sin (-x) = -\sin x$ and $\cos (-x) = \cos x$

In calculus, angles are measured in radians.

$1^o = 0.01745$ radian

1 radian $= 57^o\ 17'\ 44.80625''$

$= 57.2957795131^o$

$\sin^2 x + \cos^2 x = 1$

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\sin (A - B) = \sin A \cos B - \cos A \sin B$

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$, $\cos 2A = \cos^2 A - \sin^2 A$

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{2}\right)$$

$$\cos x = \sin\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} - x\right)$$

$\sin (\pi - x) = \sin x$, $\cos (\pi - x) = -\cos x$ ...(9)

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x),\ \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

$2 \cos A \sin B = \sin (A + B) - \sin (A - B)$

$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

$2 \sin A \sin B = -\cos (A + B) + \cos (A - B)$

$$\sin C + \sin D = 2 \sin\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right)$$

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\cos D - \cos C = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

Tangent, Cotangent, Secant, Cosecant

$$\tan x = \frac{\sin x}{\cos x}, \cot x = \frac{\cos x}{\sin x}, \sec x = \frac{1}{\cos x}, \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}, \tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Circular and Hyperbolic functions

$$e^{ix} = \cos x + i \sin x, e^{-ix} = \cos x - i \sin x$$

$$\sin x = \frac{e^{ix} - e^{ix}}{2i}, \cos x = \frac{e^{ix} - e^{ix}}{2}$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}), \cos hx = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sin hx}{\cos hx}, \quad \cot hx = \frac{\cos hx}{\sin hx}$$

$$\cosh x + \sin hx = e^x, \cos hx - \sin hx = e^{-x}$$

$$\cosh^2 x - \sin h^2 x = 1$$

$$\sinh^2 x = \frac{1}{2}(\cos h2x - 1), \cos h^2 x = \frac{1}{2}(\cos h2x + 1)$$

$$\sinh(x \pm y) = \sin hx \cos hy \pm \cos hx \sin hy$$

$$\cosh(x \pm y) = \cos hx \cos hy \pm \sin hx \sin hy$$

$$\tanh(x \pm y) = \frac{\tan hx \pm \tan hy}{1 \pm \tan hx \tan hy}$$

CALCULUS

Limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \qquad \lim_{x \to 0} \cos x = 1, \qquad \lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e, \qquad \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{\frac{1}{x}} = e$$

Differentiation

1. $\frac{d}{dx}(c) = 0$ (c constant)
2. $\frac{d}{dx}(cu) = c\frac{du}{dx}$ (c constant)
3. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$
4. $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$
5. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
6. $\frac{d}{dx}(x^n) = n\,x^{n-1}$
7. $\frac{d}{dx}(a^x) = a^x \log_e a$
8. $\frac{d}{dx}(e^x) = e^x$
9. $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$
10. $\frac{d}{dx}\log_e x$ or $\frac{d}{dx} lnx = \frac{1}{x}$
11. $\frac{d}{dx}\sin x = \cos x$
12. $\frac{d}{dx}\cos x = -\sin x$
13. $\frac{d}{dx}\tan x = \sec^2 x$
14. $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

15. $\frac{d}{dx} \sec x = \sec x \tan x$

16. $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

17. $\frac{d}{dx} \sin hx = \cos hx$

18. $\frac{d}{dx} \cos hx = \sin hx$

19. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$

20. $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1 - x^2}}$

21. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$

22. $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1 + x^2}$

23. $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2 - 1}}$

24. $\frac{d}{dx} \operatorname{cosec}^{-1} x = -\frac{1}{x\sqrt{x^2 - 1}}$

25. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (Chain rule)

Integration

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

2. $\int \frac{1}{x} \, dx = ln \mid x \mid + c$

3. $\int a^x \, dx = \frac{a^x}{\log_e a} + c$

4. $\int e^x \, dx = e^x + c$

5. $\int \sin x \, dx = -\cos x + c$

6. $\int \cos x \, dx = \sin x + c$

7. $\int \sec^2 x \, dx = \tan x + c$

8. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

9. $\int \sec x \tan x \, dx = \sec x + c$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

11. $\int \tan x \, dx = \log \mid \sec x \mid + c$

12. $\int \cot x \, dx = \log \mid \sin x \mid + c$

13. $\int \sec x \, dx = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$
$= \log (\sec x + \tan x) + c$

14. $\int \operatorname{cosec} x \, dx = \log \tan \frac{x}{2} + c$
$= \log (\operatorname{cosec} x - \cot x) + c$

15. $\int \sin hx \, dx = \cos hx + c$

16. $\int \cos hx \, dx = \sin hx + c$

17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$

19. $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log \left| \frac{x + a}{x - a} \right| + c$

20. $\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c$

21. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{(x^2 + a^2)} \right| + c$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{(x^2 - a^2)} \right| + c$

23. $\int \sqrt{x^2 + a^2} \, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{(x^2 + a^2)} \right| + c$

24. $\int \sqrt{x^2 + a^2} \, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{(x^2 - a^2)} \right| + c$

25. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

26. $\int uv \, dx = u \int v dx - \int \left[\frac{du}{dx} \int v \, dx \right] dx + c$

27. $\int e^x [f(x) + f'(x)] \, dx = e^x f(x) + c$

28. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$
$= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin (bx - \phi) + c, \quad \phi = \tan^{-1} \frac{b}{a}$

29. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$
$= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos (bx - \phi) + c, \; \phi = \tan^{-1} \frac{b}{a}$

30. $\int_a^b f(x) \, dx = [f(x)]_a^b = F(b) - F(a)$

31. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

32. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

33. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

34. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

35. $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f(x) is even function
$= 0$, if f(x) is odd function.

36. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

37. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

PROGRESSION AND SERIES

1. Arithmetic Prgression

$a, a+d, a+2d....T_n = a + (n-1)d$, $S_n = \frac{n}{2}[2a + (n-1)d]$

2. Geometric Progression

$a, ar, ar^2, ..., ar^{n-1}, ...$ $\quad T_n = ar^{n-1}$, $\quad S_n = \frac{a(r^n - 1)}{r-1}$ if $r > 1$

$= \frac{a(1-r^n)}{1-r}$ if $r < 1$.

3. (i) $1 + 2 + 3 + ... + n = \Sigma n = \frac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + ... + n^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3 + 2^3 + 3^3 + ... + n^3 = \Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2$

GEOMETRIC FORMULAE

1. Volume of the sphere $= \frac{4}{3}\pi r^3$

Surface area of the Sphere $= 4\pi r^2$

2. Volume of the cylinder $= \pi r^2 h$

Surface area of the cylinder $= 2\pi rh$

3. Volume of the right circular cone $= \frac{1}{3}\pi r^2 h$

Surface area of the right circular cone $= \pi r\sqrt{r^2 + h^2}$

4. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Area of the ellipse $= \pi ab$

5. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $b^2 = a^2(1 - e^2)$

6. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $b^2 = a^2(e^2 - 1)$

7. General equation of the

(i) Straight line : $ax + by + c = 0$

(ii) Plane : $ax + by + cz + d = 0$

(iii) Circle : $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre : $(-g, -f)$,

Radius : $\sqrt{g^2 + f^2 - c}$

(iv) Sphere : $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

Centre : $(-u, -v, -w)$

Radius : $\sqrt{u^2 + v^2 + w^2 - d}$