Note :

(1) Probability of electron occupying an energy level is given by $f(E) = 1$.

(2) Probability of electron not occupying an energy level is given by $1 - f(E)$.

(3) Relation between fermi energy E_F, fermi velocity V_F and temperature T_F, is g

$$V_F = \sqrt{\frac{2E_F}{m}} \qquad T_F = \frac{E_F}{K_B}$$

Numericals

(12) Evaluiate fermi function for an energy k_BT above fermi energy.

$$E - E_F = k_BT\,, \quad f(E) = ?$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/K_BT}}$$

$$= \frac{1}{1+e^{K_BT/K_BT}}$$

$$= \frac{1}{1+e} = \frac{1}{1+2.78}$$

$\therefore \ f(E) = 0.269$ **Answer**

(13) Use fermi function to obtain the value of $f(E)$ for $E - E_F = 0.010$ eV at 200K.

$E - E_F = 0.01 \text{ eV} = 0.01 \times 1.6 \times 10^{-19} J$

$T = 200$ K, $f(E) = ?$

$$f(E) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

$$= \frac{1}{1+e^{\left(\frac{0.01 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}\right)}}$$

$$= \frac{1}{1+e^{0.5791}}$$

$$\therefore \ f(E) = \frac{1}{1+1.784} = \frac{1}{2.784}$$

$\therefore \ f(E) = 0.35913$ **Answer**

(14) Calculate the fermi velocity and mean free path for conduction electrons, given that its fermi energy is 11.63 eV and relaxation time for electrons is 7.3×10^{-15} sec.

$E_F = 11.63 \text{ eV} = 11.63 \times 1.6 \times 10^{-19} J$

$T = 7.3 \times 10^{-15}$ sec

$V_F = ?, \ \lambda = ?$

Fermi velocity

$$V_F = \sqrt{\frac{2E_F}{m}}$$

$$\therefore \ V_F = \sqrt{\frac{2 \times 11.63 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}}$$

$$= \sqrt{4.085 \times 10^{12}}$$

$$\therefore V_F = 2.02 \times 10^6 \; m/sec$$

Answer

The mean free path

$$\lambda = \tau V_F$$

$$= (7.3 \times 10^{-15})(2.02 \times 10^6)$$

$$\therefore \lambda = 1.47 \times 10^{-8} m$$

$$\therefore \lambda = 14.75 \, nm$$

Answer

(15) Calculate the fermi energy and fermi temperature in a metal. The fermi veocity of electrons in the metal is 0.86×10^6 m/sec.

$$V_F = 0.86 \times 10^6 \; m/sec$$

$$E_F = ?, \quad T_F = ?$$

We know, $E_F = \frac{1}{2} m {V_F}^2$

$$\therefore E_F = \frac{1}{2}(9.11 \times 10^{-31})(0.86 \times 10^6)^2$$

$$\therefore E_F = 3.36 \times 10^{-19} J$$

or

$$E_F = 2.105 eV$$

Answer

Fermi temperature,

$$T_F = \frac{E_F}{k_B}$$

$$= \frac{3.36 \times 10^{-19}}{1.38 \times 10^{-23}}$$

Answer

$$\therefore T_F = 24.41 \times 10^3 K$$

(16) Calculate the fermi temperature and fermi velocity for sodium whose fermi level is 3.2 eV.

$$E_F = 3.2 \, eV = 3.2 \times 1.6 \times 10^{-19} J$$

$T_F = ?, \quad V_F = ?$

$$V_F = \sqrt{\frac{2E_F}{m}}$$

$$= \sqrt{\frac{2\times3.2\times1.6\times10^{-19}}{9.11\times10^{-31}}}$$

$$\therefore V_F = \sqrt{1.12\times10^{12}}$$

$$\therefore V_F = 1.06\times10^{6}\, m/\sec$$

Answer

Fermi temperature

$$T_F = \frac{E_F}{k_B}$$

$$= \frac{3.2\times1.6\times10^{-19}}{1.38\times10^{-23}}$$

$$\therefore T_F = 3.7\times10^{4}\ K$$

Answer

(17) Using fermi function, evaluate the temperature at which there is 1% probability that an electron in a solid will have an energy 0.5 eV above E_F of 5 eV.

$E = 5.5$ eV, $\quad E_F = 5$ ev

$$\therefore\ E - E_F = 5.5 - 5 = 0.5\ \text{eV}$$

$$\therefore\ E - E_F = 0.5\times1.6\times10^{-19}\ J$$

$f(E) = 1\% = 0.01$

$T = ?$

We know,

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{k_BT}\right)}}$$

$$\therefore f(E) = \left[1+e^{\left(\frac{E-E_F}{k_BT}\right)}\right] = 1$$

$$f(E) + f(E)\cdot e^{\left(\frac{E-E_F}{k_BT}\right)} = 1$$

$$\therefore f(E)e^{\left(\frac{E-E_F}{k_BT}\right)} = 1 - f(E)$$

$$\therefore e^{\left(\frac{E-E_F}{k_BT}\right)} = \frac{1-f(E)}{f(E)}$$

Taking logarithm on both sides

$$\frac{E-E_F}{k_BT} = \ln\left[\frac{1-f(E)}{f(E)}\right]$$

$$= \ln[1-f(E)] - \ln[f(E)]$$

$$\therefore \frac{1}{k_BT} = \frac{\ln[1-f(E)] - \ln[f(E)]}{(E-E_F)}$$

$$\therefore k_BT = \frac{(E-E_F)}{\ln[1-f(E)] - \ln[f(E)]}$$

$$\therefore T = \frac{(E-E_F)}{k_B \ln[1-f(E)] - \ln[f(E)]}$$

$$= \frac{0.5\times1.6\times10^{-19}}{(1.38\times10^{-23})[\ln(1-0.01) - \ln(0.01)]}$$

$$= \frac{8\times10^{-20}}{(1.38\times10^{-23})[-0.01005 - (-4.6051)]}$$

$$= \frac{8\times10^{-20}}{(1.38\times10^{-23})(4.595)}$$

$$= \frac{8\times10^{-20}}{6.341\times10^{-23}}$$

$$\therefore T = 1261.597$$

$$\therefore T = 1.261\times10^3\ K$$ **Answer**

(18) The fermi level in potassium is 2.1 eV. What are the energies for which the probabilities of occupancy at 300K are 0.99, 0.01 and 0.5 ?

$$E_F = 2.1\ \text{eV} = 2.1\times1.6\times10^{-19}\ J$$

$T = 300$ K

$f(E_1) = 0.99$, $E_1 = ?$

$f(E_2) = 0.01$, $E_2 = ?$

$f(E_3) = 0.5$, $E_3 = ?$

We know,

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{k_B T}\right)}}$$

$$\therefore f(E)\left(e^{\left(\frac{E-E_F}{k_B T}\right)}+1\right) = 1$$

Taking Logarithm on both sides,

$$\frac{E-E_F}{k_B T} = \ln[1-f(E)] - \ln[f(E)]$$

$$\therefore E - E_F = k_B T[\ln(1-f(E)) - \ln(f(E))]$$

$$\therefore E = E_F + k_B T[\ln(1-f(E)) - \ln(f(E))]$$

To find E_1, E_2, E_3

(1) At $f(E_1) = 0.99$

$$\therefore E_1 = E_F + k_B T[\ln(1-f(E)) - \ln(f(E))]$$

$$= (2.1\times1.6\times10^{-19})[(1.38\times10^{-23})300[\ln(1-0.99) - \ln(0.99)]$$

$$\therefore E_1 = 3.36\times10^{-19} + 4.14\times10^{-21}[-4.6051 - (-0.01005)]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}(-4.595)$$

$$\therefore E_1 = 3.36\times10^{-19} - 1.902\times10^{-20}$$

$$\therefore E_1 = 3.16\times10^{-19} J$$

or

$E_1 = 1.98 eV$ **Answer**

(2) For $f(E) = 0.01$

$$E_2 = E_F + k_B T[\ln(1 - f(E)) - \ln(f(E))]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}[\ln(0.99) - \ln(0.01)]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}[-0.01005 - (-4.605)]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}(4.595)$$

$$= 3.36\times10^{-19} + 1.902\times10^{-20}$$

$$\therefore E_2 = 3.55\times10^{-19}\ J$$

or

$$\therefore E_2 = 2.21 eV$$

Answer

(3) for $f(E) = 0.5$

$$E_3 = E_F + k_B T[\ln(1 - f(E)) - \ln(f(E))]$$

$$= 3.36\times10^{-19} + 4.14\times10^{-21}[\ln 0.5 - \ln 0.5]$$

$$= 3.36\times10^{-19}\ J$$

or

$$\therefore E_3 = 2.1\ eV$$

Answer

(19) In a solid, consider an energy level lying 0.1 eV above fermi level. What is the probability of this level not being occupied by an electron at room temperature.

$E - E_F = 0.1$ eV $= 0.1\times1.6\times10^{-19}\ J$

$T = 300$ K

The probability of unoccupancy is given by

$$1 - f(E) = 1 - \left[\frac{1}{1 + e^{\left(\frac{E-E_F}{k_B T}\right)}}\right]$$

$$= 1 - \left[\frac{1}{1 + e^{\left(\frac{0.1\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}\right)}}\right] = 1 - \left(\frac{1}{1 + e^{3.864}}\right)$$

$$= 1 - \left(\frac{1}{1 + 47.69}\right) = 1 - \frac{1}{48.69}$$

$\therefore 1-f(E)=1-0.0205$

$\therefore 1-f(E)=0.9794$

Answer

(20) Find the probability with which an energy level 0.02 eV above fermi level will be occupied at room temperature of 300 K and at 1000 K.

$$E-E_F=0.02eV=0.02\times1.6\times10^{-19}J$$

Probability of occupancy at 300 K

$$f(E)=\frac{1}{1+e^{\left(\frac{E-E_F}{K_BT}\right)}}$$

$$=\frac{1}{1+e^{\left(\frac{0.02\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}\right)}}$$

$$=\frac{1}{1+e^{0.7729}}$$

$$=\frac{1}{1+2.166}$$

$\therefore f(E)=0.315$ **Answer**

Probability of occupancy at 1000 K

$$f(E)=\frac{1}{1+e^{\left(\frac{E-E_F}{k_BT}\right)}}$$

$$=\frac{1}{1+e^{\left(\frac{0.02\times1.6\times10^{-19}}{1.38\times10^{-23}\times1000}\right)}}$$

$$=\frac{1}{1+e^{0.2318}}=\frac{1}{1+1.2609}$$

$\therefore f(E)=0.442$ **Answer**