Sardar Vallabhbhai Patel Institute of Technology, Vasad
B. E. First Sem (Mathematics 1)

Tutorial-6

1 Sketch the region of integration and change the order of integration & evaluate.

a) $\int_0^2 \int_x^{4-x^2} \frac{xe^{2y}}{4-y}\,dydx$

b) $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2+y^2}} dy\,dx$

c) $\int_0^1 \int_{-\sqrt{1-y^2}}^{1-y} y\,dx\,dy$

d) $\int_0^{\infty} \int_{-y}^{y} (y^2-x^2)e^{-y}\,dydx$

2 Evaluate $\iint (y-x)dxdy$ over the region E in the XY-plane bounded by the straight line $y=x-3, y=x+1, 3y+x=5, 3y+x=7.$

3 Under the transformation $u=3x+2y$, $v=x+4y$ evaluate the integral $\iint_R (3x^2+14xy+8y^2)\,dx\,dy$ for the region R in the first quadrant bounded by the lines $y=-\frac{3}{2}x+1$, $y=-\frac{3}{2}x+3$, $y=-\frac{1}{4}x$ and $y=-\frac{1}{4}x+1.$

4 By changing into the polar coordinates evaluate the following integrals:

a) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)\,dydx$

b) $\int_0^1 \int_0^x \sqrt{x^2+y^2}\,dydx$

c) $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dy\,dx}{\sqrt{a^2-x^2-y^2}}$

5 Evaluate (i) $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2+y^2)\,dy\,dx$ (ii) $\int_0^2 \int_0^x ydydx$ (iii) $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2-y^2}{x^2+y^2}\,dxdy$

6 Evaluate $\iint_D e^{-x^2-y^2}\,dA$ where D is the region bounded by the semi-circle $x=\sqrt{4-y^2}$ and the y-axis.

7 Evaluate

(i) $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz\,dy\,dx$

(ii) $\int_0^1 \int_0^{\sqrt{z}} \int_0^{2\pi} (r^2\cos^2\theta + z^2) r\,d\theta\,dr\,dz$

(iii) $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$

(iv) $\int_1^3 \int_{1/x}^{1} \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$

8 Evaluate using cylindrical coordinates:

(i) Find the volume bounded by the XY-plane, the paraboloid $2z=x^2+y^2$ and the cylinder $x^2+y^2=4$.

(ii) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} x^2\,dz\,dy\,dx$

9 Evaluate using spherical coordinates:

i) $\iiint_D z(x^2+y^2+z^2)\,dV$ through the volume of the sphere $x^2+y^2+z^2=1$

(ii) $\iiint_D \sqrt{x^2+y^2+z^2}\,dV$ where D is the region bounded by the plane $z=3$ and the cone $z=\sqrt{x^2+y^2}$.

10 (i) Find the area of the region that lies inside the cardioid $r=1+\cos\theta$ and outside the circle r=1.

(ii) Find the area of the region one loop of the rose $r=\cos 3\theta$.

11 Evaluate $\iiint_V 2x\,dv$, where V is the solid region under the plane $2x+3y+z=6$ that lies in the first octant.