Hypothesis Testing

Green Belt - Six Sigma MOOC

Course 4 Module 1

ANOVA and Chi-Square Tests

Video 5

One-Way ANOVA Test

A one-way ANOVA (ANalysis Of VAriance) is used to compare three or more population means and variances. We draw simple random samples from each population and use the data to test the null hypothesis that the population means are all equal. The manual method of calculating is too complicated to show here, so we use statistical tools such as Microsoft Excel or Minitab to assist us.

Assumptions for a One-Way ANOVA test:

- One-Way means there is one factor that describes the cause of the variation in the data
- ANOVA consists of two parts, the variation between treatment means, and the variation within the treatments
- The populations must be normally distributed
- The samples must be independent from one another
- Each population must have the same variance

One-Way ANOVA Test

The procedure using Excel is as follows:

1. Setup hypothesis H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = . . . \mu_k$ and H_1: all are not equal, this will be a right-tailed test
2. Pick a confidence level or α
3. Enter the data in Excel in each column of a blank spreadsheet
4. Select Data Analysis from the Data tab (or install using Excel Add-Ins)
5. Select ANOVA: Single Factor from the Data Analysis dialog box
6. Set up the ANOVA dialog box with data range, α level, etc.
7. Interpret the results

In video question v5q1

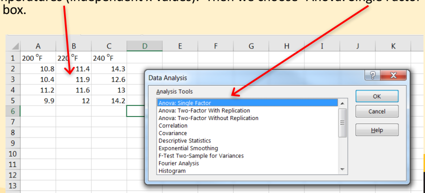
Which is not an assumption or characteristic for an ANOVA test?

- A. ANOVA consists of two parts, the variation between treatment means, and the variation within the treatments
  - Incorrect. This is a true characteristic of an ANOVA test
- B. The samples must be independent from one another
  - Incorrect. This is a true assumption of an ANOVA test
- C. The populations must be normally distributed
  - Incorrect. This is a true assumption of an ANOVA test
- D. A One-Way ANOVA test means there is only way to calculate the solution
  - Correct! One-Way means there is one factor that describes the cause of the variation in the data

One-Way ANOVA Using Excel - Example

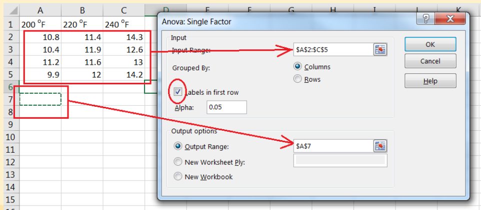
A casting process can be run at 200 °F, 220 °F, or 240 °F. Does the temperature significantly affect the moisture content? Use α-=.05.

We enter the data into the table. This data is the moisture values (dependent y-values) for the oven temperatures (independent x-values). Then we choose "Anova: Single Factor" from the Data Analysis box.



One-Way ANOVA Using Excel - Example

In the Anova: Single Factor dialog box, we enter ranges, include the checkmark, and enter OK.



One-Way ANOVA Using Excel - Example

The resultant values include the variances within the sample groups and between the sample groups. The p-value of 0.004813 is compared to the α level of 0.05.

Since p-value ≤ α (0.004813 ≤ 0.05), we reject H_0 and conclude that at the α=.05 confidence level, the data indicate that temperature does not have an impact on moisture content.

200 °F	220 °F	240 °F
10.8	11.4	14.3
10.4	11.9	12.6
11.2	11.6	13
9.9	12	14.2

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
10.8	3	31.5	10.5	0.43
11.4	3	35.5	11.83333	0.043333
14.3	3	39.8	13.26667	0.693333

ANOVA

ce of Varia	SS	df	MS	F	P-value	F crit
Between (	11.48667	2	5.743333	14.76857	0.004813	5.143253
Within Gr	2.333333	6	0.388889			
Total	13.82	8				

Chi-Square (X^2) Test

The Chi-Square statistic is used in instances where comparing the population variance becomes more important than comparing population means. The two most popular cases to apply chi-square is

1) Comparing variances when the variance of the population is known, and
2) Comparing observed and expected frequencies of test outcomes when there is no defined population variance.

The hypothesis tests for chi-square is as follows:

- H_0: $\sigma^2_x = \sigma^2_0$ and H_1: $\sigma^2_x \neq \sigma^2_0$ or
- H_0: $\sigma^2_x \leq \sigma^2_0$ and H_1: $\sigma^2_x > \sigma^2_0$ or
- H_0: $\sigma^2_x \geq \sigma^2_0$ and H_1: $\sigma^2_x < \sigma^2_0$

These hypothesis test compare a population variance σ^2_x with a fixed value σ^2_0

Chi-Square (X^2) Test

The test statistic is given by:

$$X^2 = \frac{(n-1)s^2}{\sigma_x^2} \qquad DF = n-1$$

Where the number of samples is n and the sample variance is s^2.

If the problem calls for the population variance to equal the fixed value or H_0: $\sigma^2_x = \sigma^2_0$ and H_1: $\sigma^2_x \neq \sigma^2_0$, then a two-tailed test is being requested.

Likewise, if the problem calls for anything less or greater than the fixed value, we use
H_0: $\sigma^2_x \leq \sigma^2_0$ and H_1: $\sigma^2_x > \sigma^2_0$ for a right one-tailed test, and
H_0: $\sigma^2_x \geq \sigma^2_0$ and H_1: $\sigma^2_x < \sigma^2_0$ for a left one-tailed test.

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Chi-Square (X^2) Test - Example

An R&D department of a steel plant has tried to develop a new steel alloy with less tensile variability. This department claims that the new material will show a 4-Sigma tensile variation less than or equal to 60 psi, 95% of the time. An eight sample test yielded a standard deviation of 8 psi. Can a reduction in tensile strength variation be validated with 95% confidence?

Solution: The best range of variation given in the problem is 60. At a 4-Sigma range this is 60/4=15 for one sigma, so σ^2_0 or $(15)^2$ is our fixed value to compare our sample to. So, the null hypothesis will test the claim as follows:

H_0: $\sigma^2_1 \geq (15)^2$ and H_1: $\sigma^2_1 < (15)^2$

From the X^2 table, with DF=8-1=7 and $X^2_{0.95}$, and left-tailed, we find X^2_c = 2.17. This is the value corresponding to the 5% that the variation will not be met.

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Chi-Square (X^2) Test - Example

Now compare with the test statistic:

$$X^2 = \frac{(n-1)s^2}{\sigma_x^2} = \frac{(7)(8^2)}{(15)^2} = 1.99 \qquad DF = 8-1=7$$

H_0: $\sigma^2_1 \geq (15)^2$ and H_1: $\sigma^2_1 < (15)^2$

Since X^2 = 1.99 is less than X^2_c = 2.17, we must reject the null hypothesis. The decreased variation in the new steel alloy tensile strength support the R&D claim.

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End video