For each of exercises 1–4, write the formula a_n that models each sequence, starting at $n = 1$.

1. $\{1, 3, 5, 7, \ldots\}$

$n=1$ $n=2$ $n=3$ $n=4$

$a_n = 1 + 2(n-1)$

2. $\left\{\frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \ldots\right\}$

+1 +1 +1 alternating

$a_n = (-1)^{n+1} \frac{2+n}{3+n}$

3. $\{2, 6, 18, 54, \ldots\}$

$a_n = 2(3)^{n-1}$

4. $\left\{\frac{2}{3}, -\frac{4}{4}, \frac{8}{5}, -\frac{16}{6}, \ldots\right\}$

×2 ×2 ×2

+1 +1 +1 alternating

$a_n = \frac{2(2)^{n-1}}{3+(n-1)} \underbrace{(-1)^{n+1}}_{\text{alternator}}$

For each of exercises 5–10, find the value to which the sequence converges or show that it diverges.

5. $a_n = \dfrac{n-2}{n+5}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n-2}{n+5} = 1$

$\therefore \{a_n\}$ converges to 1

6. $b_n = \dfrac{n^2 + 4n - 8}{3n^4 + 5}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n^2+4n-8}{3n^4+5} = 0$

$\therefore \{b_n\}$ converges to 0

7. $a_n = \dfrac{n^3 - 8n^2 + 12n - 5}{9n^2 + 5n - 12}$

$\lim_{n\to\infty} a_n = \infty$

because $\lim_{n\to\infty} a_n$ does not exist, $\{a_n\}$ diverges

8. $\{-2, 2, -2, 2, \ldots\}$

diverges (sequence keeps oscillating; will never settle to a final value.)

9. $\left\{\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \frac{81}{256}, \ldots\right\}$

$\{a_n\} = \{\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \frac{81}{256}, \ldots\} \Rightarrow a_n = \left(\frac{3}{4}\right)^{n-1}$ for $n \geq 1$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{3}{4}\right)^{n-1} = 0 \Rightarrow$ sequence converges to 0

10. $\{0, -3, -6, -9, \ldots\}$

diverges (terms will decrease to $-\infty$)

For each of exercises 11–16, determine whether or not the divergence test establishes divergence for the series.

→ if $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ diverges

11. $\displaystyle\sum_{n=2}^{\infty} \frac{2n^2}{5n^2 + 11n - 7}$

$\lim_{n\to\infty} \frac{2n^2}{5n^2+11n-7} = \frac{2}{5} \neq 0 \Rightarrow$ sum diverges

12. $\displaystyle\sum_{n=0}^{\infty} \cos(n)$

$\lim_{n\to\infty} \cos(n)$ does not exist $\Rightarrow$ sum diverges

13. $\displaystyle\sum_{n=1}^{\infty} e^n \cdot 3^{-n}$

$= \sum_{n=1}^{\infty} \left(\frac{e}{3}\right)^n$

$\lim_{n\to\infty} \left(\frac{e}{3}\right)^n = 0 \Rightarrow$ divergence test is inconclusive

14. $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

$= 2(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots)$

terms decrease to 0, so test is inconclusive.

15. $3 - 3 + 3 - 3 + 3 + \cdots$

diverges – terms do not approach 0.

16. $\displaystyle\sum_{n=10}^{\infty} \frac{n!}{n^{170870} - 2}$

$\lim_{n\to\infty} \frac{n!}{n^{170870}-2} = \infty$, so the sum diverges.

note: factorials grow faster than polynomials

17. Consider the family of sums $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$.

a. Find formulas for a_n and b_n such that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge and $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$ diverges.

an example: $a_n = n^2,\ b_n = n!$

$\sum_{n=1}^{\infty} n^2$ and $\sum_{n=1}^{\infty} n!$ both diverge, and $\sum_{n=1}^{\infty} n^2 + \sum_{n=1}^{\infty} n!$ also diverges.

b. Find formulas for a_n and b_n such that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both diverge and $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$ converges.

an example: $a_n = (-1)^n,\ b_n = -(-1)^n$

$\sum_{n=1}^{\infty} a_n = \underbrace{-1+1-1+1-1+\cdots}_{\text{divergent}}$, $\sum_{n=1}^{\infty} b_n = \underbrace{1-1+1-1+1+\cdots}_{\text{divergent}} \Rightarrow \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

$= (-1+1)+(1-1)+(-1+1)+\cdots$

$= \underbrace{0+0+0+\cdots}_{\text{converges}}$

18. Determine whether the following statement is true or false: If $\{a_n\}$ converges, then so does $\sum a_n$.

false — if $\lim_{n\to\infty} a_n = L$, where $L \neq 0$, then $\{a_n\}$ converges but

$\sum a_n$ diverges ($\lim_{n\to\infty} a_n = L \neq 0$) by the divergence test.

19. Determine whether the following statement is true or false: If $\{a_n\}$ diverges, then so does $\sum a_n$.

true — $\{a_n\}$ diverges if $\lim_{n\to\infty} a_n$ does not exist.

furthermore, $\sum a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$ (by the divergence test).

thus, both $\{a_n\}$ and $\sum a_n$ diverge.