For each of exercises 1–12, evaluate the integral or show that it diverges.

1. $\displaystyle\int_2^{\infty} \frac{1}{x+9}\,\mathrm{d}x$

2. $\displaystyle\int_1^{\infty} \frac{3}{2\sqrt{x}}\,\mathrm{d}x$

3. $\displaystyle\int_3^{\infty} \frac{1}{x\sqrt[4]{x}}\,\mathrm{d}x$

4. $\displaystyle\int_2^{5} \frac{1}{x-4}\,\mathrm{d}x$

5. $\displaystyle\int_{-\infty}^{\pi/2} \cos(x)\,\mathrm{d}x$

6. $\displaystyle\int_0^{\infty} \frac{1}{(x+3)^2}\,\mathrm{d}x$

7. $\displaystyle\int_{-2}^{4} \frac{1}{(x-1)^4}\,\mathrm{d}x$

8. $\displaystyle\int_0^{\pi/2} \sec(\theta)\tan(\theta)\,\mathrm{d}\theta$

9. $\displaystyle\int_{\pi/3}^{\infty} 2(\sin(\theta)+2)\cos(\theta)\,\mathrm{d}\theta$

10. $\displaystyle\int_4^{\infty} \frac{3}{x\ln(x)}\,\mathrm{d}x$

11. $\displaystyle\int_{-\pi/2}^{2} f(x)\,\mathrm{d}x,\ f(x) = \begin{cases} \sin(x) & \text{for } x \le 0 \\ x & \text{for } x > 0 \end{cases}$

12. $\displaystyle\int_{-2}^{3} g(x)\,\mathrm{d}x,\ g(x) = \begin{cases} x+5 & \text{for } -2 \le x \le 0 \\ \ln(x) & \text{for } 0 < x \le 4 \end{cases}$

For exercises 13–21, determine whether the following integrals are proper or improper. If an integral is improper, classify the type (i.e., Type I or Type II).

13. $\displaystyle\int_1^3 \frac{2}{x-3}\,\mathrm{d}x$

14. $\displaystyle\int_0^e \ln(x+1)\,\mathrm{d}x$

15. $\displaystyle\int_{-3}^{-2} \frac{1}{x^3+1}\,\mathrm{d}x$

16. $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x$

17. $\displaystyle\int_2^4 \frac{3}{x+1}\,\mathrm{d}x$

18. $\displaystyle\int_0^{1/2} \frac{-2}{x^9-1}\,\mathrm{d}x$

19. $\displaystyle\int_0^8 f(x)\,\mathrm{d}x, f(x) = \begin{cases} 3 & \text{for } 0 \le x \le 2 \\ -7 & \text{for } x > 2 \end{cases}$

20. $\displaystyle\int_{-1}^3 f(x)\,\mathrm{d}x, f(x) = \begin{cases} 2x & \text{for } x \le 0 \\ x^2 & \text{for } x > 0 \end{cases}$

21. $\displaystyle\int_{-\pi}^{\infty} f(x)\,\mathrm{d}x, f(x) = \begin{cases} \cos(x)+1 & \text{for } x \le 0 \\ x^2+1 & \text{for } x > 0 \end{cases}$

22. Find all the values of n for which $\displaystyle\int_2^{\infty} \frac{3}{x^{1-3n}}\,\mathrm{d}x$ converges.

23. Find all the values of n for which $\displaystyle\int_1^{\infty} \frac{1}{x^{5n-1}}\,\mathrm{d}x$ converges.

24. Find all the values of n for which $\int_1^\infty \frac{1}{x^{n^2-4n+4}}\,\mathrm{d}x$ diverges.

25. It is known that $0 < f(x) \le \frac{1}{x^{4/3}}$, where f is continuous for all x. Show that $\int_1^\infty f(x)\,\mathrm{d}x$ converges and is no greater than 3.

26. It is known that $0 < \frac{1}{x^{5/9}} \le g(x)$, where g is continuous for all x. Show that $\int_1^\infty g(x)\,\mathrm{d}x$ is divergent.

27. For a continuous function $g(x)$, it is known that $g(0) = 0$ and $g(4) = 12$. Evaluate $\int_0^4 \frac{g'(x)}{g(x)}\,\mathrm{d}x$ or show that it diverges.

28. It is known that $g(x) \le q(x)$, where g and q are continuous for all x. It is known that $\int_2^\infty q(x)\,\mathrm{d}x$ is finite. Explain why $\int_2^\infty g(x)\,\mathrm{d}x$ could be divergent.

29. Indicate whether the following statement is true: If $f(x)$ is continuous, positive, and decreasing for all $x \ge 1$, and $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f(x)$ converges.

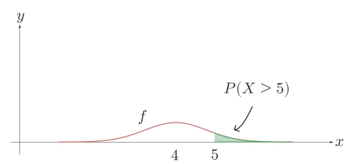
30. The rate at which people like an Instagram post is given by $i(t) = 200e^{-t/10}$, where t is time, measured in hours. What is the maximum number of likes that the Instagram post will receive?

31. The density of trees in a forest, T, in terms of the distance, r, surrounding a campsite is given by $T(r) = \dfrac{290}{r^3 + 8r^2 + 16r}$, where r is measured in meters and $T(r)$ is measured in trees per square meter. The number of trees within r meters of the campsite is given by $2\pi \int_0^r u\,T(u)\,\mathrm{d}u$. Explain why the total number of trees in the forest is less than 500.

32. A particle continuously moves along the x-axis at a velocity given by $v(t) = \dfrac{44}{t^{2/3}}$ for $t > 1$, where $v(t)$ is measured in meters per second and t is measured in seconds. Show that the particle will travel an infinite distance.

A graphing calculator is required for exercise 33.

33. In statistics, a Normal distribution has many applications—one of which is probability modeling. The integral of a probability density function (PDF) yields the probability of a random variable X falling between the bounds; that is, $\int_a^b f(x)\,\mathrm{d}x = P(a \leq X \leq b)$, where f is a PDF. Suppose that waiting times at the grocery store checkout lane are Normally distributed, with a mean of 4 minutes and a standard deviation of 0.8 minute. The PDF that models this scenario is given by $f(x) = \frac{1}{0.8\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-4}{0.8}\right)^2}$. What is the probability that a customer will wait in line for more than 5 minutes? (Hint: Large numbers, such as 9999, suffice for approximations of infinity.)



34. A freefalling object will accelerate until it reaches a terminal velocity—the maximum speed at which the object can fall. A drag force, which can be modeled by $F_D = kv$, where k is the object's drag coefficient and v is the object's velocity, acts on the object as it falls. The force due to gravity is given by $F_g = mg$, where m is the object's mass and g is the constant of gravitational acceleration, $9.8\mathrm{m}/\mathrm{s}^2$ on Earth. Newton's Second Law gives the equation $mg - kv = m\frac{\mathrm{d}v}{\mathrm{d}t}$. Performing separation of variables, an equation that can be used to solve for v_T, the terminal velocity of the object, is $\int_0^{v_T} \frac{1}{g - \frac{k}{m}v}\,\mathrm{d}v = \int_0^{\infty} \mathrm{d}t$. Show that $v_T = \frac{mg}{k}$. (Hint: m, k, and g are constants.)