For each of exercises 1–10, evaluate the infinite geometric series or show that it diverges.

1. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$

2. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

3. $\displaystyle\sum_{n=0}^{\infty}\frac{3}{5^n}$

4. $\displaystyle\sum_{n=0}^{\infty}\frac{5^n}{e^{n+1}}$

5. $3-\dfrac{3}{5}+\dfrac{3}{25}-\dfrac{3}{125}+\cdots$

6. $2\pi+2\pi^2+2\pi^3+2\pi^4+\cdots$

7. $\displaystyle\sum_{n=1}^{\infty}\frac{e^{n+3}}{\pi^n}$

8. $\displaystyle\sum_{n=0}^{\infty}\frac{2^{3n-1}}{7^n}$

9. $\displaystyle\sum_{n=0}^{\infty}\frac{5^{2n}}{e^{n+1}}$

10. $\dfrac{2}{3}-\dfrac{2e}{9}+\dfrac{2e^2}{27}-\dfrac{2e^3}{81}+\cdots$

For each of exercises 11–16, determine the values k for which the geometric series converges.

11. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2k}{3}\right)^n$

12. $\displaystyle\sum_{n=0}^{\infty}\frac{k^n}{3^{n+2}}$

13. $\displaystyle\sum_{n=1}^{\infty}\left(k^2-6k+10\right)^n$

14. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2}{k^2-8k+16}\right)^n$

15. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2k}{k-8}\right)^n$

16. $\displaystyle\sum_{n=10}^{\infty}\left(\frac{k}{2k+3}\right)^n$

For each of exercises 17–22, express the repeating decimal as an infinite geometric series.

17. $0.387\overline{2}$

18. $0.\overline{4}$

19. $5.3\overline{2}$

20. $6.23\overline{1}$

21. $2.8\overline{5}$

22. $4.6\overline{7}$

23. A ball is dropped and bounces on the floor. Each time the ball hits the floor, it rebounds to a height one-third of that from which it fell. A ball is dropped from a height of 100 feet and follows this model. What is the total vertical distance traveled by the ball?

24. In economics, individuals' marginal propensity to consume (MPC) is a number, from 0 to 1, that measures the proportion of their income, on average, that they spend. For example, a country with a MPC of 0.7 indicates that the population, on average, spends 70 percent of their income and saves 30 percent. The total economic activity produced by a purchase of 1000 dollars would be represented by the geometric series $1000 + 1000(0.7) + 1000(0.7)^2 + 1000(0.7)^3 + \cdots$, as money will change hands indefinitely under this model. Fiscal policy utilizes this number to close recessionary gaps. Suppose that the United States is in a recessionary gap of 400 billion dollars and its citizens have a MPC of 0.5. How much money must the government spend to create 400 billion dollars of economic activity, thus closing the recessionary gap?