Network Simulation

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I/ The basic model:

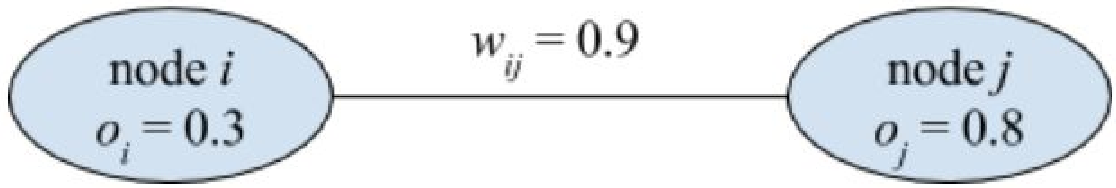
The basic model was introduced in section 7.2 of the class CS166: Modeling, Simulation and Decision Making, implemented by Prof. Scheffler at Minerva Schools at KGI. Here is the description of the model:

"Social dynamics are driven by two factors

- People prefer forming social relationships with others who share their preferences, opinions or interests.
- People's preferences, opinions or interests tend to become similar to those of other people in their social circle.

We model these two processes on a small-world network to see how both opinions and relationships change over time. Think of people's opinions on controversial topics like politics or religion. We represent each person as a node in a network with an opinion attribute associated with it. The opinion attribute can take on values between 0 and 1. People with similar values for their opinion are in agreement on the topic and people with very different values disagree strongly. Here are two nodes with differing opinion (difference = 0.5).

We represent the existence of a social relationship between two people as an edge between two nodes in the network and assign a weight to each edge, with values ranging from 0 to 1. A weight close to 0 means that the relationship between two people is weak and that those two people's opinions do not influence each other much. A weight close to 1 means that two people have a very strong relationship and will tend to adjust their opinions to be closer to each other. Below is a network with a strong relationship between two nodes.



Update rules:

The network dynamics have 3 parts.

1. People change their opinions to more closely match those of people they have a strong relationship with. We model this by selecting a random edge from the network — that is, two people who have an existing social relationship — and letting them interact. Think of this interaction as having a conversation about the topic on which everyone has an opinion. We update the opinion of each person to move a bit closer together. The stronger their relationship, the more they will move their opinions closer to each other.

The change in opinion of Person i when talking to Person j is

$$\Delta o_i = \alpha \, w_{ij} \, (o_j - o_i) \tag{1}$$

Person j 's opinion also changes, but in the opposite direction to Person i 's, thus bringing their opinions closer together. In the equation above $\alpha \in (0, 0.5]$ is a parameter of the model. The larger

α is, the faster people change their opinions to match other people's. The closer α is to 0, the more stubborn (less likely to change their opinions) people are.

2. People strengthen or weaken their relationships depending on whether they agree or disagree, respectively. During the same interaction as in Step 1, the weight of the edge connecting nodes i and j is also changed. The change in weight is

$$\Delta w_{ij} = \beta\, w_{ij}\,(1 - w_{ij})\,(1 - \gamma\,|o_i - o_j|\,) \qquad (2)$$

Here $\beta \in (0, 1)$ and $\gamma > 0$ are parameters of the model. If $\gamma \leq 1$ then all weights will converge to 1 over time since differing opinions don't matter enough to decrease edge weights. If $\gamma > 1$, the weight between two nodes will decrease if the opinions of the nodes are different enough — if |o | . i − oj > γ −1

As a final step when updating weights, we remove an edge from the network if its weight drops below 0.05. This models a social relationship that has broken down. 3. Finally we model new social connections between random people who are not yet connected. This is a relatively rare occurrence, so we do Steps 1 and 2 above 99% of the time and Step 3 only 1% of the time. Think of this process as randomly meeting someone new and forming a friendship with them. The edge weight is initially set to 0.5 in these cases. Over time the weight will increase towards 1 or decrease towards 0 depending on whether the two people have similar or differing opinions, using Steps 1 and 2. We use the parameter values $\alpha = 0.1$, $\beta = 0.3$, $\gamma = 3$ here, but feel free to experiment with different parameter settings."

In this project, I improved this basic model on Adaptive Networks by proposing my own changes to the model to make the dynamics more realistic, implemented my improvements in a simulation, and analyzed the expected (theoretical) and actual (experimental) effects of my changes on the model.

II/ Proposal of 2-3 modifications

I made 2 modifications to the basic model:

- Multiple topics: I modified my model to incorporate multiple topics that each node may have. Therefore, in initialization, each node's opinion would be represented as a list, each opinion is assigned randomly to be either 0 or 1. In each interaction, I first select a topic for the two nodes to have conversations about, and update the weight accordingly based on the difference in opinions about that specific topic.
- Constructive and destructive relationships: I modified the edge weight to take on the values ranging from -1 to 1 rather than 0 to 1. People have an edge of weight 1 have a positive relationship in the sense that their opinions will become more similar after every interaction. On the other hand, people have an edge of weight -1 have a negative relationship in the sense that their opinions will become more different after each interaction. People have an edge of weight 0 have no impact on each other opinions; therefore, I also modified the rule such that when the absolute value of an edge weight is less than 0.05 (close to 0), the edge would be removed.

III/ Local Analysis

Some review on the parameters of the basic model:

$\beta \in (0, 1)$

$\alpha \in (0, 0.5]$

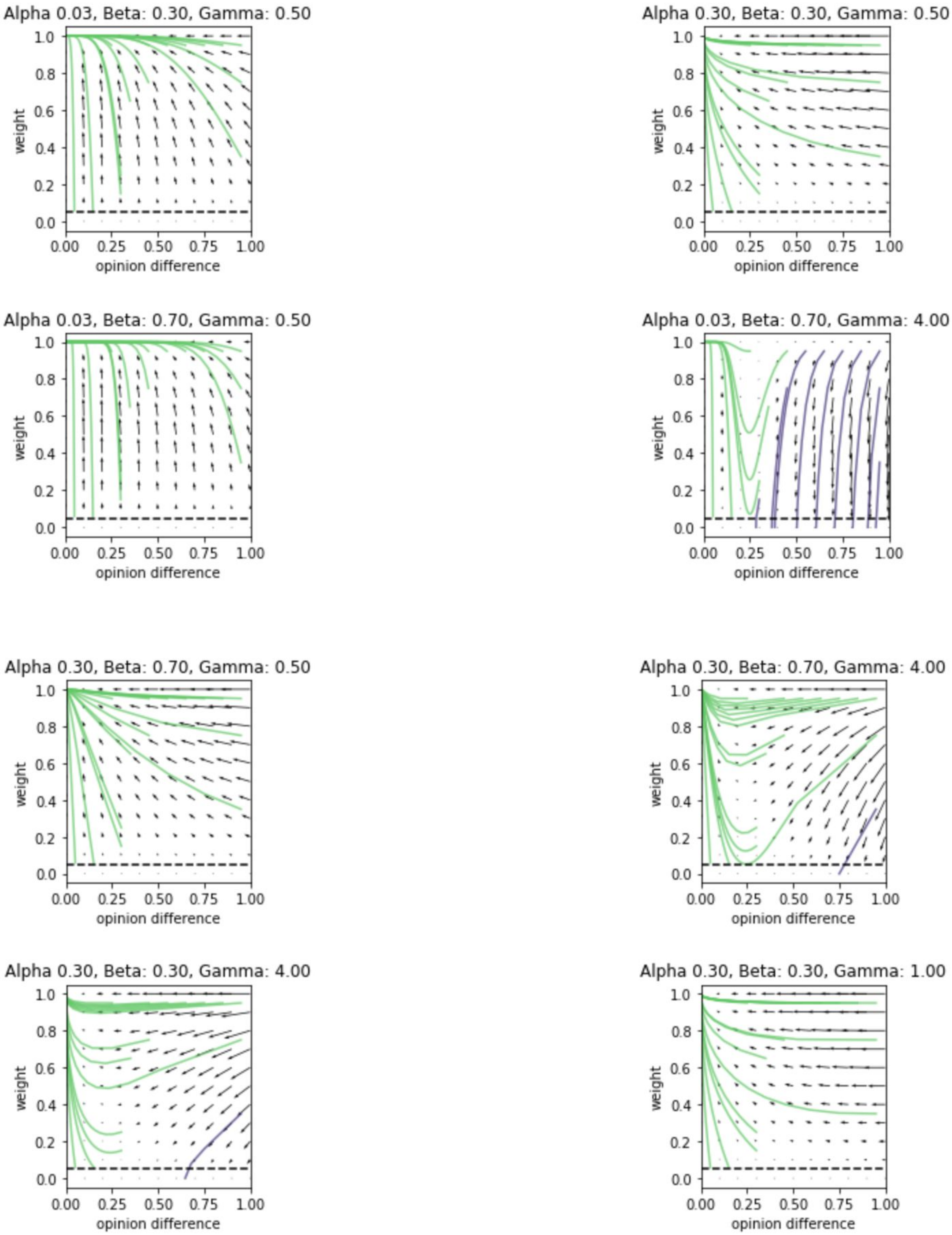
$\gamma > 0$

α represents how open a person in changing their opinions. The larger α is, the faster people change their opinions to match other people's. The closer α is to 0, the more stubborn (less likely to change their opinions) people are.

γ represents how easily the relationships between 2 nodes deteriorate due to a difference in their opinions. The higher γ is ($\gamma > 1$), the more likely one edge is removed if two nodes are different in opinion. The lower γ is ($\gamma < 1$), the less likely one edge is removed if two nodes are different in opinion.

Comparing the two cases: $\alpha = 0.3, \beta = 0.3, \gamma = 0.5$ and $\alpha = 0.03, \beta = 0.3, \gamma = 0.5$, we see that in both cases, nodes with very small difference in opinions tend to converge at (opinion: 0, weight: 1). With low value of α, no matter how strong the initial weight is, nodes with a very difference in opinion will eventually have their edge decreased in weight and finally removed. However, when we increase α from 0.03 to 0.3, while keeping β and γ the same, nodes with very different opinions but strong initial edge weight still eventually converge at (opinion: 0, weight: 1).

Comparing the two cases: $\alpha = 0.03, \beta = 0.7, \gamma = 0.5$ and $\alpha = 0.03, \beta = 0.7, \gamma = 0.4$ we see that with large value of gamma, edge weights would be decreased more quickly while with small value of gamma, edge weights would be increased more quickly and approach 1.


Alpha 0.03, Beta: 0.30, Gamma: 0.50
Alpha 0.30, Beta: 0.30, Gamma: 0.50
Alpha 0.03, Beta: 0.70, Gamma: 0.50
Alpha 0.03, Beta: 0.70, Gamma: 4.00
Alpha 0.30, Beta: 0.70, Gamma: 0.50
Alpha 0.30, Beta: 0.70, Gamma: 4.00
Alpha 0.30, Beta: 0.30, Gamma: 4.00
Alpha 0.30, Beta: 0.30, Gamma: 1.00
weight
opinion difference

Destructive vs Constructive Relationships:

We have the equation for updating the weight of an edge:

$$\Delta w_{ij} = \beta w_{ij}(1 - w_{ij})(1 - \gamma \left| o_i - o_j) \right|)$$

Now the model that incorporates both destructive and constructive relationships, there are two possible scenario: $w_{ij} > 0$ (constructive), $w_{ij} = 0$ and $w_{ij} < 0$ (destructive).

If $w_{ij} = 0$, we have $\Delta w_{ij} = 0$, which means difference in opinions does not affect the edge weight.

If $w_{ij} > 0$, we have $\beta w_{ij}(1 - w_{ij}) > 0$:

If $\gamma \leq 1$: the weight always increases as $1 - \gamma \left| o_i - o_j) \right| > 0$

If $\gamma > 1$:

If $\left| o_i - o_j) \right| > \gamma^{-1}$:

The weight decreases if the opinions of the nodes are different enough

If $\left| o_i - o_j) \right| < \gamma^{-1}$:

The weight increases if the opinions of the nodes are not different enough

If $w_{ij} < 0$, we have $\beta w_{ij}(1 - w_{ij}) < 0$:

If $\gamma \leq 1$: the weight always decreases as $1 - \gamma \left| o_i - o_j) \right| > 0$

If $\gamma > 1$:

If $\left| o_i - o_j \right| > \gamma^{-1}$:

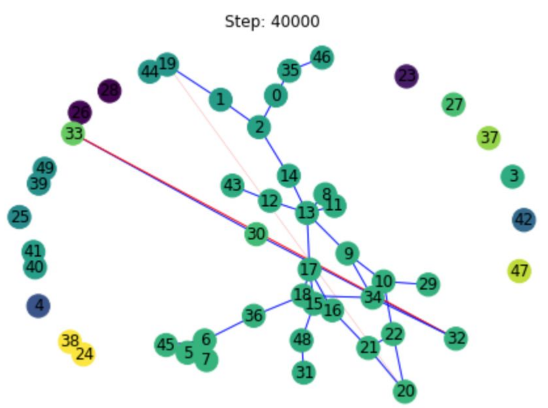
The weight increases if the opinions of the nodes are different enough. This may sound quite counter-intuitive, but it means that the weight tends to approach 0, the relationship becomes too destructive that it tends to be deleted between two nodes.

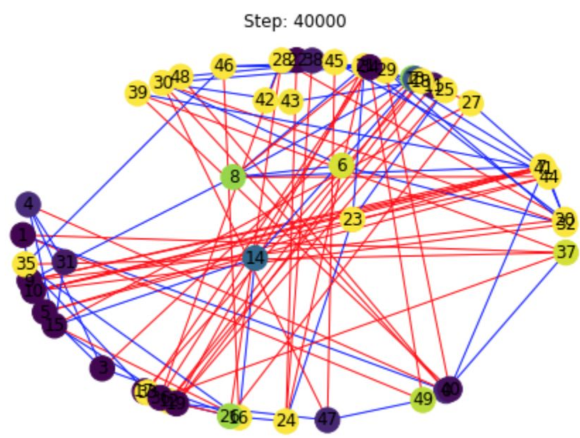
If $\left| o_i - o_j \right| < \gamma^{-1}$:

The weight decreases if the opinions of the nodes are different but not too much.

This means that the difference in opinions makes the relationship more destructive. When gamma is too large, it is always the case that $\left|o_i - o_j)\right| > \gamma^{-1}$; therefore, edges are removed very quickly. Also, opinions are homogenized, which means that people tend to have very similar opinions on a certain topic and their relationship statuses are positive.

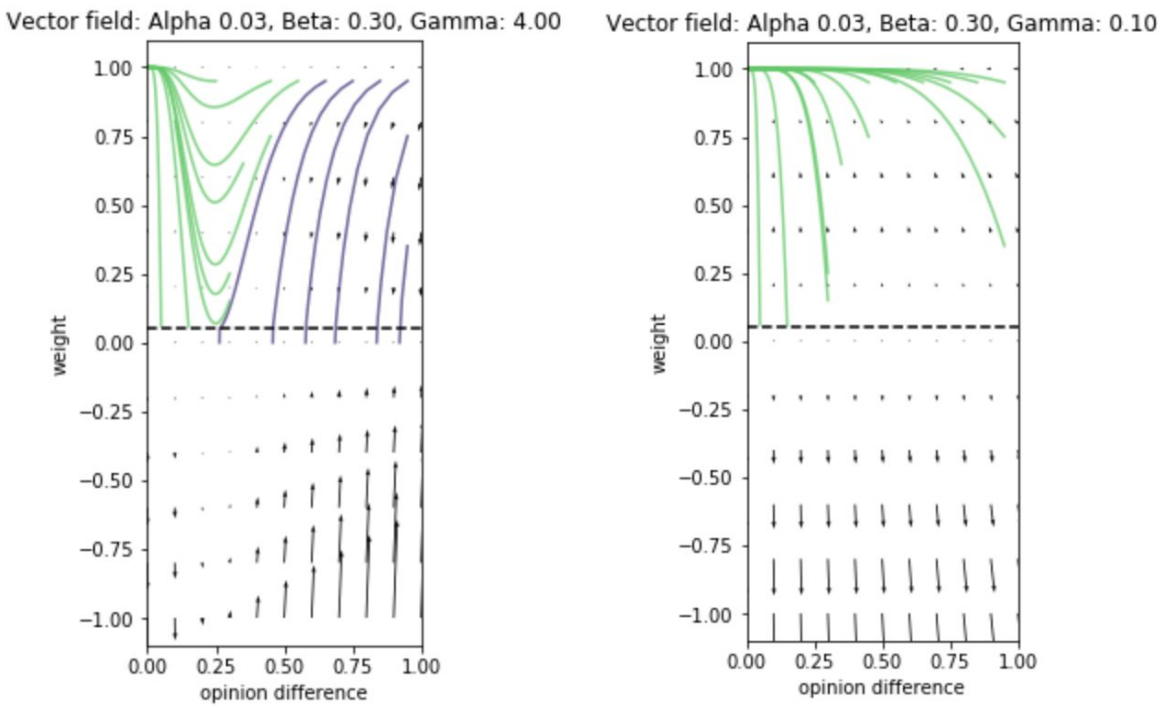
On the other hand, when gamma is smaller than 1, we observe that the colors could only get darker, which means constructive relationships tend to get better and destructive relationships tend to get worse (but not sufficient to the point that they are removed).



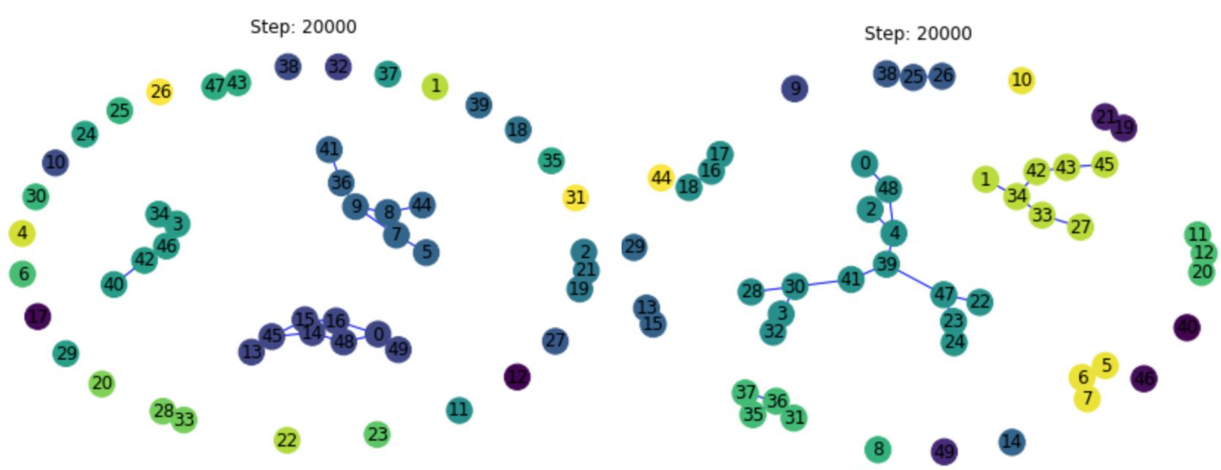


Gamma = 10

Gamma = 0.1



Therefore, if we want to observe cluster, we can keep a gamma somewhere in the middle between 1 and 10, for example 5. In that case, the edges are only removed for nodes with very different opinions and the intra-group relationships are more likely to be sustained than the inter-group relationships. As seen below, we could see some clusterings with $\gamma=4$ or $\gamma=5$



Gamma = 5 Gamma = 4

Multiple Topics:

With multiple topics, the edge weight is now affected by the difference in opinions in multiple topics. Even though we still keep the formula of updating edge weight the same, the relationship between two people would be more dynamic because the topic is chosen randomly at each interaction. We can observe this clearly in a larger scale, as discussed in the last part - Implementation Analysis.

IV/ Implementation

Watts-Strogatz graph vs Barabasi-Albert graph:

Watts-Strogatz graph is generated by randomly rewiring edges starting from a structured network (a ring). This network represents a very typical phenomenon that we call “small world phenomenon", but it doesn't capture the “heterogeneity of connectivity", which means that all nodes have almost the same degree (number of connections).

Barabasi-Albert graph, on the other hand, could capture this fact because it is a scale-free network that is created by preferential attachment. There are very few nodes with super high degree and a lot of nodes with low degree.

I incorporated both types of graph in my model and it turned out that the difference in behavior is not significant between the two types of graph.

Extreme vs Non-extreme Initialization:

Considering the situation where $w_{ij} > 0$:

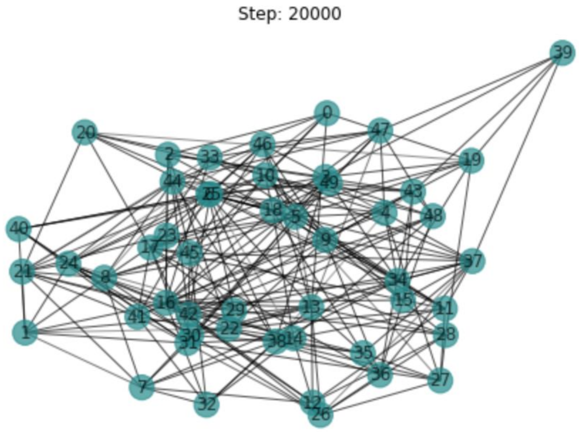
$|o_i - o_j)|$ is close to 0.

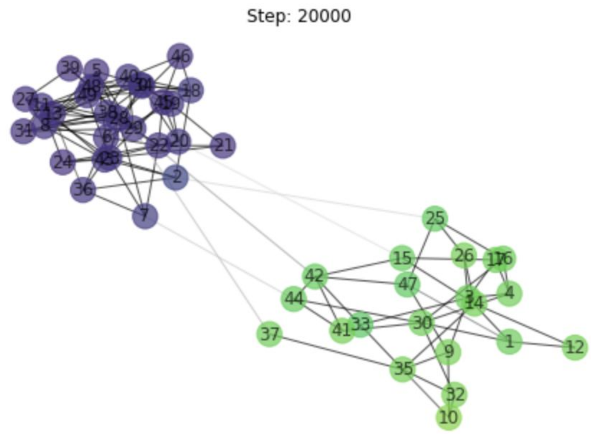
$\Delta o_i = \alpha w_{ij}(o_j - o_i)$

$\Delta o_j = \alpha w_{ij}(o_i - o_j)$

In the case of the extreme initialization, as illustrated in the provided example code, nodes can only take opinion of either 0 or 1. From the random initialization, nodes that are too different in their opinions (0 vs 1) tend to weaken their relationships and nodes that have almost same opinions (0 vs 0 or 1 vs 1) tend to strengthen their relationships. The stronger the relationships, the more quickly nodes can match their opinions with each other. This creates a positive feedback loop, which in turns causes the clustering of opinions in the network.

As a modification, instead of extreme initialization - each node can only take either 0 or 1, I sample the opinion from a beta distribution with parameters $\alpha = 0.5$ and $\beta = 0.5$. The distribution is bimodal in such a way that the high probabilities concentrate around 0 and 1, but there are still probabilities across the range. We see that now because nodes are not too different from each other from the beginning, and our γ remains the same, the relationships are more likely to be strengthened rather than weakened and removed. This leads to the fact that nodes are not really separated into clusters as the relationships between nodes are more resilient - the difference is not enough for them to be separated into groups.





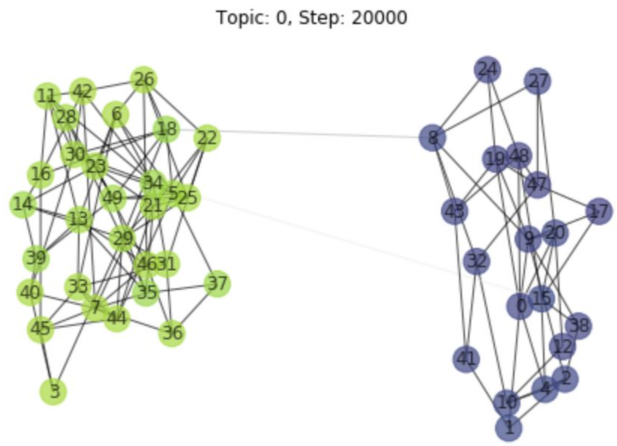
Non-extreme Initialization Extreme Initialization

V/ Simulation Analysis

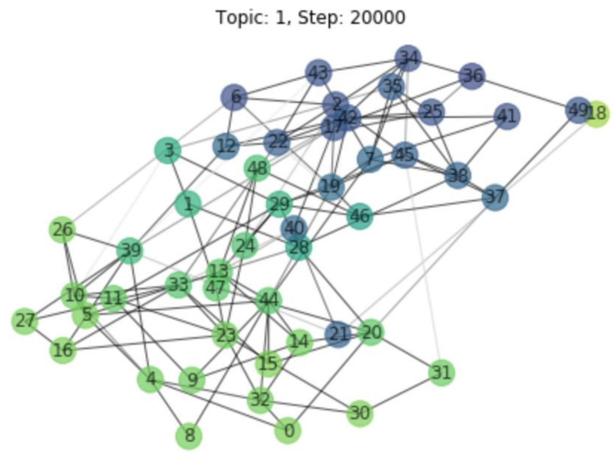
Constructive vs Destructive Relationship:

When we have destructive type of relationship in our model, we see that there are more single nodes that stand alone and not connected to any other node, and belong to any society. This makes sense because now the difference in two nodes can both gradually destroy a constructive edge and a destructive edge. There is a feedback loop going on here - the more destructive the relationship is, the more different their opinions are and that in turn makes the relationship more destructive. A node that is disconnected from other nodes will be isolated from all the clusters. This goes together with the fact the clusters are smaller in size because there are more independent nodes.

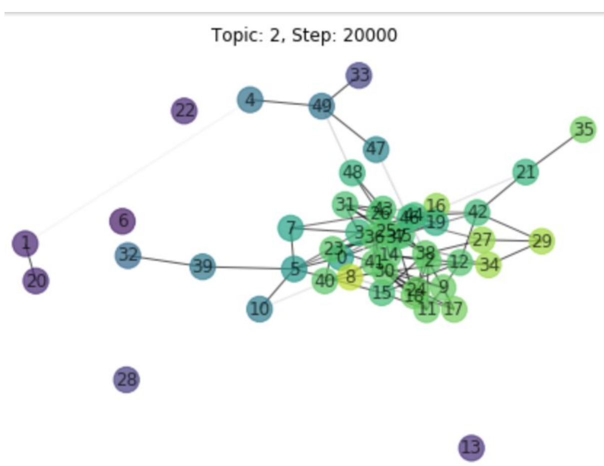
Multiple Topics:



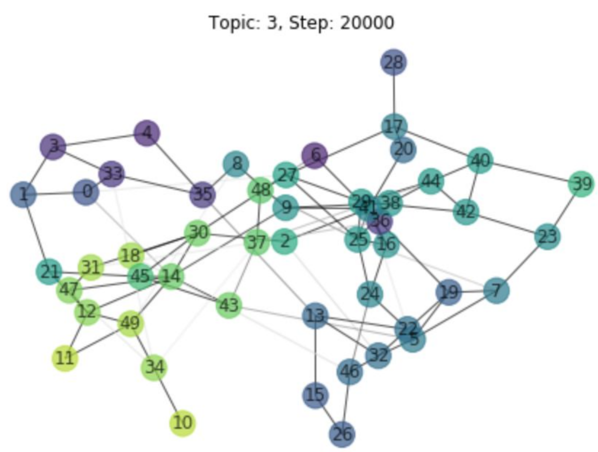
One Topic



Two Topics



Three Topics



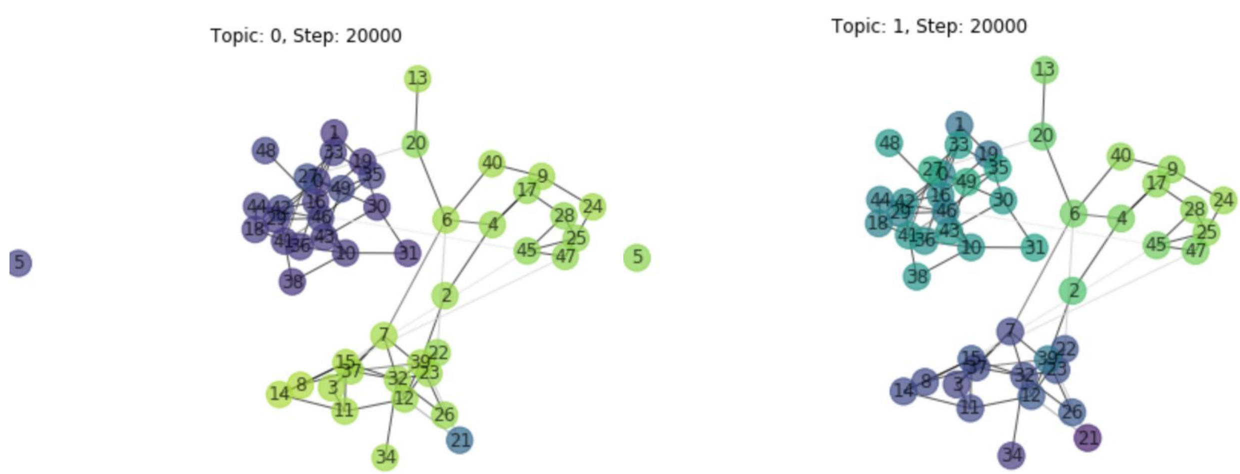
Four Topics

We observe that with one topic, we have two polars of opinions: 0 and 1, thereby leading to two big clusters in the network. The more topics we have, the less divergent the opinions become and the more but smaller-sized clusters would appear in our network. This is because the weight of an edge now depends on more topics and each of them could be similar or different. This makes sense in real life also as people form social groups also based on their similarity in opinions of certain topic.

Correlation in topics:

We also observe that even though in the beginning, we have topics that are not correlated to each other. Eventually, they become correlated in a sense that nodes belong to one cluster tend to have similar opinions in all of the topics, not just one. This is because the weight has a strong effect on all of the topics. Similarity in the opinions of one topic pulls the weight higher after one interaction, and in turn pulls the opinions on the same or other topic closer in the next interaction. It became a feedback loop and finally in-group nodes tend to converge in their opinions in all of the topics. This is a common thing that we observe in real life. Usually, people initially made friend with each other based on a few commonalities despite the differences in other topics, but then when they become closer, they affect each other in other ways as well and gradually converge in their opinions and way of thinking.

Two topics:



Three topics: