Traffic Simulation

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This report is about building cellular automata to model traffic flow. This report contains 4 parts. The first part is model of traffic jams on a circular road adapted from the paper "A cellular automaton model for freeway traffic by Nagel, K., Schreckenberg, M. and the second part is model of traffic jams on multi-lane highways illustrated in the paper "Two lane traffic simulations using cellular automata. Physica A: Statistical Mechanics and its applications" by Rickert, M., et al. The third part would be about the comparison between the two models. Finally, there would the last part dedicating to application and future work.

Part 1: Traffic jam on a circular road

A. Model description

This model has each state represented as an array containing multiple sites (default=100) with periodic boundary conditions. The value of each element of the array would correspond to the velocity of the car or equal to -1 if there is no car on that site at that particular time step. Assume the maximum velocity that a car could reach is 5. One main factor that decides how the car would change its velocity is its distance from the car ahead; this distance is calculated as the number of empty sites between two cars plus 1. The state of this model after one time step would be updated based on the following rules:

1/ Acceleration: if the velocity of a car is less than maximum allowed velocity and the distance from it to next car is larger than its current velocity[1] plus 1, then the car will advance its speed by 1.

2/ Deterministic deceleration: if the distance from a car to the next car is less than or equal to its current velocity, the car will decrease its speed to the value of the distance minus 1.

3/ Random deceleration: the velocity of each car will decrease by 1 with an assigned probability p

4/ Car motion: each car would be advanced by a number of sites that equals to its updated velocity

B. States visualization

[1] It may not sound formal to compare distance with velocity because they are not of the same unit, but in this context I meant to compare the absolute values, and not the units.

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.......3.................................0...............2..........................................
..........3..............................0..................3.......................................
..............4...........................1....................3....................................
..................4.........................2......................4................................
......................4........................3.......................4............................
...........................5......................3........................4........................
................................5.....................4........................4....................
.....................................5....................4.........................5...............
..........................................5....................5.........................5..........
...............................................5....................5.........................5.....
....................................................5....................5........................4.
...5.....................................................5...................4......................
........5.....................................................5..................4..................
.............5.....................................................5.................4..............
..................5....................................................4.................4..........
.......................5....................................................5.................5.....
...........................4.....................................................5.................5
....5..........................4.....................................................4..............
........4..........................4.....................................................4..........
.............5..........................5.....................................................5.....
.................4...........................5.....................................................5
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Figure 1 - Simulated traffic at a low density of 0.03 cars per site. Each new line shows the traffic lane after one further complete velocity-update and after the car motion. Empty sites are represented by a dot, sites which are occupied by a car are represented by the value of its velocity. We observe that cars flow completely freely at this low density.

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.....3.5....4........................3...........5........4...3..0....0..............3..............
......1...3.....4........................4............5......3..2.1....1.................4..........
.......1......4......5.......................4.............5..1.0...2...1.....................5.....
........1..........5......5.......................5..........2.10.....2...2........................5
....5.....2.............5......5.......................5......100.......2....3......................
........4....3...............5.....4........................5.00.1........2.....3...................
............4...3................4.....4.....................100..1.........2......3................
..............2.....4.................5.....5................000...1...........3.......4............
.................3.......5................4.....4............00.1....2.............4........5.......
.....................4........5................5.....5.......0.1.1......3..............4........4...
.5.......................4.........5...............4......5..0.0...2........4...............5.......
.....4.......................4.........4................5...2.10.....2...........5...............5..
.4........5.......................5........4..............2.0.00........3.............5.............
......5........5.......................5........5..........1.10.1..........3...............5........
...........5.......4........................5.......4......0.0.1.1.............4...............4....
5...............5......4.........................5.......5..10..1..2...............4................
.....5..............4......4.........................4.....20.1...2..2.................4............
.........4...............5.....4.........................4.00..1....2...3..................4........
.............4................5.....5....................0.00....2.....3....4..................4....
..................5...............4......5................100......2.......4....4..................4
...4..................4...............4......4............00.1........3........4....4...............
```

Figure 2 - Simulated traffic at a higher density of 0.1 cars per site. We can notice that at this density, traffic jam tends to form and there is a slightly backward motion of the traffic jam.

```
2..3....4..50.....3.4.1....2.1..2...3..2.0...50..3..4....1..34.3.0..12...0.1..........3......1..5..
..2....4..200.....0..1..2...1.1...2...2.10...0.1...2...3...20.1.10..0..2..1..2.............4....2..2
1....3...200.1.....1...2..2..1.1....2..10.1..0...2...2...2.0.10.0.1.0...1..1....3..............4..2.
.1.....2.00.1.1......2...2.1.0..1.....200..1..1.....3..2.0.0.0.1.10.0....1...2......4...........1..1
...2...0.00.0..1........3.10..1...2...00.1..1..1.....1.0.0.0.0..10.10......2....3.......4........1.0
1.....30.00..1...2......0.00....2...2.0.1..2..2.1.....1.10..10..0.10.1........3....3.........5....1.
.1....0.100...1....2.....10.1......3.1.1..2..20..1....0.0.1.0.1..100..1..........3.....4.........4.1
..1....100.1....2....2...00..1......1.1.1...20.1...2..0..10..1.1.00.1...2............4......5....0.0
....2..000..1......3...2.0.1...2.....1.1.1..00...2..1.0..00..0.0.0.1..2...2...............5....3..10
.....1.000...1.......2..10..1.....3...1.1..20.1....2.10..0.1..1.10...2..2...2................3...200
1.....1000.....2......1.0.1..1.......3.1.1.0.1.1...0.00..0...20.0.1...1...2....3................300.
.1....000.1......2.....10...2..2.....0..10..1.1..2.0.0.1..1..00..1.1...1....2......4............000.
...2..00.1..2......2...0.1...1....3...1.0.1.0...20..10..1..1.0.1.0..1....2.....3.......4........000.
....1.00..1...2......2.0...2..1......3.10..1.1..00..00...1.0..10..1...2....2......3........4....000.
....0.0.1...2....3...0..1...1..1......10.1..1.1.0.1.00....10..0.1...2...2.....3......3.........4000.
....0..1.1....2.....3.1..1....2..2....0.1..20..10.0.00....0.1..1..2....3...3......4.....3......000.1
.2..0...1.1.....2...0..1...2....2...3..1..20.1.0.10.00.....1..2.1...2....2....3........5....4..000..
..1..1..0..1.......30...1.....3...2...2..200.0.0.0.100......1.0...2....3...2.....3.........4..200.1.
....2..20....2.....00.....2......3..2...2000..10..1000.......10.....2.....3...3.....3.......1.000..1
.2...1.00.......3..00........3....1...2.0000..00..000.1......00.......2......3...3.....3.....1000...
...2.0.0.1.......1.00............4.1..0.000.1.00..00.1.1.....00.........2......2....3.....3..0000...
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Figure 3 - Simulated traffic at an even higher density of 0.3 cars per site. We notice that more groups of traffic jam are formed.

C. Analysis of traffic flow rate versus traffic density:

To calculate traffic flow rate, I chose the two sites as the two boundaries and counted the average number of cars that get out from one end and enter the other end at each time step. The car density is more obvious: it is the average number of cars per site across the whole lane and this is an input parameter of our model.

To generate the plot, I ran the experiment 10 times for each of 30 different densities ranging from 0 to 1. I plotted the mean result and also 95% confidence interval of traffic flow rate from all the 10 experiments.

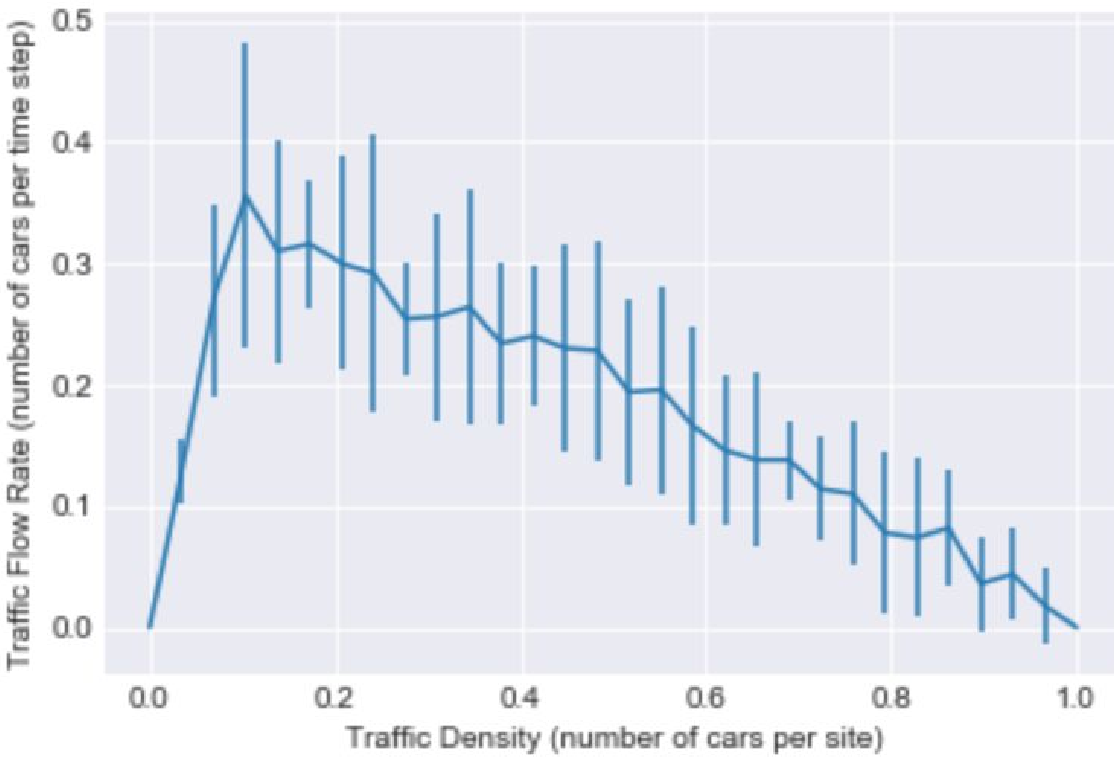

Figure 4: Traffic flow rate vs traffic density from simulation results. The line graph connecting all the mean values of traffic flow rates. The horizontal lines represent 95% confidence intervals.

We can see from figure 4 that the average flow rate increased dramatically when traffic density changed from 0 to around 0.1. It was because with low density there were only a few cars on the lane and the number of cars passing by the two ends at each time step of course would be small. Therefore, the low flow rates here doesn't mean that there are traffic jams, but it just means that there are not enough cars. After reaching its peak at around 0.35 when the density is 0.1, traffic flow rate starts to decrease gradually.

Part 2: Traffic jam on multi-lane highways

A. Model description:

First, let's look at the simplest multi-lane model - the two-lane one. We have each state represented as an array of two arrays, each represents one lane. There are three distances that we need to calculate before implementing the lane-switching rule:

“gap” is the number of empty sites between this car and the closest car in front of it on the same lane

“gap0” is the number of empty sites between this car and the closest car in front of it on the other lane "gapb" is the number of empty sites between this car and the closest car behind it on the other lane.

A car on one lane can switch to the same position on another lane if all of these conditions are fulfilled:

its current velocity plus one is larger than gap

its current velocity plus one is smaller than gap0

the maximum velocity is less than gapb

with a random chance of p, the car will switch

After a car decides to switch lane or not, it will advance its position. The rule for car motion in this model is very similar to that in the one-lane model, except now we use "gap" instead of "distance" and gap would be equal to distance minus 1.

1/ Acceleration: if a car is not yet at the maximum velocity, and the gap is larger than its current velocity, then it increases its velocity by 1

2/ Deterministic deceleration: if the gap is less than the car's current velocity, then it decreases its speed by 1

3/ Random deceleration: a car will slow down by 1 with an assigned probability

4/ Car motion: each car would be advanced by a number of sites that equals to its updated velocity

With this two-lane model, the traffic density would be determined for both lanes combined. We started with random initial conditions: if there are N cars randomly distributed on both lanes, we have density $N/2L$ with L the number of sites on each lane. For example, a density of 0.03 would mean that there are 6 cars in total and the cars would be distributed randomly on both lanes (which means there may be 4 cars on one, and 2 on the other).

B. States visualization:

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Initial State:
....1........................0......................................................................
..............5..........................2......................1..............4....................
Step 1
......2.......................1.....................................................................
...................5........................3....................1.................4................
Step 2
.........3.....................1....................................................................
........................5......................3..................1....................4............
Step 3
.............4..................1...................................................................
............................4.....................3................1.......................4........
Step 4
.................4...............1..................................................................
................................4.....................4..............2.........................4....
Step 5
......................5............2................................................................
....................................4.....................4............2...........................4
Step 6
..........................4...........3.............................................................
...4....................................4.....................4..........2..........................
Step 7
..............................4...........4.........................................................
........5....................................5.....................5........3.......................
Step 8
...................................5..........4.....................................................
............4.....................................5....................4........4...................
Step 9
........................................5.........4.................................................
................4......................................5....................5........5..............
Step 10
............................................4..........5............................................
.....................5.....................................4.....................5.......4..........
Step 11
................................................4..........4........................................
.........................4......................................5....................4.......4......
Step 12
....................................................4..........4....................................
.............................4.......................................5...................4........5.
Step 13
........................................................4...........5...............................
...5..............................5.......................................5..................4......
Step 14
............................................................4............5..........................
.......4...............................5......................................4...................5.
Step 15
................................................................4............4......................
..4........4...............................4......................................4.................
Step 16
.....................................................................5............5.................
......4........4...............................4......................................4.............
Step 17
..........................................................................5............5............
...........5.......4...............................4.......................................5........
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Figure 5 - Simulated traffic with two lanes at a low density of 0.03. Cars flow freely at this density. This situation is similar to what we saw with the density of 0.03 for one-lane model.

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Initial State:
................55......5..3...0.................1.................3........1.......1..............1
...........5..............0......5......................3..0....4..............3..0......0..........
Step 1
1...............0....4...1....30...................2.......3..........3......1........2.............
................5.........0...........5....................0........4...........1.0.......1.........
Step 2
.1..............0......2..1...0.1....................2.........4.........3.....2........2...........
.....................5.....1...............5................1...........4.......0..1.......1........
Step 3
...2............0.......1..1...1.1.....................2............5........4....3........3........
.........................4..1...................5............1..............4...0...1.......1.......
Step 4
......3..........1.......1...2.0..1.......................3..............5.......4....4........4....
..........................1..1......................4..........2...............3.1....2.......2.....
Step 5
5.........4........2.......2.0..1..1.........................3...............4.......4.....5........
............................2.1..........................5........3.............1..2.....3.......3..
Step 6
.....5.........5.....2......10....2..2...........................4...............4.......4......5...
3............................1..2............................4........4...........2...3.....3.......
Step 7
.5.......4.............2....0.1....1...2..............................5...............5......4......
....4..............4...........2...3.............................4.........5.........3...3......4...
Step 8
......5.......5...........3..1.1....1.....3...............................4................5.....4..
4........5..............5........2.....4..............................5.........5......2.....4......

Step 9
..5.......4.......4.........20..1.....2......3.................................5...............4....
.....5........5..............5.....2........5.............................4.........4.....3......4..
Step 10
5.....4.......4........5....0.1...2.....2........4..................................5...............
..5.......5.......4..............4....3.........4..............................5........4....3......
Step 11
....4......5.......5.......4.1.1.....3....2..........4..................................4...........
......4.......4........5.............4....4.........4..............................4........4....4..
Step 12
........4.......5......4....1.1..2.......4...3...........4..................................4.......
..5........5.......5........5............4.....5........4..............................4........4...
Step 13
............4........5......5...2..2.......2....3............4..................................4...
4......5........5.......5....1................5.....5.......4..............................4........
Step 14
4................5........5....3..2...3.......3.....4............4..................................
.....5......5........5......4..2..................4......5.......5.............................4....
Step 15
.....5................5.......4.1....3...3........4......5............5.............................
..........5......5........5...2..2.....................5......5......4.............................4
Step 16
.........4.................5..0..1......3...3..........5.....4............4.........................
...4...........5......5.....2...2...3.......................5.....4.......5.........................
Step 17
.............4..............1.0...1........3...3...........4.....4............4.....................
.......4...........4......4....3...3....4.......................4......5......4.....................
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Figure 6 - Simulated traffic with 2 lanes at a higher density of 0.1 cars. At this point, traffic jam started to form, but not much and only limited to one lane.

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Initial State:
.0.........3.4.5.....1..2.....0..1....1..44.....3.......2...1....02.2.5......32..5...0.44.21..3..4..
......45.4..04..33...02...3....0.0..3.......43114......25.5....1.0.........0...33.......3.....5.1...
Step 1
.0..........10......5..2..2...0....2....20....4.....4.....2..1...0.1.1....4..0.1....3.100.0..2.1...2
......0.1.1.0.1.0..2.0..2.....4.1.1.....4...0000.....5.0.1....4.10..........1..0....4.......4..1..2.
Step 2
.0.......1..0.1......1..1...2..1......3.00.........5.....5..2.1...1.1..2....20..1...0.00.1.1.0..1..0
2......1..0.0..10...1.1...2...0.0...2.....2.0000.....0..1..2..0.00............2.1........5....2..2..
Step 3
.0.........2.1..2.....1...2..1...2.....100.............4...2.1.1..0..1....3.0.1...2.0.00....1.1...20
..2......20..1.0.1..0...2....30.0......3..0.000.1.....1...2..2.100............0..1........1....1...2
Step 4
.0.........0...2...3....2...2..2...2...00.1...............30.0...2.1...2..0.0..1..0.0.0.1...0...2.00
.2...3...00...10...2.1....2..0.1.1.......20.000...2.....2..1..1000.............1...2.......1....1...
Step 5
10.........0......3...3...2..1....3..2.00..1..............00.0....1..2..1..1.1..1.0..10...2..1..0.0.
....3...30.1..00....1.1....1..1.1..2.....0.100.1.....3....2.1.000.1.............1....2......1....1..
Step 6
00.........0.........3..2...2.1.....2.10.1..1.............0.1.1....1...2..2...1....1.0.1...1...20.0.
......2.00..1.00....0...2....20..1....3..0.000..1........40..100.1.1.........2....2....2.....1....1.
Step 7
00.......1.0..........1....30..1....0.0.1..2..2............1.1..2....2..1....3..2..0.0..1...1..0.10.
2.......2.1.0.0.1....1.....3.00...1.....20.000....2......0.1.000.0..1..........2.....3....3...1.....
Step 8
00........1.1..........1...0.1..1....1.1.1..1...2...........1.1....3...2.1.....2..20.0....2..1.0.0.1
...3.....1.10..1.1.....2....100....1....00.00.1.....2.....1.100.10....2..........2......3....3..2...

Step 9
00.........1..2.........1...1.1...2...10...2.1....2.........0...2....2.0..1......200..1.....2.10..10
......3...100..0...2.....2..00.1....1...00.00..1.......3...100.100......2.........2........4.1...2.
Step 10
00..........1...2.........2..1..2....30.1...1.1.....2.......0......3.0..1...2....000....2....100..00
.3.......300.1..1.....3...1.0.1..2....2.0.100....2........3.10.000.......2..........2.......1..2...
Step 11
0.1...........2...2.........2.1....3.00...2.0...2......3.....1......1.1..1.....3....1......3.000..00
....3....00.1.1...2......2.0.0...2..2..0.0.00.1.....2.......1.0.00.1..........3...0.1...2.......2..2.
Step 12
.1.1.....0.......3...3.......1..2...10.1...10.....2........4..1.....0..1...2.......4..2.....1000..00
2.......4.0..1..2....3...1.1.1...1...2.1.100...2.....2......1.10.1.1...........2.0...2....3.....1...
Step 13
1.1.1.....1........2....3.....1....30.1..2.0.1.......3.......2.1....0....2....3......2...3..000.1.0.
...3....0..1...2..2.....3.10...2...2.0..1000......3.....3....10.10..1..........0.0......3.....4..1..
Step 14
0.0...2....1.........2.....3....2..0.1..2.10...2........3.....1.1....1......3....3......3..2000..10.
.......40...1....2...3..0.0.1.....3.10..0000..........4.....40.100....2.........10.........3....2.1.
Step 15
.10......3..1...........3.....3..1..1..20.0.1.....3........3...1..2...1........3....3....1.000.1.0.1
2......00....1.....2..1..10...2....10.1.0000...............50.100.1......3......0.1...........3..1..
Step 16
.00.......1..1.......2.....3....2..2..20.10...2......3........3..2...3.1...........4....4...0.1.1.10
..2....00.....1.........20.1.....3.0.10.000.1..............00.000...2.......3...0..1.....0..1..1..1.
Step 17
100.........2.1.........3......4.1...20.10.1....2.......3.......2...30..1.............3....30..10.0.
2...2..0.1.....1........00..1....0..10.100.1.1.............00.000......3......2.0...1....0...1...2..
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Figure 7: Simulated traffic with 2 lanes at very high density of 0.3. At this point, we see some traffic jams, but not as much compared to the situation at the same density for one lane (0.3 for one lane in the model in part 1). With one lane, we observed that traffic jams were really long and seemed not to be resolved quickly, but with two lanes, traffic jams do not really accumulate and have higher potential to be resolved.

C. Analysis of traffic flow rate versus traffic density:

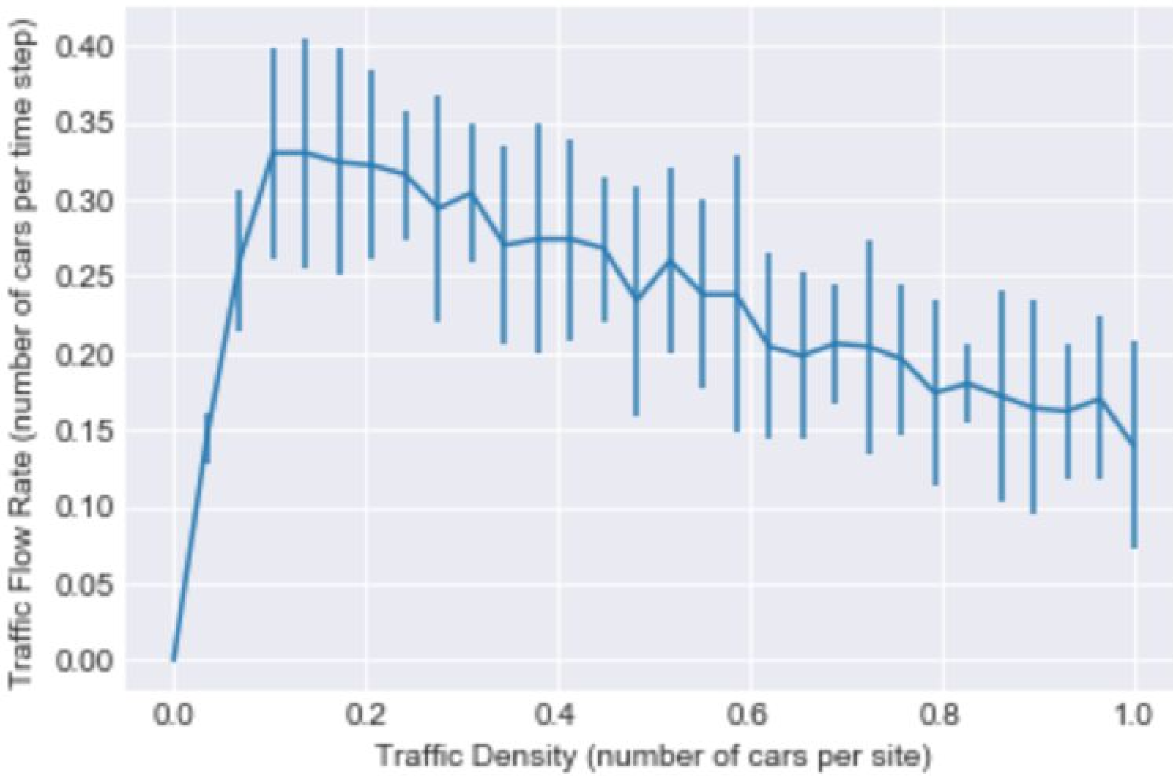
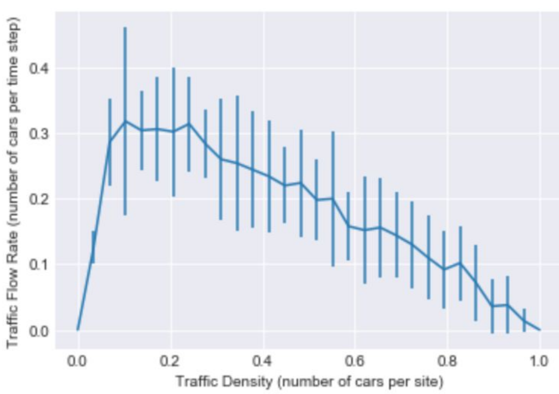


Figure 8: Traffic flow rate vs traffic density from simulation results. In this case the traffic density would be doubled for two lanes in the simulation itself, but still shown as the average density for each lane on this graph for the sake of comparison between two models. To calculate the average flow rate, we would sum up the number of cars passing by the boundaries for both lanes, and divided that number by two before dividing by the number of time steps. The line graph connecting all the mean values of traffic flow rates. The horizontal lines represent 95% confidence intervals.

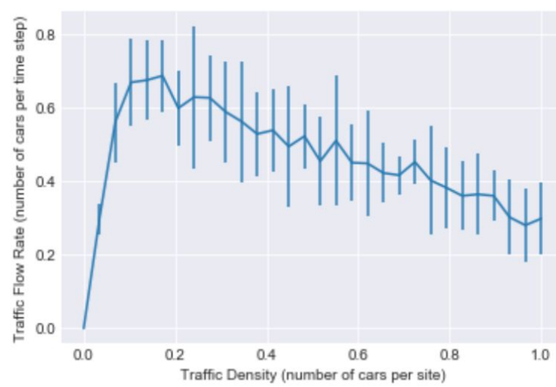
Part 3: Model comparison

In this part, we would analyze how much more traffic can flow through a multi-lane road, compare to a single lane road, at the same traffic density. For the sake of comprehensive comparison, I also implemented the three-lane road model. The rule of this model is very similar to the two-lane model, except the switching rule for car would need further considerations. Specifically, cars from the middle lane (lane 1) can switch to either of the other two lanes (lane 0 and lane 2). Therefore, if a car from the middle lane can potentially switch to both lanes (based on the original conditions), then it would switch to one of the two lanes with a random chance of 0.5. Similarly, cars from the two outside lanes can both switch to the middle lane, so each of them will potentially switch with a random chance of 0.5

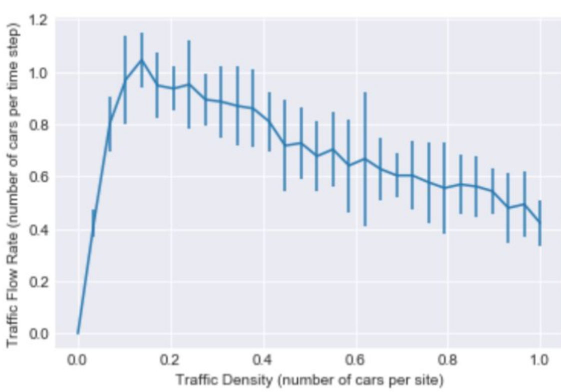
Intuitively, if we calculate the traffic flow using the accumulative traffic flow for both lanes in a multi-lane model, then the flow would be doubled for a two-lane road simply because it can facilitate double the number of cars. (as seen in Figure 9)



One-lane Model



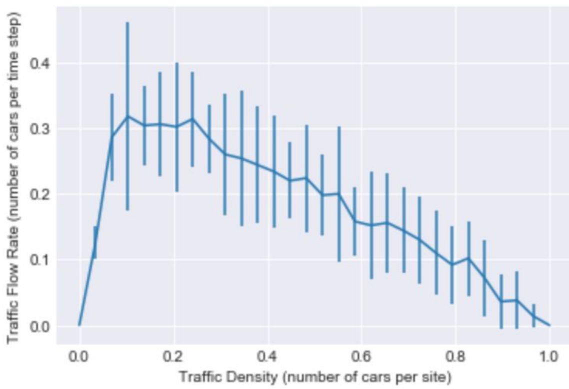
Two-lane Model



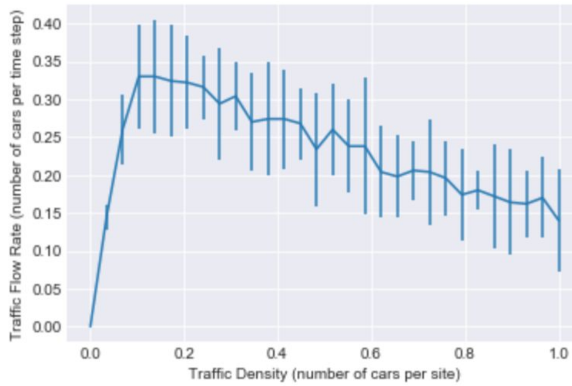
Three-lane Model

Figure 9: Comparing three graphs from three models without averaging the traffic flow for one lane

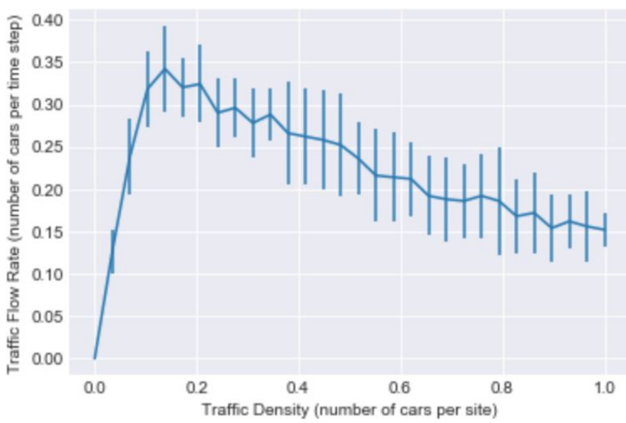
However, if we compare the traffic flow for each lane only, we can see more interesting results.



One-lane Model



Two-lane Model



Three-lane Model

Figure 10: Overall average traffic flow rate varies with traffic density for 1-lane, 2-lane and 3-lane roads

From figure 10, we can observe the similar trend of increasing traffic flow rate when traffic density increases from 0 to around 0.1. As discussed in Part 1, it's because there were very few cars on the road at the point. To answer the question "How much more traffic can flow through a 2-lane road compared to a 1-lane road at the same traffic density?", we can look at the later part of the graph when density increases from 0.1 to 1. We see that for the two-lane model, traffic flow rates decrease at a slower pace, and the minimum flow rate it could reach is 0.15, while for the one-lane model, traffic flow rates decrease very quickly and reach 0 when the whole road is occupied.

One more interesting point is that the graph for the three-lane road is not very different from the two-lane road. This may be due the assumptions I used for the switching rule. But from here we can say that even though there is a strong improvement from one-lane to two-lane regarding traffic flow rates, but with more than 2 lanes, the traffic flow would not improve much compared to the 2-lane model. This differences could be seen clearly from Figure 11.

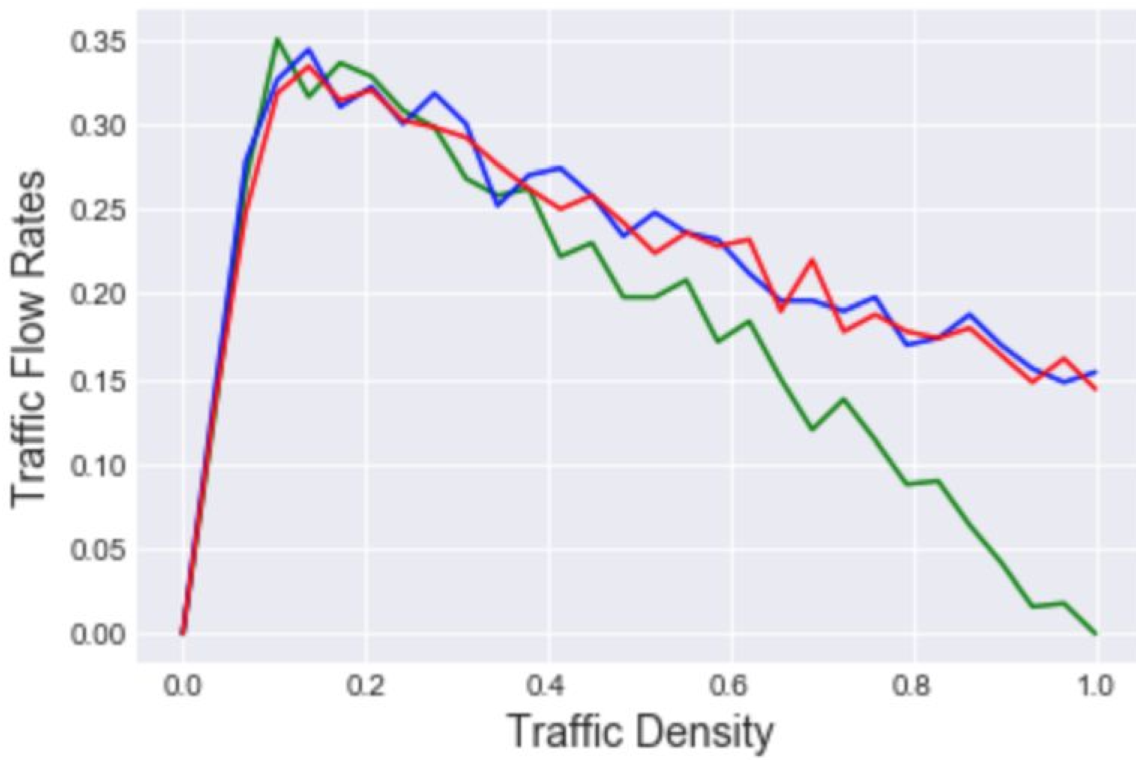


Figure 11: Overall average traffic flow rate varies with traffic density for 1-lane, 2-lane and 3-lane roads.
1-lane: Green
2-lane: Blue
3-lane: Red

Part 4: Application and Future Work

How applicable is this model to traffic in Hyderabad?

From a conversation with a local, I found out that on Friday evening, workers in the center of the city usually travel to their places (suburban or other cities), which causes a lot of traffic jams on the highway. This model can be applied to traffic in Hyderabad in a way that we can propose this policy to increase the number of lanes so that we can increase the traffic flow rates under the same traffic density. However, this model may not be that useful/ applicable for traffic inside the city overall because drivers here drive really randomly and most people violate the rules (going the wrong side,...). Therefore, we may need to incorporate more randomness into driver behavior to simulate better the traffic in Hyderabad.

Future Work :

In this report I just devised the simplest rules of traffic based on the two papers. There are multiple ways that I can further extend my models to represent more realistic traffic situations

1/ Finding other ways of measuring whether traffic is good or bad:

Example: Measure length and duration of jams, by checking for sequences of zeros

2/ Finding extensions to the traffic model to make it more realistic:

Example: Imposing starting and ending points (modeling intersections at which cars enter and exit), large vehicles may take up multiple cells and have a greater acceleration lag, random obstacles on the road, road blockages, accidents.

3/ Extensions to the driver behavior model to make it more realistic:

Example: random hard stops (velocity changes to 0) , acceleration/decelerations can be random, as opposed to by increments of 1, recklessly fast drivers and random police officers to stop them, good behavior (driving always in the middle),...

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