⑤ ⓐ $p =$ Probability of flipping a head

$$P(X=2) = p^2$$

$$P(X=3) = {}^{2}C_{1}(1-p)p^2 = 2(1-p)p^2$$

$$P(X=4) = {}^{3}C_{2}(1-p)^2 p^2 = 3(1-p)^2 p^2$$

$$\vdots \qquad \qquad \vdots$$

$$P(X=x) = {}^{x-1}C_{1}(1-p)^{x-2}p^2 = (x-1)(1-p)^{x-2}p^2$$

ⓑ

$$P(X \le x) = \sum_{x=2}^{x} (x-1)(1-p)^{x-2}p^2$$

$$= \frac{p^2}{(1-p)^2} \sum_{x=2}^{x} (x-1)(1-p)^x$$

$$= \frac{p^2}{(1-p)^2} \sum_{k=2}^{x} (k-1)(r)^k$$

$$= \frac{p^2}{(1-p)^2}\left[\frac{(1-p)^2 - x(1-(1-p))(1-p)^{x+1} - (1-p)^{x+2}}{(1-(1-p))^2}\right]$$

$$\cdot x \left[(1-p)^2 - xp(1-p)^{x+1} - (1-p)^{x+2} \right.$$

$$= \frac{\cancel{p}}{(1-p)^2}\left[\frac{(1-p)\ \cdots}{\cancel{p^2}}\right]$$

$$= \frac{(1-p)^2}{(1-p)^2}\left[1 - xp(1-p)^{x-1} - (1-p)^x\right]$$

$$= 1 - xp(1-p)^{x-1} - (1-p)^x$$

$$= 1 - \left[(1-p)^x\left(\frac{xp}{1-p} + 1\right)\right]$$

$$\Rightarrow P(X \le x) = 1 - \left[(1-p)^x\left(\frac{xp}{1-p} + 1\right)\right]$$

(c) $X_1 \rightarrow$ Attempt when first child flips head.

$X_2 \rightarrow$ Attempt when second child flips head.

$$f(x_1) = (1-p)^{x_1-1}p$$

$$f(x_2) = (1-p)^{x_2-1}p$$

$f(x_2) = (1-p \ldots$

$E[x_1] = \frac{1}{p}$ & $E[x_2] = \frac{1}{p}$

The expectation of both children flipping heads $= E[x_1] + E[x_2]$

$= \frac{1}{p} + \frac{1}{p} = \frac{2}{p}$

Same way, $Var[x] = Var[x_1] + Var[x_2]$

$= \frac{1-p}{p^2} + \frac{1-p}{p^2}$

$= \frac{2(1-p)}{p^2}$