(2) $Q = P(X=x_1) \cdot P(X=x_2) \cdots P(X=x_n)$

$$= \alpha x_1 e^{-\alpha x_1^2/2} \cdot \alpha x_2 e^{-\alpha x_2^2/2} \cdots \alpha x_n e^{-\alpha x_n^2/2}$$

$$= \alpha^n x_p \cdot e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} = Q(\alpha)$$

$$Q'(\alpha) = x_p \left[n\alpha^{n-1} e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} + \left(\alpha^n e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} \times -\frac{\sum_{i=1}^{n} x_i^2}{2} \right) \right]$$

$$= 0$$

$$x_p = \prod_{i=1}^{n} x_i$$

$$\Rightarrow n\alpha^{n-1} e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} = \frac{1}{2} \times \alpha^n e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} \times \sum_{i=1}^{n} x_i^2$$

$$\Rightarrow n\alpha^{-1} = \frac{1}{2} \sum_{i=1}^{n} x_i^2$$

$$\Rightarrow \alpha_{ML} = \frac{2n}{\sum_{i=1}^{n} x_i^2}$$

(b) $\mu = \int_0^\infty x \cdot f(x) \cdot dx$

$$\int^\infty \alpha x e^{-\alpha x^2/2} \cdot dx$$

$$= \int_0 x \cdot \alpha x$$

$$= \int_0^\infty \alpha x^2 e^{-\alpha x^2/2} \cdot dx$$

$x = 0 \Rightarrow u = 0$

$x = \infty \Rightarrow u = \infty$

$\sqrt{\alpha}\, x = u$

$\sqrt{\alpha}\, dx = du$

$$\Rightarrow \int_0^\infty \alpha x^2 e^{-\alpha x^2/2} \cdot dx = \int_0^\infty u^2 e^{-u^2/2} \cdot \frac{1}{\sqrt{\alpha}} \cdot du$$

$$= \frac{1}{\sqrt{\alpha}} \int_0^\infty u^2 e^{-u^2/2} \cdot du$$

$$\mu = \frac{1}{\sqrt{\alpha}} \times \sqrt{\frac{\pi}{2}}$$

$$\Rightarrow \mu = \sqrt{\frac{\pi}{2\alpha}}$$

$$\Rightarrow \mu^2 = \frac{\pi}{2\alpha}$$

$$\Rightarrow \alpha = \frac{\pi}{2\mu^2}$$

2,

$$\Rightarrow \alpha_{SM} = \frac{\pi}{2\overline{X}^2}, \quad \overline{X} = \text{Sample Mean}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$= \frac{\pi}{2\times\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2}$$

$$= \frac{n^2\pi}{2\times\left(\sum_{i=1}^{n} x_i\right)^2}$$