ESE 5030 - Simulation Modeling & Analysis (Homework #1)

Spring Semester, 2024 M. Carchidi

Problem #1 (15 Points) - Some Triangle Problems

Suppose that three points (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) are chosen so that X_k and Y_k are standard uniform random variables for $k = 1, 2, 3$ and consider the triangle formed by using the three points (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) as vertices.

a.) (**3 points**) Use 5000 Monte-Carlo Simulations to estimate the average perimeter of this triangle.

b.) (**3 points**) Use 5000 Monte-Carlo Simulations to estimate the average area of this triangle.

c.) (**9 points**) Use 5000 Monte-Carlo Simulations to estimate the probability that this triangle is obtuse.

Problem #2 (15 Points) - Real Roots For A Quadratic

a.) (**10 points**) Suppose that A, B and C are three standard uniform random variables and consider the quadratic equation

$$Ax^2 + Bx + C = 0.$$

Use 5000 Monte-Carlo Simulations to estimate probability that both roots of this equation are real.

b.) (**5 points**) Use at least 5000 Monte-Carlo Simulations to estimate these two roots, given that they are both real.

Problem #3 (15 points) - A Needle and Circle Problem

Suppose that the ends of a line segment are given by (X_1, Y_1) and (X_2, Y_2) where X_1, Y_1, X_2 and Y_2 are standard uniform random variables and suppose that a circle having center (X_C, Y_C) and radius R with X_C, Y_C and R all standard uniform random variables so that the entire line segment lies in the unit square $0 \le x \le 1$, $0 \le y \le 1$ and the center of the circle lies in the same unit square. Use 5000 Monte-Carlo Simulations to estimate the probability that the line intersects the circle. *Hint*: You may use the fact that the line-segment having points (X_1, Y_1) and (X_2, Y_2) as endpoints is given by

$$t(X_1, Y_1) + (1-t)(X_2, Y_2) = (tX_1 + (1-t)X_2, tY_1 + (1-t)Y_2)$$

for $0 \le t \le 1$.

Problem #4 (15 points) - Loan Size Distributions

A college student is making financial plans for next year. Expenses include the following known costs: Tuition at $8,400 and Dormitory at $5,400. There are also the following uncertain expenses: (a) Meals, ranging between $900 and $1,350, (b) Entertainment, ranging between $600 and $1,200, (c) Transportation, ranging between $200 and $600, and (d) Books, ranging between $400 and $800. The student foresees income to pay for next year. The only certain amounts are a State Scholarship at $3,000 and Parental Contributions at $4,000. Other income, which is variable, include: (a) Waiting Tables, ranging between $3,000 and $5,000, and (b) Library Aide, ranging between $2,000 and $3,000. Treat any amounts that are variable as uniformly distributed over the range indicated. The student predicts that a loan will be needed. Use simulation (1000 trials) to forecast the probability distribution for the size of the loan, from $1,500 to $5,500 in steps of $500, so that you want to fill in the following table.

Loan Amount	Probability	Loan Amount	Probability
[$0, $1500)		[$3500, $4000)	
[$1500, $2000)		[$4000, $4500)	
[$2000, $2500)		[$4500, $5000)	
[$2500, $3000)		[$5000, $5500)	
[$3000, $3500)		[$5500, ∞)	

Problem #5 (10 points) - An Arrival Problem

A bus arrives every 20 minutes at a specified stop beginning at 6:40 AM and continuing until 7:40 AM. A certain passenger does not know the schedule, but arrives randomly (uniformly distributed) between 6:50 AM and 7:30 AM every morning.

a.) **(5 points)** Using Excel, or any other program, run a 1000-day simulation and determine from this simulation the probability that the passenger will have to wait more than 5 minutes for a bus.

b.) **(5 points)** Use your simulation in part (a) to also determine the average time that the passenger will wait for a bus.

Problem #6 (15 points) - Two Intersecting Line Segments

Suppose that the four endpoints of two line-segments are standard uniform random variables given by (X_{11}, Y_{11}), (X_{12}, Y_{12}) for the first line-segment and given by (X_{21}, Y_{21}), (X_{22}, Y_{22}) for the second line-segment. Use 5000 Monte-Carlo Simulations to estimate the probability that the two line-segments intersect.

Problem #7 (15 points) - Two Intersecting Disks

Suppose that the centers (X_1, Y_1) and (X_2, Y_2) of two disks are chosen so that X_1, Y_1, X_2 and Y_2 are standard uniform and suppose that the radius (R_1) of the first disk is uniformly distributed in a way that makes sure this entire disk lies in the unit square, and suppose that the radius (R_2) of the second disk is uniformly distributed in a way that makes sure this entire disk lies in the unit square. Note that this says that

$$R_1 \sim U[0, \min(X_1, 1 - X_1, Y_1, 1 - Y_1)$$

and

$$R_2 \sim U[0, \min(X_2, 1 - X_2, Y_2, 1 - Y_2).$$

Use 5000 Monte-Carlo Simulations to estimate the probability that the two disks intersect.