(6) $X \sim U[a,b)$; $Y \sim U[a,b)$

$Z = XY$

$$f_Z(z) = \int_p^q \frac{1}{x} f_{X,Y}(x, z/x) \cdot dx$$

$$= \int_p^q \frac{1}{x} \times \frac{1}{(b-a)^2} \cdot dx$$

$$= \frac{1}{(b-a)^2} \ln\left(\frac{\min(b, z/a)}{\max(a, z/b)}\right)$$

$a < x < b$; $a < \frac{z}{x} < b$

$\Rightarrow \frac{1}{a} < \frac{x}{z} < \frac{1}{b}$

$\Rightarrow \frac{z}{a} < x < \frac{z}{b}$

$\therefore x < \min(b, z/a)$

& $x > \max(a, z/b)$

That is how we get the bounds of the integrals.

min $(b, z/a)$ is a when $z < ab$

min $(b, z/a)$ is b when $z > ab$.

Therefore,

$$f_Z(z) = \begin{cases} \ln\left(z/a^2\right)/(b-a)^2, & a^2 \le z \le ab \\ \ln\left(b^2/z\right)/(b-a)^2, & ab \le z \le b^2 \end{cases}$$

The cumulative density function is found as follows,

$$F_Z(z) = \int_{a^2}^{z} \frac{\ln(t/a^2)}{(b-a)^2} \cdot dt$$

$$= \frac{z \ln\left(b^2/z\right) - z + a^2 - 2ab}{(b-a)2}$$

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$$F_Z(z) = \begin{cases} \dfrac{z \ln\left(z/a^2\right) - z + a^2}{(b-a)2}, & a^2 \le z \le ab \\ \dfrac{z \ln\left(b^2/z\right) + z + a^2 - 2ab}{(b-a)2}, & ab \le z \le b^2 \end{cases}$$

We know that $E(z) = E(XY) = E(X) \times E(Y)$

Question mentioned

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