

$$\textcircled{2} \quad f(x) = e^{-4x} + 3xe^{-2x^2}$$

$$F(x) = \int_{-\infty}^x f(x) \cdot dx = \int_0^x e^{-4x} + 3xe^{-2x^2} \cdot dx$$

$$= -\frac{3}{4} e^{-2x^2} - \frac{e^{-4x}}{4} + 1$$

$$F(x) = R \Rightarrow g(x_n) = F(x_n) - R \quad \& \quad g'(x_n) = f(x_n)$$

$$X_{n+1} = X_n - \frac{g(x_n)}{g'(x_n)}$$

$$= X_n - \frac{(F(x_n) - R)}{f(x_n)}$$

$$= X_n - \frac{\left(-\frac{3}{4} e^{-2x_n^2} - \frac{e^{-4x_n}}{4} + 1 - R \right)}{e^{-4x_n} + 3x_n e^{-2x_n^2}}$$

$$E(X) = \int_0^{\infty} x \cdot f(x) \cdot dx$$

$$= \int_0^{\infty} (e^{-4x} + 3xe^{-2x^2}) \cdot x \cdot dx$$

$$= 0.532493$$

$$E(X^2) = \int_0^{\infty} x^2 \cdot f(x) \cdot dx$$

$$= 0.40625$$

$$\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ &= 0.40625 - (0.532493)^2 \\ &= 0.40625 - 0.28355 \\ &= 0.1227 \end{aligned}$$

$$\sigma(x) = 0.3502$$

The results obtained here closely matches with the simulation results.

I got to know that we may have to use the acceptance - rejection technique and trying that as well in the last minute.

$$f(x) = e^{-4x} + 3x e^{-2x^2}$$

$$g(x) = 1$$

$$f'(x) = -4e^{-4x} + 3 \left(x(-4x) e^{-2x^2} + e^{-2x^2} \cdot 1 \right)$$

$$\frac{d}{dx} \left(\frac{1}{g(x)} \right) = -4e^{-4x} + 3(-4x^2 e^{-2x^2} + e^{-2x}) = 0$$

$$\Rightarrow x = 0.082, 0.395$$

$$f(0.395) = 0.0015139$$

$$f(0.082) = -0.0011361$$

$$c = f(0.395) = 0.0015139$$