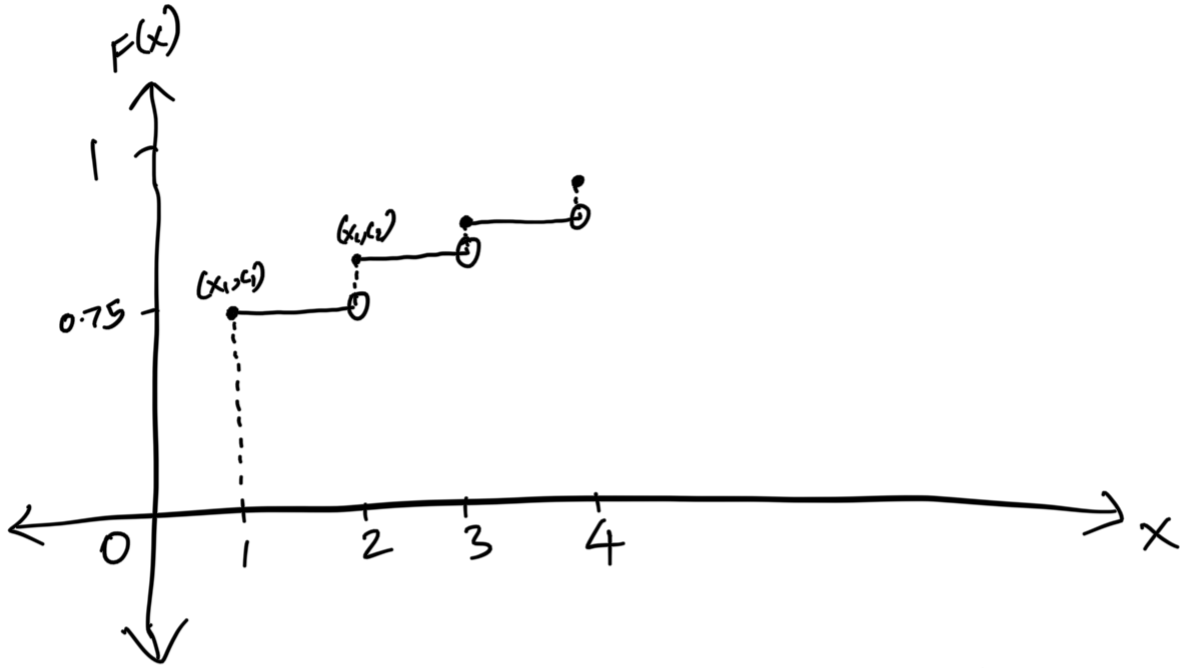
③ $x = 1, 2, 3, \cdots$

$$p(x) = \frac{2x+1}{x^2(x+1)^2}$$

$$F(x) = \sum_{x=1}^{x} \frac{2x+1}{x^2(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

$$F(x) = \begin{cases} 0, & 0 \le x < 1 \\ 3/4, & 1 \le x < 2 \\ 8/9, & 2 \le x < 3 \\ \vdots & \vdots \end{cases}$$

$F(x) = R \Rightarrow X = F^{-1}(R)$



$\cdots \quad 0 < c_k \Rightarrow X = x_k$

c_{k-1} < R <

$X = \{a+b,\ a+2b,\ a+3b, \cdots\}$

In our case, $a=0$ and $b=1$.

$X = \{0+1,\ 0+2,\ 0+3, \cdots\} = \{1, 2, 3, \cdots\}$

$X_1 \Rightarrow k=1$ & $X_2 \Rightarrow k=2$

$$X_{k-1} < F^{-1}(R) < X_k$$

$$a+(k-1)b < F^{-1}(R) < a+kb$$

$$k-1 < \frac{F^{-1}(R)-a}{b} < k$$

$$k = \left\lceil \frac{F^{-1}(R)-a}{b} \right\rceil$$

$$X = a + \left\lceil \frac{F^{-1}(R)-a}{b} \right\rceil b$$

$$= \left\lceil F^{-1}(R) \right\rceil$$

$$F(x) = \frac{x(x+2)}{(x+1)^2} = R$$

$$E(x) = \sum_{x=1}^{\infty} x \cdot f(x)$$

$$= \sum_{x=1}^{\infty} x \cdot \frac{2x+1}{x^2(x+1)^2}$$

$$= \sum_{x=1}^{\infty} \frac{2x+1}{x(x+1)^2} = 1.6449$$

$$F(x) = \frac{x(x+2)}{(x+1)^2} = R$$

$$\Rightarrow \frac{x^2+2x+1-1}{(x+1)^2} = R$$

$$\Rightarrow \frac{(x+1)^2 - 1}{(x+1)^2} = R$$

$$\Rightarrow 1 - \frac{1}{(x+1)^2} = R$$

$$\Rightarrow \frac{1}{(x+1)^2} = 1-R$$

$$\Rightarrow x+1 = \sqrt{\frac{1}{1-R}}$$

$$\Rightarrow x = -1 + \sqrt{\frac{1}{1-R}}$$

$\Rightarrow x$ $\sqrt{1-R}$

$$F^{-1}(x) = -1 + \frac{1}{\sqrt{1-R}}$$

$$X = \lceil F^{-1}(x) \rceil = \left\lceil -1 + \frac{1}{\sqrt{1-R}} \right\rceil$$