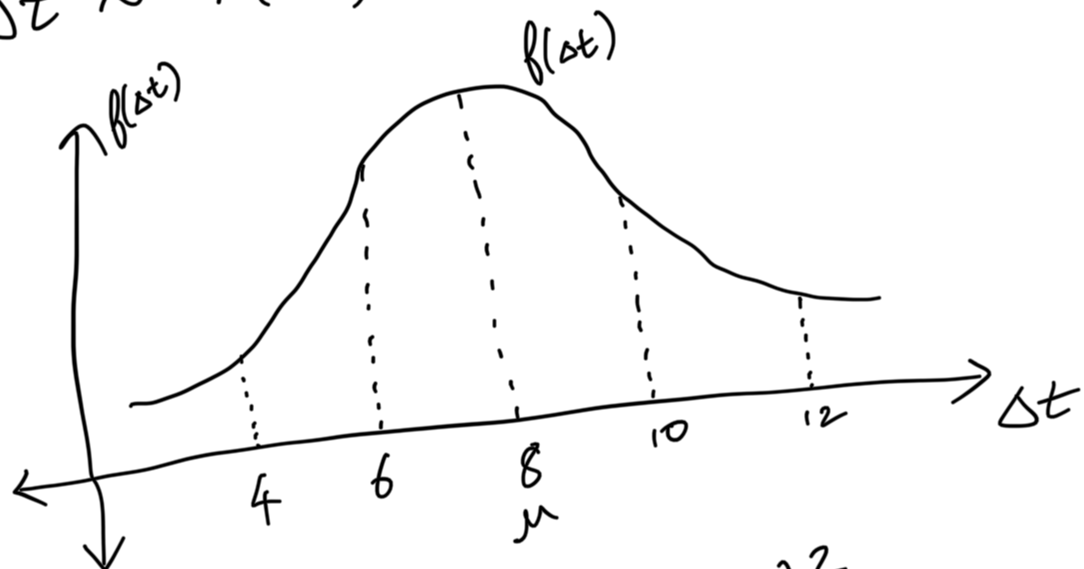
ESE 5030

HW4

Final answer for (b) in the end

1 (a) $\Delta t \sim N(\mu, \sigma^2) = N(8, 4)$



$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$P(\Delta t < 0) = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{\Delta t - \mu}{\sigma}\right)^2} \cdot \Delta t$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{1}{2}\left(\frac{\Delta t - 8}{2}\right)^2} \cdot \Delta t$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{1}{8}(\Delta t - 8)^2} \cdot \Delta t$$

$$-(\Delta t - 8)^2 / 8 \Big]^{0}$$

$$= \frac{1}{2\sqrt{2\pi}} \left. \frac{e}{-1/8 \times 2x(\Delta t - 8)} \right]_{-\infty}$$

$$= \frac{1}{2\sqrt{2\pi}} \quad \frac{e^{-8}}{2} - 0$$

$$= \frac{1}{2\sqrt{2\pi}} \times 0.00016$$

$$= 3.3457 \times 10^{-5}$$

Since $P(\Delta t < 0)$ is very very low, it is unnecessary to truncate Δt so that it is always non-negative.

(b) To compute the interarrival time between the 9^{th} and the 16^{th} customer, we can use the fact that the sum of their arrival times would follow a normal distribution

When we add up multiple independent normally distributed random variables, the resulting mean is the sum of their means and the same goes for variance.

$$\mu_{diff} = 7 \times \mu = 7 \times 8 = 56 \text{ minutes.}$$

$$\sigma_{diff}^2 = 7 \times \sigma^2 = 7 \times 4 = 28 \text{ minutes}^2$$

$$\sigma_{diff} = \sqrt{28} \text{ minutes}$$

$$P(\Delta t_{diff} < 55) = \int_0^{55} f(\Delta t_{diff}) \cdot \Delta t_{diff}$$

$$= \int_0^{55} \frac{1}{\sigma_{diff}\sqrt{2\pi}} e^{-\left(\frac{1}{2}\left(\frac{\Delta t_{diff} - \mu_{diff}}{\sigma_{diff}}\right)^2\right)} \cdot \Delta t_{diff}$$

$$= \int^{55} \frac{1}{\quad} e^{-\left(\frac{1}{2}\left(\frac{\Delta t_{diff} - 56}{\sqrt{28}}\right)^2\right)} \cdot \Delta t_{diff}$$

$$\int_0 \frac{}{\sqrt{2\pi 28}}$$

$$= \frac{1}{\sqrt{2\pi 28}} \int_0^{55} e^{-\frac{(\Delta t_{diff}-56)^2}{56}} \cdot \Delta t$$

$$= \frac{1}{\sqrt{2\pi 28}} \left[\frac{e^{-(\Delta t_{diff}-56)^2/56}}{\frac{-2(\Delta t_{diff}-56)}{56}} \right]_0^{55}$$

$$= \frac{56}{2\sqrt{2\pi 28}} \left[\frac{e^{-1/56}}{1} - \frac{e^{-56}}{-56} \right]$$

$$= \frac{28}{\sqrt{2\pi 28}} \left[\frac{56 e^{-1/56} + e^{-56}}{56} \right]$$

$$= \frac{28}{\sqrt{2\pi 28}} \left[\frac{55.0089}{56} \right]$$

$$= \frac{\cancel{28}}{\sqrt{2\pi 28}} \times \frac{55.0089}{\cancel{56}2}$$

55.0089

$$= \frac{55}{26.5276}$$

$$= 2.0736$$

I feel I made some mistake somewhere.

Since $\mu_{diff} = 56$ & $\sigma_{diff} = \sqrt{28}$, let us use the Z-values from normal distribution to compute $P(\Delta t_{diff} < 55)$.

$$Z = \frac{\Delta t_{diff} - \mu_{diff}}{\sigma_{diff}}$$

$$P(\Delta t_{diff} < 55)$$

$$= P(\mu_{diff} + \sigma_{diff} Z < 55)$$

$$= P\left(Z < \frac{55 - \mu_{diff}}{\sigma_{diff}}\right)$$

o_{diff}

$$= P\left(Z < \frac{55-56}{\sqrt{28}} \right)$$

$$= P\left(Z < -0.189 \right) = 0.425$$