③ ⓐ $$P(X|p) = \begin{cases} p, & x=1 \\ 1-p, & x=0 \end{cases}$$

$$P(X=1 \mid p) = p$$

$$P(X=0 \mid p) = 1-p$$

$$P(X=1) = \int_0^1 P(X=1 \mid p)\, f(p) \cdot dp$$ where $f(p)$ is the pdf of "p."

$$P(X=0) = \int_0^1 P(X=0 \mid p)\, f(p) \cdot dp$$

$$E[p] = \int_0^1 p\, f(p) \cdot dp$$

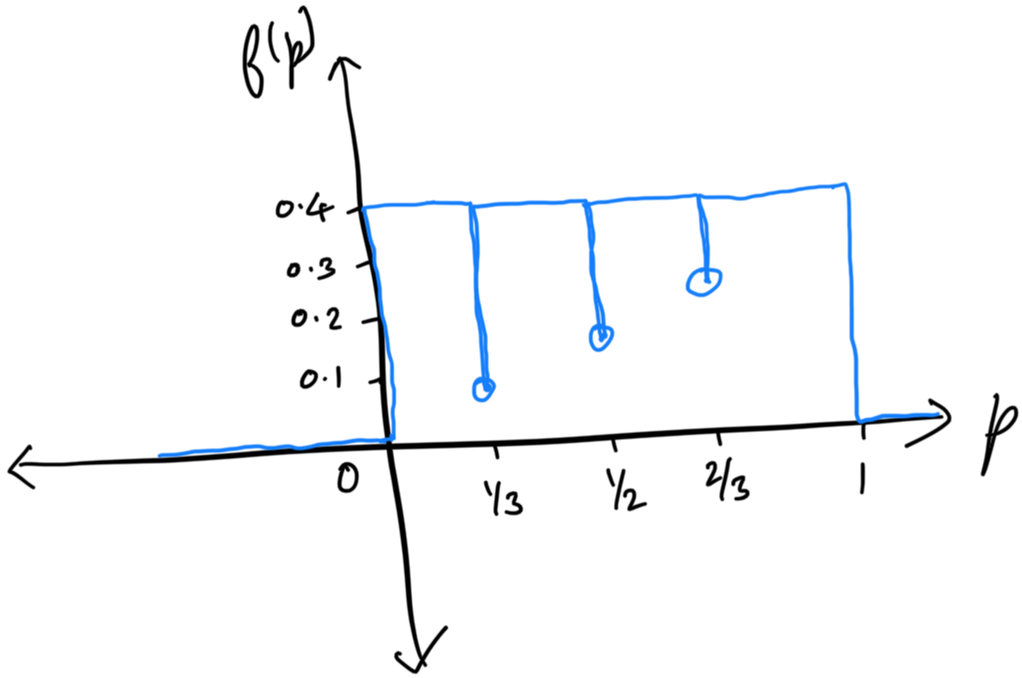
$$\therefore \quad P(X=1) = \int_0^1 p\, f(p) \cdot dp = E[p]$$

$$\& \quad P(X=0) = \int_0^1 (1-p)\, f(p) \cdot dp = \int_0^1 f(p) \cdot dp - \int_0^1 p\, f(p) \cdot dp$$
$$= 1 - E[p]$$

$E[p], \quad x=1$

$$p(x) = \begin{cases} E[p], \\ 1 - E[p], & x = 0 \end{cases}$$

(b)



$$E[p] = \int_{-\infty}^{\infty} p\, f(p) \cdot dp$$

$$= \int_{0}^{1} p\, f(p) \cdot dp$$

$$E[p]_{discrete} = \left(\frac{1}{3} \times 0.1\right) + \left(\frac{2}{3} \times 0.3\right) + \left(\frac{1}{2} \times 0.2\right)$$

$$E[p]_{continuous} = \int_{0}^{1} p\, f(p) \cdot dp$$

$$= \int_0^1 0.4\, p \cdot dp$$

$$= 0.4 \left. \frac{p^2}{2} \right]_0^1$$

$$= 0.4\left(\frac{1}{2} - 0\right) = 0.2$$

$$E[p] = E[p]_{discrete} + E[p]_{continuous}$$

$$= 0.3333 + 0.2$$

$$= 0.5333$$