(5) $$f(x,\alpha) = \frac{\alpha(\alpha+2)}{2} x^{\alpha-1}(1-x^2)$$

$$f'(x) = \left(\frac{1-x^2}{2}\right) \frac{d}{d\alpha}\left(\alpha \times (\alpha+2) \times x^{\alpha-1}\right)$$

$$= \left(\frac{1-x^2}{2}\right)\left(\left(\alpha \times (\alpha+2) \times x^{\alpha-1}\ln x\right) + \alpha x^{\alpha-1}(1) + (\alpha+2)x^{\alpha-1}\right)$$

$$= \left(\frac{1-x^2}{2}\right)\left(\alpha(\alpha+2)x^{\alpha-1}\ln x + \alpha x^{\alpha-1} + (\alpha+2)x^{\alpha-1}\right)$$

$$\sum_{i=1}^{N} \frac{f'(x,\alpha)}{f(x,\alpha)}$$

$$= \sum_{i=1}^{N} \frac{\left(\frac{1-x_i^2}{2}\right)\left(\alpha(\alpha+2)x_i^{\alpha-1}\ln x_i + \alpha x^{\alpha-1} + (\alpha+2)x^{\alpha-1}\right)}{\frac{\alpha(\alpha+2)}{2}x_i^{\alpha-1}(1-x_i^2)}$$

$$= \sum_{i=1}^{n} \frac{\alpha(\alpha+2)x_i^{\alpha-1}\ln x_i + \alpha x^{\alpha-1} + (\alpha+2)x^{\alpha-1}}{\alpha(\alpha+2)x_i^{\alpha-1}}$$

$$= \sum_{i=1}^{n} \ln x_i + \frac{1}{\alpha+2} + \frac{1}{\alpha} = 0$$

$$\sum^{n} \ln x_i$$

$$\sum^{n} \ldots \alpha + 2 \quad = \quad -$$

$$\Rightarrow \sum_{i=1} \frac{\alpha + \alpha + 1}{\alpha^2 + 2\alpha} - \quad \sum_{i=1}$$

$$\Rightarrow \sum_{i=1}^{n} \frac{2(\alpha+1)}{\alpha^2+2\alpha} = -\left(-1.832 - 0.261 - 0.733 - 0.371 - 1.560\right)$$

$$\Rightarrow \sum_{i=1}^{n} \frac{2(\alpha+1)}{\alpha^2+2\alpha} = 4.757$$

$$\Rightarrow \frac{2(\alpha+1)}{\alpha^2+2\alpha} = \frac{4.757}{5} \qquad (n=5)$$

$$\Rightarrow 2(\alpha+1) = 0.9514\alpha^2 + 2 \times 0.9514\alpha$$

$$\Rightarrow \alpha + 1 = 0.4757\alpha^2 + 0.9514\alpha$$

$$\Rightarrow 0.4757\alpha^2 + 0.0486\alpha - 1 = 0$$

From wolfram alpha and since $\alpha > 1$,

$$\alpha = 1.501$$

(b) $$E(X) = \int_0^1 x \cdot f(x) \cdot dx$$

$$= \int_0^1 \frac{\alpha(\alpha+2)}{2} \times x^{\alpha-1+1} (1-x^2) \cdot dx$$

$$= \frac{\alpha(\alpha+2)}{2} \int_0^1 x^\alpha (1-x^2) \cdot dx$$

$$= \frac{\alpha(\alpha+2)}{2} \left[\int_0^1 x^\alpha \cdot dx - \int_0^1 x^{\alpha+2} \cdot dx \right]$$

$$= \frac{\alpha(\alpha+2)}{2} \left[\frac{1}{\alpha+1} - \frac{1}{\alpha+3} \right]$$

$$= \frac{\alpha(\alpha+2)(\alpha+3-\alpha-1)}{2(\alpha+1)(\alpha+3)}$$

$$\mu = \frac{\alpha(\alpha+2)(2)}{2(\alpha+1)(\alpha+3)}$$

$$= \frac{\alpha(\alpha+2)}{(\alpha+1)(\alpha+3)}$$

$$\text{Sample mean} = \frac{0.16+0.77+0.48+0.69+0.21}{5}$$

$$= \frac{2.31}{5}$$

$$= 0.462$$

$$\therefore \frac{\alpha(\alpha+2)}{\ldots} = \mu = 0.462$$

$(\alpha+1)(\alpha+3)$

$\alpha_{SM} = 1.47$