7/2/23

ESE 5030

HW1

① Calculate the lengths of all three sides of the triangle.

$$S_{12} = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

$$S_{23} = \sqrt{(X_3 - X_2)^2 + (Y_3 - Y_2)^2}$$

$$S_{13} = \sqrt{(X_3 - X_1)^2 + (Y_3 - Y_1)^2}$$

ⓐ $P = S_{12} + S_{23} + S_{13}$

ⓑ Area is obtained using the shoelace formula found online.

$$A = \frac{1}{2} \left| x_1 y_2 + x_2 y_3 + x_3 y_1 - x_2 y_1 - x_3 y_2 - x_3 y_1 \right|$$

ⓒ We check whether the triangle is obtuse or not using the pythagoras theorem.

For an obtuse triangle,

$$\underbrace{A^2 + B^2}_{\text{Legs}} > \underbrace{C^2}_{\text{Hypotenuse}}$$

If $\max(A^2, B^2, C^2) > A^2 + B^2 + C^2 - \max(A^2, B^2, C^2)$,

then the triangle is obtuse.

② ⓐ The roots are real for the quadratic equation,

$$AX^2 + BX + C = 0$$

if, $B^2 - 4AC \geq 0$

We average the 5000 simulations to obtain the probability of the roots being real.

ⓑ The roots of the quadratic equation

is given by,

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

(3) We first compute the shortest distance from the centre of the circle to the line segment and check whether it is smaller than the radius of the circle or not.

Vector from point 1 to point 2: $\Delta x = x_2 - x_1$
$\Delta y = y_2 - y_1$

Vector from point 1 to the centre: $\Delta x_1 = x_c - x_1$
$\Delta y_1 = y_c - y_1$

We compute $T = \dfrac{\Delta x_1 \cdot \Delta x + \Delta y_1 \cdot \Delta y}{(\Delta x)^2 + (\Delta y)^2}$

$t = \max(0, \min(1, T))$

The closest x & y- coordinates are,

$$X = x_1 + t\cdot\Delta x \; ; \; Y = y_1 + t\cdot\Delta y$$

$$\text{Shortest distance} = \sqrt{(X - X_c)^2 + (Y - Y_c)^2}$$

If the shortest distance is less than the radius, we check if the distance from either endpoints of the line segment is greater than the radius or not, and if so, the line segment intersects the circle.

⑤ ⓐ The bus arrives at time 1, 21, 41 & 61.

Passenger arrives at $T = 11 + t\,(51 - 11)$, $t \sim U[0,1)$

Waiting time is found by,

$$\left(\ldots\ |T-21|,\ |T-41|,\ |T-61|\right)$$

$\mathrm{MIN}(|T-1|, |T-2|, \ldots$

We check if the above value is greater than 5 and assign a flag of 1 or zero otherwise. We use the flag and average over the simulations to obtain the probability.

⑥ The parametric representations of the two line segments are as follows,

$$1: \quad x_1 = X_{11} + t_1 (X_{12} - X_{11})$$
$$\quad\;\; y_1 = Y_{11} + t_1 (Y_{12} - Y_{11})$$

$$2: \quad x_2 = X_{21} + t_2 (X_{22} - X_{21})$$
$$\quad\;\; y_2 = Y_{21} + t_2 (Y_{22} - Y_{21})$$

We equate $x_1 = x_2$ & $y_1 = y_2$ and obtain two equations with two unknowns.

$$(X_{12} - X_{11}) t_1 + (X_{21} - X_{22}) t_2 + (X_{11} - X_{21}) = 0$$
$$(Y_{12} - Y_{11}) t_1 + (Y_{21} - Y_{22}) t_2 + (Y_{11} - Y_{21}) = 0$$

$(y_2 - y_1)$ ...

We can easily assimilate the coefficients and solve using np.linalg.solve in python but the code uses the following formula

$$t_1 = \frac{\left((x_3 - x_1) \times (y_4 - y_3)\right) - \left((y_3 - y_1) \times (x_4 - x_3)\right)}{\left((x_2 - x_1) \times (y_4 - y_3)\right) - \left((y_2 - y_1) \times (x_4 - x_3)\right)}$$

$$t_2 = \frac{\left((x_3 - x_1) \times (y_2 - y_1)\right) - \left((y_3 - y_1) \times (x_2 - x_1)\right)}{\left((x_2 - x_1) \times (y_4 - y_3)\right) - \left((y_2 - y_1) \times (x_4 - x_3)\right)}$$

We check if the values of t_1 & t_2 lie between 0 & 1 and if they do, the line segments intersect.

7.

For this problem, we just find the distance between the centres of the circular disks and check if they are lower than the sum of radii of the two disks, and if they are, they are overlapping.

(4) Basically, I use the rand() property to calculate the range of income and expenses accordingly and I subtract the income from the expenses and calculate the loan amount required. I use if conditions and flag

the ranges in the newly assigned cells and I find the average to estimate the probability of the loan amount being in that range over a 1000 simulations.