

**ESE 5030 - Simulation Modeling & Analysis (Homework #2)**

Spring Semester, 2023

M. Carchidi

---

**Problem #1 (20 points) - Single-Channel Series Queue**

Consider a small-town Pep Boys store which consist of a single server at the counter to get car parts for customers and another single server at the checkout counter for the customers to pay for their merchandise. Customers arrive so that interarrival times are distributed as shown in the following table. Also shown in this table are the distributions of service times for the server at the counter and the server at the checkout.

Interarrival Times (Minutes)	$P$	Counter Times (Minutes)	$P$	Checkout Times (Minutes)	$P$
1	0.05	1	0.10	1	0.2
2	0.10	2	0.15	2	0.4
3	0.15	3	0.20	3	0.3
4	0.20	4	0.30	4	0.1
5	0.20	5	0.20	—	—
6	0.15	6	0.05	—	—
7	0.10	—	—	—	—
8	0.05	—	—	—	—

Simulate the arrival of 500 customers assuming a FIFO discipline for both lines, and use your simulation to answer the following.

- (5 points)** Determine the probability that a customer waits in the line for service at the counter and determine the average waiting time for this service per customer.
  - (5 points)** Determine the probability that a customer waits in the line for checkout and determine the average waiting time for checkout per customer.
  - (5 points)** Determine the probability that a customer waits at all.
  - (5 points)** Determine the fraction of the time each server is idle.
-

---

**Problem #2 (20 points) - A Reliability Problem**

A certain component of a machine has a lifetime that is exponentially distributed with a mean of 10 hours. When this component fails, the machine is stopped and the component must be replaced. Suppose that the replacement time is uniformly distributed between 25 and 35 minutes. In addition, company policy is to replace the component after 15 hours of operation even if it has not failed. Run 10 simulations to mimic the replacement of 100 such components. Use your simulations to determine:

- a.) (5 points) the average total time of the simulation,
- b.) (5 points) the average number of components that must be replaced during the first five days of operation,
- c.) (5 points) the average time (in hours) that a component is used, and
- d.) (5 points) the average percent of the time the machine is idle.

*Hint:* You may use the fact that a sample  $X$  from an exponential distribution with mean  $\mu$  can be computed from a random number  $R \sim U[0, 1)$  using  $X = -\mu \ln(R)$ .

---

**Problem #3 (15 points) - Another Reliability Problem**

Whenever an electric heater turns on, the heating coil has a probability 0.4 of failing. When this coil fails, the heater does not run until the coil is replaced and the time to replace the coil can be either: 1 minute, 2 minutes or 3 minutes, with probabilities 0.2, 0.5 and 0.3, respectively, and right after the coil is replaced, the heater is set to run. Whenever the heater is running (either because the above coil did not fail, or it was just replaced), the heater will run (for a time before automatically turning off) in accordance to the run-time distribution given in the following table.

Run Time (minutes)	1	2	3	4	5	6	7	8
Probability	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05

Whenever the heater is turned off and operational, it will be called upon to turn on in a time governed by a between call-time distribution given in the following table.

Between Call Time (minutes)	1	2	3	4	5	6
Probability	0.10	0.15	0.25	0.25	0.15	0.10

Run a 50-hour simulation to estimate: (a) the number of times the heater was called upon to turn on, and (b) the number of times the coil on this heater had to be replaced.

---

---

**Problem #4 (15 points) - Another Queueing Problem**

Consider a single-channel queue with two customer interarrival-time distributions (morning and afternoon) as presented in the table below. The system contains a single server and the service-time distribution is also presented in the table below.

Morning Interarrival Times (Minutes)	$P$	Afternoon Interarrival Times (Minutes)	$P$	Service Times (Minutes)	$P$
1	0.05	1	0.250	1	0.10
2	0.10	2	0.200	2	0.20
3	0.15	3	0.175	3	0.30
4	0.20	4	0.125	4	0.25
5	0.20	5	0.100	5	0.10
6	0.15	6	0.075	6	0.05
7	0.10	7	0.050	—	—
8	0.05	8	0.025	—	—

Simulate the arrival of customers for 1 day from 8:00 AM to 5:00 PM assuming a FIFO discipline for both the morning and afternoon lines and the simulation stops once service ends past 5:00 PM. Use your simulation to answer the following:

- (3 points)** Estimate the probability that a customer waits in the line for service and determine the average waiting time per customer.
  - (3 points)** Estimate the average waiting time per customer who has to wait.
  - (3 points)** Estimate the average time a customer spends in the system.
  - (3 points)** Estimate the fraction of the time the server is idle.
  - (3 points)** Estimate the average number of customers that enter the system in 1 day.
-

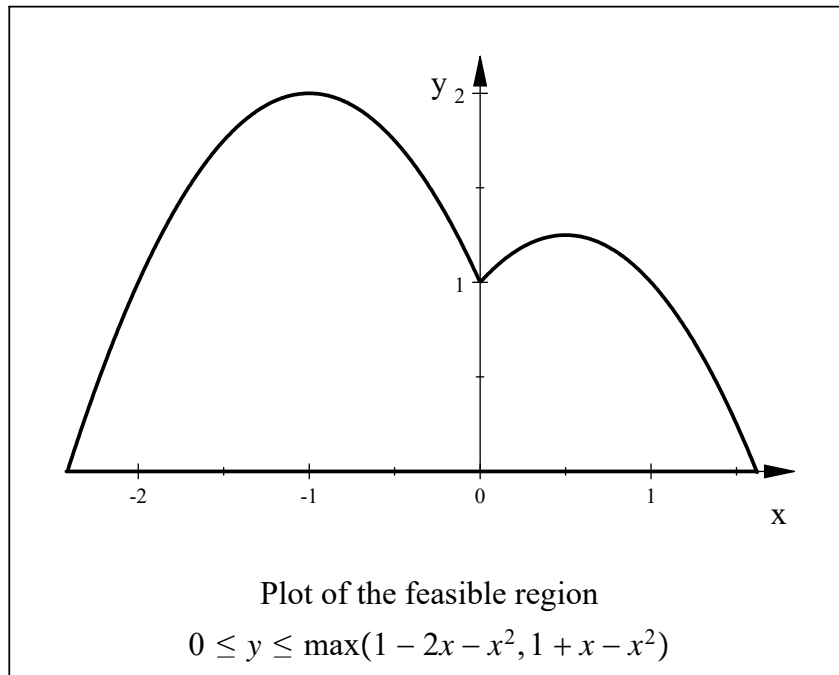
---

**Problem #5 (15 points) - A Problem in Optimization**

Use 5000 Monte-Carlo Simulations to solve the following optimization problem which seeks to maximize the expression  $z = x^2 + (y - 1)^3$ , subject to the constraint that

$$0 \leq y \leq \max(1 - 2x - x^2, 1 + x - x^2)$$

which is the region within the bold lines shown below.



---

**Problem #6 (15 points) - A Problem in Integration**

Use 5000 Monte-Carlo Simulations to estimate the value of

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} (x + y + z) e^{xyz} dz dy dx.$$

---