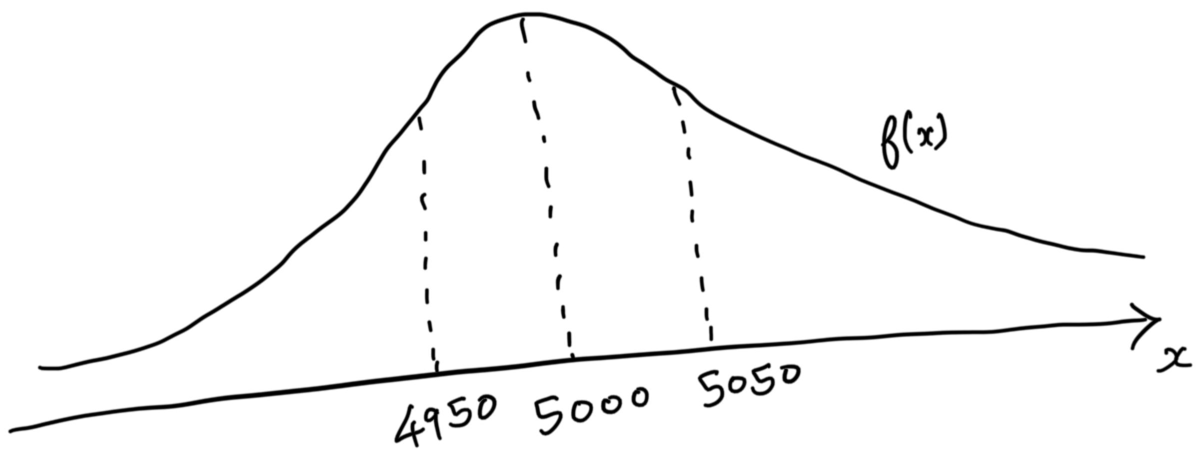
② $X_i \sim N(5000, 2500)$

$\mu = 5000;\ \sigma^2 = 2500$

$\Rightarrow \sigma = 50$



ⓐ Let us compute for a single investment first.

$$P(X_i > 5022) = \int_{5022}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} . dx$$

$$= \int_{5022}^{\infty} \frac{1}{\sqrt{2\pi} \times 50} \times e^{-\frac{1}{2}\left(\frac{x-5000}{50}\right)^2} . dx$$

$$= \frac{1}{50\sqrt{2\pi}} \int_{5022}^{\infty} e^{-\frac{1}{2 \times 2500}(x-5000)^2} . dx$$

$(x-5000)^2/5000$

$$= \frac{1}{50\sqrt{2\pi}} \int_{5022}^{\infty} e^{-(x-5000)^2/5000} \, .dx$$

$$= \frac{1}{50\sqrt{2\pi}} \left[\frac{e^{-(x-5000)^2/5000}}{-\frac{2(x-5000)}{5000}} \right]_{5022}^{\infty}$$

$$= \frac{1}{50\sqrt{2\pi}} \quad 0 - \frac{e^{-22^2/5000}}{-\frac{2\times 22}{5000}}$$

$$= \frac{1}{50\sqrt{2\pi}} \times \frac{e^{-484/5000}}{44/5000}$$

$$= \frac{1}{50\sqrt{2\pi}} \times \frac{0.9077}{0.0088}$$

$$= 0.008 \times 103.152$$

$$= 0.8252$$

$$- P(X>5022)$$

$$P(X_1 > 5022) = P(X_2 > 5022) = P(X_3 > 5022) = P(X_4 > 5022)$$

$$= 0.8252 = p$$

$$P(\text{Either of them exceeding}) = {}^4C_4\, p^4 (1-p)^0 + {}^4C_3\, p^3 (1-p)^1 + {}^4C_2\, p^2 (1-p)^2 + {}^4C_1\, p^1 (1-p)^3$$

$$= p^4 + 4p^3(1-p) + 6p^2(1-p)^2 + 4p(1-p)^3$$

$$= (0.8252)^4 + 4(0.8252)^3(0.1747) + 6(0.8252)^2(0.1747)^2 + 4(0.8252)(0.1747)^3$$

$$= 0.4636 + 4 \times 0.562 \times 0.1747 + 6 \times 0.6809 \times 0.0305 + 4 \times 0.8252 \times 0.0053$$

$$= 0.4636 + 0.3927 + 0.1246 + 0.0175$$

$$= 0.9984$$

(b) $P(\text{all four investments exceeding } 5022)$

$$= {}^4C_4\, p^4 (1-p)^0$$

$= 1 \times (0.8252)^4 \times 1$

$= 0.4636$

(c) $P(\text{average of four investments} > 5022)$

$= P\left(\frac{X_1 + X_2 + X_3 + X_4}{4} > 5022\right)$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} . dx$$

$X = F^{-1}(R)$

$$F(x) = \frac{1}{50\sqrt{2\pi}} \times \left[\frac{e^{-(x-5000)^2/5000}}{\frac{-2(x-5000)}{5000}} \right]_{-\infty}^{x}$$

$$\Rightarrow F(x) = \frac{1}{50\sqrt{2\pi}} \left(\frac{e^{-\frac{(x-5000)^2}{5000}}}{-\frac{2(x-5000)}{5000}} - \right.$$

Another attempt.

Another

$$\frac{X_1 + X_2 + X_3 + X_4}{4}$$ will have a

new distribution with the same mean but a different variance.

$$\mu_{avg} = 5000 \; ; \; \sigma_{avg}^2 = \frac{2500}{4} = 625$$

$$\sigma_{avg} = 25$$

$$Z = \frac{X_{avg.} - \mu}{\sigma}$$

$$P(X_{avg} > 5022)$$

$$= P(\mu + \sigma Z > 5022)$$

$$= P\left(Z > \frac{5022 - \mu}{\sigma}\right)$$

$$= P\left(Z > \frac{5022 - 5000}{25}\right)$$

$$= P\left(Z > 22/25\right)$$

$$= 1 - P\left(Z \leq 22/25\right)$$

$$= 0.1894$$