④ Interarrival times $0 \leq x \leq 5$ minutes are given by the pdf,

$$f(x) = \frac{6x^2(125 - x^2)}{15625}$$

$$F(x) = \sum_{x=0}^{x} f(x)$$

$$= -\frac{x^3(x^3 - 250)}{15625}$$

$$g(x) = F(x) - R$$

$$g'(x) = f(x)$$

$$X_{n+1} = X_n - \frac{g(x)}{g'(x)} = X_n - \frac{(F(X_n) - R)}{f(X_n)}$$

Now we do the same thing for service times.

$$g(x) = \frac{2x^2(2x+3)}{135}$$

$$G(x) = \int_0^x g(x) \cdot dx = \frac{x^3(x+2)}{135}$$

$$X_{n+1} = X_n - \frac{G(X_n) - R}{g(X_n)}$$

For interarrival times,

$$F(x) = R$$

$$\frac{-x^3(x^3-250)}{15625} = R$$

$$-x^6 + 250x^3 = 15625R$$

$$x^6 - 250x^3 + 15625R = 0$$

Let $u = x^3$,

$$u^2 - 250u + 15625R = 0$$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$; $b = -250$; $c = 15625R$

$$u = \frac{250 \pm \sqrt{62500 - 4 \times 15625R}}{2}$$

$$= \frac{250 \pm \sqrt{62500(1-R)}}{}$$

$$= \frac{250 \pm 250\sqrt{1-R}}{2}$$

$$\Rightarrow x^3 = 125 \pm 125\sqrt{1-R}$$

$$\Rightarrow x = \sqrt[3]{125 \pm 125\sqrt{1-R}}$$

$$= 5\sqrt[3]{1 \pm \sqrt{1-R}}$$

Since the newton's iterative method results in values greater than 5 for very large values of R, I use the minimum of the two possible values given by the above formula.

The simulation is done in the same way as previous homeworks.