Simulation 5

ⓐ $\Delta t_k \sim Exp(\lambda)$, $k = 1, 2, 3, \dots, n$

The cdf is defined by $F(\Delta t) = 1 - e^{-\lambda \Delta t}$

We have to find the probability that no two customers arrive within a time, $\theta > 0$, of each other. This is essentially the probability that $\Delta t > \theta$.

$$\begin{aligned} P(\Delta t_k > \theta) &= 1 - P(\Delta t_k < \theta) \\ &= 1 - F(\theta) \\ &= 1 - (1 - e^{-\lambda\theta}) \\ &= e^{-\lambda\theta} \end{aligned}$$

Since we have to compute for $k = 1, 2, 3, \dots, n$,

we get $P(\Delta t_1 > \theta) \times P(\Delta t_2 > \theta) \times \dots \times P(\Delta t_n > \theta)$

$$= e^{-\lambda\theta} \times e^{-\lambda\theta} \times \dots \times e^{-\lambda\theta}$$

$$= (e^{-\lambda\theta})^n$$

$$= e^{-n\lambda\theta}$$

We do not have to include the first person because they would be our reference point.

So, our final answer would be $e^{-(n-1)\lambda\theta}$

(b) Using numpy in python, I use np.random.exponential with an extra parameter of $n=10$ to generate 10 samples. I check if all of them are greater than θ, and this is checked for a thousand times, and the successes variable is incremented accordingly. The probability would be the ratio of successes to the number of runs, which

in our case.

is 1000 and

The theoretical probability is 0.2369

the simulated probability is 0.234.