MATH 420-001 Fall 2021

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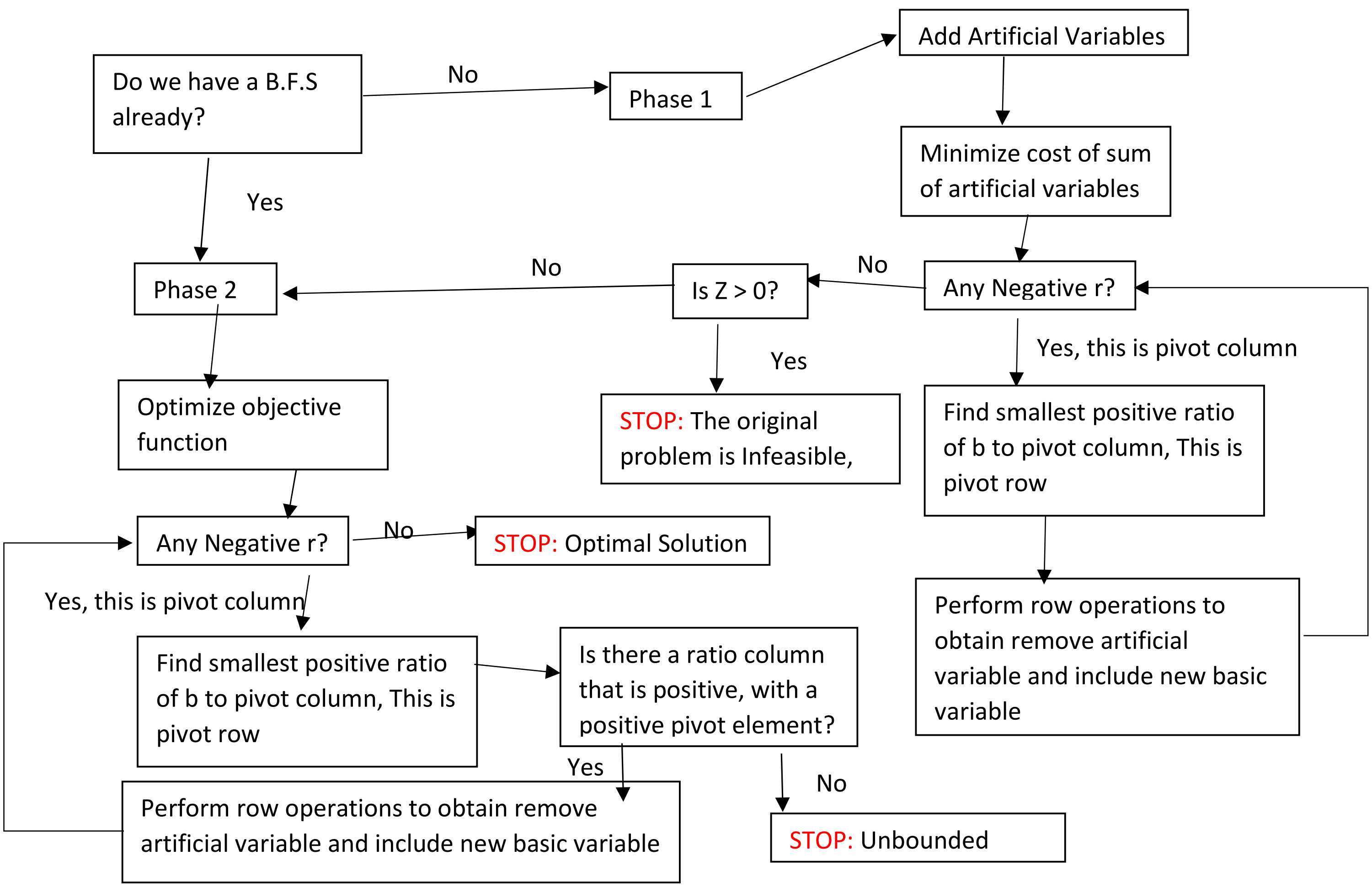
Computer HW Identification: Two-Phase Simplex Method

Computer Program Language: MATLAB

Section 1. Description of the Optimization Methods:

- We will make use of the Two-Phase simplex procedure in order to find an optimal solution for a Linear Program , if one exists.
- First the Program will check if **Surplus** or **Slack** Variables are needed, in order to convert the problem into standard form.

Flowchart of the Two-Phase Simplex Procedure:



Section 2. Structure of Computer Program:

Main program file: The program will run the entire Two-Phase Simplex Procedure

Name: Run.m

Sample Input file (Must be called "Input.txt"):

```
type
2
m n
2 2
A
1 -1
1 1
b
2
6
max
-1
1
```

Special data structure(s) used:

Program makes use of **matrices** and **vectors** within MATLAB to implement all the steps within the Two-Phase Simplex Procedure.

Instructions to use the program:

Type 1, all constraints are equality

Type 2, all constraints are <= (Slack)

Type 3, all constraints are >= (Surplus)

Edit Input.txt to specify your problem.

Navigate to the correct directory (MA420ComputerHW1_9Padmanabhan_Vyas_Oct27) within MATLAB and call Run from the Command Window. This will produce an output within the command window as well as printing the output to the file "output.txt".

Sample Output file (Will be called "output.txt"):

```
Adding Slack variables, since type 2

The standard form is:
A =
        1       -1      1       0
        1       1       0       1

b =
        2
        6

C =
        1
        -1
        0
        0

Phase I not required.

Our original BFS is X1 = 0, X2 = 0,

Phase II started.

Phase II Iteration 1

Current X:
X1 = 0, X2 = 6, Current Z = 6


Optimal Solution Reached.
Optimal X:
X1 = 0, X2 = 6, Maximum Z = 6


For Constraint 1:
LHS = -6.000000 <= 2.000000 = b(1), Correct.

For Constraint 2:
LHS = 6.000000 <= 6.000000 = b(2), Correct.

The solution is feasible (all Xi >=0)


Time Taken: 0.022953 seconds
```

Section 3. Examples and Test Results:

Example 1:

Source: Textbook problem 3.9 (Spare Parts problem)

Input.txt:

type

2

m n

2 5

A

0.02 0.01 0.03 0.03 0.01

0.03 0.02 0.02 0.01 0.01

b

700

1000

max

0.3

0.2

0.4

0.25

0.10

output.txt:

```
Adding Slack variables, since type 2

The standard form is:
A =
       0.02    0.01    0.03    0.03    0.01    1       0
       0.03    0.02    0.02    0.01    0.01    0       1

b =
       700
       1000

C =
       -0.3
       -0.2
       -0.4
       -0.25
       -0.1
       0
       0

Phase I not required.

Our original BFS is X1 = 0, X2 = 0, X3 = 0, X4 = 0, X5 = 0,

Phase II started.

Phase II Iteration 1

Current X:
X1 = 0, X2 = 0, X3 = 2.333333e+04, X4 = 0, X5 = 0, Current Z = 9.333333e+03

Phase II Iteration 2

Current X:
X1 = 0, X2 = 40000, X3 = 1.000000e+04, X4 = 0, X5 = 0, Current Z = 12000


Optimal Solution Reached.
Optimal X:
X1 = 0, X2 = 40000, X3 = 1.000000e+04, X4 = 0, X5 = 0, Maximum Z = 12000


For Constraint 1:
LHS = 700.000000 <= 700.000000 = b(1), Correct.

For Constraint 2:
LHS = 1000.000000 <= 1000.000000 = b(2), Correct.

The solution is feasible (all Xi >=0)


Time Taken: 0.039670 seconds
```

Test results:

Optimal x* = (0, 40000, 10000, 0, 0)

Optimal z* = 12000

Comments: This example showcases the ability to use a decimal value without any issues.

Example 2:

Source: Made up example

Input.txt:

type

3

m n

2 2

A

2 1

1 7

b

4

7

min

1

1

output.txt:

```
Adding Surplus variables, since type 3

The standard form is:
A =
        2       1       -1      -0
        1       7       -0      -1

b =
        4
        7

C =
        1
        1
        0
        0

Phase I required.


Phase I Iteration 1


Current X:
X1 = 0, X2 = 1,

Phase I Iteration 2


Current X:
X1 = 1.615385e+00, X2 = 7.692308e-01,

Phase I Complete.

Our original BFS is X1 = 1.615385e+00, X2 = 7.692308e-01,

Phase II started.


Optimal Solution Reached.
Optimal X:
X1 = 1.615385e+00, X2 = 7.692308e-01, Minimum Z = 2.384615e+00


For Constraint 1:
LHS = 4.000000 >= 4.000000 = b(1), Correct.

For Constraint 2:
LHS = 7.000000 >= 7.000000 = b(2), Correct.

The solution is feasible (all Xi >=0)


Time Taken: 0.034947 seconds
```

Test results:

Optimal x* = (1.615385, 0.7692308)

Optimal z* = 2.384615

Comments: This example showcases the ability to solve a type 3 problem with surplus variables

Example 3:

Source: Made up example

Input.txt:

type

1

m n

2 2

A

1 1

1 1

b

1

2

max

1

1

output.txt:

```
The standard form is:
A =
        1       1
        1       1

b =
        1
        2

C =
        -1
        -1

Phase I required.


Phase I Iteration 1


Current X:
X1 = 1, X2 = 1,

Phase I Complete.


The Orginal problem is infeasible (Constraint Violated)

Time Taken: 0.014007 seconds
```

Test results:

Optimal x* = None

Optimal z* = None

The original problem was infeasible.

Comments: This example showcases the ability to accurately identify infeasible problems.

Example 4:

Source: Made up example

Input.txt:

type

3

m n

2 2

A

1 1

1 -1

b

1

2

max

1

1

output.txt:

```
Adding Surplus variables, since type 3

The standard form is:
A =
        1       1       -1      -0
        1       -1      -0      -1

b =
        1
        2

C =
        -1
        -1
        0
        0

Phase I required.


Phase I Iteration 1


Current X:
X1 = 1, X2 = 0,

Phase I Iteration 2


Current X:
X1 = 2, X2 = 0,

Phase I Complete.

Our original BFS is X1 = 2, X2 = 0,

Phase II started.

Phase II Iteration 1

Problem is Unbounded. Maximum Z is Infinity


Time Taken: 0.016188 seconds
```

Test results:

Optimal x* = None

Optimal z* = Infinity

The original problem was unbounded

Comments: This example showcases the ability to accurately identify unbounded problems.

Example 5:

Source: Similar to Sample Input

Input.txt:

type

2

m n

2 2

A

1 -1

1 1

b

2

6

max

-1

1

5

output.txt:

```
Adding Slack variables, since type 2

The standard form is:
A =
        1       -1      1       0
        1       1       0       1

b =
        2
        6

C =
        1
        -1
        0
        0

Phase I not required.

Our original BFS is X1 = 0, X2 = 0,

Phase II started.

Phase II Iteration 1

Current X:
X1 = 0, X2 = 6, Current Z = 11


Optimal Solution Reached.
Optimal X:
X1 = 0, X2 = 6, Maximum Z = 11


For Constraint 1:
LHS = -6.000000 <= 2.000000 = b(1), Correct.

For Constraint 2:
LHS = 6.000000 <= 6.000000 = b(2), Correct.

The solution is feasible (all Xi >=0)


Time Taken: 0.039579 seconds
```

Test results:

Optimal x* = (0, 6)

Optimal z* = 11

Comments: This example showcases the ability to deal with constants in the Objective Function.