

QUOTIENT GRAPHS

Definition 0.1. If Γ is a graph with automorphism group $\text{Aut } \Gamma = G$, the quotient graph of G is the graph Γ/G having vertex set $V(\Gamma/G) = \{X \mid X = \text{Orb}_G(x), \exists x \in V(\Gamma)\}$, in which vertices X and Y , corresponding to orbits X and Y of G on Γ , are adjacent if $x \sim_\Gamma y$, for some $x \in X$ and $y \in Y$.

Lemma 0.2. *If X and Y are two orbits of the automorphism group of a graph Γ , and a vertex $x \in X$ is adjacent to a vertex $y \in Y$, then every vertex x' of X is adjacent to some vertex of Y .*

Proof. Let $x' \in X$. Since $X = \text{Orb}(x)$ is the orbit of x under the automorphism group of Γ , there is some automorphism ϕ of Γ such that $\phi(x) = x'$. Then, $\phi(y) = y'$, $\exists y' \in Y = \text{Orb}(y)$. Since $x \sim y$, it follows that $x' \sim y'$, as required. \square

Theorem 0.3.