QUOTIENT GRAPHS

Lemma 0.1. *If X and Y are two orbits of the automorphism group of a graph Γ, and a vertex $x \in X$ is adjacent to a vertex $y \in Y$, then every vertex x' of X is adjacent to some vertex of Y.*

Proof. Let $x' \in X$. Since $X = \mathrm{Orb}(x)$ is the orbit of x under the automorphism group of Γ, there is some automorphism ϕ of Γ such that $\phi(x) = x'$. Then, $\phi(y) = y'$, $\exists y' \in Y = \mathrm{Orb}(y)$. Since $x \sim y$, it follows that $x' \sim y'$, as required. $\square$