QUOTIENT GRAPHS

Definition 0.1. If Γ is a graph with automorphism group $\operatorname{Aut}\Gamma = G$, the quotient graph of G is the graph Γ/G having vertex set $V(\Gamma/G) = \{X \mid X = \mathrm{Orb}_G(x), \exists x \in V(\Gamma)\}$, in which vertices X and Y, corresponding to orbits X and Y of G on Γ, are adjacent if $x \sim_\Gamma y$, for some $x \in X$ and $y \in Y$.

Lemma 0.2. *If X and Y are two orbits of the automorphism group of a graph Γ, and a vertex $x \in X$ is adjacent to a vertex $y \in Y$, then every vertex x' of X is adjacent to some vertex of Y.*

Proof. Let $x' \in X$. Since $X = \mathrm{Orb}(x)$ is the orbit of x under the automorphism group of Γ, there is some automorphism ϕ of Γ such that $\phi(x) = x'$. Then, $\phi(y) = y'$, $\exists y' \in Y = \mathrm{Orb}(y)$. Since $x \sim y$, it follows that $x' \sim y'$, as required. $\square$

Theorem 0.3. *Every cycle in the quotient graph Γ/G corresponds to a cycle in the graph Γ.*