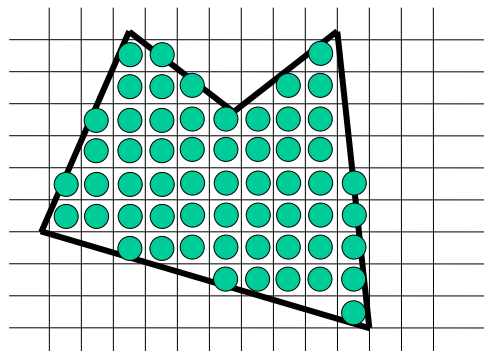

Lecture 6

Rasterizing polygons

1

Rasterizing Polygons

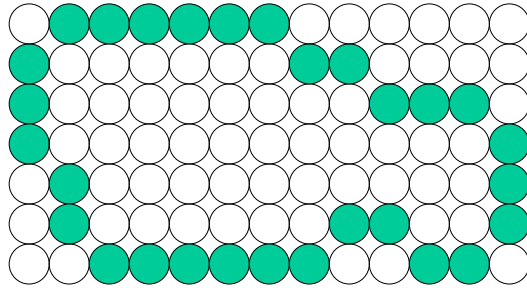
Given a set of vertices and edges,
find the pixels that fill the polygon.



2

Flood Fill

First, rasterizing its edges into the frame buffer using Bresenham's algorithm

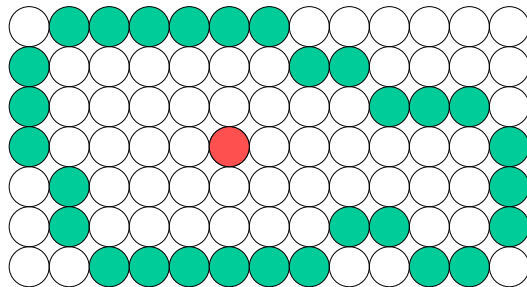


How to fill polygons whose edges are already drawn?

3

Flood Fill

First, rasterizing its edges into the frame buffer using Bresenham's algorithm

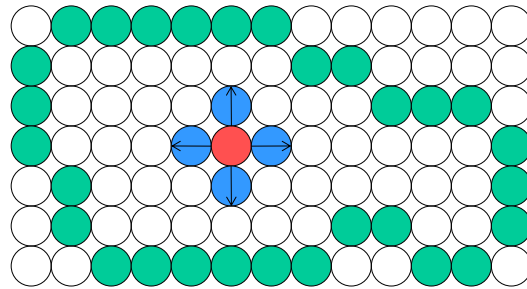


Choose a point inside, and fill outwards

4

Flood Fill

First rasterizing its edges into the frame buffer using Bresenham's algorithm



Choose a point inside, and fill outwards

5

Flood Fill

Fill a point and recurse to all of its neighbors

```
floodFill(int x, int y, color c)
{
    if(stop(x,y,c))
        return;

    setPixel(x,y,c);
    floodFill(x-1,y,c);
    floodFill(x+1,y,c);
    floodFill(x,y-1,c);
    floodFill(x,y+1,c);
}

int stop(int x, int y, color c)
{
    return colorBuffer[x][y] == c;
}
```

6



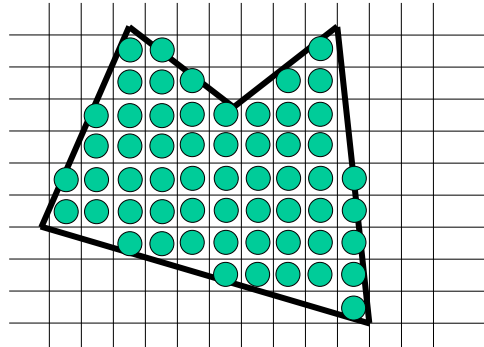
7



8

Rasterizing Polygons

Given a set of vertices and edges,
find the pixels that fill the polygon.



9

Rasterizing Polygons

vList is an ordered list of the polygon's vertices

```
fillPoly(vertex vList[ ])
    boundingBox b = getBounds(vList);
    int xmin = b.minX;
    int xmax = b.maxX;
    int ymin = b.minY;
    int ymax = b.maxY;

    for(int y = ymin; y <= ymax; y++)
        for(int x = xmin; x <= xmax; x++)
            if(insidePoly(x,y,vList))
                setPixel(x,y);
```

10

What does 'inside' mean?

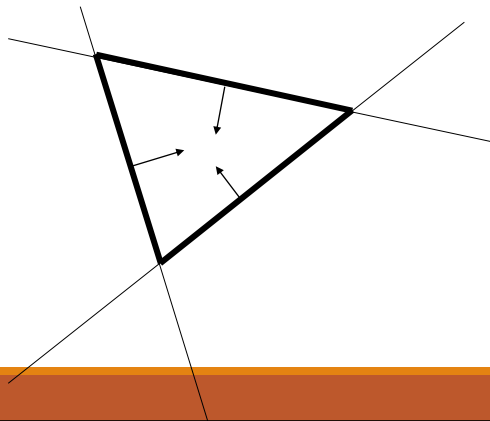
How to test if a point is inside a polygon

1. Half-space tests
2. Jordan Curve Theorem (even/odd or $+1/-1$)
3. Winding number test

11

Half Space Tests

Given the edges of a triangle, the inside is the intersection of half-spaces defined by the edges

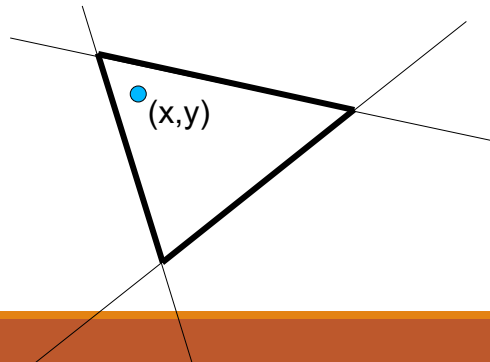


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Half Space Tests

Easily computable:

$$l(x,y) = ax + by + c < 0 \quad \text{Iff } (x,y) \text{ is inside}$$



15

Half Space Tests

lineEq computes the implicit line value for 2 vertices & a point

```
fillTriangle(vertex vList[3])
  //-- get the bounding box as before --//
  float e1 = lineEq(vList[0],vList[1],xmin,ymin);
  float e2 = lineEq(vList[1],vList[2],xmin,ymin);
  float e3 = lineEq(vList[2],vList[0],xmin,ymin);
  int xDim = xmax - xmin;

  for(int y = ymin; y <= ymax; y++)
    for(int x = xmin; x <= xmax; x++)
      if(e1<0 && e2<0 && e3<0)
        setPixel(x,y);
      e1 += a1; e2 += a2; e3 += a3;
      e1 += -xDim*a1+b1; e2 = -xDim*a2+b2; e3 = -xDim*a3+b3
```

16

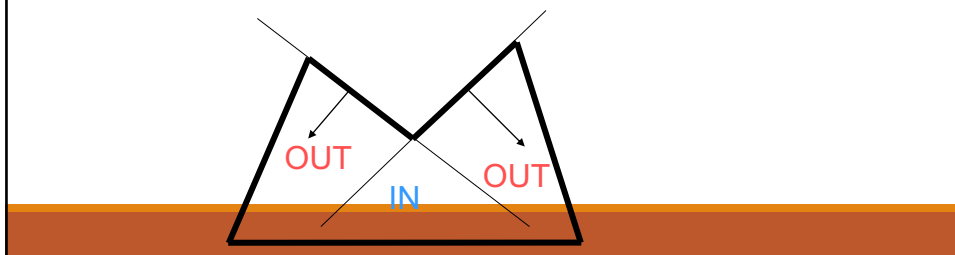
Half Space Tests

Easily computable:

$$l(x,y) = ax + by + c < 0 \quad \text{Iff } (x,y) \text{ is inside}$$

Doesn't work on concave objects!!

→ triangulate



18

What does 'inside' mean?

How to test if a point is inside a polygon

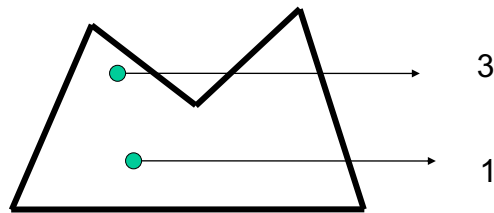
1. Half-space tests
2. Jordan Curve Theorem (even/odd or +1/-1)
 - Self-intersecting polygon OK
3. Winding number test

19

Jordan Curve Theorem

Even/odd approach

Hit test: inside or outside based on the number of intersected edges is even or odd



Any ray from a point **inside** a polygon will intersect the polygon's edges an **odd** number of times

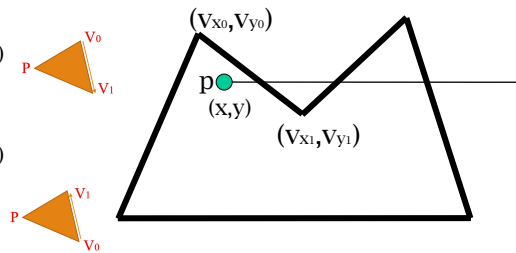
20

Jordan Curve Theorem

vList is an ordered list of the **n** polygon vertices

```
int jordanInside(vertex vList[ ], int n, float x, float y)
{
    int cross = 0;
    float x0, y0, x1, y1;

    x0 = vList[n-1].x - x;    y0 = vList[n-1].y - y;
    for(int i = 0; i < n; i++)
    {
        x1 = vList[i].x - x;    y1 = vList[i].y - y;
        if(y0 > 0)
        {
            if(y1 <= 0)
            {
                if( x1*y0 > y1*x0 )
                    cross++;
            }
        }
        else
        {
            if(y1 > 0)
            {
                if( x0*y1 > y0*x1 )
                    cross++;
            }
        }
        x0 = x1; y0 = y1;
    }
    return cross & 1;
}
```



21

upward edge

downward edge

where $V_i = (x_i, y_i)$

$$2A(\Delta) = \begin{vmatrix} x_1 - x_0 & x_2 - x_0 \\ y_1 - y_0 & y_2 - y_0 \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

$$= (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$$

i.e., $x_0y_1 - x_1y_0$

$V_i = (x_i, y_i)$

The signed area will be
 -positive if the triangle is oriented counterclockwise
 -negative if the triangle is oriented clockwise

<http://geomalgorithms.com/a03-inclusion.html>
<http://geomalgorithms.com/a01-area.html>

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upward edge

downward edge

$$2A(\Delta) = \begin{vmatrix} x_1 - x_0 & x_2 - x_0 \\ y_1 - y_0 & y_2 - y_0 \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

$$= (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$$

The signed area will be
 -Positive, if the triangle is oriented counterclockwise
 - $x_0y_1 > x_1y_0$
 -negative, if the triangle is oriented clockwise
 - $x_1y_0 > x_0y_1$

$2A = x_0y_1 - x_1y_0$

where
 $x_0 = v_0.x - p.x$
 $y_0 = v_0.y - p.y$

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$$2\text{Area} = x_0y_1 - x_1y_0$$

$A > 0$, if the triangle is oriented counterclockwise

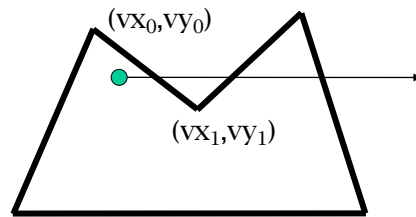
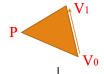
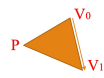
$A < 0$, if the triangle is oriented clockwise

Jordan Curve Theorem

vList is an ordered list of the n polygon vertices

```
int jordanInside(vertex vList[], int n, float x, float y)
{
    int cross = 0;
    float x0, y0, x1, y1;

    x0 = vList[n-1].x - x;    y0 = vList[n-1].y - y;
    for(int i = 0; i < n; i++)
    {
        x1 = vList[i].x - x;    y1 = vList[i].y - y;
        if(y0 > 0)
            if(y1 <= 0)
                if(x1*y0 > y1*x0)
                    cross++;
        else
            if(y1 > 0)
                if(x0*y1 > y0*x1)
                    cross++;
        x0 = x1; y0 = y1;
    }
    return cross & 1;
}
```



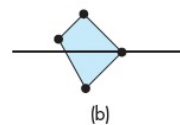
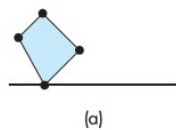
24

Jordan Curve Theorem

What if it goes through a vertex?

Treat these two cases differently:

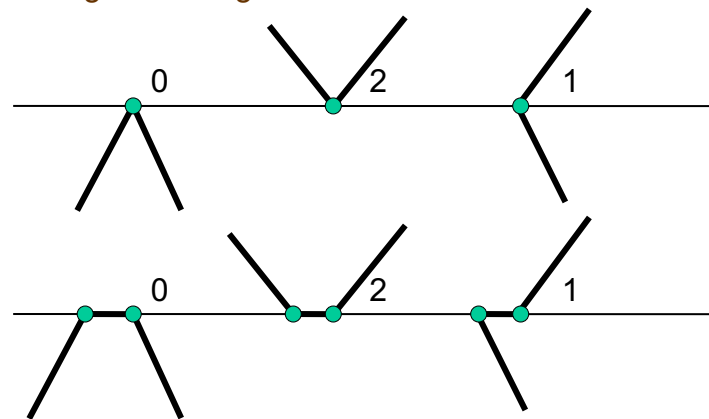
- case (a): the vertex-scanline intersection is counted as either **zero** or **two** edge crossings
- case (b): the vertex-scanline intersection must be counted as **one** edge crossing.



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Jordan Curve Theorem

What if it goes through a vertex?

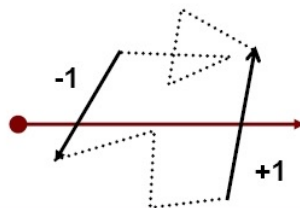


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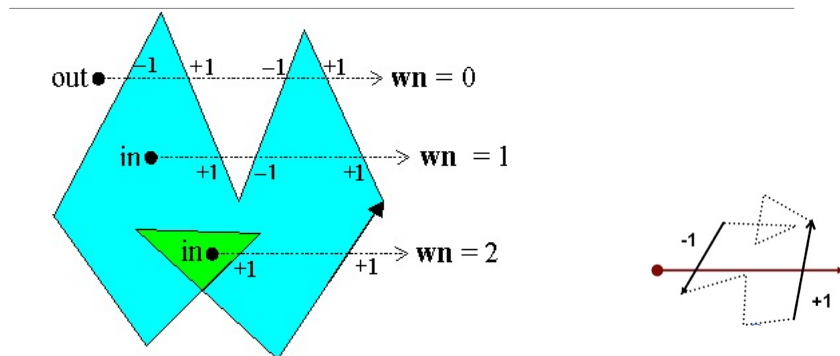
Jordan Curve Theorem

■ Non-zero winding rule:

- Draw a line from the test point to the outside
- Count +1 if you cross an edge in an anti-clockwise sense
- Count -1 if you cross an edge in a clockwise sense



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count +1, if you cross an edge in an anti-clockwise sense
 count -1, if you cross an edge in a clockwise sense
 Inside point : **non-zero**

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What does 'inside' mean?

How to test if a point is inside a polygon

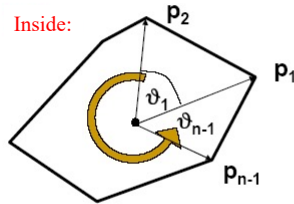
1. Half-space tests
2. Jordan Curve Theorem (even/odd or +1/-1)
3. **Winding number test**

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Winding Number Test-Method I

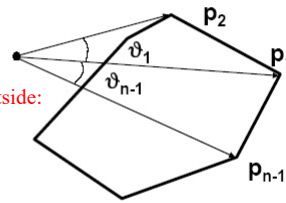
- Sum the angle subtended by the vertices

Inside:



$$\sum_{i=1}^n \vartheta_i = 2\pi$$

Outside:



$$\sum_{i=1}^n \vartheta_i \neq 2\pi$$

30

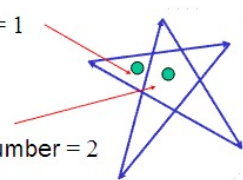
Winding Number Test – Method II

The number of times it is encircled by the edges of the polygon

- +1: if clockwise encirclements
- 1: if counterclockwise encirclements
- a point is **inside** the polygon if its winding number is **not zero**

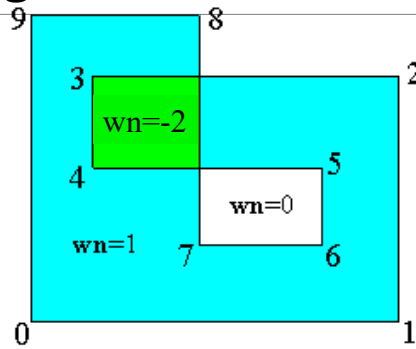
winding number = 1

winding number = 2



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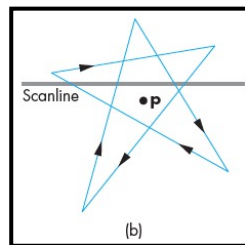
Winding Number Test II



+1: if clockwise encirclements
 -1 : if counterclockwise encirclements
 a point is inside the polygon if its winding number is not zero

32

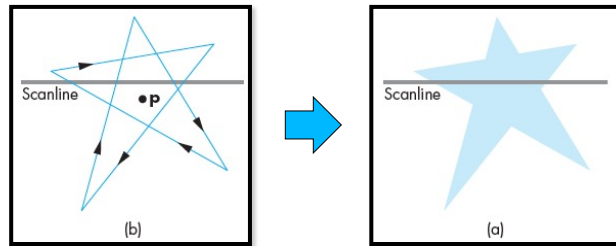
Winding Number Test – Method II



Filling with the Winding Number Test ?

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Winding Number Test

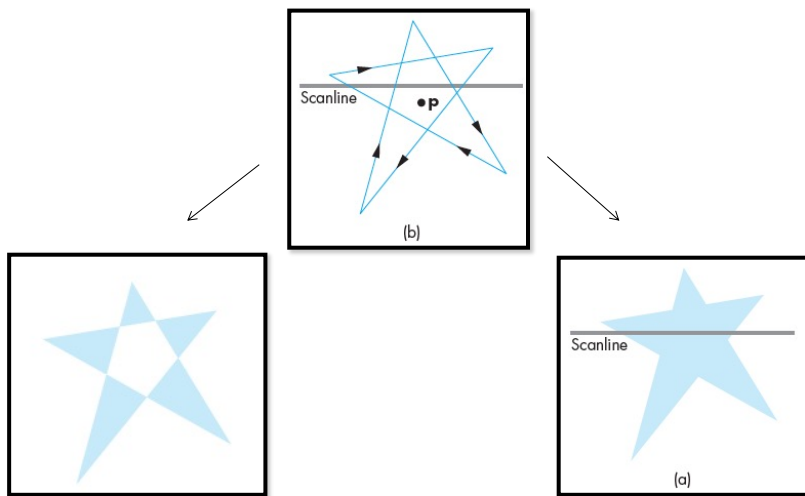


Filling with Winding Number Test II

Filling with the odd-even test ?

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Winding Number Test



Filling with the odd-even test

Filling with Winding Number Test

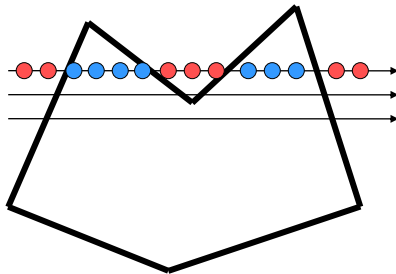
35

rasterization algorithms

36

Scan Line Algorithms

Take advantage of coherence in “insided-ness”

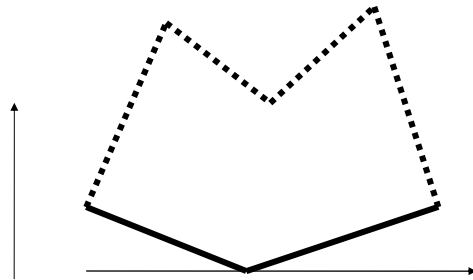


Inside/outside can only change at edge events
Current edges can only change at vertex events

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Scan Line Algorithms

Create a list of the edges intersecting the first scanline



Sort this list by the edge's x value on the first scanline

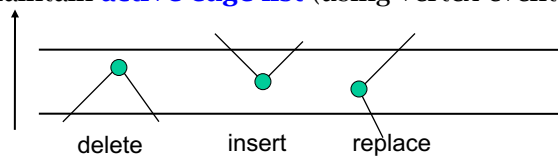
Call this the **active edge list**

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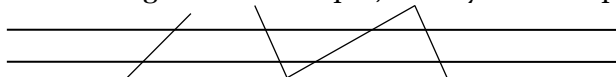
Scan Line Algorithms

For each scanline:

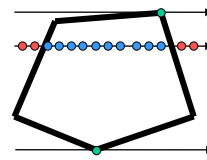
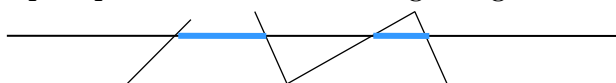
1. Maintain **active edge list** (using vertex events)



2. Increment edge's x-intercepts, sort by x-intercepts



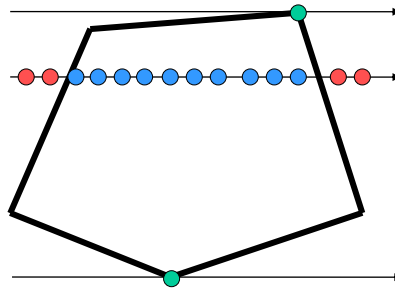
3. Output spans between left and right edges



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Convex Polygons

Convex polygons only have 1 span



Insertion and deletion events happen only once

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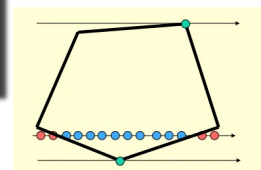
Crow's Algorithm

Step1: Find the vertex with the smallest y value to start

```
crow ( vertex vList[], int n)
```

```
    int iMin = 0;  
    for(int i = 1; i < n; i++)  
        if(vList[i].y < vList[iMin].y)  
            iMin = i;
```

```
    scanY(vList,n,iMin);
```



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Crow's Algorithm

Step2: Scan upward maintaining the **active edge list**

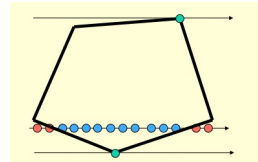
```

scanY(vertex vList[], int n, int i)
    int li, ri; // left & right upper endpoint indices
    int ly, ry; // left & right upper endpoint y values
    vertex l, dl; // current left edge and delta
    vertex r, dr; // current right edge and delta
    int rem; // number of remaining vertices
    int y; // current scanline

    li = ri = i;
    ly = ry = y = ceil(vList[i].y);

```

- (1) for(rem = n; rem > 0)
- (2) // find appropriate left edge
// find appropriate right edge
- (3) // while l & r span y (the current scanline)
// draw the span



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Crow's Algorithm

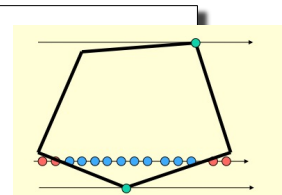
Find the appropriate next left edge

```

(2) while( ly <= y && rem > 0)
    rem--;
    i = li - 1;
    if(i < 0)
        i = n-1; // go clockwise

    ly = ceil( v[i].y );
    if( ly > y ) // replace left edge
        differenceY( &vList[li], &vList[i], &l, &dl, y);
    li = i; // index of the left endpoint

```



all vertices are in
counter-clockwise order
index: 0~(n-1)

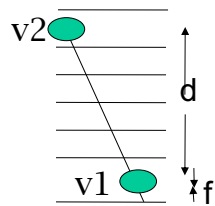
44

Crow's Algorithm

Calculate delta and starting values

```
differenceY(vertex *v1, vertex *v2, vertex *e, vertex *de, int y)
    difference(v1, v2, e, de, (v2.y - v1.y), y - v1.y);
```

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
    de.x = (v2.x - v1.x) / d;
    e.x = v1.x + f * de.x;
```



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Crow's Algorithm

Draw the spans

```
(3)for( ; y < ly && y < ry; y++)
    // scan and interpolate edges
    scanX(&l, &r, y);
    increment(&l,&dl);
    increment(&r,&dr);
```

Increment the x value

```
increment(vertex *edge, vertex *delta)
    edge.x += delta.x;
```

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Crow's Algorithm

Draw the spans

```
scanX(vertex *l, vertex *r, int y)
    int x, lx, rx;
    vertex s, ds;

    lx = ceil(l.x);
    rx = ceil(r.x);
    if(lx < rx)
        for(x = lx, x < rx; x++)
            setPixel(x,y);
```

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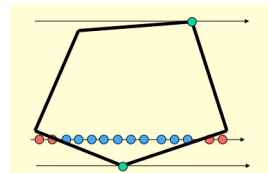
Crow's Algorithm

Step2: Scan upward maintaining the **active edge list**

```
scanY(vertex vList[], int n, int i)
    int li, ri; // left & right upper endpoint indices
    int ly, ry; // left & right upper endpoint y values
    vertex l, dl; // current left edge and delta
    vertex r, dr; // current right edge and delta
    int rem; // number of remaining vertices
    int y; // current scanline
```

```
    li = ri = i;
    ly = ry = y = ceil(vList[i].y);
```

- (1) for(rem = n; rem > 0)
- (2) // find appropriate left edge
// find appropriate right edge
- (3) // while l & r span y (the current scanline)
// draw the span



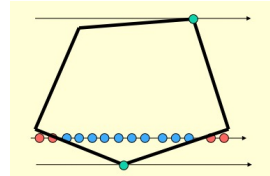
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Crow's Algorithm

Step2: Scan upward maintaining the **active edge list**

```
scanY(vertex vList[], int n, int i)
//...
li = ri = i;
ly = ry = y = ceil(vList[i].y);

(1) for( rem = n; rem > 0)
    // find appropriate left edge
(2) while( ly <= y && rem > 0)
    //...
    // find appropriate right edge
    while( ry <= y && rem > 0)
    //...
(3) for( ; y < ly && y < ry; y++) // while l & r span y (the current scanline)
    scanX(&l, &r, y); // draw the span
    increment(&l,&dl);
    increment(&r,&dr);
```



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Crow's Algorithm

Draw the spans

```
scanX(vertex *l, vertex *r, int y)
int x, lx, rx;
vertex s, ds;

lx = ceil(l.x);
rx = ceil(r.x);
if(lx < rx)
    differenceX(l, r, &s, &ds, lx);
    for(x = lx, x < rx; x++)
        setPixel(x,y);
        increment(&s,&ds);
```

Interpolating other values
E.g, colors

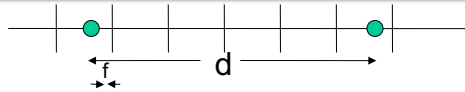
50

Crow's Algorithm

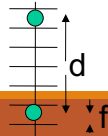
Calculate delta and starting values

```
differenceX(vertex *v1, vertex *v2, vertex *e, vertex *de, int x)
    difference(v1, v2, e, de, (v2.x - v1.x), x - v1.x);
```

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
    de.x = (v2.x - v1.x) / d;
    e.x = v1.x + f * de.x;
```



```
differenceY(vertex *v1, vertex *v2, vertex *e, vertex *de, int y)
    difference(v1, v2, e, de, (v2.y - v1.y), y - v1.y);
```



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Crow's Algorithm

Draw the spans

```
scanX(vertex *l, vertex *r, int y)
    int x, lx, rx;
    vertex s, ds;
    lx = ceil(l->x);
    rx = ceil(r->x);
    if(lx < rx)
        differenceX(l, r, &s, &ds, lx);
    for(x = lx, x < rx; x++)
        setPixel(x,y);
        increment(&s,&ds);
```

```
increment(vertex *edge, vertex *delta)
    edge.x += delta.x;
```

```
differenceX(vertex *v1, vertex *v2, vertex *e, vertex *de, int x)
    difference(v1, v2, e, de, (v2.x - v1.x), x - v1.x);
```

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
    de->x = (v2.x - v1.x) / d;
    e->x = v1.x + f * de.x;
```

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Crow's Algorithm

Interpolating other values

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
    de.x = (v2.x - v1.x) / d;
    e.x = v1.x + f * de.x;
    de.r = (v2.r - v1.r) / d;
    e.r = v1.r + f * de.r;
    de.g = (v2.g - v1.g) / d;
    e.g = v1.g + f * de.g;
    de.b = (v2.b - v1.b) / d;
    e.b = v1.b + f * de.b;
```

```
increment( vertex *v, vertex *dv)
    v.x += dv.x;
    v.r += dv.r;
    v.g += dv.g;
    v.b += dv.b;
```