

# Assignment 1

## Background Subtraction in Video Streams

DUE: Tuesday, February 7, 2023 at 11:59 pm EST  
Late assignments will **NOT** be accepted

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Use the Dynamic Mode Decomposition method on the video clips **ski\_drop.mov** and **monte\_carlo.mov** containing a foreground and background object and separate the video stream to both the foreground video and a background.

The DMD spectrum of frequencies can be used to subtract background modes. Specifically, assume that  $\omega_p$ , where  $p \in \{1, 2, \dots, \ell\}$ , satisfies  $\|\omega_p\| \approx 0$ , and that  $\|\omega_j\| \forall j \neq p$  is bounded away from zero. Thus,

$$\mathbf{X}_{\text{DMD}} = \underbrace{b_p \boldsymbol{\varphi}_p e^{\omega_p \mathbf{t}}}_{\text{Background Video}} + \underbrace{\sum_{j \neq p} b_j \boldsymbol{\varphi}_j e^{\omega_j \mathbf{t}}}_{\text{Foreground Video}} \quad (1)$$

Assuming that  $\mathbf{X} \in \mathbb{R}^{n \times m}$ , then a proper DMD reconstruction should also produce  $\mathbf{X}_{\text{DMD}} \in \mathbb{R}^{n \times m}$ . However, each term of the DMD reconstruction is complex:  $b_j \boldsymbol{\varphi}_j \exp(\omega_j \mathbf{t}) \in \mathbb{C}^{n \times m} \forall j$ , though they sum to a real-valued matrix. This poses a problem when separating the DMD terms into approximate low-rank and sparse reconstructions because real-valued outputs are desired and knowing how to handle the complex elements can make a significant difference in the accuracy of the results. Consider calculating the DMD's approximate low-rank reconstruction according to

$$\mathbf{X}_{\text{DMD}}^{\text{Low-Rank}} = b_p \boldsymbol{\varphi}_p e^{\omega_p \mathbf{t}}.$$

Since it should be true that

$$\mathbf{X} = \mathbf{X}_{\text{DMD}}^{\text{Low-Rank}} + \mathbf{X}_{\text{DMD}}^{\text{Sparse}},$$

then the DMD's approximate sparse reconstruction,

$$\mathbf{X}_{\text{DMD}}^{\text{Sparse}} = \sum_{j \neq p} b_j \boldsymbol{\varphi}_j e^{\omega_j \mathbf{t}},$$

can be calculated with real-valued elements only as follows...

$$\mathbf{X}_{\text{DMD}}^{\text{Sparse}} = \mathbf{X} - \left| \mathbf{X}_{\text{DMD}}^{\text{Low-Rank}} \right|,$$

where  $|\cdot|$  yields the modulus of each element within the matrix. However, this may result in  $\mathbf{X}_{\text{DMD}}^{\text{Sparse}}$  having negative values in some of its elements, which would not make sense in terms of having negative pixel intensities. These residual negative values can be put into a  $n \times m$  matrix  $\mathbf{R}$  and then be added back into  $\mathbf{X}_{\text{DMD}}^{\text{Low-Rank}}$  as follows:

$$\begin{aligned} \mathbf{X}_{\text{DMD}}^{\text{Low-Rank}} &\leftarrow \mathbf{R} + \left| \mathbf{X}_{\text{DMD}}^{\text{Low-Rank}} \right| \\ \mathbf{X}_{\text{DMD}}^{\text{Sparse}} &\leftarrow \mathbf{X}_{\text{DMD}}^{\text{Sparse}} - \mathbf{R} \end{aligned}$$

This way the magnitudes of the complex values from the DMD reconstruction are accounted for, while maintaining the important constraints that

$$\mathbf{X} = \mathbf{X}_{\text{DMD}}^{\text{Low-Rank}} + \mathbf{X}_{\text{DMD}}^{\text{Sparse}},$$

so that none of the pixel intensities are below zero, and ensuring that the approximate low-rank and sparse DMD reconstructions are real-valued. This method seems to work well empirically.