Signals & Systems

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Overview

- Signal
- Signal processing
- Signal Classification
  - Continuous time signals
  - Discrete time signals
  - Digital signals
  - Analog signals
- Systems
  - Impulse response
  - System properties
  - Linear time invariant (LTI systems)

Overview

- Sampling
- Frequency domain (Fourier Analysis)
- Discrete time fourier transform (DTFT)
    - DTFT important properties
- Aliasing
    - Aliasing in time domain
    - Aliasing in frequency domain
- Nyquist theorem
- Frequency Response

Overview

- Filter design
  - Filter type
  - FIR vs IIR
  - Filter order
  - Filter characteristics
- Summary

Signal

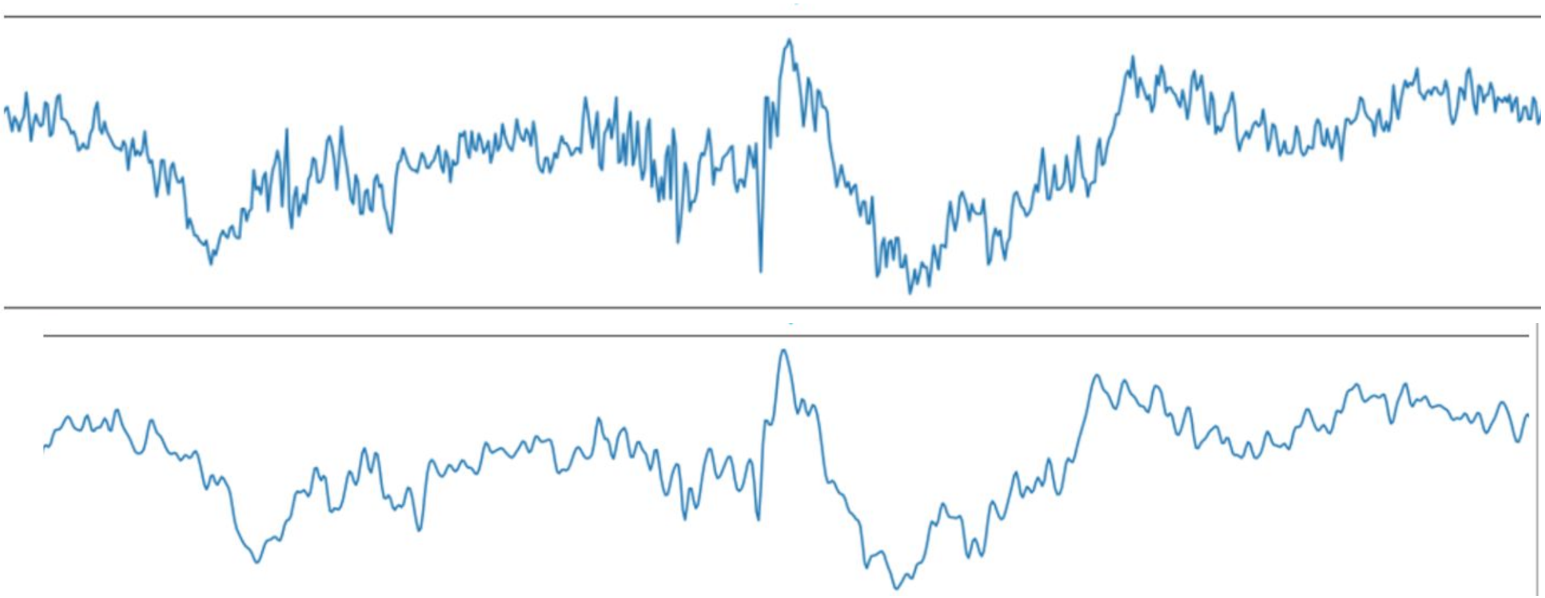
- Signal is something that conveys information
- Is used for communicating information between humans or between humans and machines
- Examples of signals:
    - Audio signals
        - sound, human voice, sound of a car, music instruments
    - Biological signals
        - EEG, EMG, ECG, etc.
    - Images, and Videos

Signal processing

- Is a subfield of mathematics and electrical engineering **[1]**
- Deals with analyzing and modifying signals **[1]**
- Application of signal processing
    - Audio and speech processing
    - Video and image processing
    - Biological signal processing
    - Biomedical imaging

Signal processing applications

- Biological signal processing



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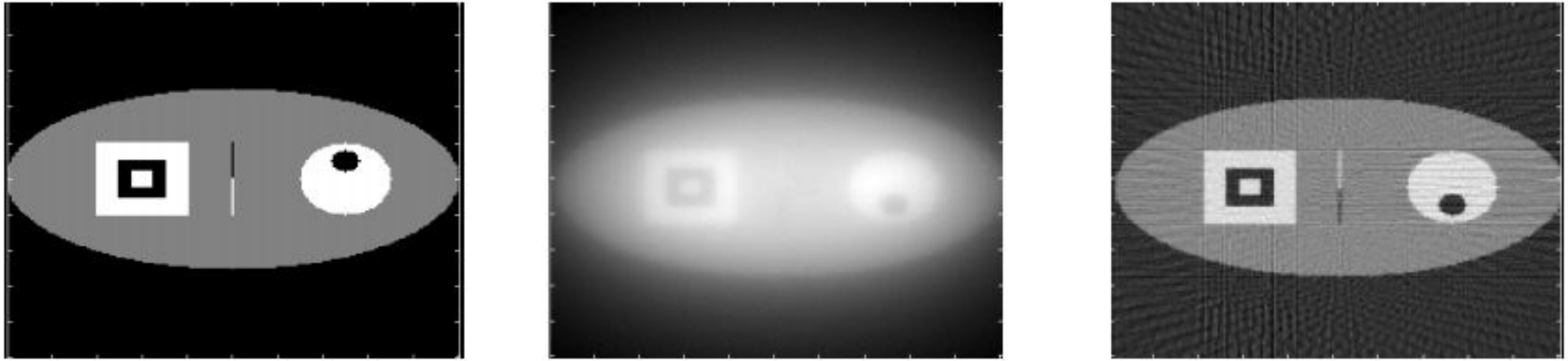
Signal processing applications

- Biomedical Image processing



Original image | Direct back-projection | Filtered back-projection

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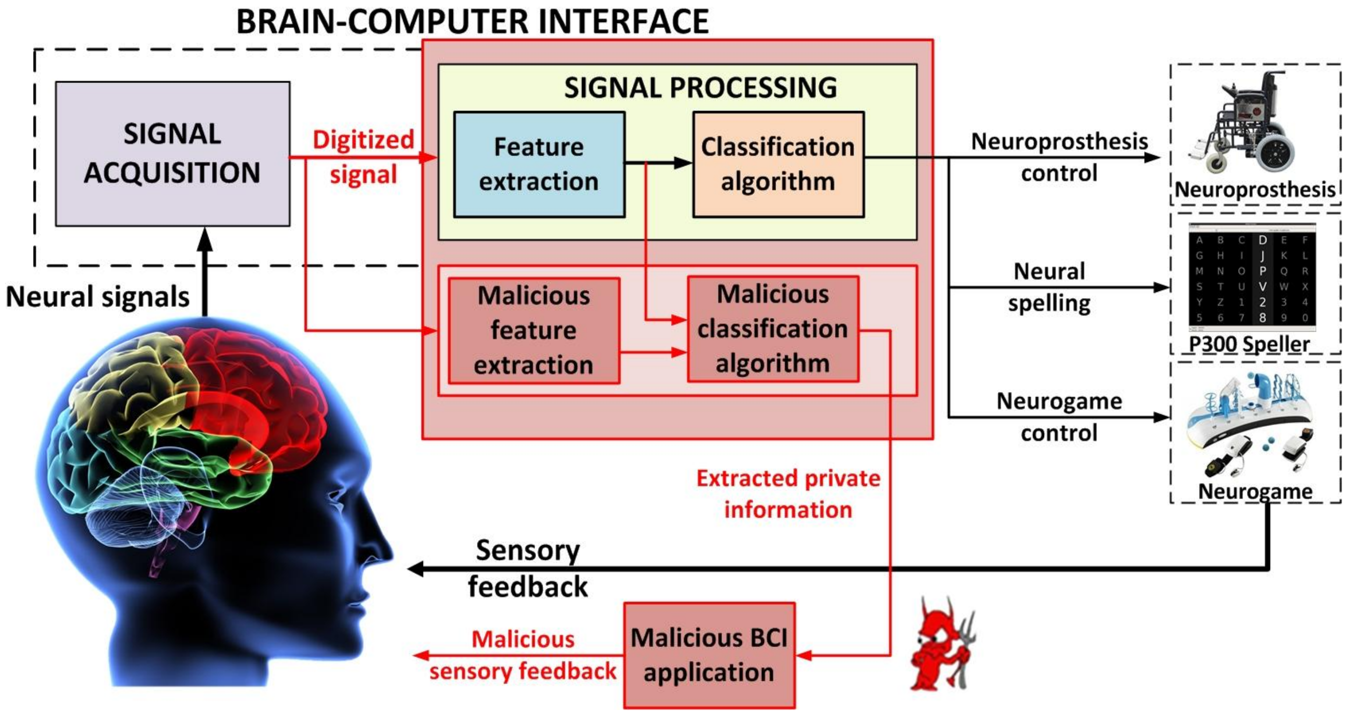
Signal processing applications



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Brain Computer Interface

https://www.cnn.com/2017/04/12/health/brain-computer-interface-partner/index.html

Signal

- A signal can be represented as a function of an ***independent variable***
- The independent variable can be time, space, etc
- Here we assume the independent variable is ***time***
- The signal amplitude is the ***dependent variable***
- Can classify signals into different groups

Signal classification

- ***<u>Continuous signal</u>***: the independent variable (time) takes continuous values
- Continuous signal is denoted by x(t)
- ***<u>Discrete signal</u>***: the independent variable (time) takes discrete values
- Discrete signal is denoted by x[n]
- ***<u>Analog signal</u>***: the dependent variable takes continuous values
- ***<u>Digital signal:</u>*** both dependent and independent variables are discrete
- In computers we deal with digital signals

Signal classification

Discrete time signals

- Represented as a sequence of numbers
- Defined at integer values, undefined everywhere else

Signal classification

- ***<u>One dimensional (1D)</u>*** signal is represented as a function of one variable (usually time)
  - x(t), x[n]
- ***<u>Two dimensional (2D)</u>*** signal is represented as a function of two variables
  - f(x,t)
  - Images

Important Signals

- One specific discrete time signal of special importance in signal processing is the ***<u>unit impulse (delta)</u>*** function

Periodic signals

- A ***periodic signal*** is a signal that repeats
- The period is the number for which the signal repeats
- A signal that is not periodic is referred to as an ***aperiodic signal***
- Is denoted by T in continuous time
- Is denoted by N in discrete time

Frequency

- Represents the number of cycles in one second
- Denoted by f and has a unit of Hz (hertz)
- We can represents signals frequency as a function of time (t)
- We can also represent signals as function of frequency (f)

Signal manipulation

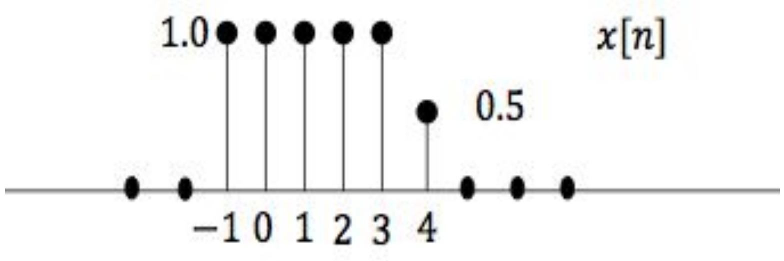
Time shift

Shift the signal by k → Replace n with n-k

x[n] → x[n-k]

Question

The following is a discrete time signal. Sketch x[n-2] !



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System

- System takes an input and maps it to an output
- Some examples of systems are:
  - Amplifiers, Filters, etc
- Continuous time systems have a continuous time input and output
- Discrete time systems have a discrete time input and output

System

Discrete time Systems

- Maps a ***discrete time input*** signal to a ***discrete time output*** signal

Impulse Response

- If the input to the system is the unit impulse signal, the output of the system is called the ***impulse response***
- The impulse response in represented by h[n]

Example

Some important properties of discrete-time systems

- ***Linearity***

Some important properties of discrete-time systems

- ***Time Invariance***: implies that if we shift the input by a value(n0) the output of the system will be shifted by the same value(n0)

LTI systems

- Time invariance and linearity property together make up an important class of systems
- These systems are referred to as ***Linear Time Invariant (LTI) systems***
- LTI systems have great importance in signal and system analysis

LTI systems

<u>Convolution</u>:

- The output of an LTI system can be represented by the convolution of the input with the impulse response of the system
- Convolution is denoted by “ * “
- Convolution is an operation like multiplication and summation

Some important operations on discrete time systems

<u>Convolution:</u>

- If the impulse response of an LTI system is known, the output corresponding to any input can be calculated

Sampling

- Most discrete signals result from sampling continuous time signals
- T denotes the ***sampling period***
- 1/T = f is the the ***sampling frequency***

Sampling

-

Fourier Analysis

- So far we talked about signals as a function of time (time domain)
- Another way to think of signals is a function of their frequency components
- Fourier analysis converts a signal from time domain to frequency domain

Fourier Analysis

https://www.coursera.org/lecture/cryo-em/1-d-sine-waves-and-their-sums-78uHF

Discrete Time Fourier Transform (DTFT)

- Recall: last week we introduced the fourier transform
- Fourier transform (FT) is the frequency representation of a continuous time signal
- The frequency representation of a ***<u>discrete</u>*** and ***<u>aperiodic</u>*** signal is called the ***<u>discrete time fourier transform (DTFT)</u>***
- The DTFT is the periodic (repetition of the fourier transform)

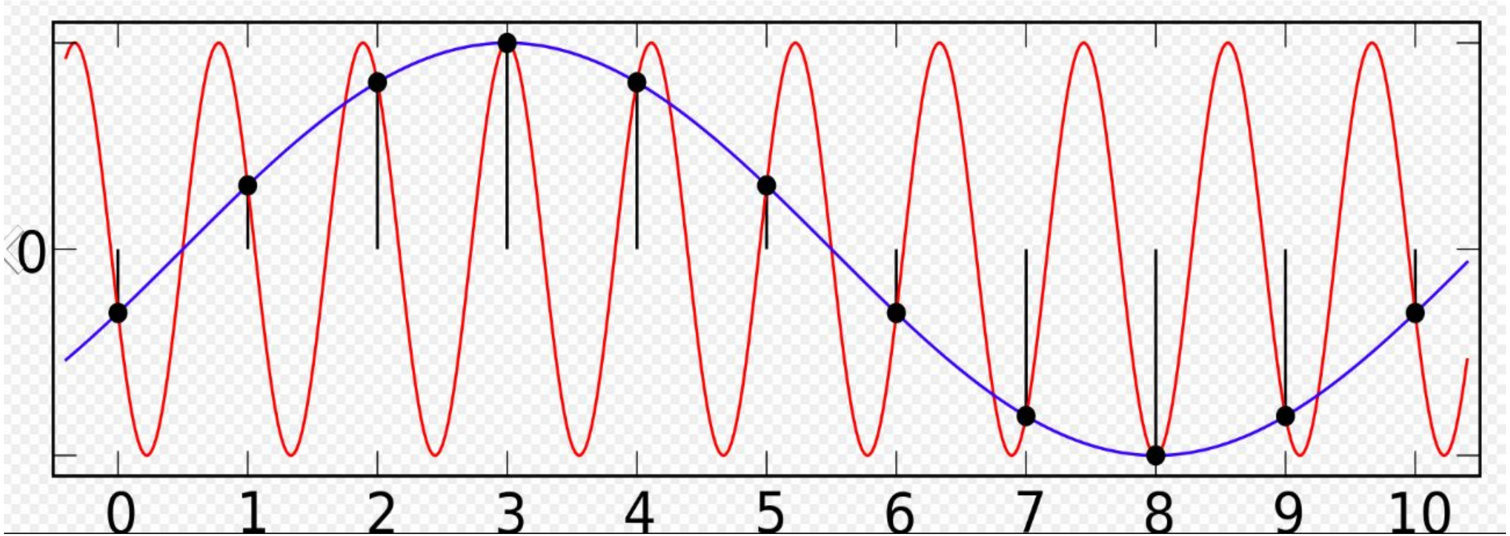
Example

Aliasing

- Assume we want to recover the original continuous time signal from the sampled discrete time signal
- If the recovered continuous signal is different from the original one ***Aliasing*** has occurred
- If multiple signals result from the reconstruction they are called aliases
- If the frequency components overlap this is called ***Aliasing***

Aliasing

- Aliasing in time domain



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Aliasing

- In frequency domain

Aliasing

[6]

Nyquist Theorem

- To prevent aliasing we utilize the ***<u>Nyquist theorem</u>***
- If highest frequency component in signal is at f_m
- The sampling rate fs has to be at least 2fm

- Also 2fm is called the ***<u>Nyquist rate</u>***

Frequency Response

- The frequency representation of the impulse response (h[n]) is called the ***<u>Frequency response</u>*** (H(w))

Important note

Time Domain		Frequency Domain
Convolution	$\longleftrightarrow$	Multiplication
Multiplication	$\longleftrightarrow$	Convolution

Filters

- One special type of ***LTI systems***
- Have many applications
- Can be used to remove, reduce, amplify the frequency content of signals
- Have to consider different parameters when designing filters
- Filters perform their operation on signals in ***time domain using convolution(*)***, and using ***multiplication in the frequency domain***

Example

- How do filters perform:

Filters

- ***Cut off frequency*** :
    - Also called the corner frequency
    - Is the frequency at which some frequency content begins to be eliminated or reduced

Filter Types

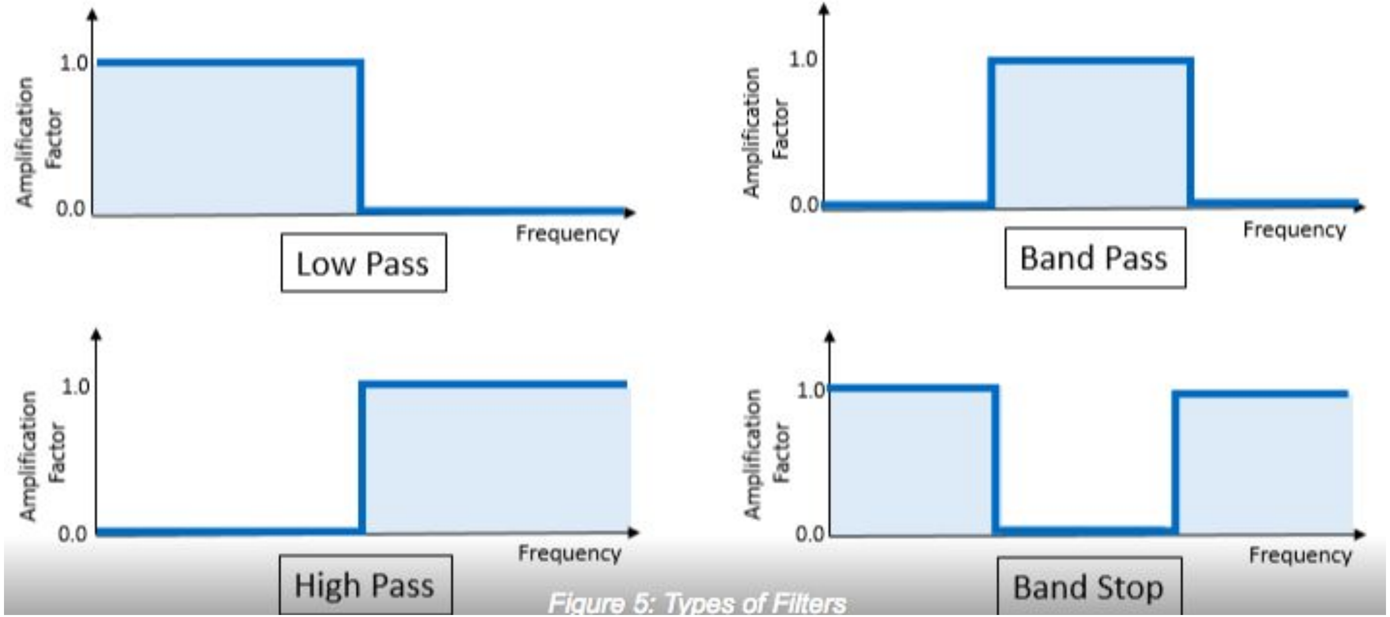

Figure 5: Types of Filters

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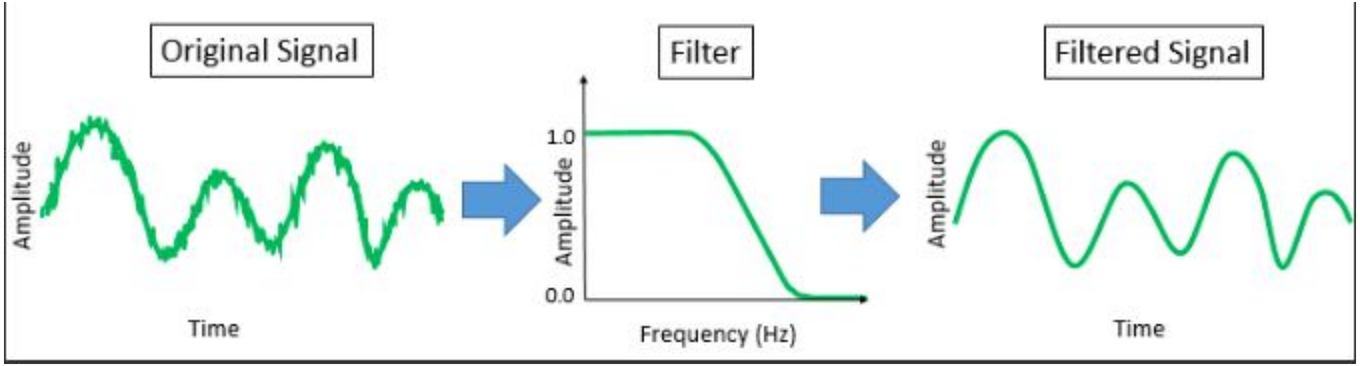
Example



Original Signal
Amplitude
Time
Filter
1.0
0.0
Amplitude
Frequency (Hz)
Filtered Signal
Amplitude
Time

[7]

Example

Filter Characteristics

- Pass Band
- Transition Band
- Stop Band

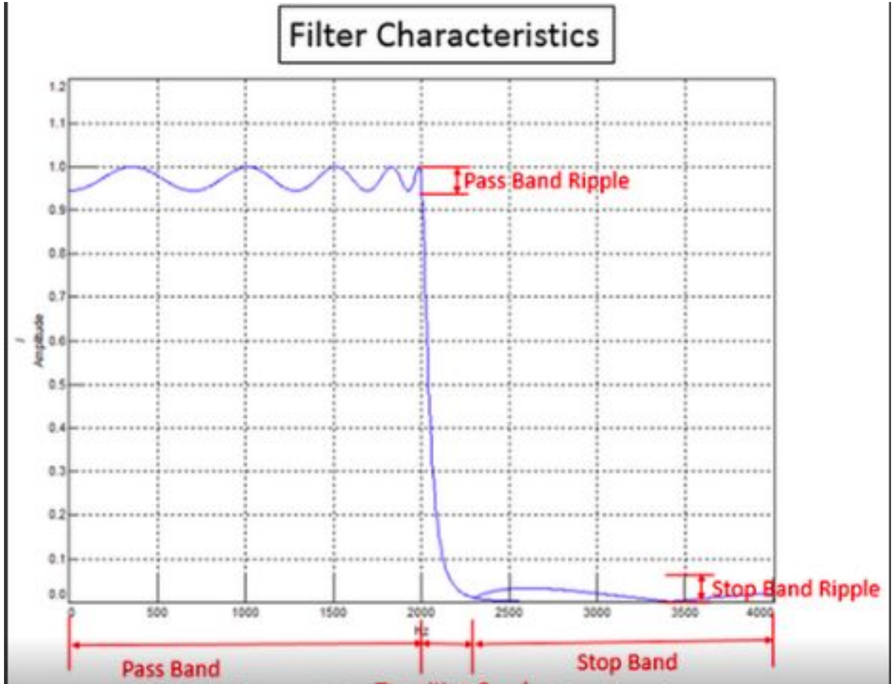


Figure 17: Pass, Transition, and Stop band.

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FIR vs IIR

- ***FIR*** stands for Finite impulse response
- ***IIR*** stands for Infinite impulse response
- Remember the impulse response (the system output to the delta function) h[n]
- In ***FIR*** filters h[n] has a finite duration
- In ***IIR*** filters h[n] does not have finite duration(it is infinite)

FIR vs IIR

FIR Filter Equation: $y(n) = \sum_{k=0}^{N} a(k)x(n-k)$

IIR Filter Equation: $y(n) = \sum_{k=0}^{N} a(k)x(n-k) + \sum_{j=0}^{P} b(j)\underbrace{y(n-j)}_{\text{Output used recursively}}$

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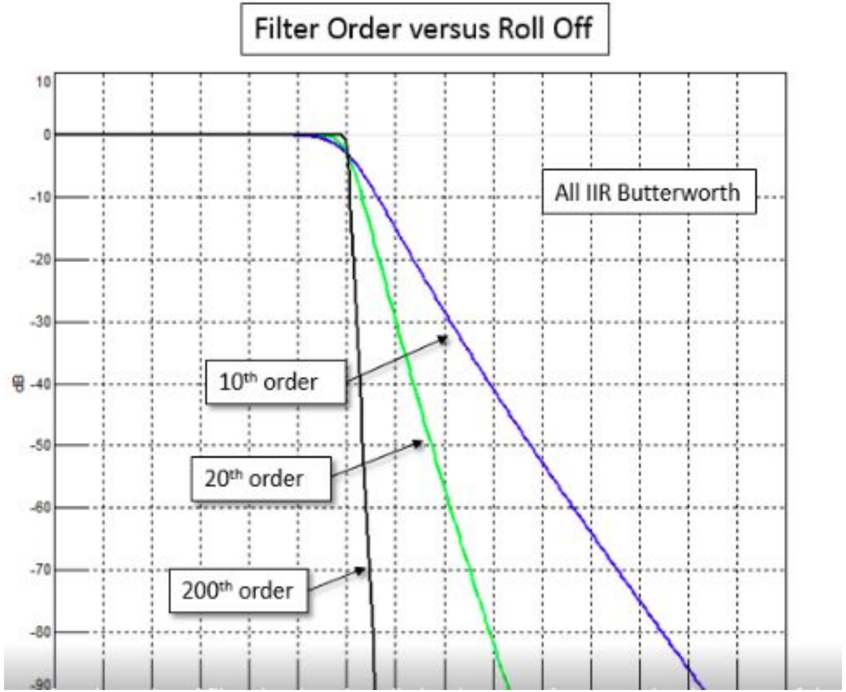
FIR vs IIR

- In ***FIR*** output corresponding to each input is calculated only using the input
- In ***IIR*** the output is calculated using the input and past output
- This is why IIR filters are computationally faster than FIR filters

Filter Order

- Refers delay in the filter
- Higher the order of the filter shaper the transition

Filter Order



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References

[1] https://en.wikipedia.org/wiki/Signal_processing

[2]https://github.com/neurotechuoft/Workshops/blob/master/workshop_2018_2019/notebooks/exercises/wk2b_intro_to_signal_processing.ipynb

[3] https://q.utoronto.ca/courses/65501/files/868141?module_item_id=150010

[4] http://brl.ee.washington.edu/neural-engineering/bci-security/

[5] Digital Signal Processing, Allan V. Oppenheim

References

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[7] https://radiopaedia.org/articles/aliasing-in-mri

[8]https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/Introduction-to-Filters-FIR-versus-IIR/ta-p/520959?fbclid=IwAR2y2k1H5grd_18LHcPI0Vvpliab2HpJzCOclb1K_AEYD7KUMxSMtYwk1bc