HW 5: Neural networks I and HW 6: Neural networks II

HW5 due Apr. 16 and HW6 due Apr. 23. Both are Sundays and both at 11:59PM central time.

Pre-compiled PDF is here. Command to compile on your own: `pandoc hw5_and_6.md -o hw5_and_6.pdf` Figures were compiled from their LaTeX source files under `figs` folder.

Please show intermediate steps for all computational problems below. Giving only the final result will result in zero point. For numerical answers, **keep 3 digits after the decimal point**.

For Problems 7 and above, write steps in matrix form as long as you can to save your time. Do NOT detail sub-matrix steps – that's a waste of time. You are encouraged to use computers to evaluate matrix operations rather than punching keys on a calculator. You are also encouraged to take advantage of the MiniNN library to do the computations for you.

How to submit: Just upload as PDF files to Canvas.

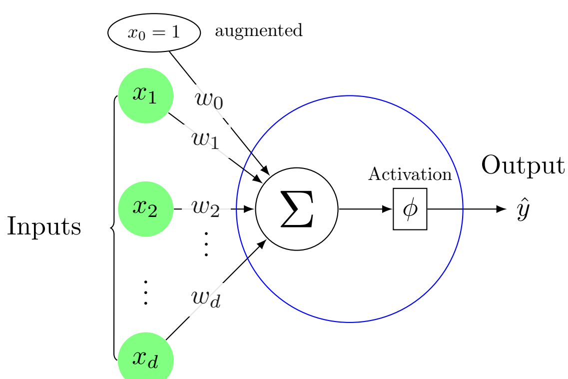
HW 5: basic and single-neuron operations [10pts plus 4 bonus pts]

1. [1pt] What is the Hadamard product $A \circ B$ between the following two matrixes?

   $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

   $B = \begin{pmatrix} 0.5 & 0.1 & 0.3 \\ -1 & -20 & 1.5 \end{pmatrix}$

2. [2pt] Continuing from Problem 1 above, what is the product AB^T? And what is the product BA^T?
3. [1pt] Continuing from Problems 1 and 2 above, is there a product AB? Why?
4. [1pt] Continuing from Problems 1, 2, and 3, above, given $f(x) = x + 1$, what is the value of $f(AB^T)$?
5. [Bonus, 2pt] In slides, to expand Eq. (2), we used negative logistic loss (also called cross entropy loss) as E and logistic activation function as ϕ. What will be the new $\frac{\partial E}{\partial w_i}$ if we use squared error loss and linear activation function? Specifically, what if $E = (\hat{y} - y)^2$ (assume just one sample) and $\phi(\mathbf{w}^T\mathbf{x}) = \mathbf{w}^T\mathbf{x}$?
6. [2pt] Here is a diagram of a neuron.

   Suppose $d = 3$. If the augmented input vector $\mathbf{x} = [x_0, x_1, x_2, x_3]^T = [1, 0, 1, 0]^T$, and the weight vector $\mathbf{w} = [w_0, w_1, w_2, w_3]^T = [5, 4, 6, 1]^T$, and the activation function $\phi(x) = x^2$ (note that in function notation, the x in $\phi(x)$ here can be any number or vector. not to be confused with the input vector $\mathbf{x}$), what is the value of the prediction $\hat{y}$?

   Hint: Eq. (1)

7. [3pt] Continuing from problem 6 above, if the loss is defined as $E = \hat{y} - y$, what is the value of $\partial E / \partial x_1$? And what is the value of $\partial E / \partial w_1$? Please treat y as a constant.

   Hint for second question: Eq. (2). And think what is the new $\frac{\partial E}{\partial \hat{y}} = \frac{\partial \hat{y} - y}{\partial \hat{y}}$?

8. [Bonus, 2pt] What is the value of $\frac{\partial E}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial E}{\partial x_0} \\ \frac{\partial E}{\partial x_1} \\ \vdots \end{pmatrix}$?

   And what is the value of $\frac{\partial E}{\partial \mathbf{w}} = \begin{pmatrix} \frac{\partial E}{\partial w_0} \\ \frac{\partial E}{\partial w_1} \\ \vdots \end{pmatrix}$?

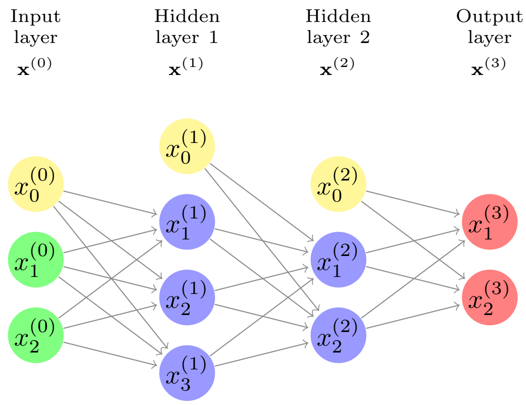
   Your answers should be two column vectors containing real values.

   Hint for second question: See the last equation on the same page with Eq. (2). But note that the E for that equation is neg log loss, not the assumed loss for Problem 7.

HW6: Operations on a neural network [10pts plus 5 bonus pts]

Hint: The slides "Recap:. . . " and "A grounded example. . . "

9. [1pt] Here is a neural network.



   Let $\mathbb{W}^{(l)}$ be the transfer matrix from layer l to layer $l + 1$, for all $l \in [0..2]$.

   What are the shapes (in terms of number of rows by number of columns, e.g., 5×4) for $\mathbb{W}^{(0)}$, $\mathbb{W}^{(1)}$, and $\mathbb{W}^{(2)}$ respectively?

10. [2pts] Continuing from Problem 9 above, if all weights in $\mathbb{W}^{(0)}$ are 0.1, all weights in $\mathbb{W}^{(1)}$ are 2, and all weights in $\mathbb{W}^{(2)}$ are 1, what are the values of all activations $\mathbf{x}^{(l)}$ for all $l \in [1..3]$? Assume the input vector $\mathbf{x}^{(0)} = [1, 1, 1]^T$, the activation function be logistic function, and bias is 1 $x_0^{(l)} = 1, \forall l \in [0..2]$. Express activations at each layer as a column vector.

11. [2.5pts] Continuing from Problems 9 and 10 above, if the target $\mathbf{y}$ is $[1, 0]^T$, what are the values of $\boldsymbol{\delta}^{(l)}$ for all $l \in \{2, 1\}$? Be sure to include $\delta_0^{(l)}$ on the bias term if applicable. Suppose we use negative logistic (cross entropy) loss, and logistic activation function. Here $\boldsymbol{\delta}^{(3)} = \hat{\mathbf{y}} - \mathbf{y}$ is 2×1 and the prediction $\hat{\mathbf{y}} = \mathbf{x}^{(3)}$.

12. [3pts] Continuing from Problems 9, 10, and 11 above, what are the values of $\nabla^{(l)} = \frac{\partial E}{\partial \mathbb{W}^{(l)}}$ for all $l \in [0..2]$?

13. [1.5pts] Finally, how should $\mathbb{W}^{(l)}$ given in Problem 9 be updated to based on $\nabla^{(l)}$ obtained in Problem 12, for all $l \in [0..2]$? Assume the learning rate $\rho = 1$.

14. [Bonus, 5pts] In the demo for Unit 5 Regression, we used a neural network with tanh as the activation function for all neurons. The range of tanh is from -1 to 1, which means that the output from that neural network is limited between -1 and 1. But in that problem, the target or the prediction ranges from 0 to 4. How do you explain? Look into the source code of scikit-learn to find out.