1 Lab: Predicates and Quantifiers

1. (3 points) Let Q(x, y) be "$x > y$" and the universe be all integers.

(a) Write the statement "For all x, there exists a y such that $x > y$" using quantifiers and Q(x, y).

(b) Write the statement "There exists an x such that for all y, $x > y$" using quantifiers and Q(x, y).

(c) Determine the truth value of the statement in (a).

2. (3 points) Determine if the following argument is valid: "All programmers are logical thinkers." "Alice is a programmer." "Therefore, Alice is a logical thinker." Explain your reasoning.

3. (2 points) Use predicates and quantifiers to express this statement: "Every student has access to at least one online resource for each course they are taking."

4. (3 points) Find a universe for variables x, y, z for which the statement $\forall x \forall y((x \neq y) \rightarrow \exists z(x < z < y))$ is true and another universe in which it is false. (Assume $<$ is a standard order).

5. (3 points) Find a counterexample, if possible, to this universally quantified statement, where the universe for all variables consists of all positive integers: $\forall x \forall y(x^2 - y^2 \neq 1)$

6. (5 points) Let P(n) be the predicate:
$P(n) : \forall n \epsilon \mathbb{N}, 2^n \geq n + 1$
Prove that $\forall n \geq 1, P(n)$ holds

7. (5 points) Prove that for all natural numbers N, if n^2 is even, then n is also even.