Basic Stats – 2

Part (a): 99% Confidence Interval Using Sample Standard Deviation x

Given data (durability in millions of characters):

{1.13,1.55,1.43,0.92,1.25,1.36,1.32,0.85,1.07,1.48,1.20,1.33,1.18,1.22,1.29}

1.Sample Mean($\bar{x}$):

$$\bar{x}=\frac{1.13+1.55+1.43+0.92+1.25+1.36+1.32+0.85+1.07+1.48+1.20+1.33+1.18+1.22+1.29}{15}$$

$$\bar{x} = \frac{17.08}{15} = 1.139$$

2.Sample Standard Deviation (s): First, find the squared deviations from the mean, sum them up, and then divide by n−1n - 1n−1, followed by taking the square root.

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$$

After calculating , we find:

$$S \approx 0.247$$

3.Determine the t-Value for 99% Confidence Interval:

For a 99% confidence interval with 14 degrees of freedom (15 - 1), the t-value can be found using a t-table or statistical software. For 99% confidence, this t-value is approximately .

$$t_{0.005,14} \approx 2.977$$

4.Construct the Confidence Interval:

CI = $\bar{x}$ ± Margin of Error = 1.2793 ± 0.126

CI = (1.1533,1.4053)

Therefore, the 99% confidence interval for the mean number of characters printed before failure is approximately (1.153, 1.405) million characters.

Part (b): 99% Confidence Interval Using Known Population Standard Deviation

Step-by-Step Calculation

1 . Determine the Population Standard Deviation

Given that the population standard deviation (σ) is known and is 0.2 million characters.

2. Compute the Confidence Interval

When the population standard deviation is known, we use the z-distribution.

For a 99% confidence interval, the critical value $Z_{0.005}$ is approximately:

$$Z_{0.005} \approx 2.576$$

The confidence interval is:

$$\text{CI} = \bar{X} \pm Z_{\propto/2} \cdot \frac{\propto}{\sqrt{n}}$$

$$\text{CI} = 1.139 \pm 2.576 \cdot \frac{0.2}{\sqrt{15}}$$

$$CI = 1.139 \pm 2.576 \cdot 0.0516$$

$$CI = 1.139 \pm 0.133$$

$$CI = (1.006, 1.272)$$

So, the 99% confidence interval for the mean durability, using the known population standard deviation, is approximately (1.006,1.272) million characters.

Rationale for Using the z-Distribution:

When the population standard deviation is known, the z-distribution is appropriate as it does not account for the additional uncertainty related to estimating the population standard deviation from the sample.

By comparing the two intervals, you can see how the confidence interval becomes narrower when the population standard deviation is known, reflecting the reduced uncertainty in the estimate.