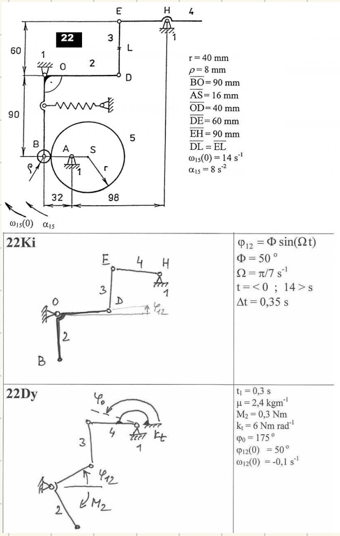
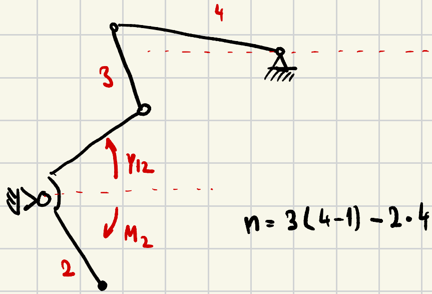
23-11

REFERÁT 1

VÍT HRDINA
MECHANIKA III

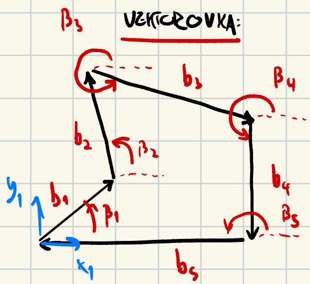
ZADÁNÍ 22





$$n = 3(4-1) - 2\cdot 4 = \underline{1^\circ}$$

VEKTOROVKA:



$$\vec{b}_1 + \vec{b}_2 + \vec{b}_3 + \vec{b}_4 + \vec{b}_5 = \emptyset$$

$b_1 = \overline{OD}$ $\qquad \beta_1 = q = \varphi_{12}$

$b_2 = \overline{DE}$ $\qquad \beta_2 = z_1$

$b_3 = \overline{EH}$ $\qquad \beta_3 = z_2$

$b_4 = \overline{DE}$ $\qquad \beta_4 = \frac{3\pi}{2}$

$b_5 = \overline{DE} + \overline{EH}$ $\qquad \beta_5 = \pi$

ROVNICE VEKTOROVKY:

$$0 = b_1 \cos\beta_1 + b_2\cos\beta_2 + b_3\cos\beta_3 + b_4\cos\beta_4 + b_5\cos\beta_5$$

$$0 = b_1 \sin\beta_1 + b_2\sin\beta_2 + b_3\sin\beta_3 + b_4\sin\beta_4 + b_5\sin\beta_5$$

$\frac{d}{dt}$

$$0 = -b_1\dot\beta_1\sin\beta_1 - b_2\dot\beta_2\sin\beta_2 - b_3\dot\beta_3\sin\beta_3$$

$$0 = b_1\dot\beta_1\cos\beta_1 + b_2\dot\beta_2\cos\beta_2 + b_3\dot\beta_3\cos\beta_3$$

$$\boxed{\underbrace{\begin{bmatrix} -b_2\sin\beta_2 & -b_3\sin\beta_3 \\ b_2\cos\beta_2 & b_3\cos\beta_3\end{bmatrix}}_{J_z}\underbrace{\begin{pmatrix}\dot\beta_2\\ \dot\beta_3\end{pmatrix}}_{\dot z} + \underbrace{\begin{bmatrix} -b_1\sin\beta_1 \\ b_1\cos\beta_1\end{bmatrix}}_{J_q}\underbrace{\begin{pmatrix}\dot\beta_1\end{pmatrix}}_{\dot q} = \emptyset}$$

$\frac{d}{dt}$

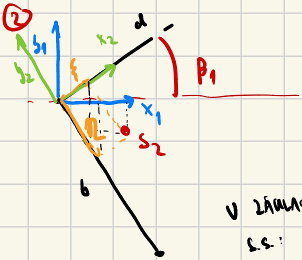
$$0 = -b_1\ddot\beta_1\sin\beta_1 - b_1\dot\beta_1^2\cos\beta_1 - b_2\ddot\beta_2\sin\beta_2 - b_2\dot\beta_2^2\cos\beta_2 - b_3\ddot\beta_3\sin\beta_3 - b_3\dot\beta_3^2\cos\beta_3$$

$$0 = b_1\ddot\beta_1\cos\beta_1 - b_1\dot\beta_1^2\sin\beta_1 + b_2\ddot\beta_2\cos\beta_2 - b_2\dot\beta_2^2\sin\beta_2 + b_3\ddot\beta_3\cos\beta_3 - b_3\dot\beta_3^2\sin\beta_3$$

$$\therefore \boxed{\underbrace{\begin{bmatrix} -b_2\sin\beta_2 & -b_3\sin\beta_3 \\ b_2\cos\beta_2 & b_3\cos\beta_3\end{bmatrix}}_{J_z}\underbrace{\begin{pmatrix}\ddot\beta_2\\ \ddot\beta_3\end{pmatrix}}_{\ddot z} + \underbrace{\begin{bmatrix} -b_1\sin\beta_1 \\ b_1\cos\beta_1\end{bmatrix}}_{J_q}\underbrace{\begin{pmatrix}\ddot\beta_1\end{pmatrix}}_{\ddot q} + \underbrace{\begin{bmatrix} -b_1\dot\beta_1^2\cos\beta_1 - b_2\dot\beta_2^2\cos\beta_2 - b_3\dot\beta_3^2\cos\beta_3 \\ -b_1\dot\beta_1^2\sin\beta_1 - b_2\dot\beta_2^2\sin\beta_2 - b_3\dot\beta_3^2\sin\beta_3\end{bmatrix}}_{\dot J_{qz}} = \emptyset}$$

$$\boxed{\ddot z = -J_z^{-1}\left(J_q\ddot q + \dot J_{qz}\right)}$$

POLOHY STŘEDŮ HMOTNOSTI

② 

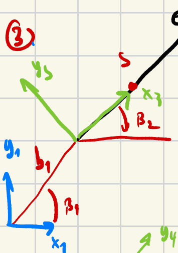
$$x_{2_S} = \frac{d \cdot \frac{d}{2} + b \cdot 0}{b+d} = \frac{d^2/2}{b+d} = \xi$$

$$y_{2_S} = \frac{d \cdot 0 + b\left(-\frac{b}{2}\right)}{b+d} = \frac{-b^2/2}{b+d} = \eta$$

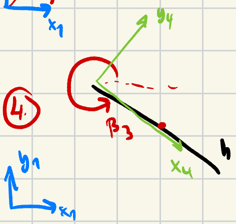
↓ ROTACE

V ZÁKLADNÍM S.S.:

$$x_{2S_I} = \xi \cos(\beta_1) - \eta \sin(\beta_1)$$
$$y_{2S_I} = \xi \sin(\beta_1) + \eta \cos(\beta_1)$$

③ 

$$x_{3_S} = \frac{e}{2} \qquad y_{3_S} = 0 \longrightarrow \begin{aligned} x_{3S_I} &= \underbrace{\frac{e}{2}\cos(\beta_2)}_{\text{ROTACE}} + \underbrace{b_1 \cos\beta_1}_{\text{POSUNUTÍ (DIAGRAM VE VEKTOROVCE)}} \\ y_{3S_I} &= \frac{e}{2}\sin(\beta_2) + b_1 \sin\beta_1 \end{aligned}$$

④ 

$$x_{4_S} = \frac{h}{2} \qquad y_{4_S} = 0 \longrightarrow \begin{aligned} x_{4S_I} &= \underbrace{\frac{h}{2}\cos(\beta_3)}_{\text{ROTACE}} + \underbrace{b_1\cos\beta_1 + b_2\cos\beta_2}_{\text{POSUNUTÍ (DIAGRAM VE VEKTOROVCE)}} \\ y_{4S_I} &= \frac{h}{2}\sin(\beta_3) + b_1\sin\beta_1 + b_2\sin\beta_2 \end{aligned}$$

DERIVACE:

① v:
$$\dot{x}_{2S_I} = -\xi \sin(\beta_1)\dot{\beta}_1 - \eta\cos(\beta_1)\dot{\beta}_1$$
$$\dot{y}_{2S_I} = \xi\cos(\beta_1)\dot{\beta}_1 - \eta\sin(\beta_1)\dot{\beta}_1$$

a:
$$\ddot{x}_{2S_I} = -\xi\cos(\beta_1)\dot{\beta}_1^2 - \xi\sin(\beta_1)\ddot{\beta}_1 + \eta\sin(\beta_1)\dot{\beta}_1^2 - \eta\cos(\beta_1)\ddot{\beta}_1$$
$$\ddot{y}_{2S_I} = -\xi\sin(\beta_1)\dot{\beta}_1^2 + \xi\cos(\beta_1)\ddot{\beta}_1 - \eta\cos(\beta_1)\dot{\beta}_1^2 - \eta\sin(\beta_1)\ddot{\beta}_1$$

② v:
$$\dot{x}_{3S_I} = -\frac{e}{2}\sin(\beta_2)\dot{\beta}_2 - b_1\sin(\beta_1)\dot{\beta}_1$$
$$\dot{y}_{3S_I} = \frac{e}{2}\cos(\beta_2)\dot{\beta}_2 + b_1\cos(\beta_1)\dot{\beta}_1$$

a:
$$\ddot{x}_{3S_I} = -\frac{e}{2}\cos(\beta_2)\dot{\beta}_2^2 - \frac{e}{2}\sin(\beta_2)\ddot{\beta}_2 - b_1\cos(\beta_1)\dot{\beta}_1^2 - b_1\sin(\beta_1)\ddot{\beta}_1$$
$$\ddot{y}_{3S_I} = -\frac{e}{2}\sin(\beta_2)\dot{\beta}_2^2 + \frac{e}{2}\cos(\beta_2)\ddot{\beta}_2 - b_1\sin(\beta_1)\dot{\beta}_1^2 + b_1\cos(\beta_1)\ddot{\beta}_1$$

3.) v: $\dot{x}_{4s_I} = -\frac{h}{2}\sin(\beta_3)\dot{\beta}_3 - b_1\sin(\beta_1)\dot{\beta}_1 - b_2\sin(\beta_2)\dot{\beta}_2$

$\dot{y}_{4s_I} = \frac{h}{2}\cos(\beta_3)\dot{\beta}_3 + b_1\cos(\beta_1)\dot{\beta}_1 + b_2\cos(\beta_2)\dot{\beta}_2$

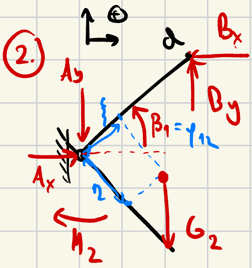
a: $\ddot{x}_{4s_I} = -\frac{h}{2}\cos(\beta_3)\dot{\beta}_3^2 - \frac{h}{2}\sin(\beta_3)\ddot{\beta}_3 - b_1\cos(\beta_1)\dot{\beta}_1^2 - b_1\sin(\beta_1)\ddot{\beta}_1 - b_2\cos(\beta_2)\dot{\beta}_2^2 - b_2\sin(\beta_2)\ddot{\beta}_2$

$\ddot{y}_{4s_I} = -\frac{h}{2}\sin(\beta_3)\dot{\beta}_3^2 + \frac{h}{2}\cos(\beta_3)\ddot{\beta}_3 - b_1\sin(\beta_1)\dot{\beta}_1^2 + b_1\cos(\beta_1)\ddot{\beta}_1 - b_2\sin(\beta_2)\dot{\beta}_2^2 + b_2\sin(\beta_2)\ddot{\beta}_2$

$$\underbrace{\begin{bmatrix}\ddot{\beta}_1\\ \ddot{\beta}_2\\ \ddot{\beta}_3\\ \ddot{x}_2\\ \ddot{y}_2\\ \ddot{x}_3\\ \ddot{y}_3\\ \ddot{x}_4\\ \ddot{y}_4\end{bmatrix}}_{a} = \underbrace{\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\\ -\frac{e}{2}\sin\beta_2 & 0\\ \frac{e}{2}\cos\beta_2 & 0\\ -b_2\sin\beta_2 & -\frac{h}{2}\sin\beta_3\\ b_2\sin\beta_2 & \frac{h}{2}\cos\beta_3\end{bmatrix}}_{V_z} \underbrace{\begin{bmatrix}\ddot{\beta}_2\\ \ddot{\beta}_3\end{bmatrix}}_{\ddot{z}} + \underbrace{\begin{bmatrix}1\\ 0\\ 0\\ -\xi\sin(\beta_1)\\ \xi\cos(\beta_1)\\ -b_1\sin(\beta_1)\\ b_1\cos(\beta_1)\\ -b_1\sin(\beta_1)\\ b_1\cos(\beta_1)\end{bmatrix}}_{V_q} \underbrace{\begin{bmatrix}\ddot{\beta}_1\end{bmatrix}}_{\ddot{q}} + \underbrace{\begin{bmatrix}0\\ 0\\ 0\\ -\xi\cos(\beta_1)\dot{\beta}_1^2 + \eta\sin(\beta_1)\dot{\beta}_1^2\\ -\xi\sin(\beta_1)\dot{\beta}_1^2 - \eta\cos(\beta_1)\dot{\beta}_1^2\\ -\frac{e}{2}\cos(\beta_2)\dot{\beta}_2^2 - b_1\cos(\beta_1)\dot{\beta}_1^2\\ -\frac{e}{2}\sin(\beta_2)\dot{\beta}_2^2 - b_1\sin(\beta_1)\dot{\beta}_1^2\\ -\frac{h}{2}\cos(\beta_3)\dot{\beta}_3^2 - b_1\cos(\beta_1)\dot{\beta}_1^2 - b_2\cos(\beta_2)\dot{\beta}_2^2\\ -\frac{h}{2}\sin(\beta_3)\dot{\beta}_3^2 - b_1\sin(\beta_1)\dot{\beta}_1^2 - b_2\sin(\beta_2)\dot{\beta}_2^2\end{bmatrix}}_{a_{qz}}$$

UVOLNĚNÍ TĚLES → $Ma = DR + Q$

2.



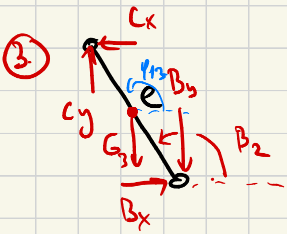
x: $m_2\ddot{x}_2 = A_x - B_x$

y: $m_2\ddot{y}_2 = -A_y + B_y - G_2$

↻: $I_{2S_2}\ddot{\varphi}_{12} = -M_2 - G_2(\xi\cos\varphi_{12} + \eta\sin\varphi_{12}) + B_y d\cos\varphi_{12} + B_x d\sin\varphi_{12}$

$\beta_1 = \varphi_{12}$

3.



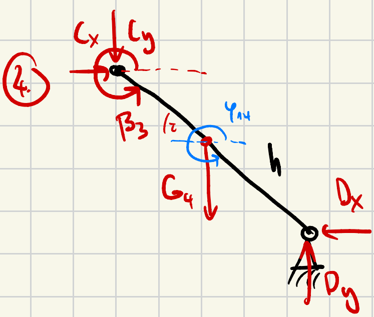
x: $m_3\ddot{x}_3 = B_x - C_x$

y: $m_3\ddot{y}_3 = -B_y + C_y - G_3$

↻: $I_{3S_3}\ddot{\varphi}_{13} = B_x\frac{e}{2}\sin\varphi_{13} + B_y\frac{e}{2}\cos\varphi_{13} + C_x\frac{e}{2}\sin\varphi_{13} + C_y\frac{e}{2}\cos\varphi_{13}$

$\beta_2 = \varphi_{13}$

4.



x: $m_4\ddot{x}_4 = C_x - D_x$

y: $m_4\ddot{y}_4 = D_y - C_y - G_4$

↻: $I_{4S_4}\ddot{\varphi}_{14} = -C_x\frac{h}{2}\sin(2\pi-\varphi_{14}) + C_y\frac{h}{2}\cos(2\pi-\varphi_{14}) - D_x\frac{h}{2}\sin(2\pi-\varphi_{14}) + D_y\frac{h}{2}\cos(2\pi-\varphi_{14})$

↳ $I_{4S_4}\ddot{\varphi}_{14} = C_x\frac{h}{2}\sin\varphi_{14} + C_y\frac{h}{2}\cos\varphi_{14} + D_x\frac{h}{2}\sin\varphi_{14} + D_y\frac{h}{2}\cos\varphi_{14}$

$\beta_3 = \varphi_{14}$

$$M = \begin{bmatrix} m_2 & & & & & & & & 0 \\ & m_2 & & & & & & & \\ & & I_{2S} & & & & & & \\ & & & m_3 & & & & & \\ & & & & m_3 & & & & \\ & & & & & I_{3S} & & & \\ & & & & & & m_4 & & \\ & & & & & & & m_4 & \\ 0 & & & & & & & & I_{4S} \end{bmatrix} \quad a = \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\varphi}_{12} \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\varphi}_{13} \\ \ddot{x}_3 \\ \ddot{y}_3 \\ \ddot{\varphi}_{14} \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & ds\varphi_{12} & dc\varphi_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{e}{2}s\varphi_{13} & \frac{e}{2}c\varphi_{13} & \frac{e}{2}s\varphi_{13} & \frac{e}{2}c\varphi_{13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{h}{2}s\varphi_{14} & \frac{h}{2}c\varphi_{14} & \frac{h}{2}s\varphi_{14} & \frac{h}{2}c\varphi_{14} \end{bmatrix} \quad R = \begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \end{bmatrix} \quad Q = \begin{bmatrix} 0 \\ -G_2 \\ -M_2 - G_2(\xi c\varphi_{12} + \eta s\varphi_{12}) \\ 0 \\ -G_3 \\ 0 \\ 0 \\ -G_4 \\ 0 \end{bmatrix}$$