# Sunday 27th of October 8:30 to 12:00

| Time | Description | Discussant |
|---|---|---|
| [15 min] | (0) Introductions and administrivia | Trikalinos |
| [25 min] | (1) DES as a composition of point processes | Alarid-Escudero |
| [30 min] | (2) NHPPPs – key properties | Trikalinos |
| [30 min] | (3) Sampling from NHPPPs | Sereda |
| [15 min] | **Break** | |
| [80 min] | (4) Guided exercise:<br>- Implement a simple cancer natural history DES for one person<br>- The many-person case<br>- Packaging | [All]<br>Chrysanthopoulou<br><br>Sereda/Alarid-Escudero<br>Trikalinos |
| [10 min] | (5) Advanced Topic Teaser on self-excitatory processes: point processes that are not NHPPPs and when you may need them | Trikalinos |
| [15 min] | General Q & A | All |

# Section 3: Sampling

# Three important properties for sampling

**Memorylessness**

You can ignore what happens outside your interval

**Composability**

You can merge two NHPPPs with intensities $\lambda_1, \lambda_2$ to get a new NHPPP with intensity $\lambda_1 + \lambda_2$.

**Transmutability (time warping)**

Any one-to-one transformation of the intensity function results in a unique NHPPP in the transformed time axis

# Overview of the sampling strategy

1. Sampling from constant rate PPP is easy
   *easy*

2. Memorylessness implies you can treat the piecewise as constant PPPs over disjoint interval
   *peasy*

3. Composability motivates an acceptance-rejection algorithm for sampling from any $\lambda(t)$
   *almost always practical*

4. Time warping allows efficient sampling if you have (cheap access to) $\Lambda(t), \Lambda^{-1}(t)$
   *sometimes possible, may be worth the hassle to get $\Lambda, \Lambda^{-1}$*

# 1. Sampling from a PPP is easy

# Constant intensity function (homogeneous PPP)

Sampling from a constant intensity function is easy.

The interarrival times have an exponential distribution.



$\lambda$ constant

$Z_{(i)} - Z_{(i-1)} \sim \text{Exponential}(\lambda)$

$Z_{(0)}$    $Z_{(1)}$    $Z_{(2)}$    $Z_{(4)}$

time
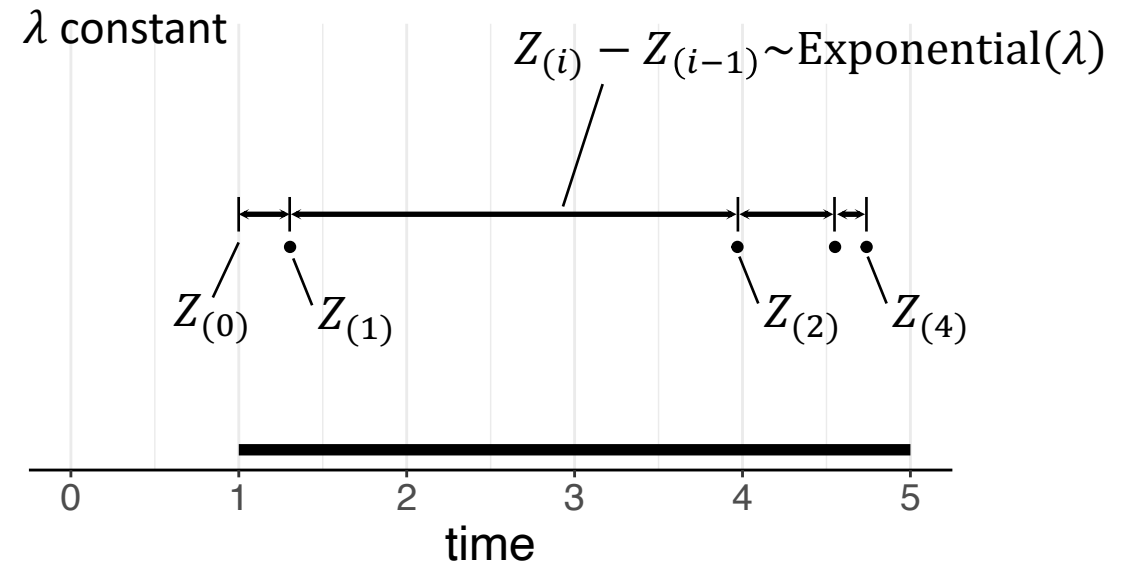
# 2. Memorylessness: Sampling from piecewise constant NHPPP is peasy

# Piecewise constant intensity function (NHPPP)

- Look at each piecewise constant interval separately

- In each interval you have a constant intensity (easy)

- Return the union of all events

Sampling from piecewise constant intensities is easy (**memorylessness**)
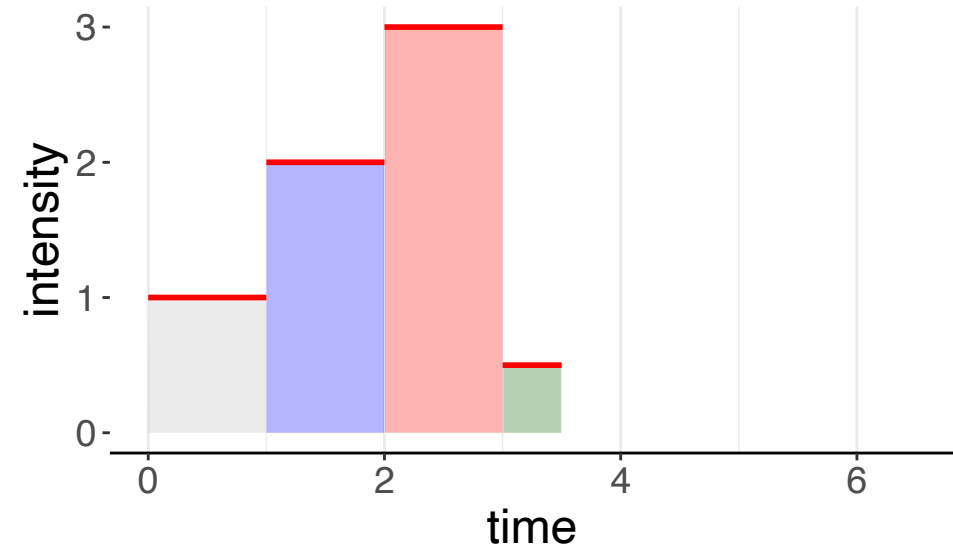
# 3. Composability: Sampling NHPPPs when you know $\lambda(t)$ reduces to sampling from a PPP (#1) or piecewise constant NHPPP (#2)*
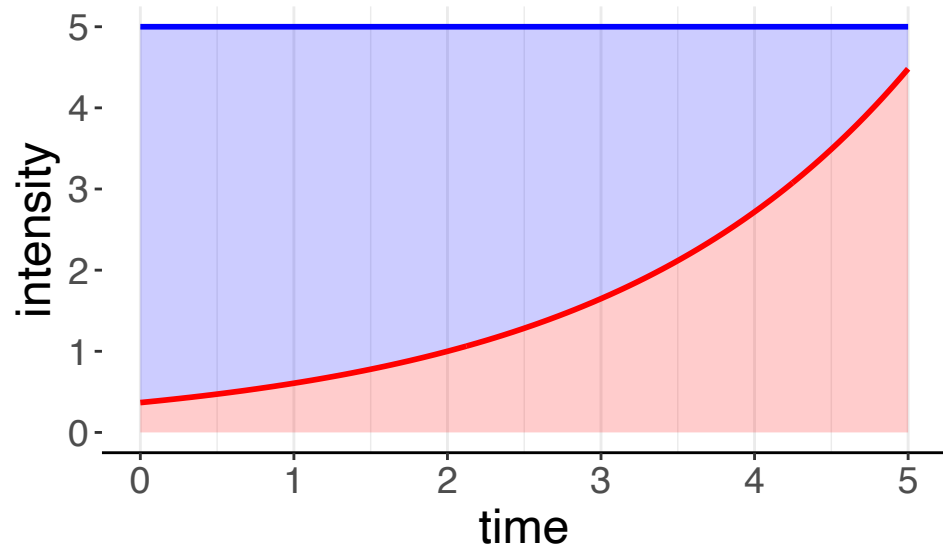
*\* You still need to find a constant or piecewise constant majorizer $\lambda_*(t)$, whose choice determines your efficiency .*
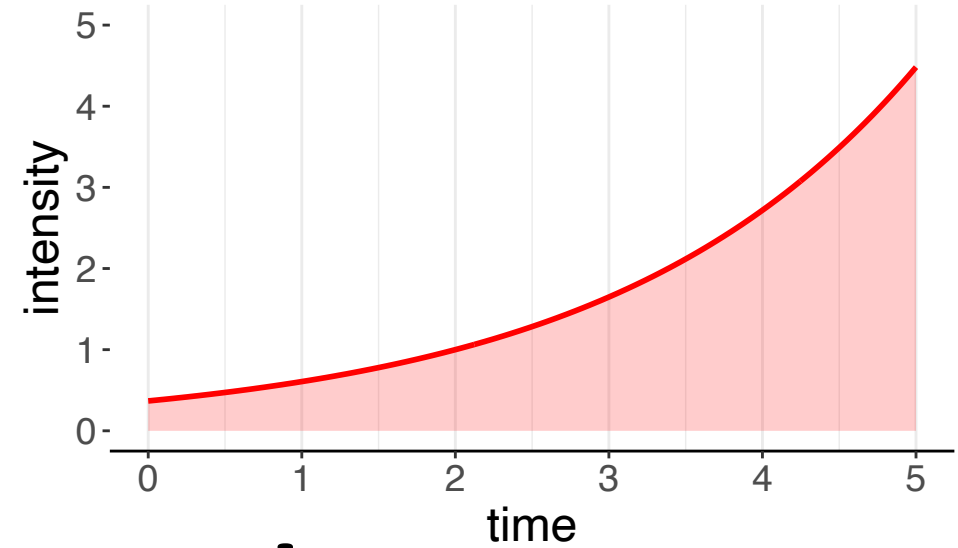*You cannot get achieve something difficult with zero effort.*
*You will put in some work.*
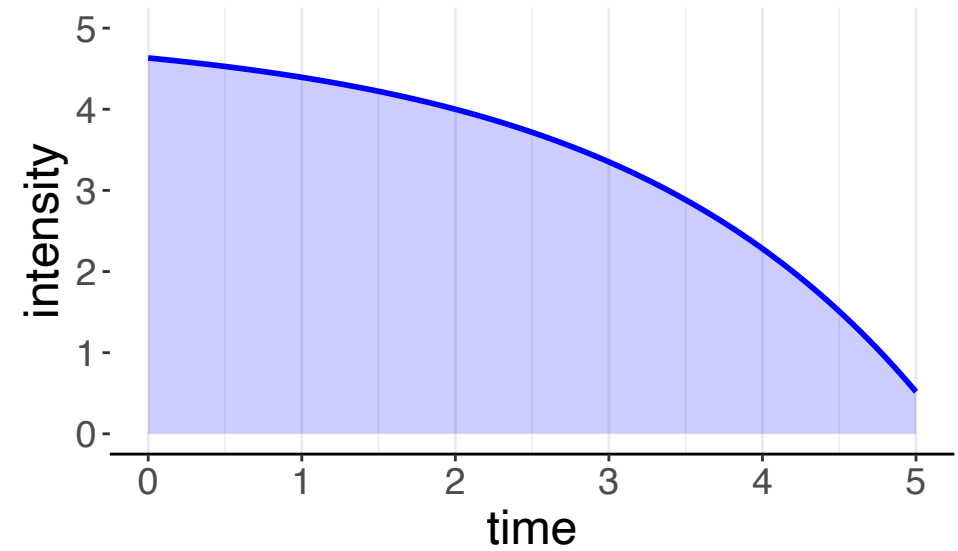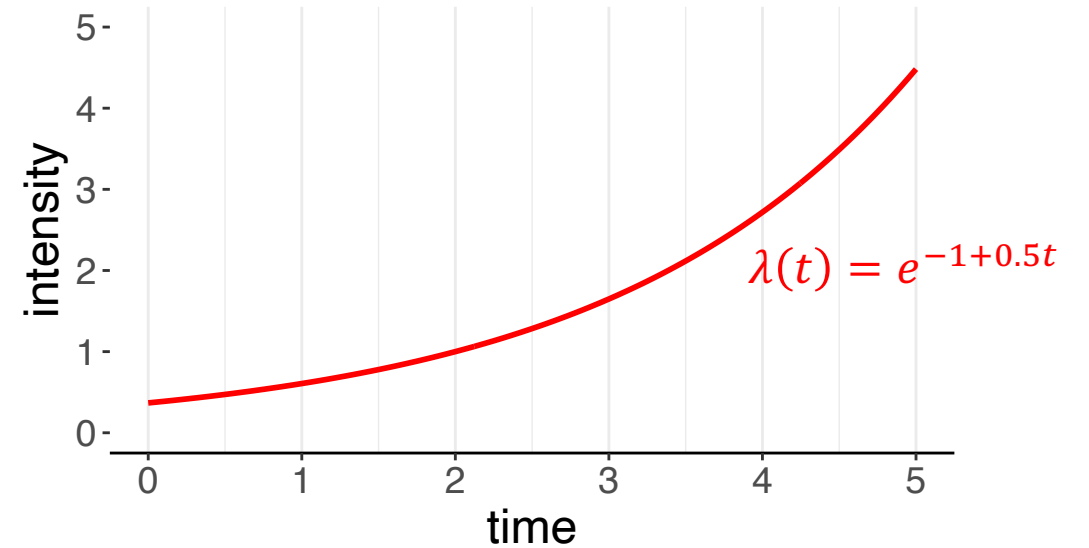*Other terms and conditions may apply.*

# Composability

# NHPPP, where you know $\lambda(t)$ : Thinning
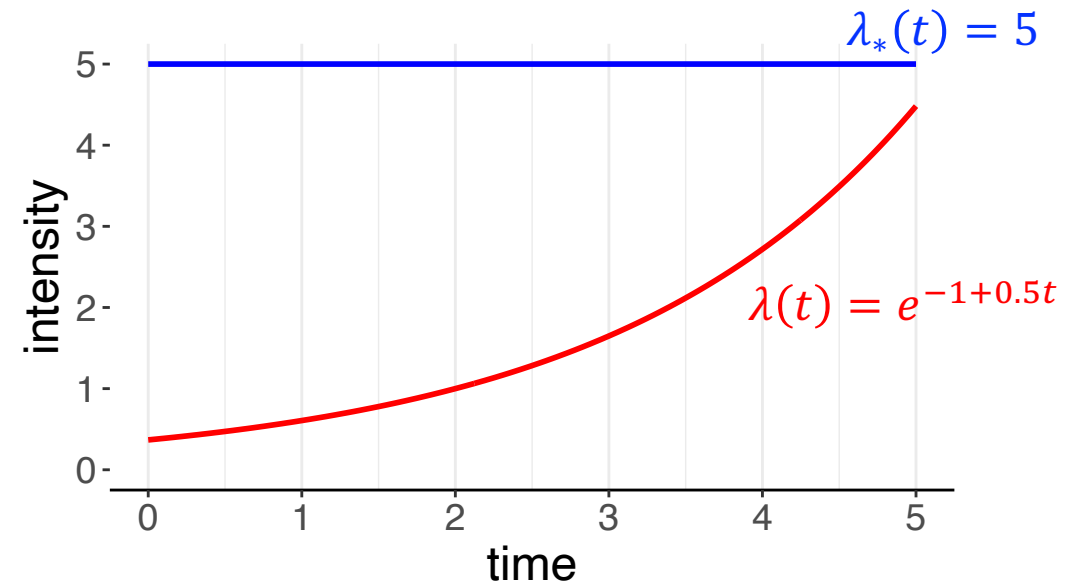
The general case is more challenging



$\lambda(t) = e^{-1+0.5t}$

# NHPPP, where you know $\lambda(t)$ : Thinning

- Find a majorizer function $\lambda_*$ that's easy to sample

*Majorizer: any function that is "taller" that $\lambda$*

$$\lambda_* \geq \lambda$$

*(and has the same support as $\lambda$)*

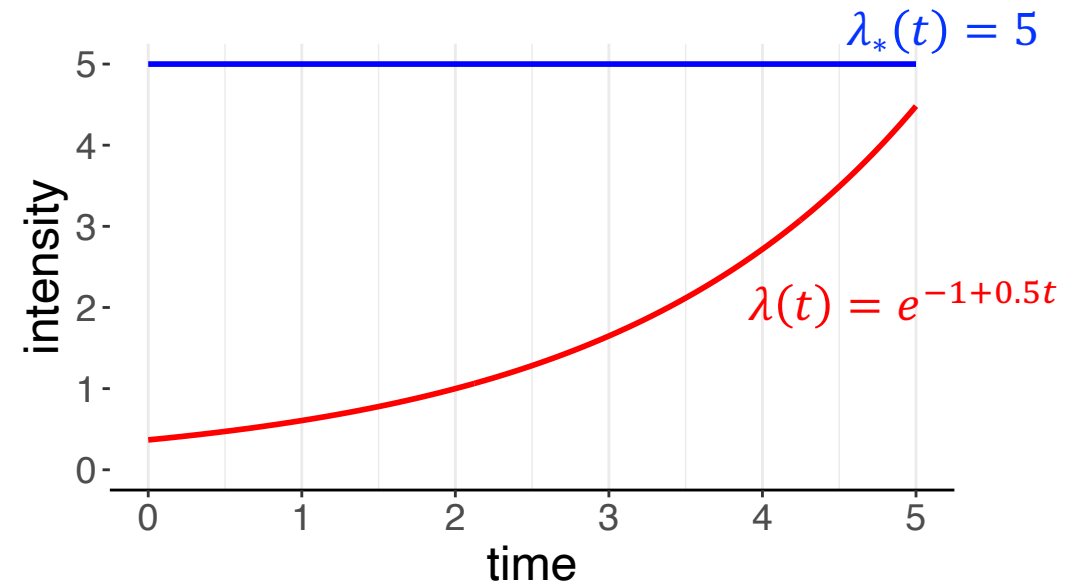# NHPPP, where you know $\lambda(t)$ : Thinning

- Find a majorizer function $\lambda_*$ that's easy to sample

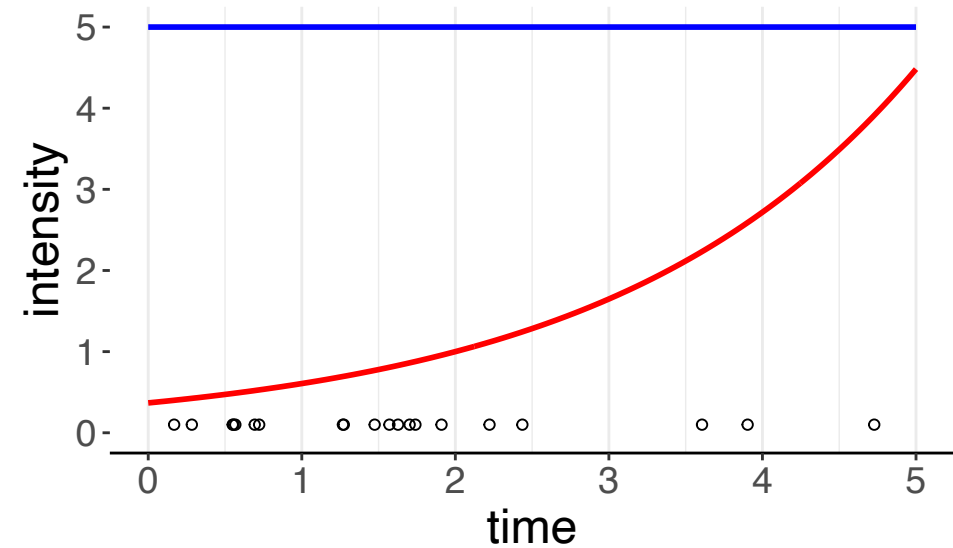$$\lambda_*(t) = \lambda(t) + [\lambda_*(t) - \lambda(t)]$$

Sample proposals from here

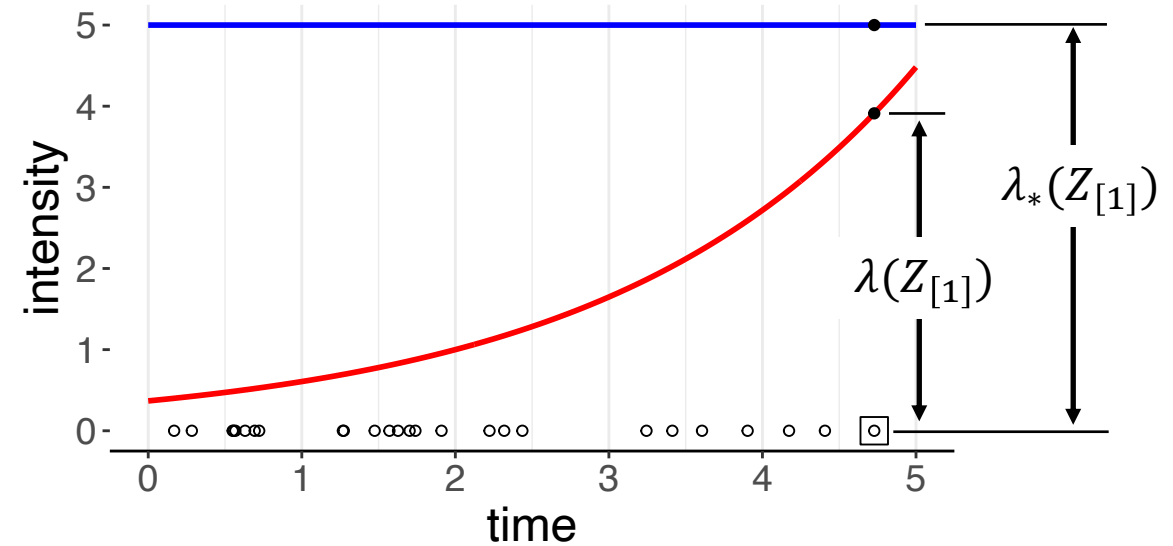Keep proposals conforming to this part

Reject the rest



$\lambda_*(t) = 5$

$\lambda(t) = e^{-1+0.5t}$

# NHPPP, where you know $\lambda(t)$ : Thinning

- Find a majorizer function $\lambda_*$ that's easy to sample

- Draw events $\{Z_{*1}, \dots\}$ from $\lambda_*$
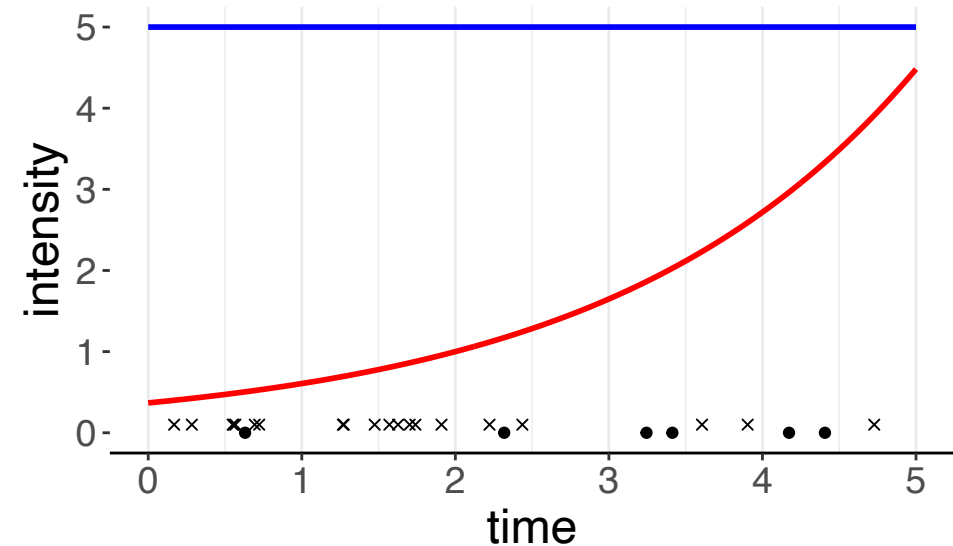
# NHPPP, where you know $\lambda(t)$ : Thinning

- Find a majorizer function $\lambda_*$ that's easy to sample

- Draw events $\{Z_1, \dots\}$ from $\lambda_*$

- Accept event $i$ with probability
  $$\frac{\lambda(Z_i)}{\lambda_*(Z_i)}$$

# NHPPP, where you know $\lambda(t)$: Thinning

- Find a majorizer function $\lambda_*$ that's easy to sample

- Draw events $\{Z_{*1}, \dots\}$ from $\lambda_*$

- Accept event $i$ with probability
$$\frac{\lambda(Z_i)}{\lambda_*(Z_i)}$$

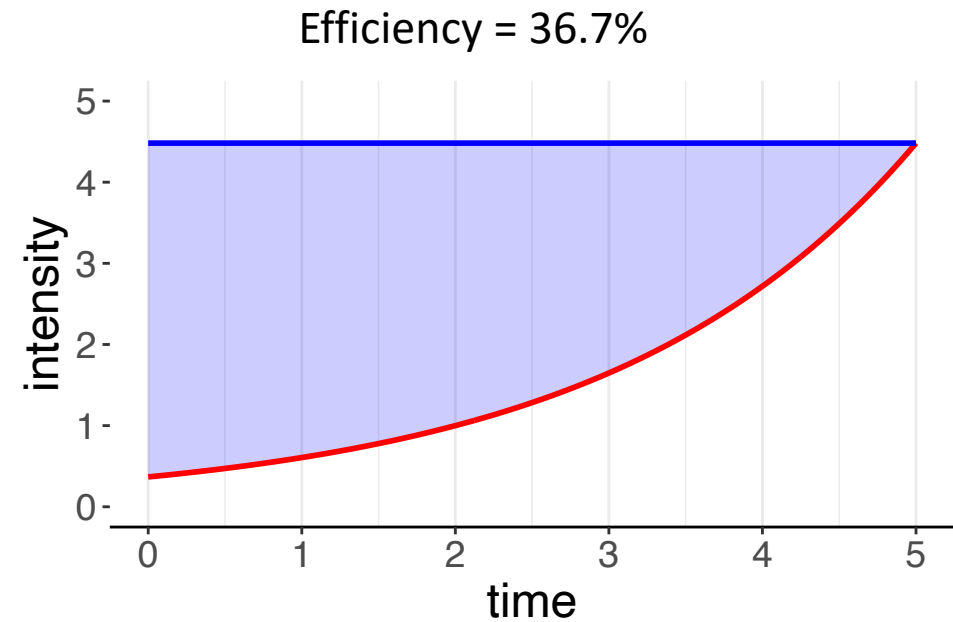- The set of accepted points is an instantiation from $\lambda(t)$

(**composability**)

# Thinning, efficiency

- Thinning efficiency: average fraction of proposals that are accepted

- Depends on the choice of $\lambda_*$

- The smaller the blue area, the better the efficiency
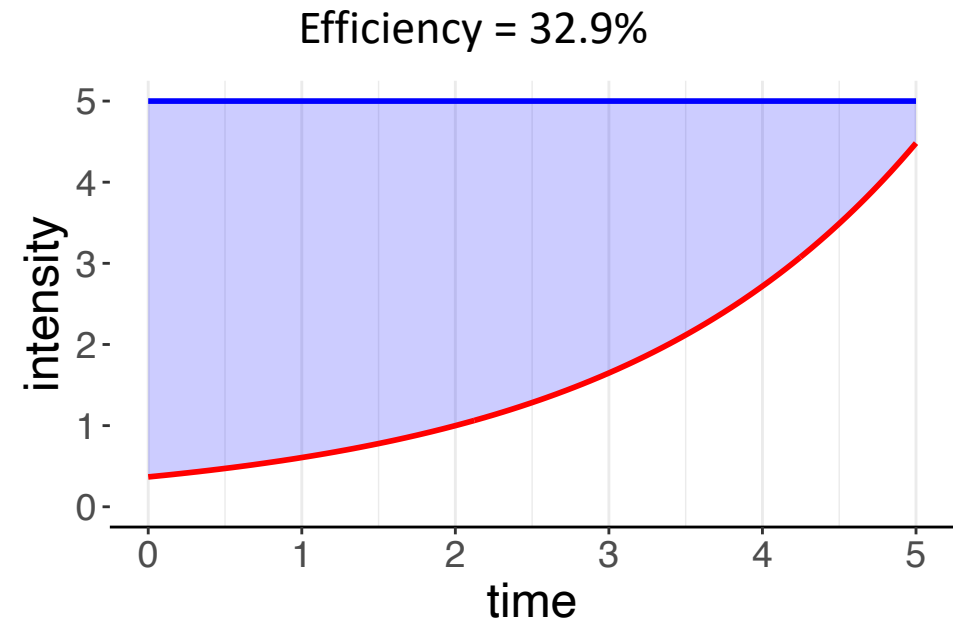


Efficiency = 32.9%

# Thinning, efficiency

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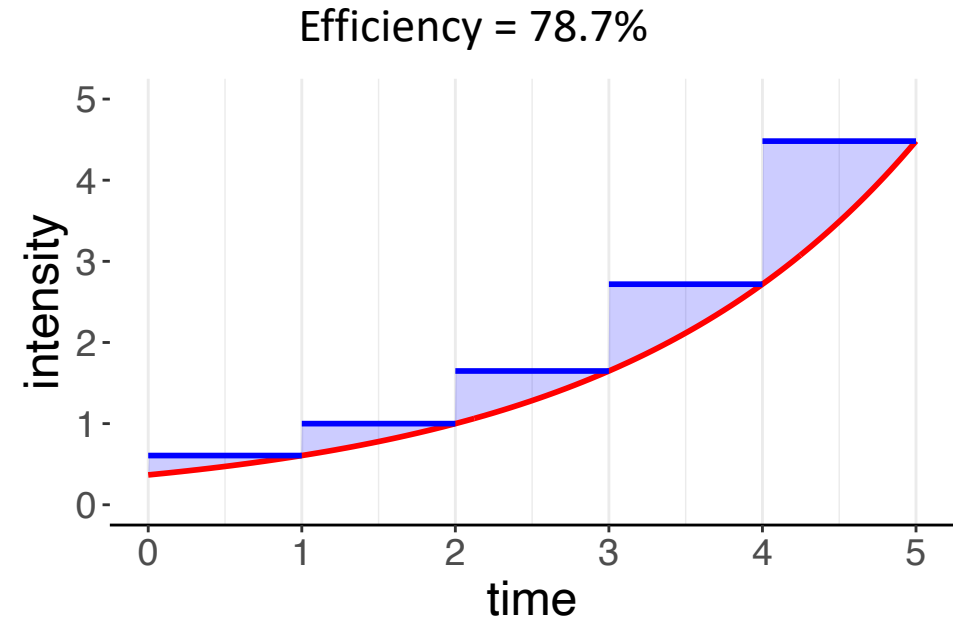
# Thinning, efficiency

- Thinning efficiency: average fraction of proposals that are accepted

- Depends on the choice of $\lambda_*$

- The smaller the blue area, the better the efficiency

# 4. Transmutability of time: Sampling NHPPPs when you know $\Lambda, \Lambda^{-1}$ reduces to sampling from a PPP with rate one (#1) *
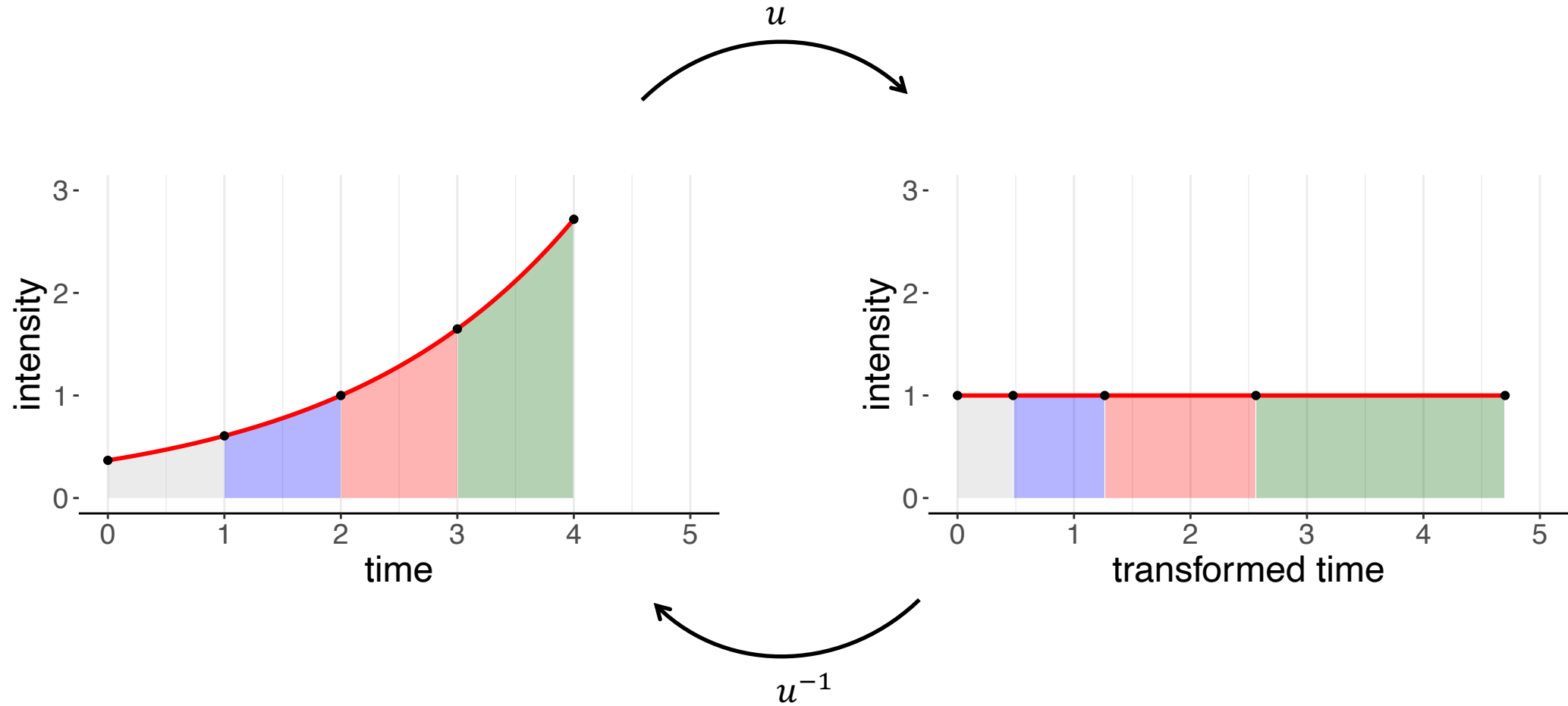
*\* You will need to do some maths to get $\Lambda, \Lambda^{-1}$. It may not be practical to do so, or even possible. In such a case, back to (#3). Even if you have $\Lambda$, you may not have a cheap $\Lambda^{-1}$.*
*You cannot achieve something difficult with zero effort. You will put in some work. Other terms and conditions may apply.*

# Transmutability
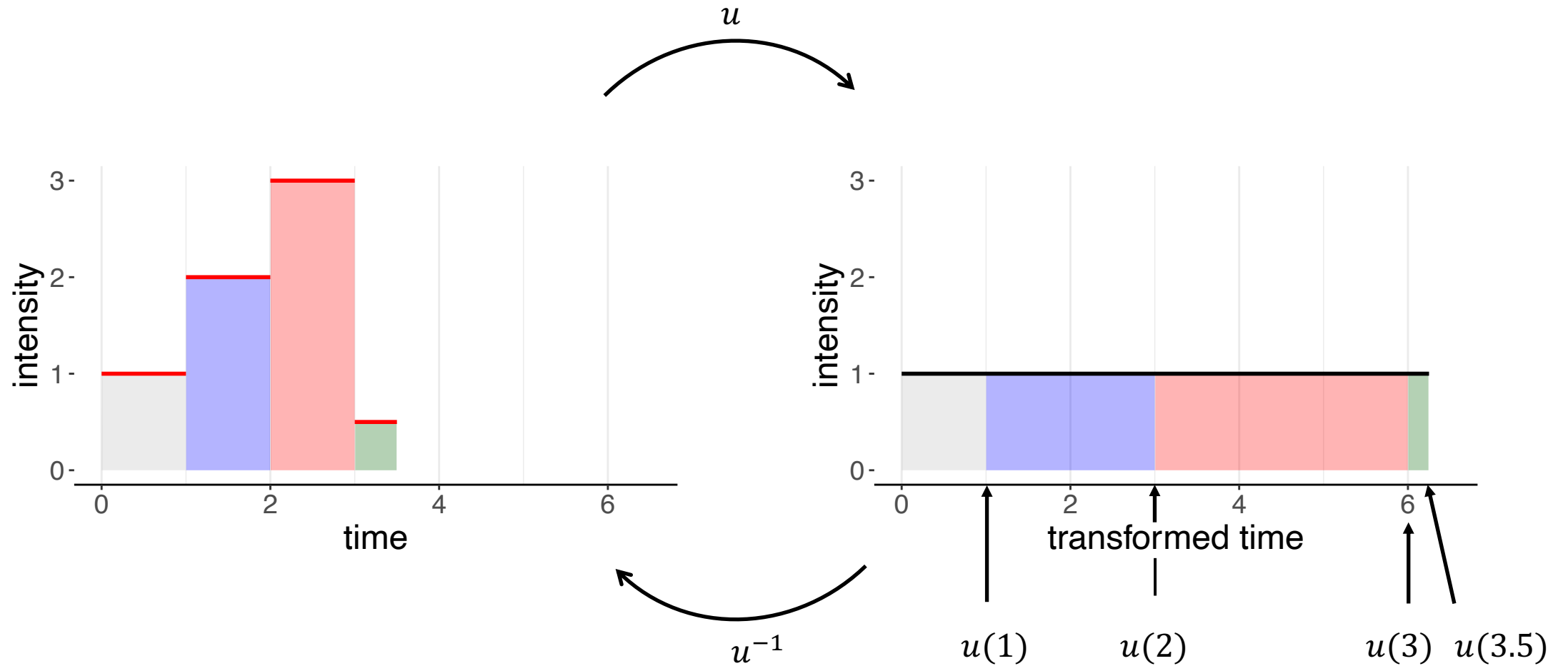
# Transmutability

# A nice $u$ is $\Lambda$ (and then $u^{-1}$ is $\Lambda^{-1}$)

Change of variable from $s$ to $u$

$$\Lambda(t) = \int_a^t \lambda(s)\, ds = \int_{u(a)}^{u(t)} \frac{\lambda(s)}{u'(s)}\, du$$

Pick $u$ so that $u' = \lambda$. Any antiderivative of $\lambda$ works. Using $u \coloneqq \Lambda$, transforms time to scale where the process has constant rate 1,

$$\int_{\Lambda(a)}^{\Lambda(t)} \frac{\lambda(s)}{\Lambda'(s)}\, du = \int_{\Lambda(a)}^{\Lambda(t)} 1\, du \,.$$

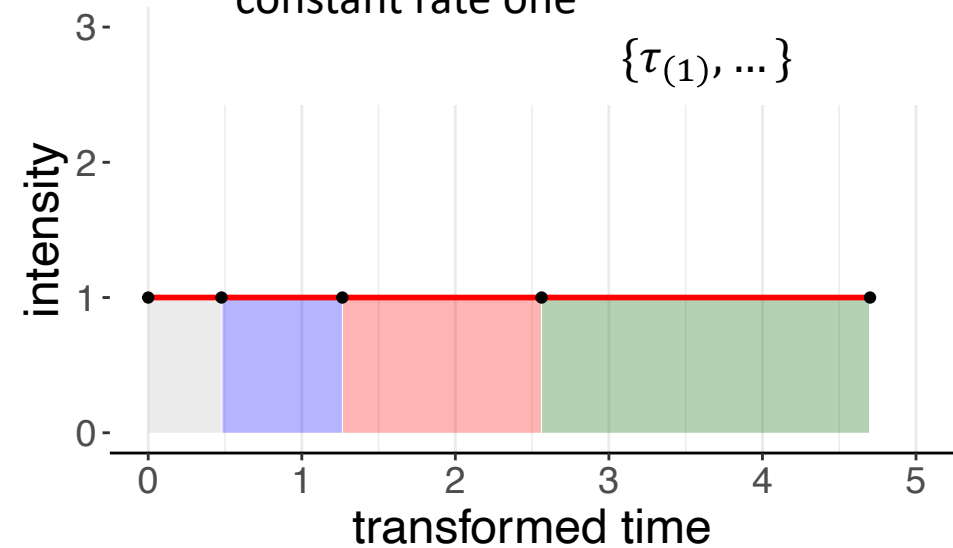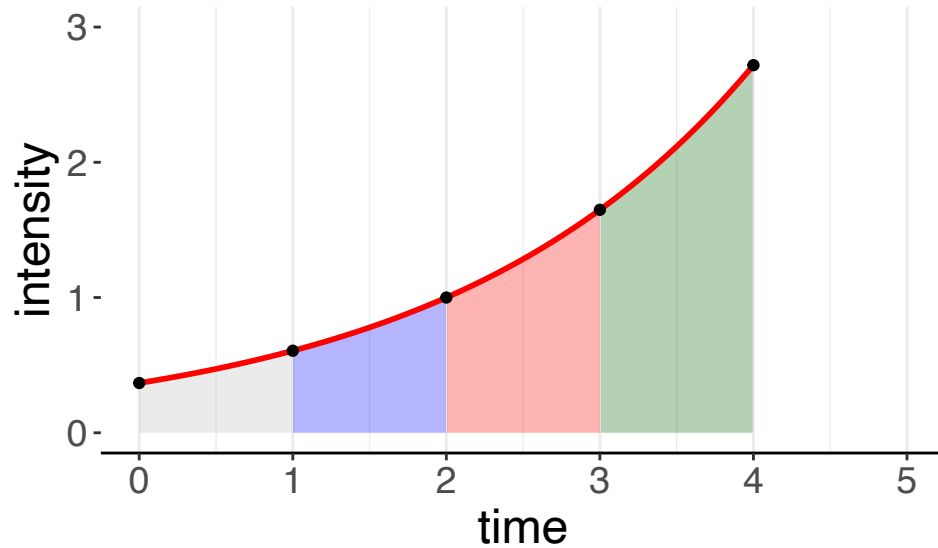*This is a sketch of the formal proof – omitting the rigorous bits*

# Transmutability



1. Find the start and stop of the transformed time interval
$\tau_{start} = \Lambda(t_{start})$ and $\tau_{stop} = \Lambda(t_{stop})$

$\Lambda$

2. Sample transformed times from a PPP with constant rate one
$\{\tau_{(1)}, \dots\}$

$\Lambda^{-1}$

3. Back-transform the instantiation to the original time scale
$\{\Lambda^{-1}(\tau_{(1)}), \dots\}$

# More in these works…

**Statistics > Computation**

## nhppp: Simulating Nonhomogeneous Poisson Point Processes in R

Thomas A. Trikalinos, Yuliia Sereda

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CSH Cold Spring Harbor Laboratory    BMJ    Yale

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## A Fast Nonparametric Sampling (NPS) Method for Time-to-Event in Individual-Level Simulation Models

David U. Garibay-Treviño, Hawre Jalal, Fernando Alarid-Escudero

# Next … Section 4: Hands-on example (simple case)

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