Fourth report

Financial mathematics for data science

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The report is divided into three sections, the first one, more theoretical, aims to show the reader the possibility to compute the Greeks values for different strike prices and times to maturity then the surface of this 3d function has been plotted.

The second part, empirical, deals with the computation of the implied volatility surface, as a function of strikes and time to maturity, for a non dividend paying asset using the call prices.

The second part aim to show the reader the inadequacy of the Black&Scholes model which relies on the assumption of a constant volatility, the falseness can be proven due to the presence of the so-called volatility smile.

Also the Greeks are plotted against the time to maturity to check their smoothness over time.

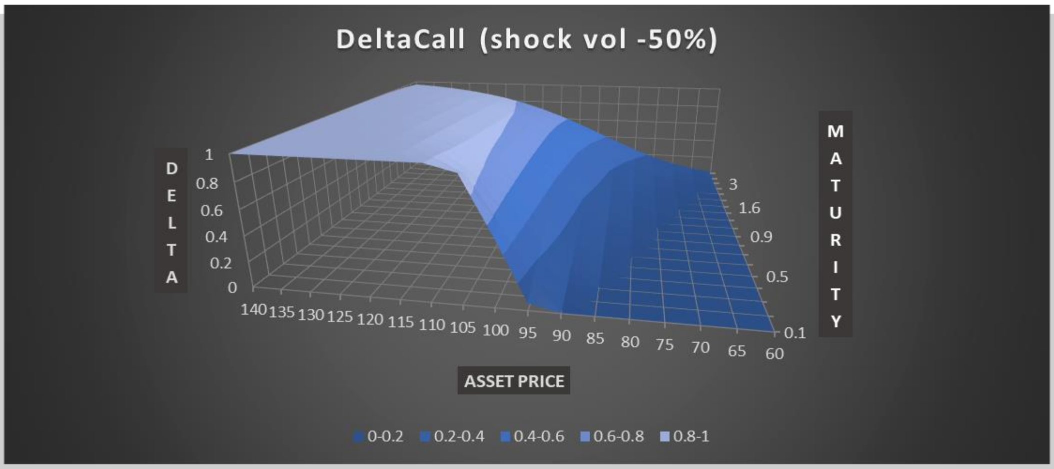
The third part resumes data about the precious to estimate the historical volatility and to compare it with the ATM implied volatility found on Yahoo Finance.

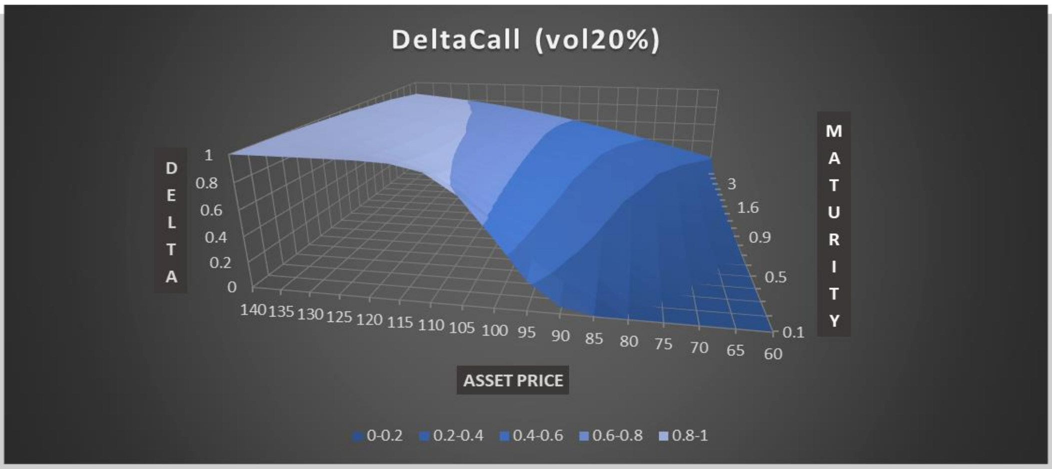
Prices of the ATM call options are computed with the Black&Scholes formula at different maturities with historical volatility and provided interest rates then compared to the actual call values.

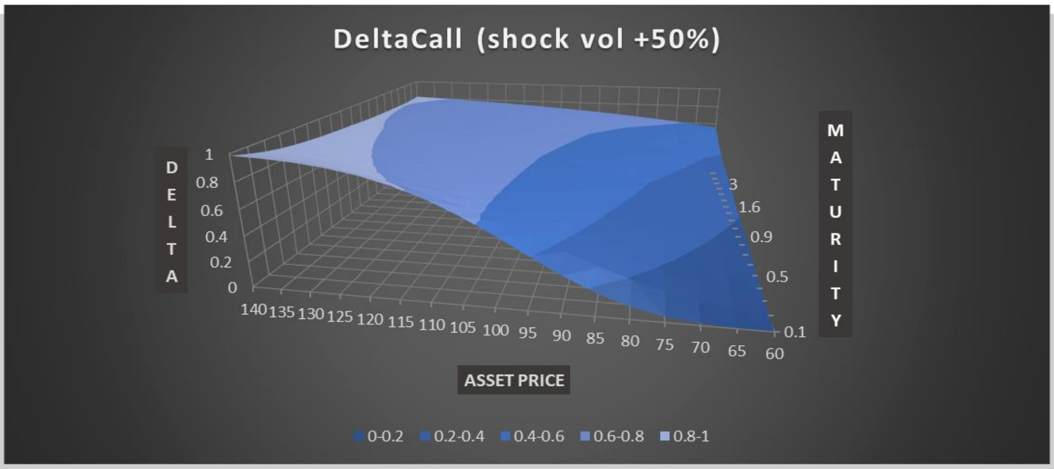
1. The Option Greek formulae express the change in the option price with respect to a parameter change taking as fixed all the other inputs.

The increase in the volatility tends to increase the smootheness of the Greeks plots and their characteristic are heavily affected over maturity.

High volatility tends to lower the guarantee of success, low probability of an ITM strikes expires ITM (delta decreases) and increase the probability of an OTM strike expires ITM (delta increases).

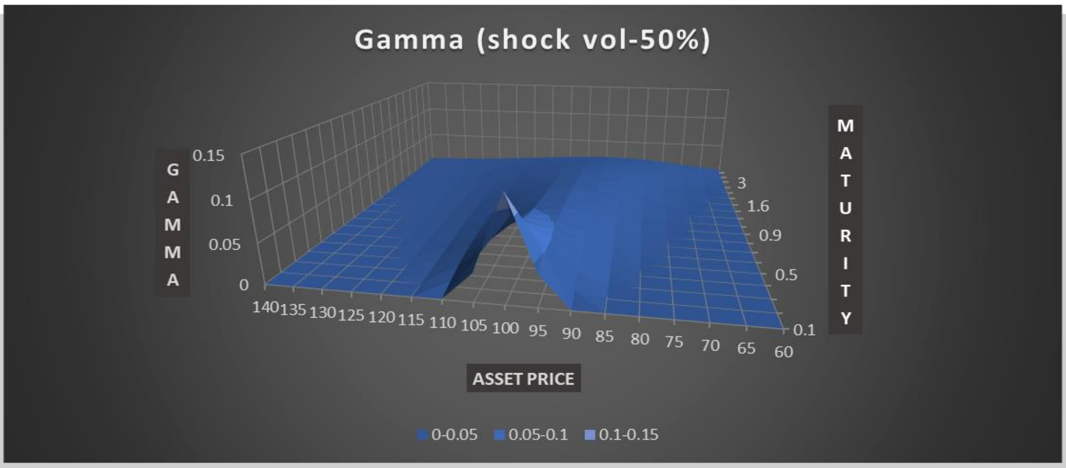


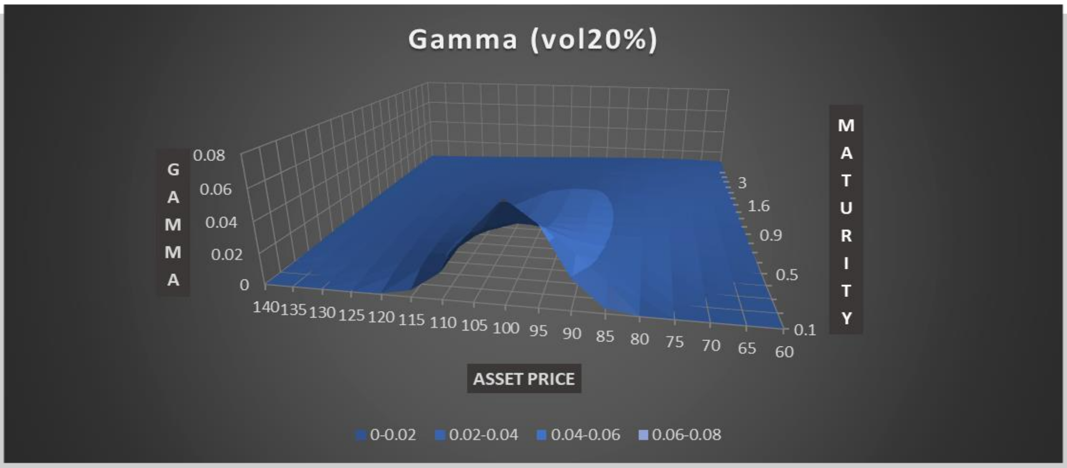


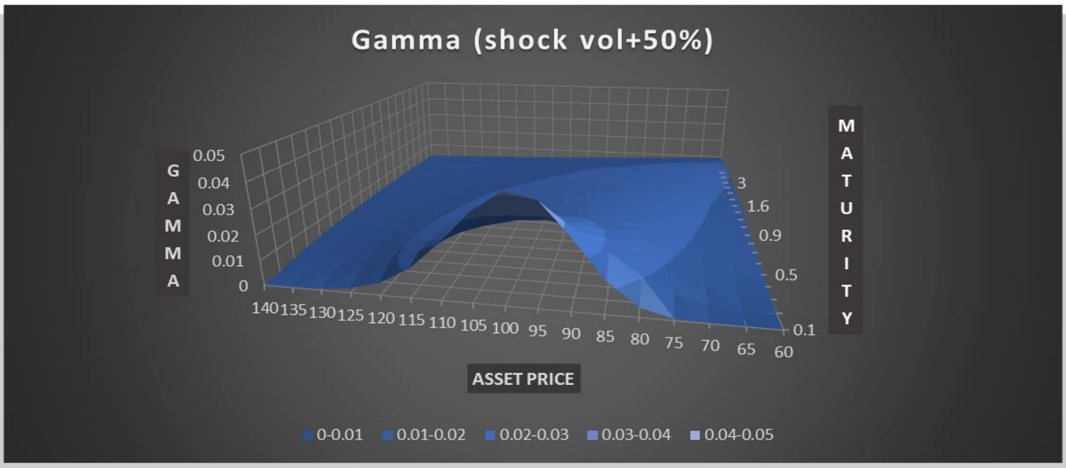


Gamma is higher for options that are ATM and closer to expiration. A front-month, ATM option will have bigger Gamma than a long-term option with the same strike.

Gamma is lower in the longer-dated options as more strikes remain possibilities for being ITM at expiration because of the amount of time remaining.





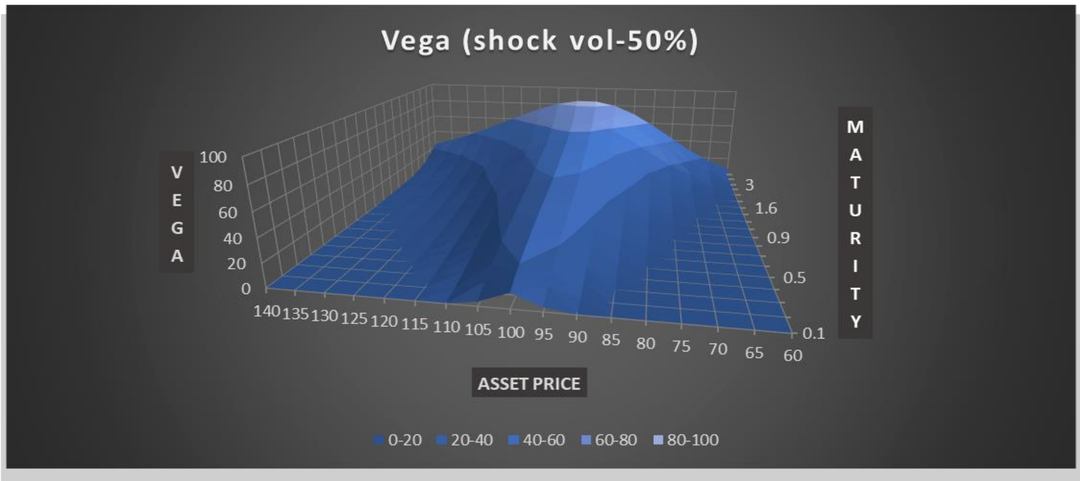


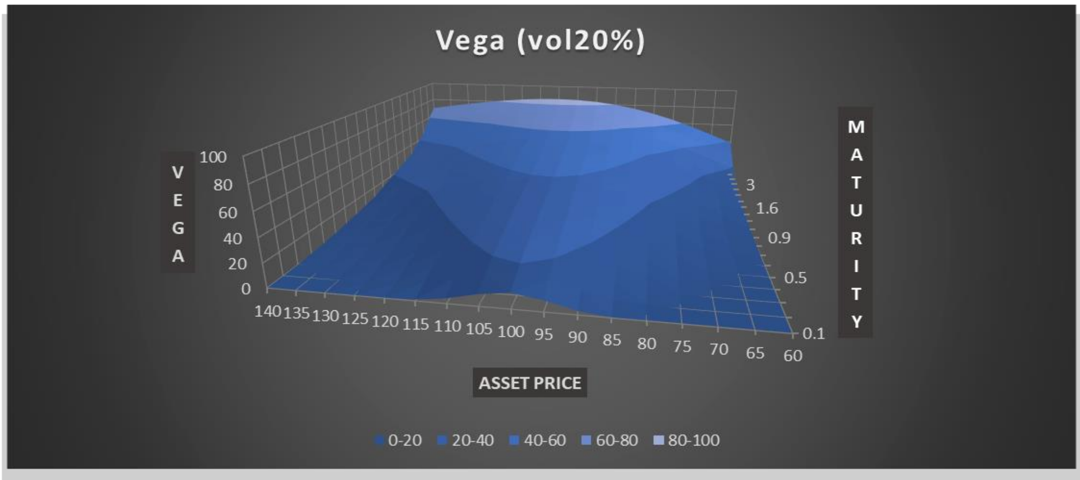
When the stock is far away from the strike, the extrinsic value is low and an increase in volatility would not affect the payoffs by much, so the vega is low.

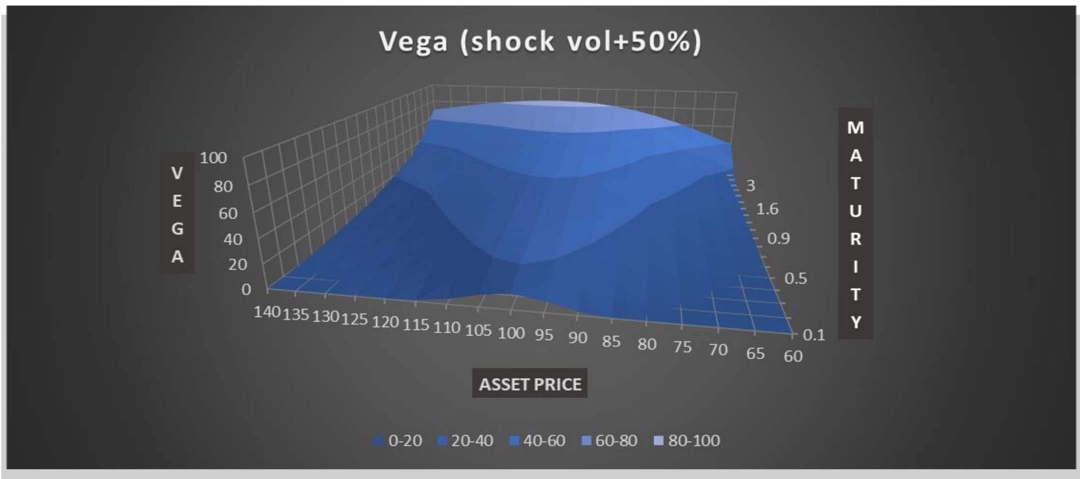
When the stock is near the strike, an increase in volatility has a direct effect on the payoffs.

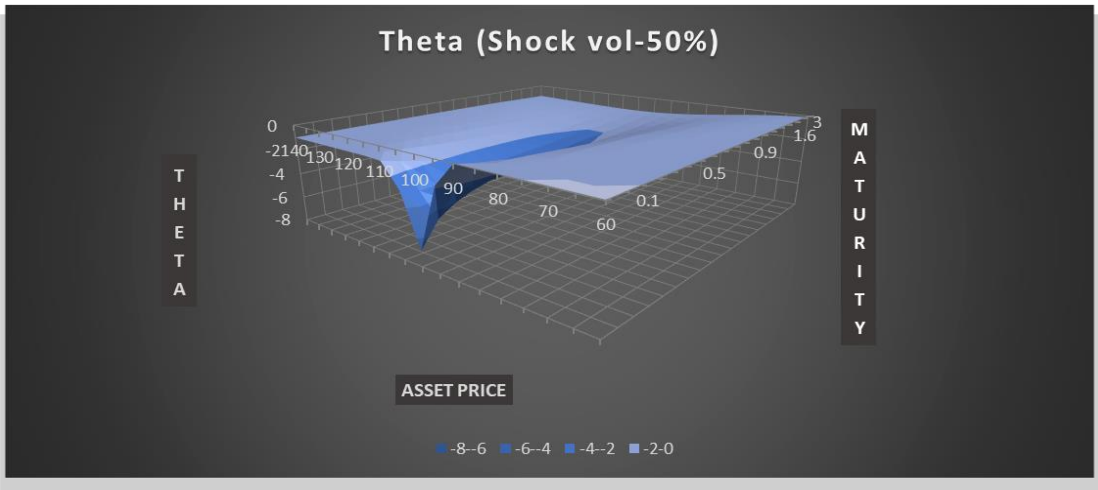
Hence the extrinsic value will increase significantly, so the vega is higher.

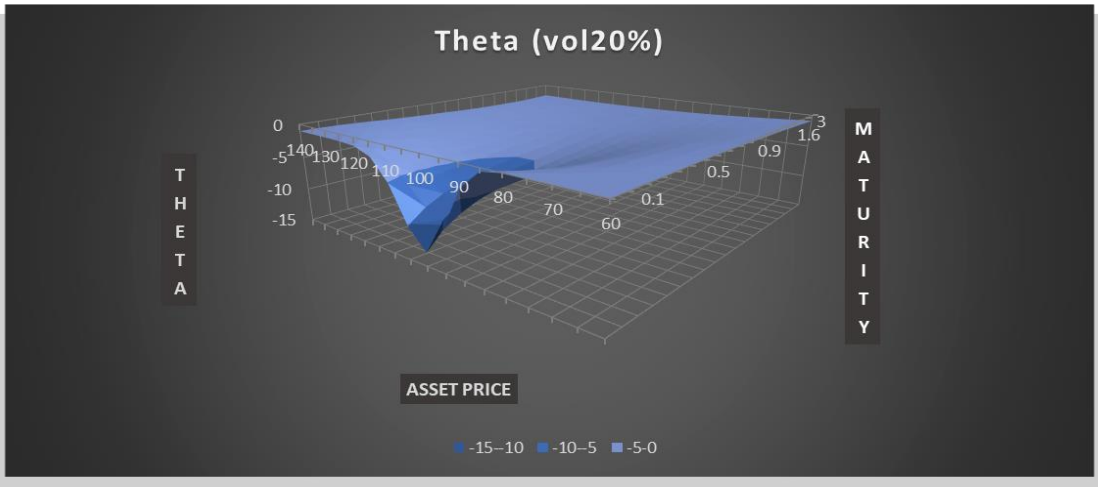
Time amplifies the effect of volatility changes, so vega is greater for long date options.

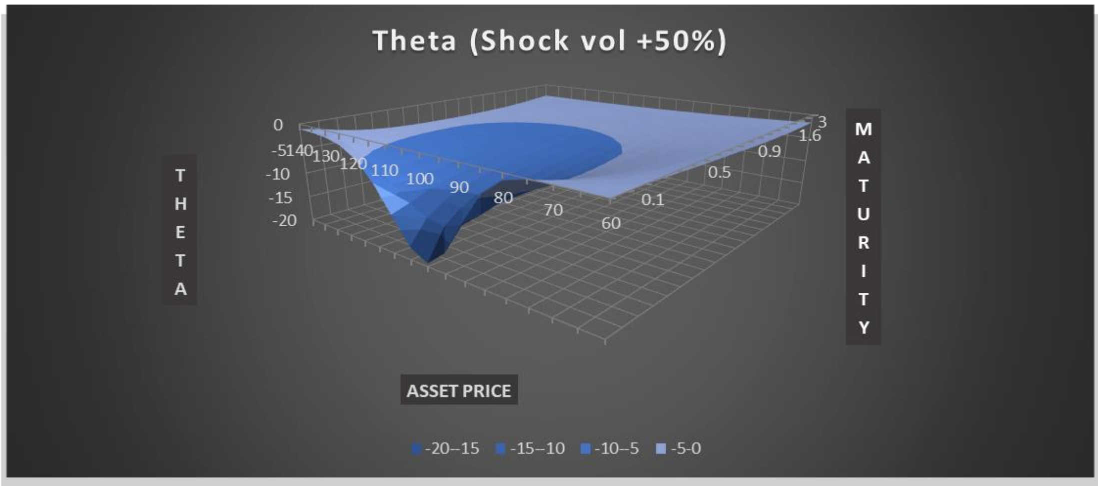

Vega (shock vol-50%)
VEGA
100
80
60
40
20
0
140 135 130 125 120 115 110 105 100 95 90 85 80 75 70 65 60
ASSET PRICE
3
1.6
0.9
0.5
0.1
MATURITY
0-20 20-40 40-60 60-80 80-100

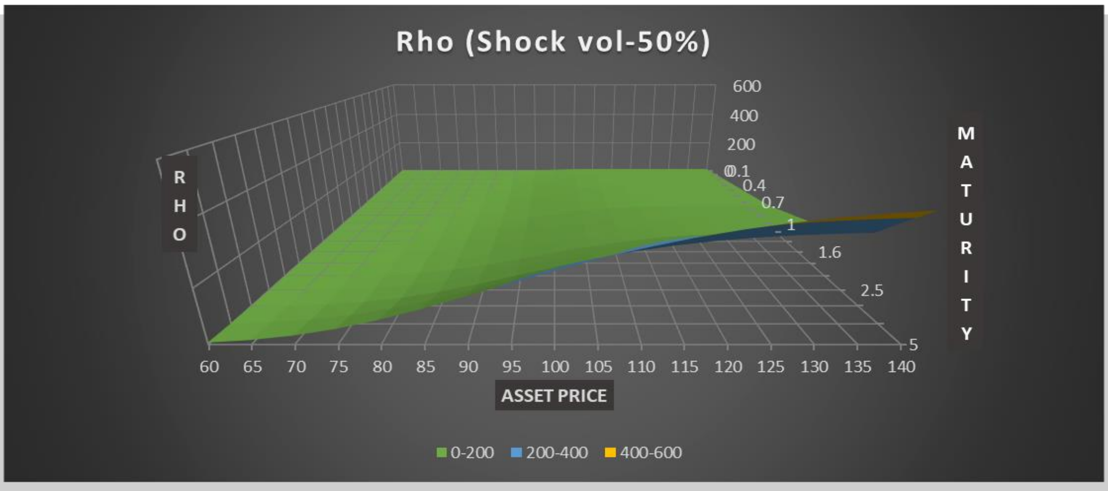

Vega (vol20%)
VEGA
100
80
60
40
20
0
140 135 130 125 120 115 110 105 100 95 90 85 80 75 70 65 60
ASSET PRICE
3
1.6
0.9
0.5
0.1
MATURITY
0-20 20-40 40-60 60-80 80-100


Vega (shock vol+50%)
VEGA
100
80
60
40
20
0
140 135 130 125 120 115 110 105 100 95 90 85 80 75 70 65 60
ASSET PRICE
3
1.6
0.9
0.5
0.1
MATURITY
0-20 20-40 40-60 60-80 80-100

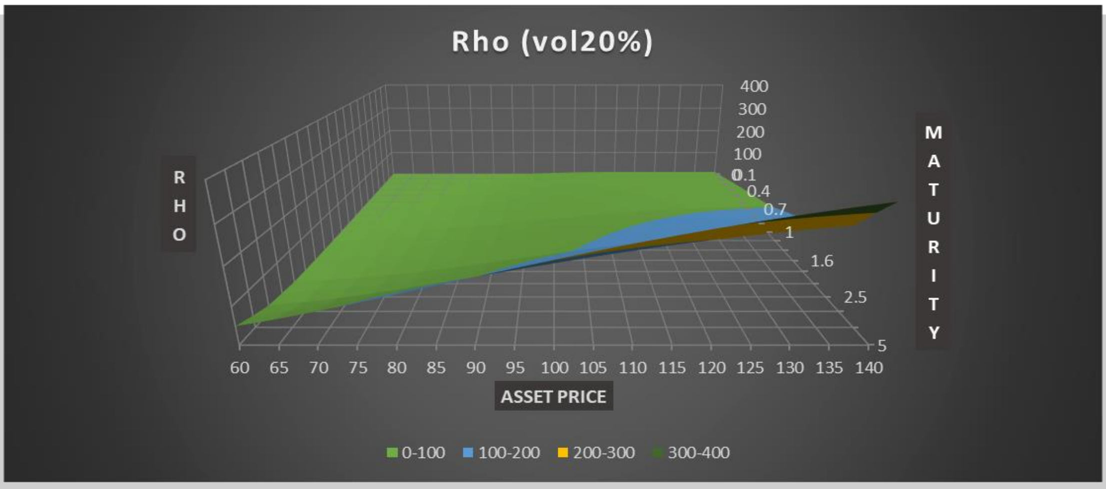

Theta (Shock vol-50%)
THETA
MATURITY
ASSET PRICE
0
-2
-4
-6
-8
140
130
120
110
100
90
80
70
60
0.1
0.5
0.9
1.6
3
-8--6
-6--4
-4--2
-2-0


Theta (vol20%)
THETA
MATURITY
ASSET PRICE
0
-5
-10
-15
140
130
120
110
100
90
80
70
60
0.1
0.5
0.9
1.6
3
-15--10
-10--5
-5-0

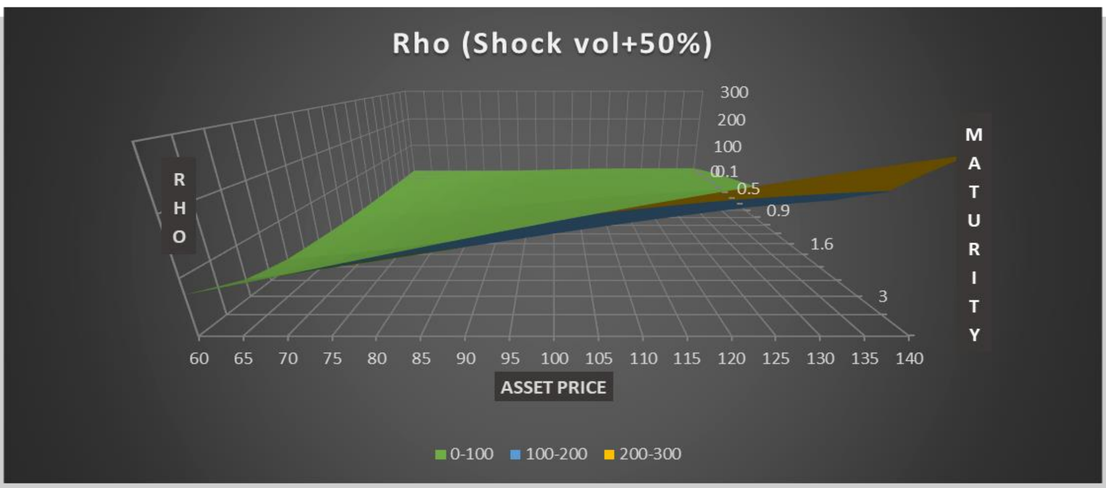

Theta (Shock vol +50%)
THETA
MATURITY
ASSET PRICE
0
-5
-10
-15
-20
140
130
120
110
100
90
80
70
60
0.1
0.5
0.9
1.6
3
-20--15
-15--10
-10--5
-5-0



Rho (Shock vol-50%)
600
400
200
0
RHO
0.1
0.4
0.7
1
1.6
2.5
5
MATURITY
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
ASSET PRICE
0-200 200-400 400-600



Rho (vol20%)
400
300
200
100
0
RHO
0.1
0.4
0.7
1
1.6
2.5
5
MATURITY
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
ASSET PRICE
0-100 100-200 200-300 300-400



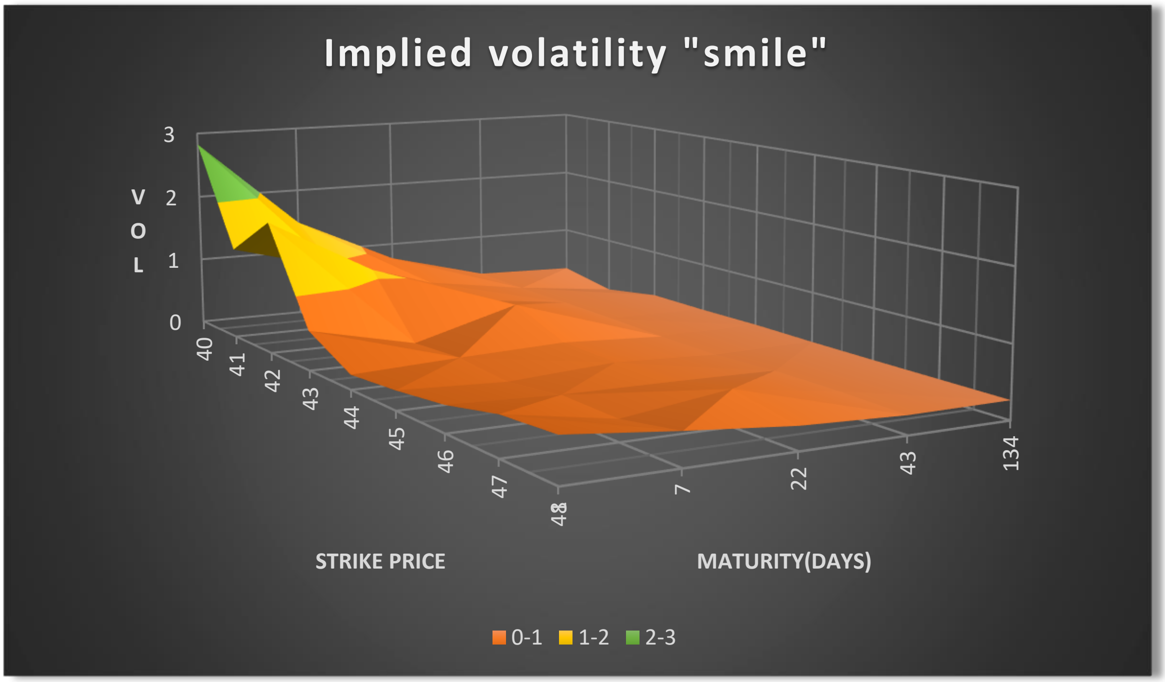
Rho (Shock vol+50%)
300
200
100
0
RHO
0.1
0.5
0.9
1.6
3
MATURITY
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
ASSET PRICE
0-100 100-200 200-300

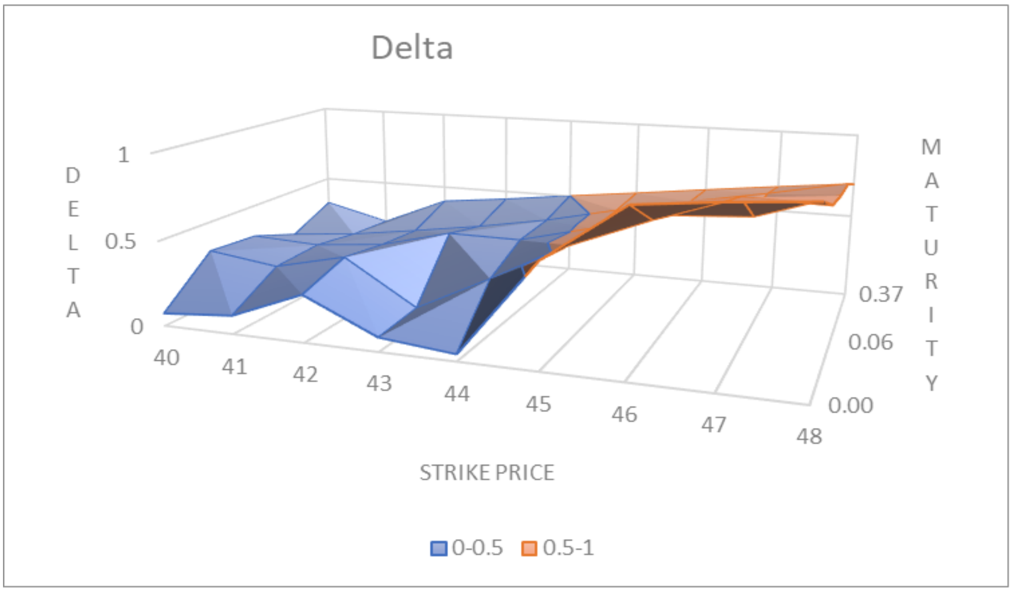
2. The non dividend paying asset chosen is Boston Scientific Corporation (BSX) with a book of American Options.

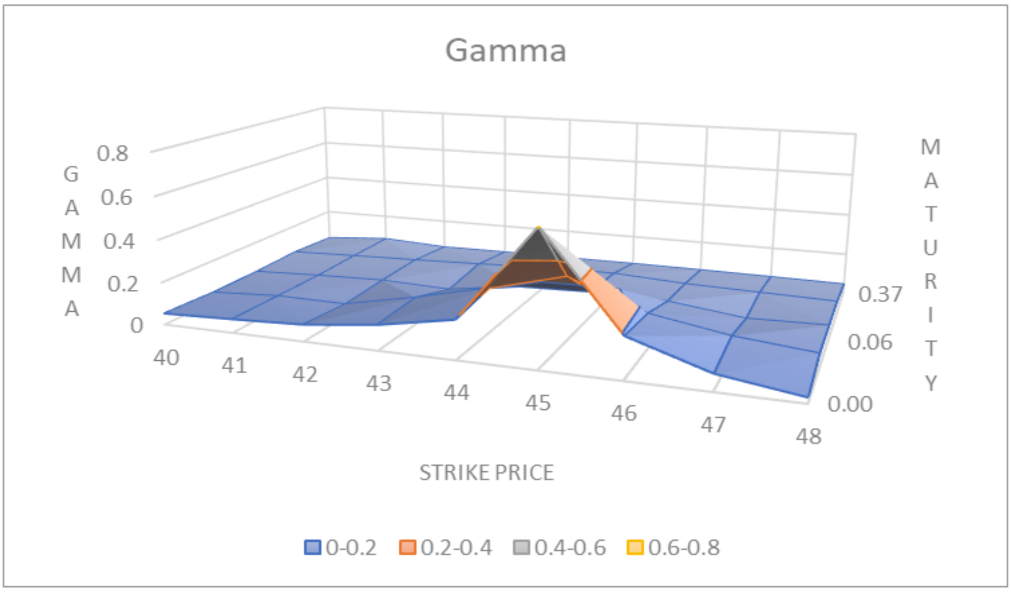
To estimate Implied Volatility is fundamental to set up a fully functional Option Pricing model.

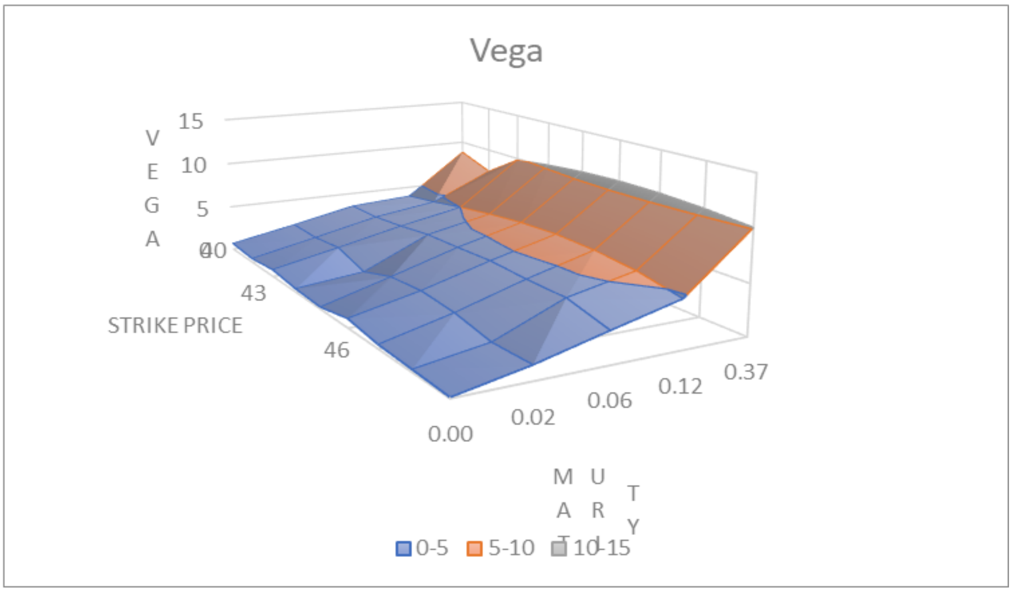
The implied volatility of an option is that value of the volatility of the underlying, when input in an option pricing model (such as Black–Scholes), will return a theoretical value equal to the current market price of that option.

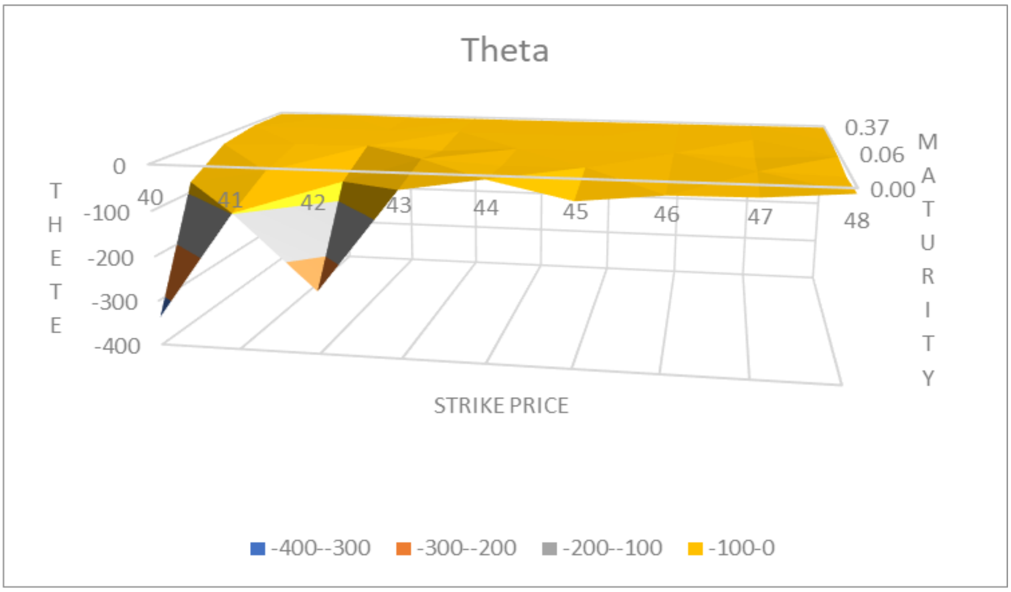
The implied volatility surface is generally set up as a 3-D plot that maps out the smile/skew/smirk and term structure of volatility implied from option chain data. This surface is normally developed by taking option market data relating to prices, bids and offers and interpolating that data using Black Scholes.



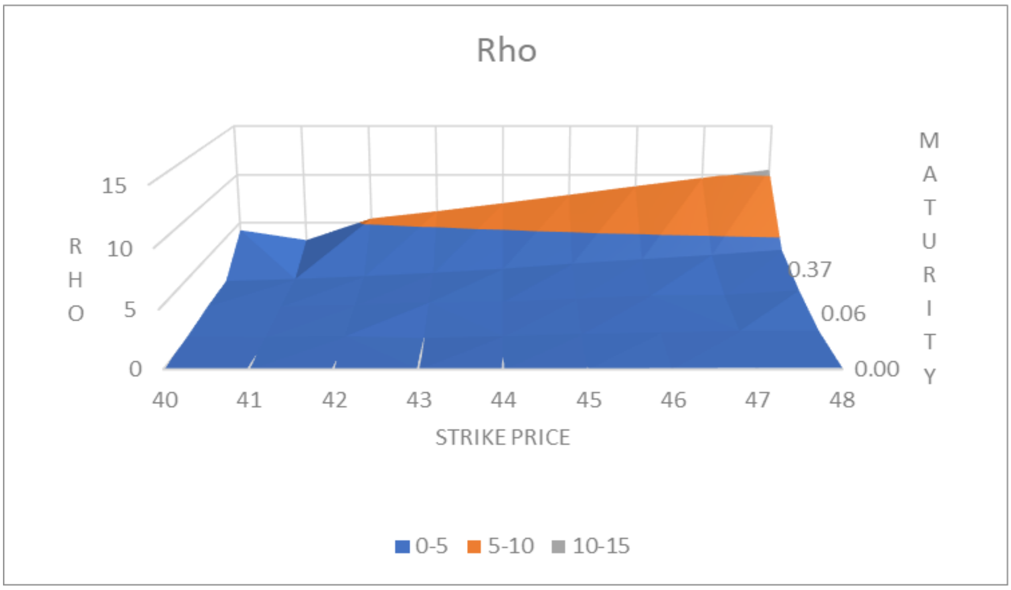

Delta
DELTA
1
0.5
0
40
41
42
43
44
45
46
47
48
STRIKE PRICE
MATURITY
0.37
0.06
0.00
0-0.5
0.5-1


Gamma
GAMMA
0.8
0.6
0.4
0.2
0
40
41
42
43
44
45
46
47
48
STRIKE PRICE
MATURITY
0.37
0.06
0.00
0-0.2
0.2-0.4
0.4-0.6
0.6-0.8


Vega
VEGA
15
10
5
0
40
43
46
STRIKE PRICE
0.00
0.02
0.06
0.12
0.37
MATURITY
0-5
5-10
10-15



Theta
0
-100
-200
-300
-400
THETE
40
41
42
43
44
45
46
47
48
0.37
0.06
0.00
MATURITY
STRIKE PRICE
-400--300
-300--200
-200--100
-100-0

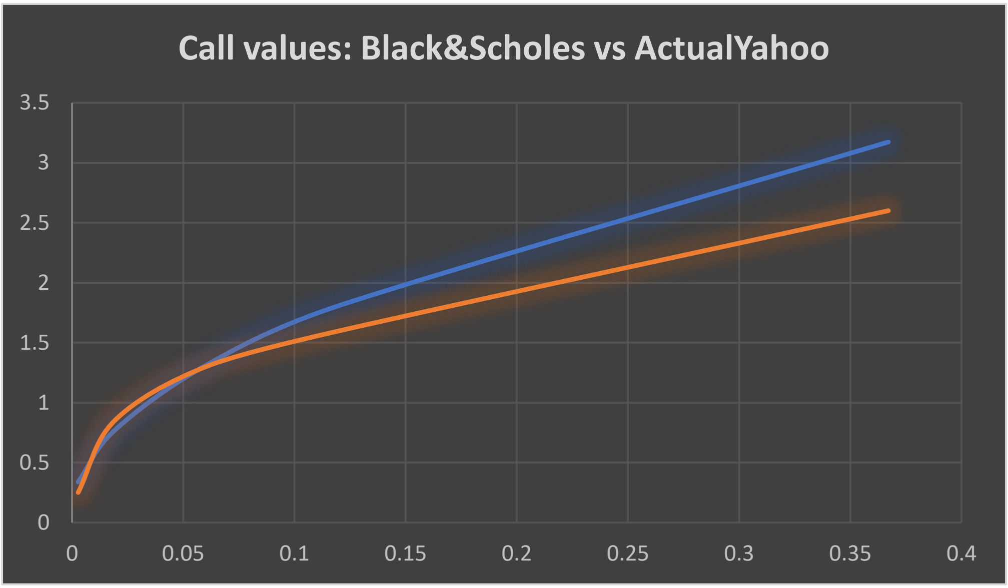


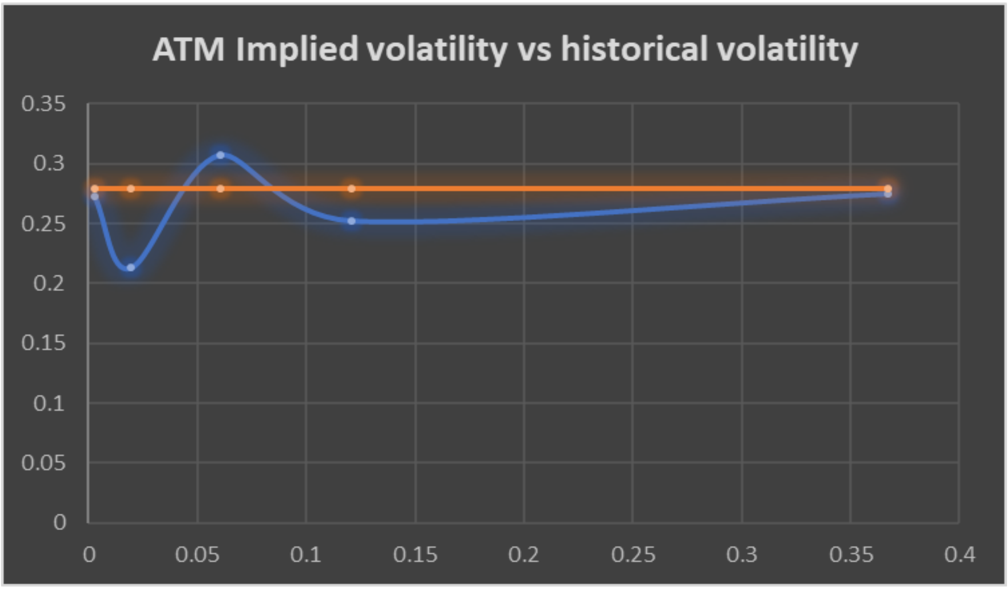
Rho
15
10
5
0
RHO
40
41
42
43
44
45
46
47
48
0.37
0.06
0.00
MATURITY
STRIKE PRICE
0-5
5-10
10-15

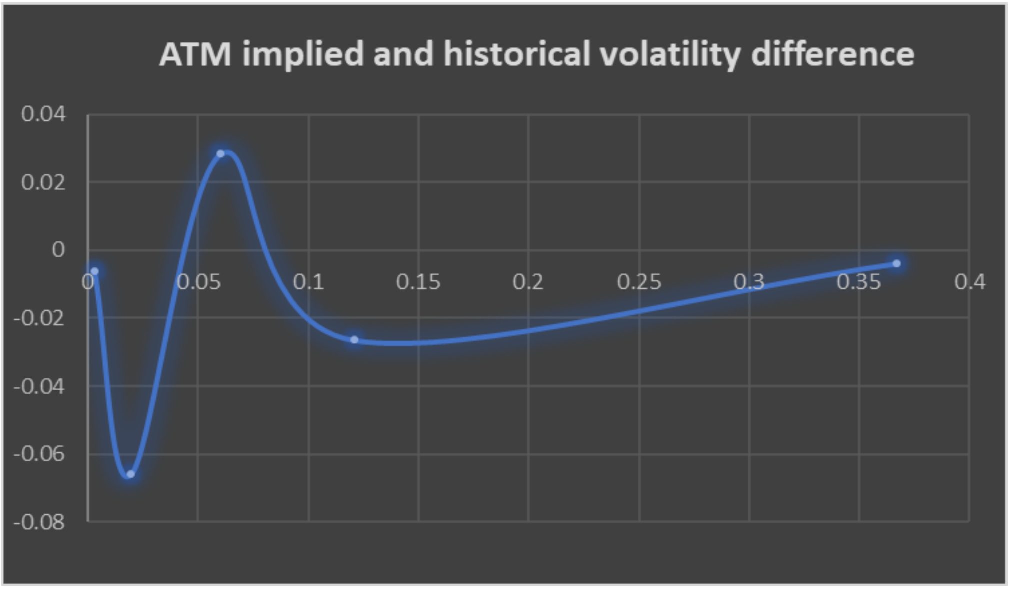
3. For the estimate of the historical volatility of the BSX asset historical data are grossed up to compute the daily returns column and with that the annual volatility, that will be compared with the implied volatilities.

Then estimate the prices for the CALL options ATM with the Black&Scholes formula which relies on the assumption of a constant volatility.

The strike price are not perfectly centered ATM so the differences between the B&S prices and the actual prices are definitely sharper, moreover if we move OTM or ITM the values clearly diverges.






ATM implied and historical volatility difference
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

1 Additional notes

The VBA scripts for the computation of the Greeks, the implied volatility and the price with BS formula of ATM call option were provided by the site VinegarHill-FinanceLabs

(https://sites.google.com/view/vinegarhill-financelabs/home?authuser=0).

The following formulae has been used to estimate the historical volatility of an asset for the third part of the report (daily returns, daily volatility and yearly volatility).

$$return_t = \frac{adj.S_t - adj.S_{t-1}}{adj.S_{t-1}} \tag{1}$$

$$\sigma_{day} = \sqrt{\frac{\sum_{i=1}^{N}(r_{t_i} - \overline{r_t})^2}{N-1}} \tag{2}$$

where N is hte number of data for the historical volatility.

$$\sigma_{year} = \sigma_{day}\sqrt{252} \tag{3}$$

Call Option Greeks

$\frac{\partial c}{\partial S}$	Delta	$e^{-qT}N(d_1)$
$\frac{\partial^2 c}{\partial S^2}$	Gamma	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$
$-\frac{\partial c}{\partial T}$	Theta	$-S_0N'(d_1)\sigma e^{-qT}/(2\sqrt{T}) + qS_0N(d_1)e^{-qT} - rKe^{-rT}N(d_2)$
$\frac{\partial c}{\partial \sigma}$	Vega	$S_0\sqrt{T}N'(d_1)e^{-qT}$
$\frac{\partial c}{\partial r}$	Rho	$KTe^{-rT}N(d_2)$

Put Option Greeks

$\frac{\partial p}{\partial S}$	Delta	$e^{-qT}[N(d_1) - 1]$
$\frac{\partial^2 p}{\partial S^2}$	Gamma	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$
$-\frac{\partial p}{\partial T}$	Theta	$-S_0N'(d_1)\sigma e^{-qT}/(2\sqrt{T}) - qS_0N(-d_1)e^{-qT} + rKe^{-rT}N(-d_2)$
$\frac{\partial p}{\partial \sigma}$	Vega	$S_0\sqrt{T}N'(d_1)e^{-qT}$
$\frac{\partial p}{\partial r}$	Rho	$-KTe^{-rT}N(-d_2)$