week2手寫作業 戴偉璿

第一題：

> $note: \lim\limits_{x\to\infty} k = k, \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$

$$\text{a. } \lim_{n\to\infty} \frac{3n+1}{n-1} = \frac{\lim\limits_{n\to\infty} (3+\dfrac{1}{n})}{\lim\limits_{n\to\infty} (1-\dfrac{1}{n})} = \frac{3}{1} = 3$$

$$\text{b. } \lim_{n\to\infty} \frac{n}{n^2+1} = \frac{\lim\limits_{n\to\infty} 1}{\lim\limits_{n\to\infty} (n+\dfrac{1}{n})} = \frac{1}{n}$$

> $note: f(n) \in O(g(n)) \Leftrightarrow \backslash\text{exist}k > 0, \backslash\text{exist}N, \forall n > N, f(n) \leq k\cdot g(n)$

$\text{c. } f(n) \in O(2^n) \Longleftrightarrow f(n) \in O(2^{n+1})\$$ (1).proof;f(n)\in O(2^n)\Longrightarrow f(n)\in O(2^{n+1})\ \because f(n)\in O(2^n)\\therefore\exist k\ge 0,\displaystyle\lim_{n\to \infty}\cfrac{f(n)}{2^n}=\displaystyle\lim_{n\to \infty}\cfrac{f(n)}{n}=k\\therefore\displaystyle\lim_{n\to \infty}\cfrac{f(n)}{2^{n+1}}=\displaystyle\lim_{n\to \infty}\cfrac{f(n)}{n}=k=\displaystyle\lim_{n\to \infty}\cfrac{f(n)}{2^n}\$

$\therefore f(n)\in O(2^{n+1})\$

$(2).proof;f(n)\in O(2^n)\Longleftarrow f(n)\in O(2^{n+1})\$

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