Fundamentals of Biomedical Image Processing HW 2

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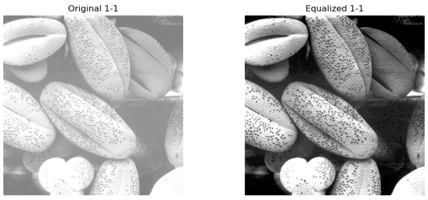
1 Theorem questions

1. Mapping the intensity values from $[r_{min}, r_{max}]$ to $[0, L-1]$ can be done using a linear transformation. Assume the original intensity value is r, the new intensity value s can be calculated by:
$$\frac{r - r_{min}}{r_{max} - r_{min}} = \frac{s}{L-1}, s = \frac{r - r_{min}}{r_{max} - r_{min}} \cdot (L-1)$$

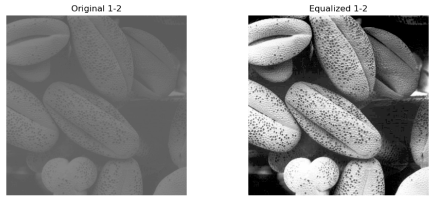
2. The given Laplacian equation is:
$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$
While the Unsharp Masking equation is:
$$g(x,y) = f(x,y) + k(f(x,y) - \overline{f}(x,y))$$
where $\overline{f}(x,y) = \dfrac{f(x,y) + f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)}{5}$
The result of subtracting Laplacian can be derived as:
$$\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f(x,y) \\
&= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)] \\
&= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] \\
&= 5\left[f(x,y) - \frac{f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)}{5}\right] \\
&= 5[f(x,y) - \overline{f}(x,y)]
\end{aligned}$$
Thus, we can see that subtracting the Laplacian is equivalent to Unsharp Masking with $k = 5$.

2 Programming exercises

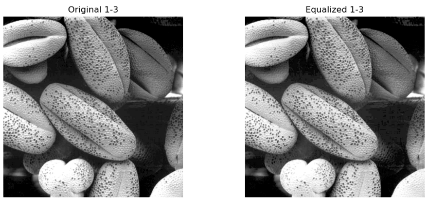
1. As Fig. 1 shows, the left column is the original images, and the right column is the histogram equalization results.



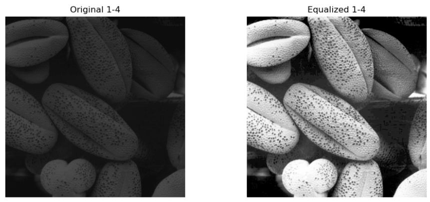
(a) Fig1-1



(b) Fig1-2



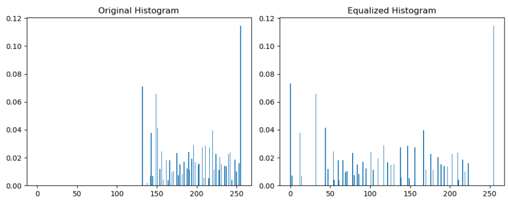
(c) Fig1-3



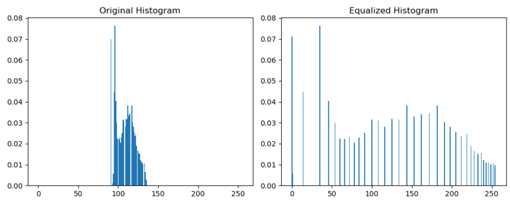
(d) Fig1-4

Figure 1: Histogram Equalization Results

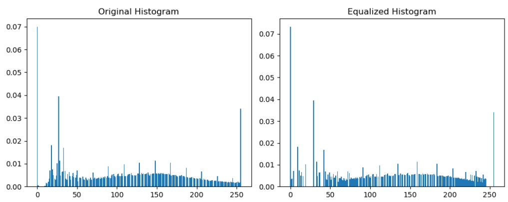
Fig. 2 are the histograms before and after histogram equalization.



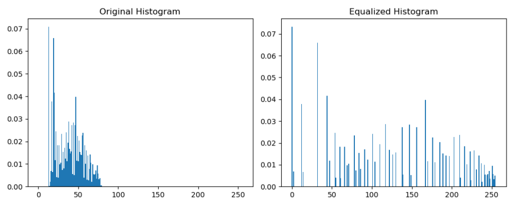
(a) Fig1-1Histogram



(b) Fig1-2Histogram



(c) Fig1-3Histogram



(d) Fig1-4Histogram

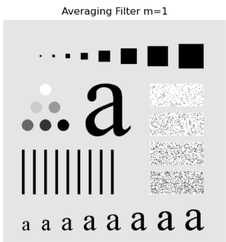
Figure 2: Histograms Before and After Histogram Equalization

Following is the code used to perform histogram equalization.

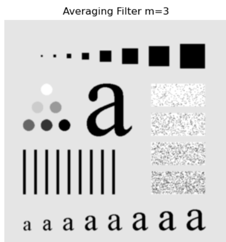
```
import os
import cv2 as cv
import numpy as np
import matplotlib
matplotlib.use('Agg')
```

```python
from matplotlib import pyplot as plt

def _hist(img):
    hist, _ = np.histogram(img.flatten(), bins=256, range=[0,256])
    hist = hist.astype(np.float64)
    hist /= hist.sum()
    return hist

def CDF(img):
    hist, cdf = _hist(img), np.zeros(256)
    cdf[0] = hist[0]
    for i in range(1,256):
        cdf[i] = cdf[i-1] + hist[i]
    return cdf

def hist_equalize(img):
    cdf = CDF(img)
    cdf_m = np.ma.masked_equal(cdf,0)
    cdf_m = (cdf_m - cdf_m.min()) * 255 / (cdf_m.max() - cdf_m.min())
    cdf = np.ma.filled(cdf_m,0).astype('uint8')
    img_eq = cdf[img]
    return img_eq

if __name__ == "__main__":
    for i in range(1,5):
        img = cv.imread(f'../src/Fig1-{i}.bmp', cv.IMREAD_GRAYSCALE)
        if img is None:
            print(f"Error loading image ../src/Fig1-{i}.bmp")
            continue
        img_eq = hist_equalize(img)
        os.makedirs('result/p1', exist_ok=True)

        # show the origin and equalized image
        plt.figure(figsize=(10,4))
        plt.subplot(1,2,1)
        plt.title(f'Original 1-{i}')
        plt.imshow(img, cmap='gray', vmin=0, vmax=255)
        plt.axis('off')
        plt.subplot(1,2,2)
        plt.title(f'Equalized 1-{i}')
        plt.imshow(img_eq, cmap='gray', vmin=0, vmax=255)
        plt.axis('off')
        plt.tight_layout()
        plt.savefig(f'result/p1/hist_equalized_1-{i}.png')

        # show the histograms
        plt.figure(figsize=(10,4))
        plt.subplot(1,2,1)
        plt.title('Original Histogram')
        hist = _hist(img)
        plt.bar(range(256), hist, width=1.0)
        plt.subplot(1,2,2)
        plt.title('Equalized Histogram')
        hist_eq = _hist(img_eq)
        plt.bar(range(256), hist_eq, width=1.0)
        plt.tight_layout()
        plt.savefig(f'result/p1/hist_equalized_1-{i}_hist.png')
```

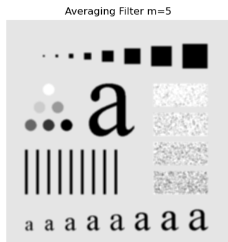
2. As Fig. 3 shows, the results of applying averaging filter with different mask sizes. To compare the effects of different mask sizes, I also include the result of mask size $m = 1$ (original image).



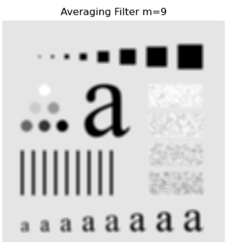
(a) m=1



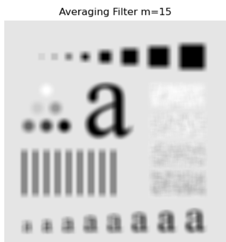
(b) m=3



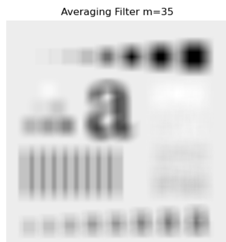
(c) m=5



(d) m=9



(e) m=15



(f) m=35

Figure 3: Averaging Filter Results with Different Mask Sizes

Following is the code used to perform averaging filter with different mask sizes.

```python
import os
import cv2 as cv
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt

img = cv.imread('../src/Fig2-1.bmp', cv.IMREAD_GRAYSCALE)
if img is None:
    raise ValueError("Image not found or could not be opened.")
os.makedirs('result/p2', exist_ok=True)

sizes = [1, 3, 5, 9, 15, 35]
for m in sizes:
    smooth = cv.blur(img, (m, m))
    plt.figure()
    plt.title(f"Averaging Filter m={m}")
    plt.imshow(smooth, cmap='gray')
    plt.axis('off')
    plt.savefig(f'result/p2/averaging_filter_m_{m}.png')
```

3. As Fig. 4 shows, the results of applying Roberts Cross operator to compute the gradient magnitude.

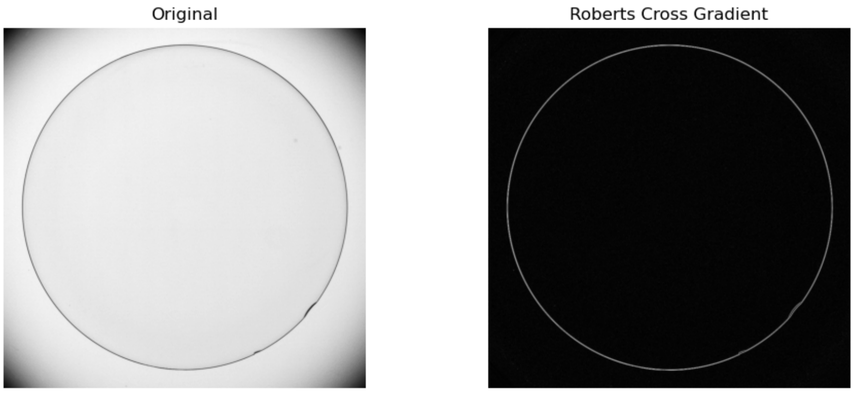


Figure 4: Roberts Cross Gradient Magnitude

Following is the code used to perform Roberts Cross operator to compute the gradient magnitude.

```
import os
import cv2 as cv
import numpy as np
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt

img = cv.imread('../src/Fig3-1.bmp', cv.IMREAD_GRAYSCALE)
if img is None:
    raise ValueError("Image not found or could not be opened.")
os.makedirs('result/p3', exist_ok=True)

# Roberts mask
Gx = np.array([[1, 0],
               [0, -1]], dtype=np.float32)

Gy = np.array([[0, 1],
               [-1, 0]], dtype=np.float32)

# Convolve with Roberts masks
grad_x = cv.filter2D(img, cv.CV_32F, Gx)
grad_y = cv.filter2D(img, cv.CV_32F, Gy)

# Compute gradient magnitude
grad = cv.magnitude(grad_x, grad_y)
grad = cv.normalize(grad, None, alpha=0, beta=255, norm_type=cv.NORM_MINMAX)
grad = grad.astype(np.uint8)

plt.figure(figsize=(10,4))
plt.subplot(1,2,1)
plt.title('Original')
plt.imshow(img, cmap='gray')
plt.axis('off')

```

```python
plt.subplot(1,2,2)
plt.title('Roberts Cross Gradient')
plt.imshow(grad, cmap='gray')
plt.axis('off')
plt.tight_layout()
plt.savefig('result/p3/roberts_cross_gradient.png')
```

4. As Fig. 5 shows, the results of each step in the image enhancement process.

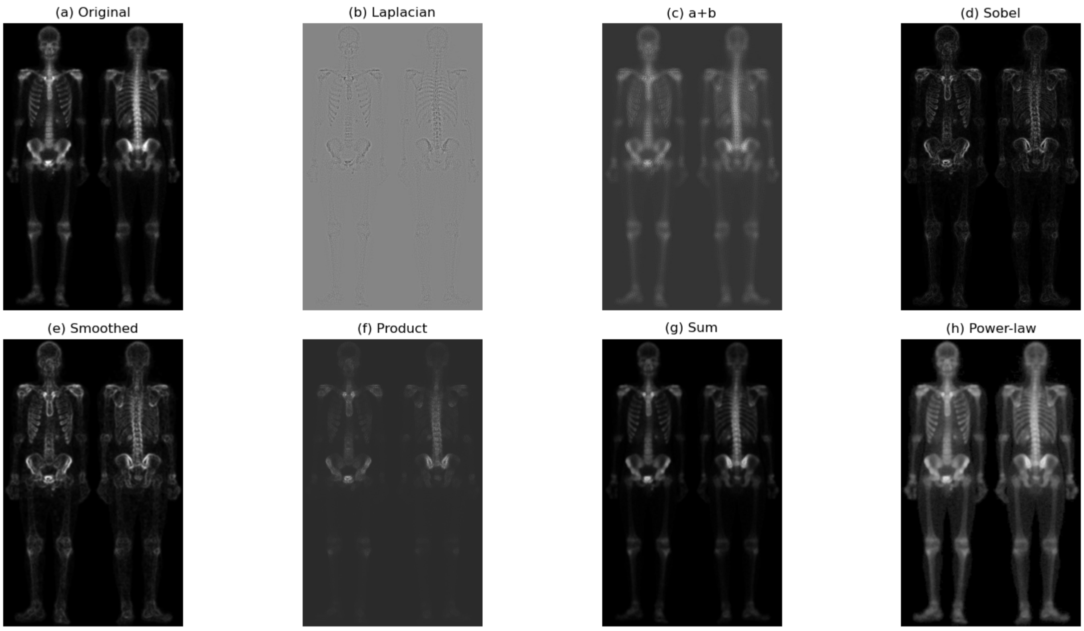


Figure 5: Image Enhancement Steps

Following is the code used to perform the image enhancement process.

```python
import os
import cv2 as cv
import numpy as np
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt

a = cv.imread('../src/Fig4-1.bmp', cv.IMREAD_GRAYSCALE)
if a is None:
    raise ValueError("Image not found or could not be opened.")
a = a.astype(np.float32)
os.makedirs('result/p4', exist_ok=True)

b = cv.Laplacian(a, cv.CV_32F, ksize=3)

c = cv.add(a, b)

gx = cv.Sobel(a, cv.CV_32F, 1, 0, ksize=3)
gy = cv.Sobel(a, cv.CV_32F, 0, 1, ksize=3)
d = cv.magnitude(gx, gy)

e = cv.blur(d, (5,5))
```

```python

f = cv.multiply(c, e, scale=1/255.0)

g = cv.add(a, f)

gamma = 0.5
g_norm = cv.normalize(g, None, 0, 1, cv.NORM_MINMAX)
power = np.power(g_norm, gamma)
h = cv.normalize(power, None, 0, 255, cv.NORM_MINMAX).astype(np.uint8)

titles = ['(a) Original', '(b) Laplacian', '(c) a+b', '(d) Sobel',
          '(e) Smoothed', '(f) Product', '(g) Sum', '(h) Power-law']
images = [a, b, c, d, e, f, g, h]

plt.figure(figsize=(16,8))
for i in range(8):
    plt.subplot(2,4,i+1)
    plt.imshow(images[i], cmap='gray')
    plt.title(titles[i])
    plt.axis('off')
plt.tight_layout()
plt.savefig('result/p4/enhancement_steps.png')
```