The Design and Analysis of Algorithms HW1

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1. (a) $\dfrac{n}{\log^3 n} = O(n)$, $64^{\sqrt{n}} = O(2^{\sqrt{6n}})$, $\log^{10} n = O(\log n)$,
   $\log(n!) = O(n \log n)$ (by Stirling's approximation), $n\sqrt{n} = O(n^{3/2})$, $\log 3^n = O(n)$,
   $n \log^2 n$

   (b) i. $\boxed{\textbf{Disprove.}}$ To prove or disprove $n^{\frac{1}{2}} = O(n^{\frac{1}{3}})$, consider the definition of Big-O notation. $n^{\frac{1}{2}} \leq cn^{\frac{1}{3}}$ where c is a constant. Dividing both sides by $n^{\frac{1}{3}}$ gives $n^{\frac{1}{6}} \leq c$. As n approaches infinity, $n^{\frac{1}{6}}$ also approaches infinity, so there is no constant c that satisfies the inequality. Therefore, $n^{\frac{1}{2}} \neq O(n^{\frac{1}{3}})$.

   ii. $\boxed{\textbf{Prove.}}$ To prove or disprove $3^n = \Omega(27^{\sqrt{n}})$, consider the definition of Big-Omega notation. $3^n \geq c27^{\sqrt{n}}$ where c is a constant. Dividing both sides by $27^{\sqrt{n}}$ gives $\dfrac{3^n}{27^{\sqrt{n}}} = 3^{n-3\sqrt{n}} \geq c$. As n approaches infinity, $n - 3\sqrt{n}$ also approaches infinity, so there exists a constant c that satisfies the inequality. Therefore, $3^n = \Omega(27^{\sqrt{n}})$.

2. (a) The outer loop runs from 1 to n, and the inner loop runs from 1 to $\sqrt{i}$. So the total number of iterations is:$\displaystyle\sum_{k=1}^{n} \sqrt{k}$. To analysis this, we can use the integral method:$\displaystyle\sum_{k=1}^{n} \sqrt{k} \approx \int_1^n \sqrt{x}\, dx = \left[\frac{2}{3}x^{3/2}\right]_1^n = \frac{2}{3}(n^{3/2} - 1) = \Theta(n^{3/2})$.