Supplementary Files of "A Universal Large-scale Many-objective Optimization Framework based on Cultural Learning"

Abstract

This document provides supplementary information for the main manuscript. Table A.1 shows the list of abbreviations for this paper. Tables A.2 and A.3 show the results of the GD and Δ_P metrics. Table A.4 shows the complexity analysis.

Appendix A. Abbreviation of this paper

Table A.1 is the list of abbreviations for this paper.

Appendix B. Results of GD and Δ_P metrics of UCLMO and the state-of-the-art algorithms on LSMOPs

Tables A.2 and A.3 show the results of the GD and Δ_P metrics obtained by the proposed UCLMO algorithm and other state-of-the-art algorithms on LSMOPs test suits.

As can be seen from the table, UCLMO achieves significantly better results in the GD and Δ_P metrics compared to CCGDE3, LMOCSO, LSMaODE, CVEA3, AMPDEA, BCE-IBEA, GLMO, DGEA, WOF, and FDV on LSMOP1-9. This demonstrates the effectiveness of the proposed UCLMO algorithm in solving large-scale many-objective optimization problems.

Table A.1: List of abbreviations for this article

Abbreviation	Full name
LMaOPs	Large-scale many-objective optimization problems
MOPs	Multi-objective optimization problems
MaOPs	Many-objective optimization problems
MaOEAs	Many-objective evolutionary algorithms
PF	Pareto optimal front
PBI	Penalty-based boundary intersection
MaxFEs	the maximum number of evaluations for population evolution
IGD	Inverted Generational Distance
UCLMO	Universal large-scale many-objective optimization framework based on cultural learning
MOEA/D	MOEA based on Decomposition
CCGDE3	Cooperative co-evolutionary GDE3
WOF	Weighted optimization framework
GLMO	Grouped and Linked Polynomial Mutation Operator
DGEA	Direction guided adaptive offspring generation method in MOEA framework
LMOCSO	Large-scale multi-objective competitive swarm optimizer algorithm
CA	Cultural algorithm
LSMaODE	Multipopulation-based differential evolution algorithm
FDV	Fuzzy decision variable framework for large-scale MOPs
CVEA3	Cost value based evolutionary algorithm 3
AMPDEA	Multi-population-driven evolutionary algorithm
BCE-IBEA	Bi-criterion evolution based IBEA

Appendix C. Analysis of complexity

The complexity analysis of UCLMO and other compared algorithms are presented in Table A.4. Here, M denotes the number of objectives, N represents the population size, D is the number of decision variables, and C denotes the computational complexity of the embedded non-Pareto algorithm. Additionally, $NumEsp$ denotes the number of subpopulations, $Gmax$ represents the number of iterations for each subpopulation, and $RefNo$ is the number of reference vectors used for offspring generation. According to the table, the algorithmic complexity of UCLMO is similar to most of the comparison algorithms (except

Table A.2: Comparison results of GD metric obtained by UCLMO and 10 state-of-the-art algorithms on LSMOPs

Problem	M	CCGDE3	LMOCSO	LSMaODE	CVEA3	AMPDEA	BCE-IBEA	GLMO	DGEA	WOF	FDV	UCLMO
LSMOP1	5	2.3495e+0 (4.43e-1) -	6.2597e-1 (1.06e+0) -	1.9807e+0 (1.59e+0) -	1.7096e-1 (2.16e-2) -	9.0716e+0 (8.30e+0) -	1.0667e+0 (7.58e-2) -	3.0887e-1 (4.89e-2) -	1.1425e+0 (6.41e-1) -	8.1074e-1 (1.36e-1) -	7.3330e-1 (8.15e-2) -	5.2082e-2 (3.97e-3)
	8	2.6154e+0 (3.33e-1) -	6.1808e-1 (1.74e+0) -	4.7415e+0 (1.85e+0) -	6.7392e-1 (7.38e-2) -	1.0565e+1 (5.96e+0) -	1.2405e+0 (8.03e-2) -	4.1105e-1 (7.93e-2) -	5.1664e-1 (2.41e-1) -	9.6185e-1 (1.29e-1) -	1.1393e+0 (7.72e-2) -	4.3629e-2 (5.09e-3)
	10	2.1780e+0 (1.15e-1) -	1.0988e+0 (2.49e+0) -	3.9591e+0 (3.39e-1) -	6.3744e-1 (8.39e-2) -	1.1144e+1 (7.64e+0) -	9.6886e-1 (6.22e-2) -	3.1222e-1 (4.07e-2) -	5.6191e-1 (2.36e-1) -	7.7790e-1 (9.07e-2) -	1.0841e+0 (8.32e-2) -	2.0054e-2 (1.02e-3)
	15	1.9983e+0 (9.52e-2) -	2.4587e+0 (3.28e+0) -	3.5097e+0 (3.09e-1) -	7.1399e-1 (5.10e-2) -	1.2306e+1 (7.89e+0) -	9.2596e-1 (7.81e-2) -	3.3254e-1 (3.78e-2) -	9.2389e-1 (2.74e-1) -	8.3954e-1 (1.09e-1) -	8.6543e-1 (5.88e-2) -	2.2452e-2 (1.42e-3)
LSMOP2	5	1.0787e-2 (2.83e-4) -	1.1066e-2 (9.19e-5) -	8.2708e-3 (2.37e-4) +	8.4494e-3 (1.67e-4) +	2.8906e-2 (8.81e-3) -	7.7457e-3 (3.42e-4) +	9.7880e-3 (1.46e-4) -	1.0297e-2 (6.12e-4) -	1.0729e-2 (2.82e-4) -	9.7207e-3 (1.26e-4) -	9.0343e-3 (1.00e-4)
	8	3.2769e-2 (2.61e-3) -	9.1620e-2 (1.17e-1) -	4.7297e-2 (2.86e-3) -	1.4621e-2 (6.57e-4) +	1.3380e-1 (1.25e-1) -	1.5598e-2 (2.57e-3) -	2.1150e-2 (8.90e-4) -	2.3619e-2 (5.66e-3) -	1.9547e-2 (1.07e-3) -	1.7949e-2 (9.31e-4) -	1.5396e-2 (3.31e-4)
	10	1.7175e-2 (6.07e-4) -	9.5865e-2 (7.99e-2) -	1.8489e-2 (6.85e-4) -	1.2188e-2 (4.06e-4) -	6.5846e-2 (3.97e-2) -	1.0769e-2 (4.06e-4) -	1.4121e-2 (2.51e-4) -	1.2716e-2 (5.72e-4) -	1.5358e-2 (3.75e-4) -	1.4369e-2 (2.30e-4) -	7.6684e-3 (3.00e-4)
	15	1.9357e-2 (1.09e-3) -	4.3905e-2 (2.47e-2) -	2.1799e-2 (8.64e-4) -	1.4809e-2 (4.78e-4) -	5.1665e-2 (2.11e-2) -	1.2072e-2 (6.80e-4) -	1.3775e-2 (3.64e-4) -	1.3762e-2 (1.00e-3) -	1.4677e-2 (5.80e-4) -	1.4220e-2 (3.34e-4) -	6.3782e-3 (2.96e-4)
LSMOP3	5	7.1379e+3 (1.25e+3) -	3.9472e+0 (1.66e+0) -	4.5427e+3 (2.88e+3) -	1.1057e+2 (4.27e+1) -	1.6644e+4 (1.03e+4) -	1.8621e+3 (4.25e+2) -	5.4859e+1 (9.47e+1) -	1.7436e+3 (3.43e+3) -	2.0362e+3 (5.50e+2) -	1.1048e+3 (3.29e+2) -	2.8130e-1 (1.07e-1)
	8	2.6539e+4 (1.08e+4) -	1.1187e+1 (1.57e+1) -	9.5046e+4 (4.18e+4) -	1.7515e+3 (7.31e+2) -	1.0404e+5 (1.10e+5) -	4.8354e+3 (5.62e+2) -	2.9879e+3 (2.19e+3) -	7.1824e+3 (1.04e+4) -	1.1928e+3 (3.69e+2) -	2.0693e+3 (4.41e+2) -	3.2995e-1 (1.32e-1)
	10	2.8226e+4 (4.78e+3) -	2.9240e+1 (3.87e+1) -	6.9179e+4 (1.85e+4) -	2.0991e+3 (4.68e+2) -	1.2004e+5 (1.41e+5) -	3.6973e+3 (3.13e+2) -	2.5374e+3 (1.33e+3) -	2.1137e+4 (1.20e+4) -	1.7088e+3 (1.04e+3) -	3.2883e+3 (1.04e+3) -	2.8365e-1 (6.97e-2)
	15	8.1867e+3 (1.12e+3) -	1.6500e+2 (6.50e+2) -	6.6148e+3 (1.05e+3) -	2.0055e+3 (5.17e+2) -	2.0632e+4 (1.59e+4) -	2.4889e+3 (2.28e+2) -	1.9660e+2 (7.75e+1) -	1.9145e+3 (6.55e+2) -	2.5453e+3 (5.09e+2) -	2.2236e+3 (6.19e+2) -	4.4283e-1 (2.82e-1)
LSMOP4	5	7.5136e-2 (5.43e-3) -	5.0756e-2 (2.62e-3) -	7.4208e-2 (1.07e-2) -	2.3482e-2 (1.55e-3) +	3.2940e-1 (2.35e-1) -	3.9551e-2 (2.92e-3) -	4.5329e-2 (2.70e-3) -	3.4261e-2 (9.12e-3) -	3.4807e-2 (2.16e-3) -	5.1579e-2 (3.01e-3) -	3.3283e-2 (8.52e-4)
	8	5.1087e-2 (3.45e-3) -	6.3770e-2 (3.11e-2) -	2.8226e-2 (4.05e-3) -	2.3246e-2 (1.28e-3) +	3.7985e-2 (1.88e-2) -	1.5722e-2 (1.53e-3) +	3.3005e-2 (1.62e-3) -	4.1984e-2 (2.29e-3) -	4.3519e-2 (1.65e-3) -	3.0213e-2 (9.83e-4) -	2.3788e-2 (6.20e-4)
	10	3.4483e-2 (3.62e-3) -	5.2452e-2 (3.83e-2) -	1.2629e-2 (6.01e-4) -	1.7250e-2 (1.14e-3) -	3.4710e-2 (1.33e-2) -	1.2311e-2 (1.05e-3) -	2.7082e-2 (1.50e-3) -	1.8707e-2 (3.59e-3) -	3.2067e-2 (1.48e-3) -	2.3090e-2 (9.33e-4) -	1.1478e-2 (5.58e-4)
	15	2.3000e-2 (4.58e-3) -	6.2700e-2 (5.02e-2) -	2.1123e-2 (7.10e-4) -	1.4538e-2 (3.24e-4) -	8.1116e-2 (4.37e-2) -	1.4119e-2 (9.12e-4) -	2.4738e-2 (6.32e-4) -	1.6222e-2 (2.22e-3) -	2.3291e-2 (1.82e-3) -	2.3627e-2 (9.22e-4) -	8.9260e-3 (1.65e-4)
LSMOP5	5	1.5439e+1 (9.34e-1) -	7.3709e-1 (2.39e-1) -	2.0029e+0 (1.99e+0) -	1.2753e+0 (2.88e-1) -	2.1593e+1 (1.69e+1) -	5.6193e+0 (6.87e-1) -	8.7432e-1 (4.64e-1) -	5.5458e-1 (1.22e+0) =	6.8614e-1 (4.00e-1) -	2.8176e+0 (3.08e-1) -	1.2847e-1 (1.33e-2)
	8	1.8809e+1 (1.06e+0) -	2.8361e+0 (3.98e+0) -	4.9098e+0 (2.24e+0) -	7.0651e+0 (1.60e+0) -	1.0562e+1 (7.03e+0) -	7.8689e+0 (2.06e+0) -	1.9612e+0 (7.05e-1) -	1.2347e+0 (1.23e+0) -	2.6942e+0 (1.01e+0) -	6.0178e+0 (1.06e+0) -	1.0049e-1 (1.15e-2)
	10	1.5008e+1 (6.91e-1) -	1.8618e+0 (2.81e+0) -	4.5818e+0 (1.13e+0) -	8.5938e+0 (8.76e-1) -	1.2670e+1 (9.02e+0) -	4.8768e+0 (1.05e+0) -	9.1821e-1 (6.63e-1) -	2.5177e+0 (1.52e+0) -	2.9264e+0 (1.44e+0) -	7.3953e+0 (5.15e-1) -	4.8502e-2 (5.28e-3)
	15	1.3646e+1 (1.25e+0) -	3.4836e+0 (2.91e+0) -	3.5487e+0 (5.69e-1) -	7.5626e+0 (7.58e-1) -	9.2719e+0 (6.30e+0) -	6.0217e+0 (3.43e-1) -	1.0967e+0 (5.18e-1) -	3.0506e+0 (1.11e+0) -	2.4722e+0 (1.07e+0) -	6.7276e+0 (2.32e+0) -	3.4775e-2 (3.18e-3)
LSMOP6	5	1.2968e+5 (3.73e+4) -	8.8845e+3 (7.22e+3) -	8.7578e+2 (1.32e+3) -	3.3736e+3 (2.16e+3) -	1.2921e+5 (1.40e+5) -	9.4943e+3 (1.74e+3) -	2.7801e+3 (3.46e+3) -	8.4585e+3 (1.53e+4) -	5.8108e+3 (2.96e+3) -	4.4524e+3 (1.10e+3) -	2.3740e-1 (7.30e-2)
	8	2.4134e+5 (2.32e+4) -	1.0005e+3 (1.90e+3) -	2.3570e+4 (1.59e+4) -	2.0085e+4 (1.16e+4) -	3.9891e+4 (5.87e+4) -	1.2743e+5 (2.33e+4) -	3.5866e+1 (1.30e+2) -	2.5768e+4 (3.32e+4) -	5.0911e+3 (9.38e+3) -	1.2782e+4 (3.25e+3) -	1.5902e-1 (2.07e-2)
	10	1.8942e+5 (1.93e+4) -	6.0965e+3 (9.59e+3) -	2.2892e+4 (2.87e+3) -	4.1628e+4 (6.98e+3) -	3.6198e+4 (6.62e+4) -	1.2425e+5 (1.78e+4) -	7.6975e+2 (1.15e+3) -	3.7698e+4 (2.68e+4) -	1.0165e+4 (6.12e+3) -	2.6235e+4 (2.42e+3) -	1.1329e-1 (2.49e-2)
	15	1.6258e+5 (3.31e+4) -	3.2566e+4 (5.12e+4) -	9.1056e+3 (3.03e+3) -	4.2475e+4 (1.24e+4) -	1.8778e+4 (1.21e+4) -	9.8942e+4 (2.10e+4) -	1.1769e+4 (2.64e+3) -	3.2414e+4 (2.35e+4) -	1.4202e+4 (1.04e+4) -	3.5440e+4 (1.89e+4) -	3.4459e-1 (9.56e-2)
LSMOP7	5	1.3726e+5 (2.60e+4) -	3.4931e+1 (1.86e+2) -	8.5877e+3 (9.60e+3) -	1.4171e+3 (3.96e+2) -	1.5472e+5 (1.58e+5) -	1.5681e+4 (4.98e+3) -	1.3139e+2 (5.44e+2) -	9.0182e+3 (1.95e+4) =	5.7759e+2 (1.57e+3) -	4.3543e+3 (1.28e+3) -	3.4956e-1 (9.01e-2)
	8	2.0653e+5 (4.55e+4) -	2.6130e+4 (1.87e+4) -	4.0831e+3 (3.52e+3) -	2.8459e+4 (1.36e+4) -	5.0133e+4 (7.63e+4) -	7.5932e+4 (2.60e+4) -	8.6636e+3 (1.08e+4) -	1.2247e+4 (1.34e+4) -	1.2051e+4 (9.26e+3) -	1.7885e+4 (8.02e+3) -	1.9611e-1 (3.34e-2)
	10	1.6904e+5 (2.72e+4) -	2.4288e+4 (4.32e+4) -	8.2107e+3 (3.35e+3) -	5.6944e+4 (1.33e+4) -	5.3046e+4 (6.38e+4) -	1.0392e+5 (1.63e+4) -	1.0519e+4 (4.50e+3) -	2.3063e+4 (1.69e+4) -	1.6386e+4 (1.48e+4) -	1.5647e+4 (9.81e+3) -	2.6150e-1 (5.76e-2)
	15	1.6630e+5 (2.86e+4) -	1.3380e+4 (1.80e+4) -	1.4981e+4 (2.07e+3) -	3.0922e+4 (6.58e+3) -	1.6543e+4 (2.47e+4) -	1.0596e+5 (1.23e+4) -	3.6827e+3 (1.90e+3) -	3.7744e+4 (1.89e+4) -	1.7585e+4 (8.62e+3) -	3.4970e+4 (1.08e+4) -	1.7347e-1 (2.72e-1)
LSMOP8	5	7.3602e+0 (8.02e-1) -	6.5062e-2 (9.31e-2) +	1.2067e+0 (9.51e-1) -	2.7082e-1 (4.68e-2) -	1.0830e+1 (9.59e+0) -	1.7476e+0 (2.02e-1) -	2.5951e-2 (2.34e-2) +	1.9563e-1 (4.71e-1) +	5.7026e-2 (7.82e-2) +	7.4466e-1 (1.09e-1) -	2.6220e-1 (4.78e-2)
	8	9.9248e+0 (4.45e-1) -	1.1347e+0 (2.05e+0) -	2.9162e+0 (1.48e+0) -	3.1168e+0 (8.05e-1) -	8.0339e+0 (5.34e+0) -	4.9057e+0 (7.70e-1) -	1.1726e+0 (1.18e+0) =	5.7000e-1 (8.09e-1) -	1.7867e+0 (7.67e-1) -	2.2556e+0 (6.70e-1) -	6.1534e-2 (1.08e-2)
	10	7.6777e+0 (4.88e-1) -	1.4146e+0 (1.24e+0) -	2.3904e+0 (5.54e-1) -	3.7631e+0 (6.35e-1) -	6.2385e+0 (3.36e+0) -	3.8950e+0 (6.87e-1) -	6.0074e-1 (4.94e-1) -	1.4853e+0 (9.24e-1) -	1.5494e+0 (5.41e-1) -	3.0589e+0 (3.62e-1) -	2.9303e-2 (3.05e-3)
	15	7.0486e+0 (6.79e-1) -	2.5345e+0 (1.45e+0) -	1.8206e+0 (1.37e-1) -	2.5563e+0 (3.47e-1) -	4.2030e+0 (3.56e+0) -	4.5403e+0 (4.29e-1) -	1.9850e-1 (1.01e-1) -	2.1445e+0 (5.21e-1) -	1.4751e+0 (6.18e-1) -	4.1586e+0 (2.50e-1) -	2.2456e-2 (5.17e-3)
LSMOP9	5	4.7047e+1 (8.69e+0) -	7.4495e-1 (2.55e-1) -	7.1262e+1 (2.58e+1) -	6.2875e-1 (1.09e-1) -	5.0397e+2 (2.24e+2) -	5.3198e-1 (2.37e-2) -	1.4369e-2 (3.93e-2) +	4.2212e+1 (1.20e+1) -	7.7734e-3 (7.03e-3) +	1.3951e+0 (1.43e-1) -	3.9431e-2 (2.86e-2)
	8	1.5162e+2 (3.21e+1) -	1.2264e+1 (1.04e+1) -	1.7570e+2 (2.44e+1) -	1.9109e+1 (5.33e+0) -	1.3784e+3 (5.54e+2) -	1.2793e+1 (1.78e+0) -	4.4819e-1 (8.37e-1) +	1.4072e+2 (6.10e+0) -	1.6235e-1 (1.47e-1) +	9.7790e+0 (1.75e+0) -	7.0837e-1 (1.80e-1)
	10	2.0904e+2 (3.04e+1) -	1.9318e+1 (5.95e+0) -	1.8708e+2 (6.99e+0) -	1.0242e+2 (1.29e+1) -	2.0621e+3 (1.04e+3) -	4.6333e+1 (1.51e+1) -	1.6793e+0 (6.21e-1) =	1.6891e+2 (5.83e+0) -	5.8815e+0 (6.17e+0) -	1.6371e+1 (2.13e+0) -	1.7714e+0 (2.61e-1)
	15	4.8081e+2 (2.53e+1) -	2.8887e+2 (2.14e+2) -	4.0067e+2 (1.13e+1) -	2.7615e+2 (2.09e+1) -	5.3471e+3 (2.52e+3) -	1.5606e+2 (1.29e+1) -	5.1509e+0 (1.21e+0) +	3.7316e+2 (9.15e+0) -	1.4981e+2 (6.00e+1) -	6.7865e+1 (7.30e+0) -	1.3535e+1 (1.22e+0)
+/-/=		0/36/0	1/35/0	1/35/0	4/32/0	0/36/0	2/34/0	4/30/2	1/33/2	3/33/0	0/36/0	

Table A.3: Comparison results of Δ_P metric obtained by UCLMO and 10 state-of-the-art algorithms on LSMOPs

Problem	M	CCGDE3	LMOCSO	LSMaODE	CVEA3	AMPDEA	BCEIBEA	GLMO	DGEA	WOF	FDV	UCLMO
LSMOP1	5	2.4341e+1 (3.30e+0) -	3.9656e+0 (2.32e+0) -	8.0452e+0 (6.84e+0) -	1.7680e+0 (1.80e-1) -	3.2611e+1 (2.79e+1) -	1.1332e+1 (7.61e-1) -	2.4415e+0 (3.54e-1) -	8.0744e+0 (3.75e+0) -	7.3652e+0 (1.19e+0) -	7.3376e+0 (6.95e-1) -	5.4669e-1 (4.33e-2)
	8	2.7121e+1 (1.86e+0) -	1.5762e+0 (2.36e+0) -	2.8296e+1 (1.27e+1) -	6.8786e+0 (7.33e-1) -	4.6054e+1 (2.62e+1) -	1.3221e+1 (8.09e-1) -	3.4175e+0 (7.47e-1) -	4.0033e+0 (1.04e+0) -	9.6922e+0 (1.24e+0) -	1.1515e+1 (5.48e-1) -	4.9230e-1 (3.77e-2)
	10	2.8568e+1 (1.47e+0) -	2.3558e+0 (3.07e+0) -	3.7590e+1 (3.80e+0) -	9.2093e+0 (9.73e-1) -	4.9015e+1 (3.38e+1) -	1.3699e+1 (6.30e-1) -	3.2105e+0 (5.01e-1) -	5.4710e+0 (1.87e+0) -	1.1233e+1 (1.43e+0) -	1.4174e+1 (6.15e-1) -	3.8051e-1 (1.10e-2)
	15	2.7483e+1 (1.16e+0) -	5.5019e+0 (6.58e+0) -	4.0335e+1 (3.83e+0) -	1.0664e+1 (7.78e-1) -	5.4853e+1 (3.52e+1) -	1.2922e+1 (1.01e+0) -	3.0013e+0 (5.06e-1) -	9.7972e+0 (2.32e+0) -	1.2293e+1 (1.64e+0) -	1.2034e+1 (6.49e-1) -	4.2432e-1 (1.46e-2)
LSMOP2	5	1.9009e-1 (6.84e-3) -	1.7382e-1 (9.41e-4) -	1.5286e-1 (2.93e-3) +	1.7294e-1 (4.39e-3) -	7.2625e-1 (1.81e-1) -	1.5031e-1 (4.26e-3) +	1.7292e-1 (6.57e-4) -	1.7190e-1 (2.62e-3) -	1.7857e-1 (8.54e-4) -	1.7292e-1 (6.86e-4) -	1.6899e-1 (5.60e-4)
	8	3.0371e-1 (9.54e-3) -	4.7628e-1 (1.93e-1) -	3.8369e-1 (2.09e-2) -	2.9156e-1 (6.28e-3) -	1.2580e+0 (6.37e-1) -	3.4681e-1 (1.07e-1) -	3.0597e-1 (2.11e-3) -	2.8515e-1 (6.30e-3) =	3.1428e-1 (2.04e-3) -	2.9897e-1 (2.54e-3) -	2.8610e-1 (1.90e-3)
	10	3.0148e-1 (9.88e-3) =	6.0427e-1 (2.17e-1) -	4.0836e-1 (2.19e-2) -	2.8261e-1 (1.09e-2) +	9.7036e-1 (3.05e-1) -	2.5905e-1 (2.75e-2) +	3.3778e-1 (1.79e-3) -	2.4307e-1 (1.02e-2) +	3.4202e-1 (5.56e-3) -	3.3622e-1 (3.47e-3) -	3.0023e-1 (3.85e-3)
	15	3.6114e-1 (2.30e-2) -	5.0352e-1 (1.38e-1) -	5.0178e-1 (2.23e-2) -	4.1263e-1 (2.16e-2) -	8.6548e-1 (1.71e-1) -	3.3808e-1 (5.14e-2) -	2.8212e-1 (1.41e-3) -	2.8692e-1 (1.50e-2) -	2.8587e-1 (6.78e-3) -	2.9032e-1 (4.20e-3) -	2.7016e-1 (1.03e-3)
LSMOP3	5	6.5178e+4 (9.88e+3) -	3.1416e+1 (9.70e+0) -	2.6130e+4 (1.96e+4) -	8.9623e+2 (3.47e+2) -	6.0377e+4 (3.80e+4) -	1.4642e+4 (3.00e+3) -	3.1935e+2 (5.42e+2) -	6.3077e+3 (1.29e+4) -	1.6233e+4 (4.37e+3) -	9.0553e+3 (2.44e+3) -	2.7268e+0 (1.00e+0)
	8	1.8979e+5 (5.85e+4) -	3.7945e+1 (4.59e+1) -	5.0015e+5 (2.84e+5) -	1.3157e+4 (5.29e+3) -	4.3375e+5 (4.53e+5) -	4.0439e+4 (5.66e+3) -	1.1615e+4 (1.26e+4) -	1.6002e+4 (2.80e+4) -	9.9228e+3 (3.33e+3) -	1.5949e+4 (3.73e+3) -	2.8531e+0 (9.20e-1)
	10	2.4200e+5 (4.04e+4) -	8.1032e+1 (9.02e+1) -	5.4806e+5 (1.82e+5) -	2.4626e+4 (4.84e+3) -	5.2744e+5 (6.19e+5) -	3.9576e+4 (4.15e+3) -	1.3112e+4 (1.20e+4) -	1.0561e+5 (8.46e+4) -	1.9209e+4 (1.20e+4) -	3.2991e+4 (7.58e+3) -	3.2758e+0 (1.02e+0)
	15	1.0178e+5 (1.39e+4) -	3.2769e+2 (1.09e+3) -	6.9676e+4 (1.38e+4) -	2.4925e+4 (7.78e+3) -	8.9820e+4 (6.91e+4) -	2.3665e+4 (2.34e+3) -	1.2754e+3 (6.37e+2) -	1.1008e+4 (4.52e+3) -	3.1163e+4 (6.24e+3) -	2.3974e+4 (7.49e+3) -	4.7577e+0 (1.63e+0)
LSMOP4	5	6.5543e-1 (3.86e-2) -	4.2352e-1 (1.39e-2) -	5.9035e-1 (7.14e-2) -	2.3667e-1 (7.31e-3) +	1.8200e+0 (9.71e-1) -	3.1991e-1 (1.75e-2) -	4.1133e-1 (1.88e-2) -	3.2656e-1 (6.81e-2) -	3.4088e-1 (2.15e-2) -	4.3996e-1 (2.04e-2) -	3.1800e-1 (5.57e-3)
	8	4.0272e-1 (2.80e-2) -	5.1392e-1 (1.80e-1) -	3.7546e-1 (2.21e-2) -	3.2219e-1 (5.57e-3) +	9.2802e-1 (1.35e-1) -	2.8475e-1 (5.35e-2) +	3.4975e-1 (3.00e-3) -	3.4478e-1 (1.36e-2) -	3.8861e-1 (1.56e-2) -	3.4645e-1 (1.99e-3) -	3.2605e-1 (2.48e-3)
	10	3.6735e-1 (2.28e-2) -	5.0436e-1 (1.79e-1) -	3.9840e-1 (2.14e-2) -	2.9924e-1 (8.45e-3) +	8.4789e-1 (1.62e-1) -	2.5650e-1 (9.20e-3) +	3.7710e-1 (2.45e-3) -	2.6937e-1 (1.49e-2) +	3.8254e-1 (1.07e-2) -	3.7004e-1 (4.47e-3) -	3.1789e-1 (3.26e-3)
	15	3.9406e-1 (2.26e-2) -	5.6445e-1 (1.67e-1) -	5.1238e-1 (1.90e-2) -	4.1847e-1 (1.82e-2) -	1.1163e+0 (3.58e-1) -	3.6249e-1 (6.72e-2) -	3.0830e-1 (1.80e-3) -	3.0214e-1 (1.73e-2) -	3.1616e-1 (1.11e-2) -	3.1515e-1 (5.12e-3) -	2.7970e-1 (2.16e-3)
LSMOP5	5	1.5686e+2 (9.46e+0) -	5.7655e+0 (9.94e-1) -	1.0396e+1 (1.17e+1) -	1.0905e+1 (1.88e+0) -	5.9864e+1 (4.59e+1) -	5.9483e+1 (7.71e+0) -	1.9847e+0 (1.53e+0) =	2.2992e+0 (4.92e+0) -	1.2503e+0 (1.05e+0) =	2.5224e+1 (3.05e+0) -	1.1138e+0 (1.23e-1)
	8	1.9639e+2 (1.10e+1) -	7.4578e+0 (1.03e+1) -	3.1242e+1 (1.64e+1) -	6.9011e+1 (1.76e+1) -	3.9235e+1 (2.48e+1) -	8.2430e+1 (2.22e+1) -	8.9700e+0 (4.43e+0) -	7.4248e+0 (9.41e+0) -	2.6462e+1 (1.25e+1) -	5.9394e+1 (1.16e+1) -	9.0796e-1 (5.54e-2)
	10	2.1259e+2 (1.05e+1) -	6.2183e+0 (8.70e+0) -	4.0609e+1 (1.13e+1) -	1.2354e+2 (1.37e+1) -	5.3305e+1 (3.79e+1) -	6.7704e+1 (1.53e+1) -	7.4004e+0 (6.09e+0) -	2.4892e+1 (1.78e+1) -	4.1419e+1 (2.09e+1) -	1.0152e+2 (7.91e+0) -	9.1727e-1 (1.28e-2)
	15	1.9796e+2 (1.79e+1) -	9.7031e+0 (8.44e+0) -	3.6327e+1 (5.99e+0) -	1.1168e+2 (1.25e+1) -	4.1151e+1 (2.81e+1) -	8.8582e+1 (5.22e+0) -	1.0222e+1 (5.12e+0) -	3.2461e+1 (1.06e+1) -	3.3753e+1 (1.53e+1) -	9.2770e+1 (3.67e+1) -	9.8765e-1 (7.06e-3)
LSMOP6	5	1.1575e+6 (3.66e+5) -	1.4976e+4 (1.15e+4) -	2.6054e+3 (3.41e+3) -	1.7656e+4 (1.51e+4) -	3.4308e+5 (3.80e+5) -	8.8927e+4 (1.70e+4) -	6.8543e+3 (1.34e+4) -	4.4540e+4 (8.25e+4) -	7.4243e+3 (4.60e+3) -	3.2818e+4 (1.10e+4) -	1.3706e+0 (8.80e-2)
	8	2.3372e+6 (2.24e+5) -	1.4717e+3 (2.44e+3) -	1.4950e+5 (1.01e+5) -	1.4042e+5 (8.64e+4) -	1.3856e+5 (1.83e+5) -	1.3412e+6 (2.61e+5) -	8.3938e+1 (2.63e+2) -	1.3702e+5 (1.97e+5) -	4.1552e+4 (8.52e+4) -	1.1037e+5 (2.50e+4) -	1.2709e+0 (2.44e-2)
	10	2.5237e+6 (2.61e+5) -	9.3387e+3 (1.27e+4) -	2.4425e+5 (3.41e+4) -	5.3403e+5 (9.92e+4) -	1.4830e+5 (2.75e+5) -	1.7793e+6 (2.72e+5) -	2.3272e+3 (3.47e+3) -	3.9913e+5 (3.02e+5) -	6.2191e+4 (5.76e+4) -	3.2551e+5 (2.93e+4) -	1.1831e+0 (2.12e-2)
	15	2.2146e+6 (4.40e+5) -	5.1439e+4 (7.37e+4) -	6.5690e+4 (1.61e+4) -	5.8417e+5 (1.90e+5) -	8.0202e+4 (5.13e+4) -	1.3850e+6 (3.03e+5) -	6.4550e+4 (1.84e+4) -	3.1044e+5 (2.24e+5) -	1.2028e+5 (9.81e+4) -	4.2295e+5 (3.19e+5) -	1.9862e+0 (1.09e-1)
LSMOP7	5	1.3185e+6 (2.28e+5) -	4.7033e+1 (2.43e+2) -	3.3143e+4 (3.87e+4) -	7.9516e+3 (1.64e+3) -	3.9207e+5 (4.02e+5) -	1.3415e+5 (4.06e+4) -	1.3246e+2 (5.44e+2) -	4.8174e+4 (1.05e+5) =	5.8111e+2 (1.57e+3) =	3.8404e+4 (1.06e+4) -	1.6952e+0 (2.97e-1)
	8	2.0167e+6 (4.44e+5) -	3.7674e+4 (3.37e+4) -	1.6902e+4 (1.76e+4) -	2.1717e+5 (1.19e+5) -	1.6813e+5 (2.71e+5) -	7.9211e+5 (2.84e+5) -	2.9356e+4 (4.09e+4) -	6.9637e+4 (8.08e+4) -	6.8223e+4 (5.75e+4) -	1.3236e+5 (8.35e+4) -	1.6595e+0 (3.31e-2)
	10	2.2644e+6 (3.60e+5) -	3.0840e+4 (5.15e+4) -	4.7686e+4 (1.49e+4) -	7.3196e+5 (2.03e+5) -	1.9634e+5 (2.29e+5) -	1.4918e+6 (2.38e+5) -	5.0156e+4 (3.09e+4) -	2.0001e+5 (1.57e+5) -	1.7179e+5 (2.01e+5) -	1.4538e+5 (1.45e+5) -	1.9764e+0 (2.92e-1)
	15	2.2803e+6 (3.83e+5) -	1.9887e+4 (2.57e+4) -	1.5658e+5 (2.08e+4) -	4.0612e+5 (9.67e+4) -	7.1579e+4 (1.08e+5) -	1.5040e+6 (1.80e+5) -	1.5642e+4 (8.96e+3) -	4.0858e+5 (2.18e+5) -	1.4973e+5 (7.66e+4) -	3.0318e+5 (1.77e+5) -	1.5358e+0 (3.27e-1)
LSMOP8	5	7.5567e+1 (7.71e+0) -	1.0030e+0 (5.49e-2) +	5.7766e+0 (4.04e+0) -	2.2895e+0 (2.84e-1) -	2.7581e+1 (2.52e+1) -	1.6486e+1 (1.97e+0) -	3.3580e-1 (1.77e-2) +	7.6890e-1 (1.20e+0) +	3.8360e-1 (6.69e-2) +	7.5431e+0 (1.06e+0) -	1.5827e+0 (1.08e-1)
	8	1.0516e+2 (5.11e+0) -	3.7733e+0 (6.38e+0) -	1.9339e+1 (1.02e+1) -	2.9793e+1 (9.07e+0) -	2.7630e+1 (1.77e+1) -	5.2638e+1 (8.18e+0) -	5.3312e+0 (5.39e+0) =	3.3992e+0 (5.20e+0) -	1.6059e+1 (8.62e+0) -	2.2005e+1 (7.35e+0) -	8.0723e-1 (1.50e-2)
	10	1.0985e+2 (6.89e+0) -	4.5231e+0 (2.99e+0) -	2.1560e+1 (5.08e+0) -	5.0732e+1 (1.04e+1) -	2.2162e+1 (1.15e+1) -	5.6313e+1 (1.06e+1) -	5.3074e+0 (4.15e+0) -	1.1782e+1 (8.48e+0) -	2.0957e+1 (7.89e+0) -	4.1298e+1 (5.18e+0) -	8.0884e-1 (5.22e-3)
	15	1.0258e+2 (9.55e+0) -	1.1194e+1 (7.25e+0) -	2.3105e+1 (2.05e+0) -	3.5665e+1 (5.62e+0) -	1.8215e+1 (1.55e+1) -	6.5174e+1 (6.34e+0) -	1.0966e+0 (4.26e-1) -	2.6414e+1 (7.45e+0) -	2.0655e+1 (8.93e+0) -	5.7309e+1 (3.20e+0) -	1.0279e+0 (3.44e-2)
LSMOP9	5	4.8803e+2 (8.83e+1) -	2.7701e+0 (7.29e-1) -	4.8554e+2 (2.07e+2) -	7.8726e+0 (6.42e-1) -	5.0678e+2 (2.24e+2) -	7.6054e+0 (3.22e-1) -	2.1906e+0 (4.62e-1) -	2.8331e+2 (1.12e+2) -	1.9634e+0 (1.57e-1) -	1.4657e+1 (1.29e+0) -	1.2123e+0 (7.27e-1)
	8	1.5473e+3 (3.18e+2) -	2.8928e+1 (2.06e+1) -	1.4662e+3 (2.60e+2) -	1.0012e+2 (2.72e+1) -	1.3835e+3 (5.54e+2) -	1.3487e+2 (1.96e+1) -	5.3314e+0 (8.49e-1) +	1.3242e+3 (7.56e+1) -	5.8015e+0 (1.60e+0) +	1.0357e+2 (1.75e+1) -	1.1910e+1 (2.11e+0)
	10	2.8653e+3 (4.10e+2) -	4.3925e+1 (9.78e+0) -	2.4001e+3 (8.02e+1) -	1.3881e+3 (1.86e+2) -	2.0685e+3 (1.04e+3) -	6.4688e+2 (1.97e+2) -	1.0100e+1 (3.17e+0) +	2.2509e+3 (9.01e+1) -	8.1067e+1 (8.00e+1) -	2.3358e+2 (2.92e+1) -	3.0666e+1 (3.02e+0)
	15	6.7587e+3 (3.90e+2) -	4.2815e+2 (2.78e+2) -	5.5142e+3 (1.68e+2) -	3.8001e+3 (3.09e+2) -	5.3590e+3 (2.52e+3) -	2.3503e+3 (1.99e+2) -	3.0347e+1 (5.70e+0) +	5.1201e+3 (1.38e+2) -	2.1635e+3 (8.75e+2) -	1.0136e+3 (1.13e+2) -	2.1724e+2 (1.66e+1)
+/-/=		0/35/1	1/35/0	1/35/0	4/32/0	0/36/0	4/32/0	4/30/2	3/31/2	2/32/2	0/36/0	

GLMO and CCGDE3). However, as described in the experimental results in the manuscript, it can be found that UCLMO achieves better optimization performance under similar algorithmic complexity.

Table A.4: Complexity analysis results of UCLMO and comparison algorithms

	Total Complexity
CCGDE3	$O(NumEsp \cdot Gmax \cdot ND)$
LMOCSO	$O(N^2D)$
LSMaODE	$O(N^2P^2)$
CVEA3	$O(MN^2)$
AMPDEA	$O(N^2D^2)$
BCE-IBEA	$\max\{O(C); O(MN^2)\}$
GLMO	$\max\{O(MOEA); O(MND)\}$
DGEA	$\max\{O(MOEA); O(RefNo \cdot MN^2)\}$
WOF	$\max\{O(MOEA); O(MN^2)\}$
FDV	$\max\{O(MOEA); O(MN^2)\}$
UCLMO	$\max\{O(MOEA); O(MN^2)\}$