Formal logic Homework. Dmitry Semenov, M3100, ISU 409537

1. For each given set of sentences, determine whether it is logically consistent (jointly satisfiable).

To determine whether the given set of sentences is logically consistent or jointly satisfiable, we have to check if there exists an assignment of truth values to the variables that satisfies all the sentences.

(a) $\neg D, (D \lor F), \neg F$

D	F	$\neg D$	$D \lor F$	$\neg F$
0	0	1	0	1
0	1	1	1	0
1	0	0	1	1
1	1	0	1	0

There aren't valuations which make all sentences true.

$\Rightarrow$ set isn't logically consistent

(b) $(T \to K), \neg K, (K \lor \neg T)$

T	K	$T \to K$	$\neg K$	$(K \lor \neg T)$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

There are valuations which make all sentences true: $T, K = 0, 0$

$\Rightarrow$ set is logically consistent

(c) $\neg(A \to (\neg C \to B)), ((B \lor C) \land A)$

A	B	C	$\neg(A \to (\neg C \to B))$	$((B \lor C) \land A)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

There aren't valuations which make all sentences true.

$\Rightarrow$ set isn't logically consistent

(d) $(C \to B), (D \lor C), \neg B, (D \to B)$

B	C	D	$C \to B$	$D \lor C$	$\neg B$	$D \to B$
0	0	0	1	0	1	1
0	0	1	1	1	1	0
0	1	0	0	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	0	1	1	1	0	1
1	1	0	1	1	0	1

1	1	1	1	1	0	1

There aren't valuations which make all sentences true.

⇒ set isn't logically consistent

2. Complete the following deductive formal proofs by filling in missing formulae and justifications

💡 All prooves has been verified using https://proofs.openlogicproject.org

(a)

1	$H \to (R \wedge C)$	Premise
2	$\neg R \vee \neg C$	Premise
3	$\neg(R \wedge C)$	$\boxed{\text{DeM}}$ 2
∴	$\boxed{\neg H}$	MT 1, 3

(b)

1	$K \wedge S$	
2	$\neg K$	
3	K	∧E 1
4	$\bot$	⊥I 2, 3
5	$\neg S$	X 4

(c)

1	$A \to \neg A$	
2	$\quad \neg\neg A$	
3	$\quad A$	DNE 2
4	$\quad \neg A$	MT 1, 2
5	$\quad \bot$	¬E 3, 4
6	$\neg A$	IP 2–5

(d)

1	$(P \wedge Q) \vee (P \wedge R)$	
2	$\quad (P \wedge Q)$	
3	$\quad P$	∧E 2
4	$\quad (P \wedge R)$	
5	$\quad P$	∧E 4
6	P	∨E 1, 2–3, 4–5

3. Symbolize the given arguments with well-formed formulae (WFFs) of propositional logic. For each argument, determine its validity using a truth table. For each valid argument, provide a deductive formal proof in Fitch notation. For each invalid argument, provide a counterexample valuation.

💡 All prooves has been verified using https://proofs.openlogicproject.org

(a) If philosophers ponder profound problems, their quandaries quell quotidian quibbles. Either their quandaries don't quell quotidian quibbles or right reasoning reveals reality (or both). Philosophers do ponder profound problems. Therefore, right reasoning reveals reality.

> (a) Если философы обдумывают глубокие проблемы, их затруднения утихают повседневные мелочи. Или их затруднения не утихают повседневные мелочи, или правильное рассуждение раскрывает реальность (или и то, и другое). Философы действительно обдумывают глубокие проблемы. Следовательно, правильное рассуждение раскрывает реальность.

$Let:$

- P — Philosophers ponder profound problems
- Q — Quandaries quell quotidian quibbles
- R — Right reasoning reveals reality

Sentences can be symbolized in WFF as follows:

- $P \to Q$
- $(\neg Q \lor R)$
- P

$\therefore R$

P	Q	R	$P \to Q$	$\neg Q \lor R$	P	$\therefore$	R
0	0	0	1	0	0	.	0
0	0	1	1	1	0	.	1
0	1	0	1	1	0	.	0
0	1	1	1	1	0	.	1
1	0	0	0	0	1	.	0
1	0	1	0	1	1	.	1
1	1	0	1	0	1	.	0
1	1	1	1	1	1	$valid!$	1

$\Rightarrow$ Argument is valid

Proof:

Construct a proof for the argument: $P \to Q$, $\neg Q \lor R$, $P \therefore R$

1	$P \to Q$	
2	$\neg Q \lor R$	
3	P	
4	Q	$\to$E 1, 3
5	│ $\neg Q$	
6	│ $\bot$	$\bot$I 4, 5
7	│ R	X 6
8	│ R	
9	│ R	R 8
10	R	$\lor$E 2, 5–7, 8–9

(b) If aardvarks are adorable, then either baby baboons don't beat bongos or crocodiles can't consume cute capybaras (or both). Baby baboons beat bongos. Aardvarks aren't adorable unless crocodiles can't consume cute capybaras. Therefore, aardvarks aren't adorable.

> Если трубкозубы очаровательны, то либо детеныши бабуинов не бьют бонго, либо крокодилы не могут есть милых капибар (или и то, и другое). Детеныши бабуинов бьют бонго. Трубкозубы не очаровательны, если только крокодилы не могут есть милых капибар. Следовательно, трубкозубы не очаровательны.

Let :

- A — aardvarks are adorable
- B — baby baboons beat bongos
- C — crocodiles can consume cute capybaras

Sentences can be symbolized in WFF as follows:

- $A \to (\neg B \vee \neg C)$
- B
- $\neg A \vee \neg C$

$\therefore \neg A$

A	B	C	$A \to (\neg B \vee \neg C)$	B	$\neg A \vee \neg C$	$\therefore$	$\neg A$
0	0	0	1	0	1	.	1
0	0	1	1	0	1	.	1
0	1	0	1	1	1	$valid!$	1
0	1	1	1	1	1	$valid!$	1
1	0	0	1	0	1	.	0
1	0	1	1	0	0	.	0
1	1	0	1	1	1	$invalid!$	0
1	1	1	0	1	0	.	0

$\Rightarrow$ Argument is invalid (c. e. valuation marked using red in truth table)

(c) If discipline doesn't defeat deficiency, then geniuses generally get good grades. If discipline defeats deficiency, then homework has harmed humanity. Therefore, geniuses generally get good grades unless homework has harmed humanity

> Если дисциплина не побеждает дефицит, то гении, как правило, получают хорошие оценки. Если дисциплина побеждает дефицит, значит, домашнее задание нанесло вред человечеству. Следовательно, гении, как правило, получают хорошие оценки, если домашнее задание не нанесло вреда человечеству

Let :

- D — Discipline defeats deficiency
- G — Geniuses generally get good grades
- H — Homework has harmed humanity

Sentences can be symbolized in WFF as follows:

- $\neg D \to G$
- $D \to H$

$\therefore H \vee G$

D	G	H	$\neg D \to G$	$D \to H$	$\therefore$	$H \lor G$
0	0	0	0	1	.	0
0	0	1	0	1	.	1
0	1	0	1	1	$valid!$	1
0	1	1	1	1	$valid!$	1
1	0	0	1	0	.	0
1	0	1	1	1	$valid!$	1
1	1	0	1	0	.	1
1	1	1	1	1	$valid!$	1

$\Rightarrow$ Argument is valid

Proof:

Construct a proof for the argument: $\neg D \to G,\ D \to H \therefore H \lor G$

1	$\neg D \to G$	
2	$D \to H$	
3	$\quad D$	
4	$\quad H$	$\to$E 2, 3
5	$\quad H \lor G$	$\lor$I 4
6	$\quad \neg D$	
7	$\quad G$	$\to$E 1, 6
8	$\quad H \lor G$	$\lor$I 7
9	$H \lor G$	TND 3–5, 6–8

(d) Crocodiles can consume cute capybaras only if incarcerating iguanas isn't illegal. Mad monkeys make mayhem and dinosaurs do disco dance, unless crocodiles consume cute capybaras. It is known that incarcerating iguanas is illegal. Therefore, dinosaurs do disco dance if and only if mad monkeys make mayhem.

> Крокодилы могут поедать милых капибар, только если содержание игуан в заключении не является незаконным. Обезумевшие обезьяны устраивают погром, а динозавры танцуют диско, если только крокодилы не едят милых капибар. Известно, что содержание игуан в заключении незаконно. Следовательно, динозавры танцуют диско тогда и только тогда, когда безумные обезьяны устраивают погром.

$Let:$

- C — Crocodiles can consume cute capybaras
- I — Incarcerating iguanas is illegal
- M — Mad monkeys make mayhem
- D — Dinosaurs do disco dance

Sentences can be symbolized in WFF as follows:

- $C \to \neg I$
- $C \lor (M \land D)$
- I

$\therefore D \leftrightarrow M$

C	I	M	D	$C \to \neg I$	$C \lor (M \land D)$	I	$\therefore$
0	0	0	0	1	0	0	.
0	0	0	1	1	0	0	.

0	0	1	0	1	0	0	.
0	0	1	1	1	1	0	.
0	1	0	0	1	0	1	.
0	1	0	1	1	0	1	.
0	1	1	0	1	0	1	.
0	1	1	1	1	1	1	$valid!$
1	0	0	0	1	1	0	.
1	0	0	1	1	1	0	.
1	0	1	0	1	1	0	.
1	0	1	1	1	1	0	.
1	1	0	0	0	1	1	.
1	1	0	1	0	1	1	.
1	1	1	0	0	1	1	.
1	1	1	1	0	1	1	.

$\Rightarrow$ Argument is valid

Proof:

Construct a proof for the argument: $C \to \neg I$, $C \lor (M \land D)$, $I \therefore D \leftrightarrow M$

1	$C \to \neg I$	
2	$C \lor (M \land D)$	
3	I	
4	C	
5	$\neg I$	→E 1, 4
6	$\bot$	⊥I 3, 5
7	$\neg C$	¬I 4–6
8	$(M \land D)$	DS 2, 7
9	M	∧E 8
10	D	∧E 8
11	M	
12	D	R 10
13	D	
14	M	R 9
15	$D \leftrightarrow M$	↔I 11–12, 13–14

4. For each given argument, construct a deductive proof in Fitch notation using only basic rules.

> 💡 All prooves has been verified using https://proofs.openlogicproject.org

(a) $\neg\neg A \therefore A$

Proof:

Construct a proof for the argument: $\neg\neg A \therefore A$

1	$\neg\neg A$	
2	$\neg A$	
3	$\bot$	⊥I 1, 2
4	A	IP 2–3

(b) $((A \to B) \to A) \therefore A$

Proof:

Construct a proof for the argument: $(A \to B) \to A \therefore A$

1	$(A \to B) \to A$	
2	$\quad \neg A$	
3	$\quad\quad A \to B$	
4	$\quad\quad A$	$\to$E 1, 3
5	$\quad\quad \bot$	$\bot$I 2, 4
6	$\quad \neg(A \to B)$	$\neg$I 3–5
7	$\quad\quad A$	
8	$\quad\quad \bot$	$\bot$I 2, 7
9	$\quad\quad B$	X 8
10	$\quad A \to B$	$\to$I 7–9
11	$\quad \bot$	$\bot$I 6, 10
12	A	IP 2–11

(c) $(\neg B \to \neg A) \therefore (A \to B)$

Proof:

Construct a proof for the argument: $\neg B \to \neg A \therefore A \to B$

1	$\neg B \to \neg A$	
2	$\quad A$	
3	$\quad\quad \neg B$	
4	$\quad\quad \neg A$	$\to$E 1, 3
5	$\quad\quad \bot$	$\bot$I 2, 4
6	$\quad B$	IP 3–5
7	$A \to B$	$\to$I 2–6

(d) $\neg(A \lor B) \therefore (\neg A \land \neg B)$

Proof:

Construct a proof for the argument: $\neg(A \lor B) \therefore \neg A \land \neg B$

1	$\neg(A \lor B)$	
2	$\quad A$	
3	$\quad A \lor B$	$\lor$I 2
4	$\quad \bot$	$\bot$I 1, 3
5	$\neg A$	$\neg$I 2–4
6	$\quad B$	
7	$\quad A \lor B$	$\lor$I 6
8	$\quad \bot$	$\bot$I 1, 7
9	$\neg B$	$\neg$I 6–8
10	$\neg A \land \neg B$	$\land$I 5, 9

(e) $(\neg A \land \neg B) \therefore \neg(A \lor B)$

Proof:

Construct a proof for the argument: $\neg A \wedge \neg B \therefore \neg(A \vee B)$

1	$\neg A \wedge \neg B$	
2	$\neg A$	∧E 1
3	$\neg B$	∧E 1
4	$\mid (A \vee B)$	
5	$\mid \mid A$	
6	$\mid \mid \bot$	⊥I 2, 5
7	$\mid \mid B$	
8	$\mid \mid \bot$	⊥I 3, 7
9	$\mid \bot$	∨E 4, 5–6, 7–8
10	$\neg(A \vee B)$	¬I 4–9

(f) $(A \to B) \wedge (\neg A \to B) \therefore B$

Proof:

Construct a proof for the argument: $(A \to B) \wedge (\neg A \to B) \therefore B$

1	$(A \to B) \wedge (\neg A \to B)$	
2	$(A \to B)$	∧E 1
3	$(\neg A \to B)$	∧E 1
4	$\mid \neg B$	
5	$\mid \mid A$	
6	$\mid \mid B$	→E 2, 5
7	$\mid \mid \bot$	⊥I 4, 6
8	$\mid \neg A$	¬I 5–7
9	$\mid B$	→E 3, 8
10	$\mid \bot$	⊥I 4, 9
11	B	IP 4–10

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5. For each given tautology, construct a deductive proof in Fitch notation.

💡 All prooves has been verified using https://proofs.openlogicproject.org

(a) $(A \to B) \vee (B \to A)$

Proof:

Construct a proof for the argument: $\therefore (A \to B) \vee (B \to A)$

1	$\mid A$	
2	$\mid \mid B$	
3	$\mid \mid A$	R 1
4	$\mid B \to A$	→I 2–3
5	$\mid (A \to B) \vee (B \to A)$	∨I 4
6	$\mid \neg A$	
7	$\mid \mid A$	
8	$\mid \mid \bot$	⊥I 6, 7
9	$\mid \mid B$	X 8
10	$\mid A \to B$	→I 7–9
11	$\mid (A \to B) \vee (B \to A)$	∨I 10
12	$(A \to B) \vee (B \to A)$	TND 1–5, 6–11

(b) $A \to (B \to A)$

Proof:

Construct a proof for the argument: $\therefore A \to (B \to A)$

1	A	
2	B	
3	A	R 1
4	$B \to A$	→I 2–3
5	$A \to (B \to A)$	→I 1–4

(c) $(\neg B \to \neg A) \to ((\neg B \to A) \to B)$

Proof:

Construct a proof for the argument: $\therefore (\neg B \to \neg A) \to [(\neg B \to A) \to B]$

1	$(\neg B \to \neg A)$	
2	$(\neg B \to A)$	
3	$\neg B$	
4	$\neg A$	→E 1, 3
5	A	→E 2, 3
6	$\bot$	⊥I 4, 5
7	B	IP 3–6
8	$(\neg B \to A) \to B$	→I 2–7
9	$(\neg B \to \neg A) \to ((\neg B \to A) \to B)$	→I 1–8

(d) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

Proof:

Construct a proof for the argument: $\therefore [A \to (B \to C)] \to [(A \to B) \to (A \to C)]$

1	$(A \to (B \to C))$	
2	$(A \to B)$	
3	A	
4	$B \to C$	→E 1, 3
5	B	→E 2, 3
6	C	→E 4, 5
7	$A \to C$	→I 3–6
8	$(A \to B) \to (A \to C)$	→I 2–7
9	$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$	→I 1–8